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# Life Tables, Canada, Provinces and Territories

2000 to 2002



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## Note of appreciation

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## Introduction

A life table represents a universally accepted demographic or actuarial model that synthesizes the mortality experience of a population, in a clear and concise manner, and enables comparative measures of expected longevity. The model used to construct these tables assumes that a hypothetical cohort of 100,000 individuals born at the same moment in time is subject to the age-sex-specific mortality rates experienced by an actual population during a specific time period.

This report contains life tables constructed from the age-sex-specific mortality rates observed for Canada, the provinces and the territories in the 2000 to 2002 period. It also explains the methods used to produce these tables, and reproduces the formulae used to derive the estimates. Life tables for the first year of life have been produced separately, by sex, for Canada as a whole. Complete life tables by single year of age, for both males and females, have been generated for Canada and for each province except Prince Edward Island. Because of their small populations and low death counts, abridged life tables with five-year age groups have been produced for both Prince Edward Island and all territories combined, by sex.

For the 1995 to 1997 tables,<sup>1</sup> we introduced some major changes to the methodology for producing the tables: (1) the use of the Coale-Kisker model for mortality at advanced ages instead constructing estimates from actual counts, (2) use of 1 January population estimates instead of stationary population modelling for mortality in children under 5 years of age. For the 2000 to 2002 tables, we therefore chose to adhere as closely as possible to the methodology used in the 1995 to 1997 tables in order to offer the highest degree of consistency and comparability between the two tables so that changes observed would be due more to actual changes in the mortality experience and less due to changes in the methodology.

### 1. Input data

Creation of the life tables involved three major steps: gathering input data, calculation of mortality rates, and calculation of other life table values. This section describes the first step. For the construction of the 2000 to 2002 life tables, the following input data were required, by sex, for Canada, the provinces and the territories.

*Input data for the infant life tables* (i.e. for the first year of life):

- The total number of births in 2000 and 2001.
- The number of births by month in 1999 and in 2002.
- The number of infant deaths observed during the 2000 to 2002 period, aggregated by life-span period (i.e. days, weeks and months of life – see Table 1a).
- The value of  $T_1$  by sex, from the corresponding complete life table for Canada (see Formulae (29.1) to (29.4)).

*Input data for the complete life tables:*

- The number of deaths of children aged 0 to 4 years, observed during the 2000 to 2002 period, by single year of age, year of birth and year of death.
- January 1 population counts, by sex, for the years 2000 to 2003, by single year of age for ages 0 through 4 years.
- Separation factors at ages 0 to 4 years (see appendix 1).
- The number of deaths observed during the 2000 to 2002 period, by single year of age up to 104 years, and a final group of deaths of persons aged 105 and older.
- The number of deaths of persons aged 87 and older, by single year of age, observed during the 2000 to 2002 period (see the Coale and Kisker model, section 2.1.6).
- July 1, 2001 population counts, by individual ages 0-99, and a final age group consisting of persons aged 100 and older.

*Input data for the abridged life tables:*

- The total number of births recorded during the 1999 to 2002 period.
- The total number of deaths observed during the 2000 to 2002 period for the following age groups: infants under 1 year (age 0), ages 1 to 4 years combined, five-year age groups for ages 5 to 9 to 95 to 99 years, and a final group of deaths of persons aged 100 and older.
- July 1, 2001 population counts for the following age groups: under 1 year (age 0), ages 1 to 4 years combined, five-year age groups for ages 5 to 9 to 95 to 99 years, and a final age group consisting of persons aged 100 years and older.
- Separation factors at ages 0 and 1 to 4 years combined.

*Input data for the separation factors:*

- The number of deaths observed in the 2000 to 2002 period of children aged 0 to 4 years by death group. The term 'death group' refers to a dichotomous variable derived from the year of birth, year of death and age at death. The death group indicates whether or not an individual had a birthday (or was born - in the case of an infant death) in the calendar year during which their death occurred. See also Appendices 1 and 2.

*Data sources:*

- Birth and death data by province or territory of residence were compiled by Health Statistics Division, Statistics Canada. These data were extracted from birth and death registrations submitted to Vital Statistics registrars in the province or territory in which the event occurred.
- Population counts, produced by Demography Division, Statistics Canada, are estimates of the July 1, 2001 population, based on the 2001 Canadian Census of Population. These counts include non-permanent residents, and are adjusted for net census under-coverage.<sup>2</sup>

Adjusted population estimates are generally higher than unadjusted estimates. The use of higher adjusted population estimates results in slightly lower mortality rates, which in turn generate slightly higher life expectancy values than would otherwise have been calculated.<sup>3</sup>

For Canada as a whole, the complete life table values are shown to age 109 for each sex. For the provinces, these tables have been truncated at ages ranging from 105 to 109 years, depending on the quality of the source data. For Prince Edward Island and the territories (i.e. Yukon, Northwest Territories and Nunavut combined), abridged estimates are presented for five-year age groups, up to age 95 to 99 years, with a final group for ages 100 years and older.

## 2. Methodology

The methodology followed in constructing the 2000 to 2002 life tables is the same as that employed previously in producing the set of life tables for the years 1995 to 1997.<sup>1</sup> Variance estimation for  $q_x$  has again been included, in order to produce coefficients of variation of life expectancies,  $e_x$ .

As was done in the 1990 to 92 tables,<sup>4</sup> the value of  $q_x$  is not set to 1 for the oldest age published in the complete life tables, but rather is reported as the probability of death estimated for that single year of age. Where data quality permitted, each complete life table was expanded up to age 109 years. The abridged life tables extend to age "100 years plus" rather than to "90 years plus" as in the 1990 to 1992 life tables.

All calculations were carried out using the Statistical Analysis System (SAS)<sup>5</sup> software. Full decimal precision was maintained until the Sirken rounding procedure,<sup>4</sup> used in the 1990 to 1992 life tables, was applied to the data at the end of the calculations. The Sirken rounding procedure sets the values of  $d_x$  and  $L_x$  equal to the difference between two consecutive rounded values of  $l_x$  and  $T_x$ , respectively. In this way, both of the following basic relationships are preserved in the published tables:  $l_x - d_x = l_{x+1}$  and  $T_x - L_x = T_{x+1}$  (see section 3 for an explanation of these symbols).

The following sections describe methodological issues specific to each set of tables.

### 2.1 Complete life tables

The procedure employed in the construction of the complete life tables is essentially that described by Greville in *United States Life Tables and Actuarial Tables, 1939-1941*.<sup>6</sup> The same methodology was used to produce the earlier sets of life tables for Canada, beginning with the years 1970 to 1972.<sup>4</sup>

The principal values of all life tables are the  ${}_nq_x$  values. They represent the probability for persons of exact age "x" of dying within the interval from the beginning of age "x" to the beginning of age "x+n". In other words,  ${}_nq_x$  is the probability of death for the hypothetical cohort of 100,000 people in the age interval [x, x+n). For the complete life tables, the age interval is one year, i.e.,  $n = 1$ . In this case, the left subscript is sometimes omitted and the life table mortality rate is referred to simply as  $q_x$ .

Sections 2.1.1 to 2.1.6 show how the  $q_x$  values were calculated. The calculation of other life table values is explained in section 3.

