RESEARCH REPORT

IMPACT OF THE HOMEBUYERS' PLAN ON HOUSING DEMAND

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Frédéric Chartrand and Mario Fortin
Economics Department
Université de Sherbrooke

June 13, 2003

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Frédéric Chartrand and Mario Fortin*
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Summary

We are studying the effect of the Home Buyers' Plan (HBP) on housing demand. The study is theoretical and is based on a continuous-time life-cycle model in which the household draws satisfaction from the use of a composite asset and from housing services. The analysis is carried out in stages. First, we integrate the Registered Retirement Savings Plans (RRSP) into the life-cycle model. One interesting result is that, based on the hypothesis that household income is taxed at a constant rate throughout life, the rate applicable to accumulation of savings becomes the net real interest rate when there are unused RRSP contributions. We then demonstrate that RRSP withdrawals permitted under the HBP result in a gain in wealth proportionate to the taxation rate; this gain remains stable for households that must borrow to contribute to an RRSP but that do not repay the RRSP. The impact of the gain in wealth is distributed equally among the use of the composite asset, the use of the home and terminal wealth. The effect on housing demand is therefore always positive. Furthermore, the household will reduce its initial debt.

Key Words: Housing demand, life-cycle

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1 INTRODUCTION

The purchase of a home is a very important expenditure to which owner-occupants allocated an average 19.6% of their budget in 1997 (Statistics Canada, 2000). Its importance is such that housing demand has gained a specific place in the theory of life cycle consumption (Artle and Varaiya, 1978). However, due to the various constraints encountered and the many elements of the cost of residential property, the decision to purchase a home has proven complex to model. The household must consider lifetime wealth, the price of renting a home instead of becoming owner-occupant, its ability to assume the common housing expenditures, the implicit cost of home equity and borrowing constraints. The latter in effect limits the value of the mortgage loan based on equity and the household's repayment capacity.

Because of the social and economic importance of access to homeownership, governments are intervening in the sector in various ways. The decision to buy a home is thus also influenced by the tax benefits offered. As theoretical literature has come mostly from the United States, there are few articles that examine the specifics of the Canadian tax system. The necessary adjustments first relate to the deductibility of mortgage interest and property taxes from taxable income. In the United States, these two expenses are deductible, except for households that claim the standard deduction, while they are not deductible in Canada. However, capital gain from a principal residence is never taxable in Canada, while it may be in the United States. These Canadian particularities were integrated into the housing demand theory by Fortin (1988). However, there are other differences, two of which we will examine in this document. First, the impact of Registered Retirement Savings Plans (RRSP) has never been integrated into the life-cycle theory. *A fortiori*, its impact on housing demand has not been studied either. Furthermore, the effect of the Home Buyers' Plan (HBP) has not been established either.

The Home Buyers' Plan is undoubtedly the most innovative element adopted by the Canadian government in the last ten years to change the terms of home financing. The purpose of the HBP is to make housing more affordable and facilitate access to the amounts that a household must amass to cover their initial down payment. In short, the HBP allows up to $20,000 to be withdrawn from RRSPs without incurring penalties if that amount is used to cover the down payment for the home. Such withdrawals must be repaid through a series of annual payments of at least

1/15 of the amount withdrawn over a maximum period of 15 years. Should households fail to make the required repayment, they must add the amount in default to their taxable income.

This tax measure is very popular. Between its introduction in February 1992 and 1998, the HBP has enabled over 777,000 individuals to withdraw close to $7.5 billion from their RRSPs to finance the purchase of their first home (Manouchehri 1999). However, no one has yet developed a firm argumentation or an adequate modeling of the advantages and drawbacks of the HBP. This issue is all the more important in that the behaviour of the participants in the plan is quite varied. In fact, despite the growing popularity of the HBP, it can be seen that a significant proportion of participants do not make the repayment required into their RRSPs according to the terms of the HBP. The Home Buyers' Plan therefore presents some excellent questions: Can the program have financial drawbacks for some households? Does the creation of the HBP permit increased consumption of residential capital? Does the possibility of early, tax-free withdrawal of funds already contributed represent a benefit that is likely to attract more fortunate households that are not subject to strict borrowing constraints? What households benefit most from the program?

The purpose of this document is to analyze the impact of the HBP on housing demand. The analysis is theoretical and makes use of the housing demand model based on the continuous-time life-cycle analysis. This model was applied to housing demand by Artle and Varaiya (1978) and has since been used many times, particularly by Wheaton (1985) in the United States and Fortin (1988) in Canada. In short, the study seeks to verify how the HBP influences housing demand. The work is structured as follows: In the next section, we identify the context and problem of RRSPs and the HBP and then examine the key factors at play in the decision to purchase a home. In section three, we examine the wealth gain resulting from the use of the HBP. In the fourth section, we develop a continuous-time life-cycle model in order to see the effect of the HBP on certain model variables. We present conclusions in the final section.

2 CONTEXT AND PROBLEM

2.1 The Home Buyers' Plan

The purchase of a home is a major investment, particularly for young families that often have difficulty in obtaining the necessary financing to cover the down payment. The HBP is designed to reduce this obstacle to purchasing. Generally, the HBP permits a household that has never owned a home to withdraw up to $20,000 from RRSPs for the purchase or construction of an eligible home without having to include the HBP-eligible withdrawals in taxable income.[1] Repayment of the amounts withdrawn from RRSPs may be spread over a maximum of 15 years, with a minimum annual repayment of 1/15 of the amount initially withdrawn. If the annual repayment is lower than the minimum, the difference is added to taxable income for the year in which the repayment was due. Since 1999, eligibility criteria have been broadened to include former homeowners, on condition that they have not owned a home for five years.

This program is very popular. In 1998 alone, the federal government's Home Buyers' Plan helped more than 110,000 individuals to become owner-occupant. In all, these individuals withdrew more than $1.1 billion from their RRSPs to purchase homes (Manouchehri 1999, 10), which, as indicated in Table 1.1, is similar to use observed in previous years.

Table 1.1
HBP Participation[2]

Period Covered	Participants	Withdrawals in $M
26/02/1992 to 1/03/1993	159,000	1,536
2/03/1993 to 1/03/1994	102,000	1,011
2/03/1994 to 31/12/1994	56,000	455
1/01/1995 to 31/12/1995	79,000	718
1/01/1996 to 31/12/1996	119,000	1,136
1/01/1997 to 31/12/1997	132,000	1,306

However, the HBP is not without its problems. In 1995, one third of the 230,000 taxpayers required to make repayments did not do so. As the unpaid amounts are treated as taxable RRSP withdrawals, 76,000 taxpayers had to pay taxes on those amounts (Frenken 1998). We will see in a later section that, in fact, not making the repayments is not very costly. Furthermore, as indicated in Table 1.2, the provincial distribution of HBP participants represents very closely the population shares of the provinces. Thus, the average withdrawal varies greatly from one province to another. The average withdrawal varied from $6,824 in Nova Scotia to $11,518 in Quebec. At first glance, the amount of the

[1] For the various eligibility criteria, see *Home Buyers' Plan (HBP) – For 1998 Participants* (1988).

[2] Source: Frenken (1998)"The RRSP Home Buyers' Plan," *Perspectives on Labour and Income* 10(2), Statistics Canada, p. 41

average withdrawal does not seem to be tied to the per-capita income or average home prices.

Table 1.2
Provincial Distribution of HBP Participation in 1998[3]

Province	No. of Participants	Withdrawal in $
Newfoundland	886	7,344
Prince Edward Island	236	7,501
Nova Scotia	1,929	7,228
New Brunswick	1,227	6,824
Quebec	33,658	11,511
Ontario	45,550	9,533
Manitoba	2,640	6,983
Saskatchewan	1,975	6,585
Alberta	11,063	7,808
British Columbia	11,699	9,344
NWT	117	10,572
Yukon	113	8,642
Total	111,063	9,376

2.2 Issue of the HBP and RRSPs relative to Housing Demand

Before a theoretical study of the HBP based on life-cycle, is it essential that the context of RRSPs be established as relates to housing demand. In Canada, Registered Pension Plans (RPP) and RRSPs were created to meet certain social and economic objectives.

> The main social goal is to provide an adequate level of income for people upon retirement (and also to reduce government expenditure for this purpose). In addition, the RRSP scheme allows individuals to average their income over their lifetimes. The economic objective is to encourage savings, and thus to increase the supply of funds available for investment (Boadway and Kitchen 1999, 120).

As RRSPs allow the contributor to defer taxes on contributions and gains until they withdraw from the plan, there are many contributors, particularly among the more fortunate. For 1992, RRSP contributors with

[3] Source: Manouchehri (1999), p. 11

income of $40,000 or more, although representing only 26% of taxpayers, accounted for 63.3% of contributions. Those earning less than $25,000, 45.4% of taxpayers, in turn only accounted for 11.5% of contributions (Ascah 1996).

A first home is usually purchased at an age when households do not have much equity. The low level of wealth in young households is often the main obstacle to purchasing and the HBP represents a way of reducing this difficulty. However, as low-income households do not contribute much to RRSPS, this obviously limits the amounts that they may withdraw from their RRSPs to finance the purchase of a first home. Furthermore, since households show a desire to be homeowners fairly early in life, since the income of young households is lower and since they have not been able to contribute for as long as older households, the amount that they can withdraw from RRSPs as part of the HBP is obviously limited. One strategy that can be used to bypass this problem is to take out a loan at a financial institution for the amount to be contributed to an RRSP and to then withdraw the funds. We will examine how this strategy is used in the rational decision-making of households.

