# Liability Management Using Dynamic Portfolio Strategies 

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This paper previously appeared in the Algorithmics Research Quarterly Volume 2 Number 3 September 1999, a limited distribution research journal.

The analysis and conclusions offered in this working paper do not represent the official views of the Bank of Canada or the Department of Finance.
Rather, they are meant to provide interested parties with information on issues under discussion to stimulate debate on these issues. Feedback from interested parties is welcome.


#### Abstract

Many institutions issue debt in both short-term markets, which implies frequent rebalancing, and long-term bond markets, which typically pay a higher coupon. The liability manager must consider this risk/cost trade-off in the face of uncertain interest rates and funding requirements. The debt management problem is characterized by long time horizons. Standard implementations of common risk management tools that do not account for portfolio effects are not suitable. This paper couples dynamic portfolio strategies with scenario generation in a simulation methodology to determine how the costs and risks of a liability portfolio evolve over time. The design of a simplified government debt program is used to illustrate the benefits of dynamic portfolio strategies.


## Sommaire

Beaucoup d'institutions émettent des titres de dette à la fois sur les marchés monétaires de court terme, ce qui suggère des ajustements fréquents, et sur les marchés obligataires de long terme, qui paient généralement un coupon plus élevé. Le gestionnaire de dette doit considérer cette relation risque/coût en tenant compte de l'incertitude sur les taux d'intérêt et les besoins d'emprunt. Le problème de gestion de la dette est caractérisé par un horizon de long terme. L'application standard des outils courants de gestion du risque qui ne tiennent pas compte des effets de portefeuille n'est pas adéquate. Cet article combine les stratégies dynamiques de portefeuille avec la génération de scénarios dans une méthodologie de simulation afin de déterminer comment les coûts et les risques d'un portefeuille de dette évoluent avec le temps. La conception d'un programme d'emprunts gouvernemental simplifié est utilisée pour illustrer les bénéfices des stratégies dynamiques de portefeuille.

## Introduction

Many institutions, especially large corporations and governments, incur liabilities by issuing debt in order to raise capital. This capital is usually raised in both short- and long-term bond markets. In raising this capital, it is necessary to consider the risk/cost trade-off between issuing short-term debt, which implies frequent rebalancing at uncertain interest rates, and long-terms bonds, which typically pay a higher coupon. In liability management, cost may be defined as the expected value of cumulative interest expenses and risk as the variability of cumulative interest expenses.

The liability management problem is characterized by long time horizons, typically several years. Over these lengthy horizons "portfolio effects" such as issuance, aging, coupon payments and maturities must be considered. To model the impact of these effects the positions must change over time. Dynamic portfolios are portfolios with positions that change over time according to dynamic portfolio strategies - decision rules that define how the positions change. These rules may be a simple deterministic, pre-specified rebalancing schedule or a more complicated set of rules that depend on future market events and/or portfolio characteristics.

Standard implementations of common risk management tools, such as RiskMetrics Value-at-Risk (VaR) (JP Morgan 1996), assume there are no portfolio effects. In fact, RiskMetrics VaR analysis is based on portfolios in which the positions do not change over the horizon of the analysis. While this assumption may be reasonable for the time horizon of traditional risk management ( 10 days, say), it is untenable for longer horizons. If these portfolio effects are unaccounted for, the estimates of future portfolio characteristics may be biased under longer time horizons.

Moreover, VaR methodologies base risk measures on "marked-to-market" values. Although suitable for many applications, the VaR methodology is not as appropriate when stakeholders of an organization are concerned about factors other than market value. For example, shareholders, rightly or wrongly, may consider price, earnings and profit indicators. In the context of an institution issuing debt, the market value is perceived to be less important, and managers typically look at the development in debt costs, as defined by some function of interest payments.

This paper couples dynamic portfolio strategies with scenario generation in a simulation methodology to determine how the interest expenses of a liability portfolio evolve over time. The scenario generation methodology is assumed to forecast risk-factor innovations that may have nonzero means. Another standard assumption in risk management is that these factors have mean zero; this assumption leads to biased results in an analysis that extends over a long time horizon. The reader is referred to Kim, Malz and Mina (1999) for a discussion of the issues associated with forecasting financial variables over a long time horizon.