### 2.1.1 Ages 0 to 4 years

By definition,

(1)  $q_x$  is the probability that a person aged exactly  $x$  years old will die before reaching exact age  $x+1$ ,

and, since  $p_x = 1 - q_x$ ,

(2)  $p_x$  is the probability that a person aged exactly  $x$  years will survive to exact age  $x + 1$ .

By commonly-used notation,<sup>7,8</sup> the probability of survival defined in (2) can be expressed in the form of the product of two other probabilities of survival:

$$(3) p_x = ({}_a p_x) ({}_s p_x),$$

where

$x$  denotes the age from 0 to 4 years,

${}_a p_x$  is the probability that a person aged exactly  $x$  years old will survive to the end of the calendar year in which age  $x$  was attained, and

${}_s p_x$  is the probability that a person alive at the end of the calendar year in which age  $x$  was attained will survive to exact age  $x+1$ .

From this,

$$(4) {}_a p_x = P_x' / E_x,$$

where

$P_x'$  is the number of persons who attained age  $x$  during the three-year period of observation, in this case, 2000 to 2002, and who were alive at the end of the year in which exact age  $x$  was attained, and

$E_x$  is the number of persons who attained age  $x$  during the period 2000 to 2002.

Similarly,

$$(5) {}_s p_x = E_{x+1} / P_x'',$$

where

$E_{x+1}$  is the number of persons who attained age  $x+1$  during the period 2000 to 2002, and

$P_x''$  is the number of persons alive at the end of the calendar year in which age  $x$  was attained and whose  $(x+1)^{\text{th}}$  birthday falls in the period 2000 to 2002.

Therefore,  $q_x$  is calculated by the following equation,

$$(6) q_x = 1 - (P_x' / E_x) (E_{x+1} / P_x''),$$

for  $x = 0$  to 4 years.

*Note:* Uppercase letters P and E are not to be confused with lowercase letters p and e that designate, respectively, the proportion of individuals surviving, and the average remaining years of life. For more explanations on how P and E were calculated, see Appendix 2.

In the provincial complete life tables, values of  $q_0$  to  $q_4$  were verified to ensure that they decreased monotonically. Generally  $q_0 > q_1$ , so ensuring that the series from  $q_0$  to  $q_4$  decreased monotonically required adjustments to the sub-series  $q_1$  through  $q_4$  only. This adjustment proceeded as follows. From definition (1) above, provincial survival probability values  $p_x$  were calculated from the  $q_x$ , and new  $p_x$  were obtained by applying national survival rates at ages 1 to 4 to the geometric mean of provincial survival rates. The new  $p_x$  is calculated as follows:<sup>9</sup>

$$(7) \quad (\text{new } p_x) = \frac{(\text{national } p_x) \left[ \prod_{k=1}^4 (\text{old } p_k) \right]^{\frac{1}{4}}}{\left[ \prod_{k=1}^4 (\text{national } p_k) \right]^{\frac{1}{4}}}$$

for  $x = 1$  to 4 years,

where “national  $p_x$ ” is the sex-specific national survival probability at age  $x$ , and “old  $p_k$ ” is the previously-calculated sex-specific provincial survival probability at age  $k$ . New values of  $q_x$  were then calculated from the new values of  $p_x$ . An attractive feature of this procedure is that both the old and the new values of  $p_x$  (and thus  $q_x$ ) produce the same number of survivors at age 5. Text table 1 shows an example of this calculation for females in New Brunswick, starting from a cohort of 100,000 live births.

**Text table 1 Adjustment of mortality rates, ages 1 to 4 years, New Brunswick females**

Age x	Expected number of deaths using old $q_x$	Expected number of deaths using new $q_x$
0	368	368
1	27	34
2	9	19
3	17	14
4	25	11
Number of survivors at age 5 years	99,554	99,554

Text table 1 shows the effect of the adjustment of probabilities of death among New Brunswick females aged 1 to 4 years. The new mortality rates at ages 1 to 4 years decrease in a monotonic fashion, and the number of survivors at age 5 remains the same for the old and the new mortality rates.

It is important to note that this procedure requires that  $q_0$  through  $q_4$  already form a monotonically decreasing sequence by sex at the national level. One adjustment was needed to achieve this. For males at the Canada level, the initial estimate of  $q_2$  was slightly lower than  $q_3$ . so in the spirit of the procedure, we simply reversed  $q_2$  and  $q_3$  so that there would be no impact on the number of survivors at age 5.

Formula (7) was applied to the data for males in all provinces except Québec and Manitoba, where this adjustment was unnecessary, because the  $q_0$  through  $q_4$  already formed a monotonically decreasing sequence. The same adjustment needed to be made to the data for females in all provinces except Ontario and Alberta.



### 2.1.2 Ages 7, 12, 17, ..., 87 years

These ages are referred to as pivotal ages because they represent the middle point of the typical age groups 5 to 9 years, 10 to 14 years, 15 to 19 years, etc. The basic equation, also known as the "Actuarial Method", is

$$(8) \quad q_x = 2 m_x / (2 + m_x),$$

where  $x = 7, 12, 17, \dots, 87$ ,

and  $m_x$  is the central age-specific death rate, defined as follows:

$$(8.1) \quad m_x = (D_x / 3) / P_x,$$

where  $D_x$  and  $P_x$  are defined by King's formula:<sup>4</sup>

$$D_x = 0.216 D'_x - 0.008 (D'_{x-5} + D'_{x+5}),$$

$$P_x = 0.216 P'_x - 0.008 (P'_{x-5} + P'_{x+5}),$$

in which

$D'_x$  is the sum of deaths observed in the period 2000 to 2002 at ages  $x-2, x-1, x, x+1$  and  $x+2$ ,

$P'_x$  is the sum of the population estimates at ages  $x-2, x-1, x, x+1$  and  $x+2$ .

Although there is uncertainty about the quality of the population estimates above age 90, population counts by single year of age were used for ages up to 99 years. An upper limit of 102 years has been deemed to be acceptable according to research conducted by Bourbeau and Lebel.<sup>10</sup> Using single year values of population counts as far as possible avoided the problem encountered in the 1990 to 1992 publication, in which the last age group of the population was 90 years and older. As a result,  $D'_{92}$  and  $P'_{92}$ , as calculated, were larger than their true values, and the final effect on  $m_{87}$  and  $q_{87}$  was considered to be "unknown" in the 1990 to 1992 publication.

Following the example of Chiang,<sup>11</sup> the variance of  $q_x$  was estimated as:

$$(9) \quad \text{var}(q_x) = q_x^2 (1 - q_x) / D^*_x,$$

where  $D^*_x$  is the average number of deaths at age  $x$  over the 3-year period 2000 to 2002,

$$D^*_x = (1/3) [ (\# \text{ deaths at age } x \text{ in } 2000) + (\# \text{ deaths at age } x \text{ in } 2001) + (\# \text{ deaths at age } x \text{ in } 2002) ].$$

In place of  $D^*_x$ , we actually use  $D_x / 3$  where  $D_x$  is defined above in King's formula. This is more consistent with the "D" term used in the calculation of  $m_x$  and  $q_x$ .

