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# Public capital and its contribution to the productivity performance of the Canadian business sector

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# Public Capital and Its Contribution to the Productivity Performance of the Canadian Business Sector<sup>\*</sup>

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<sup>&</sup>lt;sup>\*</sup> The Canadian Productivity Accounts produce a set of non-parametric multifactor productivity estimates that accord with international best practice as outlined by the OECD Productivity Manual (OECD 2001). However, in order to keep abreast of new developments and to provide quality control for the databases that are used to produce these estimates, the productivity group also experiments with alternate methods of measuring productivity. The estimates in this paper are derived from one such attempt to explore a new domain—one that tries to quantify the contribution of public capital to multifactor productivity growth. The estimates in this paper are experimental and will differ from the official estimates that are listed in Statistics Canada's CANSIM database.

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### Abstract

This paper quantifies the contribution of public capital to productivity growth in the Canadian business sector. The approach developed here incorporates demand and supply forces, including the contribution of public capital, which may affect productivity performance. We estimate the model using disaggregated data composed of 37 industries in the Canadian business sector for the period 1961-2000. The results indicate that the main contributors to productivity growth, both at the industry and aggregate levels are technical change and exogenous demand (representing the effect of aggregate income and population growth). Public capital contributed for about 18% of the overall business sector multifactor productivity growth over the 1961-2000 period. This is somewhat lower than the figures reported in the literature. However, the magnitudes of the contribution of public capital to productivity growth vary significantly across industries, with the largest impact occurring in transportation, trade and utilities.

Keywords: public capital, productivity, infrastructure, externalities.

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# Executive Summary

The main goal of this paper is to examine the contributions of public capital to economic growth and productivity growth of the Canadian business sector and its constituent 37 industries over the 1961-2000 period. Public capital is defined as the engineering construction component of public administrations' capital stock (federal, provincial and territorial, and local) and includes primarily transportation systems, such as subways and highways, mass transit, water supply, and wastewater treatment facilities. The study provides empirical evidence of the positive impacts of public capital on business sector costs of production. It also evaluates the effects of capital investment on the business sector's demand for labour, capital formation, and intermediate inputs; estimates the marginal benefits of public capital; and identifies the contribution of public capital and other economic factors to the productivity growth rate in the Canadian business sector.

The study examines in detail three sets of questions:

1. What are the effects of public capital on the business sector production costs, level of output, and demand for labour, capital, and intermediate goods?

A principal conclusion of this study is that an increase in the services of public capital has an initial direct productivity effect: it reduces the total cost of producing a given level of output in almost all industries. The cost-reducing 'productivity effect' of public capital varies in magnitude across industries. The size of the public capital productivity effect on each of the 37 industry sectors comprising the Canadian business sector is indicated by the 'cost elasticity' measure.

The cost elasticity value indicates the percentage change in the total private cost of producing a given level of output that is associated with a 1% change in the value of the public capital services. It is derived from the econometric estimation of the industry cost function by taking the first partial derivative of the total cost function with respect to public capital. A negative sign indicates that an increase in public capital results in total cost reduction. Cost reductions are relatively large in industries such as transportation (-0.15), wholesale (-0.12), other utility (-0.09) and retail (-0.12). The cost elasticities range between -0.002 to -0.06 in the manufacturing sector and between -0.001 to -0.05 in the primary sector.

To obtain a business sector level estimate of the cost-reducing impact of public capital investment, industry cost elasticity measures are weighted by the industry's share of business sector's nominal gross output and summed. The average cost elasticity with respect to public capital for the Canadian business sector during the period 1961 to 2000 is about -0.062.

The economic impact of public capital on the various industries does not stop with the direct productivity effect. Cost reductions permit products to be sold at lower prices and lower prices can be expected to lead to output growth. This is termed the 'output effect' of public capital. The size of industry output expansion depends on the nature of the demand for products and therefore varies across industry sectors. Of course, at higher production levels, a producer's total costs will increase because of the additional labour, capital and intermediate

inputs that are required to make the additional output. An important empirical finding of the current study is that the higher total production costs associated with the output expansion effect are 'financed' almost entirely by the cost-saving productivity gains of public capital.

Given the cost-reducing and output-expanding impacts of public capital, it is not surprising that public capital has a significant effect on the business sector's demand for labour, capital, and intermediate inputs. The magnitude of the effect, which is termed 'conditional factor demand,' varies among the three inputs (labour, capital, and intermediate inputs), across industries and whether we are examining industry's demand for resources in the context of the 'productivity effect' alone (i.e. when output level is held fixed) or after allowing for the 'output effect' (i.e. when the output level is allowed to increase in response to the cost-saving/price-reducing effects of public capital).

We find that the initial productivity effect of an increase in public capital results in a reduction in the demand for labour and intermediate inputs, but an increase in the demand for private capital in all industries.

We also evaluate changes in the production sector's demand for labour, capital and intermediate inputs when industry production levels vary (increase) due to the 'output effect' of public capital. The direction of the impacts on business demand for labour, capital, and intermediate inputs when the output effect is considered are the same as under the productivity effect alone, (i.e. public capital increases result in reductions in demand for labour and intermediate inputs but increases in demand for private capital). However, when industry output expansion is considered, the magnitude of the change in demand for labour and intermediate inputs is substantially reduced while the demand for private capital increases significantly. That is, the output effect of public capital leads to an even larger 'crowding in' of private capital formation.

We can generally conclude that the productivity and output effects of public capital substantially change the input ratios of the production function in all industries, point toward an important role for public capital spending in contributing to investment-led economic expansions, and imply that public capital is a complement to private capital.

2. What are the marginal benefits to industry sectors and the aggregate business sector of an increase in public capital?

The marginal benefit of public capital is measured in terms of its initial cost-reducing impact (i.e. the productivity effect). The magnitude of cost reduction depends on the industry's elasticity of cost with respect to public capital and the industry's total costs of production relative to the size of the public capital stock. This study indicates that the marginal benefits of public capital are positive in all industries and the lie within reasonably tight confidence intervals, thereby reflecting reliable estimates. Marginal benefit estimates can be interpreted as a measure of producers' 'willingness to pay' for an additional unit of public capital, and vary considerably across industries and over time. For industries such as construction, transportation, wholesale, retail, communication and other utility, the marginal benefits of a \$1.00 increase in public capital range between 19 cents and 42 cents. Industry marginal

benefit estimates can be translated into a dollar value of cost reduction in each industry for a given amount of public capital spending. The simplest way to do this is to multiply the measure of marginal benefit in each industry by the net increase in public capital for a particular year or period.

The calculation of the marginal benefit of public capital investment at the business sector level assumes that the use of the public capital system by one industry does not preclude or reduce the value of its use to any other industry (i.e. we assume non-rivalrous consumption of the public good). Therefore, industry marginal benefits are additive across the 37 sectors. The weighted sum of marginal benefits across all industries is about 0.17. That is, a \$1.00 increase in the net capital stock generates, on average, approximately 17 cents of 'cost saving' producer benefits per year for the business sector. Assuming the depreciation charge against public spending levels in subsequent periods are sufficient to maintain the net capital stock value, benefits can be thought of as continuing over the design life of the underlying public capital improvement.

3. What is the contribution to productivity growth of public capital?

The contribution of public capital to productivity growth is positive in all industries. The current results show a pervasive influence of public capital on industry productivity growth but its magnitudes vary across industries. In some industries, such as wholesale, the effect can be quite large. At the aggregate level, public capital's contribution to multifactor productivity growth is about 18%, thus confirming our previous finding that the main contributors to productivity growth, both at the industry and aggregate levels, are technical change and exogenous demand (representing the effects of aggregate income and population growth). Nonetheless, the contribution of public capital remains non negligible.

## I. Introduction

Transportation systems, such as roads, highways, railways, ports, bridges and airports; streets, water and sewer systems and dams are public capital assets that have long been recognized as an integral and important part of the measurement of the wealth of nations. It is only recently, however, that economists began to quantify the effect of public capital on economic growth and productivity performance. For example, a well-constructed highway allows a truck driver to avoid back roads and to transport goods to market in less time. The reduction in required time means that the producer incurs a lower cost and the truck experiences less wear and tear. Hence, public investment in a highway enables private companies to produce their products at a lower cost. The condition of the highway, of course, is just as important as its existence. Similar stories can be told for mass transit, water and sewer systems, and other components of public capital.<sup>1</sup>

Although the first attempt to investigate empirically the significance of public capital in an aggregate production function was due to Ratner (1983), it was not until Aschauer's (1989) study that the issue became a topic of heated debate among economists. Aschauer concluded, on the basis of his results, that "a significant weight should be attributed to public investment decisions...when assessing the role the government plays in the course of economic growth and productivity improvement" (p. 197).

Aschauer's study sparked off an extensive empirical literature that largely focused on the U.S. economy.<sup>2</sup> Despite its importance in the policy debate, the role of public capital in the productivity performance of the private sector has attracted less attention in Canada (Wylie 1996 and Harchaoui 1997 are the few exceptions).<sup>3</sup> In view of the different methodologies employed by these studies, the results are not easily comparable, thereby making it difficult to form a definitive view on the role of public capital in Canada's economic performance. There is, therefore, considerable scope for investigating the effects of public capital within a unified framework using new modelling techniques and a comprehensive industry dataset to answer the following public policy issues:

<sup>&</sup>lt;sup>1</sup> For an analysis of public infrastructure trends in Canada, see Harchaoui *et al.* (2003).

<sup>&</sup>lt;sup>2</sup> There are two alternate ways to examine the contribution of public capital to productivity performance. The first wave of empirical research in this area, based on a Cobb-Douglas production function estimated with aggregated time-series data, is known as the primal approach. The results derived from this methodology have overstated the magnitude of the effects of public capital on output and productivity growth. Because the results produced by the primal approach are 'too large to be credible,' the production structure of the firm has subsequently been used to model private inputs, public capital and output decisions in a cost-minimization, or dual, framework. The studies that used the dual approach to analyse the effect of public capital on output and productivity growth have produced more reasonable estimates. The literature includes Morrison and Schwartz (1996), Nadiri and Mamuneas (1994a, b), Lynde and Richmond (1992) and Deno (1988) who investigated the impact of public capital on the cost structure of the US industries. Lynde and Richmond (1993), Berndt and Hansson (1992), Shah (1992) and Conrad and Seitz (1994) performed similar studies for other countries. Demetriades and Mamuneas (2000) examined the importance of public capital in the production structure of OECD countries. In this paper, we apply the dual approach in a way similar to Nadiri and Mamuneas (1994b).

<sup>&</sup>lt;sup>3</sup> Wylie's approach is based on a production function while Harchaoui's makes use of the cost-minimization framework. The latter study imposed constant returns to scale on the technology structure but it has a broader industry coverage and a split of public capital by level of jurisdiction—federal, provincial, and local.

- 1. The effect of public capital on the private sector's total cost and demand for labour, capital, and intermediate inputs;
- 2. The marginal benefits to specific industries of an increase in public capital;
- 3. The contribution to productivity growth of public capital.

The analytical framework used in this paper considers:

- 1. The effect of aggregate demand on the productive behaviour of individual industries; that is, the effects of changes in aggregate income and population on industry demand which in turn affect the output and productivity growth of the industry;
- 2. The contribution of changes in real factor prices, including wages and capital rental prices, on productivity growth;
- 3. The impact of both federal and non-federal public capital on the demand for labour, intermediate inputs and private sector physical capital;
- 4. The marginal benefits of public capital to various industries and its contribution to the output and productivity growth.

This paper is organized as follows. Section II provides the general analytical framework of our study specifying demand and cost functions for individual industries. The analytical structure allows estimation of the structural parameters for each industry's output demand and cost functions and provides a framework for decomposing multifactor productivity growth into several components, including the contribution of public capital. This methodology allows us to trace the effects of growth of aggregate output (GDP), population, real factor price changes, technical change, and public capital on multifactor productivity growth in each industry.

Section II also describes briefly the sources of data, some descriptive statistics and construction of the variables used to estimate the model. The primary data are a panel of prices and quantities of output and inputs for 37 industries, for which we believe output is well measured, over the period 1961-2000. These industries collectively cover 71% of the Canadian business sector and provide a basis to estimate the contributions of various factors to the growth of output and productivity for the overall business sector. Also, data on public capital stock and some aggregate data series, such as GDP, population and the GDP deflator for this period are included.

In Section III, we discuss the estimation approach for the demand and cost functions and the underlying concepts, using the parameter estimates of the econometric model, to calculate the characteristics of the industry production structure, marginal benefits of public capital and a decomposition of multifactor productivity growth. This section also reports the results of sensitivity tests that examine the stability of the econometric model. Criticism aimed at time-series econometric models designed to quantify the contribution of public capital to economic growth and productivity are specifically addressed.

Section IV presents empirical estimates of the effect of public capital on industry production costs and input demand when the level of output is assumed to be given. Estimates of the effect of an increase in public capital on the derived demand for inputs such as labour, capital and intermediate inputs are presented. We present estimates of induced output expansion effects due to an increase in public capital and calculate the total effects of an increase in public capital investment on industry cost and demand for labour, intermediate inputs and capital when the level of output is allowed to vary. Estimates of the marginal benefits of a change in the level of public capital to each industry, along with their degree of reliability, are also provided. Finally, we also examine the results of a decomposition of multifactor productivity growth into its various components, including public capital, by industry.

Section V presents measures of the contribution of the public capital stock to the whole business sector's output and productivity growth by aggregating industry-specific estimates. We also discuss the decomposition of multifactor productivity growth at the business sector level. Since the results pertaining to recent years are of interest for policy purposes, we discuss very briefly, the contribution of public capital in the last decade (i.e., 1988-2000) to the growth of output, labour input, capital input and productivity growth. These results are based on the parameter estimates of the basic model. Section V provides a brief summary and conclusion followed by a mathematical appendix.

# II. Analytical Framework

### 1. Set up

The basic methodology employed in this study involves the estimation of industry demand and cost functions. The approach, based on Nadiri and Mamuneas (1994b), identifies the contribution of output demand, relative input prices, technical change and publicly financed capital to multifactor productivity growth. Analyzing the relative contribution of these components in the context of a unified framework helps to answer policy questions regarding the extent and significance of public capital's effect on the productivity performance.

Using these industry parameter estimates, we deduce the corresponding estimates for the aggregate business sector. We evaluate the demand and cost equations separately and then we use their estimated parameters to decompose multifactor productivity growth and to calculate the marginal benefits of public capital for each industry.

The critical parameter estimates for the decomposition of multifactor productivity growth are the price and income elasticities, obtained by estimating the output demand function, and the degree of scale and input substitutions, obtained by estimating the cost function. The estimates of marginal benefits, the impact of public capital on demand for labour, private capital, and intermediate inputs are based on estimating the model for the 37 industries and sectors for the sample period, 1961-2000.

#### 1.1 The Cost Function

We write the cost function for the *ith* industry as  $C_i(w_i, Y, t; G)$  where  $C_i$  is a twice continuously differentiable, normalized cost function;  $w_i$  is an n-1 dimensional vector of relative variable factor prices, Y is the quantity of output, t is an index of time representing disembodied technical change, and G represents public capital services, a quasi-fixed input.

