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# **A Preliminary Review of Nonspectral Wave Properties: Grouping, Wave Breaking, and "Freak" Waves**

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A PRELIMINARY REVIEW OF NONSPECTRAL WAVE PROPERTIES:  
GROUPING, WAVE BREAKING, AND "FREAK" WAVES

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## ABSTRACT

This report presents a review of some aspects of wind-waves which are not usually included in the specification of the sea-state: wave groups, "freak" waves, deep-water breaking, and the occurrence of extreme wave slopes. Available observations pertinent to each one of these topics are briefly reviewed together with an outline of relevant theoretical ideas. The relevance of each type of phenomenon to offshore design conditions is discussed.

An assessment of the state of theory and observations on all four topics shows that, although much recent progress has been made, remaining gaps in information and understanding are too wide to allow a precise account of their influence to be made in offshore design wave estimation.

Because of their limited instrumental response, Waverider buoys cannot yield data which can provide clear answers to questions pertaining to the very strongly nonlinear conditions associated with breaking and extreme wave slopes. We do however make some suggestions for some data analysis which may contribute to the understanding of wave slope statistics and groupiness characterization.

## RÉSUMÉ

Le présent rapport est une récapitulation générale de quelques aspects de la houle causée par le vent qui ne sont pas ordinairement compris dans l'évaluation de l'état de la mer: groupe de houles, vague "monstre," déferlement abyssal et pentes extrêmes des vagues. Les données disponibles pertinentes à chacun de ces sujets sont brièvement exposées, accompagnées des grandes lignes des idées théoriques s'y rattachant. On traite du rapport entre chaque type de phénomène et la conception des structures utilisées en haute mer.

Une évaluation de l'état des données et de la théorie concernant ces quatre thèmes démontre que, même s'il y a eu de récents progrès, les lacunes dans l'information et la compréhension sont trop grandes pour pouvoir avancer une explication précise de leur influence sur l'estimation des vagues dans la conception du matériel hauturier.

Les houlographes, à cause de la réaction limitée de leurs instruments, ne peuvent pas produire des données qui fourniraient des réponses aux questions relatives aux conditions fortement nonlinéaires associées au déferlement et aux pentes extrêmes des vagues. Nous faisons toutefois quelques suggestions pour l'analyse de données qui pourront contribuer à la compréhension des statistiques sur les pentes des vagues et des caractéristiques des regroupements.



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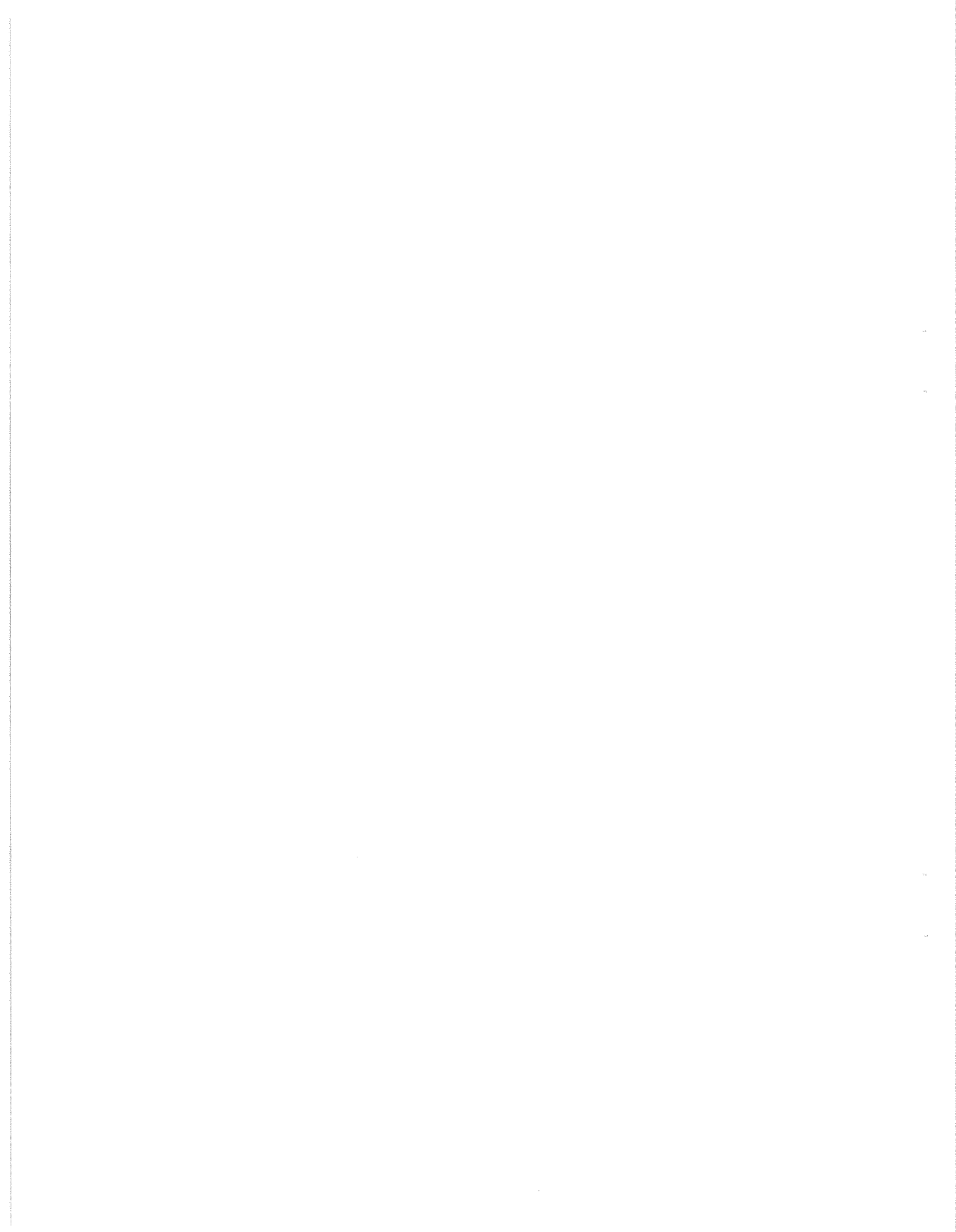
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1. INTRODUCTION

Standard characterizations of sea-state are often made in terms of a relatively small number of parameters, such as significant wave height and peak period. Even the more extensive descriptions in terms of one or two-dimensional spectra are also considerably simplified from the original time-series from which they are derived. There is always information in the original wave data which is not included in any statistical parameterization. This report is concerned with some wave properties which are normally not revealed by statistical descriptions of the sea-state. These are:

- i - Groups: Sequences of larger-than-average waves.
- ii - Unique Events: Rare, extremely high waves, sometimes referred to as "episodic", or "freak" waves.
- iii - Wave Breaking: The tendency for deep-water waves to break.
- iv - Extreme Wave Steepness: The tendency for large waves to steepen more than smaller waves in a sea-state.

The information available in the published literature on each one of these topics will be briefly reviewed here, together with relevant theoretical ideas, and a preliminary assessment will be made of their relevance to offshore design-wave determinations. Recommendations will be made regarding possible use of available Waverider data to increase knowledge of the above phenomena.

## 2. WAVE GROUPS

### 2.1 Questions Arising from Wave Group Occurrence

A sea-state is often represented as a random process, wherein successive waves are not strictly predictable from their predecessors, although they have similar periods and amplitudes, as characterized, for example, by a significant wave height  $H_S$  and spectral peak period  $T_P$ . The wave height amplitudes obey, to a good approximation, a Rayleigh distribution, so that the probability,  $P$ , that a certain wave height  $H$  exceeds a value  $H_C$  is

$$P(H \geq H_C) = P(H_C) = \exp [-2H_C^2/H_S^2] \quad (1)$$

where  $H_S$  is the significant wave height.

There is then some finite probability  $P_N$  that  $N$  successive waves may exceed a chosen wave height  $H_C$ . Such groups of waves larger than those which surround them are indeed observed: a striking example of a wave group is seen in Figure 1.

A number of questions immediately arise concerning the importance of such groups in the description of the sea-state. For example: do groups occur with the frequency predicted by Rayleigh statistics? It is clear that a structure which is exposed to a group of much higher than average waves may encounter forces which are also well in excess of those corresponding to a sea-state which is made up, on the average, of smaller waves than those found in the group. This has been demonstrated, for example by Johnson et al. (1978) in their study of the effect of wave grouping on breakwater stability. One would thus like to know how common wave grouping is in the ocean, as a function of sea-state or perhaps also of storm type, and whether existing theories are capable of explaining the observations of groupiness.

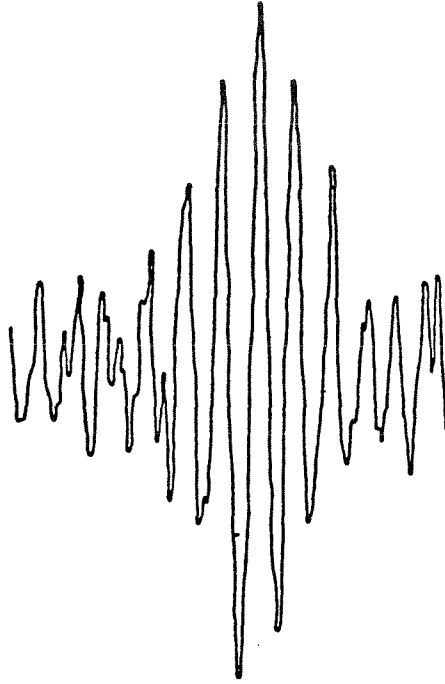


Figure 1. An example of a group of large waves, from a wave record obtained under stormy conditions in the North Sea. From Rye (1974).

## 2.2 Observations

A wave group may be defined as a succession of waves with heights exceeding a certain preset value. Wave groups are readily recognized by visual inspection of a time-series of sea level displacement (Figure 1). Other more quantitative specifications are discussed in the following sections.

Investigations on the presence and length of groups (properties usually termed "groupiness") in measured ocean waves have been reported from the Pacific Ocean (Goda, 1970; Goda and Nagai, 1974), the Gulf of Mexico (Nolte and Hsu, 1972), both sides of the North Atlantic (Wilson and Baird, 1972; Sawhney, 1963, on the western side; Burcharth, 1981, off Cornwall) and the North Sea (Rye, 1974; Siefert, 1976; Burcharth, 1981). The geographical distribution of these observations is broad enough to ensure that the grouping properties have a certain degree of universality and are not peculiarities of waves occurring in certain corners of the globe.

### 2.2.1 Characterization of Groupiness

#### Counting Waves

The simplest way to characterize a wave time-series as to its groupiness is simply to count the number of consecutive waves with heights exceeding the chosen level. Wave heights may be defined as the difference in sea-level between crest and trough following a zero up-crossing or between trough and crest following a zero down-crossing, as shown in Figure 2. Different definitions of wave height lead to different series of numbers. The reference wave height  $H_c$  chosen for defining a group may be any value of  $H$  up to the maximum value in the sample  $H_{max}$ . The counting process is illustrated graphically in Figure 3. In practice, the



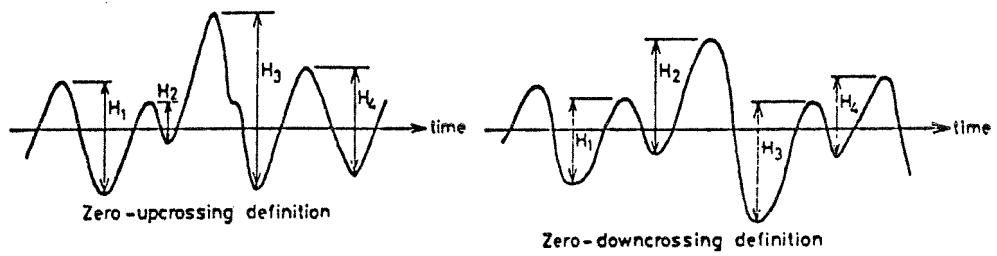


Figure 2. Zero up- and down-crossing definitions, as used by Burcharth (1981).

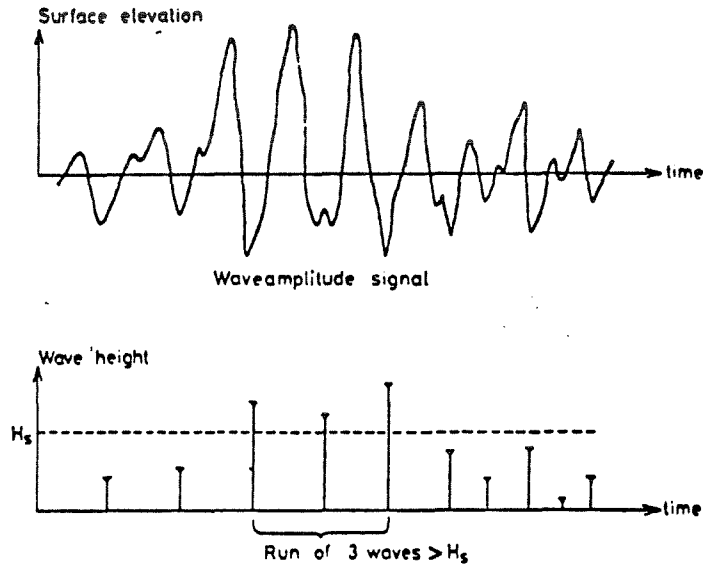


Figure 3. A wave group, as seen in the time-series of sea level variation (above) and as it appears in the sequence of wave heights (below).

significant wave height  $H_s$  is often chosen as the exceedance level. In counting rare events such as sequences of three or more large waves, greatly different results may be obtained for different wave height definitions. An example of this effect is shown in Figure 6 accompanying the later discussion of absolute probabilities.

