Labour or Total Factor Productivity:

Do We Need to Choose?

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Abstract

Two competing measures of productivity are commonly used by both academics and policy makers. These are labour productivity—output per hour—and total factor productivity (TFP)—which measures productivity net of the contribution of capital. Which measure is the 'best' has been the subject of recent debate in academic and policy circles. In this paper, we argue that both measures have their place, and that neither tells the whole story. TFP is more useful over the long run, assuming that one is confident about the underlying growth process and the quality of capital stock data, whereas labour productivity is more reliable in the short run, when there is doubt about the underlying growth process, or when capital stock data are unreliable.

I.   Introduction

Canada's rate of productivity growth is currently attracting great interest, not only from academics but also from journalists, policymakers and the Canadian public in general. Why productivity has been so slow in the 1990s, whether growth is now picking up, and whether Canada is falling behind the United States are all issues of obvious importance. Yet those following these debates could be forgiven for being confused about what exactly is the subject of debate. As with measures such as unemployment, most people have a vague idea of what productivity is, but unlike unemployment, there are actually two different official measures of productivity: labour productivity (output per unit of labour) and total factor productivity (TFP)[1].

Which measure is the 'best' has been the subject of some debate in academic and policy circles. On the one hand there are those who argue that TFP is the appropriate measure of productivity growth, and that labour productivity is a much cruder measure[2]. On the other hand, there are those who argue that TFP depends too much on arbitrary assumptions, and that labour productivity is more closely related to current living standards, which is what society ultimately cares about.

In this paper, we shall argue that both measures have their place, and that neither tells the whole story. We begin by explaining how productivity measures are derived conceptually within the context of the neoclassical growth model, and then discuss how these theoretical concepts match up with the empirical measures produced by statistical agencies such as Statistics Canada. We then examine the usefulness of these empirical measures in neoclassical growth models that incorporate capital-embodied technical change, and in endogenous growth models. Finally we discuss under what circumstances one would want to rely more on TFP growth as a guide to trends in productivity, and under what circumstances one might prefer to rely more on labour productivity. We conclude by arguing that both measures have their place in the analysis of trends in productivity.

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[1] Statistics Canada refers to TFP as 'Multifactor Productivity', or MFP.

[2] See for example May (2000).

II.   What the Neoclassical Model tells us about Productivity: Theory and Facts

We begin by outlining the basic neoclassical model of growth and then we relate it to the empirical estimates published by statistical agencies such as Statistics Canada.

Basic Theory Behind the Neoclassical Model

For simplicity we will assume a Cobb-Douglas production function of the form

$$Y = AK^{\alpha}L^{1-\alpha}, \ \ 0<\alpha<1, \tag{1}$$

where Y is output, K is capital input, L is labour input and A is TFP, a parameter that governs the relation between the inputs, capital and labour, and output. TFP is commonly identified with the level of technology but it actually incorporates a wide variety of factors, such as the internal organization of firms and the level of worker effort.

Dividing equation (1) by labour input (L), gives the following expression,

$$\dot{y} = \dot{A} + \alpha \cdot \dot{k} \tag{2}$$

where lower case letters denote quantities per unit of labour input, so that y expresses labour productivity or output per unit of labour input while k denotes the level of capital intensity or the capital–labour ratio. The dots indicate the rate of change of each variable.

Equation (2) implies that the rate of growth of labour productivity ($\dot{y}$) is equal to the rate of growth of TFP ($\dot{A}$) plus α times the rate of growth of capital intensity. When using the neoclassical framework, one should keep in mind that the capital stock is not an exogenous determinant of growth. Rather, it is an endogenous variable that depends on TFP growth. In a long run steady state—a situation where all per capita variables are growing at a constant rate—one can show that the growth of capital intensity is the same as the rate of growth of labour productivity, that is,

$$\dot{k} = \dot{y} = \frac{\dot{A}}{1-\alpha} \tag{3}$$

The expression on the right-hand side is sometimes referred to as the Harrodian rate of technical progress (see Cas and Rymes, 1991) or the 'adjusted' rate of TFP growth. Equation (3) says that in the long run *all* of labour productivity growth is ultimately the result of TFP growth. In other words, in the long run, if TFP were to stop growing, capital intensity would also stop growing, and there would be no further labour productivity growth and ultimately no further improvements in standards of living.

What drives this conclusion? Diminishing returns to capital are responsible. With constant returns to capital (i.e., $\alpha = 1$), the growth rate of capital intensity would not depend on the size of the capital stock (K), and it could grow forever without any growth in TFP (i.e., growth in A). However, with diminishing returns to capital, the growth rate

of capital intensity does depend on the size of the capital stock. Each additional unit of capital adds less and less to output so that the only way to offset this process in the long run is if TFP grows, which means that a given capital stock is becoming more productive over time.