However, one of the difficulties in studying the HBP is that RRSPs have yet to be integrated into the life-cycle theory and, *a fortiori*, in that of housing demand. This lack of theoretical basis does not stop personal finance specialists from arguing for or against the use of the HBP and RRSPs in home purchases. Before developing our argument on this matter, we will in the next section see the key factors in deciding to become an owner-occupant. This will allow us to identify what elements are changed by the RRSP and HBP.

2.3 Key Factors in the Decision to Purchase a Home

Many factors play in the decision to purchase a home. The first is, of course, the service costs for an owner-occupied home as opposed to the cost of residential rental because, if buying is cheaper than renting, this encourages people to become homeowners. That said, residential property does not present a large cost benefit over rental. In effect, as demonstrated by Fortin (1991), many elements influence the relative cost of both types of occupancy: the real interest rate, inflation rates, the amount of the mortgage debt compared to financing with equity and various tax considerations (capital cost deduction, capital gains taxation rates, and investment income taxation rates). We will not examine all of these elements in depth, but will instead lend our attention to those elements that are modified by the HBP and RRSPs.

The equity invested in a home involves an implicit cost equal to the net-of-tax return that those funds would have created had they been invested in the most competitive investment. A household with unused RRSP contributions can receive an investment benefit for which taxation is deferred until withdrawal from the plan. The benefit relinquished by the household is then higher than that sacrificed by the household that used all contributions because, in the latter case, investment income is taxable annually. The service costs for owner-occupied homes are therefore higher when a household can contribute to an RRSP.

Total income and its temporal distribution both play a role in the decision to purchase. In effect, total income, i.e., the current value of lifetime household income, must be sufficient to meet the overall financial obligations of the residential property. The distribution of income over time, on the other hand, has an effect because households face two borrowing constraints when purchasing a home. The wealth constraint exists because financial institutions cannot grant an ordinary mortgage loan that exceeds 75% of the value of the home or a loan insured under the *National Housing Act* that exceeds 95% of the value of the home. In concrete terms, this means that a household cannot purchase a home worth more than 20 times the equity at their disposal. Thus, before even purchasing a home, the household must be able to save from their own everyday income the equity required by the mortgage lender. In addition to the wealth constraint, the loan is also subject to an income restraint, as the amount of the loan must not be such that the borrower allocates more than a certain percentage of everyday income to housing costs. This second constraint effectively limits the maximum amount of the loan, based on a formula that uses the nominal interest rate on the mortgage and the duration of the amortization plan selected. Thus, a household with income that is concentrated late in life may purchase a home later than a household with an identical overall income that is earned more quickly.

Artle and Varaiya (1978) were the first to model the decision to purchase a major non-liquid asset, in this case a residential home, using a life-cycle model. They demonstrate that the household's temporal consumption profile is distorted when borrowing constraints are binding because there is forced saving prior to purchase. Inflation compounds these effects. Kearl (1979) was one of the first to study the impact of distortion of mortgage payments due to inflation. Without developing a formal model, he nonetheless notes that inflation, through higher nominal interest rates, increases mortgage payments at the beginning of home ownership. The gradual increase in prices and nominal income, however, then progressively reduces the real value of the payments. In some cases, inflation can result in households delaying the purchase of a home, or completely abandoning the idea. In addition, although the explicit modeling of the consumer problem is faulty, Kearl concludes that increased inflation reduces the value of the home to be bought by the household when liquidity is limited. Wheaton (1985) was the first to develop a formal model for studying the effect of inflation. As he notes, inflation causes three types of distortion that can influence the decision to become a homeowner. "Inflation raises mortgage payments through higher interest rates, [it] causes such payments to fall rapidly over time in real terms, and it creates a real growth in housing equity, as borrowed debt is leverage against the inflating value of homes" (Wheaton 1985, 161).

Two opposing effects of inflation are involved. When borrowing constraints related to household income are binding, the amount that can be borrowed decreases as anticipated inflation increases. However, this acceleration of inflation also reduces the net real rate of return on a household's investments if tax is paid on investment income, reducing the home service costs.[4] Thus, the property has greater fiscal benefit when the monthly payments are most difficult. Fortin (1989) uses simulations to demonstrate that the impact of amortization distortions is negligible when inflation is low, but becomes more significant when inflation rises. Thus, an increase in low inflation rates favours increased housing demand, while an increase tends to have an opposite effect when rates are already high. It should be noted, however, that if return on equity is not taxed each year, as is the case with RRSPs, inflation then has no impact on the net return on equity. The only remaining effect of inflation is that of distorting amortization. Inflation, then, cannot have any positive impact on owner-occupied housing demand for taxpayers with unused RRSP contributions.

Before presenting the housing demand model, we will, in the next section, examine the effect of the HBP on wealth.

3 IMPACT OF THE HBP ON WEALTH

3.1 Hypotheses and Definition of Wealth

To clearly identify the potential monetary benefit of the HBP, we will avoid discussing the presence of liquidity constraints to see how the changes to initial distribution of assets and debts made possible by the HBP result in financial gains. Most obvious is that, because the HBP

[4] The reasoning put forth in this paragraph is based on the hypothesis that real interest rates before taxes remain unchanged while inflation varies.

consists of reducing the mortgage debt by withdrawing from an RRSP, the financial benefit of the withdrawal can be changed at will by postulating a difference between the mortgage and RRSP interest rates. In order to disregard this, our analysis presupposes a nominal interest rate i that is identical for the mortgage and the RRSP. In practice, due to the banking margin, a situation in which the return on the RRSP is higher than the mortgage interest rate is probably not likely if we only consider investments with a risk level comparable to that of the mortgage. Our hypothesis thus consists of ignoring this margin and thus giving an edge to RRSP investments compared to the actual situation.

Furthermore, an advantage can also be given to or taken away from the RRSP by presupposing a taxation rate at the time of the contributions that differs from that applied at the time of withdrawal more specifically, by presupposing a lower (higher) rate at the time of withdrawal than at the time of contribution, a benefit (disadvantage) as relates to the RRSP investment. It is very difficult to justify one hypothesis over the other. For example, we can assume that withdrawals will be made at a time when the taxpayer is not working. As the tax system is progressive, we would be justified in believing that the withdrawals would be taxed at a lower rate. However, old age security benefits are subject to a recovery provision. If the RRSP withdrawals result in the household attaining a sufficient level of income for this provision to apply, the effective taxation rate for the withdrawals will be considerably higher. We will disregard these situations and assume a tax rate that is constant throughout life.

Finally, the household's situation when the contributions are made can vary, as the household may or may not need to borrow to contribute. As we are disregarding the effect of the bank margin, we will assume that the household can borrow to contribute at a minimal gross nominal interest rate of i, which is also the interest rate for the mortgage and the return rate for the RRSPs. We will study two situations, one in which the household borrows to make RRSP contributions and one in which the contributions come instead from liquid assets. We will also distinguish between a situation in which the household makes the required repayments and that in which it does not repay the RRSP and instead pays the tax penalty.

3.2 Impact of the HBP on Wealth

Suppose that a household has sufficient initial wealth to contribute to an RRSP without borrowing. Their wealth can be invested in three ways: in a liquid financial asset with the real value represented by a, in an active RRSP with the real value represented by w, or finally, in a home. The housing stock is represented by h with the real unit price represented by P. The real value of the owned home is therefore Ph, from which we subtract the real value of the mortgage debt, D, to obtain the net value of the home: $\alpha = Ph - D$. We suppose that the household's income is subject to a constant marginal tax rate of m. Having contributed w to an RRSP, the household benefits from a tax reduction of mw, such that it has only cost the household $(1 - m)w$ in liquid assets to obtain an RRSP worth w. Conversely, as RRSP withdrawals are taxable, withdrawing the full RRSP will only increase liquid assets by $(1 - m)w$. Thus, the household's initial net real wealth, R, is represented by equation (1).

$$R = a + (Ph - D) + (1-m)w \tag{1}$$

Equation (1) defines the household's wealth if the HBP is not used. We assume that the HBP permits an amount, $k\theta$ to be withdrawn from the participant's RRSP to be applied to the net value of the home, on condition that the withdrawn amount to recontributed, without a tax deduction, in k equal instalments worth θ each. If we use R^* to designate the initial wealth of a household withdrawing under the HBP, this is equal to the right side of equation (2).

$$R^* = a + (Ph - D + k\theta) + (1-m)(w - k\theta) \tag{2}$$

The effect of the HBP on wealth is calculated by taking the difference between R^* and R. However, this difference must reflect the negative impact on the potential accumulation resulting from the obligation to either recontribute to the RRSP without benefiting from the tax deductions or to pay taxes on the default repayment. To this end, we compare (1) and (2) after the RRSP has been fully repaid or the tax penalties have been fully paid. In our reasoning, we assume that the gross nominal interest rate on the liquid assets, the active RRSP and the debt is i. As indicated previously, we thus eliminate the possibility of arbitrarily creating an advantage for one situation by assuming that one asset has a higher return rate. As interest income from investments other than RRSPs is taxable, the net return on liquid assets is only $(1-m)i$. With an inflation rate of π, the real net return rate is $n = (1-m)i - \pi$ The real return rate on the RRSP and mortgage debt is $r = i - \pi$. It should be noted that the difference between the net interest rate for the RRSP and that of liquid assets is $r\text{-}n = mi$, thus proportional to the proceeds of the marginal taxation rate and the nominal interest rate.