#### The Envelope Method

Another method of describing groupiness, discussed by Nolte and Hsu (1972) and by Goda (1976), involves a construction of a wave envelope (Figure 4) which smoothly connects the crests and troughs of zero-crossing waves. This technique eliminates the need to worry about which definition of wave-height to use. The envelope of sea-level may then be interpreted as a continuous function of time which smooths out some of the smaller-wave noise in the sea-state. The presence and duration of groups may be detected by finding whether the envelope function, or perhaps a rectified form thereof, lies above a certain level, and for how much of the time.

#### Smoothed Instantaneous Wave Energy History

The envelope method is, however, not free from criticism. In their study of groupiness, Funke and Mansard (1979) found that envelope functions were often quite rough, i.e. varying too quickly in time to be representative wave height averages, and that they sometimes did not readily identify the presence of a group. A more appropriate definition was needed, one in particular which would not be subject to the problems of wave-height definition and that would reflect the increased wave energy present in groups. The Smoothed Instantaneous Wave Energy History,  $E(t)$  [SIWEH] is a form of running average of sea level variance, proportional to wave energy, defined by

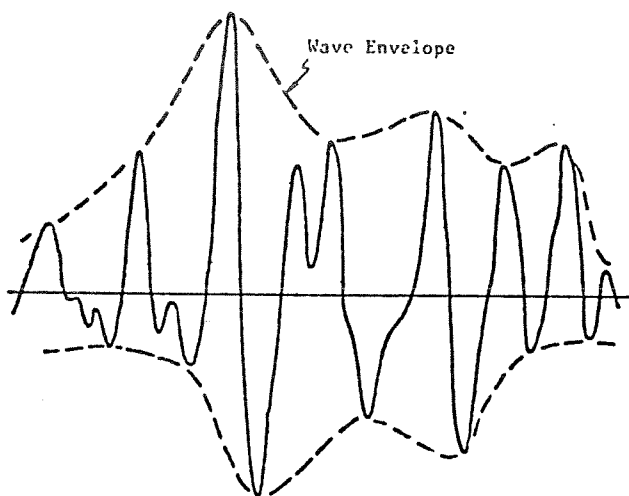


Figure 4. Construction of a wave envelope by joining successive wave crests and troughs.

$$E(t) = \frac{1}{T_p} \int_{\tau=-\infty}^{\infty} \eta^2(t + \tau) Q_k(\tau) d\tau, \quad (2)$$

where  $T_p$  is the wave period which corresponds to the peak of the variance spectrum,  $\eta(t)$  is the instantaneous sea level displacement from its average level and  $Q_k(\tau)$  is a smoothing function which tapers quickly enough away from  $\tau = 0$  that only about one period  $T_p$  is included in the running average process.

Results of the application of the SIWEH smoothing process to a wave record are compared to envelope results in Figure 5. Being tapered, the Bartlett smoothing function

$$\begin{aligned} Q_1(\tau) &= 1 - |\tau|/T_p; \quad |\tau| \leq T_p \\ &= 0 \quad \quad \quad |\tau| > T_p \end{aligned} \quad (3)$$

is not very sensitive to an exact matching of the dominant wave period, which may vary slightly through a wave record. This smoothing function is that adopted by Funke and Mansard (1979).

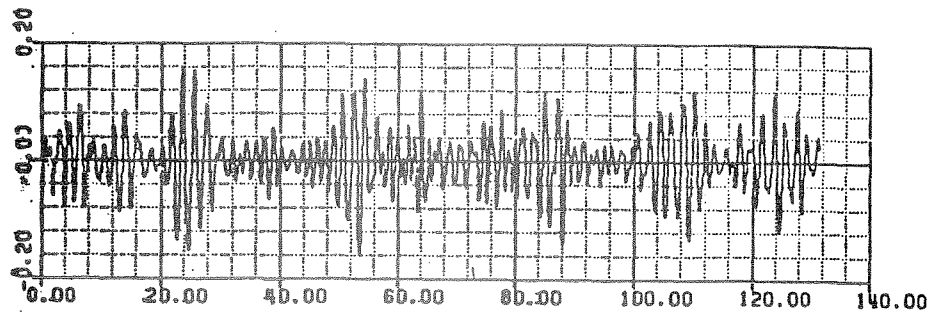
It can be seen from Figure 5 that the SIWEH is a sharper indicator of wave groups than the two forms of wave envelope shown. The SIWEH is however proportional to the average energy, not to the wave height, so that one cannot simply estimate the number of waves above a certain height by monitoring the level of the SIWEH and its duration above a certain level. For a sinusoidal wave of height  $H$  such that

$$\zeta = \frac{H}{2} \sin \frac{2\pi t}{T_p} \quad (4)$$

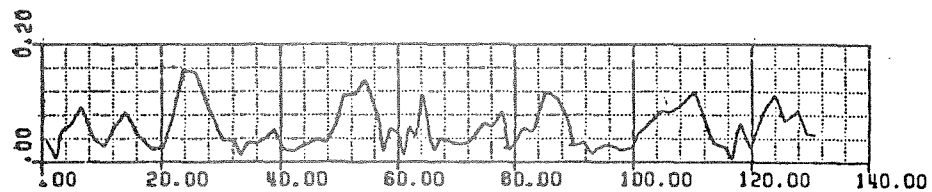
where  $\zeta$  is the wave amplitude,  $E(t)$  may be found by integration of

$$E(t') = \frac{H^2}{8\pi} \int_{-2\pi}^{2\pi} \sin^2(z + t') \left(1 - \frac{|z|}{2\pi}\right) dz, \quad (5)$$

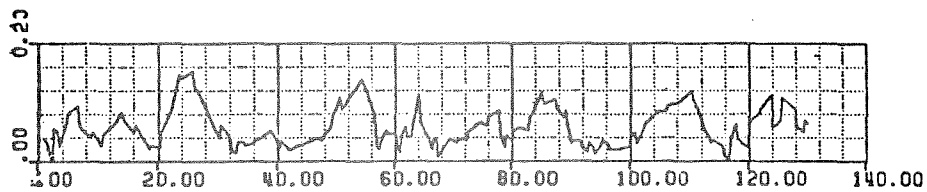
with  $t' = 2\pi t/T_p$ ,  $z = 2\pi\tau/T_p$ .



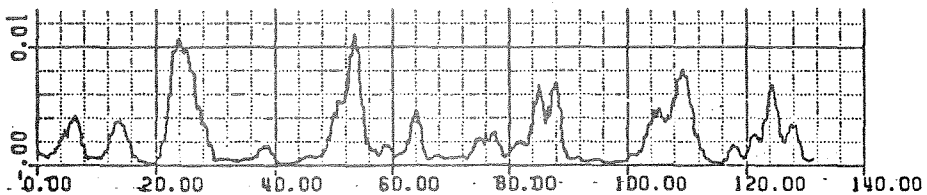
A WAVE RECORD



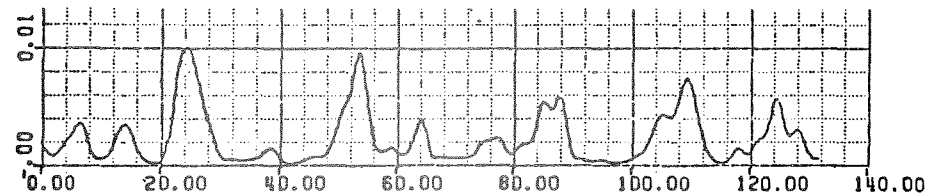
THE HALF-WAVE RECTIFIED ENVELOPE



THE FULL-WAVE RECTIFIED ENVELOPE



THE SMOOTHED INSTANTANEOUS WAVE ENERGY HISTORY  
USING RECTANGULAR SMOOTHING



THE SMOOTHED INSTANTANEOUS WAVE ENERGY HISTORY  
USING BARTLETT SMOOTHING

Figure 5. Application of the SIWEH method to wave group characterization for two types of filters  $Q_k$  and comparison with envelope methods. From Funke and Mansard (1979).

The levels of  $E(t)$  can then be related to values of  $H$ , although at the price of assuming the wave form (4). Although the SIWEH provides a powerful diagnostic tool for investigations of groupiness, it is nevertheless a derived quantity which cannot unambiguously be related back to the original sea level displacement record. The SIWEH method has not yet been applied to the study of natural wave groups. All observations summarized below draw on wave counting or envelope estimates of group properties.

### 2.2.2 Groupiness Statistics

#### Absolute Probabilities

How many groups are there within a typical sea-state? This answer is found in terms of "absolute probabilities" of wave groups, such as those shown in Figure 6. From all the waves observed in the field records at Hanstholm, about 8% were found to equal or exceed  $H_S$  in height (as measured using the down-crossing definition) without being followed by another one like it: those are "groups" of one; reading along the field data curve in Figure 6, for example, about 3% are doublets (two values of  $H \geq H_S$  in succession), 0.8% triplets and 0.15% quartets. The total percentage of waves occurring in groups of  $H \geq H_S$  is then the sum  $\sum_j S_j$  where  $j$  is the number of waves in a group and  $S_j$  its probability. The Hanstholm data show that about 12% of the waves lie in such groups, i.e. from one to four waves in sequence. From Rayleigh statistics (1) we expect that 13.5% of the waves would have heights  $H \geq H_S$ . It is not surprising to find such good agreement: Rayleigh statistics are a good approximation to observed wave height distributions. A more revealing kind of observation would concern the relative abundances of groups containing different numbers of waves.

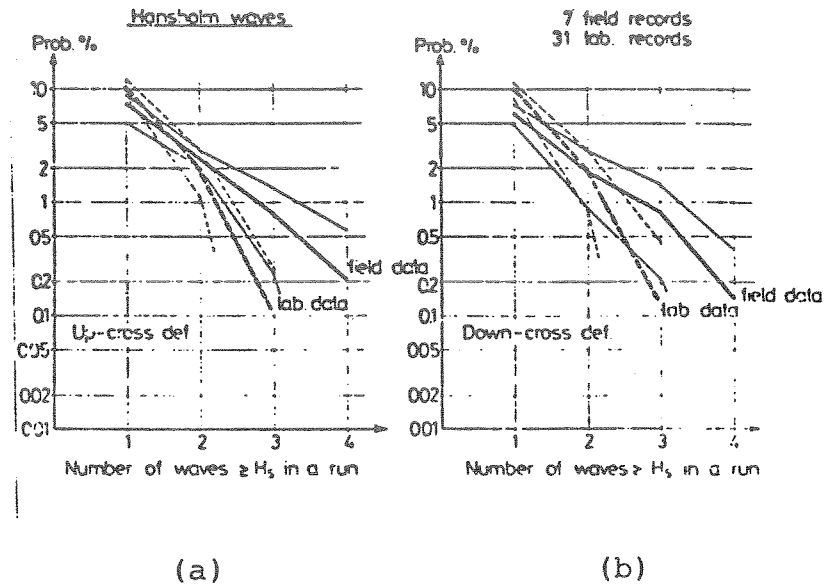


Figure 6. Absolute probabilities of waves of height greater than  $H_s$  occurring in groups as a function of the number of waves in the group. Hanstholm is a site on the northwest coast of Denmark, in 20 m of water. Note the difference in the results when up- or down-crossing definitions of wave height are used. The thinner lines above and below the heavy mean "field data" and "laboratory data" curves are one standard deviation on either side of the mean. From Burcharth (1981).



### Relative Probabilities

Relative probabilities of occurrence of groups of  $j$  waves within the  $N$  waves with  $H \geq H_s$  are estimated by calculating the ratios  $n_j/N$ , where  $n_j$  is the number of runs of  $j$  waves in the sample. An example of the frequency distribution of the run length of waves, extracted from Goda (1976) is given in Table I. The observed relative probabilities are compared to the probabilities which would be expected to be obtained in a sample of uncorrelated waves obeying Rayleigh statistics, whereby one expects that the probability of a run of  $j$  waves with height  $H \geq H_s$  is

$$P_{1j} = P(H_s)^{j-1} (1-P(H_s))$$

with  $P(H_s)$  given by (1).

The most striking result found in most of the observations (Wilson and Baird, 1972; Rye, 1974; Goda, 1976; Siefert, 1976; Burcharth, 1981) is that there are more groups of many waves (3 and more) than would be expected from theory. The only exception is found in the observations of Nolte and Hsu (1972). We shall return to a discussion of possible reasons behind the unexpected frequency of long groups in Section 2.3. The excess relative probability of long groups with respect to the theoretical expectation stands out most clearly in graphical form (Figure 7). The mean length of runs observed is about 1.4 waves, compared to a theoretical expectation of 1.16.

Another relevant observation mentioned by Goda (1976) is that runs which contain  $H_{\max}$ , the largest wave in a sample, contain on average more waves than runs of waves with  $H > H_s$  (2.4 waves compared to 1.4 waves). In other words, the highest wave is often accompanied by several other high waves.

TABLE I

Observed frequency distribution of lengths of runs of wave height  $H \geq H_S$ .

	$n_j$	$n_j/N$	$P_{1j}$	$n_j^*$
$j = 1$	1327	0.708	0.865	1619
2	374	0.199	0.116	218
3	122	0.065	0.015	29
4	37	0.019	0.002	4
5	9	0.005	$3 \times 10^{-4}$	0.6
6	2	0.001	-	-
7	1	-	-	-

Total number of waves,  $M = 20,051$

Number of waves with  $H \geq H_S$ :  $N = \sum j n_j = 2653$ ;  $N/M = 0.132$

Total number of groups,  $G = \sum n_j = 1872$

Mean number of waves in a group (run length) =  $N/G = 1.4$

$P_{1j}$  is the expected relative probability from Rayleigh statistics

and  $n_j^*$  is the number of waves in groups of  $j$  expected from  $P_{1j}$ .