By adopting the neoclassical approach to calculating TFP growth, one is more than just calculating the relative importance of capital in explaining labour productivity growth, one is adopting a *theory* of growth, and one in which causation runs *from* TFP growth *to* capital stock growth. This matters for which measure of productivity growth is most useful, because it means that, in the long run, capital stock growth will tend to follow TFP growth, and not the other way around.

So, for example, if labour productivity growth is well above the adjusted TFP growth, the neoclassical model implies that capital is growing faster than its long run sustainable rate, and so labour productivity growth will have to fall if TFP does not rise. In this sense, TFP growth, not labour productivity growth, is the more informative predictor of future trends in productivity in the neoclassical model.

Basic Facts about TFP and Labour Productivity in Canada

There are many ways to estimate productivity growth[3]. Equation (2) is usually at the heart of how TFP is actually measured by economists and statistical agencies. TFP growth is calculated as a residual by subtracting the contribution of growth in the capital–labour ratio from labour productivity growth. To do this, one requires an estimate of α, the marginal productivity of capital. Under perfect competition and constant returns to scale, this parameter is equal to capital's share in output, denoted by $\hat{\alpha}$. TFP can then be calculated according to the formula where LP is labour productivity.

$$\dot{TFP} = \dot{LP} - \hat{\alpha} \cdot \dot{k} \tag{4}$$

In the short run, this means that capital accumulation in practice has an independent role in the calculation of TFP growth. Table 1 shows the results of a growth accounting exercise of this kind for Canada over the period 1966–1998. Focusing first on the short-run relationship between labour productivity and TFP, TFP growth explains about two-thirds of labour productivity growth in the period 1966–1973, whereas after 1973 increases in capital intensity and growth in TFP each explain about half of labour productivity growth.

In the long run, the neoclassical model implies that TFP growth explains *both* growth in labour productivity and growth in capital intensity, and that both of these growth rates should be equal to the rate of adjusted TFP growth. However, between 1973 and 1988,

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[3] Other methods include index numbers.

the average rate of growth of capital intensity was much higher than the average rate of adjusted TFP growth (Table 1). According to the neoclassical model, the increase in capital intensity over and above the rate of adjusted TFP growth was unsustainable, as it would have implied a diminishing marginal product of capital. Indeed, the rate of growth of capital intensity slowed considerably, to 1.1 per cent, during the last decade (1988–1998).

Table 1. Average Annual Growth Rate in Productivity, Business Sector, 1966-98

		1966-1973	*1973-1979*	*1979-1988*	*1988-1998*
$\dot{Y}$	Growth of real GDP	4.9	3.6	3.2	2.1
$\dot{L}$	Growth in Labour	1.9	2.2	1.9	1.1
$\dot{K}$	Growth in Capital	4.3	4.3	3.8	2.2
$\hat{\alpha}$	Share of capital in output	0.38	0.38	0.42	0.36
$\dot{k} \equiv \dfrac{\dot{K}}{L}$	Growth in Capital Intensity	2.4	2.1	1.9	1.1
$\dot{TFP}$	Growth in TFP	2.1	0.6	0.5	0.6
$\dot{LP}$	Growth in Labour Productivity	3.0	1.4	1.3	1.0
$\hat{\alpha} \cdot \dot{k}$	Adjusted Growth in Capital Intensity	0.9	0.8	0.8	0.4
$\dfrac{\dot{TFP}}{(1-\hat{\alpha})}$	Adjusted Growth in TFP	3.4	1.0	0.9	0.9

Note: Average growth rates correspond to overlapping years.

Source: Statistics Canada (http://www.statcan.ca/Daily/English/991222/d991222e.htm).

The preceding is the basic neoclassical interpretation of recent slow growth in labour productivity. However, accurate estimates of TFP require accurate measures of the capital stock. Unfortunately, the capital stock is generally difficult to measure and poses serious difficulties in estimating TFP growth. Different assumptions to estimate capital stock and even cyclical variations directly affect the estimates.

One reason why official estimates of capital are difficult to measure is because of different assumptions about depreciation rates and aggregation. For example, different depreciation rates would lead to estimates of the capital stock that are not comparable, especially across countries[4]. Also, there is also the question of how to aggregate different components of the capital stock. While statistical agencies typically use historical cost to weight different kinds of capital good, Jorgensen and Griliches (1967) argue that the more appropriate technique from a conceptual standpoint is to weight assets by their factor shares. In this way measures of the capital stock can capture increases in the quality of capital resulting from substitution towards capital goods with higher marginal products. Failure to do so will bias downwards estimates of the capital stock and so bias upwards measures of TFP growth.