Let us first examine the situation in which a household does not participate in the program and calculate the household's wealth after k years. Liquid assets will have increased at a rate of n, permitting an actual amount of $a(1 + n)^k$ to be accumulated. As to the mortgage debt, it compounds at an interest rate r. The net value of the home after k years will thus be only $Ph - (1 + r)^k D$. As regards to the RRSP assets, they will be worth $(1 - m)w(1 + r)^k$ after k years, as the amount withdrawn from the RRSP is taxable when the funds are withdrawn. Thus, wealth without the use of the HBP will be the sum of the three assets, as follows:

$$R = a(1 + n)^k + Ph - D(1 + r)^k + (1-m)w(1 + r)^k \tag{3}$$

Let us now examine a situation in which the household participates in the HBP. Following an initial withdrawal of $k\theta$, the household must deduct the θ amount from its liquidities each year after the end of the first year. After k years, the household will have accumulated liquidities in the amount of $a(1+n)^k - \theta\left[\frac{(1+n)^k - 1}{n}\right]$, less than if the household does not participate in the HBP due to the annuity required for making repayments (see Appendix 1). On the other hand, the net worth of the home will have increased more rapidly, as the initial mortgage debt was lower. Thus, after k years, the net worth of the home reaches $Ph - (D - k\theta)(1 + r)^k$. As for RRSP assets, they are also less than they would have been without the withdrawal, because of the loss of accumulated return between the time of the initial withdrawal and the repayment. After k years, the accumulated value reaches $(1 - m)\left[\ (w - k\theta)(1 + r)^k + \theta\left(\frac{(1+r)^k - 1}{r}\right)\ \right]$. The real accumulated worth after k years of a household participating in the HBP is the sum of the three previous elements as follows:

$$\begin{aligned} R^* = {} & a(1+n)^k - \theta\left[\frac{(1+n)^k - 1}{n}\right] + Ph - (D - k\theta)(1+r)^k \\ & + (1-m)(w - k\theta)(1+r)^k + \theta(1-m)\left[\frac{(1+r)^k - 1}{r}\right] \end{aligned} \tag{4}$$

To conclude that HBP participation results in a wealth gain, one needs only see if R^* - R> 0, where R^* is wealth after k years with participation in the program. This difference is obtained as follows:

$$\frac{R^* - R}{\theta} = \left[\frac{(1+r)^k - 1}{r}\right] - m\left[\frac{(1+r)^k - 1}{r}\right] + km(1+r)^k - \left[\frac{(1+n)^k - 1}{n}\right] \tag{5}$$

Thus, by analyzing the equations, we see that the difference is null if $m = 0$ and is positive if $m > 0$ (see Appendix 1). This result is very easy to explain. When the government grants a household the right to a tax-free RRSP withdrawal, it is as though it were agreeing to lend the RRSP tax credit interest-free. The amount of the gain thus increases with the household's taxation rate (because the "loan" by the government is greater) and with the nominal interest rate (because the "loan" is proportional to nominal income at any given tax rate). From the financial standpoint, it is best to delay the repayment as late as possible in order to benefit for a longer period of time from the interest-free loan from the government. Thus, all households with a positive taxation rate financially benefit from using the HBP, but those with the highest taxation rate, i.e., the highest income, benefit the most. Thus the HBP does not favour a progressive tax system.

Analysis also shows that households experience a wealth gain even when they do not repay the required amounts, but pay the tax on the unpaid amounts instead. In effect, if repayment is not made, there is an accumulation of liquid assets after k years in the amount of $a(1+n)^k - m\theta\left[\frac{(1+n)^k - 1}{n}\right]$, while that of the active RRSP and the net value of the home are represented respectively by $(1 - m)(w - k\theta)(1 + r)^k$ and $Ph - (D - k\theta)(1 + r)^k$. Once again, if we examine the difference in wealth when the HBP is used and when it is not, we obtain $R^* - R = m\theta k(1+r)^k - m\theta\left[\frac{(1+n)^k - 1}{n}\right]$ (see Appendix 1), a difference that is positive if $m > 0$. Again, the household receives a gain from participating in the HBP if its income is taxable. Note that the gain is less than in the previous example, as the household saves less in its RRSP. On the other hand, as it is paying only $m\theta$ each year instead of θ, it disposes of a higher available income for consuming other goods. The reason such a high percentage of households do not repay their RRSPs is undoubtedly because the choice to recontribute or pay the taxes on the contributions not made is reduced to a decision whether or not to save at a rate of r.

We have also examined situations in which households make the repayments under the HBP, but must borrow the amounts necessary for the repayments at an interest rate of r. Let us assume that in this case the household does not initially have any liquidity. Note that the accumulation of RRSP assets and the net value of the home is identical to that of the first case, as nothing has changed in this regard. The change relates to liquid assets because, after k years, it will have decreased by $-\theta\left[\frac{(1+r)^k - 1}{r}\right]$. We can see that the difference in wealth is $R^* - R = mk\theta(1 + r)^k > 0$. This latter expression is very simple and corresponds to the capitalized value of the "loan" by the government at a rate of r after k years. In the previous examples, the expression was more complex because we needed to take into account the fact that the initial contribution and the repayment to the RRSP were made by drawing from liquid assets with a before-tax return rate of just n instead of r.

Because of its simplicity and the fact that it applies to the less favourable situation, this latter expression lends itself well to an approximate evaluation of the present value of the financial gain for participating households, and the cost of the program for the government. In effect, the present value is simply $mk\theta$. Since the maximum withdrawal is \$20,000, this present value is \$8,000 for a household with a marginal taxation rate of 40%. If we apply this same marginal rate to the \$7.5 billion that were withdrawn under the HBP between 1992 and 1998, the program provided \$3 billion in direct financial assistance from the government to households purchasing homes. To this, we must eventually add the loss in tax revenues associated with RRSP repayments from liquidity for which the return would have been taxable.

In short, the HBP provides a financial benefit to all taxed households, whether or not they must borrow to contribute to the RRSP prior to withdrawing the amounts, whether or not they borrow to repay, or whether or not they make the repayments. This gain is greater for higher-taxed households and for those that can contribute to an RRSP and repay it without borrowing. The reason many participants do not repay their RRSPs according to the terms set forth is that the decision to repay is justified by the desire to save at a rate of r. A household that has borrowed to contribute to an RRSP for the sole purpose of withdrawing under the HBP was not initially a lender. It borrows to take advantage of the tax benefit, but will probably decide not to repay the RRSP and to pay the tax on the payments not made. Finally, note that the value of the gain increases with the income taxation rate and the nominal interest rate. The recent fall in interest rates is therefore likely to make the Home Buyers' Plan less attractive.

4 INFLUENCE OF THE HBP ON HOUSING DEMAND

4.1 Variables and Hypotheses of the Base Model Without RRSP

In this section, we integrate RRSPs and the HBP into a continuous-time life-cycle model to see the possible effects of adding RRSPs and the HBP on residential capital consumption. Since RRSPs has never been studied using such a model, we first develop a basic model inspired by Fortin (1988) that includes neither an RRSP nor the HBP. We then progressively add these elements, first RRSPs and then the HBP, to see how each changes the optimal choice.

Let us consider a household that wishes to maximize at the date t_1 the function $\int_{t_1}^{t_2} e^{-\delta t} U(c(t), h)dt$ plus a certain terminal value function that depends on the wealth in t_2. In the model, t_1 and t_2 are, respectively, the dates on which the home is bought and sold, while δ represents the pure rate of time preference, $c(t)$ represents the instantaneous consumption rate of the Hicksian aggregate that serves as numeraire and h represents the flow of services drawn from the residential capital stock. We consider that h is directly proportional to the residential capital stock, such that it designates both the housing stock and the flow of services from them. We disregard the temporal indicator for h because we assume that this variable is initially chosen by the household, but must remain fixed throughout the period in which the household owns and occupies the home. These two arguments are put forth as separable and the utility function is also assumed to be additively separable and strictly concave, which means that the marginal rate of substitution is decreasing, but also that the marginal consumption utility is positive and diminishing in both arguments.