### 2.1.3 Ages 92, 97, 102, 107 and 112 years

At pivotal ages 92 to 112 years, the value of  $q_x$  was obtained by extrapolation using Nagnur's "Equation (14)".<sup>12</sup>

$$(10) \quad q_x = \min \{ (4 q_{x-5} - 6 q_{x-10} + 4 q_{x-15} - q_{x-20}), 1 \},$$

and the corresponding variance estimation (ignoring covariances):

$$(10.1) \quad \text{var}(q_x) = 16 \text{var}(q_{x-5}) + 36 \text{var}(q_{x-10}) + 16 \text{var}(q_{x-15}) + \text{var}(q_{x-20}),$$

for  $x = 92, 97, 102, 107$  and  $112$ .

### 2.1.4 Intermediate ages from 13 to 106 years

Intermediate  $q$ -values were derived from the  $q$ -values at pivotal ages according to Karup-King's third-difference tangential formula subject to an upper bound of 1. This formula preserves the  $q$  values calculated so far at pivotal ages.

$$(11) \quad q_{x+1} = \min \{ (-0.064 q_{x-5} + 0.912 q_x + 0.168 q_{x+5} - 0.016 q_{x+10}), 1 \},$$

$$(12) \quad q_{x+2} = \min \{ (-0.072 q_{x-5} + 0.696 q_x + 0.424 q_{x+5} - 0.048 q_{x+10}), 1 \},$$

$$(13) \quad q_{x+3} = \min \{ (-0.048 q_{x-5} + 0.424 q_x + 0.696 q_{x+5} - 0.072 q_{x+10}), 1 \},$$

$$(14) \quad q_{x+4} = \min \{ (-0.016 q_{x-5} + 0.168 q_x + 0.912 q_{x+5} - 0.064 q_{x+10}), 1 \},$$

where  $x = 12, 17, 22, \dots, 102$ .

For example, to obtain  $q_{106}$ , use the formula for  $q_{x+3}$  with  $x = 102$ , whereby one would find that the pivotal age values  $q_{97}, q_{102}, q_{107}$  and  $q_{112}$  are required.

Variance estimation follows the strategy of the previous section, so for example, for  $q_{x+1}$ :

$$\text{var}(q_{x+1}) = (0.064)^2 \text{var}(q_{x-5}) + (0.912)^2 \text{var}(q_x) + (0.168)^2 \text{var}(q_{x+5}) + (0.016)^2 \text{var}(q_{x+10})$$

### 2.1.5 Intermediate ages from 5 to 11 years

Different formulae were used for the intermediate ages between 5 and 11 years. They are adapted from Nagnur, with a few changes in the signs.<sup>12</sup>

For ages 8 to 11, the following set of four equations was derived from Jenkins' fifth difference osculatory non-reproducing formula:

$$(15) \quad q_8 = -(217 q_2 - 3,892 q_7 - 966 q_{12} + 140 q_{17} + q_{22}) / 4,500,$$

$$(16) \quad q_9 = -(296 q_2 - 3,056 q_7 - 1,968 q_{12} + 220 q_{17} + 8 q_{22}) / 4,500,$$

$$(17) \quad q_{10} = -(279 q_2 - 2,124 q_7 - 2,862 q_{12} + 180 q_{17} + 27 q_{22}) / 4,500,$$

$$(18) \quad q_{11} = -(208 q_2 - 1,228 q_7 - 3,054 q_{12} + 40 q_{17} + 64 q_{22}) / 4,500.$$

This produces a smoother curve than Karup-King's formula which was used for the intermediate ages from 13 to 105 years.<sup>13</sup>

Finally, for ages 5 and 6:

$$(19) \quad q_5 = -0.3 q_3 + q_4 + 0.5 q_7 - 0.2 q_8,$$

$$(20) \quad q_6 = -0.2 q_3 + 0.5 q_4 + q_7 - 0.3 q_8.$$

Note that since these formulae use  $q_2, q_3$  and  $q_4$ , but some adjustments are made to these estimates in order to ensure that the series  $q_0$  through  $q_4$  decreases monotonically as was described in Section 2.1.1. Nonetheless, the values of  $q_5, q_6$  and  $q_8$  through  $q_{11}$  calculated based on the original unadjusted values of  $q_1$  through  $q_4$  were retained.

### 2.1.6 Model-based replacement estimates of national and provincial mortality rates at advanced ages

As noted in Section 2.1.2, the quality of population estimates becomes very uncertain at advanced ages. Coale and Kisker<sup>14</sup> found significant problems in the accuracy of population estimates for advanced ages in the U.S., in part due to misreporting of age. As a result, they developed a model-based alternative for

estimating the central death rates for advanced ages. This method produces a more realistic evolution of mortality rates at very old ages, since recent observations show a deceleration in the rate of increase of mortality rates. The method follows these steps:

Compute age-specific central death rates for  $x = 82, 83, \dots, 86$  years:

$$M_x = D^*_x / P_x,$$

where, as in Section 2.1.2 above,  $D^*_x$  is the average number of deaths at age  $x$  over the 3-year period 2000 to 2002. (This time, we did not use the King's formula, which produces  $D_x$ .)

Take the average of  $M_x$  around  $x = 84$ :

$$M_{avg} = (1/5) (M_{82} + M_{83} + M_{84} + M_{85} + M_{86});$$

Define two constants ( $K_{85}$  and  $S$ ) next:

$$K_{85} = (1/4) \log(M_{86} / M_{82});$$

$$S = (-1/325) [ \log(M_{avg} / M_{110}) + 26 K_{85} ].$$

Set  $M_{110} = 1$  for males and 0.8 for females.

For age values of  $x$  from 87 years onward, the new CK (Coale-Kisker) estimates for age-specific death rates are calculated as follows:

$$(21) M_{CK}(x) = M_{avg} \text{EXP}\{ (x - 84)[ K_{85} + (x - 85)(S/2) ] \}.$$

For ages (87 years)  $x$  (117 years), the new CK estimates for the probability of dying before the next birthday were calculated as in the basic equation (8):

$$(22) Q_{CK}(x) = ( 2 M_{CK}(x) ) / ( 2 + M_{CK}(x) ).$$

The variance estimation on the new CK estimates used  $Q_{CK}(x)$  in place of  $q_x$  in Formula (9).

It was then necessary to decide at what age to replace the value of  $q_x$  calculated from observed data by Equation (8) with the new model-based value of  $Q_{CK}(x)$ . We repeated the decision taken for the 1995 to 1997 tables here. For the province-level estimates, the value of  $q_x$  was kept up to the pivotal age  $x = 87$  years, and then the value of  $Q_{CK}(x)$  was used starting with  $x = 88$  years. At the national level, however, the value of  $q_x$  was kept up to  $x = 92$  years, and then replaced with the value of  $Q_{CK}(x)$  starting at  $x = 93$  years.

This raised another question, however. Recalling the computation of  $q$  values for intermediate ages in Section 2.1.4, it can be seen, for example, that the computation of  $q_{88}$  through  $q_{91}$  requires the use of  $q_x$  at the pivotal ages 77, 82, 87, 92 and 97. Now that new values of  $q_x$  at age 97 years at the national level, and at ages 92 and 97 years at the provincial level have been calculated, the decision needed to be made whether to re-compute the values of  $q_{88}$  through  $q_{91}$  with these new pivotal age  $q_x$  values. Since the smoothness of the  $q_x$  series, produced with the original  $q_{88}$  through  $q_{91}$  values combined with the new pivotal age  $q_x$  values, was satisfactory, no further adjustments were made.