Public capital services affect the cost structure of an industry in several ways. First, an increase in quantity (or better quality) of public capital services shifts the cost per unit of output downward in an industry. We call this the 'productivity effect.' Second, firms will adjust their demand for labour, intermediate inputs, and physical capital stock if public capital services are either substitutes for, or complements to, the inputs in the private sector. That is, the effects of public capital may not be neutral with respect to private sector input demand decisions. We call this the 'factor demand effect.' Third, the cost reduction induced by the increase in public capital investment may lead to a reduction in the price of output which in turn may result in an increase in the demand for output. We call this the 'output expansion effect.' This output expansion effect is an indirect effect of public capital. The increased capital leads to greater output production, which in turn leads to an increase in the demand for labour, intermediate inputs and private capital. The net effect of public capital on total cost and its structure will be the combination of the productivity, factor demand, and output expansion effects.

We assume that the technology of the industry is represented by a translog cost function of the following form:

$$\begin{split} \ln \tilde{C} &= \alpha_{o} + \alpha_{K} \ell n \tilde{w}_{K} + \alpha_{L} \ell n \tilde{w}_{L} + \alpha_{Y} \ell n Y + \alpha_{G} \ell n G + \alpha_{t} t \\ &+ \frac{1}{2} \Big[ \alpha_{KK} \left( \ell n \tilde{w}_{K} \right)^{2} + \alpha_{LL} \left( \ell n \tilde{w}_{L} \right)^{2} + \alpha_{YY} \left( \ell n Y \right)^{2} + \alpha_{GG} \left( \ell n G \right)^{2} + \alpha_{tt} t^{2} \Big] \\ &+ \alpha_{KL} \ell n \tilde{w}_{K} \ell n \tilde{w}_{L} + \alpha_{KY} \ell n \tilde{w}_{K} \ell n Y + \alpha_{KG} \ell n \tilde{w}_{K} \ell n G + \alpha_{Kt} \ell n \tilde{w}_{K} t \\ &+ \alpha_{LY} \ell n \tilde{w}_{L} \ell n Y + \alpha_{LG} \ell n \tilde{w}_{L} \ell n G + \alpha_{Lt} \ell n \tilde{w}_{L} t + \alpha_{YG} \ell n Y \ell n G + \alpha_{Yt} \ell n Y t \\ &+ \alpha_{Gt} \ell n G t, \end{split}$$

and the share equations

$$S_{K} = \alpha_{K} + \alpha_{KK} \ell n \tilde{w}_{K} + \alpha_{YK} \ell n Y + \alpha_{KL} \ell n \tilde{w}_{K} + \alpha_{KG} \ell n G + \alpha_{Kt} t$$

$$S_{L} = \alpha_{L} + \alpha_{KL} \ell n \tilde{w}_{K} + \alpha_{YL} \ell n Y + \alpha_{LL} \ell n \tilde{w}_{L} + \alpha_{LG} \ell n G + \alpha_{Lt} t,$$
(2)

where  $\tilde{C}$  is total cost normalized by the price of intermediate inputs,  $w_M \cdot \tilde{w}_K \left(=\frac{q_K}{w_M}\right)$  and  $\tilde{w}_L \left(=\frac{w_L}{w_M}\right)$  are the relative prices of capital and labour, respectively. The production cost is given by  $C = q_K K + w_L L + w_M M$ , where  $q_K \left[=\left(\frac{1-u-z\theta}{1-u}\right)w_K \left(r+\delta-\frac{\dot{w}_K}{w_K}\right)+\tau\right]$  is the rental price of capital.  $w_K$  denotes the acquisition price of capital, r is the (external) rate of interest,  $\delta$  is the rate of depreciation and  $\frac{\dot{w}_K}{w_K}$  is the price change of capital goods,  $\frac{1-u-\theta z}{1-u}$  the tax price (u is the corporate income tax rate, z is the present value of the depreciation,  $\theta$  is the investment tax credit rate,  $\tau$  is the property tax rate). Y is the level of output, G the level of public capital, t is an index of technical change. Finally,  $S_K$  and  $S_L$  are the cost shares of labour and capital.

As stated above, *G* is the stock of public capital. This is a public good, and as a result, no market prices can be related to the services it provides. However, it is possible to determine the shadow price or willingness to pay for these services as the private production cost savings associated with *G*. The marginal benefit of public capital is measured as  $-\frac{\partial \tilde{C}}{\partial G}$ , that is, an additional unit of public capital  $\partial G$ , result in a cost reduction,  $\partial \tilde{C}$ . For example, a better network of highways reduces driving time and saves labour, fuel and other operating costs.

Aside from the direct productivity effect of public capital indicated by  $-\frac{\partial \tilde{C}}{\partial G}$ , there are factor demand adjustment effects that arise out of the complementarity and substitutability of private inputs (such as labour, private capital and intermediate inputs) for public capital. These effects can be calculated as  $-\frac{\partial X_f}{\partial G}$  where  $X_f$  is the quantity of the (private) input f. If the expression  $-\frac{\partial X_f}{\partial G}$  is greater than, equal to or less than zero, then public capital has a negative, zero, or positive effect on the demand for that particular input.

Finally, the effect of an increase in public capital on the rate of technical change can be calculated using the following expression,  $\frac{\partial \tilde{C}}{\partial t \partial G} = \frac{\left(\frac{\partial \tilde{C}}{\partial t}\right)}{\partial G}$  which indicates that an additional unit of public capital results in a productivity increase or cost decrease due to technical change.

#### **1.2 Demand Function Estimates**

The model has been characterised to trace the effects of public capital on multifactor productivity growth. The decomposition of multifactor productivity growth into various components requires two sets of parameter estimates: First, estimates of the cost elasticities of public capital and other parameters of the cost function (1); second, parameter estimates of the output demand function which relate growth of output demand to changes in price of output and per capita income.

The output demand equation for each industry, i, is specified as a log linear function

$$\dot{Y} = \lambda + \alpha \left( \dot{P}_Y - \dot{P}_D \right) + \beta \dot{Z} + (1 - \beta) \dot{N} .$$
(3)

The demand function is estimated in growth rates. According to equation (3), the output growth rate in each industry is regressed on a constant, the growth rate of its output price normalized by the GDP deflator and the growth rate of real GDP per capita (Z and N are, respectively, GDP and population). Thus, changes in quantity demanded in an industry are related to its own price movement in comparison to the GDP deflator and changes in the level of aggregate income and population of the economy.

We are interested in two of the estimated parameters of the demand function (3). They are: (a) the price elasticity of output demand, which is measured by the coefficient  $\alpha$  ( $\alpha = 0$  implies demand is perfectly inelastic;  $\alpha = 1$  implies demand is unitary elastic; and  $\alpha > 1$  implies demand is elastic); and (b) the per capita income elasticity of output demand, which is measured by the coefficient  $\beta$  (same definitions as for  $\alpha$ ).

#### 1.3 Cost Reduction, Scale Elasticities and Output Expansion

From the estimates of the demand and cost functions, we derive the critical demand and cost elasticities that are necessary to measure the impact of public capital on the cost structure and productivity growth of each industry. The critical elasticities are the following:

$$\eta_{\tilde{C}G} = -\left(\frac{\partial \ell n \tilde{C}}{\partial \ell n G}\right)$$
 represents the private cost elasticity with respect to public capital;

 $\eta = \frac{\partial \ell n \tilde{C}}{\partial \ell n Y}$  is the cost elasticity with respect to output;

$$\eta^* = \frac{\eta}{(1 - \eta_{\tilde{C}G})}$$
 is the cost elasticity of output when all inputs, including public capital are included.

The increase in output due to the cost-reduction effect of an increase in public capital measured in terms of elasticities is  $\eta_{YG} = -\frac{\eta_{\tilde{C}G}}{\eta}$ ; that is, the cost elasticity of public capital multiplied by the estimated internal degree scale for each industry.<sup>4</sup>

$$L(w,Y,G,t,\varphi) = C(w,Y,G,t) - \varphi[F(K,G,L) - Y],$$

where  $\varphi$  is the Lagrange multiplier. Applying the envelope theorem, we have

$$\frac{\partial L}{\partial G} = \frac{\partial C}{\partial G} - \varphi \frac{\partial F}{\partial G} = 0,$$

and

$$\frac{\partial L}{\partial Y} = \frac{\partial C}{\partial Y} - \varphi = 0.$$

Substituting the second condition into the first and multiplying the result by  $\frac{G}{Y}$  gives the relationship between public capital output elasticity and public capital cost elasticity

$$\frac{\partial \ell n Y}{\partial \ell n G} = \frac{-\left(\frac{\partial \ell n C}{\partial \ell n G}\right)}{\frac{\partial \ell n C}{\partial \ell n Y}}.$$

The latter provides the linkage between the primal approach and the dual approach. This condition can be used to recover the public capital output elasticities from the public capital cost elasticities.

<sup>&</sup>lt;sup>1</sup> Under cost-minimization, the Lagrangian is given by

#### 1.4 Multifactor Productivity Growth Decomposition

Multifactor productivity growth,  $M\dot{F}P$ , can be decomposed as follows (see the Appendix):

$$M\dot{F}P = A[\alpha\dot{\eta} + \alpha(1+\theta)] + A\alpha \left[\sum_{f} (\hat{\pi}_{f}\dot{w}_{f} - \dot{P}_{D})\right] + A[\lambda + \beta\dot{Z} + (1-\beta)\dot{N}] + \left(A\alpha - \frac{1}{\kappa B}\right)\eta_{CG}\dot{G} + \left(A\alpha - \frac{1}{\kappa B}\right)\dot{T},$$
(4)

where

$$A = \frac{\left(\frac{\kappa - \eta^{2}}{\kappa}\right)}{\left[1 - \alpha(\eta - 1)\right]}.$$

We obtain the relevant parameter estimates from the estimates of the industry demand and the cost functions. The parameters of equation (4) are defined as follows:  $\alpha$  is the output elasticity of demand,  $\beta$  is the income elasticity of output demand,  $\theta$  is the markup over  $\cot\left(\frac{P_y-MC}{MC}\right)$ ,  $\dot{Z}$  and  $\dot{N}$  are respectively the growth of GDP and population,  $\dot{w}_f$  and  $\dot{P}_D$  are respectively the growth rates of the industry input prices f and the GDP price deflator,  $\dot{\eta}$  is the change in degree of scale,  $\kappa$  is the ratio of output price  $P_y$  to average  $\cot\left(\frac{\tilde{C}}{Y}, \hat{\pi}_f = \frac{w_f X_f}{\sum_f w_f X_f}\right)$  is the share of the *fth* input in private cost, C,  $\eta_{\tilde{C}G}$  is the cost elasticities with respect to public capital,  $B = 1 - \eta_{\tilde{C}G}$ ,  $\dot{G}$  the change in the public capital and  $\dot{T}$  is the change in level of technology.

Nadiri and Mamuneas (1994b) provided the following heuristic interpretation to the different components of (4):

- (i) the exogenous demand effect,  $A[\lambda + \beta \dot{Z} + (1 \beta) \dot{N}];$
- (ii) the factor price effect,  $A\alpha \left[\sum_{f} (\hat{\pi}_{f} \dot{w}_{f} \dot{P}_{D})\right];$
- (iii) the public capital effect,  $\left(A\alpha \frac{1}{\kappa B}\right)\eta_{\tilde{C}G}\dot{G}$ ; and
- (iv) the disembodied technical change  $\left(A\alpha \frac{1}{\kappa B}\right)\dot{T}$ .

The parameter estimates of both the cost function (1) and the demand function (3) are critical for the decomposition of multifactor productivity growth. In particular,  $\alpha$  and  $\beta$ , the price and income elasticities of demand,  $\eta$  and  $\eta_{\tilde{C}G}$ , the output cost elasticity and elasticity of cost with respect to an increase in public capital *G*, play critical roles in the decomposition of multifactor productivity growth.

The public capital effect can be decomposed further into direct and indirect effects. For example, the direct effect of public capital *G*, is given by  $\left(\frac{\eta_{\tilde{C}G}}{\kappa\beta}\right)\dot{G}$  while its indirect effect is given by  $A\alpha\eta_{\tilde{C}G}\dot{G}$ . Thus, an increase in public capital initially increases multifactor productivity by reducing the private cost of production, which in turn leads to a lower output price and higher output growth. Changes in output growth in turn lead to changes in multifactor productivity growth.

The important parameters in (4) are the price and income elasticities of demand and the cost elasticities of the private cost function. Note that if the demand function is completely inelastic ( $\alpha = 0$ ), then shifts in the cost function due to real factor price changes, public capital, or disembodied technical change have no effect on output and hence no indirect effect on multifactor productivity. In addition, if the technology exhibits constant returns to scale with respect to all inputs, including public capital inputs, (i.e.  $\eta^* = \kappa = 1$ ), then equation (4) collapses to:

$$M\dot{F}P = -\frac{1}{\beta} \left( \eta_{\tilde{C}G} \dot{G} - \dot{T} \right).$$
<sup>(5)</sup>

#### 2. Data Construction, Industry Prices and Cost Structure

The model detailed in the previous section is estimated using data for 37 two-digit industries of the Canadian business sector during the period 1961 to 2000.

The data set we use comes from three sources. One part of the data is the KLEMS database from the Canadian Productivity Accounts, which contains information on the value of gross output and the costs of labour, capital services and intermediate inputs, as well as their corresponding price and quantity indices, for all industries. The second part is data on public capital, and the third part consists of aggregate series on population and the implicit price of GDP. A brief description of the data set is in order. The KLEMS database provides hours worked by industry. Household survey data are used to disaggregate total hours into hours worked by different types of workers, classified by demographic variables such as sex, age, and education. Assuming that workers are paid proportionately to the value of their marginal products, Gu *et al.* (2003) calculate labour input as a weighted sum of hours worked by different types of workers, weighted by relative wage rates. Annual growth in the labour input for the business sector as a whole from 1961-2000 averaged 2.1%; hours grew an average 1.3% per year; and labour composition increased an average of 0.8%.

Harchaoui and Tarkhani (2003) also estimate capital input adjusted for compositional changes. In order to perform this adjustment, the rental price of nineteen types of capital are needed. Because the rental price is not directly observable, they obtain total payments to capital as property compensation, a residual after all other inputs have been paid. Using these data, they derive the implied rental rates for each asset type of capital based on knowledge of this stock and the depreciation rates for each asset type, and tax parameters such as the corporate income tax and investment tax credits. Over the 1961-2000 period, capital composition grew at an average rate of 1.70%.

Data on net public capital stock are from Statistics Canada's Investment and Capital Stock Division. The aggregate public capital stock used in this paper is a chain Fisher index of engineering construction of the federal, provincial, and municipal administrations.

We present certain selective descriptive statistics on the cost and prices of the 37 industries in our data. In Table 1, we provide the average levels of total cost and average annual growth rates of real output, prices of real inputs and cost shares for the period 1961-2000 for the 37 industries.

As is clear from the descriptive statistics, the size of the industries, measured by total cost or gross nominal output, varies considerably. Trade, construction, transportation equipment, food and transportation are among the largest industries in the business sector defined in this paper.<sup>5</sup> Other industries such as mining, tobacco, furniture and fixtures, and leather and allied products are relatively small.

In addition, factor cost shares vary considerably among the 37 industries. For example, labour compensation share ranges from a low of about 0.07 in crude petroleum and natural gas to a high of 0.56 in retail trade. Capital compensation's share of total cost also varies considerably across industries, ranging from 0.07 in construction to 0.74 in crude petroleum and natural gas. Generally, capital compensation's share of total cost, with a few exceptions—most notably mining, crude petroleum and natural gas, quarry and sand pit, pipeline transport, other utility, beverage, tobacco, and fishing and trapping—is less than labour compensation's share. Intermediate inputs on the other hand, have the largest share in total cost in almost all industries, ranging from 0.18 in pipeline transport to 0.90 in refined petroleum and coal products.