Consumer choices are limited by constraints regarding total resources and cash flow. Households receive an exogenous constant real liquid income stream at a rate of y. This income can be consumed or saved. The accumulated value of liquid savings is indicated by $a(t)$ and this asset is capitalized at a rate of n, where n is the net real interest rate. The total net flow of return of the liquid asset in t is thus indicated by $na(t)$, which is added to the income y to give the stream of new liquid asset availability. Following the approach developed by Artle and Varaiya (1978) in studying the effect of liquidity constraints on housing demand, we apply a non-negativity constraint on $a(t)$. Such a constraint hinders a household from borrowing because it pledges its future earnings. Given the low level of unsecured borrowing ability of households compared to the actual value of future income, such a hypothesis better complies with households' actual borrowing abilities.[5]

To establish the instantaneous liquidity variation, we must take into account disbursement of liquid assets. In addition to $c(t)$ there are expenditures related to housing services. As we are examining the period of the plan in which the household owns a home, we have chosen to take into account only mortgage capitalization costs. We are therefore disregarding other housing-related expenses (maintenance costs, property taxes, etc.), as including them would have no qualitative effect on the results of the analysis. The symbol $v(t)$ designates the real value of the flow of mortgage disbursements for each capital unit, while h represents the number of residential capital units purchased by households. As we are assuming that the mortgage loan is amortized over an annuity period of s with a constant nominal value, the real mortgage disbursements decrease in real terms due to inflation. If each residential capital unit has an asset price of P, the cost of acquiring the h units is Ph. We will assume that the initial mortgage lending value is $D(t_1)$. The expression governing the flow of mortgage disbursements over the period t is thus:

$$\begin{aligned} v(t) &= (r+\pi)D(t_1)e^{-\pi(t-t_1)}(1-e^{-(r+\pi)(s-t_1)})^{-1} \\ v(t) &= v(t_1)e^{-\pi(t-t_1)} \end{aligned} \tag{6}$$

By subtracting disbursements from available liquid resources, we obtain an expression regarding the rate of change for liquid assets if $a(t)$ is positive, as follows:

$$\dot{a}(t) = na(t) + y - c(t) - v(t)h \tag{7}$$

In addition to liquid assets, a household can accumulate wealth in the form of the net value of a home that it owns and occupies. This net value in the period t, indicated as $\alpha(t)$, is obtained by subtracting the value of the residual mortgage debt $D(t)$ from the home's asset price Ph, thus $\alpha(t) = Ph - D(t)$, or $D(t) = Ph - \alpha(t)$. If we assume that interest is collected

[5] This statement is easy to understand. Letting the loan be limited only by lifetime financial resources is paramount to recognizing that a 25-year old household that expects to have a real annual income of $50,000 for 40 years would be able to borrow close to $857,000 at an interest rate of 5% with no other guarantee but the promise to repay. Clearly, lenders offer unsecured credit for only a small fraction of this amount.

on the value of the residual debt at a rate of r, i.e., the gross real interest rate, then the following relation must be verified:

$$rD(t) = r(Ph - \alpha(t)) = rPh - r\,\alpha(t) \tag{8}$$

The rate of change of the real home equity is given by the difference between the mortgage payment $v(t)h$ and the previous expression, that is:

$$\dot{\alpha}(t) = r\alpha(t) + v(t)h - rPh \tag{9}$$

This expression is non-null for owner-occupant and shows that the implicit return rate on home equity is the gross real interest rate r, since it is the amount saved on mortgage debt interest charges when the household repays a dollar of debt. We assume that $\alpha(t)$ is non-liquid and indivisible. The household cannot therefore sell or rent a part of the home; we can call this a hypothesis of mutual exclusivity of occupation.

The household thus faces constraints when accumulating wealth. In view of these constraints, the household must decide as to the number of capital units h that it wishes to purchase. Note that, due to the non-liquid and indivisible nature of the home, the only way for a household to have access to the net value of the home is to sell it. The date t_2 corresponds to the time when the household decides to sell the home. This optimal control problem can thus be formulated as follows:

$$\max_{(c(t),h)} \int_{t_1}^{t_2} e^{-\delta t} U(c(t),h)dt + V(a(t_2)+\alpha(t_2),t_2)$$

subject to

$$\dot{a}(t) = na(t) + y - c(t) - v(t)h$$
$$\dot{\alpha}(t) = r\alpha(t) + v(t)h - rPh$$
$$a(t_1) = \alpha(t_1 -) - (Ph - D(t_1))$$
$$\alpha(t_1) = Ph - D(t_1)$$
$$a(t) \geq 0,\ t_1 \leq t \leq t_2$$
$$a(t_2) \text{ free}$$
$$\alpha(t_2) \text{ free}$$
$$t_2 \text{ free}$$

and where $V(a(t2) + \alpha(t_2), t_2)$ is the terminal value function. The additive formulation $a(t_2) + \alpha(t_2)$ means that, at the time of sale, the household is indifferent as to the makeup of its wealth. This hypothesis is easy to understand because, immediately following the sale, all wealth is converted into liquid assets. The expression $a(t_1\text{-})$ designates liquidities immediately prior to the date t_1 on which the home is purchased.

At this optimal control problem can be associated the following Hamiltonian function and necessary conditions (see Kamien and Schwartz 1981). First, for the Hamiltonian function, we have:

$$H = e^{-\delta t}\, U(c(t),h) + \gamma_a\,(na(t)+y-c(t)-v(t)h) + \gamma_\alpha\,(ra(t)+v(t)h-rPh) + \mu a(t)$$

As for the necessary conditions, they are represented by:

$$U_c\, e^{-\delta t} = \gamma_a \tag{10}$$

$$\int_{t_1}^{t_2} U_h e^{-\delta t} dt = \int_{t_1}^{t_2} (\gamma_a[v(t) - n\tfrac{da(t_1)}{dh}] + \gamma_\alpha[rP - r\tfrac{d\alpha(t_1)}{dh} - v(t)])dt \tag{11}$$

or

$$\int_{t_1}^{t_2} U_h e^{-\delta t} dt = \int_{t_1}^{s} (\gamma_a[v(t) - n\tfrac{da(t_1)}{dh}] + \gamma_\alpha[rP - r\tfrac{d\alpha(t_1)}{dh} - v(t)])dt + \int_{s}^{t_2} \gamma_a nP dt \tag{12}$$

$$\dot{a}(t) = na(t) + y - c(t) - v(t)h \tag{13}$$

$$\dot{\alpha}(t) = r\alpha(t) + v(t)h - rPh \tag{14}$$

$$\dot{\gamma}_\alpha + n\gamma_\alpha = -\mu \tag{15}$$

$$\dot{\gamma}_\alpha + r\gamma_\alpha = 0 \tag{16}$$

$$\mu_a = 0,\ \mu \geq 0,\ a \geq 0 \tag{17}$$

$$a(t_2)[\gamma_a(t_2) - \partial V / \partial a(t_2\text{-})] = 0, \quad a(t_2) \geq 0, \quad \gamma_a(t_2) \geq \partial V / \partial a(t_2\text{-}) \tag{18}$$

$$\gamma_a(t_2) - \partial V / \partial \alpha(t_2\text{-}) = 0 \tag{19}$$

$$[H(t_2) + \partial V / \partial t_2][T - t_2] = 0, \quad H(t_2) \geq \partial V / \partial t_2,\ T \geq t_2 \tag{20}$$

A similar problem was solved in Fortin (1988): we will not re-examine the solution here. We should note, however, that the relevant rate for determining the consumption path is the rate r as long as the mortgage is not fully amortized but becomes rate n after the complete amortization of the mortgage loan. If the house is sold before the loan if fully repaid the rate remains r throughout the entire ownership period and the first-order condition that applies to optimize the choice of h is then 11. However, if the mortgage loan is amortized before the sale of the home, condition 12 applies to choose optimally h. In the next section, we will add the possibility of saving in an RRSP and see what solution emerges.

4.2 Model With RRSP Asset

We now assume that the household can, in addition to the asset elements seen earlier, accumulate wealth in an RRSP asset, for which the real value in period t is indicated by $w(t)$. Withdrawals from the RRSP are always taxable at a rate of m, while contributions result in a tax reduction equal to a fraction, m, of the contribution. Thus the net impact on liquid assets of a contribution flow to the RRSP asset $\rho(t)$ is $(1-m)\rho(t)$. We assume that the capitalization rate for the RRSP asset is identical to the interest rate on the mortgage debt, rate r, such that the flow of accumulation in t will be $rw(t)$. The movement equation for the RRSP asset will thus be $\dot{w}(t) = rw(t) + \rho(t)$, while that of the liquid assets is modified to become $\dot{a}(t) = na(t) + y - c(t) - v(t)h - (1-m)\rho(t)$. There is no change to the equation of $\dot{\alpha}(t)$.

The initial purchasing conditions are the same as with the base model in terms of a and α, but for w we now have $w(t_1) = w(t_1\text{-})$, as the amount accumulated in the RRSP prior to the purchase date is considered as given. Finally, we must also modify the terminal value function, which becomes $V(a(t_2) + \alpha(t_2) + (1-m)w(t_2), t_2)$ to reflect the taxation of the RRSP asset when it is transformed into liquidity on the sale date t_2. We therefore assume that, at t_2, the household withdraws the full RRSP.

With these differences from the base model, the optimal control problem with RRSP assets is formulated as follows:

$$\max_{(c(t),h)} \int_{t_1}^{t_2} e^{-\delta t} U(c(t),h)dt + V(a(t_2) + \alpha(t_2) + (1 - m)w(t_2), t_2)$$

subject to:

$$\dot{a}(t) = na(t) + y - c(t) - v(t)h - (1-m)\rho(t)$$

$$\dot{w}(t) = rw(t) + \rho(t)$$

$$\dot{\alpha}(t) = r\alpha(t) + v(t)h - rPh$$

$a(t_1) = a(t_1\text{-}) - (Ph - D(t_1))$
$w(t_1) = w(t_1\text{-})$
$\alpha(t_1) = Ph - D(t_1)$
$a(t) \geq 0$
$a(t_2)$, $w(t_2)$, $\alpha(t_2)$ and t_2 free

with $v(t) = (r+\pi)D(t_1)e^{-\pi(t-t_1)}(1 - e^{-(r+\pi)(s-t_1)})^{-1}$ where $D(t_1)$ represents the initial mortgage debt.