## 2.2 Abridged life tables

In an abridged life table, the move from one row to the next usually involves a gap of more than a single year of age. For this reason, in this section the notation  ${}_nq_x$  is used for the probability that a person of exact age  $x$  years will die before reaching exact age  $x+n$ .

Abridged life tables were required for the province of Prince Edward Island and for all northern territories combined because their smaller populations prevented meaningful construction of complete life tables for these areas. Hence, following the procedure adopted for the 1970 to 1972 life tables, abridged life tables were constructed separately for males and females. The methodology has been explained in a technical paper by Silins and Zayachkowski.<sup>15</sup> Whereas the life tables for the 1990 to 1992 publication followed this methodology closely, some enhancements were made for the 1995 to 1997 publication, and these have been followed for the 2000 to 2002 publication.

The first step was to define 22 age groups: 0 years, 1 to 4 years, 5 to 9 years, 10 to 14 years, 15 to 19 years, 20 to 24 years, 25 to 29 years, 30 to 34 years, 35 to 39 years, 40 to 44 years, 45 to 49 years, 50 to 54 years, 55 to 59 years, 60 to 64 years, 65 to 69 years, 70 to 74 years, 75 to 79 years, 80 to 84 years, 85 to 89 years, 90 to 94 years, 95 to 99 years, and 100 years and older.

Next, the following calculations were performed at the provincial level:

$$(23) m_x = D_x / (3 P_x),$$

where

$m_x$  is the age-specific central death rate,

$x$  designates the lower bound of each age group:  $x = 0, 1, 5, 10, \dots, 85, 90, 95, 100$  years,

$D_x$  is the number of deaths in the age group starting with age  $x$  during the period 2000 to 2002, and

$P_x$  is the population count in the age group starting with age  $x$  on July 1, 2001, adjusted for net census under-coverage and including non-permanent residents (see Section 1).

$$(24) F_0 = g_2 / (g_1 + g_2),$$

where

$F_0$  is the separation factor at age 0 years, and

$g_k$  is the number of deaths at age 0 years in death group  $k$ ; group 1 contains individuals who died in the calendar year in which they were born; group 2 contains individuals who died in the calendar year following their birth year (see Appendix 1 for more details).

As explained in detail in Appendix 1, separation factors were calculated separately by sex, but not necessarily by province. Data for PEI and the northern territories were included with other provinces into "regions" for greater stability in the computation of separation factors for the abridged life tables. For PEI, separation factors were calculated with data for all four Atlantic provinces combined. For the three northern territories combined, separation factors were calculated with data for the territories in combination with all of western Canada (from Manitoba westward).

For the first row of the Abridged Table, which represents age 0 to 1:

$$(25.1) {}_1q_0 = D_0 / [B_{99-01} (F_0) + B_{00-02} (1 - F_0)],$$

where

$D_0$  is the number of deaths at age 0 years observed during the period 2000 to 2002,

$B_{99-01}$  is the total number of births observed during the period 1999 to 2001,

$B_{00-02}$  is the total number of births observed during the period 2000 to 2002.

For the second row of the Abridged Table, which represents age 1 to 4:

$$(25.2) {}_4q_1 = m_1 / \beta_1, \text{ known as Greville's method,}^3$$

where

$m_1$  is defined by Equation (23), and

$$\beta_1 = (1 / w_1) + m_1 [(1 - F_1) + (w_1/12) (m_1 - k)],$$

in which

$$w_1 = 4 \text{ (the width of the age 1 to 5 years interval),}$$

$F_1$  is the separation factor for the 1 to 4 years age group, calculated by sex and by region as explained above, and

$$k = (1/45) \ln (m_{85} / m_{40}).$$

For all of the subsequent rows of the Abridged Table, which cover 5-year age groups:

$$(25.3) \quad {}_5q_x = m_x / \beta_x,$$

where

$$x = 5, 10, \dots, 85, 90, 95,$$

$m_x$  is defined by Equation (23), and

$$\beta_x = (1 / w_x) + m_x [0.5 + (w_x/12) (m_x - k)],$$

in which

$$w_x = 5 \text{ (the width of age intervals for } x = 5 \text{ through } 95), \text{ and}$$

$$k = (1/45) \ln (m_{85} / m_{40}).$$

For all values of  ${}_nq_x$  up to this point, the following variance estimate is used:

$$(25.4) \quad \text{Var}(q_x) = {}_nq_x^2 (1 - {}_nq_x) / D_x.$$

$$(25.5) \quad q_{100} = 1$$

Note that  $\text{var}(q_{100}) = 0$ , whether because the value of  $q_{100}$  is set to a constant, or whether the variance estimation formula above is used. An unfortunate consequence is that the value of  $\text{var}(e_{100}) = 0$ .

$$(26.1) \quad l_0 = 100,000$$

$$(26.2) \quad {}_1d_0 = {}_1q_0 l_0$$

$$(27.1.1) \quad l_1 = l_0 - {}_1d_0$$

$$(27.1.2) \quad {}_4d_1 = {}_4q_1 l_1$$

$$(27.2.1) \quad l_5 = l_1 - {}_4d_1$$

$$(27.2.2) \quad {}_5d_5 = {}_5q_5 l_5$$

$$(27.3.1) \quad l_x = l_{x-5} - {}_5d_{x-5} \text{ for } x = 10, 15, \dots, 100$$

$$(27.3.2) \quad {}_5d_x = {}_5q_x l_x \text{ for } x = 10, 15, \dots, 100$$

$$(28.1) \quad {}_1L_0 = l_0 - (1 - F_0) {}_1d_0$$

$$(28.2.1) \quad {}_4L_1 = {}_4d_1 / {}_4m_1 \text{ if } {}_4m_1 \neq 0$$

$$(28.2.2) \quad {}_4L_1 = 4 l_1 \text{ if } {}_4m_1 = 0$$

$$(28.3.1) \quad {}_5L_5 = {}_5d_5 / {}_5m_5 \quad \text{if } {}_5m_5 \neq 0$$

$$(28.3.2) \quad {}_5L_5 = 5 l_5 \quad \text{if } {}_5m_5 = 0$$

$$(28.4.1) \quad {}_5L_x = 2.5 (l_x + l_{x+5}) + (5/24) ({}_5d_{x+5} - {}_5d_{x-5})$$

$$\text{if } {}_5m_x \neq 0, \text{ for } x = 10, 15, \dots, 90$$

$$(28.4.2) \quad {}_5L_x = 5 l_x \quad \text{if } {}_5m_x = 0, \text{ for } x = 10, 15, \dots, 90$$

$$(28.5.1) \quad {}_5L_{95} = {}_5d_{95} / {}_5m_{95} \quad \text{if } {}_5m_{95} \neq 0$$

$$(28.5.2) \quad {}_5L_{95} = 5 l_{95} \quad \text{if } {}_5m_{95} = 0$$

$$(28.6.1) \quad L_{100+} = d_{100+} / m_{100+} \quad \text{if } m_{100+} \neq 0$$

$$(28.6.2) \quad L_{100+} = 4 l_{100+} \quad \text{if } m_{100+} = 0$$

$$(29.1) \quad T_{100+} = L_{100+}$$

$$(29.2) \quad T_x = T_{x+5} + {}_5L_x \quad \text{for } x = 95, 90, \dots, 5 \quad (\text{values of } x \text{ descending})$$

$$(29.3) \quad T_1 = T_5 + {}_4L_1$$

$$(29.4) \quad T_0 = T_1 + {}_1L_0$$

$$(30) \quad e_x = T_x / l_x \quad \text{for } x = 0, 1, 5, 10, \dots, 100$$

For the variance estimation of  $e_x$ , the procedure illustrated in Chiang<sup>11</sup> for the variance estimation of  $q_x$  was followed, using the formula:

$$(30.1) \quad \text{var}(e_x) = \frac{1}{l_x^2} \sum_{i=x}^{N-1} l_i^2 [(1 - f_0) w_i + e_{i+1}]^2 \text{var}(q_x)$$

where,

$N$  is the total number of rows in the table, and

$w_x$  is the width of the age interval for which  $x$  is the lower bound.