The other reason why official estimates of capital may depart from the theoretically ideal measure is that not all capital goods are used at full capacity at all points during the business cycle. Just as the expression for labour input should only reflect the number of persons employed, so too should the measure of the capital stock include only capital employed. Although in the long run changes in capacity utilisation should even out, in the short term, failure to adjust for changes capacity utilisation will tend to impart a pro-cyclical bias to measured TFP[5].

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[4] See Coulombe (2000) for more on this topic in the context of U.S.–Canada comparisons.

[5] See Wilkins (1992) *et al.* for a measure of TFP that adjusts for changes in capacity use.

III.   A Neoclassical Model with Capital-Embodied Technical Change

This section examines how TFP growth may change when we include capital-embodied technical change. The key conclusion is that extending the neoclassical model to allow for quality adjustments of capital may produce lower observed TFP growth measures, even if the underlying rate of technical change is the same.

A Model with Capital-embodied Technical Change

Models of capital-embodied technical change, such as that of Solow (1960), postulate that much of productivity growth can be attributed to increases in the *quality* of capital goods. In these models technological change is said to be *embodied* in new capital goods. Models of capital-embodied technical change are often called 'vintage' models, because capital goods of more recent vintages are more productive—of greater quality—than older capital goods. Thus for example, a computer today may cost the same as a computer bought ten years ago, but the newer computer will be much more productive than the old one as a result of technical change over this period. An appealing feature of these models is that new investment is required before the economy can take advantage of this kind of technological change.

To formalize a model of capital-embodied technical change, we assume that the aggregate production function takes the same Cobb Douglas form as before, where E is the *effective* capital stock. This quality-adjusted capital is related to the conventionally-measured capital stock K by the following relationship

$$Y = A' E^{\alpha} \cdot L^{1-\alpha}, \text{ where } E_t = \sum_{v=0}^{t} K_{vt} \cdot B_v \quad (5)$$

where K_{vt} is the quantity of capital (net of physical depreciation) built in year v still in use at time t, and B_v is the level of technology embodied in capital goods produced in year v. The relationship between growth of effective capital stock and the growth of the conventionally measured capital stock can be approximated by the relationship, $\dot{E} = \dot{K} + \dot{B} - \dot{B} \cdot \Delta H$, where ΔH is the change in the average age of the capital stock (Nelson 1964)[6]. In this case, the effective capital stock grows faster than the conventional measure of capital stock, as long as the capital stock is not ageing too quickly.

Incorporating capital-embodied technical change into the neoclassical model provides an additional source of growth in the short and long run. In the short run, labour

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[6] Nelson (1964) shows that if the conventionally-measured capital stock is calculated net of *economic* depreciation, then $\dot{E} = \dot{K}^{net} + \dot{B}$. This occurs because economic depreciation, the fall in the economic value of capital goods over time as cheaper equivalents become available, is equivalent to the 'vintage effect' picked up by the term $\left(-\dot{B} \cdot \Delta H \right)$. Without increasing quality of capital goods over time, there would be no economic (as opposed to physical) depreciation, because older capital goods would be just as efficient and therefore valuable as new capital goods.

productivity growth depends on disembodied technological change, $\dot{A}'$, capital-embodied technical change $\dot{B}$, and the growth in the conventionally-measured capital stock $\dot{k}$. In the long run, as with the simple neoclassical model, the rate of capital accumulation should not be thought of as an exogenous determinant of productivity growth. Rather, it is determined by the rate of technological change. This is because there is still diminishing returns to capital. Even though the quality of new capital goods is increasing, this is a purely exogenous process, and is not affected by an increase in investment. In the long run, one can show that labour productivity growth depends on the rate of disembodied technological change ($\dot{A}'$) and the rate of capital-embodied technical change ($\dot{B}$), so that

$$\dot{y} = \frac{\dot{A}' + \alpha \cdot \dot{B}}{1 - \alpha} \qquad\qquad (6)$$

Productivity Estimates with Capital-Embodied Technical Change

How should we interpret standard measures of TFP when there is capital-embodied technical change? The answer depends crucially on the extent to which capital stock measures are adjusted by statistical agencies for quality changes. In the case where there is no adjustment made for quality changes, measured TFP growth will pick up both the disembodied portion of technological change and the embodied portion, as in equation (6). Thus the conventional growth accounting procedure gives the relevant measure of productivity growth for understanding long-run changes in labour productivity growth. What it does not do is give the portion of long-run growth that is due to capital-embodied technical change.