The introduction of dynamic strategies enables all the tools of risk management, including Monte Carlo simulation, VaR and stress testing, to be applied to asset/liability management problems. Precise valuation models that account for portfolio effects improve the accuracy of the calcula-
tions. Conversely, it also allows the tools of asset/liability management, for example, risk/return analysis, to be applied to risk management problems.

The usefulness of dynamic portfolio strategies is illustrated through an extended example. In the example, we consider the problem of designing a government's debt program. This problem can be viewed as a constrained optimization problem in which an asset/liability manager must minimize risk subject to a cost constraint and also, in this case, a constraint that the government maintains a given cash balance in its account.

The example is organized as follows. First, the problem is described in detail to illustrate how a simple dynamic portfolio strategy can be used to meet the government's objective under a particular nominal interest rate scenario. In this case, 3-month bills and 5-year bonds are used by the government to maintain a given cash balance. The results of this analysis are stress tested. Then, in order to develop a better understanding of the risk/cost trade-off, the analysis is repeated for different dynamic portfolio strategies and cost levels. The result is an efficient frontier. Since there are only two types of instruments, this efficient frontier is simply a locus of risk/cost points and all strategies are efficient. To generate a more interesting set of results, the problem is enriched by the introduction of 2-year bonds. The set of dynamic portfolios on the new efficient frontier strictly dominates any others. Finally, a more complicated dynamic portfolio strategy is considered and its effect is discussed in terms of the risk/cost trade-off.

A priori, the example is expected to illustrate the conventional wisdom that short-term debt is cheap but risky and long-term debt is expensive but safe. Indeed, the results strongly support this. However, even this rather simple analysis suggests some shortcomings with this intuition. In particular, the results suggest that issuing too much long-term debt can actually increase risk. The increased risk associated with financing the higher interest payments of the long-term debt more than offsets any reduction in risk incurred by issuing long-term debt.

Although asset/liability issues can be addressed using other techniques, dynamic portfolio strategies offer a more intuitive and easily implemented approach that is capable of dealing with practical problems in a real-world setting. Before launching into the example, the next section provides a brief overview of other methods of addressing asset/liability management problems and how they relate to dynamic portfolio strategies.

## Dynamic portfolios

Although not yet prevalent in risk management practices, the history of dynamic portfolios is extensive. In its simplest form, single-period portfolio selection can be thought of as a dynamic portfolio problem. Traditional Markowitz mean-variance analysis $(1952,1987)$ can be used to determine the optimal holdings in a portfolio of risky assets such that the optimal, rebalanced portfolio yields the minimum variance for a given rate of return. However, this approach allows for only a single rebalancing within the time horizon examined.

More interesting multi-stage dynamic portfolio problems may be modeled as the outcome of a dynamic stochastic programming problem. Dynamic stochastic programming, the study of procedures for decision making over time in a stochastic environment, deals well with uncertainty; however, the problem size escalates dramatically even for very small numbers of financial assets, time periods, possible return outcome values and constraints the government wishes to impose.

Eppen and Fama $(1968,1971)$ model two- and three-asset problems using this technique and Daellenbach and Archer (1969) extend their work to include one liability. These models consider uncertainty of return and are dynamic, but only problems with a very small number of financial instruments can be analyzed simultaneously; hence, they are of limited use in practice.

Wolf (1969), Bradley and Crane $(1972,1980)$ and Lane and Hutchinson (1980) use stochastic decision tree models. Bradley and Crane apply their dynamic stochastic model to bond portfolio management. Their model, while useful for small problems, again becomes computationally unwieldy with even a few periods and possible outcomes. Kusy and Ziemba (1986) discuss a stochastic linear program under the uncertainty approach and compare it with Bradley and Crane's models. They show by simulation that the stochastic linear programming approach is superior to the decision tree dynamic programming approach developed by Bradley and Crane; however, the Kusy and Ziemba model does not account for final period effects nor is it truly dynamic since it is solved two periods at a time in a rolling fashion.