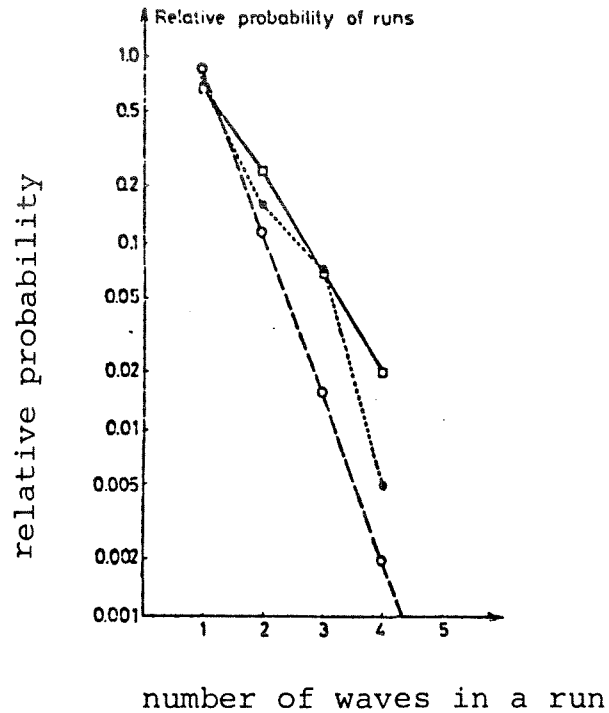


Figure 7. Relative probabilities of runs of waves of height greater than  $H_s$  as a function of the number of waves which they contain. Squares refer to data from Hansthalm (in the North Sea, off northern Denmark in 20 m of water); full circles from Perran Bay (on the north side of Cornwall, in the Celtic Sea, in 22 m of water) and open circles to the expected values from Rayleigh statistics. The wave heights in the field observations were defined by the zero up-crossing technique. From Burcharth (1981).

### Dependence on Spectral Peakiness

Goda (1970) noted that long wave groups tend to be more frequent in very peaky spectra. His theoretical work on this problem will be reviewed in Section 2.3. Rye (1974) also noted that long groups were more frequent during growing sea conditions in the North Sea, with peaky wave spectra of the JONSWAP type, than during wave decay, characterized by flatter spectral peaks (Figure 8). Rye also found that there was a non-zero correlation between successive waves (+0.24 on the average at one wave lag) and that the correlation was larger (0.3) during wave growth than during decay (0.2).

Data presented by Goda (1976) on the mean length of runs of large waves (Figure 9) as a function of the peakiness factor  $Q_p$  show some trend but also a large amount of scatter. The peakiness factor of the spectrum  $S(f)$  is defined as

$$Q_p = \frac{2}{m_0^2} \int_0^{\infty} f S^2(f) df \quad (6)$$

with  $m_0^2 = H_s^2/16$ .

### Dependence on Water Depths

Siefert (1976) has presented measurements of relative probabilities of wave groups in very shallow water, near the mouth of the Elbe River, in depths ranging from 12 m to the surf zone itself. His results indicate that in very shallow water the probability of long runs is diminished from that observed in deep water and is nearly the same as that calculated by Goda (1970) using Rayleigh statistics. Low frequency motions identified as "surf beat" in the nearshore zone

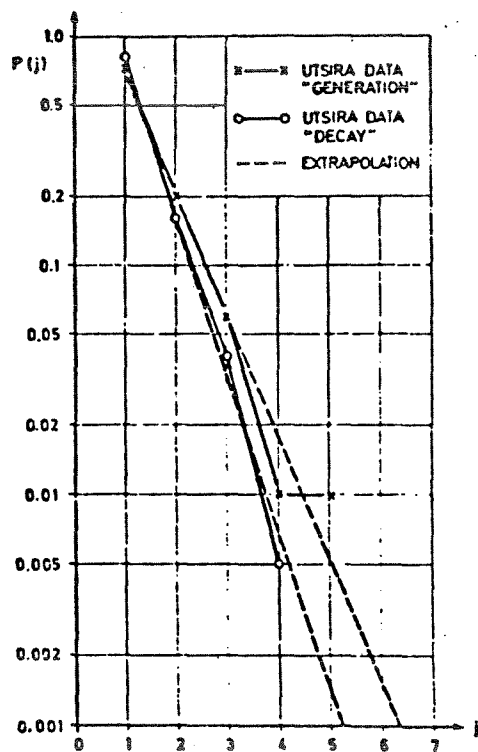


Figure 8. Relative probabilities of occurrence  $P(j)$  of groups of waves with heights greater than  $H_S$  as a function of the number of waves in a group,  $j$ , for conditions of wave generation and wave decay at Utsira, in a water depth of 100 m off the southwest coast of Norway. From Rye (1974).

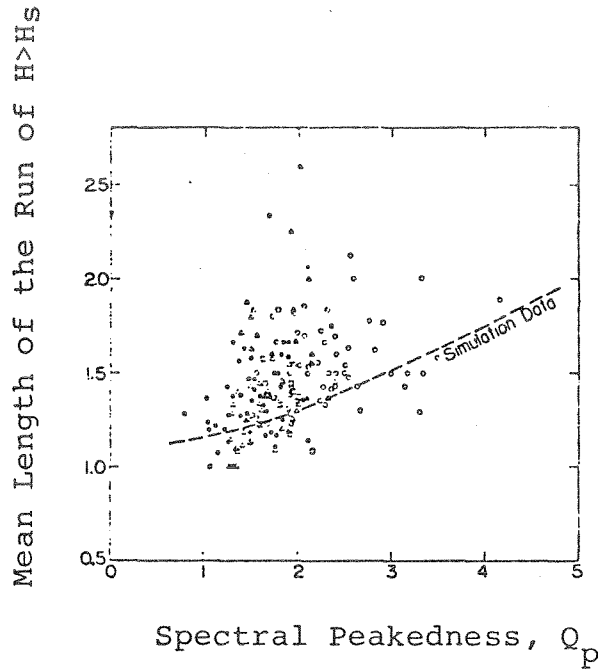


Figure 9. The mean number of waves in groups of waves of height greater than  $H_s$  as a function of the spectral peakedness factor  $Q_p$  defined in (6). From Goda (1976).

may well be related to deep-sea wave groups. Holman et al. (1978) have found sharp increases in low-frequency gravity wave energy during the passage of a storm: they associate this increased activity with the arrival of groups of waves generated by the storm. Shore based observations might thus provide some confirmation of Rye's (1974) results on higher groupiness in growing seas.

### 2.2.3 Summary of Observed Results

The essential results based on observed wave groups are:

- 1) Runs of many high waves occur more frequently than predicted by simple Rayleigh statistics.
- 2) Groups which contain the highest wave in a record contain more waves than those which do not.
- 3) Groups appear to be more frequent in growing seas, and in sea-states characterized by more highly peaked spectra.
- 4) Long runs of high waves are rarer in shallow than in deep water.

## 2.3 Some Theoretical Considerations

Theoretical studies of wave groups have followed three main avenues: i) - the search for a proper method of characterizing wave groups in a time-series; ii) - probabilistic studies of group frequency; iii) - modulation of sea-waves and propagation of envelopes as wave packets.

### 2.3.1 Group Recognition in Time-Series

Following on techniques of group characterization by wave counting or envelope formation, the SIWEH concept introduced above is the latest method of recognizing

the presence of groups of waves in a time-series of sea level displacement. The SIWEH was introduced to explicitly identify the presence of groups in wave records and to be able to simulate waves with an equivalent degree of groupiness. A groupiness factor, GF, was introduced by Funke and Mansard (1979) to describe wave group activity:

$$GF = \left\{ \frac{1}{T_n} \int_0^{T_n} (E(t) - \bar{E})^2 dt \right\} / \bar{E}, \quad (7)$$

where  $E(t)$  is defined as in (2),  $\bar{E}$  is the average of  $E(t)$  over the wave record of duration  $T_n$ . Examples of wave records with different values of GF are shown in Figure 10. These authors have also discussed the use of the SIWEH in the synthesis of wave trains in laboratory scale modelling applications of appropriate groupiness. The SIWEH method is clearly a useful tool in these types of wave studies - there has, however, been to date no published application of the technique to the analysis of ocean waves, so that it is difficult to relate SIWEH results to field measurements using other types of wave group characterization.

### 2.3.2 Statistical Studies

A straightforward calculation of the expected relative probability of occurrence of runs of  $j$  waves (Goda, 1970, 1976) based on the assumption of complete independence of successive waves (i.e. Rayleigh statistics), showed that long groups (more than 2 waves) occur more frequently than theory would predict. The observations of Rye (1974), that there is some correlation between successive waves, does explain in part why Rayleigh statistics would fall short in predicting the frequency of the longer runs. The assumption of complete indepen-



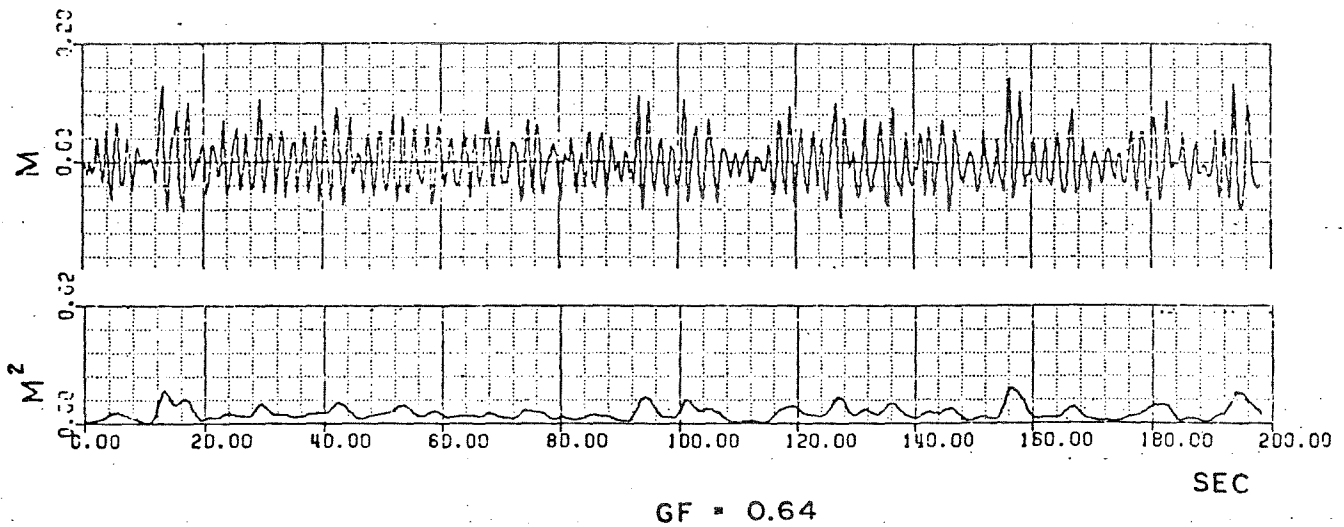
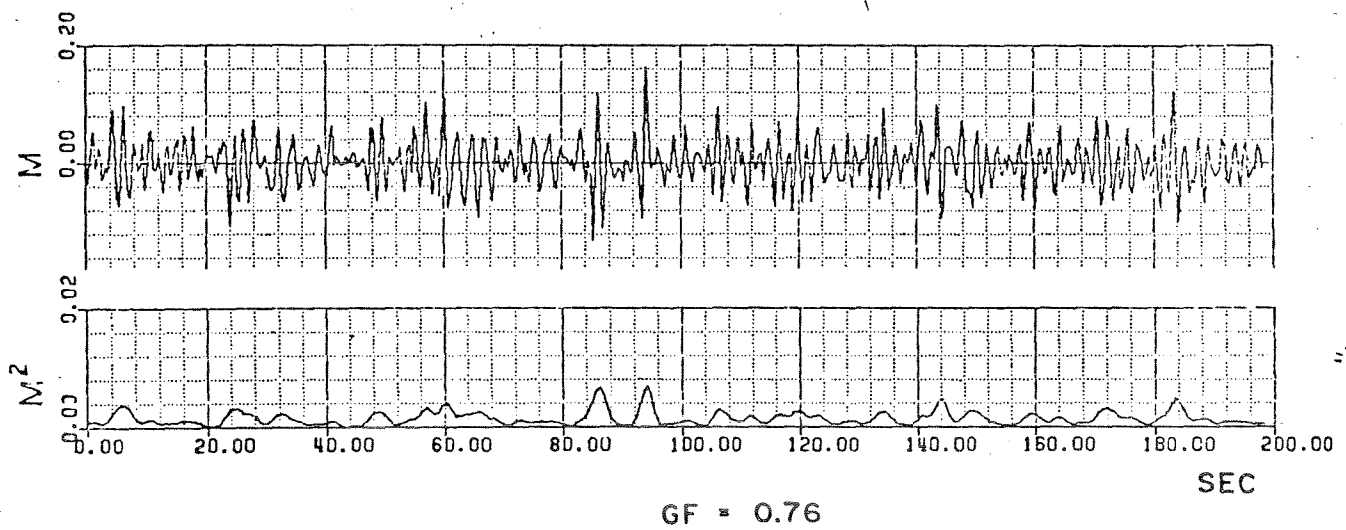
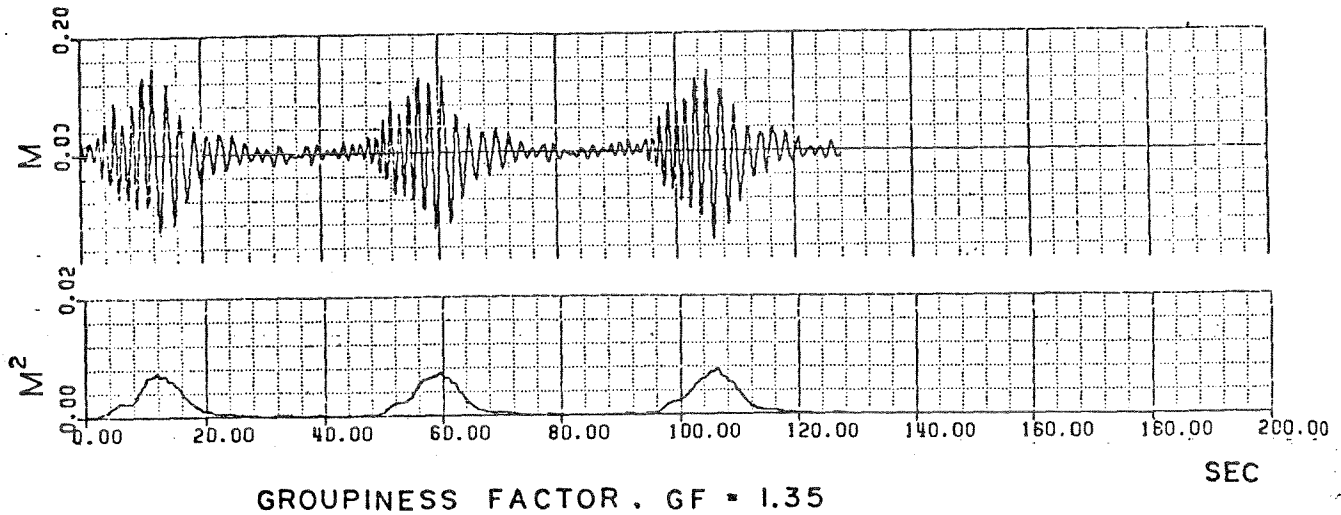