The rates of growth of output  $(\dot{Y})$  and inputs  $(\dot{K}, \dot{L} \text{ and } \dot{M})$  shown in Table 1 also vary among industries over the period 1961-2000. In leather and allied products the growth of output was negative, while in tobacco and tobacco products, fishing and trapping, clothing and storage and warehousing, output growth was modest, ranging from 0.27% to 2.3% per year. Some industries in manufacturing and service sectors experienced impressive gains in output; the growth rates for these industries ranged from approximately 3.8% in transportation to about 8.6% in plastic products. The diversity in the growth pattern of output and inputs across industries suggests that different industries may have experienced different degrees of change in their input mix and output and productivity growth. Similar patterns of lower and more rapid growth rates are visible in the growth rates of labour, capital and intermediate inputs. The growth rates of output price and input prices with few exceptions were all positive but varied considerably across industries. Real public capital stock grew at an average of 2.8%, GDP and its implicit price deflator growth rates were 3.7% and 5.0%, respectively, and that of the population almost 1.4% in the period 1961-2000.

<sup>&</sup>lt;sup>5</sup> Recall that we retained only the industries for which real output is reasonably accurately measured. Finance and real estate, insurance, amusement and recreational service, accommodation and food services, health and social service, business service, personal and household service and educational service have been excluded.