The Russell-Yasuda Kasai model, developed for the Yasuda Fire and Marine Insurance Co., Ltd. and described in Carino and Ziemba (1998) and Carino, Myers and Ziemba (1998), builds on this previous research to develop a large scale dynamic model with possibly dependent scenarios, final period effects, and all the relevant institutional and policy constraints of Yasuda Kasai's business enterprise. Although the Russell-Yasuda Kasai model is one of the first genuine commercial applications of dynamic stochastic programming, the complexity of the model limits its extensibility and adaptability by other institutions.

An alternative to modeling dynamic portfolios using stochastic dynamic programming is to use decision rules. These rules have several advantages over other methods: they are simple to communicate, they can truly capture the nature of the firm's behaviour and they are robust in the face of uncertainty. However, they may not be optimal in the sense that the dynamic portfolio resulting from a set of decision rules may or may not replicate the portfolio resulting from a stochastic dynamic programming exercise. Decision rules can vary dramatically from straightforward rules such as a "buy-and-hold" rule to very complex rules involving derivative securities.

In the context of debt issuance, debt portfolio decision rules may vary widely in complexity. For instance, small institutions with access to well-developed capital markets may have the flexibility of choosing to issue debt at any maturity as their participation may have little impact on the overall market. In this case, their decision rules may be solely based on the trade-off between risk and cost. One possible decision rule, though not necessarily the optimal one, may simply be to issue debt at the lowest cost maturity along the term structure.

In contrast, the issuance behaviour of larger institutions may affect the overall market. Thus, their decision rules may have to account for additional criteria and therefore may be more complicated. For example, due to liquidity constraints, a large institution may not be able to issue all its debt at a single maturity, as a small institution can, but may have to spread the issuance across the term structure. Even so, it may not always be able to issue as much debt as required without affecting the debt price. Taken together, these problems imply that the decision rules for a large institution must account for both the concentration of issuance at a particular maturity as well as the absolute amount of issuance.

The decision rules used in this paper range from simple rules that do not depend on any future events to more complex, dynamic rules that depend on future events as they unfold. The complex
rules are easily derived by modifying the simple decision rules. These rules are applied to the debt issuance problem introduced in the next section.

## Debt issuance

An example of government debt issuance is used to illustrate the benefits of dynamic portfolios. A stylized program of debt issuance is as follows. Each year the government's budget determines expected borrowing requirements net of any maturities of outstanding debt. Actual requirements are met by issuing bonds and bills using a bond issuance program that is pre-announced at the beginning of the horizon and a bill issuance program that dynamically adjusts to meet outstanding borrowing requirements. Any changes in borrowing requirements arising from, for example, changes in fiscal policy or changes in interest rates, are accommodated by an offsetting adjustment to the bill program.

The size of the pre-announced bond program relative to overall borrowing requirements is determined by considering the risks and costs inherent in the overall borrowing strategy. In this example, cost is measured by interest expenses cumulated over the entire horizon and risk is measured by the variability of the cumulative interest expenses. These definitions suggest that in an environment where the yield curve slopes, on average, upward, issuing short-term debt will often be cheaper than issuing long-term debt. However, short-term debt must be refinanced each period and as the interest rate at which this will occur is not known in advance, the variability in interest payments-which determines risk-associated with issuing short-term debt is higher than that associated with issuing long-term debt.

The value of the cash account today is equal to the value of the cash carried over from the previous period, plus any issuance in bonds and bills, less the value of interest payments and any settlements into the cash account from bonds and bills that mature in this period:

$$
\begin{equation*}
\text { cash }_{t}+\text { cash }_{t-1}+\text { bills }_{t}+\text { bonds }_{t}+\text { maturities }_{t}+\text { interest }_{t} \tag{1}
\end{equation*}
$$

where cash $h_{t}$ represents the government's cash account at time $t$, cash $_{t-1}$ represents the cash account from the previous period, bills $s_{t}$ and bonds $s_{t}$ represent new bill and bond issuance, respectively, and maturities ${ }_{t}$ and interest $_{t}$ represent principal repayments and interest expenses.