Figure 10. Examples of wave time-series and SIWEH for different values of the groupiness factor  $GF$ , as defined by (7). From Funke and Mansard (1979).

dence between waves is equivalent to representing the waves by a very wide bandwidth spectrum. Nagai (1973) and Ewing (1973) have looked into wave grouping probability in narrow band spectra. Goda (1976) has used envelope theory and results from Longuet-Higgins' (1957) studies to find the dependence of the mean length of wave runs on spectral width, as represented by the peakedness factor  $Q_p$  defined in (6). Results are shown in Figure 11 and are seen to compare well with simulated data of spectral form  $Af^{-m}\exp[-Bf^{-n}]$  (to have confidence in these results, one would, however, like to see comparisons with real data as well).

A different kind of statistical study is that presented by Hamilton et al. (1979). These authors proposed a nonspectral statistical model of the sea-state consisting of sequences of uncoupled wave groups. The model is founded on a representation of the sea-state as a series of phase-locked oscillations travelling at the same speed, which contrasts with the random superposition of independent Fourier components underlying spectral models; the basis for this different viewpoint will be discussed below. The idealized wave group structure presented in this model helps one to understand the origin of statistical errors in the spectrum and to develop methods for masking unwanted spikes in it. The authors do not claim that their model is a faithful, or even realistic description of the sea surface; it does however account for the observed waviness of the tail of the correlation function and provides some empirical guidance in reducing errors in spectral estimates.

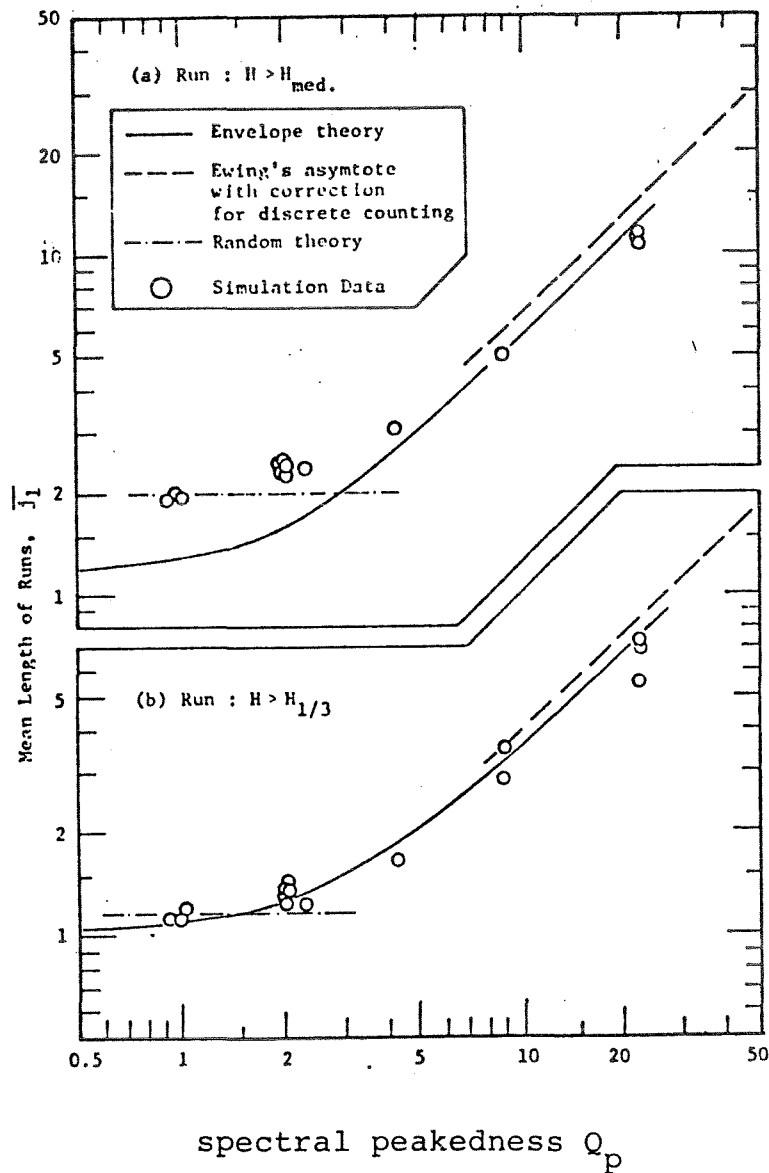


Figure 11. Comparison of the results of Goda's (1976) envelope theory calculations of the mean number of waves in a group as a function of spectral peakedness  $Q_p$ . Random theory refers to the results obtained for a very broad band process using Rayleigh statistics; the asymptote for very narrow spectra is based on Ewing (1973). From Goda (1976).

### 2.3.3 Groups as Wave Envelopes

The presence of long groups in excess of what is expected from a purely random theory, suggests that for the narrow spectra which characterize real sea-states, mechanisms may be at work which favour the formation and the preservation of wave groups.

Studies of the stability of surface gravity waves have yielded a series of new results on the evolution of wave trains; an excellent review of the subject is that of Yuen and Lake (1980).

Early theoretical investigations of the evolution of weakly nonlinear waves by Lighthill (1965) and Benjamin and Feir (1967) showed that deep-water wave trains were subject to a modulational instability, whereby a modulated sea-state would grow out of a uniform wave train, as shown in Figure 12. The ratio of the modulation frequency to the dominant wave frequency  $f_e/f$  was calculated by Longuet-Higgins (1980c) as a function of wave steepness. Theoretical and laboratory results are compared in Figure 13. The evolution of the modulation itself was first found by Zakharov (1968) to satisfy a nonlinear Schroedinger equation, possessing envelope soliton solutions which can propagate without changing their form. Conditions are thus favourable to the formation as well as to the coherent propagation of wave groups. Laboratory experiments by Lake et al. (1977) showed, however, that a modulated wave train would eventually return to its initial, nearly uniform configuration. This return to the original condition is known as a Fermi-Pasta-Ulam recurrence (cf. Scott et al., 1973) and is characteristic of systems obeying the non-linear Schroedinger equation. The influence of randomness in the wavefield on modulational instability has been investigated by Crawford et al. (1980), who

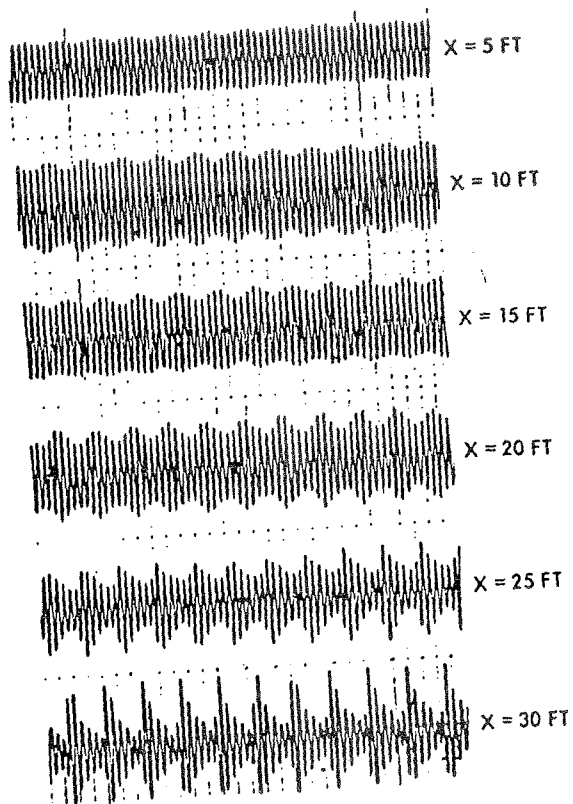


Figure 12. The disintegration of a regular wave train, showing the growth of modulational instability in a wave tank ( $x$  indicates the distance from the wavemaker). From Yuen and Lake (1980).

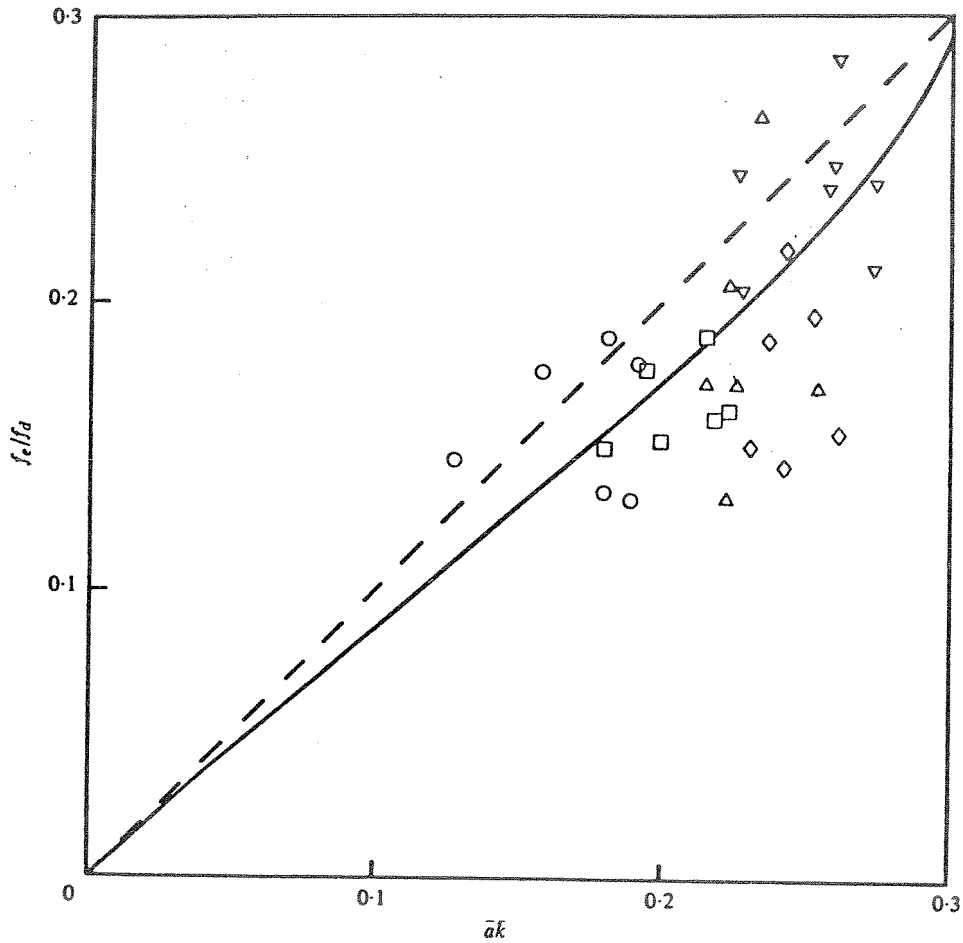


Figure 13. The ratio of the modulation frequency ( $f_e$ ) to the dominant wave frequency ( $f_d$ ) as a function of wave slope  $ak$ . Observation points (circles, etc...) are from laboratory data from Lake and Yuen (1978). Oceanic data would be found at lower values of slope, i.e.  $ak \approx 0.1$ . From Longuet-Higgins (1980c).

found that the reversible transfer of energy leading to modulation and recurrence occurs much more rapidly than the irreversible spectral redistribution of energy which leads to peak steepening, as in the JONSWAP spectra.

Further research has revealed the possibility of existence of more complex envelope patterns: two- and three- dimensional soliton patterns of varied structure (Dysthe, 1979; Hui and Hamilton, 1979; Saffman and Yuen, 1980; McLean et al., 1981).