all and finited service $[69 0, 02 0, 02 0, 03 0, 03 0, 03 5, 18 1, 70 6, 40 2, 26 6, 40 10, 70 0, 70$		C	$S_K$	$S_L$	$S_M$	Ķ	Ĺ	Ņ	Ý	$\dot{w}_{K}$	$\dot{w}_L$	$\dot{W}_M$	$P_{\gamma}$
and trapping         0.9         0.34         0.25         0.36         0.73         0.06         5.18         1.70           croblem and matural gas         0.7         0.04         0.25         0.35 <t< td=""><td>Agricultural and related service</td><td>16.9</td><td>0.22</td><td>0.27</td><td>0.51</td><td>0.9</td><td>-1.38</td><td>4.52</td><td>3.35</td><td>9.52</td><td>6.41</td><td>4.17</td><td>3.91</td></t<>	Agricultural and related service	16.9	0.22	0.27	0.51	0.9	-1.38	4.52	3.35	9.52	6.41	4.17	3.91
(and forearry)         50         0.14         0.37         0.53         0.33         2.16         4.12         2.68           erolerm and natural gas         12,9         0.27         0.23         0.33         2.11         0.66         3.92         2.46           and sand pit         23,9         0.10         0.15         0.27         0.23         0.34         4.12         2.56           incidential to mineral extraction         23,9         0.10         0.15         0.75         2.44         0.06         2.29         2.23           incidential to mineral extraction         23,9         0.10         0.13         0.25         0.63         0.9         3.06         0.71         2.29         2.23           products         3,1         0.13         0.25         0.63         0.9         3.06         0.21         2.13           ordinets         1,1         0.31         0.23         0.58         0.23         0.71         2.29         2.13           ordinets         1,1         0.33         0.58         0.23         0.71         2.26         2.73         2.71           ordinets         1,1         0.33         0.56         5.01         1.13         2.75 <td>Fishing and trapping</td> <td>0.9</td> <td>0.39</td> <td>0.25</td> <td>0.36</td> <td>0.7</td> <td>0.02</td> <td>5.18</td> <td>1.70</td> <td>6.46</td> <td>7.68</td> <td>5.04</td> <td>6.79</td>	Fishing and trapping	0.9	0.39	0.25	0.36	0.7	0.02	5.18	1.70	6.46	7.68	5.04	6.79
7.6         0.44         0.25         0.35         2.1         0.64         3.92         2.45           incidential for mineral extraction         0.7         0.32         0.74         0.07         3.92         2.564         5.39           incidential for mineral extraction         2.3         0.21         0.32         0.27         0.41         1.8         5.01         3.53         3.23           produces         3.5         0.22         0.13         0.22         0.31         0.38         0.08         0.071         2.99         2.39         2.37           produces         3.1         0.13         0.22         0.13         0.23         0.36         0.27         3.041         4.66           ordences         3.1         0.13         0.25         0.63         2.3         0.07         1.33         0.36           ordences         3.1         0.13         0.23         0.63         2.3         0.07         1.33         2.1         3.06         2.2         3.09         3.06         2.1         3.05         2.1         3.05         2.1         3.06         2.1         3.05         3.06         3.06         3.06         3.06         3.06         3.06 <t< td=""><td>Logging and forestry</td><td>5.0</td><td>0.14</td><td>0.37</td><td>0.50</td><td>0.3</td><td>-0.86</td><td>4.12</td><td>2.68</td><td>8.32</td><td>7.61</td><td>4.95</td><td>5.33</td></t<>	Logging and forestry	5.0	0.14	0.37	0.50	0.3	-0.86	4.12	2.68	8.32	7.61	4.95	5.33
y and sample in antimal gas         129         0.74         0.07         0.19         3.4         5.75         9.04         4.67           y and sample in antimal gas         12.9         0.74         0.07         0.19         3.4         0.06         2.29         2.211         2.211 <td< td=""><td>Mining</td><td>7.6</td><td>0.40</td><td>0.25</td><td>0.35</td><td>2.1</td><td>-0.64</td><td>3.92</td><td>2.45</td><td>2.88</td><td>7.21</td><td>4.69</td><td>4.28</td></td<>	Mining	7.6	0.40	0.25	0.35	2.1	-0.64	3.92	2.45	2.88	7.21	4.69	4.28
The second set of the second	Crude petroleum and natural gas	12.9	0.74	0.07	0.19	3.4	5.57	9.04	4.67	9.39	7.42	5.35	6.65
cs incidental to mineral extraction         2.4         0.21         0.32         0.47         5.1         5.02         5.64         5.39           age         23.3         0.11         0.15         0.15         0.15         0.16         0.29         2.19           componducts         1.3         0.21         0.13         0.28         0.5         5.2         5.01         8.10           componducts         1.3         0.13         0.25         0.55         5.2         5.01         8.10         8.60           crand allied products         1.3         0.13         0.25         0.55         5.01         1.23         2.13           eproducts         1.8         0.11         0.26         0.25         3.91         1.11           eproducts         1.8         0.11         0.26         3.91         1.01         4.19         3.65           in ad lifed products         1.3         0.11         0.23         0.58         3.91         1.01         3.96         3.50         1.16         3.75         2.66         3.59           used metal products         1.4         0.23         0.33         0.66         3.55         2.66         5.75         5.66 <td>Quarry and sand pit</td> <td>0.7</td> <td>0.32</td> <td>0.27</td> <td>0.41</td> <td>1.8</td> <td>-0.15</td> <td>3.21</td> <td>3.18</td> <td>3.98</td> <td>7.96</td> <td>4.94</td> <td>4.18</td>	Quarry and sand pit	0.7	0.32	0.27	0.41	1.8	-0.15	3.21	3.18	3.98	7.96	4.94	4.18
age         23.9         0.10         0.15         0.75         24         0.06         229         219           are products         1.3         0.22         0.21         0.31         0.33         0.68         0.23 <td< td=""><td>Services incidental to mineral extraction</td><td>2.4</td><td>0.21</td><td>0.32</td><td>0.47</td><td>5.1</td><td>5.02</td><td>5.64</td><td>5.39</td><td>4.24</td><td>6.67</td><td>4.87</td><td>5.16</td></td<>	Services incidental to mineral extraction	2.4	0.21	0.32	0.47	5.1	5.02	5.64	5.39	4.24	6.67	4.87	5.16
35         0.27         0.22         0.51         0.8         -071         2.99         2.36           1.3         0.11         0.22         0.15         0.68         0.68         0.27           3.1         0.11         0.25         0.65         5.0         1.32         0.13         0.58         0.27         0.58         0.27         0.58         0.27         1.32         -1.132 </td <td>Food</td> <td>23.9</td> <td>0.10</td> <td>0.15</td> <td>0.75</td> <td>2.4</td> <td>-0.06</td> <td>2.29</td> <td>2.19</td> <td>6.84</td> <td>6.70</td> <td>4.55</td> <td>4.70</td>	Food	23.9	0.10	0.15	0.75	2.4	-0.06	2.29	2.19	6.84	6.70	4.55	4.70
1.3         0.22         0.15         0.63         0.9         -3.08         0.68         0.27           1.9         0.11         0.31         0.28         2.5         1.26         4.12         8.60           3.1         0.13         0.25         0.63         1.2         2.69         1.76         2.27           1.8         0.11         0.25         0.63         1.2         2.69         1.76         2.27           1.8         0.11         0.25         0.63         1.2         2.69         1.76         2.27           1.8         0.11         0.23         0.55         5.0         1.61         3.39         3.39           ucts         13.7         0.14         0.23         0.56         5.0         1.66         3.39           ucts         13.7         0.14         0.23         0.56         5.0         1.66         3.36           ucts         13.7         0.14         0.23         0.56         5.0         1.66         3.36         3.33           ucts         13.7         0.14         0.23         0.55         5.0         1.76         5.26           ucts         144         3.3         3	Beverage	3.5	0.27	0.22	0.51	0.8	-0.71	2.99	2.36	7.25	7.23	4.67	4.64
19         0.11         0.31         0.58         2.5         126         4.12         4.60           all         0.13         0.25         0.63         1.2         -2.69         1.76         -2.37           18         0.011         0.25         0.63         2.8         0.82         3.79         4.14           18         0.011         0.26         0.63         2.8         0.82         3.79         4.14           18         0.011         0.26         0.63         2.8         0.82         3.79         4.14           9.1         0.09         0.33         0.56         5.0         1.63         3.96         3.69           18         0.11         0.24         0.10         0.33         0.56         5.0         1.63         3.96         3.66           ucts         13.7         0.14         0.24         0.06         3.9         1.017         2.98         2.717         6.62           hund allied         6.8         0.12         0.30         0.56         5.0         1.66         3.96         3.55         2.66         5.75         4.75           hund allied         10.5         0.12         0.30         0.58<	Tobacco products	1.3	0.22	0.15	0.63	0.9	-3.08	0.68	0.27	9.03	9.12	4.69	5.72
3.1         0.13         0.25         0.62         5.2         5.01         8.10         8.60           aducts         0.8         0.08         0.33         0.58         0.2         -3.20         -1.1.32         -1.1.33           1.1         0.11         0.26         0.63         1.2         -2.697         1.80         1.81           1.8         0.11         0.26         0.63         2.8         0.82         3.79         4.14           1.8         0.11         0.26         0.63         3.96         3.96         3.96           9.1         0.13         0.23         0.65         5.0         1.63         3.96         3.95           1.8.7         0.14         0.24         0.05         5.0         1.63         3.96         3.95           ucts         10.5         0.33         0.56         5.0         1.63         3.96         3.69           ucts         10.5         0.13         0.23         0.65         5.1         -0.21         2.71           ucts         10.5         0.36         0.56         4.5         2.34         5.75         4.79           ucts         1.12         0.13         0.29 <td>Rubber products</td> <td>1.9</td> <td>0.11</td> <td>0.31</td> <td>0.58</td> <td>2.5</td> <td>1.26</td> <td>4.12</td> <td>4.60</td> <td>7.37</td> <td>6.62</td> <td>4.11</td> <td>3.46</td>	Rubber products	1.9	0.11	0.31	0.58	2.5	1.26	4.12	4.60	7.37	6.62	4.11	3.46
oducts         0.8         0.08         0.33         0.58         0.2         -3.20         -1.32         -1.32         -1.32         -1.32         -1.32         -1.32         -1.32         -1.32         -1.32         -1.33         -2.27         -1.33         -2.27         -1.33         -2.27         -1.33         -2.27         -1.33         -2.27         -1.33         -2.27         -1.33         -3.69         1.76         -2.27         -3.59         -3.69         1.76         -2.27         -3.69         1.76         -2.27         -3.69         1.76         -2.27         -3.69         1.76         -2.27         -3.69         1.76         -2.27         -3.69         1.76         -2.27         -3.69         1.76         -2.27         -3.69         1.76         -2.27         -3.69         1.76         -2.27         -2.77         -2.77         -2.77         -2.77         -2.77         -2.77         -2.76         -2.77         -2.75         -2.76         -2.77         -2.75         -2.76         -2.77         -2.75         -2.76         -2.77         -2.76         -2.77         -2.76         -2.76         -2.75         -2.76         -2.76         -2.76         -2.76         -2.76         -2.76         -2.76	Plastic products	3.1	0.13	0.25	0.62	5.2	5.01	8.10	8.60	11.47	6.66	3.95	3.68
2.1         0.13         0.25         0.63         1.2         -2.69         1.76         2.27           1.8         0.01         0.26         0.63         2.8         0.82         3.79         4.14           9.1         0.09         0.28         0.65         5.0         1.63         3.96         3.69           1.8         0.09         0.28         0.65         5.0         1.63         3.96         3.69           1.8         0.14         0.24         0.16         0.33         0.56         5.0         1.63         3.96         3.69           1.3         0.14         0.23         0.56         5.0         1.63         3.96         3.69         3.66           1.47         0.08         0.21         0.71         0.25         2.8         1.40         3.32         2.86           10.5         0.12         0.30         0.56         4.9         0.49         3.71         6.65         2.55         4.77           110         0.13         0.29         0.58         4.9         0.49         7.17         6.65         5.57         4.79           111         30.7         0.13         0.28         0.55         <	Leather and allied products	0.8	0.08	0.33	0.58	0.2	-3.20	-1.32	-1.33	4.62	6.14	4.74	4.54
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	Primary textile	2.1	0.13	0.25	0.63	1.2	-2.69	1.76	2.27	6.33	7.27	3.27	2.72
4.0         0.09         0.33         0.58         2.2         -0.97         1.80         1.81           ucts         2.4         0.10         0.33         0.56         5.0         16.3         3.96         3.93           ucts         13.7         0.14         0.23         0.56         5.0         16.3         3.96         3.83           und allied         6.8         0.15         0.38         0.46         3.9         1.46         3.55         2.62           and         14.7         0.08         0.21         0.71         2.22         -0.17         2.98         2.86           ant         6.1         0.13         0.30         0.56         4.5         2.34         2.71           actical mach)         6.1         0.13         0.30         0.55         4.4         2.34         2.71           actical products         11.2         0.13         0.29         0.55         4.4         2.4         4.7           al products         11.1         0.13         0.29         0.55         4.1         4.45           al products         11.1         0.28         0.55         1.7         -0.03         2.62         2.55 <td< td=""><td>Textile products</td><td><math>\frac{1.8}{1.8}</math></td><td>0.11</td><td>0.26</td><td>0.63</td><td>2.8</td><td>0.82</td><td>3.79</td><td>4.14</td><td>7.86</td><td>6.43</td><td>3.57</td><td>3.33</td></td<>	Textile products	$\frac{1.8}{1.8}$	0.11	0.26	0.63	2.8	0.82	3.79	4.14	7.86	6.43	3.57	3.33
9.1         0.09         0.28         0.05         5.9         1.01         4.19         5.96         3.96         3.96         3.96         3.96         3.96         3.96         3.96         3.96         3.96         3.96         3.96         3.96         3.96         3.96         3.96         3.67         3.96         3.67         3.96         3.67         3.96         3.67         3.96         3.67         3.96         3.67         3.96         3.67         3.96         3.67         3.96         3.67         3.68         3.91         1.46         3.55         2.62         3.34         2.71         6.88         3.31           etrical mach)         6.1         0.13         0.20         0.55         1.7         2.03         0.55         1.7         2.98         2.26         4.77         6.88           inic products         11.2         0.13         0.29         0.55         1.7         -0.03         2.62         2.35         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65         6.65	Clothing	4.0 2	0.0 0	0.33	0.58	2.2	-0.97	1.80	1.81	6.70	6.13 7 20	3.53	3.75
ucts         2.4         0.10         0.33         0.56         5.0         1.65         3.96         3.69	Wood	1.6	60.0	0.28	0.63	<u>5.9</u>	10.1	4.19	3.90	8.23	20.7	5.21	5.20
ucts         13.7         0.14         0.24         0.62         2.1         -0.21         5.34         2.11           ucts         1.47         0.08         0.21         0.71         2.22         -0.17         2.98         2.86           abitics         1.17         0.08         0.21         0.71         2.22         5.55         2.66           abitics         0.15         0.13         0.20         0.56         4.5         2.29         5.55         2.66           abitic products         11.12         0.13         0.20         0.56         4.5         2.34         6.77         6.85           abitic products         11.12         0.13         0.29         0.56         4.5         2.34         6.77         6.85           abitic products         11.1         0.11         0.06         0.90         0.85         1.7         0.03         2.62         2.51         6.77         6.85           ad coal products         11.1         0.13         0.28         0.36         3.57         1.17         6.62         2.53         2.56         6.57         6.65         2.51         2.51         2.51         2.51         2.51         2.51         2.51	Furniture and fixture	2.4	0.10	0.33	0.56	5.0	1.63	3.96 3.01	3.69	7.88	6.27	4.55	4.72
and allied         6.8         0.15         0.38         0.46         3.9         1.46         3.55         2.62           allects         14.7         0.08         0.21         0.71         2.2         0.17         2.98         2.86           allects         6.1         0.13         0.30         0.56         4.5         2.29         5.55         4.77           extrical mach)         6.1         0.13         0.29         0.55         4.5         2.34         6.77         6.85           incip products         11.2         0.13         0.29         0.56         4.7         6.82         3.31           and         aoal products         11.10         0.04         0.06         0.90         0.88         -0.25         2.78         2.51         6.45           all products         11.10         0.04         0.06         0.90         0.88         -0.25         2.78         2.51           all products         11.10         0.19         0.63         0.65         4.17         4.45           all products         12.1         0.23         0.65         1.12         3.29         3.50           all products         12.1         0.13	Paper and allied products	13.7	0.14	0.24	0.62	2.1	-0.21	3.34	2.71	5.48	7.24	4.82	5.02
Inters         14.1         0.08         0.21         0.11         2.2         -0.11         2.98         2.80           ducts         10.5         0.12         0.30         0.56         4.5         2.29         5.25         4.79           ment         30.7         0.08         0.20         0.75         6.4         2.34         6.71         6.83           ment         30.7         0.08         0.29         0.55         1.7         -0.03         2.62         2.55         4.79           ment         30.7         0.08         0.29         0.55         1.7         -0.03         2.62         2.55         2.55         3.31           products         11.0         0.04         0.06         0.90         0.57         5.7         1.12         2.62         2.55           al products         11.0         0.04         0.06         0.90         0.57         5.7         1.12         3.60           al products         12.8         0.19         0.63         3.8         1.21         2.89         3.60           3.65         0.55         0.45         0.30         0.57         5.7         1.12         3.80         3.60	rinting, publishing and allied	6.8 1	0.15	0.38	0.46	6.0 0.0	1.46 0.17	3.55	2.62	6.68 7 10	6.17	4.71	96.6 17
Incress         10.5         0.12         0.30         0.58         2.8         1.40         5.32         5.31           ment         30.7         0.08         0.20         0.56         4.5         2.29         5.55         4.79           mic products         11.2         0.13         0.29         0.56         4.5         2.34         6.77         6.68           ic products         11.2         0.17         0.28         0.55         1.7         -0.03         2.65         2.55         4.79           al coal products         11.0         0.04         0.06         0.90         0.8         0.25         2.34         6.67         6.65           al coal products         11.0         0.04         0.06         0.90         0.8         2.57         1.17         4.45           al products         12.8         0.16         0.33         0.59         3.8         1.21         2.89         2.56           3.0         0.8         0.73         0.10         0.18         1.2         3.81         1.41         4.45           3.0         0.8         0.73         0.10         0.18         1.2         3.89         2.56         6.65         6.18 <td>rimary metal</td> <td>14.7</td> <td>0.08</td> <td>0.21</td> <td>0./1</td> <td>7.7</td> <td>-0.17</td> <td>2.98</td> <td>2.80</td> <td>6.13 7 20</td> <td>7.08 1.08</td> <td>4.68</td> <td>10.4</td>	rimary metal	14.7	0.08	0.21	0./1	7.7	-0.17	2.98	2.80	6.13 7 20	7.08 1.08	4.68	10.4
ectrical mach) $b_{0.1}$ $0.13$ $0.20$ $0.20$ $0.72$ $6.4$ $2.29$ $5.25$ $4.70$ ment $30.7$ $0.08$ $0.20$ $0.72$ $6.4$ $2.34$ $6.77$ $6.85$ inc products $11.2$ $0.13$ $0.28$ $0.55$ $1.7$ $-0.03$ $2.62$ $2.35$ inc products $11.0$ $0.04$ $0.06$ $0.90$ $0.8$ $-0.25$ $2.78$ $2.51$ $6.62$ al products $11.0$ $0.04$ $0.06$ $0.90$ $0.8$ $0.65$ $4.17$ $6.65$ al products $11.0$ $0.012$ $0.30$ $0.57$ $5.7$ $1.12$ $3.22$ $2.78$ $2.52$ $250.5$ $0.071$ $0.33$ $0.53$ $0.53$ $0.65$ $1.11$ $2.32$ $2.78$ $2.52$ $2.58$ $2.567$ $6.18$ $8.025$ $0.36$ $0.31$ $0.13$ $0.23$ $2.11$ $2.32$ $2.66$ <td>abricated metal products</td> <td>10.5</td> <td>0.12</td> <td>0.30</td> <td>0.58</td> <td>7.8</td> <td>1.40</td> <td>3.32</td> <td>3.31</td> <td>6.78</td> <td>6.12</td> <td>4.56</td> <td>4.49</td>	abricated metal products	10.5	0.12	0.30	0.58	7.8	1.40	3.32	3.31	6.78	6.12	4.56	4.49
ment $30.1$ $0.08$ $0.20$ $0.1/2$ $0.4$ $2.34$ $0.1/1$ $0.85$ l products $11.2$ $0.13$ $0.29$ $0.58$ $1.7$ $-0.03$ $2.62$ $2.35$ a coal products $11.0$ $0.04$ $0.06$ $0.90$ $0.8$ $-0.25$ $2.78$ $2.51$ $2.53$ al products $11.0$ $0.04$ $0.06$ $0.90$ $0.8$ $-0.25$ $2.78$ $2.51$ $2.53$ al products $11.0$ $0.012$ $0.33$ $0.57$ $5.7$ $1.12$ $3.60$ $2.52$ $20.5$ $0.077$ $0.33$ $0.59$ $3.61$ $2.567$ $6.18$ $2.52$ $21.8$ $0.73$ $0.10$ $0.18$ $1.2$ $3.97$ $5.67$ $6.18$ $8.07$ $0.13$ $0.11$ $0.23$ $0.445$ $0.28$ $2.29$ $2.50$ $2.56$ $6.18$ $8.07$ $0.13$ $0.12$ $0.20$ <t< td=""><td>vlachinery (except electrical mach)</td><td>0.1</td><td>0.13</td><td>0.30</td><td>00.0</td><td>4. V</td><td>67.7</td><td>07.0</td><td>4./9</td><td>0.90</td><td>6.40 7 80</td><td>c/.4</td><td>4.8.4 2.00</td></t<>	vlachinery (except electrical mach)	0.1	0.13	0.30	00.0	4. V	67.7	07.0	4./9	0.90	6.40 7 80	c/.4	4.8.4 2.00
Inc products         11.2         0.13         0.29         0.28         0.49         0.11         0.02           products         4.2         0.17         0.28         0.55         1.7         -0.03         2.62         2.35           ad coal products         11.0         0.04         0.06         0.90         0.8         -0.25         2.78         2.51           ad products         12.8         0.19         0.63         3.5         0.65         4.17         4.45           50.5         0.07         0.33         0.59         3.8         1.21         2.89         2.52           23.3         0.16         0.39         0.45         1.5         0.86         4.15         3.80           1.8         0.73         0.10         0.18         1.2         3.97         5.67         6.18           1.8         0.73         0.10         0.18         0.23         4.1         2.50         6.92           1.12         0.36         0.41         0.23         4.1         2.36         5.67         6.18           1.8         0.73         0.10         0.23         4.1         2.56         6.69         2.62         2.24 <tr< td=""><td>I ransportation equipment</td><td>30.7 11.0</td><td>0.08</td><td>0.20</td><td>0.72</td><td>0.4 4.0</td><td>2.34</td><td>0.//</td><td>C8.0</td><td>00.1</td><td>0.80</td><td>4.95</td><td>4.50 2.50</td></tr<>	I ransportation equipment	30.7 11.0	0.08	0.20	0.72	0.4 4.0	2.34	0.//	C8.0	00.1	0.80	4.95	4.50 2.50
Iproducts         4.2         0.11         0.28         0.55         1.7         -0.05         2.62         2.53           ad coal products         11.0         0.04         0.06         0.90         0.8         -0.25         2.78         2.51           al products         12.8         0.18         0.19         0.63         3.5         0.65         4.17         4.45           50.5         0.07         0.33         0.59         3.8         1.21         2.89         2.52           23.3         0.16         0.39         0.45         1.5         0.86         4.15         3.81           1.8         0.73         0.10         0.18         1.2         3.97         5.67         6.18           1.8         0.73         0.10         0.18         1.2         3.97         5.67         6.18           1.12.6         0.36         0.41         0.23         4.1         2.36         6.65         6.92           121.8         0.50         0.21         0.20         2.36         6.65         6.92           12.8         0.52         0.30         0.32         4.1         2.36         6.65         6.92           24.3	Electrical and electronic products	11.2	0.15 0	0.29	0.28	י <del>ג</del> זי	0.49	.1.	20.0	50.C	10.0	7.00	7.24 7.24
al products 11.0 0.04 0.00 0.90 0.63 -0.25 2.78 2.51 2.41 4.5 2.1 2.8 0.18 0.19 0.63 3.5 0.65 4.17 4.45 2.5 2.5.5 0.07 0.33 0.59 3.8 1.21 2.89 2.52 3.60 5.0.7 0.33 0.59 3.8 1.21 2.89 2.52 3.81 1.8 0.73 0.10 0.18 1.2 3.97 5.67 6.18 1.2 0.8 0.25 0.45 0.30 3.0 0.43 2.29 2.26 1.8 0.25 0.41 0.23 4.1 2.36 6.65 6.92 1.2.6 0.36 0.41 0.23 4.1 2.36 6.65 6.92 1.2 8 0.59 0.21 0.20 2.9 2.50 6.61 5.16 2.4.3 0.18 0.50 0.32 4.0 2.29 2.50 6.61 5.16 2.4.3 0.13 0.56 0.31 4.3 1.93 4.30 3.97 5.67 6.18 2.4.3 0.18 0.50 0.32 4.0 2.29 2.50 6.61 5.16 0.24 0.26 0.31 4.3 1.93 4.30 3.97 5.66 0.21 0.20 2.9 2.50 6.61 5.16 0.24 0.50 0.32 4.0 2.82 5.90 5.42 0.24 0.31 4.3 1.93 4.30 3.97 0.51 0.51 0.51 0.50 0.31 4.3 1.93 4.30 3.97 3.97 3.97 3.97 3.97 3.97 3.97 3.97	Non-metallic mineral products	4.7	0.17	0.28	cc.0	1./ 0.0	-0.03	79.7	CS.2	0.02	70.0 70.0	4.80	4 v 7 f
The products 12.8 0.18 0.19 0.03 5.7 0.05 4.17 4.43 4.43 4.0 0.12 0.30 0.57 5.7 1.12 3.29 3.60 5.62 50.5 0.07 0.33 0.59 3.8 1.21 2.89 2.52 3.81 1.8 0.73 0.10 0.18 1.2 3.97 5.67 6.18 1.2 3.97 5.67 6.18 0.8 0.25 0.45 0.30 3.0 0.43 2.29 2.26 1.12 8 0.59 0.21 0.23 4.1 2.36 6.65 6.92 12.8 0.59 0.21 0.20 2.9 2.50 6.61 5.16 24.3 0.18 0.50 0.31 4.1 2.36 6.65 6.92 2.26 2.4 0.24 0.22 2.9 2.50 6.61 5.16 24.3 0.13 0.56 0.31 4.3 1.93 4.30 3.97 3.61 5.16 24.3 0.13 0.56 0.31 4.3 1.93 4.30 3.97 3.97 1.00 1.00 1.00 0.00 0.53 0.00 0.53 0.00 0.50 5.42 2.00 5.42 2.00 5.42 2.00 5.42 2.00 5.42 2.00 5.50 0.51 0.51 0.20 2.9 2.50 6.61 5.16 0.51 0.50 0.50 0.50 0.50 0.50 0.50 0.50	Kennea petroleum and coal products	0.11	0.04	0.00	06.0	0.0 2	C7-0-	8/.7	10.7	06.1 11	1.50		11.0
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	chemical and chemical products	12.8	0.18	0.19	0.63	Ω U I	C0.U	4.17	4.45 2,00	1.14	0.71	4.80	4.30
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Other manufacturing	4.0	0.12	0.30	0.57	5.7	1.12	3.29	3.60	5.88	6.17	4./8	4. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Construction	50.5	0.07	0.33	0.59	3.8	1.21	2.89	2.52	5.28	6.22	4.79	4.97
1.8 $0.73$ $0.10$ $0.18$ $1.2$ $3.97$ $5.67$ $6.18$ $0.8$ $0.25$ $0.45$ $0.30$ $3.0$ $0.43$ $2.29$ $2.26$ $12.6$ $0.36$ $0.41$ $0.23$ $4.1$ $2.36$ $6.65$ $6.92$ $12.6$ $0.36$ $0.21$ $0.23$ $4.1$ $2.36$ $6.65$ $6.92$ $12.8$ $0.59$ $0.21$ $0.20$ $2.9$ $2.50$ $6.61$ $5.16$ $24.3$ $0.18$ $0.50$ $0.32$ $4.0$ $2.82$ $5.90$ $5.42$ $28.7$ $0.13$ $0.56$ $0.31$ $4.3$ $3.97$ $3.97$ $380.7$ $0.17$ $0.30$ $0.53$ $3.31$ $1.21$ $4.45$ $3.96$ cost; $S_K = capital compensation share; S_L = labour compensation share; S_M = share of intermediate inputs; \vec{K} = \vec{V} = capital compensation share; \vec{N} = capital compensation share; \vec{N} = capital compensation share; \vec{N} = ca$	Iransportation	23.3	0.16	0.39	0.45	1.5	0.86	4.15	3.81	5.42	6.47	5.02	4.13
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Pipeline transport	1.8	0.73	0.10	0.18	1.2	3.97	5.67	6.18	7.53	7.28	4.91	3.06
tion 12.6 0.36 0.41 0.23 4.1 2.36 6.65 6.92 ade 24.3 0.59 0.21 0.20 2.9 2.50 6.61 5.16 24.3 0.18 0.50 0.31 4.0 2.82 5.90 5.42 28.7 0.13 0.56 0.31 4.3 1.93 4.30 3.97 ector 380.7 0.17 0.30 0.53 3.31 1.21 4.45 3.96 average total cost; $S_k = \text{capital compensation share; } S_L = \text{labour compensation share; } S_L = \text{labour compensation share; } S_M = \text{share of intermediate inputs; } \dot{K} =$	Storage and warehousing	0.8	0.25	0.45	0.30	3.0	0.43	2.29	2.26	2.86	6.78	4.63	4.42
ade       12.8       0.59       0.21       0.20       2.9       2.50       6.61       5.16         ade       24.3       0.18       0.50       0.32       4.0       2.82       5.90       5.42         28.7       0.13       0.56       0.31       4.3       1.93       4.30       3.97         ector       380.7       0.17       0.30       0.53       3.31       1.21       4.45       3.96         average total cost; $S_k = \text{capital compensation share; } S_L = \text{labour compensation share; } S_M = \text{share of intermediate inputs; } \dot{K} =$ 3.96	Communication	12.6	0.36	0.41	0.23	4.1	2.36	6.65	6.92	6.16	6.57	4.63	2.93
24.3 0.18 0.50 0.32 4.0 2.82 5.90 5.42 28.7 0.13 0.56 0.31 4.3 1.93 4.30 3.97 <b>380.7 0.17 0.30 0.53 3.31 1.21 4.45 3.96</b> e total cost; $S_k$ = capital compensation share; $S_L$ = labour compensation share; $S_M$ = share of intermediate inputs; $\dot{K}$ = $content rate of hence editors from control and the state of intermediate inputs; \dot{K} = content rate of hence editors from control and the state of intermediate inputs; \dot{K} = content rate of hence editors from control and the state of intermediate inputs; \dot{K} = content rate of hence editors from control and the state of intermediate inputs; \dot{K} = content rate of hence editors from control and the state of intermediate inputs; \dot{K} = content rate of hence editors inverses in \dot{M} = content and content and \dot{V} = content content vectors of intermediate inputs; \dot{V} = content vectors $	Other utility	12.8	0.59	0.21	0.20	2.9	2.50	6.61	5.16	7.58	6.78	4.71	4.61
28.7 0.13 0.56 0.31 4.3 1.93 4.30 3.97 <b>380.7 0.17 0.30 0.53 3.31 1.21 4.45</b> 3.96 we total cost; $S_K =$ capital compensation share; $S_L =$ labour compensation share; $S_M =$ share of intermediate inputs; $\dot{K} =$	Wholesale trade	24.3	0.18	0.50	0.32	4.0	2.82	5.90	5.42	6.54	6.22	4.33	4.08
380.7   0.17   0.30   0.53   3.31   1.21   4.45   3.96 e total cost; $S_K$ = capital compensation share; $S_L$ = labour compensation share; $S_M$ = share of intermediate inputs; $\dot{K}$ = $\dot$	Retail trade	28.7	0.13	0.56	0.31	4.3	1.93	4.30	3.97	4.53	5.87	4.55	4.03
= average total cost; $S_K$ = capital compensation share; $S_L$ = labour compensation share; $S_M$ = share of intermediate inputs; $\dot{K}$ = - average annual arouth rate of have adjusted for labour composition; $\dot{M}$ = average annual arouth rate of intermediate inputs; $\dot{V}$	Business Sector	380.7	0.17	0.30	0.53	3.31	1.21	4.45	3.96	6.33	6.60	4.68	4.48
– avanaa amuud aravuth mta af haura dinatad far laham aammacitian. M – avanaa amuud aravuth mta af intammadiata imuuta. V	$C = average total cost; S_k = capital c$	compensation	1 share; $S_{I=1}$		pensation sh		share of int	ermediate in	Ш	ital services av	verage annu	ial growth	rate;
= average annual growth rate of nours adjusted for labour composition: <i>N</i>	$\dot{L}$ = average annual growth rate of ho	ours adiusted	for labour c	omposition:		ee annual er	rowth rate o	f intermedia	te inputs: $\dot{Y} = g$	eross output av	verage annu	al prowth	rate:
f = average annual growth of the price of the input ( $f = K$ , L and M). W		۲		, . •		2				-	2	0	