If future capital requirements are known in advance and interest rates are deterministic, then all the terms of Equation 1 are known and the equation can be solved for total issuance at each time $t$ $\left(\right.$ bills $_{t}+$ bonds $\left._{t}\right)$. Designing a debt issuance program would be straightforward-a pre-announced schedule for both bonds and bills could be designed to meet the necessary requirements.

In a more realistic environment where capital requirements are not necessarily known in advance and interest rates are stochastic, the problem becomes more complicated and the program must be designed to accommodate future changes in these variables. In this case, given forecasts of future borrowing requirements and interest rates, Equation 1 can still be used to find an expected issuance program, but the program will depend on the actual (unknown) path of future interest rates and capital requirements.

To illustrate how dynamic portfolio strategies may be used in this debt issuance environment, an extended example is presented that begins with a very simple set of dynamic portfolio strategies.

## A simple example

Following the stylized debt issuance program outlined above, the first example in this paper uses a simple set of dynamic portfolio strategies to develop a new debt issuance program. This issuance program is developed using a given yield curve referred to as the nominal scenario; this nominal scenario also serves as a reference curve for the sets of scenarios generated for stress analyses in later sections

At the beginning of the horizon, the government's outstanding debt portfolio consists solely of treasury bills. The government initiates a bond program in an effort to reduce its interest cost and risk exposure. In light of this, the government wishes to analyze the risk and cost implications of this new program over a five-year horizon. The debt program in this example may be summarized as follows:

- Each quarter the government has a capital requirement that is met by issuing short- and longterm debt using treasury bills and bonds. A quarterly capital requirement exists to maintain a cash balance of $\$ 100,000$.
- Part of this requirement is met by issuing bonds according to a pre-announced quarterly schedule. Each quarter, $\$ 5,000$ worth of 5 -year, fixed-rate coupon bonds are issued at par.
- The remainder of the requirement is raised using treasury bills. These are issued in a dynamic fashion such that in each quarter enough bills are issued to meet the overall capital requirement. In particular, sufficient bills are issued to cover any interest expenses and maturities in excess of new bond issuance. Bills are assumed to be 3-month bills (zero-coupon bonds).
- At the beginning of the five-year horizon, the institution has previously issued enough 3month bills so that its current cash balance is $\$ 100,000$.

From Equation 1, the assumption that the cash balance remains constant implies that interest payments evolve according to

$$
\text { interest }_{t}=\text { bills }_{t}+\text { bonds }_{t}-\text { maturities }_{t}
$$

or equivalently, if maturities are identified as being either bond and bill maturity

$$
\begin{equation*}
\text { interest }_{t}=\text { netbillissuance }_{t}+\text { netbondissuance }_{t} \tag{2}
\end{equation*}
$$

Figures 1 and 2 illustrate the evolution of the bill and bond program in an environment with an upward sloping yield curve. At the beginning of the horizon, there is $\$ 100,000$ of outstanding 3month bills that mature at the end of the first quarter. At that time, the principal repayment and interest due on these bills must be refinanced to maintain the required cash balance. Bonds equalling $\$ 5,000$ are issued and the remaining refinancing requirement is met using bills. This can be seen in Figure 1. In the first quarter, bonds with a net worth of $\$ 5,000$ are issued, interest expenses are approximately $\$ 1,000$, and, using Equation 2, net bill issuance is $\$ 1,000-\$ 5,000=-\$ 4,000$. In other words, $\$ 100,000$ worth of matured bills and approximately $\$ 1,000$ in interest are refinanced using $\$ 5,000$ worth of bonds and approximately $\$ 96,000$ worth of bills. After the first quarter, as
illustrated in Figure 1, the bond program continues to issue $\$ 5,000$ per quarter and net bill issuance remains negative. Over time, as shown in Figure 2, the higher bond issuance compared with bill issuance leads bonds to dominate the composition of government debt. Note also from Figure 2 that total debt increases over time as interest payments are financed through new issuance.