The Hamiltonian function is $H = e^{-\delta t}U(c(t),h) + \gamma_\alpha\,(\text{na(t)} + y - c(t) - v(t)h - (1-m)\rho(t)) + \gamma_\alpha(r\alpha(t) + v(t)h - rPh) + \gamma_w(rw(t) + \rho(t)) + \mu a$

and the necessary conditions are:

$$U_c e^{-\delta t} = \gamma_a \tag{21}$$

$$\int_{t_1}^{t_2} U_h e^{-\delta t}dt = \int_{t_1}^{t_2}(\gamma_a\left[v(t) - n\tfrac{da(t_1)}{dh}\right] + \gamma_\alpha\left[rP - r\tfrac{d\alpha(t_1)}{dh} - v(t)\right])dt \tag{22}$$

or

$$\int_{t_1}^{t_2} U_h e^{-\delta t}dt = \int_{t_1}^{s}(\gamma_a[v(t) - n\tfrac{da(t_1)}{dh}] + \gamma_\alpha[rP - r\tfrac{d\alpha(t_1)}{dh} - v(t)])dt + \int_{s}^{t_2}\gamma_\alpha nPdt \tag{23}$$

$$\dot{a}(t) = na(t) + y - c(t) - v(t)h - (1-m)\rho(t) \tag{24}$$

$$\dot{\alpha}(t) = r\alpha(t) + v(t)h - rPh \tag{25}$$

$$\dot{w}(t) = rw(t) + \rho(t) \tag{26}$$

$$\dot{\gamma}_a + n\gamma_a = -\mu \tag{27}$$

$$\dot{\gamma}_\alpha + r\gamma_\alpha = 0 \tag{28}$$

$$\dot{\gamma}_w + r\gamma_w = 0 \tag{29}$$

$$\mu a = 0,\quad \mu \geq 0,\quad a \geq 0 \tag{30}$$

$$a(t_2)[\gamma_\alpha(t_2) - \tfrac{\partial V}{\partial a(t_2 -)}] = 0, \quad a(t_2) \geq 0, \quad \gamma_a(t_2) \geq \tfrac{\partial V}{\partial a(t_2 -)} \tag{31}$$

$$\gamma_\alpha(t_2) - \tfrac{\partial V}{\partial \alpha(t_2 -)} = 0 \tag{32}$$

$$\gamma_w(t_2) - \tfrac{\partial \mathrm{V}}{\partial w(t_2 -)} = 0 \tag{33}$$

$$H(t_2) + \tfrac{\partial V}{\partial t_2} = 0 \tag{34}$$

Condition 21 indicates the evolution of the marginal consumption utility of $c(t)$ in time. Conditions 22 and 23 have identical interpretations, but differ according to whether the house is sold before or after the complete amortization of the mortgage loan. They indicate that consumption of housing services must satisfy a relationship of equality between the present value of marginal service flow and that of the real net costs of those services. Equations 24, 25 and 26 give us, respectively, the rate of change of a, α and w, while the rate of change of the implicit price of these same assets is given, respectively, by equations 27, 28 and 29. We therefore know that the implicit price of the net value of the home and the RRSP asset decrease at a rate of r. However, μ appears in the determination of the implicit price of the liquid asset due to the constraint of non-negativity in equation 30. Thus, the implicit price of liquid assets must decrease at a rate of n if $a > 0$, but could follow a different path if $a = 0$. Equations 31, 32 and 33 indicate the conditions of transversality that must be satisfied for the value of the three assets to be optimal when the home is sold. Finally, condition 34 determines the optimal date for the sale of the home.

This model permits us to demonstrate that, if a household has unused RRSP contributions, it will not hold liquid assets. We do so by showing that $a(t) > 0$ is incompatible with an optimal solution. In effect, $a(t) > 0$ impliess, in equation 30, that $\mu = 0$ and, as a result, in equation 27, $\dot{\gamma}_\alpha + n\gamma_\alpha = 0$. In addition, we know from equations 28 and 29 that $\dot{\gamma}_\alpha + r\gamma_\alpha = 0$ and $\dot{\gamma}_\omega + r\gamma_\omega = 0$, such that $\gamma_\alpha(t)$ and $\gamma_\omega(t)$ will be respectively represented by $\gamma_\alpha(t) = \gamma_a(t_1)e^{-n(t-t_1)}$ and $\gamma_w(t) = \gamma_w(t_1)e^{-r(t-t_1)}$ between t_1 and t_2. Thus, the implicit price of a decreases at rate n, while those of α and w decrease at rate r. It remains to determine the absolute level of these prices, which is made by the conditions of transversality. But if the non-negativity constraint on $a(t)$ is non-binding, we know, by equation 31, that $\gamma_a(t_2) = \frac{\partial V}{\partial_a (t_2 -)}$, while equations 32 and 33 show that $\gamma_\alpha(t_2) = \frac{\partial V}{\partial_\alpha (t_2 -)}$ and

$\gamma_w(t_2) = \frac{\partial V}{\partial_w(t_2\text{-})}$. Furthermore, as $a(t_2) = a(t_2\text{-}) + \alpha\,(t_2\text{-}) + (1-m)w(t_2\text{-})$, this means that $\frac{\partial_a(t_2)}{\partial_a(t_2\text{-})} = 1$, $\frac{\partial_a(t_2)}{\partial_\alpha(t_2\text{-})} = 1$ and $\frac{\partial_a(t_2)}{\partial w(t_2\text{-})} = (1 - m)$. Thus the chain rule allows us to write:

$$\frac{\partial V}{\partial a(t_2\text{-})} = \frac{\partial V}{\partial a(t_2)}\frac{\partial a(t_2)}{\partial a(t_2\text{-})} = \frac{\partial V}{\partial a(t_2\text{-})}$$
$$\frac{\partial V}{\partial \alpha(t_2\text{-})} = \frac{\partial V}{\partial a(t_2)}\frac{\partial a(t_2)}{\partial \alpha(t_2\text{-})} = \frac{\partial V}{\partial a(t_2)}$$
$$\frac{\partial V}{\partial w(t_2\text{-})} = \frac{\partial V}{\partial a(t_2)}\frac{\partial a(t_2)}{\partial w(t_2\text{-})} = (1 - m)\frac{\partial V}{\partial a(t_2)}$$

We therefore find the following relationships between the implicit prices at date t_2:

$$\gamma_a(t_2) = \gamma_\alpha(t_2) = \tfrac{1}{(1\text{-}m)}\gamma_w(t_2)$$

At the precise time of sale, adding a dollar to the net value of the home has the same impact on the value of the plan as adding a dollar of liquidity. However, the value of the RRSP asset is less because conversion of RRSP into liquidity is taxed at a rate of *m*.

What about the implicit prices of assets prior to the sale? For the $[t_1, t_2]$ period, the prices of assets are given by:

$$\gamma_a(t) = \gamma_a(t_2)e^{n(t_2\text{-}t)} \tag{35}$$
$$\gamma_\alpha(t) = \gamma_\alpha(t_2)e^{r(t_2\text{-}t)} \tag{36}$$
$$\gamma_w(t) = \gamma_w(t_2)e^{r(t_2\text{-}t)} \tag{37}$$

As γ_α and γ_w both decrease at the same rate *r* and $\gamma_\alpha(t_2) = \frac{1}{(1\text{-}m)}\gamma_w(t_2)$, we immediately see that $\gamma_\alpha(t) = \frac{1}{(1\text{-}m)}\gamma_w(t)$ for $t \in [t_1,t_2]$. The implicit price of the RRSP asset is always in a $(1-m)$ proportion of that of the net value of the home due to the tax treatment of RRSP contributions and withdrawals. As for the price of liquid assets, knowing that $\gamma_a(t_2) = \gamma_\alpha(t_2)$, we see in equation 36 that $\gamma_\alpha(t_2) = \gamma_\alpha(t)e^{-r(t_2\text{-}t)}$. By substituting in 35, we find that $\gamma_a(t) = \gamma_\alpha(t)e^{(n\text{-}r)(t_2\text{-}t)}$. Thus, as long as $n < r$, that is $m > 0$, $\gamma_a(t) < \gamma_\alpha(t)$, $t \in [t_1,t_2]$. The paths of the implicit prices are indicated in Figure 1. As such, if $\gamma_a(t) < \gamma_\alpha(t)$, the value of the plan is improved by decreasing $a(t)$ to increase $\alpha(t)$. As this is possible if $a(t) > 0$, we are faced with a contradiction: the hypothesis that the constraint of non-negativity on liquidity is non-binding is not compatible with an optimal plan if contributions rights to RRSP are unused. As a result, it is necessary that $a(t) = 0$ if the household has unused contributions. Thus, and this is a relevant result for the rest of the HBP analysis, the household will always be in a situation where $\dot{a}(t) = a(t) = 0$ if there are unused RRSP contributions. When savings are possible, they will be made in the form of RRSP assets and the $\rho(t)$ contribution will be made by resolving equation 24 when $\dot{a}(t) = a(t) = 0$. We will thus have a contribution determined by the equation $0 = y - c(t) - v(t)h - (1 - m)\rho(t)$, which is the same as saying that RRSP contributions will be $\rho(t) = \frac{y\text{-}c(t)\text{-}v(t)h}{(1\text{-}m)}$.