The substantial diversity in the growth of output and the structure of costs among the industries over the period 1961-2000 provides a rich body of data to test econometrically the impact of different variables on growth of output and productivity. The diversity pattern noted here implies that the response of various industries to changes in variables such as public capital, real GDP and population growth are likely to be very diverse. Therefore, we would expect the estimated elasticities, marginal benefits of public capital and multifactor productivity growth rates calculated for different industries, using the parameter estimates of our econometric model, will vary considerably across industries.

### III. Econometric Implementation

#### 1. Model Estimation

We estimated the demand function (3) for each industry. Initial estimation revealed that in a few industries the price elasticities had incorrect signs. A different formulation of the demand functions was attempted by estimating the model with the industry panel data; we also formulated alternative specifications of (3), introducing other variables in the demand function such as the interest rate, the unemployment rate, and the price of exports. The results of these alternative specifications did not differ much from those reported in Table 2. The results indicate that the price elasticities of demand and the per-capita income elasticities of demand vary across industries. The price elasticity of output demand is negative, less than one, and statistically significant in most industries. The per-capita income elasticity is positive and significant in the majority of industries as well.<sup>6</sup>

The parameters of the underlying cost function are estimated using the cost function (1) and the share equations (2). These equations depend on private input prices, the level of industry output Y, the time trend t, and the level of capital stock G. Hulten (1990) argues that the intensity of public capital usage fluctuates over time. For example, there are variations in the utilization of highways, evidenced for example, by the ratio of vehicle miles travelled to the capital stock of roads. Also, public capital is a collective input which firms must share with others and therefore is subject to congestion (see Deno 1988). Firms might have some control over the use of the public stock (see Shah 1992 and Fernald 1999). For instance, a firm may have no influence on the level of highways provided by the government, but it can vary its use of existing highways by choosing different routes. Adjustment for utilization of highway capital could indeed affect both the magnitudes and the time pattern of industry marginal benefits. However, it is difficult at present to obtain a reliable and appropriate measure of highway usage by each industry over the period 1961-1997. Therefore, at this stage of our analysis, we have not made any adjustment for utilization of the public capital stocks G.

<sup>&</sup>lt;sup>6</sup> The confidence intervals for the aggregate price elasticity of output demand and per-capita income elasticity at 5% level are, respectively, -.2512; -.3852 and .4412; .5718.

	α	β
Agricultural and related service	-0.1844 (2.2561)	0.2941 (2.0331)
Fishing and trapping	-0.1423 (1.7514)	0.1458 (1.0614)
Logging and forestry	-0.1185 (1.6831)	0.3396 (1.5143)
Mining	-0.2596 (1.5614)	0.3694 (2.3361)
Crude petroleum and natural gas	-0.2214 (2.2532)	0.3381 (2.1497)
Quarry and sand pit	-0.1729 (1.6533)	0.1185 (1.0532)
Services incidental to mineral extraction	-0.1589 (1.9744)	0.2569 (1.1436)
Food	-0.3658 (3.1675)	0.6814 (2.3314)
Beverage	-0.3074 (2.5591)	0.3961 (2.6971)
Tobacco products industry	-0.2582 (1.9596)	0.0981 (1.9914)
Rubber products	-0.2932 (2.0264)	0.4121 (1.7234)
Plastic products	-0.2012 (1.9208)	0.2591 (1.8896)
Leather and allied products	-0.2545 (1.6617)	0.5698 (2.3612)
Primary textile	-0.2121 (1.7150)	0.4485 (3.6471)
Textile products	-0.1893 (2.0669)	0.3561 (1.7569)
Clothing	-0.2534 (2.5591)	0.3279 (2.6598)
Wood	-0.4125 (2.4179)	0.8474 (1.6674)
Furniture and fixture	-0.2914 (1.6641)	0.4659 (1.6541)
Paper and allied products	-0.3261 (1.9514)	0.5141 (1.5524)
Printing, publishing and allied	-0.1844 (2.2561)	0.2941 (2.0331)
Primary metal	-0.1423 (1.7514)	0.1458 (1.0614)
Fabricated metal products	-0.1185 (1.6831)	0.3396 (1.5143)
Machinery ind. (except electrical mach)	-0.2596 (1.5614)	0.3694 (2.3361)
Transportation equipment	-0.2214 (2.2532)	0.3381 (2.1497)
Electrical and electronic products	-0.1729 (1.6533)	0.1185 (1.0532)
Non-metallic mineral products	-0.1589 (1.9744)	0.2569 (1.1436)
Refined petroleum and coal products	-0.3658 (3.1675)	0.6814 (2.3314)
Chemical and chemical products	-0.3074 (2.5591)	0.3961 (2.6971)
Other manufacturing	-0.2582 (1.9596)	0.0981 (1.9914)
Construction	-0.2932 (2.0264)	0.4121 (1.7234)
Transportation	-0.2012 (1.9208)	0.2591 (1.8896)
Pipeline transport	-0.2545 (1.6617)	0.5698 (2.3612)
Storage and warehousing	-0.2121 (1.7150)	0.4485 (3.6471)
Communication	-0.1893 (2.0669)	0.3561 (1.7569)
Other utility	-0.2534 (2.5591)	0.3279 (2.6598)
Wholesale trade	-0.4125 (2.4179)	0.8474 (1.6674)
Retail trade	-0.2914 (1.6641)	0.4659 (1.6541)
Business Sector	-0.3261 (1.9514)	0.5141 (1.7524)

 Table 2 - Estimation of Demand Function Mean Values: 1961-2000

Note: The following demand function  $\dot{Y} = \lambda + \alpha (\dot{P}_Y - \dot{P}_D) + \beta \dot{Z} + (1 - \beta) \dot{N}$  is estimated using the fixed effect model to control for inter-industry differences. The specification assumes that the changes in quantity demanded in an industry are related to its own price movement in comparison to the GDP deflator and changes in the level of aggregate income and population of the economy. The estimates have been corrected for the autocorrelation of residuals; *t*-statistics are between parentheses.

The sample consists of pooled time-series cross-section data for 37 two-digit industries of the 1980 standard industrial classification during the period 1961-2000. In order to capture industry specific effects, we assume the parameters  $\alpha_K$ ,  $\alpha_L$ ,  $\alpha_M$  and  $\alpha_G$  are industry specific. Thus, we assume  $\alpha_{iij} = \alpha_{ii} + h_{ij}D_j$ , where the parameters are normalized with respect to the k - th industry

 $(h_{ik} = 0)$ ,  $D_{ij}$  is an industry dummy variable taking values either 1 or 0, and *j* is an industry identification index. We estimate the model using an iterative seemingly unrelated regression approach (ISUR). Initial estimation revealed serial correlation in the residuals. Therefore, the equations were re-estimated with a correction for first order serial autocorrelation.

In Table 3, we present parameter estimates for the translog cost function (1). The estimated factor demand system satisfies all the required regularity conditions: the estimated cost function is shown to be non-decreasing in output, linearly homogeneous in input prices, and concave in factor prices. The results show that the cost model is well specified and that the parameter estimates are statistically significant. Most of the coefficients of the industry dummy variables, not shown in Table 3, are also statistically significant, which suggests significant differences in the cost structure across industries. The squares of the correlation coefficients between the actual and predicted values are high, and the standard errors of each equation of the model are small. The estimated value of the autocorrelation parameter,  $\rho$ , is about 0.90.

		Parameter Estimate		t-statistic
$lpha^*$		0.1245		1.7514
$lpha_K^*$		0.1541		2.1641
$lpha_L^*$		0.4438		3.1457
$\alpha_Y^*$		1.1447		2.5579
$\alpha_G^*$		-0.0554		2.0147
$\alpha_t^*$		-0.0538		2.7251
$\alpha_{KK}$		0.0524		3.1472
$lpha_{LL}$		0.0941		2.8516
$lpha_{\scriptscriptstyle YY}$		-0.0091		1.5842
$lpha_{GG}$		-0.0037		1.8513
$lpha_{tt}$		-0.0008		1.7513
$lpha_{KL}$		-0.0258		3.0173
$lpha_{\scriptscriptstyle KY}$		-0.0447		2.9142
$lpha_{\scriptscriptstyle KG}$		0.0574		2.3671
$lpha_{Kt}$		0.0018		3.1592
$lpha_{LY}$		-0.0216		1.7632
$lpha_{LG}$		-0.0055		1.6671
$lpha_{Lt}$		-0.0011		2.5513
$lpha_{GY}$		-0.0744		2.0164
$lpha_{Gt}$		0.0118		3.4176
Equation	Standard Error	$R^2$	ρ	DW
Labour-Material	0.0254	0.92	0.92	1.59
Capital-Intermediate Inputs	0.0514	0.88	0.94	1.87
Total Cost Function	0.0145	0.93	0.89	1.96

Note: \* Dummy variables were included.

To estimate the effect of an increase in public capital stock on industry cost and demand for inputs, we need estimates of the cost elasticity with respect to public capital, G, and the cost elasticity of output for each industry. Before getting those estimates, we need to address the question of the reliability of the estimates by performing various econometric tests on the model, focusing specifically on public capital.

#### 2. Reliability of the Estimates

This section assesses the reliability of our econometric model. We first provide some statistical tests and we then examine the robustness of our empirical results.

#### 2.1 Hypothesis Tests

We used the estimation results from equation (1) to econometrically test a number of hypotheses concerning the technology structure. Log-likelihood ratios are used for the tests and the results are presented in Table 4. The likelihood ratio tests suggest a decisive rejection of the joint hypothesis that the coefficients of the industry dummies are zero, suggesting that strong interindustry differences are present in the cost structure of the industries under consideration. Also, the hypothesis that the coefficient of public capital is zero in the cost function is also rejected (see Table 4, row 2). We also tested for constant returns to scale, as well as for hypotheses of no technical change. These hypotheses were rejected as indicated by the  $\chi^2$  test statistics shown in the table (rows 3 and 4).

	Table 4 - Hypothesis Tests							
Hypothesis	Parameter Restriction	Log of Likelihood	d.f.	$\chi^2$	Test Result			
No industry dummies	$\underline{h}_{o} = \underline{h}_{K} = \underline{h}_{L} = \underline{h}_{Y} = \underline{h}_{G} = \underline{h}_{I} = 0$	7952.5	216	439.5	Reject			
No public capital effects	$\alpha_G = \alpha_{GG} = \alpha_{KG} = \alpha_{LG} = \alpha_{GY} = \alpha_{Gt} = 0$	1910.3	216	555.7	Reject			
Constant returns to scale technology	$\alpha_Y = 1$ and	459.4	5	92.9	Reject			
	$\alpha_{YY} = \alpha_{KY} = \alpha_{LY} = \alpha_{GY} = 0$							
No technical change	$\alpha_t = \alpha_{tt} = \alpha_{Kt} = \alpha_{Lt} = \alpha_{Gt} = 0$	339.6	5	77.5	Reject			

Note: <u>h</u> is a vector of dummy parameters. The critical values of  $\chi^2$  at 99% level of confidence for 5 and 216 degrees of freedom (d.f.) are, respectively 15.1 and 323.4

#### 2.2 Sensitivity Analysis

The literature that estimates the effect of public capital on the production structure based on time series has been vigorously challenged on both conceptual and methodological grounds. Two types of criticism of these types of models can be identified.