Figure 1: Future issuance patterns


Figure 2: Future outstanding debt
Figures 3(a) and 3(b) graph interest expenses on a quarterly and cumulative basis. Again, it can be seen that interest expenses are increasing. More interestingly, it can also be seen that interest expenses vary over time. The interest curve evolves through the forward rates and because the
interest curve is upward sloping, future spot rates change. This affects both the cost of raising funds using bills and, through its affect on the coupon rate, the cost of raising funds using bonds. As the gross issuance of bills is greater than the gross issuance of bonds, interest expenses due to bill issuance tend to be more volatile over time. (As these are 3-month bills, gross bill issuance is equal to the amount of bills outstanding which exceeds the $\$ 5,000$ of gross bond issuance - see Figure 2)


Figure 3(a): Quarterly interest expense


Figure 3(b): Cumulative interest expense

## Alternative issuance patterns

To explore the impact on cost of various issuance patterns, the previous analysis is repeated with alternative issuance patterns. The alternative patterns range from a strategy of issuing no bonds (implying that all refinancing is done with 3-month bills) to one that calls for issuing $\$ 10,000$ bonds, in increments of $\$ 2,500$. Table 1 summarizes the results.

| Bond program <br> (\$) | Cumulative <br> interest expense <br> (\$) |
| :---: | :---: |
| 0 | 25,182 |
| 2,500 | 26,339 |
| 5,000 | 27,496 |
| 7,500 | 28,653 |
| 10,000 | 29,810 |

Table 1: Alternative issuance patterns
Recall that an upward sloping yield curve is used to calculate the interest exposure. It is apparent from Table 1 that portfolios with a higher bond issuance have higher cost, measured in cumulative interest payments. However, what is not apparent from examining the table is how the risk, measured by the volatility of these interest payments, changes with the amount of bond issuance. The risk associated with different issuance patterns is explored, first with a simple stress test and then in a Monte Carlo simulation.

## A simple stress test

The first simple test stresses the portfolio to assess the sensitivity of cumulative interest expense to the nominal interest rate scenario. The stress test is a $1 \%$ parallel shift in the yield curve that occurs in the second year of the planning horizon. Figures 4(a), 4(b), and 4(c) present the results corresponding to a $\$ 5,000$ bond program.



Figure 4(b): Parallel shift in interest rates - Interest expense from bills
Not surprisingly, the increase in interest rates leads to an increase in interest expenses. What is more relevant is that most of the increase in interest expenses, at least in the near term, is due to interest payments associated with the bill program. Recall that each quarter the entire bill program is "rolled over" (that is, each quarter all the bills mature and are then reissued). Thus, the higher interest rates are felt immediately. The interest expenses associated with the bond program are less sensitive to the curve shift because there is less new bond issuance. Note, however, that interest expenses associated with the bond program increase over time as more bonds are issued at the higher rates.


Figure 4(c): Parallel shift in interest rates - Interest expense from bonds
Table 2 presents the results for the range of issuance strategies presented in Table 1. Portfolios with a higher bond issuance are less affected by the increase in interest rates. Again, portfolios
comprising more bills will incur higher costs as the bills are rolled over in the higher interest rate environment.

| Bond <br> program <br> (\$) | Cumulative interest expense (\$) |  |  |
| ---: | ---: | ---: | ---: |
|  | Nominal | Stress <br> Test | Difference |
| 0 | 25,182 | 29,846 | 4,665 |
| 2,500 | 26,339 | 30,450 | 4,111 |
| 5,000 | 27,496 | 31,053 | 3,557 |
| 7,500 | 28,653 | 31,656 | 3,001 |
| 10,000 | 29,810 | 32,259 | 2,449 |

Table 2: Interest rate stress test
The results of the stress test suggest that issuing more bonds leads to less variability in cumulative interest payments, albeit at a higher cost.

## An efficient frontier

In this section we examine the risk associated with different issuance patterns in a Monte Carlo setting. The issuance patterns are those used in the previous stress test. The innovations to the nominal interest rate scenario are generated using a standard two-factor affine yield model similar to a model first proposed in Vasicek (1977). The model was calibrated using empirical moments estimated from the McCulloch and Kwon (1993) dataset.