$$(31) \quad {}_n p_x = 1 - {}_n q_x; \quad n = 1 \text{ for } x = 0; \quad n = 4 \text{ for } x = 1; \quad n = 5 \text{ for } x = 5, 10, \dots, 95$$

After the above calculations were performed, values were rounded according to Sirken's<sup>4</sup> method, which consists of the following steps:

- round  $l_x$  and  $T_x$  to the nearest integer, for  $x = 0, 1, 5, 10, \dots, 100$
- set  ${}_1d_0 = l_0 - l_1$
- set  ${}_4d_1 = l_1 - l_5$
- set  ${}_5d_x = l_x - l_{x+5}$  for  $x = 5, 10, \dots, 95$
- set  ${}_5d_{90} = l_{90}$
- set  ${}_1L_0 = T_0 - T_1$
- set  ${}_4L_1 = T_1 - T_5$
- set  ${}_5L_x = T_x - T_{x+5}$  for  $x = 5, 10, \dots, 95$
- set  ${}_5L_{90} = T_{90}$
- round  ${}_n p_x$ ,  ${}_n q_x$  and  $e_x$  for  $x = 0, 1, 5, 10, \dots, 100$  ( ${}_n p_x$  and  ${}_n q_x$  are rounded to the fifth decimal place;  $e_x$  is rounded to the second decimal place).

### 2.3 Infant life tables

This is the seventh time that sets of life tables for subdivisions of the first year of life have been produced for Canada. As was the case with respect to the 1990 to 1992 life tables, the method described in detail by Sirken<sup>16</sup> was employed in constructing these tables. The basic underlying assumption in the production of these tables is that a closed cohort of 100,000 live births is subject to the mortality rates of each subdivision of one year of age, but for the first year of life only.

Vital statistics data files of deaths (years 2000 to 2002) and births (years 1999 to 2002) were used to produce infant life tables. The age at death was calculated as the number of completed days or months of life. For deaths occurring within 24 hours of birth, the age was reported as the number of minutes or hours lived. For these deaths, age was coded as 0 days (i.e., less than one day lived). Other infant deaths were coded as the number of days or months lived as reported on the death data files.

The following 21 subdivisions of the first year of life were used to calculate probabilities of death in infant life tables: the first seven days, weeks 2 to 4, and months 2 to 12. In addition to those subdivisions, infant life tables include two more rows: one for the first week (summing deaths from the first seven days) and one for the first month (summing deaths from the first four weeks).

**Text Table 2 Subdivisions of the first year of life**

Subdivision Number (s)	Age Interval	Subdivision Number (s)	Age Interval
1	≥ 0 and < 1 day	12	≥ 2 and < 3 months
2	≥ 1 and < 2 days	13	≥ 3 and < 4 months
3	≥ 2 and < 3 days	14	≥ 4 and < 5 months
4	≥ 3 and < 4 days	15	≥ 5 and < 6 months
5	≥ 4 and < 5 days	16	≥ 6 and < 7 months
6	≥ 5 and < 6 days	17	≥ 7 and < 8 months
7	≥ 6 and < 7 days	18	≥ 8 and < 9 months
8	≥ 1 and < 2 weeks	19	≥ 9 and < 10 months
9	≥ 2 and < 3 weeks	20	≥ 10 and < 11 months
10	≥ 3 and < 4 weeks	21	≥ 11 and < 12 months
11	≥ 4 weeks and < 2 months		

#### 2.3.1 Probabilities of death

The probabilities of death for the 21 subdivisions of the first year of life were calculated in two steps, as follows.

##### Step 1: Calculate the number of births exposed to the risk of death

For each of the 21 subdivisions of the first year of life,  $\beta_s$ , the number of births in subdivision  $s$  that were exposed to the risk of death, was calculated from the following formulae. The formulae are attributable to Sirken and are expressed differently but equivalently by other authors.<sup>7, 11, 15</sup>

For subdivision 21 (corresponding to the age interval 11 to 12 months):

$$(32) \quad \beta_{21} = \left\{ \frac{B_{1999,1}}{2} + \sum_{m=2}^{12} B_{1999,m} \right\} + \left\{ \sum_{m=1}^{12} (B_{2000,m} + B_{2001,m}) \right\} + \left\{ \frac{B_{2002,1}}{2} \right\},$$

where  $\beta_{y,m}$  is the number of births observed in month  $m$  of year  $y$ .

For subdivisions 20, 19, ..., 11:

$$(33) \quad \beta_{21-(i-1)} = \left\{ \frac{B_{1999,i}}{2} + \sum_{m=i+1}^{12} B_{1999,m} \right\} + \left\{ \sum_{m=1}^{12} (B_{2000,m} + B_{2001,m}) \right\} + \left\{ \left( \sum_{m=1}^{i-1} B_{2002,m} \right) + \frac{B_{2002,i}}{2} \right\}$$

for  $i = 2$  to 11.

For subdivisions 10, 9, ..., 1:

$$(34) \quad \beta_{21-(i-1)} = (f_i B_{1999,12}) + \left\{ \sum_{m=1}^{12} (B_{2000,m} + B_{2001,m}) \right\} + \left\{ \left( \sum_{m=1}^{11} B_{2002,m} \right) + (1-f_i) B_{2002,12} \right\}$$

for  $i = 12$  to 21, i.e.,  $i - 1 = 11$  to 20, where  $f_i$  is the fraction given in Text Table 3.

**Text Table 3 Fractions used in the calculation of births exposed to the risk of death, for subdivisions 10 to 1**

i	12	13	14	15	16	17	18	19	20	21
Subdivision no. $s = 21 - (i-1)$	10	9	8	7	6	5	4	3	2	1
$f_i$	49/62	35/62	21/62	13/62	11/62	9/62	7/62	5/62	3/62	1/62

Fractions  $f_i$  are used for  $i = 12$  to 21. This corresponds to subdivisions 10 to 1, respectively, which are the subdivisions for the first month of life, as shown in Text Tables 2 and 3. For example,  $f_{21}$  is used in calculating the number of births exposed to the risk of death in subdivision no. 1 (which corresponds to the first day of life). In this case,  $f_{21} B_{1999,12} = (1/62) B_{1999,12}$  represents half of the births observed during the last day of 1999 (December 31, 1999):  $(0.5) (1/31) = 1/62$ . For  $i = 20$ ,  $s = 2$  and  $f_{20} B_{1999,12} = (3/62) B_{1999,12}$  represents all births observed during the last day of 1999, plus half of the births observed on the day before (December 30, 1999):  $(1/31) + (0.5) (1/31) = 3/62$ .