Modulational instability leads to a redistribution of spectral energy: as the nonlinear wave train evolves, the spectrum widens and contracts as recurrent uniform and modulated states succeed each other. Figure 14 shows an example of this phenomenon. Since modulational instability occurs on a rapid time scale, one might then speculate (Lake & Yuen, 1978) that the spectral form of observed sea-states might be due as much to this phenomenon as to the combination of energy input and exchange mechanisms in a random wave superposition (the usual picture). In the random wave model each wave propagates independently, at the phase speed appropriate to its frequency; in the nonlinear modulation model, all waves with frequencies above that of the spectral peak are locked-in with the dominant one and travel at the same speed. Measurements of wind waves in a laboratory set-up (Figure 15a) (Ramamonjiarissoa, 1974) and some field observations (Figure 15b) (Ramamonjiarissoa and Giovanangeli, 1978; Von Zweck, 1969) have been interpreted in favour of the modulation model (Ramamonjiarissoa and Mollo-Christensen, 1979). This interpretation has however been severely challenged by Huang (1981), so that there is no convincing evidence to date of phase locking of the high frequency part of the spectrum

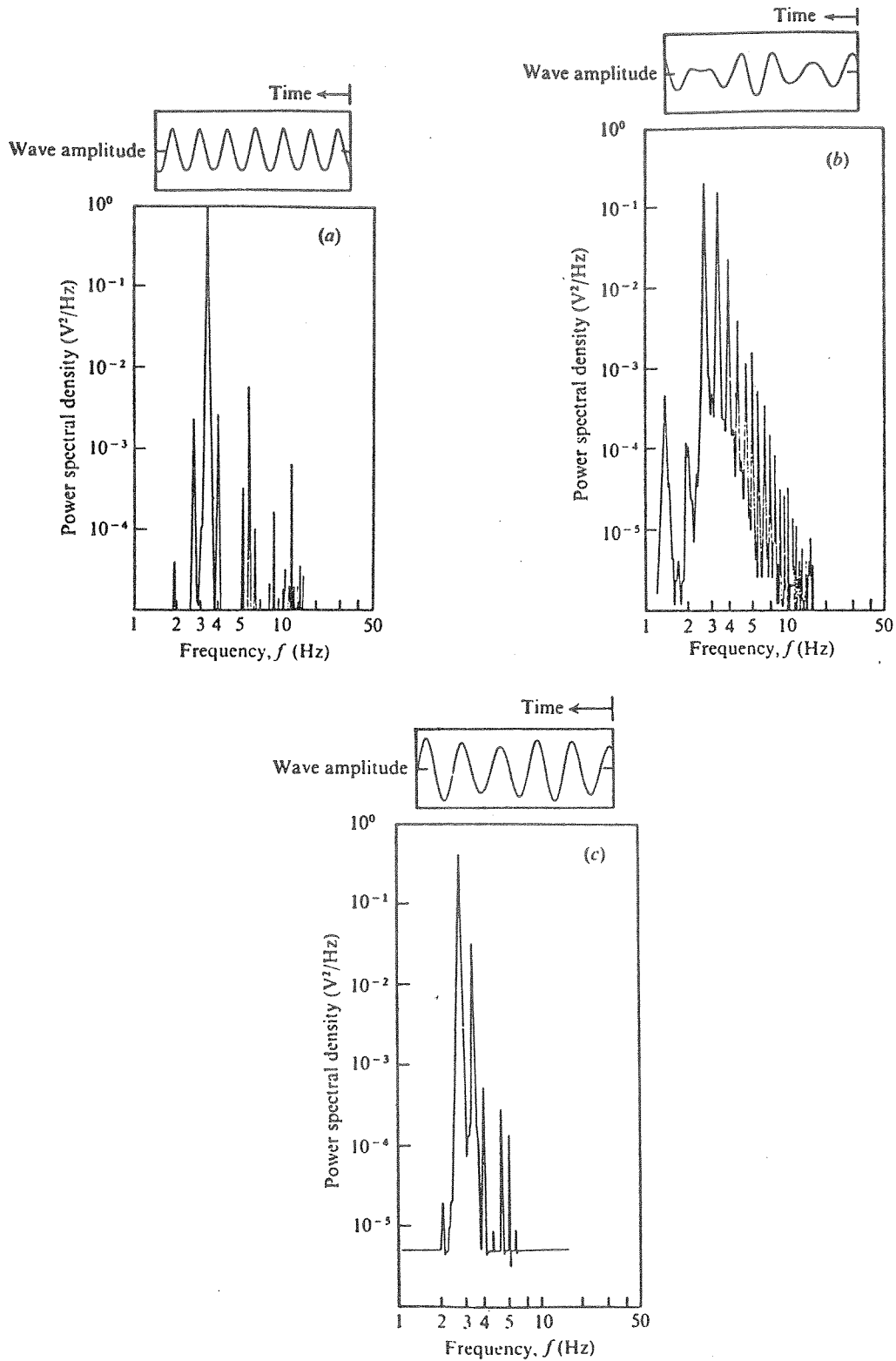


Figure 14. Spectra of laboratory waves at different stages of the modulational instability recurrence cycle: characteristic segments of wave form are shown above the spectra. From Lake et al. (1977).



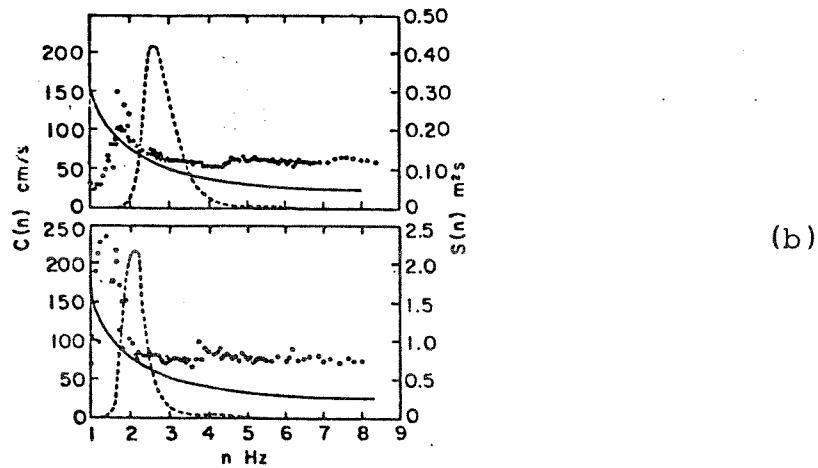
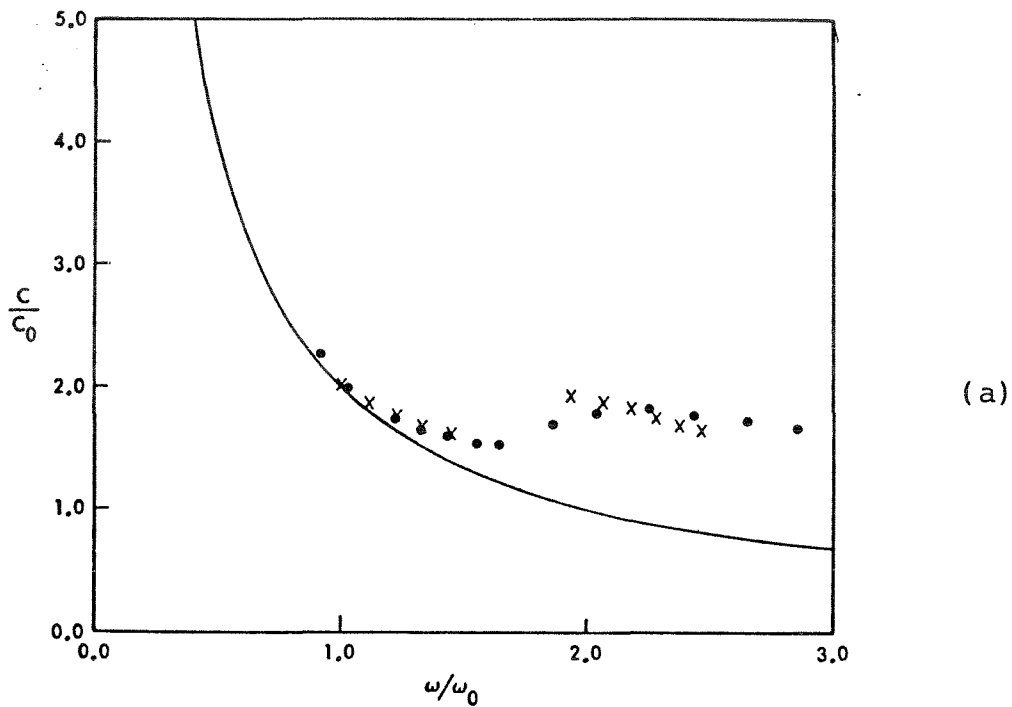


Figure 15. The phase speed of a) mechanically generated laboratory waves (no wind), (from Yuen and Lake, 1980) and b) real waves observed at sea (from Ramamonjariarisoa, 1974). In both cases, frequencies above the spectral peak travel at the same speed as the dominant wave rather than at the lower speed corresponding to their frequency. The solid curves in both graphs show the dispersion relation for free waves. The double dashed curves in b) show the energy spectrum.

with the peak frequency wave in nature. Nevertheless, Mollo-Christensen and Ramamonjariisoa's (1978) model of wave groups in a random wave field may still be of some relevance to actual sea-states. In this model, a number of wave-envelope solitons are superimposed to reproduce the sea-state: a characteristic feature of the spectra corresponding to this superposition of groups is the presence of higher harmonics of the peak frequency. The authors show some unfiltered spectra which show peaks at approximately the right frequencies, but admit themselves that the evidence is far from conclusive.

According to these new ideas, groups are a very essential part of the observed sea-state. Mollo-Christensen and Ramamonjariisoa (1978) go as far as representing the complete sea-state in term of a superposition of a particular type of groups. While the evidence still heavily favours the random Fourier superposition model of sea-state, it is possible that a better representation might include some aspects of both models.

#### 2.4 Relevance to Offshore Design

Roberts (1981) has explained the crucial role which low frequency sea-level oscillations can play on the response of large floating structures with long resonance periods. This role, coupled with the observation that the largest waves to be encountered are likely to occur within a group, make knowledge of the groupiness of sea-surface extremely important in estimating the worst conditions to be met by offshore structures. In this light, the present knowledge of the distribution and frequency of groups of waves on the sea-surface appears very meager. We know nothing of the spatial distribution of wave groups on the plane, i.e. of the low frequency two-dimensional modulations

of the sea-surface in a storm. Theoretical developments are not sufficiently advanced to suggest answers, although further analysis is likely to reveal what may be expected in a realistic directional sea-state.

The controversy on the nature of the sea-state (random wave field vs. phase-locked modulation) might seem rather abstract and unrelated to wave forces on offshore structures, since both models are likely to yield essentially the same spectral form. This is an unjustifiably optimistic view: the energy spectrum does not contain phase information and does not tell the whole story. Phase locking on the peak frequency might affect wave steepness significantly. If higher waves are in random phase with respect to the spectral peak frequency, more or less steep waves may result depending on how these waves superimpose; phase-locking on the other hand imposes definite phase relations which may make the forward face of large waves much steeper on the average than if phase relations were random.

Theoretical studies of groupiness and of the relative roles of modulational instability and irreversible spectral energy transfers should be encouraged: they are relatively inexpensive, compared to observational work. Observational programs to measure groups could include analysis of existing punctual data but would be difficult to extend to two-dimensions. Were it but possible to extract from these tantalizing SEASAT synthetic aperture radar images a reliable estimate of a two-dimensional SIWEH! Some progress in that direction has actually been reported recently by Larsen (1982).

3. UNIQUE EVENTS

Under "unique events", we include observations of waves of gigantic size and unexpected proportions, sometimes referred to as "freak" or "episodic" waves. The very rarity of such events constrains the description of the observations to the anecdotal level. Furthermore theories of their formation are very difficult to verify.

3.1 Observations

Very large waves are encountered at sea during any strong storm. Reports indicate however that during a storm there sometimes occur, unexpectedly, a wave of truly enormous size. Reviews of incidents involving encounters of large passenger liners and bulk carriers with such waves have been presented by Dawson (1977) and Britton (1978). The Queen Mary (81,000 tons) nearly capsized during a North Atlantic storm in 1943 when it was caught broadside by a gigantic wave which tore gear off the decks and smashed glass windows in the bridge (Figure 16). The 632 ft. tanker Texaco Oklahoma was broken in two by a giant wave in the North Atlantic in 1971. Another tanker, the 258,000 ton Svealand had its bulbous bow wrenched off by a wave off the east coast of South Africa. Many smaller ships have disappeared without warning or subsequent trace, perhaps following encounters with such waves. Fixed structures are perhaps even more vulnerable to impact from very large waves, since they can't maneuver out of the storm area: the disappearance of the "Texas Tower" radar platform off New York City a few years ago, or the more recent tragedy of the semisubmersible Ocean Ranger on the Grand Banks are but two possible examples. The SEDCO 135-F, drilling off the B.C. coast in 1968 survived a