First, time series data on output and public capital have common trends and therefore, the significant positive relationship between productivity and public capital reported in the literature may be spurious (false) due to the presence of a common trend. One way to deal with this problem is to use some form of differencing of these variables. When Hulten and Schwab (1991) and Tatom (1991) first-differenced their macroeconomic time series, the marginal product of public capital was much smaller and almost always statistically insignificant.

Second, at the aggregate level, it is not clear whether a decrease in public capital expenditure is due to a decrease in the level of aggregate output or vice versa. In other words, public capital could either be an endogenous variable or an independent variable that explains the growth of output. Therefore, the issue of simultaneity between output and public capital must be dealt with econometrically.

#### 2.2.1 Spurious Correlation

The presence of common trend among variables in the time series models of public capital is a serious econometric issue. This criticism is equally applicable to production and cost function studies, whether they include public capital or not. It is true that private sector variables such as output, labour, intermediate inputs and private capital stock are highly correlated over time and may share a common trend. There is nothing particular about public capital in this respect.

One method for removing a common trend is to estimate the model in a first-difference form. Estimation of this form eliminates a potential influence of trend which may be an over correction and not appropriate when we are seeking to trace the effect of public capital on the productivity performance. Nonetheless, equations (1) and (3) are estimated in 'first-difference' form by setting the serial correlation parameter  $\rho$  to unity. The parameter estimates (not reported here but available on request) indicate that the models fit the data very well. The parameter estimates of the first-differenced models have the correct sign and magnitudes in comparison to those associated to the models estimated in level form. This should not come as a surprise as the values of the serial correlation coefficients  $\rho$ , shown in Table 3, are close to unity.

#### 2.2.2 Causality

The problem of simultaneity between output and public capital is more severe in production function studies than in cost function studies as both output and public capital appear as explanatory variables in the cost function. Nonetheless, we performed a number of causality tests and the results suggest that public capital can be considered as an exogenous variable in our industry cost function. Moreover, we reestimated the model using a three-stage least squares technique with lagged values of all exogenous variables as instruments. The results are similar to those reported in Table 3.<sup>7</sup>

<sup>&</sup>lt;sup>7</sup> This does not come as a surprise as the model underlying the estimates of Table 3 makes use of lagged exogenous variables as instruments to correct the autocorrelation of residuals. It appears that these instruments correct for both the autocorrelation of residuals problem and the endogeneity problem.

### IV. Empirical Results

#### 1. Contribution of Public Capital at the Industry Level

#### 1.1 Cost Reduction of Public Capital

The average elasticities of cost with respect to public capital  $\eta_{\tilde{c}G}$  for our 37 industries are shown in column 1 of Table 5. They indicate that an increase in public capital reduces costs (given the level of output) in all industries. The results reported in Table 5 show that in all industries, the elasticities have the expected negative sign. The magnitudes of these elasticities vary across industries, and range from -0.00125 for fishing and trapping and -0.12 for retail trade. However, these estimates lie within reasonably tight confidence intervals.<sup>8</sup> The elasticities are relatively large in trade, transportation, construction, communication and other utility. These industries are probably the most intensive users of public capital. In most of the manufacturing industries, the elasticities range from -0.0020 (other manufacturing) to -0.0571 (transportation equipment). The industries with fairly small elasticities are, among others, textile products, furniture and fixtures, and leather and allied products.

Elasticities  $\eta$  and  $\eta^*$  shown in Table 5 have a return to scale interpretation. The inverse of  $\eta$ , or  $\frac{1}{\eta}$ , represents internal returns to scale, or the effect on output of an equal proportional increase in all inputs except public capital. That is, if  $\frac{1}{\eta} > 1$  an equal proportional increase in labour, capital, and intermediate inputs, holding public capital fixed, yields a proportional increase in output.<sup>9</sup> For example, in retail trade, the degree of returns to scale to private inputs is approximately 1.06. However, an equal proportional increase in all inputs, including public capital, yields a proportional increase in output or total returns to scale of 1.22. The results show that both  $\eta$  and  $\eta^*$  are less than one for all industries, suggesting that increasing internal and particularly total returns to scale prevail in all industries. These scale elasticities are robust; that is, the magnitudes do not change with different estimation methods. The magnitudes of these scale estimates vary across industries. The degree of internal returns to scale in each industry is smaller than the degree of total returns to scale. This is expected because total returns to scale account for the positive contribution of public capital.

The output elasticities in different industries with respect to an increase in public capital,  $\eta_{YG}$ , are presented in column 4 of Table 5. The magnitudes of the elasticities vary considerably across industries. The patterns of these elasticities are similar, as expected, to those of the cost elasticities of public capital shown in column 1 of the Table 5. The output expansion effect of an increase in public capital, ranges from approximately 0.129 for retail to 0.00123 for fishing and trapping. The industries with the largest output elasticity with respect to public capital are the

<sup>&</sup>lt;sup>8</sup> The cost elasticity estimate of public capital at the aggregate level lies within the following confidence intervals: -0.05981 and -0.071412.

<sup>&</sup>lt;sup>9</sup> The confidence intervals for the elasticities  $\frac{1}{\eta}$  and  $\frac{1}{\eta^*}$  at the aggregate level at 5% level are, respectively, 1.0385; 1.07145 and 1.1547; 1.1978.

transportation industries, the trade industries, the utility industries, the construction industries, and some manufacturing industries.

Table 5 - Translog Cost Function Elasticities							
	$\eta_{ ilde{C}G}$	$\frac{1}{\eta}$	$rac{1}{\eta^*}$	$\eta_{YG}$			
Agricultural and related service	-0.04681	1.0711	1.2241	0.05153			
Fishing and trapping	-0.00125	0.9812	1.0243	0.00123			
Logging and forestry	-0.01420	1.0123	1.0914	0.01438			
Mining	-0.02514	1.0531	1.1543	0.02648			
Crude petroleum and natural gas	-0.03746	1.0914	1.1932	0.04089			
Quarry and sand pit	-0.01015	0.9122	1.0124	0.00926			
Services incidental to mineral extraction	-0.01201	0.9457	1.0285	0.01135			
Food	-0.03718	1.0257	1.1413	0.03813			
Beverage	-0.03521	1.0441	1.1591	0.03676			
Tobacco products industry	-0.01901	0.9841	1.0432	0.01871			
Rubber products	-0.03032	1.0374	1.0674	0.03145			
Plastic products	-0.01725	1.0471	1.0934	0.01806			
Leather and allied products	-0.01100	1.0223	1.0341	0.01125			
Primary textile	-0.02015	1.0218	1.1014	0.02058			
Textile products	-0.01601	1.0541	1.1463	0.01688			
Clothing	-0.02102	1.0614	1.0874	0.02231			
Wood	-0.03110	1.0335	1.0532	0.03214			
Furniture and fixture	-0.01300	1.0228	1.0642	0.01330			
Paper and allied products	-0.03403	1.0674	1.1247	0.03633			
Printing, publishing and allied	-0.03027	1.0647	1.1396	0.03223			
Primary metal	-0.05208	1.0468	1.1574	0.05452			
Fabricated metal products	-0.04904	1.0749	1.1712	0.05271			
Machinery ind. (except electrical mach)	-0.05302	1.1247	1.2341	0.05963			
Transportation equipment	-0.05711	1.0973	1.1774	0.06266			
Electrical and electronic products	-0.00302	1.1462	1.2412	0.00346			
Non-metallic mineral products	-0.02231	1.0334	1.0974	0.02306			
Refined petroleum and coal products	-0.04182	1.0974	1.1531	0.04588			
Chemical and chemical products	-0.03511	1.0579	1.1973	0.03714			
Other manufacturing	-0.00202	1.0124	1.0741	0.00205			
Construction	-0.06951	1.0335	1.2231	0.07184			
Transportation	-0.09314	1.0456	1.2785	0.09739			
Pipeline transport	-0.05231	1.0121	1.1894	0.02338			
Storage and warehousing	-0.01501	1.0224	1.0863	0.01534			
Communication	-0.06861	1.0974	1.1241	0.07529			
Other utility	-0.06142	1.0121	1.0874	0.06216			
Wholesale trade	-0.11846	1.0547	1.1913	0.12494			
Retail trade	-0.12130	1.0631	1.2213	0.12896			
Business Sector	-0.06203	1.0575	1.1760	0.06575			

Table 5 - Translog Cost Function Elasticities

Note:  $\eta_{\tilde{C}G}$  = the private cost elasticity with respect to public capital;  $\frac{1}{\eta}$  = the internal return to scale, or the effect on output of an equal proportional increase in all inputs except public capital. That is, an equal proportional increase in labour, capital, and intermediate inputs, holding public capital fixed, yields a proportional increase in output;  $\frac{1}{\eta^*}$  = overall return to scale, or the effect on output of an equal proportional increase in all inputs including public capital;  $\eta_{YG}$  = Marginal productivity of public capital.

#### 1.2 Effects of Public Capital Stock on Demands for Labour, Capital and Intermediate Inputs

Public capital has both direct and indirect effects on the productivity of the private sector. The direct effect is a consequence of the positive marginal product of public capital, i.e., an increase in public capital services decreases private sector production costs. The indirect effect arises because private and public capital are complements in production, i.e., the partial derivative of the marginal product of private capital with respect to public capital is positive. If private capital and public capital are complements, an increase in public capital raises the marginal productivity of private capital, and, given the rental price of capital, private capital formation increases, further raising private sector output. The same will occur with labour and demand for intermediate inputs, depending on whether they are substitutes or complements with public capital in the production process.

If all private inputs are substitutes with public capital, then an increase in public capital is always cost saving. The inverse, of course, is not true. The review of available literature on cost functions supports the hypothesis that cost savings are associated with an increase of public capital. Hence, if one of the private inputs is a complement to public capital then cost savings can arise only if the substitution effects of the other private inputs outweigh its own complementary effect. It is clear, a priori, that no sign can be assigned to the effect of public capital on the inputs of production. The direction and magnitude of the effect is an empirical question. Estimates in the literature support the hypothesis that labour and public capital are substitutes. However, the relationship between public capital and private capital is not as clear. For instance, Conrad and Seitz (1994) and Lynde and Richmond (1992) find that public capital and private capital are complements, while Shah (1992), Nadiri and Mamuneas (1994a, b), and Morrison and Schwartz (1996) find they are substitutes.

Table 6a presents average values of the elasticities of conditional input demands with respect to public capital for the period 1961-2000. Conditional input demands are the demand for labour, capital, and intermediate inputs holding output constant. We calculate these elasticities according to  $\frac{\partial X_f}{\partial G}$ . The magnitudes of the elasticities of labour, private capital and intermediate inputs with respect to public capital vary across industries. They suggest that in all industries, the demand for labour and intermediate inputs is reduced as the investment in public capital is increased. However, private capital and public capital are complements in most industries. This complementarity effect is relatively large in trade industries and some manufacturing industries such as refined petroleum and coal products, primary metals and food industries.

	$\eta_{\scriptscriptstyle KG}$	$\eta_{LG}$	$\eta_{MG}$
Agricultural and related service	0.0214	-0.0854	-0.1125
Fishing and trapping	0.0041	-0.0219	-0.0052
Logging and forestry	0.0123	-0.0321	-0.0231
Mining	0.1039	-0.0018	-0.0259
Crude petroleum and natural gas	0.1514	-0.0101	-0.0754
Quarry and sand pit	0.0011	-0.0162	-0.0204
Services incidental to mineral extraction	0.0091	-0.0014	-0.0417
Food	0.1133	-0.1123	-0.1241
Beverage	0.0752	-0.0975	-0.0849
Tobacco products industry	0.0631	-0.0521	-0.0454
Rubber products	0.1839	-0.0663	-0.0715
Plastic products	0.0123	-0.0621	-0.0513
Leather and allied products	0.1585	-0.0024	-0.0174
Primary textile	0.1693	-0.0672	-0.0579
Textile products	0.1471	-0.0711	-0.0673
Clothing	0.1102	-0.0659	-0.0254
Wood	0.0913	-0.0366	-0.0493
Furniture and fixture	0.1658	-0.0248	-0.0367
Paper and allied products	0.0671	-0.0743	-0.0857
Printing, publishing and allied	0.0541	-0.0884	-0.0941
Primary metal	0.1236	-0.0814	-0.0957
Fabricated metal products	0.1132	-0.0717	-0.0884
Machinery ind. (except electrical mach)	0.1213	-0.0661	-0.0831
Transportation equipment	0.1453	-0.0814	-0.0775
Electrical and electronic products	0.1485	-0.0418	-0.0549
Non-metallic mineral products	0.1101	-0.0331	-0.0276
Refined petroleum and coal products	0.2041	-0.1108	-0.0731
Chemical and chemical products	0.0512	-0.0836	-0.0971
Other manufacturing	0.0941	-0.0151	-0.0241
Construction	0.0914	-0.0337	-0.0841
Transportation	0.0452	-0.0749	-0.1134
Pipeline transport	-0.0143	-0.0831	-0.0519
Storage and warehousing	0.0541	-0.0667	-0.0575
Communication	0.0524	-0.0314	-0.0749
Other utility	-0.0367	-0.0171	-0.0452
Wholesale trade	0.1141	-0.1274	-0.1478
Retail trade	0.0971	-0.1087	-0.1295
Business Sector	0.0929	-0.0683	-0.0882

Table 6a - Direct Elasticities of Conditional Input Demand

Note:  $\eta_{fG}(f = K, L, M)$  = elasticities of conditional input demands with respect to public capital. Conditional input demands are the demand for labour, capital, and intermediate inputs holding output constant.

Table 6b presents the total effect of an expansion in public capital on demand for labour, capital and intermediate inputs in different industries for the period 1961-2000. The total effect is the sum of the effects shown in Table 6a (when the output level is fixed) and the induced output expansion effect. The latter effect measures the increased demand for the inputs in response to the increase in output induced by the initial cost reduction of the public capital. The output expansion effect on input demands is positive for all three inputs but their magnitudes vary across industries and among inputs. The expansion effect reduces the magnitude of the

substitution effect shown in Table 6a for both labour and intermediate inputs. That is, the effects of an increase in the public capital on labour and intermediate inputs are still negative, but the magnitudes of these elasticities are much smaller than the elasticities reported in Table 6a. The magnitudes of these effects vary considerably among industries. The total effect of an increase in public capital on demand for capital is positive and larger than the elasticities reported in Table 6a.