Table 3 presents the results. Again, the mean cumulative interest expenses (the cost of the issuance patterns) increase with the amount of bond issuance. In contrast, however, the standard deviation (variability) of interest charges (the risk of the issuance pattern) decreases with the amount
of bond issuance. This happens because long-term rates are less volatile than short-term rates and, more importantly, because there is less rollover risk associated with a large bond program.

|  | Cumulative interest <br> expense (\$) |  |
| :---: | ---: | ---: |
| Bond program (\$) | Mean | Standard <br> deviation |
| 0 | 22,580 | 4,238 |
| 2,500 | 24,551 | 3,523 |
| 5,000 | 26,521 | 2,896 |
| 7,500 | 28,492 | 2,425 |
| 10,000 | 30,463 | 2,215 |

Table 3: Risk and cost of alternative issuance patterns
Figure 5 displays the results for a large number of bond programs. Point A corresponds to a portfolio with a $\$ 10,000$ bond program and point B to a portfolio with a $\$ 0$ bond program. Points along the line connecting point A and point B represent portfolios with both bonds and bills in some combination. Moving from point $A$ to point $B$ corresponds to reducing the size of the bond program. As the bond issuance decreases, risk increases and cost decreases.


Figure 5: A simple efficient frontier
Figure 5 is a simple example of an efficient frontier-the locus of risk and cost combinations offered by portfolios of risky assets that yield the minimum risk for a given cost.

The efficient frontier may be used to determine the appropriate issuance rule for an institution given its risk appetite. Given the amount of risk that the organization wishes to bear, the bond program that the institution should employ may be found by vertically mapping the chosen risk level
to the efficient frontier. This will lead to a bond program with the lowest possible expected cost for this level of risk; the actual cost may be found by horizontally mapping the point on the frontier to the vertical axis.

The efficient frontier in Figure 5 is simply a line -there are no interior points. This is because there are only two instruments available; in the next section, following the introduction of a third instrument, the frontier becomes a region in the risk/cost space.

## An increased opportunity set

In this section, 2-year, fixed-rate coupon bonds may be issued in addition to 3-month bills and 5year bonds. As with 5 -year bonds, the amount of issuance may vary between $\$ 0$ and $\$ 10,000$.

Figure 6 presents the results of the exercise superimposed on the previous results. The line that joins point A (all 5-year bonds) and point B (all 3-month bills) corresponds to the efficient frontier of the previous section. Point C corresponds to a portfolio of only 2 -year bonds and point D to a portfolio of \$10,000 worth of 2-year bonds and \$10,000 worth of 5 -year bonds. The feasible region is defined by the lines that connect point B with point C , point C with point D and point D with point B . Note that point A lies inside the feasible region.

Adding 2-year bonds to a portfolio of bills only (point B), moves the portfolio toward point C. The line BC is below BA , implying that, at the margin, the cost of reducing risk using 2-year bonds is less than the cost of using 5-year bonds. When issuing 2-year bonds rather than 5 -year bonds, the increase in risk is more than offset by the decrease in cost.


Figure 6: A more complicated efficient frontier
Moving along the frontier, from point C toward point $\mathrm{D}, 5$-year bonds are added to a portfolio that comprises $\$ 10,000$ worth of 2 -year bonds. At the margin, this addition reduces risk and increases cost -up to a point. Eventually, the bond program becomes so large that risk starts to increase because the larger bond program generates more cash than required and thus, bill issuance is actu-
ally negative. Risk increases as the magnitude of the bill issuance becomes increasingly more negative.

As can be seen, the new efficient frontier dominates the old frontier everywhere- strategies that include 2-year bonds have both reduced risk and cost over any strategy that uses only bills and 5year bonds. The portfolios that lie between the original frontier and the new frontier comprise bills and bonds, both 2-year and 5-year. The portfolios in the region bounded by ABC (labelled with a "*") correspond to portfolios that have a total bond program of less than $\$ 10,000$. The porftolios in the region between line AC and the new efficient frontier represent bond programs of more than $\$ 10,000$. The portfolios worth less than $\$ 10,000$ have less risk because of the different risk/cost profile of 2-year bonds compared to 5-year bonds; portfolios worth more than $\$ 10,000$ have less risk because programs with larger bond issuance tend to reduce risk simply because bonds are less risky.