The assumption underlying the use of the above fractions is that births were uniformly distributed during the months of December 1999 and 2002. Similarly, the fraction 1/2 is used in Formulae (32) and (33) above under the assumption that births in the corresponding months were uniformly distributed.

**Step 2: Calculate probabilities of death**

The probability of death for subdivision  $s$  was then calculated according to the following equations:

$$(35) \quad q_1 = d_1/\beta_1$$

$$(36) \quad q_s = d_s / \left\{ \beta_s - \sum_{k=1}^{s-1} d_k \right\}$$

where,

$d_1$  is the number of deaths in subdivision 1, and

$d_s$  is the number of deaths in subdivision  $s$ , for  $s = 2$  to 21.

For variance estimation, the formula from Chiang<sup>11</sup> was used again:

$$(37) \quad \text{Var}(q_s) = q_s^2 (1 - q_s) / d_s.$$



### 2.3.2 Stationary population

The number of person-years in the stationary population was calculated as follows:

$$(38) L_s = c_s [ l_s - d_s/2 ] \quad \text{for } s = 1 \text{ to } 21,$$

where,

$c_s$  represents the duration of the age interval in years: for a period of one day ( $s = 1$  to  $7$ ),  $c_s$  is  $1/365$ ; for a period of one week ( $s = 8$  to  $10$ ),  $c_s$  is  $7/365$ ; for the second month of life ( $s = 11$ ),  $c_s$  is  $357/4,015$ ; for months 3 to 12 of life ( $s = 12$  to  $21$ ),  $c_s$  is  $335/4,015$ ;

$l_s$  is the number of persons alive at the beginning of subdivision  $s$ ; and

$d_s$  is the number of deaths in subdivision  $s$ .

For the first day of life, the number of deaths is usually multiplied by a factor greater than 0.5. The usual formula is:

$$(39) L_1 = c_1 [ l_1 - (1 - f_0) d_1 ],$$

where

$c_1 = 1/(365+1/3)$ , because there were on average  $365 + 1/3$  days in each of the years 2000, 2001 and 2002, and

$f_0$  denotes the fraction of deaths in the first day of life (i.e., deaths within 24 hours after birth) that occurred on the calendar day following birth.

The fraction  $f_0$  can be expressed as:

$$(40) f_0 = \delta_2 / (\delta_1 + \delta_2)$$

where

$\delta_1$  is the number of deaths in the first day of life that occurred on the day of birth, and

$\delta_2$  is the number of deaths in the first day of life that occurred on the calendar day following the day of birth.

On average, since death in the first day of life occurs more often within the first few hours following birth,  $\delta_1$  should be greater than  $\delta_2$ . This implies that  $f_0$  should be less than 0.5 and that  $1 - f_0$  should be greater than 0.5. The values of  $f_0$  at the Canada level were calculated to be 0.10882 for males and 0.11873 for females.

### 3. Explanation of life table column headers

This document contains three types of life tables: infant life tables (i.e. life tables for the first year of life), complete life tables, and abridged life tables. Estimates have been generated for males and females separately for all of these tables. In the case of the infant life tables, data have been produced at the Canada level only. Complete life tables have been constructed for Canada and all of the provinces, except Prince Edward Island. Abridged tables have been derived for Prince Edward Island and the combined territories (i.e. Yukon, Northwest Territories and Nunavut) only.

#### Age x column: Age interval

The major visual difference between the various life tables found in this publication lies in the age groupings for which estimates have been produced. In the case of infant life tables, age groupings are of the form  $[x, x+n)$ , where the first age,  $x$ , is included in the interval while the second age,  $x+n$ , is not. In other words, the

first age value indicates the number of *completed* lifetime units (in days or months). For example, the interval “0-1 day” refers to deaths that occur in the 24-hour interval starting at a live birth (stillbirths are excluded) and ending at the end of day 1 (i.e. 23 hours and 59 minutes after birth, which is reported as 23 completed hours). The interval “1 to 2 days” represents the interval spanning the beginning of the second day to the end of the second day of life; that is, it comprises the deaths of infants who completed the first day of life but died before completing the second day.

In the case of *complete life tables*, there is only one age value per line, which indicates the number of completed years lived. The intervals in this table represent the interval between two exact ages. In other words, age  $x$  represents an interval of one year, starting at the beginning of the day an individual reaches exact age  $x$  and ending at the end of the day prior to that individual's next  $(x+1)$  birthday. For example, death at age 30 years means that the death occurred on or after the 30<sup>th</sup> birthday but before the 31<sup>st</sup> birthday.

In *abridged life tables*, age intervals are of the form  $[x, x+n-1]$ ; that is, both ages  $x$  and  $x+n-1$  are included in the interval. For example, age interval  $[40, 44]$  comprises deaths occurring among 40 to 44 year-olds (i.e., the interval starts at age 40 and ends just before reaching the 45<sup>th</sup> birthday). Most age intervals in abridged life tables span five years. The exceptions occur in the first two rows of these tables and the last row: the first row (age 0) represents a one-year interval and the second row, a four-year interval (ages 1 to 4); the last row, however, is an open age interval, which comprises all deaths occurring at ages 100 and older.

**$l_x$  column: Number of survivors to exact age  $x$**

This column represents an estimate of the number of persons in an initial cohort of 100,000 live births who are still alive at the beginning of each subsequent age interval (i.e. at the attainment of exact age  $x$ ). The expected number of survivors, which slowly drops as the cohort ages, is assumed to depend on the age-sex-specific mortality rates experienced in 2000 to 2002. Progressive values of  $l_x$  are derived by the successive application of  ${}_nq_x$  mortality rates to the remainder of the original cohort of 100,000 live births still alive at the beginning of each interval.

One very useful application of this column is that it can be used to estimate probabilities of surviving to age  $y$  given that the person has so far lived to age  $x$ , where  $y > x$ . From page 156 of Chiang<sup>11</sup>, we have:

$$\text{Pr}(\text{survive to age } y \mid \text{lived to age } x) = l_y/l_x$$

For example, from the Canada table for females, from a cohort of 100,000 live births, we expect 99,411 of them to still be alive at age 10, and 99,197 of them to still be alive at age 20. We can therefore estimate the probability that a 10-year-old girl will live to see her 20<sup>th</sup> birthday as:

$$\text{Pr}(\text{survive to age } 20 \mid \text{lived to age } 10) = l_{20}/l_{10} = 99,197/99,411 = 0.9978$$

**${}_nd_x$  column: Number of persons dying between exact age  $x$  and age  $x+n$**

This column shows the number of persons dying in each successive age interval who were alive at the beginning of the interval. These values are obtained by first multiplying  $l_x$  by the corresponding value of  ${}_nq_x$  (i.e.,  ${}_nd_x = l_x \cdot {}_nq_x$ ). Then, after calculating all life table values using the full decimal precision provided by SAS,  $l_x$  values are rounded to the nearest integer, and  ${}_nd_x$  values are set equal to the difference between consecutive rounded values of  $l_x$  (i.e. Sirken's rounding method<sup>3</sup> is used, where  ${}_nd_x$  is ultimately assigned the value of  $l_x - l_{x+n}$ ).