	$\eta_{\scriptscriptstyle KG}$	$\eta_{LG}$	$\eta_{MG}$
Agricultural and related service	0.0468	-0.0503	-0.0674
Fishing and trapping	0.0166	-0.0186	-0.0006
Logging and forestry	0.0494	-0.0107	-0.0107
Mining	0.1495	-0.0008	-0.0135
Crude petroleum and natural gas	0.1782	-0.0091	-0.0365
Quarry and sand pit	0.0022	-0.0036	-0.0408
Services incidental to mineral extraction	0.0182	-0.0002	-0.0055
Food	0.1585	-0.0758	-0.1117
Beverage	0.1123	-0.0716	-0.0428
Tobacco products industry	0.0956	-0.0270	-0.0325
Rubber products	0.2235	-0.0198	-0.0228
Plastic products	0.0450	-0.0367	-0.0217
Leather and allied products	0.1950	0.0101	0.0122
Primary textile	0.2058	-0.0414	-0.0127
Textile products	0.1868	-0.0342	-0.0194
Clothing	0.1361	-0.0193	0.0198
Wood	0.1234	-0.0241	-0.0131
Furniture and fixture	0.2110	-0.0119	-0.0160
Paper and allied products	0.0930	-0.0284	-0.0361
Printing, publishing and allied	0.0872	-0.0425	-0.0353
Primary metal	0.1495	-0.0488	-0.0833
Fabricated metal products	0.1584	-0.0352	-0.0625
Machinery ind. (except electrical mach)	0.1578	-0.0203	-0.0385
Transportation equipment	0.1790	-0.0355	-0.0130
Electrical and electronic products	0.2549	-0.0172	-0.0704
Non-metallic mineral products	0.1470	-0.0206	-0.0170
Refined petroleum and coal products	0.2497	-0.0651	-0.0279
Chemical and chemical products	0.0775	-0.0379	-0.0634
Other manufacturing	0.1397	-0.0051	-0.0096
Construction	0.1467	-0.0086	-0.0389
Transportation	0.1003	-0.0491	-0.0769
Pipeline transport	-0.0040	-0.0307	-0.0394
Storage and warehousing	0.0862	-0.0214	-0.0238
Communication	0.1178	-0.0188	-0.0490
Other utility	-0.0242	-0.0068	-0.0296
Wholesale trade	0.1567	-0.1011	-0.0793
Retail trade	0.1230	-0.0717	-0.0843
Business Sector	0.1334	-0.0387	-0.0508

**Table 6b - Total Elasticities of Input Demand** 

Note: The total effect is the sum of the effects shown in Table 6a (when the output level is fixed) and the induced output expansion effect. The latter effect measures the increased demand for the inputs in response to the increase in output induced by the initial cost reduction of the public capital.

We can generally conclude that increases in public capital substantially change the input ratios in all industries. The effects of these changes vary considerably across industries and among inputs. Increases in public capital save labour and intermediate inputs, but these increases also raise the demand for private capital in all of the industries that we consider.

#### 1.3 Marginal Benefits

The marginal benefit of public capital is measured in terms of its private cost-reducing impact. The magnitude of cost reduction depends on the industry's elasticity of cost with respect to public capital  $\left(\frac{\partial \ell n \tilde{C}}{\partial \ell n G}\right)$  and the industry's cost of production relative to the size of public capital stock  $\left(\frac{\tilde{C}}{G}\right)$  using (1). Put differently, the marginal benefit of public capital is defined to be the negative of the partial derivative of the cost function (1) with respect to public capital.<sup>10</sup> This derivative can be interpreted as the marginal willingness to pay function. We measure this for each industry using the following expression

$$-\frac{\partial \tilde{C}_i}{\partial G} = \frac{\tilde{C}_i}{G} \left[ \alpha_G + \alpha_{GG} \ell n G + \alpha_{GY} \ell n Y_i + \alpha_{KG} \ell n \tilde{w}_K + \alpha_{LG} \ell n \tilde{w}_L + \alpha_{Gt} t \right]$$
(6)

In this expression, the marginal benefit of public capital, *G*, is measured in terms of cost reduction. The magnitude of the marginal benefit depends on the ratio of an industry's cost to the size of the public capital stock,  $\frac{\tilde{C}_i}{G}$ . Other factors that determine the magnitude of the marginal benefit for the industry are the level of output,  $Y_i$ , and the relative input prices of labour, capital and intermediate inputs. The stocks of public capital, *G*, and the level of technology, also affect the measure of industry marginal benefit.

Table 7 lists the average marginal benefits of an increase in the public capital using (6) for each industry over the period 1961-2000. These benefits indicate the 'willingness to pay' for an additional unit of public capital services by each industry. This 'willingness to pay' is exclusive of the income taxes, gasoline taxes, fees and interest payment on bonds, etc. that are used to construct and operate public capital. These are measures of the public capital's externality benefits to various industries.

For the business sector, the marginal benefits of a \$1.00 increase in public capital range between 0.01 cents for other manufacturing and 2.6 cents for transportation and retail trade.<sup>11</sup> The magnitudes of the marginal benefits of public capital vary considerably across industries. The largest benefits occur in transportation, trade and other utility. The marginal benefits are moderately high in transportation equipment, agriculture and primary metal. The marginal benefits are very small in printing and publishing, and logging and forestry.

<sup>&</sup>lt;sup>10</sup> The marginal benefits estimates are constructed as follows: first, we estimate (1) and then we take the antilog of the predicted value of  $\ell n \tilde{C}$  to get  $\tilde{C}$ ; second, from (1), we estimate the cost elasticity of public capital  $\frac{\partial \ell n \tilde{C}}{\partial \ell n G}$ ; third, we calculate the marginal benefits of public capital using the outcomes of step 1 and 2 and the level of public capital stock in constant prices.

<sup>&</sup>lt;sup>11</sup> Industry marginal benefit estimates can be translated into a dollar value of cost reduction in each industry for a given amount of public capital spending. The simplest way to do this is to multiply the measure of marginal benefit in industry by the net increase in public capital for a particular year period.

	٦Ĉ
	$\frac{\partial \tilde{C}}{\partial G}$
Agricultural and related service	0.60
Fishing and trapping	0.00
Logging and forestry	0.05
Mining	0.14
Crude petroleum and natural gas	0.36
Quarry and sand pit	0.00
Services incidental to mineral extraction	0.02
Food	0.65
Beverage	0.09
Tobacco products industry	0.02
Rubber products	0.04
Plastic products	0.04
Leather and allied products	0.01
Primary textile	0.03
Textile products	0.02
Clothing	0.06
Wood	0.21
Furniture and fixture	0.02
Paper and allied products	0.34
Printing, publishing and allied	0.15
Primary metal	0.56
Fabricated metal products	0.38
Machinery ind. (except electrical mach)	0.24
Transportation equipment	1.29
Electrical and electronic products	0.02
Non-metallic mineral products	0.07
Refined petroleum and coal products	0.34
Chemical and chemical products	0.33
Other manufacturing	0.01
Construction	2.58
Transportation	2.56
Pipeline transport	0.07
Storage and warehousing	0.01
Communication	0.64
Other utility	0.81
Wholesale trade	2.11
Retail trade	2.56
Business Sector	17.44

#### Table 7 - Marginal Benefits of Public Capital (Mean Values, 1961-2000)

Note: Marginal benefit of public capital is defined to be the negative of the partial derivative of the cost function with respect to public capital. This derivative can be interpreted as the marginal willingness to pay function.

The calculation of the marginal benefit of public capital at the business sector level assumes that the use of public capital by one industry does not preclude or reduce the value of its use to any other industry (i.e. we assume non-rival consumption of public capital). Therefore, industry marginal benefits are additive across the 37 sectors. The weighted sum of marginal benefits across all industries is about 0.17. That is, a \$1.00 increase in the net capital stock generates approximately 17 cents of 'cost saving' producer benefits per year.

#### 1.4 Industry Multifactor Productivity Growth Decomposition

One of the fundamental goals in analyzing the effect of public infrastructure is to determine its contribution to productivity growth. As indicated at the outset of this report, this issue provides the rationale for much of the literature in this area. For example, Aschauer (1989) attributes almost all of the slowdown in the rate of aggregate productivity growth to the slowdown in the growth of public capital. To examine this issue further, we calculate the contribution of public capital to multifactor productivity growth at the disaggregated industry level using the parameter estimates of our econometric model.

Table 8 lists the decomposition of multifactor productivity growth for each industry based on (4). Changes in exogenous demand are large in several industries such as electrical and electronic equipment, machinery, transportation and food. In other industries, particularly manufacturing industries and utilities, changes in exogenous demand are a major contributor to the multifactor productivity growth.

The sign of the contribution of relative input prices to multifactor productivity growth depends on whether the changes in an industry's factor price exceeds that of the general price level in the economy. Productivity growth in an industry is hampered when its input price inflation exceeds the national inflation rate, measured by the GDP deflator. As Table 8 shows, the growth in relative input prices contributes negatively to multifactor productivity growth in some industries while contributes positively in many others. The magnitude of this effect varies across industries ranging from -0.0166 in other manufacturing to -0.0549 in the beverage industry. However, compared to the contribution of exogenous demand, the contribution of changes in the relative input price to multifactor productivity growth is very small.

Public capital's contribution to multifactor productivity growth is positive in all industries. In some industries, its contribution is relatively large, accounting for more than 2/3 of multifactor productivity growth in construction, transportation and trade industries, but in most industries its contribution to changes in multifactor productivity is fairly modest. When the effects of demand, relative input price changes, and public capital are taken into account, the rate of technical change is much smaller than conventionally calculated. In general, the main contributors to multifactor productivity growth in almost all industries are technical change, the exogenous shifts in demand and public capital.

	Exogenous Demand	Relative Input Prices	Public Capital	Disembodied Technical	MFP
A ' 1, 1 1 1, 1 '	0.4415	0.0457	0.1014	Change	1 221 6
Agricultural and related service	0.4415	0.0457	0.1814	0.7544	1.3316
Fishing and trapping	0.0514	-0.0021	0.0015	-0.0348	0.0160
Logging and forestry	0.4361	-0.0124	0.0102	0.5573	0.9912
Mining	0.5614	-0.0341	0.1756	0.5145	1.2174
Crude petroleum and natural gas	0.5912	-0.0257	0.2622	0.6964	1.5241
Quarry and sand pit	0.2256	-0.0032	0.0013	0.6275	0.8512
Services incidental to mineral extraction	0.3263	0.0033	0.0025	-0.1624	0.1697
Food	0.4121	-0.0514	0.2541	0.0484	0.6632
Beverage	0.3713	-0.0549	0.2784	0.8469	1.4417
Tobacco products industry	0.3144	0.0127	0.1301	0.3969	0.8541
Rubber products	0.2452	0.0032	0.1123	0.8741	1.2347
Plastic products	0.1891	0.0012	0.0851	1.0644	1.3398
Leather and allied products	0.2934	0.0131	0.0412	0.0996	0.4474
Primary textile	0.2411	0.0451	0.1247	1.0384	1.4493
Textile products	0.1955	0.0237	0.1033	0.8792	1.2017
Clothing	0.2933	0.0541	0.0293	0.2926	0.6693
Wood	0.5312	-0.0123	0.0833	0.089	0.6912
Furniture and fixture	0.1576	-0.0252	0.1013	0.6262	0.8597
Paper and allied products	0.2569	0.0219	0.0810	-0.0230	0.3368
Printing, publishing and allied	0.3591	-0.0125	0.0915	-0.2089	0.2287
Primary metal	0.2629	-0.0259	0.0813	1.0907	1.4091
Fabricated metal products	0.2984	-0.0359	0.1127	0.8341	1.2093
Machinery ind. (except electrical mach)	0.4696	-0.0412	0.1451	0.6806	1.2541
Transportation equipment	0.4940	0.0143	0.2412	0.9193	1.6687
Electrical and electronic products	0.4219	-0.0174	0.0412	2.0114	2.4571
Non-metallic mineral products	0.2145	-0.0129	0.0708	0.6788	0.9512
Refined petroleum and coal products	0.3012	0.0512	0.1451	-0.0149	0.4465
Chemical and chemical products	0.4123	0.0241	0.2041	0.5831	1.2236
Other manufacturing	0.1455	-0.0166	0.0104	0.3192	0.4585
Construction	0.2902	-0.0457	0.3641	-0.0512	0.5574
Transportation	0.6541	-0.0125	0.3152	1.0573	2.0141
Pipeline transport	0.2142	-0.0085	0.1451	1.6477	1.9985
Storage and warehousing	0.2362	-0.0114	0.1108	0.001	0.3366
Communication	0.5485	-0.0375	0.0729	2.4445	3.0286
Other utility	0.3465	-0.0121	0.2125	0.5167	1.0636
Wholesale trade	0.2895	-0.0133	0.3694	0.4729	1.1185
Retail trade	0.2563	-0.0251	0.3971	1.2291	1.8574
Business Sector	0.2303	-0.0187	0.2223	0.6405	1.2239

#### Table 8 - Decomposition of Multifactor Productivity Growth by Industry

Note: Exogenous demand captures growth of real domestic income and aggregate population; relative input price captures the growth in relative input prices; highway capita captures the combined direct and indirect effects of the growth of public capital; disembodied technical change, or adjusted multifactor productivity growth (*MFP*), captures the effect of

exogenous technical change, derived as the difference between the standard measure of MFP (the final column of this table), and the exogenous demand, relative input price and public capital components.

#### 2. Contribution of Public Capital at the Business Sector Level

To obtain the business sector's average elasticity of aggregate cost with respect to public capital, we weight the individual industry estimates by their respective industry cost shares. For example, let us define the cost elasticity of public capital for industry *i* as  $\eta_{\tilde{C}G} = \left(\frac{\partial \tilde{C}_i}{\partial G}\right) \left(\frac{G}{\tilde{C}_i}\right)$ . We obtain the 'aggregated' cost elasticity by using the expression

$$\eta_{\tilde{C}G} = \sum_{i} \eta_{\tilde{C}G_{i}} \left( \frac{\tilde{C}_{i}}{\sum_{i} \tilde{C}_{i}} \right).$$
(7)

That is, the 'aggregated' cost elasticity is a cost share weighted average of individual industry elasticities.

#### 2.1 Aggregate Cost and Output Elasticities

The aggregate cost and output elasticities with respect to public capital stock, the scale elasticities, and the output elasticities of inputs—labour, intermediate inputs and capital—are reported in the bottom of Tables 5 through 7. The scale measures for the business sector shown in Table 5 suggest a degree of scale with respect to private input of 1.06. This suggests a modest degree of increasing returns to scale. However, the total scale measure is approximately 1.18. This measure reflects the contribution of public capital as unpaid inputs in the production function.

Either the aggregate cost elasticity  $\bar{\eta}_{\tilde{C}G}$  or the aggregate output elasticity  $\bar{\eta}_{YG}$  shown in Table 5 can measure the productivity effect of public capital. These elasticities imply the average cost reduction or output increase due to an increase in public capital investment. For example, the magnitude of the marginal productivity of public capital  $\bar{\eta}_{YG}$  suggests that a 1% increase in public capital leads to approximately a 0.066% average cost reduction for the business sector. The magnitude of this elasticity is much smaller than the elasticities reported in the literature. In particular, our result contrasts markedly with the elasticity estimates reported in Wylie (1996). Specifically, our estimates of cost and output elasticities with respect to increases in public capital are 8 times lower than his estimates for total infrastructure capital, largely reflecting methodological differences and data sources.

The estimated cost and output elasticities can be used to calculate the total marginal benefit of an increase in public capital for the business sector. The sum of the marginal benefits over all 37 industries is shown in the bottom of Table 7. This sum suggests that an increase of 1% in net public capital generates a total benefit of 17 cents at the business sector.

These estimates of the total benefit measure the externality benefits of public capital, exclusive of payments toward the construction and operation of this capital already paid by the producers. The producer payments are included in our basic dataset as the expenses for taxes and intermediate inputs as part of factor costs. In addition, these externality benefits of public capital are exclusive of the benefits of public capital in the consumption sector of the economy which may be large and not accounted for here.

As noted above, we report that the total marginal benefit of public capital is 0.17. Increased public capital is labour and intermediate inputs saving at the business sector level as well as at the industry level. That is, an increase in public capital investment reduces the demand for both labour and intermediate inputs. In addition, public capital investment also has a strong positive effect on the demand for private capital. That is, private capital and public capital are complements. Public capital investment leads to 'crowding in' of private capital formation. These conclusions hold, as shown in the bottom of Tables 6a and 6b, whether the level of output is fixed or variable. In fact, the induced output expansion effect on demand for inputs are all positive and fairly large. Increases in public capital save on inputs such as labour and intermediate inputs, but they also increase demand for private capital investment. This pattern, as was noted earlier, also holds for the business sector.