Let us now examine what happens to the paths of the implicit prices when there is room for savings and $\dot{a}(t) = a(t) = 0$. By means of equation 31, we know that $\gamma_a(t_2) \geq \frac{\partial V}{\partial_a(t_2\text{-})}$. By combining this with the other conditions of transversality, we obtain $\gamma_{\alpha}(t_2) \geq \gamma_{\alpha}(t_2) = \frac{1}{(1\text{-}m)}\gamma_w(t_2)$. However, if there are savings, they must be in RRSP assets, which means that $w(t_2\text{-})$ is positive. However, if $w(t_2\text{-}) > 0$, we cannot have $\gamma_{\alpha}(t_2) > \frac{1}{(1\text{-}m)}\gamma_w(t_2)$ because, if liquid assets have an implicitly larger value, adjusted for taxation, than that of the RRSP asset, it is then better to slightly decrease the RRSP asset to add some to the liquid assets. We thus cannot have strict inequality and the prices of the assets at t_2 comply with the double equality $\gamma_a(t_2) = \gamma_{\alpha}(t_2) = \frac{1}{(1\text{-}m)}\gamma_w(t_2)$.

As regards the value of assets prior to the sale date, the path of the RRSPs implicit price is a parallel evaluation of $\gamma_{\alpha}(t)$ because both prices decrease at rate r. In the path of the liquid assets' implicit price, we can, through what was demonstrated previously, state that μ will be adjusted to have an identical path to that of the implicit price of the home's net value. If not, it would be possible to improve the plan by reallocating assets to increase the retention of liquid assets and the plan would no longer be optimal.

Figure 2 indicates the situation when $\mu > 0$ and when there is room for savings in the model. We can see that the $\mu > 0$ relationship is such that the path of the liquid assets' implicit price is adjusted to that of the net value of the home. Obviously, if $U_c e^{-\delta t} = \gamma_a$, this means that the consumption path will be changed to ensure a decrease in the implicit price of the liquid asset at a rate of r. The case where liquidities are tight and there are no savings by the household is of slight interest to us at this time, as we will also have $\dot{a}(t) = a(t) = 0$.

In brief, faced with an RRSP, the relevant rate for accumulation of savings by the household and for the choice of a consumption path is no longer n as it is when the RRSP asset is not integrated into the model, but instead becomes r, the gross real interest rate. Note that this permits us to simplify the model somewhat. Thus, in the model without RRSPs, it was important to distinguish between a case where $s < t_2$ and a case where $s \geq t_2$ because the relevant accumulation rate was r until the mortgage was amortized and then became n. As such, there is no longer any reason to make such a distinction because the household can save in RRSPs. In the next section, we will simplify the presentation of the model by assuming that $s = t_2$, which will permit us to lighten the presentation without sacrificing its generality. Note that this will result in $\alpha(t_2) = Ph$.

4.3 Model With RRSP Assets and HBP

4.3.1 Description of the Model With RRSP Assets and HBP

It is important here to briefly review the HBP mechanism to show the difference between this model and the model with only an RRSP. The HBP permits a tax-free amount to be withdrawn from the participant's RRSP to be allocated to the net value of the home, insofar as the participant makes yearly repayments under the HBP. To simplify the matter, we will assume that the HBP repayment period corresponds to that of the mortgage amortization s that, in turn, corresponds to the date on which the home is sold. We will also not allow the participant to choose to not to make the repayments and to instead pay the tax on the default payments.[6] Finally, we will limit our analysis to cases in which there are unused contributions.

With these hypotheses, the differences between this model and the model examined in the previous section are as follows. First, because the HBP allows participants to withdraw amounts from their RRSPs without penalty, this changes the initial condition of the RRSP asset, which becomes $w(t_1) = w(t_1\text{-}) - s\theta$, where $s\theta$ is the amount withdrawn under the HBP and θ is the periodic repayment. As this amount reduces what must be taken from liquid assets to make the down payment when purchasing the home, the initial condition of liquid assets becomes $a(t_1) = a(t_1\text{-}) - (Ph - D(t_1) - s\theta)$.[7] The last two changes are the result of RRSP

[6] Although interesting, this possibility complicates the model and we were not able to study its implications.

[7] Note that $D(t_1)$ is not necessarily identical to that of the previous problem.

repayments. Thus, the $\dot{a}(t)$ equation becomes $\dot{a}(t) = na(t) + y - c(t) - v(t)h - (1-m)\rho(t) - \theta$, while the $\dot{w}(t)$ equation will be $\dot{w}(t) = rw(t) + \rho(t) + \theta$. The rest of the model is similar to that without RRSP assets as seen in the previous section, including the hypotheses that were developed. It is therefore not necessary to repeat them. The overall model is thus:

$$\max_{(c(t),h)} \int_{t_1}^{t_2} e^{-\delta t} U(c(t),h)dt + V(a(t_2) + \alpha(t_2) + (1-m)w(t_2), t_2)$$

subject to:

$$\dot{a}(t) = na(t) + y - c(t) - v(t)h - (1-m)\rho(t) - \theta$$
$$\dot{w}(t) = rw(t) + \rho(t) + \theta$$
$$\dot{a}(t) = r\alpha(t) + v(t)h - rPh$$
$$a(t_1) = a(t_1\text{-}) - (Ph - D(t_1) - s\theta)$$
$$w(t_1) = w(t_1\text{-}) - s\theta$$
$$\alpha(t_1) = Ph - D(t_1)$$
$$a(t) \geq 0$$
$a(t_1)$, $w(t_2)$, $\alpha(t_2)$ and t_2 free

The Hamiltonian function associated with this problem and the necessary conditions are somewhat modified to become:

$$\max_{(c(t),h)} \int_{t_1}^{t_2} e^{-\delta t} U(c(t),h)dt + V(a(t_2) + (1-m)w(t_2), t_2)$$

$$U_c e^{-\delta t} = \gamma_a \tag{38}$$

$$\int_{t_1}^{t_2} U_h e^{-\delta t} dt = \int_{t_1}^{t_2} (\gamma_a[v(t) - n\tfrac{da(t_1)}{dh}] + \gamma_\alpha[rP - r\tfrac{d\alpha(t_1)}{dh} - v(t)])dt \tag{39}$$

$$\dot{a}(t) = na(t) + y - c(t) - v(t)h - (1-m)\rho(t) - \theta \tag{40}$$

$$\dot{\alpha}(t) = r\alpha(t) + v(t)h - rPh \tag{41}$$

$$\dot{w}(t) = rw(t) + \rho(t) + \theta \tag{42}$$

$$\dot{\gamma}_a + n\gamma_a = -\mu \tag{43}$$

$$\dot{\gamma}_\alpha + r\gamma_\alpha = 0 \tag{44}$$

$$\dot{\gamma}_w + r\gamma_w = 0 \quad (45)$$

$$\mu a = 0, \quad \mu \geq 0, \quad a \geq 0 \quad (46)$$

$$a(t_2)[\gamma_a(t_2) - \tfrac{\partial V}{\partial a(t_2\text{-})}] = 0, \; a(t_2) \geq 0, \; \gamma_a(t_2) \geq \tfrac{\partial V}{\partial a(t_2\text{-})} \quad (47)$$

$$\gamma_\alpha(t_2) - \tfrac{\partial V}{\partial \alpha(t_2\text{-})} = 0 \quad (48)$$

$$\gamma_w(t_2) - \tfrac{\partial V}{\partial w(t_2\text{-})} = 0 \quad (49)$$

$$H(t_2) + \tfrac{\partial V}{\partial t_2} = 0 \quad (50)$$

4.3.2 Model Solution

These modifications do not change some properties of the model. Thus, although we will not re-demonstrate it here, it is clear that it is still optimal to contribute to an RRSP when savings are possible so that the relevant interest rate for the rhythm of change to consumption is r rather than n. In this way, we will undoubtedly have a relationship of $\dot{a}(t) = a(t) = 0$. The effect on residential capital demand is studied by analyzing the effect of a change in the HBP amount on the optimal choice of h. In fact, this comes down to studying the wealth gain resulting from the HBP. We will show how this wealth gain is distributed based on two situations, when there are savings and when there are not.

As $\dot{a}(t) = a(t) = 0$, we know from equation 40 that $y = c(t) + v(t)h + (1–m)\rho(t) + \theta$. To demonstrate the adjustments that must occur, we use a contradiction by assuming that no change takes place in $c(t)$ or in h because of the RAP. But we previously demonstrated that there is a wealth gain associated with the HBP. It thus follows that the HBP will have allowed greater wealth to be accumulated at t_2. As $a(t) = 0$ and $\alpha(t_2) = Ph$, which has not changed in our hypothesis, this additional wealth must necessarily be in the RRSP. As a result, $w(t_2\text{-})$ has increased. However, if $w(t_2\text{-})$ has increased, equation 48 is such that $\gamma_w(t_2)$ must decrease, as $\frac{\partial V}{\partial_w(t_2\text{-})}$ is concave. As such, if $\gamma_w(t_2)$ diminishes without $\gamma_\alpha(t_2)$ changing, this violates the transversality conditions determining that $\gamma_\alpha(t_2) = \frac{1}{(1\text{-}m)}\gamma_w(t_2)$. It does not therefore comply with an optimal plan of not increasing $c(t)$ or h. As such, to re-establish the condition regarding the respective price values for these two assets at t_2, the savings accumulated in the RRSP must be accompanied by an increased accumulation of wealth in the residential capital. It is therefore necessary that h increases proportionally to the increased wealth seen in t_2. The effect of the HBP on housing demand is then positive. We must now verify whether or not this increased wealth, in combination with increased consumption of residential capital, leads to an increase in $c(t)$.