When prices of capital goods are adjusted for quality change, the situation becomes more complicated. Statistical agencies such as Statistics Canada and the Bureau of Labor Statistics now adjust downwards the prices of computers and related capital goods to reflect quality change. This means that the real capital stock of these machines is now measured in efficiency units, to give a measure of the effective capital stock (E). In this case standard growth accounting procedures will only reveal the disembodied part of technological change ($\dot{A}'$). To calculate the part of technological change that is embodied in capital goods, one must use the difference between the non hedonically-adjusted price index and the hedonically-adjusted price, as in Greenwood *et al.* (1997).[7] This will give an empirical estimate of $\dot{B}$ to calculate the long-run growth path of the economy.

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[7] The complication arises in hedonic pricing methods used to calculate not only the effective capital stock, but also to the real value of output. Greenwood *et al.* (1997) argue that the appropriate way to measure real output is to use the price of consumption goods, not investment goods, to deflate the value of real investment. Failure to do so biases upward real output and therefore labour productivity and disembodied TFP, and in consequence reduces the relative importance of capital-embodied technical change.

IV.   Productivity in the New Growth Models

While the neoclassical model assumes TFP to be determined outside the model, 'new' growth theory, or endogenous growth theory, attempts to explain the evolution of the TFP. All new growth theories stress the importance of ideas as the underlying 'engine' of growth. However, they differ in what is posited to be the primary determinant of ideas: some authors stress investment in human capital, other stress investment in new capital goods, or spending on research and development.

What does new growth theory imply for growth accounting? If we assume that TFP depends on investment in human capital, or that part of spending on R&D not measured as investment, then the growth accounting—shown in previous sections—largely carries through. Because there are still diminishing returns to physical capital, the (physical) capital–labour ratio cannot grow faster than adjusted TFP in the long run, and so TFP growth is still a good guide to long-term growth trends. The pattern of causation still runs from TFP to physical capital: the only change is that now TFP is determined by some other factor, such as investment in human capital or R&D spending.

In the traditional neoclassical growth model, the ability of government policies to influence long-run living standards growth is limited. For example, if policy succeeded in raising the investment rate, according to the neoclassical model the result would be a one-time improvement in living standards, following which the economy would return to its original steady state growth rate. In some new growth theory models, in contrast, a rise in the investment rate can result in a *permanent* rise in the growth rate of output, leading to ongoing improvements in living standards. The reason for the ongoing effect is that, according to the new growth theory, more output not only raises living standards but also leads to more human capital formation and innovation, which increase productivity and output, which increase human capital and innovation again, and so on in a virtuous circle. Thus, in the new growth theory models, the impact of an increase in the investment rate in the long run can be much greater than that predicted by the traditional neoclassical growth model.

A New Growth Model with Physical Capital

An example of a new growth model with capital as the engine of growth is found in Romer (1987). In a standard aggregate production function such as the one shown in equation (1), capital can be defined as

$$K^{\alpha} = \sum_{i=1}^{M} X_i^{\alpha} \qquad\qquad (7)$$

where X_i is a distinct kind of capital good, and *M* represents the different types of capital good. With $\alpha < 1$, there are diminishing returns to the addition of more of an existing kind of capital good, but constant returns to the addition of new types of capital good. The total amount of capital goods produced at any given time is limited by what Romer calls 'primary' capital[8]. In equilibrium, Romer shows that the same amount of each kind of capital good will be produced (that is, $X_i = \dfrac{K}{M}$) so that labour productivity in the standard production function can be defined as:

$$\dot{y} = \dot{A}{''} + (1-\alpha) \cdot \dot{M} + \alpha \cdot \dot{k} \qquad (8)$$

Although the expression for labour productivity in this new growth model looks very similar to the one found before in the neoclassical models, the central difference is that *M* is now an endogenous source of growth. *M* depends on the amount of resources society is willing to devote to the production of capital (Romer's 'primary capital'). Intuitively, the idea is that as the economy gets bigger, the extent of specialization can increase because of the fixed resource cost of introducing new capital goods.

New Growth Models, New Productivity Estimates?

If we apply the standard growth accounting procedure in the context of Romer's growth model, we find that

$$T\dot{F}P = \dot{L}P - \hat{\alpha} \cdot \dot{k} = \dot{A}{''} + (1-\alpha) \cdot \dot{M} \qquad (9)$$

Unlike the neoclassical model, TFP is no longer a better guide to long term trends in productivity than labour productivity. This is because, as mentioned above, TFP now follows capital accumulation, rather than being a cause of it.