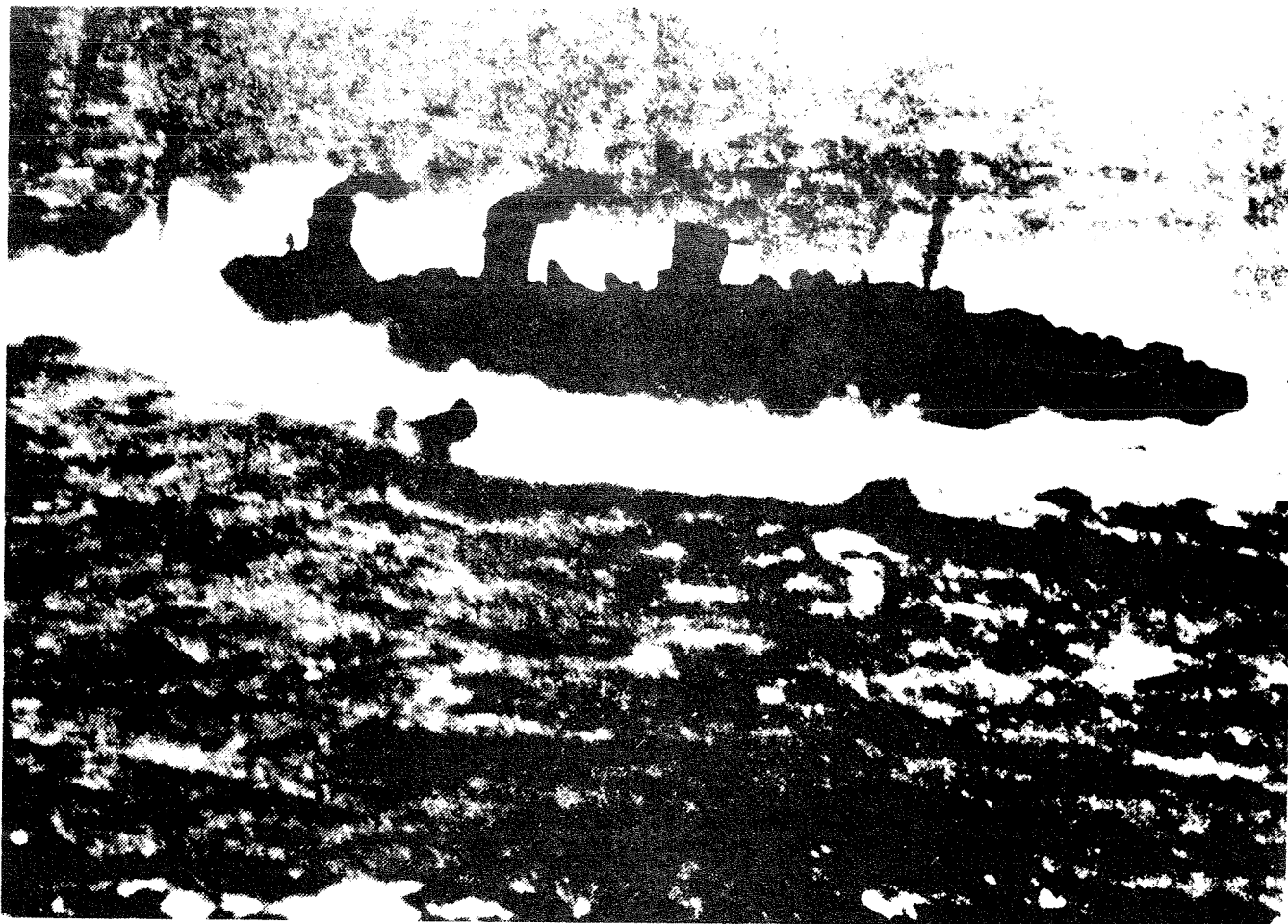


Figure 16. Photograph of the Queen Mary encountering a very large wave broadside. This occurred during a North Atlantic crossing in 1943. From Teagle (1978).

gigantic 95 ft wave (Anon, 1969).

Reports of encounters with giant waves have come from a number of areas. The North Atlantic, perhaps because of the high traffic density across it, has contributed many incidents. Kjeldsen et al. (1980) have reported that freak waves also seem to occur off Nova Scotia, near the Bermuda Rise, as well as off Greenland and the Norwegian Coast, where 25 Norwegian vessels were lost from 1970-77 due to capsizing. One area of the ocean which seems particularly prone to dangerous waves is the edge of the continental shelf on the east coast of South Africa, in the south-flowing Agulhas current. Mallory (1974) has documented eleven cases of vessels having encountered abnormal wave conditions or having foundered as a result of storm waves in that area. The abnormal wave conditions were usually met just offshore of the 100 m fathom line; none were reported inshore of that depth contour. The worst wave situations reported by Mallory (1974) and Sanderson (1974) consist of an unexpectedly long wave trough, a "hole in the sea", followed by an extremely sharp slope to the next crest. These conditions are illustrated in Figure 17, and seem to indicate that it is not so much the height of the wave, as its extreme slope which makes it so dangerous. This is indeed the point on which theoretical explanations focus.

#### Summary of Observations

From what little information is available there seems to exist two populations of unique events, or "freak" waves. Specimens of the first population are encountered in the deep ocean, far from coasts or currents, in heavy storm seas. They are characterized by their height as well as their slope. Members of the second group are found in a region of strong shear flow, the Agulhas Current, and seem to be distinguished by their

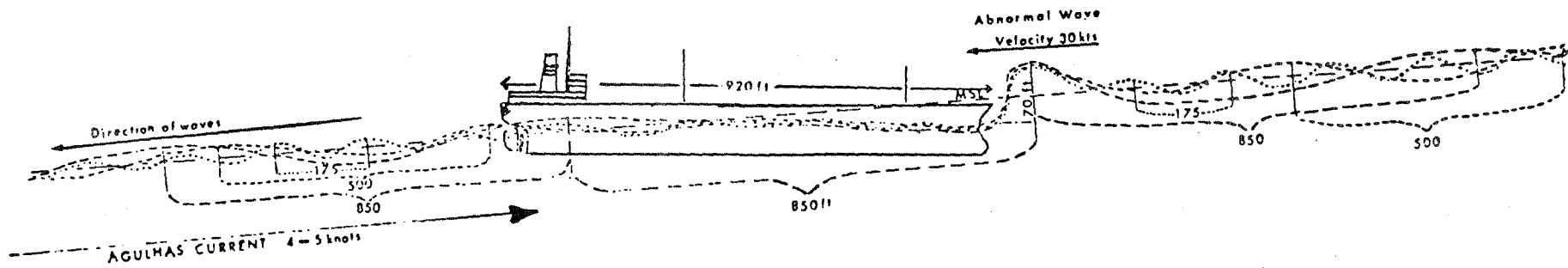


Figure 17. The wave superposition mechanism proposed by Mallory (1979).

*Seacorsulf*

extreme slope rather than by their height.

### 3.2 Theoretical Explorations

The simplest, elementary explanation of the formation of freak waves with high amplitudes and extreme slopes is based on the idea of interference between waves of different lengths which could temporarily add up in the proper fashion, as shown in Figure 17. This picture has the advantage of simplicity: it is however difficult to reconcile with the standard view of a sea-state with a peaky spectrum, wherein the large waves are found in a narrow band of wave lengths. It is not clear how waves of comparable amplitudes and quite different wavelengths can interfere in the manner shown in Figure 17 for a real sea-state. If the occurrence of extremely steep slopes in high waves is a matter of random interference, their probability could be evaluated in a fairly straightforward fashion to provide a more solid basis for Mallory's (1979) interference explanation. This has not been done yet. We shall return to this point in our discussion of wave slopes in the next section. As it is, we cannot at present judge whether some or any of the freak waves reported are possible extreme values of sea conditions which could be deduced from the methods of extreme analysis or whether they do represent very special conditions.

A more quantitative theoretical explanation of the waves observed in the Agulhas Current has been presented by Smith (1976). Long swell propagating against a sheared current is refracted and steepened while gaining energy from the mean flow. Smith calculates a possible amplification factor of 4 for the wave height of 12 sec waves on the Agulhas Current. The highest waves would occur in a narrow region near a caustic where the waves are reflected, in agreement



with Mallory's (1974) reports of the presence of a narrow region of abnormal wave observations.

### 3.3 Consequences for Offshore Design

The reported existence of extremely large and steep waves is, of course, a concern for offshore operations. One should distinguish however between the two populations of abnormal waves.

It may well be that the freak waves reported in the deep sea, i.e. those of the first population, are probable rare extremes for which provision is already made in the design process. More attention should perhaps be given to the joint probability of extreme slopes and extreme wave heights in this process. On the other hand, the presence of abnormal waves in the Agulhas Current, together with a theoretical model which explains how such waves can arise there, infers that similar wave conditions could occur in other currents (the Labrador Current, for example) and that it would be prudent to extend Smith's (1976) model to such waters.

## 4. DEEP-WATER WAVE BREAKING

The nonlinearity of the free-surface boundary conditions is what makes surface gravity waves complicated and interesting. Breaking is the ultimate manifestation of strong nonlinearity, and is one of the least understood aspects of water waves. We focus our attention on wave breaking in deep-water, which is unrelated to shoaling effects when gravity waves interact with the sea bottom.

### 4.1 Observations

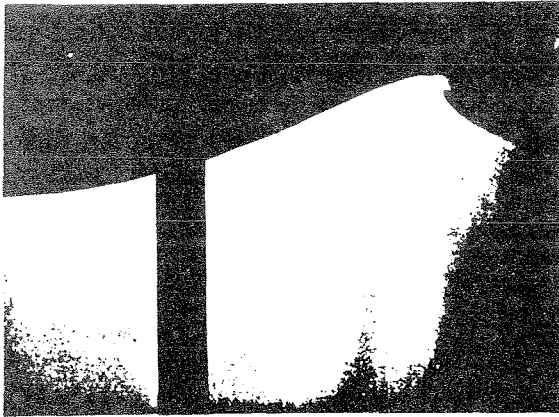
#### 4.1.1 Breaker Types

A breaking wave is one in which water particles near

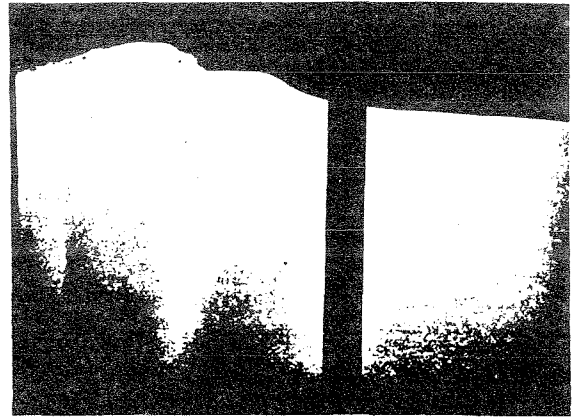
the wave crest move forward at a speed greater than the propagation speed of the wave profile. There are two main types of breaking waves in deep-water (Mason, 1952): 'plunging breakers', in which the wave crest curls forward and plunges deeply into the forward slope of the wave, some distance from the crest, and 'spilling breakers', in which the broken water spills more gently into the forward slope as a quasi-steady whitecap. Kjeldsen and Myrhaug (1978a, 1979), in a series of laboratory experiments, have also observed breaking in the form of deep-water bores, but the spilling breaker is by far the more common type. The three types of breakers observed by Kjeldsen and Myrhaug (1979) are shown in Figure 18; the developments of each type is illustrated in Figure 19.

The spilling breaker is characterized by the fact that the wave breaks at the very top of the crest, with fluid elements sliding down the leading slope (Figure 19c). When the breaking is vigorous, an extensive whitecap is seen, in which air is entrained into the spilled water. For weak breaking, however, spilling may take place very gently, without whitecapping. As pointed out by Banner and Phillips (1974) breaking itself may thus be far more widespread than indicated by the occurrence of whitecaps.

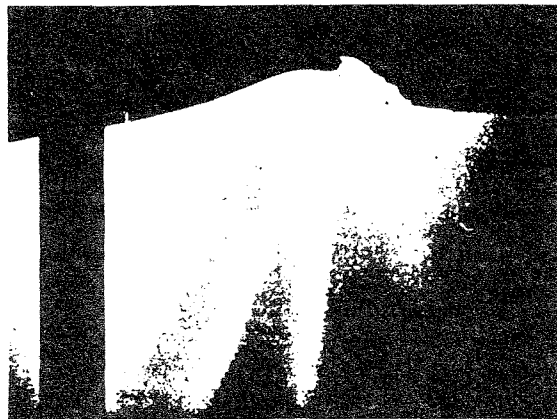
The deep-water bore appears as a highly nonlinear wave-wave interaction, wherein one wave overtakes another. Kjeldsen and Myrhaug (1979) describe this breaker as being characterized by a foam zone covering up to one third of the total wave height, over which air entrainment occurs. The top part of the wave proceeds over the lower part in a way similar to the travel of a tidal bore in a river (Figure 19b).



(a)



(b)



(c)

Figure 18. Profiles of water surface in deep-water breaking laboratory experiments of Kjeldsen and Myrhaug (1979), showing in a) a plunging breaker, b) a deep-water bore, c) a spilling breaker.