If residential capital consumption h increases, the marginal utility drawn from residential capital consumption will decrease. In other words, according to equation 39, we will see $\int_{t_1}^{t_2} U_h e^{-\delta t}$ falling, which suggests that γ_α or γ_a, or both, will decrease. Suppose that it is γ_α that decreases but that γ_a remains the same. At that time, in particular, $\gamma_\alpha(t_2) \neq \gamma_a(t_2)$. However, we have shown that, in cases where households have access to savings, there must be equality between these two implicit prices at t_2, which contradicts our assumption. Thus, it is easy to see that the only solution in this case is that the two implicit prices decrease together to be equal throughout the entire interval $[t_1, t_2]$. Knowing that the implicit price of the liquid assets is decreasing, we know by equation 38 that the marginal utility drawn by the household from consumption of $c(t)$ must also decrease. This leads to a conclusion that consumption of c(t) increases. In fact, as the relative price of housing services has not changed, the increased consumption of the Hicksian aggregate must be sufficient to maintain constancy in the average ratio of the flows of housing service consumption and the Hicksian aggregate. In short, there will be an increase in housing consumption and demand proportional to the increased wealth at the end of the plan.

Let us pursue the analysis further and examine how the HBP affects mortgage decisions. As $w(t_2\text{-})$ has increased, but $w(t_1)$ has decreased, the flow of RRSP contributions $\rho(t)$ must be greater in the $]t_1, t_2]$ period than the flow of contributions $\rho(t)$ for the model without the HBP. Indeed, not only must reimbursements θ be made, but also it is necessary to compensate for the interest that would have been earned in the RRSP had the withdrawal not been made. We also know that $c(t)$ and h also increase. As a result, in the constraint $y = c(t) + v(t)h + (1\text{–}m)\rho(t) + \theta(t)$, the only element that can be lowered in the right-hand side to ensure that equality is respected is $v(t)$. It is therefore clear, by means of equation 6, that this requires a decrease of $D(t_1)$.

5 CONCLUSION

The purpose of this work was to determine the impact of the HBP on housing demand. We chose to study this based on the framework of a continuous-time life-cycle model. This approach led us to first study the financial benefit of the HBP. We have shown that, based on the hypothesis of an identical interest rate for all assets and debts, a household participating in the HBP benefits from a wealth gain proportional to its taxation rate. This wealth gain is the result of the fact that the HBP consists of receiving an interest-free loan from the government, with the principal equal to the tax to which the RRSP withdrawal would normally have been subject, with a duration which extends the date at which the household must recontribute to the RRSP. The exact value of the loan varies depending on whether or not the household must borrow to make contributions. The gain favours households with higher income, as they have the highest marginal taxation rate.

When the time came to integrate the analysis into the framework of a life-cycle model, we first studied how the HBP changes choices. The main difference is that the possibility of making savings grow at rate r changes the consumption path. By adding the HBP, we then demonstrated that the HBP decreases the initial mortgage loan. In addition, due to the wealth effect associated with the HBP, we see an increase among participants of consumption of housing capital and other non-housing goods. This point is in line with the comment by Théroux (1999), who states that amounts of money that should be systematically invested to secure [the household's retirement] are quite often used to instead purchase a more valuable home. However, the purchase of a more expensive home and increased consumption of composite goods are not an obstacle to accumulating wealth because we show that terminal wealth also increases. Finally, as repayment of RRSP withdrawals is a decision whether or not to save at rate r, there is no reason a household cannot participate in the HBP to benefit from the wealth effect without wishing to save. Households that do not save therefore have no incentive to repay the withdrawal.

One further development of the model that would be interesting would be to make the purchase date endogenous. As Fortin (1988) showed that purchase takes place as soon as the household meets the borrowing criteria, we would expect to demonstrate that the HBP permits an earlier purchase of the home, as it facilitates accumulation of the down payment.

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APPENDIX 1: DEMONSTRATION OF THE HBP GAIN

Calculation of the Gain with HBP Repayment and No Loan

Let us begin by examining what happens as regards the accumulation of the three assets when a household does not use the HBP. Initially, we have a wealth of $R = a + (Ph - D) + w(1-m)$, where R is wealth, a represents liquidity, $Ph - D$ is the net value of the home, m is the household's personal taxation rate and w is the RRSP asset.

The accumulation of liquid assets capitalized at a rate of n over k years is expressed by $a(1 + n)^k$. The mortgage debt D is capitalized at rate r. Thus, the net value after k years is expressed by $Ph - D(1 + r)^k$. The RRSP asset, also capitalized at rate r, is worth $w(1 + r)^k$ after k years. These formulae are well-known in the field of financial analysis. The household's wealth after k years, without use of the HBP, will thus be $R = a(1 + n)^k + Ph - D(1 + r)^k + (1-m)w(1 + r)^k$.

Next, let us see what accumulation occurs in the three assets when the HBP is used. Initially, we have a wealth of $R^* = a + (Ph - D + k\theta) + (w - k\theta)$, where R^* is wealth, a is liquidity, $Ph - D$ is the net value of the home, $k\theta$ is the amount withdrawn under the HBP such that θ is the required repayment and w is the RRSP asset.

Accumulation of liquid assets after:

One year: $a(1 + n) - \theta$

Two years: $a(1 + n)^2 - \theta(n + 2)$

Three years: $a(1 + n)^3 - \theta(n^2 + 3n + 3)$

Four years: $a(1 + n)^4 - \theta(n^3 + 4n^2 + 6n + 4)$

We can see a general pattern developing. The term assigned to θ in the equations, stemming from the HBP repayments, can be simplified with the equivalent term $\theta\,[\frac{(n+1)^k - 1}{n}]$, which is the formula for an annuity capitalized at rate n over k years. If we continue the calculations, we will see that, over k years, the accumulation of liquid assets will be obtained by $a(1+n)^k - \theta\,[\frac{(n+1)^k - 1}{n}]$.

Net value of the home after:

One year: $Ph - (D - k\theta)(1 + r)$

Two years: $Ph - (D - k\theta)(1 + r)^2$

Three years: $Ph - (D - k\theta)(1 + r)^3$

It is easy to see that, generally, we will have $Ph - (D - k\theta)(1 + r)^k$

Accumulation of RRSP assets after:
One year: $(w - k\theta)(1 + r) + \theta$
Two years: $(w - k\theta)(1 + r)^2 + \theta(r + 2)$
Three years: $(w - k\theta)(1 + r)^3 + \theta(r^2 + 3r + 3)$

We see, similar to what we have seen previously, that the term assigned to θ can be reduced to $\theta\,[\frac{(1+r)^k - 1}{r}]$, such that the general pattern of RRSP asset accumulation is obtained by:

$$(1 - m)[(w - k\theta)(1 + r)^k + \theta\,[\tfrac{(1+r)^k - 1}{r}]]$$

Thus, total wealth when using the HBP will be expressed by:

$$R^* = a(1+n)^k - \theta\,[\tfrac{(1+n)^k - 1}{n}] + Ph - (D - k\theta)(1+r)^k + (1 - m)(w - k\theta)(1+r)^k + \theta\,[\tfrac{(1+r)^k - 1}{r}](1 - m)$$

We therefore need only see if $R^* - R > 0$ to find that, in this case, the HBP results in a wealth gain compared to a situation in which a household does not participate. Thus:

$$\tfrac{R^* - R}{\theta} = [\tfrac{(1+r)^k - 1}{r}] - m\,[\tfrac{(1+r)^k - 1}{r}] + km(1+r)^k - [\tfrac{(1+n)^k - 1}{n}]$$

Thus, if $m = 0$, the two centre elements of the right-side are zero. Furthermore, in such a case, $n = r$, such that the first and last part cancel each other out. Thus $R^* = R$ if $m = 0$. If $m > 0$, this implies that $r > n$. By means of combinatorial analysis, we then see that $\frac{(1+r)^k - 1}{r} > \frac{(1+n)^k - 1}{n}$ and that $km(1+r)^k > m[\frac{(1+r)^k - 1}{r}]$. Thus, $R^* - R > 0$ if the taxation rate is positive and the household experiences a gain from using the HBP compared to the alternative situation. Demonstration of the other situations is done in a similar manner.

Calculation of Gain Without HBP Repayment and Without Borrowing

When a household decides to participate in the HBP but does not make the repayments required under the HBP, the accumulation equations are changed. Note first that the equations for a situation in which a household decides not to participate in the program remain unchanged, identical to the previous demonstration. It is thus at the HBP participation level that equations will change.

According to the terms of the HBP, the household must make annual repayments. If these repayments are not made, the amounts are taxed, as they are then considered income. Let us look at what not making the repayments changes in the accumulation equations for the three assets in question.

Accumulation of liquid assets after:

One year: $a(1+n) - m\theta$

Two years: $[a(1+n) - m\theta](1 + n) - m\theta = a(1 + n)^2 - m\theta(n + 2)$

Three years: $[a(1+n)^2 - m\theta](n+2)(1+n) - m\theta = a(1+n)^3 - m\theta\,(n^2+3n+3)$

In general, after k years, we see $a(1+n)^k - m\theta\,[\frac{(n+1)^k - 1}{n}]$.