**${}_np_x$  column: Life table survival rate (or proportion surviving to age  $x+n$ )**

This column represents the probability that a person exact age  $x$  will survive to reach exact age  $x+n$ , that is, the proportion of cohort survivors at the beginning of an age interval who are expected to survive to the beginning of the next age interval. The value of  ${}_np_x$  “the proportion surviving” is the complement from 1 of  ${}_nd_x$ , “the proportion dying” (i.e.,  ${}_np_x = 1 - {}_nd_x$ ).

**${}_nq_x$  column: Life table mortality rate (or proportion dying between exact age x and age x+n)**

This column represents the probability that a person of exact age x will die before reaching age x+n, that is, the proportion of cohort survivors at the beginning of an age interval who are expected to die before reaching the beginning of the next age interval. *This is the most important column of the life table, because it is the basis for the entire table structure.* Specifically, this is the initial column in the generation of a life table (i.e. derived from the observed data), and from which other columns are derived, on the basis of interdependent relationships.

 **$cv({}_nq_x)$  column: The coefficient of variation of the variable  ${}_nq_x$** 

The coefficient of variation (cv) associated with an estimated value for a variable (such as q) is a measure used to compare the variability of that estimate. This measure is derived from the variance estimate for the value of the variable via an intermediate step. (Variance estimates for q are described in Section 2.) Once the variance for q has been calculated, the estimated standard error (se) of q is calculated simply by taking its square root:

$$se(q) = \sqrt{\text{var}(q)}$$

Thus se is a measure of variation in the same scale as q.

The following step transforms se into a coefficient of variation, which is a relative measure of variation. The formula below shows that a given se measurement will have a greater impact on a smaller q estimate than it will on a larger q value.

$$cv(q) = se(q) / q$$

The coefficients of variation in the life tables are expressed in percentages. For example, in Table 2a,  $q_7$  for females at the Canada level has a value of 0.000071, with an accompanying cv of 26.6%. This means that the estimated standard error of the  $q_7$  estimate is the product of the cv and  $q_7$ , or  $se(q_7) = cv(q_7) * q_7 = 0.266(0.000071) = 0.000019$ . Then, using a normal distribution approximation to the underlying distribution for estimates for  $q_7$ , there is:

- a 68% probability that the interval [ 0.000052, 0.000090 ] covers the true  $q_7$  value, where this interval is built from one standard error above and below the  $q_7$  estimate
- a 95% probability that the interval [ 0.000033, 0.000109 ] covers the true  $q_7$  value, where this interval is built from two standard errors above and below the  $q_7$  estimate
- a 99% probability that the interval [ 0.000023, 0.000119 ] covers the true  $q_7$  value, where this interval is built from 2.5 standard errors above and below the  $q_7$  estimate

Estimates with a coefficient of variation exceeding 33.3% are to be used with great caution, since they are highly variable. Although q estimates with coefficients of variation equal to or greater than 100.0% are published here, the coefficients of variation themselves have been suppressed.

 **${}_nL_x$  column: Stationary population (number of life years lived in the age interval)**

${}_nL_x$  indicates the number of life years lived by persons in the stationary population in the age interval [x, x+n). If one assumes that a) a cohort of 100,000 persons is being born every year for an indefinite period of time, b) the proportion dying in each age interval is fixed (as determined by the values of  ${}_nq_x$ ), c) deaths are evenly distributed over time within age intervals; d) there is no migration, and e) the births are evenly distributed over the calendar year, then the survivors of these successive cohorts constitute a "stationary population." The term "stationary" is used because the number of persons living in any given age group does not change over time, and the number entering any given age group equals the number leaving the group due to death or aging (i.e. passage from one age group to the next). The number of deaths each year equals the number of births, which equals 100,000. In other words, the assumptions involved imply that column  ${}_nL_x$  remains fixed from one year to the next and thus stationary.

The derivation of the values of  ${}_nL_x$  does vary at young ages, however, due to the unequal distribution of deaths throughout the youngest age intervals (i.e. there is a higher risk of death during the earlier part of these intervals and a progressively reduced risk in the later part). To compensate for this problem, at ages 0 to 4 years, the following formulae are used in the calculations of the complete life tables:

$$(41) L_x = l_x - (1 - F_x) d_x$$

for  $x = 0, 1,$

$$(42) L_x = l_x - (1 - F_x) d_x - (d_{x-1} - d_{x+1}) / 24$$

for  $x = 2$  to 4,

where  $F_x$  represents the separation factor at age  $x$ , that is, the proportion of individuals dying in age interval  $[x, x+1)$  who have lived in excess of half the interval. (See Appendix 1 for actual  $F_x$  values used.)

At ages five years and older, it is considered sufficiently accurate to use the following approximate formula (equivalent to Equation (41) with  $F_x = 0.5$ ):

$$(43) L_x = l_x - 0.5 d_x$$

for  $x = 5$  to the maximum age in the life table.

**$T_x$  column: Cumulative stationary population (total number of life years lived beyond age  $x$ )**

$T_x$  shows the total number of years lived by persons in the stationary population in the indicated age interval and all subsequent age intervals.

$$(44) T_x = \sum_{k=x}^{\omega} L_k$$

for  $x = 0$  to  $\omega$

where  $\omega$  is the maximum age in the life table (note: at age  $\omega$ ,  $T\omega = L\omega$ ).

**$e_x$  column: Life expectancy at age  $x$  (average remaining lifetime)**

Life expectancy at age  $x$  represents the average number of years remaining to be lived by persons surviving to exact age  $x$ , based on a given set of age-sex-specific mortality rates (such as the 2000 to 2002 mortality rates) from that age onwards.

Life expectancy at age  $x$  is calculated by dividing the  $T_x$  value (the total number of person-years lived at that age and subsequent ages) by the corresponding  $l_x$  value (the number of survivors at that age):

$$(45) e_x = T_x / l_x$$

Note: Following the recommendation in Chiang<sup>11</sup> to simplify the notation, the symbol  $e_x$  is used instead  $e_x^\circ$  denote full life expectancy at age  $x$ .

For example, in the 2000 to 2002 complete life tables for Canada (Tables 2a and 2b), the average number of years of life remaining for 60-year-old Canadian males is 20.84 years, for an average age at death of 80.84 years. The corresponding estimate for 60-year-old Canadian females is 24.72 remaining years of life (on average), with death occurring at 84.72 years (on average).

**$cv(e_x)$  column: The coefficient of variation of the variable  $e_x$**

The variance estimation of  $e$  for the complete life tables mirrors that of the abridged tables [see Section 2, Equation (30.1)], with the following two modifications: a) in each row,  $w_x = 1$ ; b) the number of rows ( $N$ ) equals 110 (for ages 0 through 109 years inclusively). Once  $var(e_x)$  is obtained,  $cv(e_x)$  is calculated in the same fashion as  $cv(q_x)$ .

For an explanation on how to interpret the  $cv(e_x)$  column, see the description for the  $cv({}_nq_x)$  column: the coefficient of variation of the variable  ${}_nq_x$ .

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## 6. Appendices

### Appendix 1.

#### Calculation of separation factors

The separation factor  $F_x$  represents the proportion of individuals dying in age interval  $[x, x+1)$  who have lived in excess of half of the interval. The value is calculated as follows:

Consider all individuals who died in age interval  $[x, x+1)$  and divide them into two groups.