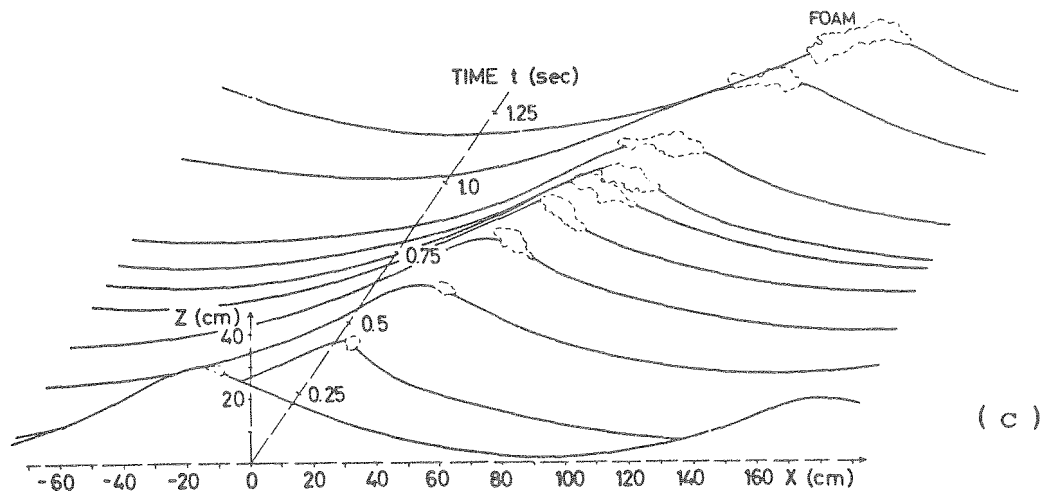
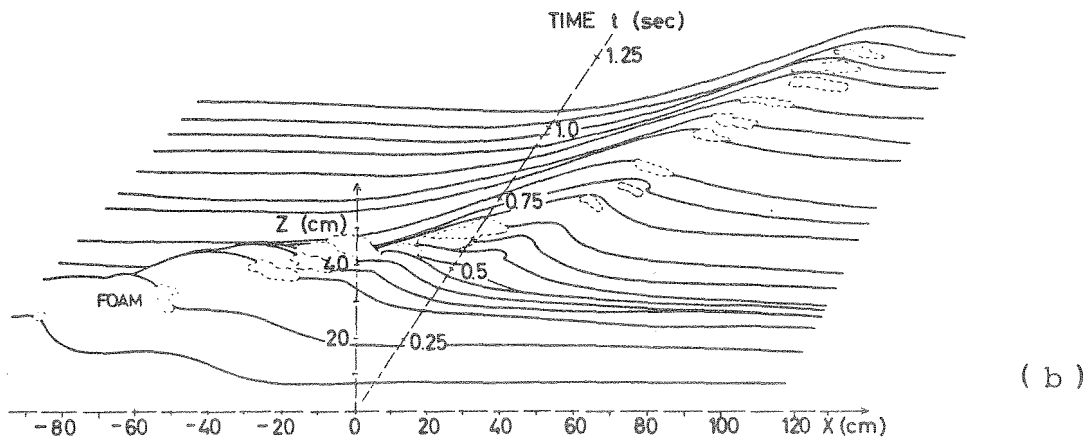
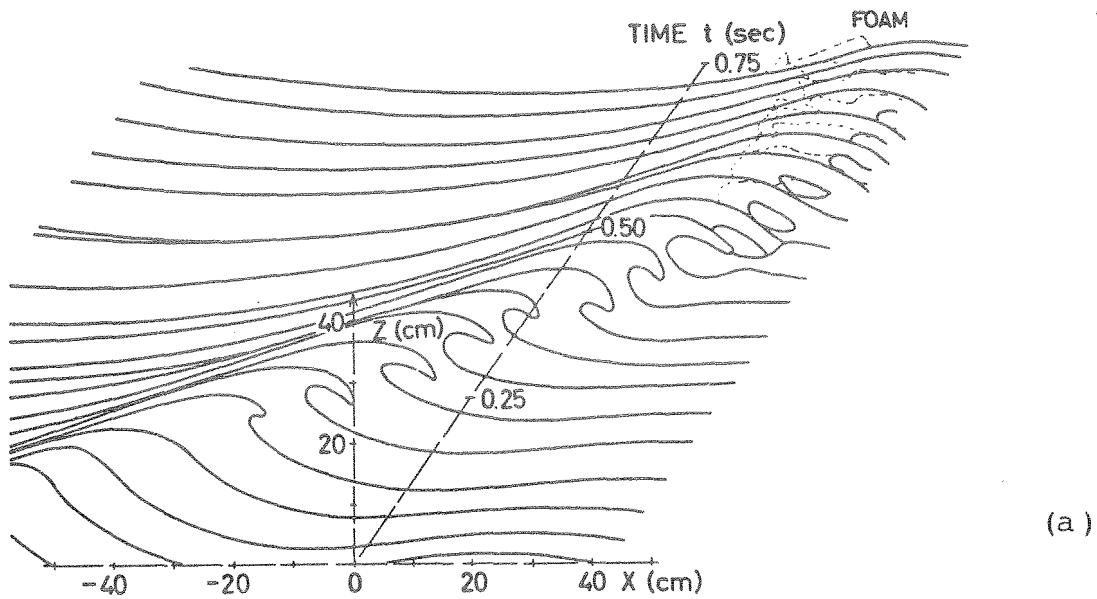


Figure 19. Synoptic film recordings of a) a plunging breaker, b) a deep-water bore and c) a spilling breaker, showing the evolution of each type of deep-water breaker in Kjeldsen and Myrhaug's (1979) experiments.

Deep-water plunging breakers were found in Kjeldsen and Myrhaug's (1979) experiments to result from strong highly nonlinear interactions of several waves in a wave train, as well as in conditions under which waves run into an opposing current. The evolution of the plunging breaker is seen in Figure 19a.

The most relevant parameter in the classification of deep-water breakers was found by Kjeldsen and Myrhaug (1979) to be the crest-front steepness  $\epsilon = \eta/L$ , with  $\eta$  = the crest elevation above mean water level and  $L$  the horizontal distance from the zero up-crossing point which immediately precedes the crest to the position of the crest itself. The secondary effects of other parameters, such as backslope steepness and asymmetry are also discussed in Kjeldsen and Myrhaug (1978a). A result of great importance to the interpretation of field data is the fact that the total wave steepness (height/wave length between successive zero crossings) is NOT uniquely defined for the asymmetric waves studied and cannot be used as a simple parameter related to wave breaking. Breaking waves in the laboratory had a value of  $\epsilon$  between 0.32 and 0.78, with plunging breakers having the highest values.

#### 4.1.2 Water Velocities in a Breaker

A consequence of wave breaking is to induce a strong shear in horizontal current near the surface. This effect has been observed in laboratory conditions by Donelan (1978); it has also been measured by Kjeldsen et al. (1980). Consequences on air-sea interaction and spilled oil drift are discussed by Donelan (1978), by Phillips and Banner (1974), and by Naess (1980). Actual extreme particle velocities reported by Kjeldsen et al. (1980) reach up to 2.8 times the (linear) wave propagation velocity in plunging breakers.

#### 4.1.3 Pressures Due to Breaking

Measurements of shock pressures produced by deep-water waves have been reported by Kjeldsen and Myrhaug (1979). Results for a typical record are shown in Figure 20. Very sharp peaks are noted on the leading edge of the wave.

#### 4.1.4 Field Data

Data from a Waverider buoy located off the coast of Norway (Tromsøflaket; 71°30'N, 19°00'E) in a water depth of 230 m were analysed by Kjeldsen and Myrhaug (1978b, 1979). The statistics of forward slope steepness of 25,000 waves with heights greater than 5 m were found (Figure 21) to be well approximated by the Rayleigh distribution

$$P(\epsilon > \epsilon_C) = \exp(-\epsilon_C^2 / \epsilon_0^2). \quad (8)$$

The probability of  $\epsilon$  exceeding a value  $\epsilon_C$  is expressed in terms of the r.m.s. value  $\epsilon_0$  (0.107 in the field data).

In view of the importance of asymmetry in wave breaking, future field data might be subjected to a bi-spectral analysis, as proposed by Masuda and Kuo (1981a). Applications to field data (Masuda and Kuo, 1981b) show that the imaginary part of the bi-spectrum is an indicator of asymmetry and of "leaning forward" of waves, especially noticeable during wave generation.

#### 4.1.5 Spatial Distribution of Wave Breaking

How much of the sea surface is occupied by breaking waves as a function of sea-state? This question is relevant to a number of physical transfer processes at the sea surface, as well as to estimation of forces to be expected on structures.

SHOCK PRESSURES AND WAVE FORM  
FOR A SPILLING BREAKER

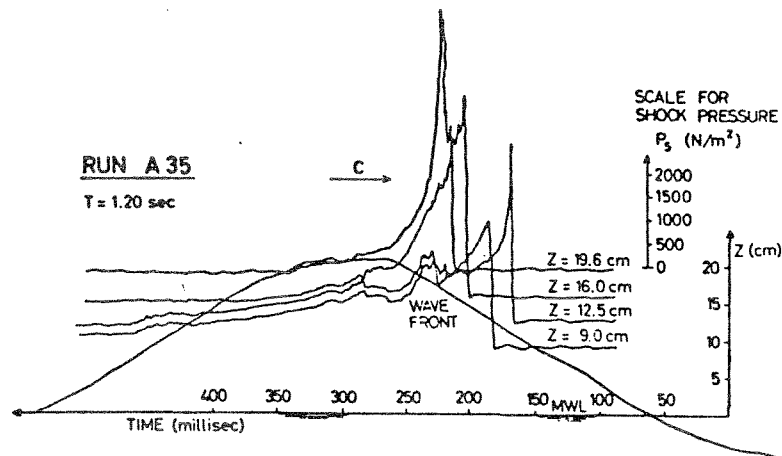


Figure 20. Shock pressures measured in the laboratory for a deep-water spilling breaker. The wave front is also indicated. Values of Z are the levels of the pressure transducers above mean water level. From Kjeldsen and Myrhaug (1979).

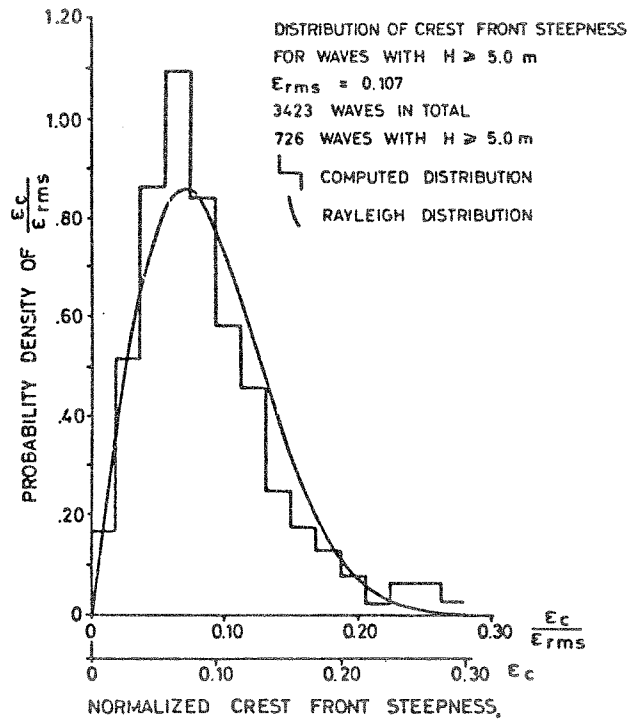


Figure 21. Distribution of forward slope steepness, from Kjeldsen and Myrhaug, 1979.



Measurements of whitecap coverage, in terms of the fraction of the sea surface  $W$  covered by whitecaps, have been made photographically in the Atlantic by Monahan, (1971) and in the East China Sea by Toba and Chaen (1973). The results of these observations have been analyzed by Wu (1979) in terms of a power-law relationship of the form

$$W = AU^B, \quad (9)$$

where  $U$  is the wind speed.

Statistical fits of all available data by Monahan and Muircheartaigh (1980) give  $A = 2.95 \times 10^{-6}$ ,  $B = 3.52$  for ordinary least square (OLS) fitting and  $A = 3.84 \times 10^{-6}$ ,  $B = 3.41$  for "robust biweight fitting". Fitted curves are seen in Figure 22. There is a lot of scatter in the data and little information at very high wind speeds. Further observations are clearly required.

Some authors (Donelan et al., 1972) have noted a certain periodicity in whitecap distribution. It was suggested by these authors, and later by Thorpe and Humphries (1980) that this regularity might be associated with wave groups. The latter propagate at the group velocity, while individual crests travel at the phase speed (twice as fast in deep-water) so that successive crests reach the top of the group one after the other and then break, giving a series of breaking points downwind.

#### 4.1.6 Summary of Observations

Little is known about deep-water wave breaking, at least in comparison with the enormous body of information available on breaking in shallow water. The work of Kjeldsen and his co-workers is extremely important in that it begins to document deep-water

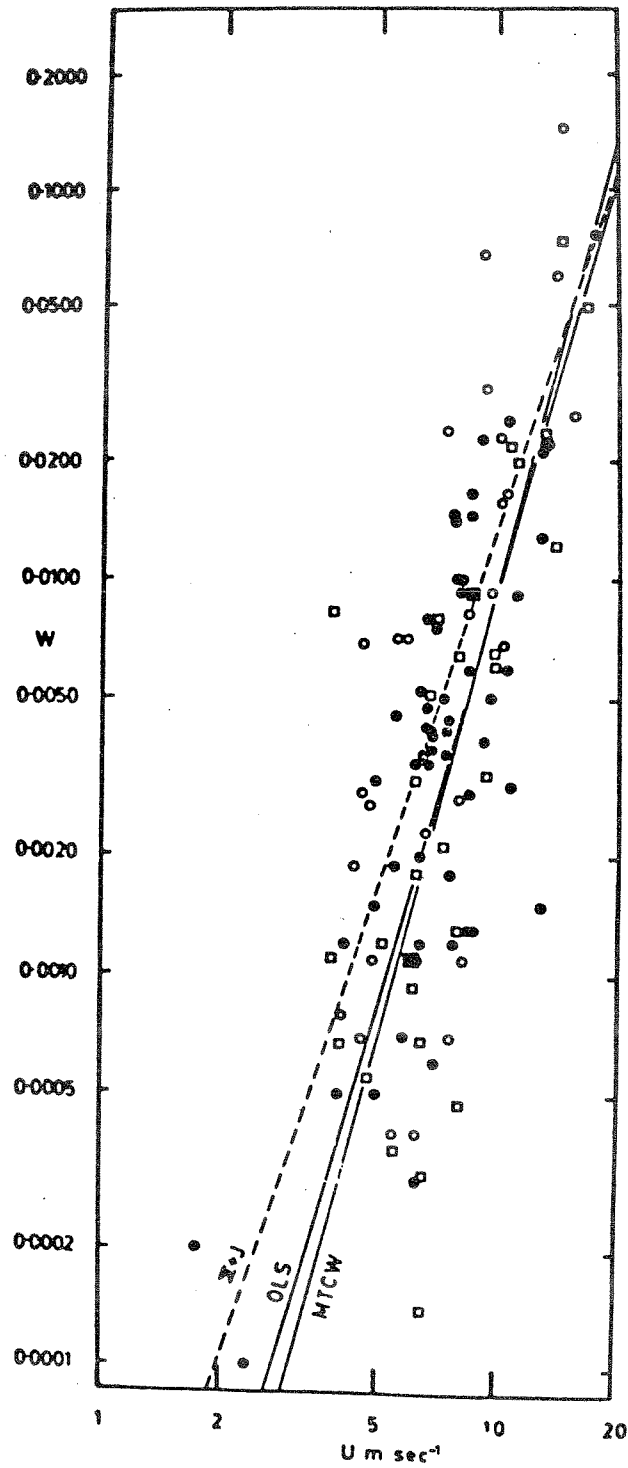


Figure 22. The fraction  $W$  of the sea surface covered by whitecaps against the wind speed  $U$  at 10 m. Full circles are Monahan's (1971) data; open squares, Toba and Chaen's (1973). The OLS line refers to ordinary least square fitting of those data;  $\Sigma + J$  includes in addition data from the JASIN experiment; MTCW is Wu's (1979) fit. From Monahan and Muircheartaigh (1980).

wave properties and their effect on structures. Spatial distributions of wave breaking are not known at high wind speeds. Presently known distributions cannot readily be converted to estimates of mean forces exerted unless more information on the strength and type of breakers is obtained.