Accumulation of the net value of the home is obtained by $Ph - (D - k\theta)(1 + r)^k$, as with the previous demonstration.

Accumulation of RRSP assets is as follows: $(1-m)(w - k\theta)(1 + r)^k$. Thus, wealth after k years as a result of having used the HBP will be:

$$R* = a(1+n)^k - m\theta[\frac{(n+1)^k - 1}{n}] + Ph - (D - k\theta)(1+r)^k + (1-m)(w - k\theta)(1+r)^k$$

We already know that, according to the previous demonstration, wealth when the household does not use the HBP is $R = a(1 + n)^k + Ph - D(1 + r)^k + (1-m)w(1 + r)^k$, such that, by differentiating between $R*$ and R, we obtain:

$$R* - R = m\theta\,[\frac{(n+1)^k - 1}{n}] + k\theta(1+r)^k - (1-m)k\theta(1+r)^k$$

$$\frac{R*-R}{\theta} = -m\,[\frac{(n+1)^k - 1}{n}] + k(1+r)^k - (1-m)k(1+r)^k$$

After simplification, this expression becomes:

$$\frac{R*-R}{\theta m} = k(1+r)^k - [\frac{(n+1)^k - 1}{n}]$$

and we know that this difference is positive, which completes this portion of the demonstration. The reader may use the same procedure to prove the gain with HBP repayment and with a loan to make the initial withdrawal.

Calculation of Gain With HBP Repayment and Loans

Finally, we will demonstrate that there is a gain even if the household makes HBP repayments and, to do so, must borrow the repayment amount at rate r. Suppose, furthermore, that the household does not initially have liquidity. Note that accumulation of α and w is similar to what we saw previously, in that, after k years, we will have $(1-m)(w-k\theta)(1+r)^k + \theta\frac{(1+r)^k-1}{r}$ for w and $Ph - (D - k\theta)(1 + r)^k$ for α. The difference is in the accumulation of a. In effect, after one year, the household must borrow θ to make the first contribution. After two years, the household owes θ plus the interest on that amount, and must again borrow θ to make the second RRSP contribution. This means that $-\theta(1 + r) - \theta = -\theta(r + 2)$. After three years, the household owes $\theta(r+2)$ plus the interest on this amount, and must again borrow θ, which, after three years, gives an equation of $-\theta(r+2)(r+1) - \theta = -\theta(r^2 + 3r + 3)$. In general, we would have the following equation after k years: $-\theta\,[\frac{(1+r)^k-1}{r}]$.

As with the two previous demonstrations, we need only demonstrate that $R^* - R > 0$. Thus,

$$R^* - R = -\theta\,[\tfrac{(1+r)^k-1}{r}] + Ph - (D-k\theta)(1+r)^k + (1-m)(w-k\theta)(1+r)^k$$

$$+\theta[\tfrac{(1+r)^k-1}{r}] - Ph + D(1+r)^k - (1-m)w(1+r)^k$$, which is equivalent to the expression $R^* - R = mk\theta(1 + r)^k > 0$.

This therefore proves that there is a gain, even in this situation. This gain is equivalent to the amount initially withdrawn from the RRSP, multiplied by the taxation rate and capitalized over k years at rate r.

APPENDIX 2: FIGURES

Implicit Prices when $a(t) > 0$

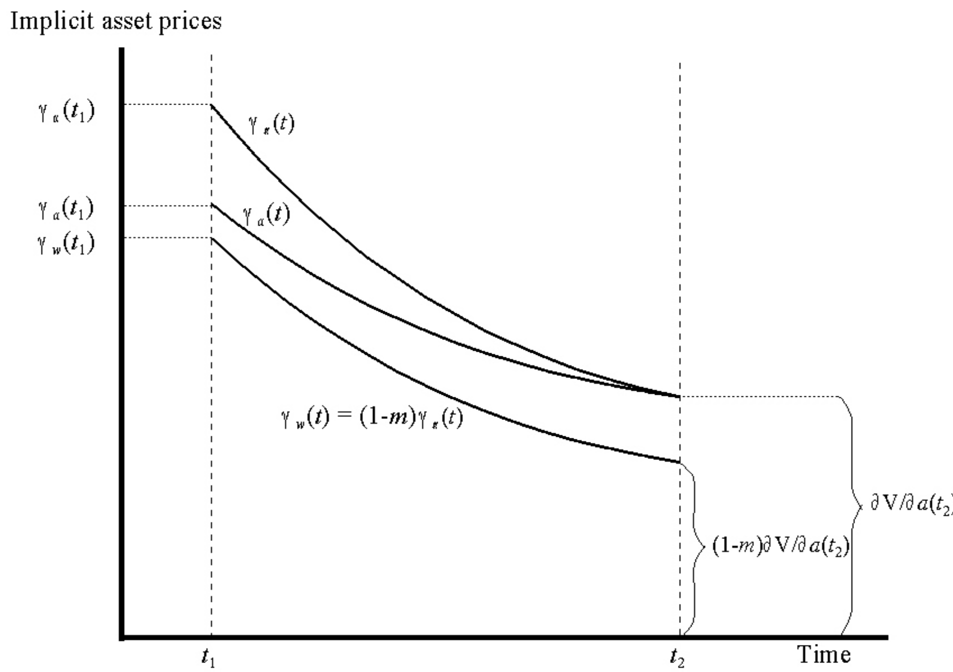

Figure 1

Implicit Prices when $a(t) = 0$

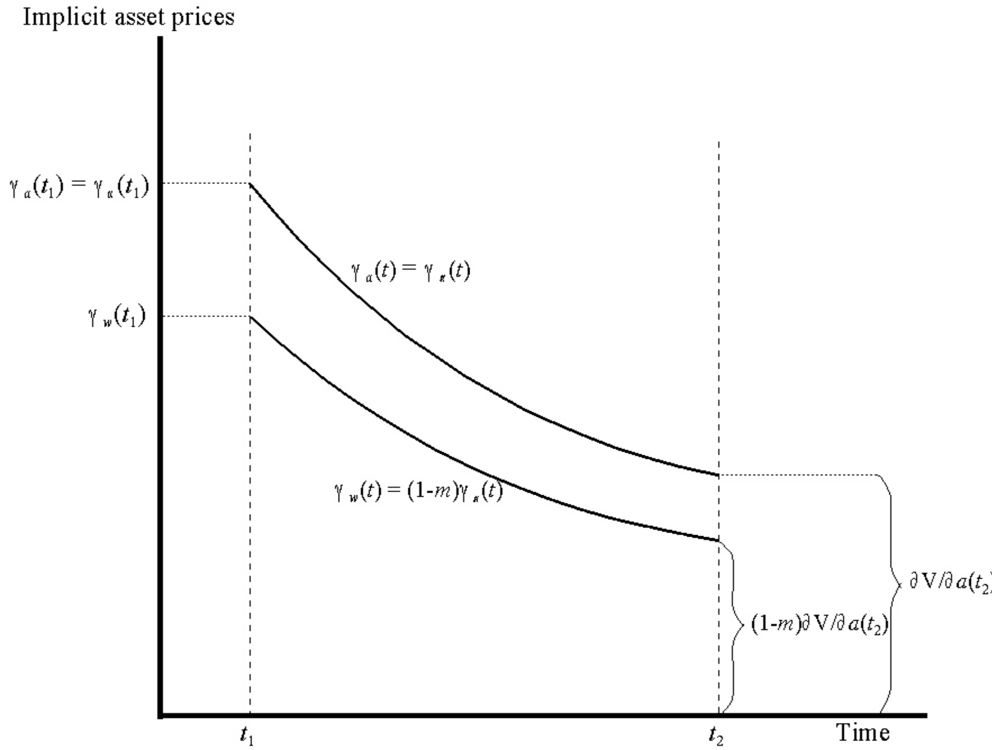

Figure 2

APPENDIX 3: LIST OF SYMBOLS

i: Nominal interest rate on liquid assets, RRSP assets and the mortgage debt
m: Marginal income taxation rate
π: Inflation rate
r: Real gross interest rate (before taxes)
n: Real net interest rate (after taxes)
a: Real value of liquid assets
w: Real value of RRSP assets
P: Real purchase price of a housing unit
h: Number of housing units purchased and the flow of housing services used
D: Real value of mortgage debt
α: Net value of home
R: Real wealth of household
θ: Periodic amount to be recontributed to the RRSP to repay the withdrawal permitted under the HBP
k: Maximum number of periods authorized for repaying the withdrawal permitted by the HBP
t_1: Beginning of plan and date on which the home is purchased
t_2: Date on which the home is sold
T: End of plan
y: Flow of real income
c: Rate of instantaneous consumption of the Hicksian aggregate used as numeraire
δ: Pure rate of time preference
v: Flow of disbursements necessary to amortize the mortgage debt for a capital unit
s: Duration of the mortgage amortization plan
$V(\cdot, t_2)$: Terminal value function, the first argument being the real wealth at t_2
γ_a: Implicit price of liquid assets
γ_α: Implicit price of the net value of the home
γ_w: Implicit price of RRSP assets
μ: Implicit price of the non-negativity constraint on liquid assets
ρ: Real value of the RRSP contribution

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