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[8] Each capital good is assumed to be produced using primary capital according to the cost function: $C(X_i) = c_0 + c_1 \cdot X_i$. The fixed cost ($C_o$) limits the number of types of capital good at any point in time. This slightly simplified version of the cost function in the Romer model is taken from Aziz (1996).

V. Conclusions

Table 2 contrasts the previous and the new estimates in the short run and in steady states according to three growth models considered in this note. In the long run or steady state, the original expression $\dot{A}/(1-\alpha)$, that is, the Harrodian rate of technical progress, indicated that *all* of labour productivity growth was ultimately the result of TFP growth. In other words, in the long run, if TFP were to stop growing, capital intensity would also stop growing and there would be no further improvements in standards of living.

The other two models presented in this note emphasize other sources of growth. On the one hand, extending the neoclassical model to allow for capital-embodied technical change implies that labour productivity growth now depends on both the rate of disembodied technological change ($\dot{A}'$) and the rate of capital-embodied technical change ($\dot{B}$), where the latter is an exogenous source of growth. On the other hand, with our example of new growth models, long-term growth in labour productivity is determined by the number of different types of capital good *M*, which is an endogenous source of growth because it depends on the amount of resources society is willing to devote to the production of capital.

Table 2. Summary of Results

Time Horizon Model	Short Run	Steady State
1. Standard Neoclassical Model	$T\dot{F}P = \dot{L}P - \hat{\alpha} \cdot \dot{k}$	$\dot{y} = \dfrac{\dot{A}}{1-\alpha}$
2. Neoclassical Model with Capital-Embodied Technical Change	$T\dot{F}P = \dot{L}P - \alpha \cdot (\dot{k} + \dot{B})$	$\dot{y} = \dfrac{\dot{A}'}{1-\alpha} + \dfrac{\alpha}{1-\alpha} \cdot \dot{B}$
3. New Growth Model with Physical Capital	$T\dot{F}P = \dot{L}P - (1-\alpha) \cdot \dot{M}$	$\dot{y} = \dfrac{\dot{A}''}{(1-\alpha)} + \dot{M}$

What then do we believe to be the most appropriate measure of productivity growth, TFP or labour productivity? Our analysis suggests that the choice should depend several factors, including the time period of interest, the quality and comparability of the capital stock data, and the growth model assumed.

The Time Period of Interest

In most of the models examined in this paper, diminishing returns to capital accumulation imply that capital intensity cannot grow faster in the long run than (adjusted) TFP growth. Any tendency for labour productivity to grow faster than TFP is merely a temporary

phenomenon. Nevertheless, this adjustment to the long run could well be prolonged. Table 1 illustrates that it is possible for capital intensity to grow more quickly than adjusted TFP for a considerable period of time. So, if the intent is to examine trends in the economy over a period of less than a decade or so, then labour productivity is a better guide. If the intent is to examine long run trends in the economy, then TFP may well be a better choice than labour productivity.

The Quality and Comparability of the Capital Stock Data

If there are important biases in the estimates of the capital stock used to construct measures of TFP growth, then it will clearly be better to rely on measures of labour productivity, which are estimated directly using readily available estimates of value added and labour inputs. This is especially relevant when making cross-country comparisons, because the procedures used by different statistical agencies to deal with, for example, depreciation and aggregation, may be quite different.

What the Underlying Model of Productivity Growth Is Assumed to Be

Any attempt to measure TFP involves an implicit assumption about the underlying model of growth. This means that simply estimating TFP growth is not enough. Understanding the growth models is more fundamental. Estimating A (or A' or A'') is not enough because long-run productivity growth may stem from a number of other factors, depending on which growth model one assumes:

- In the *neoclassical model of growth*, when no adjustments are made for changing capital quality, measured TFP growth reflects both disembodied and embodied technological change, and remains the best guide to long-run trends in productivity. When adjustments are introduced, measured TFP (i.e., growth in A') only reflects disembodied technological change, and additional data is required to construct a measure of embodied technological change (i.e., growth in B).

- In the *new growth models* in which physical capital accumulation is the engine of growth, capital intensity drives TFP growth, not the other way around. Therefore, trends in capital accumulation are more relevant for examining the growth process than are trends in TFP growth.

Given that both the short and the long run matter to policy makers, and that there is considerable debate over how to measure the capital stock, and over which is the 'right' model of economic growth, it seems reasonable to argue that both labour productivity and TFP are useful indicators of growth trends in the economy. But neither should be relied upon exclusively.

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