Group 1 consists of those individuals who died in a given year on or after their birthday. For these individuals, birth year plus age equals death year. For example, someone born in April 1940 who died at age 60 in June 2000:  $1940 + 60 = 2000$ .

Group 2 consists of those individuals who died in a given year before their birthday. For these individuals, birth year plus age equals death year minus 1. For example, someone born in April 1940 who died at age 60 in January 2001:  $1940 + 60 = 2001 - 1$ .

Then,  $F_x = g_{2,x} / (g_{1,x} + g_{2,x})$ , where  $g_{k,x}$  is the number of deaths at age  $x$  in death group  $k$  during the 2000 to 2002 period.

Text tables A1 and A2 present a list of the separation factors at ages 0 to 4 years used in the calculation of the 2000 to 2002 life tables. Complete life tables use separation factors for each age from 0 to 4 years, while abridged life tables use one separation factor at age 0 and another one for age 1 to 4 years. Furthermore, as explained in Section 2.2, regional separation factors were calculated for the 2000 to 2002 tables, as was done for the 1995 to 1997 tables.

**Text table A1 Separation factors by region, sex and age - Complete life tables**

Region	Males					Females				
	Age (years)					Age (years)				
	0	1	2	3	4	0	1	2	3	4
Atlantic (N.L., P.E.I., N.B., N.S.)	0.08621	0.50000	0.16667	0.57143	0.28571	0.14286	0.33333	0.50000	0.50000	0.66667
Quebec	0.09076	0.66667	0.48276	0.48148	0.50000	0.10000	0.39394	0.61111	0.56250	0.68421
Ontario	0.10243	0.46269	0.39474	0.47619	0.39130	0.10819	0.50000	0.45455	0.52778	0.37037
Western (Man., Sask., Alta., B.C.) and Northern Territories	0.13006	0.42254	0.51282	0.52941	0.53659	0.13889	0.42857	0.34286	0.58333	0.31250
Canada	0.10882	0.48649	0.44643	0.50427	0.45614	0.11873	0.44068	0.44762	0.55000	0.47059

**Text table A2 Separation factors by region, sex and age - Abridged life tables**

Region	Males		Females	
	Age (years)		Age (years)	
	0	1 - 4	0	1 - 4
Atlantic (N.L., P.E.I., N.B., N.S.)	0.08621	0.43902	0.14286	0.48148
Quebec <sup>1</sup>	0.09076	0.54128	0.10000	0.53488
Ontario <sup>1</sup>	0.10243	0.43523	0.10819	0.47273
Western (Man., Sask., Alta., B.C.) and Northern Territories	0.13006	0.48648	0.13889	0.42105

1. Separation factors for Quebec and Ontario are provided for information purposes only; no abridged life tables are published for these two provinces.

## Appendix 2.

### Calculation of life table mortality rates at ages 0 to 4 years in the complete life tables

From section 2.1.1, recall that  $q_x$  was calculated as

$$(6) \quad q_x = 1 - \left( \frac{P_x'}{E_x} \right) \left( \frac{E_{x+1}}{P_x''} \right),$$

for  $x = 0$  to 4 years.

In order to explain this formula, define the following terms:

$E_x^z$  is the number of persons attaining age  $x$  in calendar year  $z$ ,

$P_x^z$  is the number of persons living at the beginning of year  $z$  who are age  $x$  years,

$D_x^z$  is the number of persons dying in year  $z$  at age  $x$ ,

${}_aD_x^z$  is the number of persons dying in year  $z$  at age  $x$  who attained age  $x$  in year  $z$  (their year of birth is  $z - x$ ); this corresponds to death group 1 mentioned in Appendix 1,

${}_sD_x^z$  is the number of persons dying in year  $z$  at age  $x$  who attained age  $x$  in year  $z - 1$  (their year of birth is  $z - x - 1$ ); this corresponds to death group 2 mentioned in Appendix 1.

Three important relationships follow from the previous definitions:

$$(46) \quad D_x^z = {}_aD_x^z + {}_sD_x^z$$

$$(47) \quad E_x^z = P_x^{z+1} + {}_aD_x^z$$

$$(48) \quad E_{x+1}^z = P_x^z - {}_sD_x^z$$

The population counts  $P_x'$  and  $P_x''$  are known; they are the January 1 population estimates by age and sex from Demography Division, Statistics Canada. The population counts of those who lived to exact age  $x$  in the period 2000 to 2002,  $E_x$  and  $E_{x+1}$ , were then calculated as follows. For example, as of January 1, 2000, there was an estimated population of 164,953 females in Canada age 0 years (i.e. they were born in 1999 and they survived past the end of December 31, 1999). This count is derived from the 1999 count of live births, from which is subtracted the deaths and emigrations and to which is added the immigrations for this birth cohort. Thus  $P_0^{2000} = 164,953$ .

Now suppose that there were 22 females in Canada who were born in 1999 but who died in 2000 before they reached their first birthday, that is  ${}_sD_0^{2000} = 22$ , we can use Equation (48) to get:

$$\begin{aligned} E_1^{2000} &= P_0^{2000} - {}_sD_0^{2000} \\ &= 164,953 - 22 \\ &= 164,931 \end{aligned}$$

as the estimated number of females who attained age 1 in 2000.

This procedure is continued until all needed  $E_x^z$  values are obtained.

Next, from the previous definitions, the life table mortality rates based on a one-year period would be calculated as  $q_x = 1 - (P_x^{z+1} / E_x^z) (E_{x+1}^z / P_x^z)$ . But since mortality rates from a three-year period (2000 to 2002) are desired, the following are first defined:

$E_x$  the number of persons who attained age  $x$  during the period 2000 to 2002,

$E_{x+1}$  the number of persons who attained age  $x+1$  during the period 2000 to 2002,



$P'_x$  the number of persons who attained age  $x$  during the period 2000 to 2002 and who were alive at the end of the year in which exact age  $x$  was attained,

$P''_x$  the number of persons alive at the end of the calendar year in which age  $x$  was attained and whose  $(x+1)^{\text{th}}$  birthday falls in the period 2000 to 2002.

It then follows that

$$(49) E_x = \sum_{z=2000}^{2002} E_x^z = \sum_{z=2000}^{2002} (P_x^{z+1} + {}_{\alpha}D_x^z)$$

$$(50) E_{x+1} = \sum_{z=2000}^{2002} E_{x+1}^z = \sum_{z=2000}^{2002} (P_x^z - {}_{\delta}D_x^z)$$

$$(51) P'_x = \sum_{z=2000}^{2002} P_x^{z+1}$$

$$(52) P''_x = \sum_{z=2000}^{2002} P_x^z$$

In the above equations,  $E_x^z$  is defined for  $x = 0$  to 4 and  $z = 2000$  to 2002. Variance estimation followed that used for other age groups:

$$\text{var}(q_x) = q_x^2 (1 - q_x) / ({}_{\alpha}D_x^z + {}_{\delta}D_x^z)$$

But note that there was no averaging of death counts in the denominator over the 3-year period. Whereas in Section 2.1.2 a 3-year average of death counts was used to replace a single year death count in the denominator of the central death rate  $m_x$ , in this estimation the 3-year totals are kept, and no averages are used to replace a single year's count.