The bottom of Table 5 presents the output elasticity of public capital. A one-percent increase in public capital leads to an average of a 0.066% increase in total output over the period 1961-2000. It is also important to note that the output elasticity of private sector capital is clearly larger than the output elasticity of public capital.<sup>12</sup> The results indicate that the contribution to the business sector's growth of a one-percent increase in private capital stock is more than twice that of a similar increase in the public capital stock.

However, it is important to note that the output elasticity with respect to public capital has been declining over time. The average elasticity of output with respect to public capital,  $\bar{\eta}_{\tilde{C}G}$ , starts out relatively high in pre-1973 period—about 0.15—but steadily declines thereafter. The average output elasticity with respect to public capital for the years, 1981-2000 is about 0.032; in 2000, the value of  $\bar{\eta}_{YG}$  is about 0.019. The reduction in the value of  $\bar{\eta}_{YG}$  during this period, to a great extent reflects the fall in the ratio of public capital to total cost and output of the business sector, i.e.,  $\frac{G}{Y}$ . In contrast, the aggregate output elasticity with respect to private capital,  $\bar{\eta}_{YK}$ , is relatively stable over the entire sample period. It ranges from about 0.16 to 0.18.

#### 2.2 Decomposition of Aggregate Multifactor Productivity Growth

A central issue in the debate on the role of public capital centers on the question of its contribution to the growth of aggregate multifactor productivity and to the deceleration of multifactor productivity growth in the period post 1973. Aschauer (1989), Munnell (1990a) and others claim that the decline in this period is mainly, if not exclusively, due to the decline in growth of infrastructure capital. Hulten and Schwab (1991), Gramlich (1994) and others have argued for no or minimal contribution of public capital to productivity slowdown.

To calculate the sources of productivity growth at the business sector level, we aggregate the industry decomposition, shown in Table 8, using the industry's share in total output as weights to calculate the effect of exogenous demand, relative prices, public capital stock, and technical change on the growth rate of multifactor productivity in the Canadian business sector during the period 1961-2000.

<sup>&</sup>lt;sup>12</sup> The elasticities of labour, private capital and intermediate inputs correspond to the shares of these inputs in total output. The output elasticity of intermediate inputs is the largest followed by that of labour and private capital.

As indicated in Table 9, the main source of multifactor productivity growth over the period 1961-2000 is technological change. It accounts for about 21% of multifactor productivity growth over the period followed by exogenous demand. Input price changes contribute less than 2.1% to multifactor productivity growth. Public capital's contribution to total factor productivity growth is about 12%. This pattern generally persists across sample periods, although its magnitude fluctuates. Public capital's contribution to multifactor productivity was high in the period before 1981. Since then, it has contributed much less.

These results stands in contrast to those reported by Wylie (1996) for Canada and the proponents of large contributions of infrastructure in United States. Furthermore, our results also differ from the results reported by those who deny any role for public capital in enhancing the growth rate of productivity. Our analysis suggests a middle course.

That is, increases in public capital stock contribute to the performance of the business sector. However, the magnitude of its contributions to growth of output and productivity are modest in comparison to the contribution of exogenous demand. Most of the contribution of public capital to productivity growth occurred in the pre-1973 period. Since 1981, public capital has made a small contribution to trend multifactor productivity.

Tuble >	Decomposition of riggle gate multiluctor rioudentity Growth				
	Exogenous	Relative	Public	Disembodied	MFP
	Demand	Input	Capital	Technical	
		Prices		Change	
1961-2000	0.3797	-0.0137	0.2816	0.8015	1.4917
1961-1973	0.4132	-0.0187	0.2203	0.6872	1.3025
1973-1981	0.3519	-0.0296	0.2233	0.5654	1.1109
1981-1988	0.3318	-0.0129	0.1674	0.5053	0.9917
1988-2000	0.3798	-0.0186	0.2232	0.6399	1.2242

#### Table 9—Decomposition of Aggregate Multifactor Productivity Growth

# V. Concluding Remarks

This paper is the second in a series that examines the contribution of public capital to productivity performance of the business sector. Since replication is a key part of the analytical process, we modified in several ways the analysis developed in Harchaoui (1997).

The main goal of this paper is to analyze and measure the contribution of public capital to business sector's productivity growth. The approach developed here explicitly incorporates demand and supply forces, including the contribution of public capital, which may affect productivity performance. We estimate the model using disaggregated data composed of 37 industries for the period 1961-2000. The data include measures of gross output, intermediate inputs, private capital and labour. We also estimate demand and supply (cost) functions for each industry. We identify the determinants of productivity growth for each industry, including the contribution of public capital, and we measure specifically the marginal benefit of public capital to each industry.

There are two differences between this study and the previous one (Harchaoui 1997):

First, we extend the industrial coverage and the sample period to cover the period 1961-2000. The data set for the 37 industries which comprises about 70% of the Canadian business sector were revised significantly and in principle lead to a new set of estimates and results. Second, using a non-constant returns to scale translog cost function, we explicitly take into account the interaction of the whole public capital with private sector inputs at the industry and macroeconomic levels.

To generate aggregate measures for the portion of the business sector that has the most reliable measures of output, we use a weighted sum of individual industry elasticities to obtain the aggregate elasticity measures for the business sector. Using these estimates, we decompose multifactor productivity growth into its various components.

The quantitative results of this report are briefly summarized as follows:

- 1. There is evidence of a mild degree of increasing returns to scale in most industries and at the business sector level. The marginal products of labour, capital and intermediate inputs vary across industries. The output elasticity of intermediate inputs is in general the largest, followed by that of labour and capital inputs. More importantly, at both the industry level and the business sector level, the elasticity of private capital is larger than that of public capital by a factor of two times for the entire period and by a factor of about four times for the period 1981-2000. This result is in sharp contrast to those reported in Aschauer (1989) and Fernald (1999) for United States. All of those studies imply that an additional dollar of public investment is substantially more productive than a corresponding dollar of private investment.
- 2. Public capital contributes significantly to economic growth and productivity at the industry and business sector levels. This contribution varies across industries and over time. The magnitude of the elasticity of output with respect to public capital at the aggregate level is about 0.066, which is much smaller than comparable estimates reported in the literature and somewhat larger than our previous results.
- 3. An increase in public capital has an initial productivity effect: it reduces total cost for a given level of output for all industries and at the aggregate level. This productivity effect induces output expansion in all industries, which in turn increases costs by requiring increases in input demands. When output level is allowed to vary, the productivity gains of public capital offset the cost increases required by the output expansion.

Public capital has a significant effect on labour, private capital formation and demand for intermediate inputs in all industries. The magnitude of these effects varies among the three inputs in a given industry and across industries. Given a level of output, an increase in public capital leads to a reduction in demand for labour and intermediate inputs and an increase in demand for private capital in all industries. These results are similar to those presented in Harchaoui (1997). The direction of the effects on the demand for labour, capital, and intermediate inputs remain the same as noted earlier. However, the magnitude of the elasticities are substantially different in each industry.

- 4. The marginal benefits of public capital are positive in all industries. The magnitudes of these benefits, which can be interpreted as a measure of producers' 'willingness to pay,' varies considerably across industries and over time. We observe that the average of marginal benefits across all industries is about 17 cents for every dollar increase of public capital.
- 5. The contribution of public capital to multifactor productivity growth is found to be positive in all industries. Our present results show a more pervasive influence on multifactor productivity growth than our previous results. However, the magnitudes of the contribution of public capital to productivity growth varies across industries. At the aggregate level, public capital's contribution to multifactor productivity growth is about 12% of total growth. Compared with estimates previously reported in the literature, this figure is relatively small, though still meaningful. The decomposition of multifactor productivity growth confirms the result reached by Harchaoui (1997) that the main contributor to productivity growth both at the industry and aggregate levels is technical change and exogenous demand (representing the effect of aggregate income and population growth), followed by public capital.
- 6. In this study, we concentrate on the so-called core public infrastructure capital. While the figures reported in this study are point estimates that are subject to uncertainty, they lie within reasonably tight confidence interval. Future work will examine the extent to which overall public capital that encompasses health and education sectors will impact on the estimates of the marginal benefits of public capital and related economic performance indicators reported in the paper.

Future work will examine the sensitivity of our findings to alternate specifications and the derivation of confidence intervals on the contribution made by public capital and other components to multifactor productivity growth.

### Appendix—Multifactor Productivity Decomposition

Assume that the production function of an industry is given by

$$Y = F(X, G, t), \tag{A.1}$$

where Y is the output of the industry, X is an n-dimensional vector of private inputs, G is public capital, and t denotes the level of disembodied technology. The traditional measure of multifactor productivity growth is defined by

$$M\dot{F}P = \dot{Y} - \sum_{f=1}^{F} \pi_f \dot{X}_f,$$
 (A.2)

where the dot denotes rate of growth, for example,  $\dot{Y} = \frac{\partial Y}{\partial t} \frac{1}{Y}$ ; and  $\pi_f = \frac{w_f X_f}{P_y Y}$  is the revenue share of the *f*-*th* private input.

Differentiating (A.1) with respect to time, and dividing by output, we obtain

$$\dot{Y} = \sum_{f=1}^{F} \frac{\partial F}{\partial X_f} \frac{X_f}{Y} \dot{X}_f + \frac{\partial F}{\partial G} \frac{G}{Y} \dot{G} + \frac{1}{Y} \frac{\partial F}{\partial t}.$$
(A.3)

Assuming cost minimization of all inputs, public capital included, and letting  $w_f$  be the price of the *f*-*th* private input and *z* the shadow price of public capital *G*, we obtain the following first-order conditions:

$$\frac{\partial F}{\partial X_f} = \frac{w_f}{\mu}$$
 and  $\frac{\partial F}{\partial G} = \frac{z}{\mu}$ , (A.4)

where  $\mu$  is the Lagrangian multiplier, together with the envelope conditions

$$\frac{\partial C^*}{\partial Y} = \mu \text{ and } -\frac{\partial C^*}{\partial t} = \mu \frac{\partial F}{\partial t},$$
 (A.5)

where  $C^* = \sum_{f=1}^{F} w_f X_f + zG = C^*(Y, w, z, t)$  is the total cost function including the shadow cost of public capital. Eliminating  $\mu$  from (A.4) and (A.5) and substituting (A.4) and (A.5) in (A.3), we obtain:

$$\dot{Y} = \sum_{f=1}^{F} \frac{w_f X_f}{\left(\frac{\partial C^*}{\partial Y}\right) Y} \dot{X}_f + \frac{zG}{\left(\frac{\partial C^*}{\partial Y}\right) Y} \dot{G} + \frac{\left(-\frac{\partial C^*}{\partial Y}\right)}{\left(\frac{\partial C^*}{\partial Y}\right) Y}.$$
(A.6)

Firms, however, do not adjust the public capital stocks—they are exogenously given. What actually is observed is that firms minimize their private production cost subject to the production function (A.1). Let the optimal private cost of production, given the output level and public capital, be  $C = \sum_{f=1}^{F} w_f X_f = C(Y, w, G, t)$ . Then the marginal benefit of an increase of public capital at the optimum will be given by

$$-\frac{\partial C}{\partial G} = z. \tag{A.7}$$

It is not difficult to show using comparative statics that the total cost elasticity,  $\eta^*$ , is given by

$$\eta^* = \frac{\partial \ell n C^*}{\partial \ell n Y} = \frac{\left(\frac{\partial \ell n C}{\partial \ell n Y}\right)}{B} = \frac{\eta}{B},$$

where  $B = 1 - \frac{\partial \ell_n C}{\partial \ell_n G} = 1 - \eta_{CG}$  and  $\eta_{CG}$  is the private cost elasticity with respect to public capital, and  $\eta$  is the private cost elasticity. The cost diminution due to technical change is  $\dot{T} = \frac{\partial \ell_n C^*}{\partial t} = \frac{\left(\frac{\partial \ell_n C}{\partial t}\right)}{B}$ .

Following Caves *et al.* (1981), total returns to scale of the production function is defined as the proportional increase in output due to an equiproportional increase of all inputs (private and public, holding technology fixed), and is given by the inverse of  $\eta^*$ . Private returns to scale, i.e., the proportional increase in output due to an equiproportional increase in private inputs, holding public inputs and technology fixed, is given by the inverse of  $\eta$ . Thus, we identify two scale effects in our study, one internal and the other total, which is the sum of internal and external scale effects. Substituting (A.7) in (A.6) and then in (A.2) we get

$$M\dot{F}P = \left(\frac{\kappa - \eta^*}{\kappa}\right)\dot{Y} - \frac{1}{\kappa B}\eta_{CG}\dot{G} - \frac{1}{\kappa B}\dot{T},$$
(A.8)

where  $\kappa = \frac{P_y Y}{C^*} = \frac{P_y}{\left(\frac{C^*}{Y}\right)}$  is the ratio of output price,  $P_y$ , to average total cost,  $\frac{C^*}{Y}$ .

According to equation (A.8), multifactor productivity growth is decomposed into three components: a gross total scale effect given by the first term; a public capital stock effect given by the second term; and the technological change effect given by the last term.

The next step is to further decompose the scale effect. We assume the output price is related to private marginal cost in the following manner:

$$P_{y} = (1+\theta)\frac{\partial C}{\partial Y},$$

where  $\theta$  is a markup over marginal cost. The markup depends on the elasticity of demand as well as on the conjectural variations held by the firms within an industry.

Using the definition of output elasticity,  $\eta$ , along with the private cost function, we obtain

$$P_{y} = (1+\theta)\eta \frac{C}{Y}.$$
 (A.9)

After time differentiating (A.9), the pricing rule implies

$$\dot{P}_{y} = (1+\theta) + \dot{\eta} + \dot{C} - \dot{Y}. \tag{A.10}$$

Differentiating the private cost function with respect to time and using Shephard's lemma yields

$$\dot{C} = \eta \dot{Y} + \sum_{f=1}^{F} \hat{\pi}_{f} \dot{w}_{f} + \eta_{CG} \dot{G} + \dot{T}, \qquad (A.11)$$

where  $\hat{\pi}_f = \frac{w_f X_f}{\sum_f w_f X_f}$  is the share of the *fth* input in private cost, *C*.

In order to obtain the equilibrium of output growth we assume a log linear demand function in growth rate form:

$$\dot{Y} = \lambda + \alpha \left( \dot{P}_{y} - \dot{P}_{D} \right) + \beta \dot{Z} + (1 - \beta) \dot{N}, \qquad (A.12)$$

where Z and N are real aggregate income and population, respectively, and  $\lambda$  reflects a demand time trend, and  $P_D$  is the GDP deflator. Substituting (A.11) in (A.10) and the result in (A.12), we obtain the reduced form function for the growth rate of multifactor productivity:

$$M\dot{F}P = A[\alpha\dot{\eta} + \alpha(1+\theta)] + A\alpha \left[\sum_{f} \left(\tilde{\pi}\dot{w}_{f} - \dot{P}_{D}\right)\right] + A[\lambda + \beta\dot{Z} + (1-\beta)\dot{N}] + \left(A\alpha - \frac{1}{\kappa B}\right)\eta_{CG}\dot{G} + \left(A\alpha - \frac{1}{\kappa B}\right)\dot{T}.$$
(A.13)

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