Note that these results depend on the specification of the interest-rate scenarios. In general, yields and the variance/covariance of yields along the curve will determine the relative risks and costs of different strategies.

Note also that the simple, predetermined issuance strategies considered so far can result in negative bill issuance. The bill program is conditional on interest rates - if interest rates are lower than expected, then, depending on the predetermined bond program, the amount of bill issuance will be lower and may become negative. Constraints can be added to the problem that restrict the size of bond issuance such that bill issuance is always positive. Alternatively, conditional decision rules can be used to address this problem.

## Conditional decision rules

In the previous section, the bond program is defined by a simple rule-a fixed amount of bonds is issued in each quarter for the duration of the planning horizon. Such simple rules are not very intuitive-institutions regularly update their bond programs in light of their current portfolio and market conditions. Such rules are also limiting in the sense that, by using more complicated issuance rules, portfolios that have less cost and less risk may be constructed.

Many institutions adjust the amount of bond issuance each quarter to maintain a pre-specified ratio of bonds to total outstanding debt. More specifically, if the actual proportion of bonds to total debt outstanding is greater than a target ratio, bond issuance is decreased and bill issuance is increased by an offsetting amount; if the proportion is less than the target the reverse happens bond issuance is increased and bill issuance is decreased by an offsetting amount. The more complex issuance rules considered in this section are based on this practice.

The target ratio of outstanding bonds to total outstanding debt varies between zero and one for the different issuance functions examined. As well, the issuance of any one bond must be positive and may not exceed $\$ 10,000$; thus, the results are comparable to the previous programs. Given these rules, in particular the limit on the maximum issuance size, the target ratio may not always be obtained immediately, especially if the target ratio is quite different from the current ratio.Figure 7 presents the results for this set of strategies superimposed on the results in Figure 6. Point E corresponds to a portfolio of $\$ 10,0005$-year bonds and $\$ 10,0002$-year bonds. Any relaxation of the predetermined issuance rules will lead to an efficient frontier that lies on or outside that depicted in Figure 6. As can be seen, the new rule allows for a reduction in both risk and cost relative to the
simple rules of predetermined issuance. More specifically, the more complicated rules allow for a reduction in risk and cost for portfolios with large bond issuance.


Figure 7: Efficient frontier with a variable bond program

## Conclusions

Dynamic portfolio strategies give risk managers the ability to realistically forecast the cost/risk relationship inherent in portfolios over longer horizons. By examining longer horizons than those analyzed in traditional risk management, managers can better assess long-term risk including risk measured on a cash flow basis as well as on a market or fair value basis. Thus, a long-term Value-at-Risk can be calculated for a dynamic portfolio in a Mark-to-Future framework.

In the paper, the problem of debt issuance in a stochastic interest rate environment is used to illustrate how dynamic portfolio strategies may be applied. The paper specifies a number of example dynamic portfolio strategies, beginning with a very simple deterministic strategy based on a prespecified rebalancing schedule applied to a portfolio of two instruments - a 3-month bill and a 5year bond. The addition of a 2-year bond to the portfolio demonstrates the benefits of expanding the universe of instruments. Finally, the complexity of the dynamic strategy is increased by specifying a rule to dynamically adjust the amount of bond issuance each quarter in order to target a pre-specified ratio of fixed-rate debt to total outstanding debt. This additional flexibility allows further reductions in both cost and risk.

The task of constructing dynamic portfolio strategies for actual applications will likely result in strategies that are more complicated than those presented in this paper. Indeed, the design of new strategies is an important area of future research. A strategy of particular interest to risk managers might encapsulate limit-based risk management by comparing a VaR measure to a limit, permitting trading only when the VaR does not exceed the limit. Thus, dynamic strategies applied to risk management policies may translate into a risk/cost trade-off for the firm.

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