#### 4.2 Theoretical Models for Breaking Waves

There has been a great amount of progress in the description and understanding of large amplitude surface gravity waves in recent years. Longuet-Higgins (1980a) has reviewed recent developments in a field where he has been one of the main contributors. In particular, the question of how breakers and white-caps actually form is now much better understood.

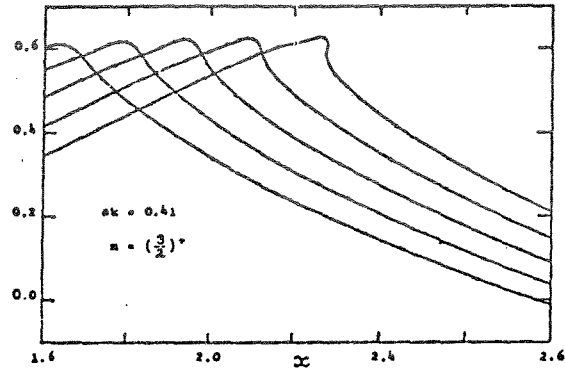
Exact forms for the steepest possible water waves of periodic (Longuet-Higgins and Fox, 1978) or solitary shapes (Longuet-Higgins, 1973) have been calculated with improved numerical techniques. Excellent simple approximations to both types of waves (Fox, 1977; Longuet-Higgins, 1974) have been discovered. It was also discovered, however, that the sharp-crested limiting waves are not necessarily the fastest or most energetic: speed as well as energy content both reach a maximum for a wave steepness below its extreme value (Longuet-Higgins and Fenton, 1974; Cokelet, 1977). It was then realized that breaking could occur before the formations of a sharp crest (Longuet-Higgins and Cokelet, 1976) in a wave raised (by the wind, perhaps) to an energy level greater than its maximum. Other mechanisms for breaking involving instabilities were discussed by Longuet-Higgins and Cokelet (1978) in a subsequent paper. Two kinds of instability enter: subharmonic instabilities,

discussed earlier as modulational instabilities, and which tend to lead to bunching of waves into groups, and local instabilities confined mainly to the wave crest and with a rapid rate of growth. This is the kind of instability which leads to rapid overturning, as verified numerically by those authors (Figure 23). Different numerical simulations of plunging breakers by Kjeldsen et al. (1980) give similar results (Figure 24). Understanding of sea-surface instabilities and the power of computers have thus combined to greatly clarify the mechanisms of wave breaking.

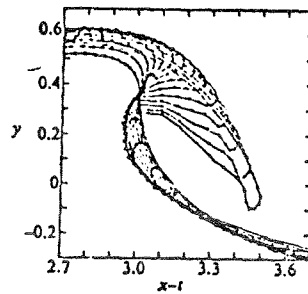
There are still many unresolved problems. Do instabilities control the formation of spilling breakers as well? What happens after spilling or plunging? Work on this question has been undertaken by Longuet-Higgins and Turner (1974) and by Longuet-Higgins (1980b). Is the frequency of breakers on the sea-surface that which one would expect from the rate of dissipation of an equilibrium wave spectrum? The theoretical and practical challenges presented by the problem of breaking waves are likely to stimulate considerable work over the years to come.

#### 4.3 Consequences for Offshore Design

The highest water velocities, accelerations and forces to be encountered by an offshore structure, be it fixed or floating, are likely to be associated with breaking waves. Theoretical and observational studies of the interaction between a structure and deep-water breakers are therefore required. The distribution of deep-water breakers over the sea surface, in terms of type and strength, are of direct importance to the evaluation of the risks which offshore structures face.

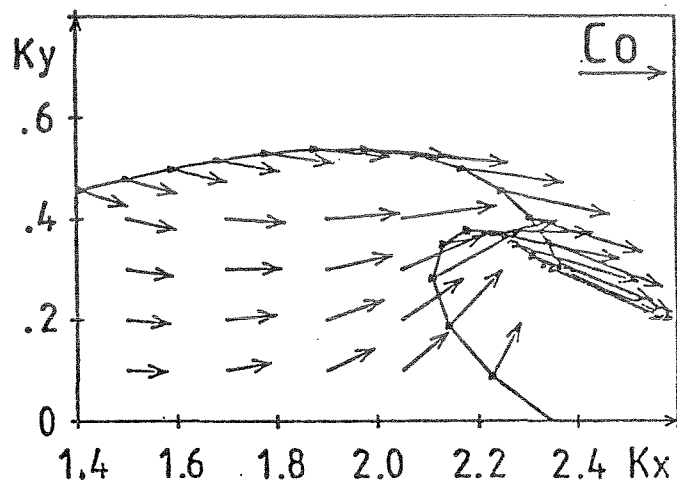


(a)

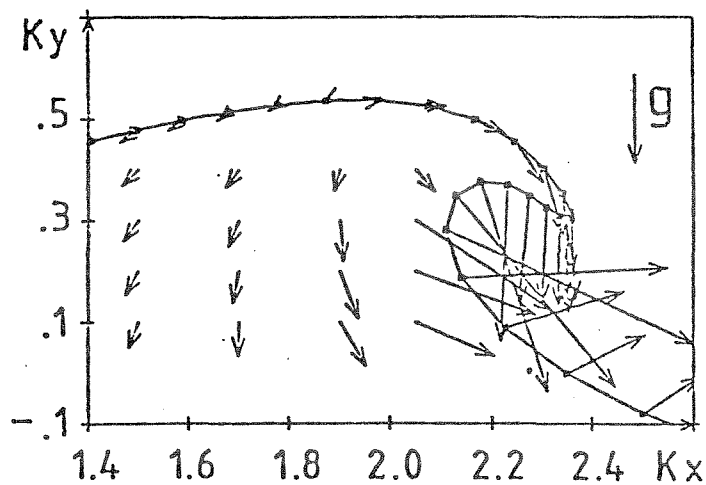


(b)

Figure 23. Successive wave crests near overturning a) and, at a later stage, b) after a plunging breaker has formed. From Longuet-Higgins (1980a).



(a)



(b)

Figure 24. Numerical simulation of a) velocity field and b) accelerations in a plunging breaker. From Kjeldsen et al. (1980).

5. UNUSUAL WAVE STEEPNESS

The occurrence of unusual steepnesses in ocean waves is intimately related to that of wave breaking and to the type of "unique" event discussed above. Direct instrumental measurements of very high wave slopes are rare and difficult to make. Indirect observations and theoretical results may provide some guidance in the discussion of this phenomenon.

5.1 Some Observed Properties

Wave slopes can be calculated from the displacement measurements from arrays of sea-level sensors with sufficient spatial resolution and they may be sensed directly with tilt-meters, such as those in pitch-roll buoys (Longuet-Higgins et al., 1963; Mitsuyasu et al., 1975). Remote sensing methods also provide slope information. The stereo photography reported by Coté et al. (1960) gives direct information on individual wave slopes, but only for relatively few waves at the time. Techniques such as the measurement of sun glitter from the sea surface (Cox and Munk, 1954) or more modern versions based on satellite sensors give slope estimates averaged over large areas of the sea surface. Both methods are likely to miss rare extreme slopes: in the first case because of the paucity of the data base, in the second because of the inherent averaging effect.

Waverider or similar types of buoys cannot provide direct information on wave slopes: they inform only on the temporal structure of sea-level variations. The wave slope statistics reported by Kjeldsen and Myrhaug (1979) in the previous section are derived quantities, obtained by converting time delays be-

tween a zero-crossing and the following crest into distances by assuming a wave speed. Which wave speed is used depends on the sea-state model adopted; that is either the spectral or phase-locked representations. The authors quoted here assumed a spectral model, wherein  $c = g/\omega$  with each wave travelling independently. A different interpretation of the nature of the sea-state, wherein waves at frequencies greater than that of the spectral peak ( $\omega_p$ ) travel at the same speed as the peak, would yield different results, apparently diminishing the slopes calculated from the spectrum, since  $c(\omega_p)$  will be larger than  $c(\omega)$  over most of the range of frequencies observed. Because of the phase-locking however (we recall that there is no explicit phase information in an energy spectrum) the waves may actually be much steeper than one would expect from random superposition.

Another source of derived information on wave slopes comes from hindcast studies, where statistics of sea-state conditions over many years are compiled by applying observed wind fields to a wave generation model. One application of this kind, using the U.S. Navy's Spectral Ocean Wave Model (SOWM), has been presented by Cummins and Bales (1980). A mean-slope parameter  $\alpha$ , calculated from the random-wave superposition as the r.m.s. value of  $kH/2$  is

$$\alpha^2 = m_4 / g^2, \quad (10)$$

(where  $m_4$  is the fourth-moment of the spectrum and  $g$  is the acceleration due to gravity). This parameter was estimated as a function of wave height (Figure 25) and of wind speed (Figure 26) in the eastern North Atlantic. According to those results wave steepness increases with wave height as well as with wind speed. Because of the random phase hypothesis, the wave slopes



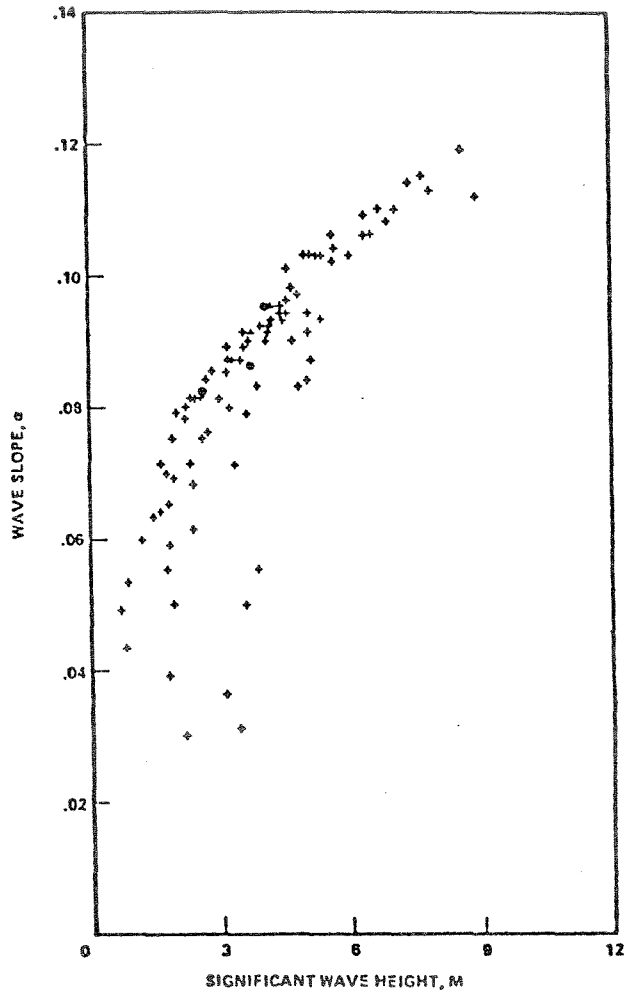


Figure 25. Root-mean-square wave slopes from a hindcast model ( $\alpha$  is defined by equation 10) against significant wave height for a four month period in the North Atlantic. From Cummins and Bales (1980).

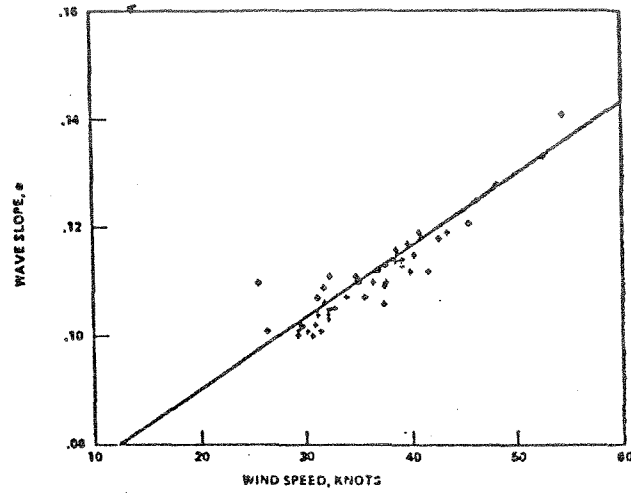


Figure 26. Relation between r.m.s. wave slope  $\alpha$  (equation 10) and wind speed in the North Atlantic from the hindcast studies of Cummins and Bales (1980).

inferred remain well below breaking levels. The largest steepness values  $\alpha = kH/2 = \pi H/\lambda$  reach only about 0.14; in comparison, the deep-water laboratory experimental results of Kjeldsen and Myrhaug (1979) give values of the forward face steepness  $\epsilon$  [in a symmetric wave profile  $\epsilon = 2H/\lambda$ ] from 0.32 to 0.78. Wave steepnesses inferred from spectral models are clearly not relevant to extreme steepness estimates!

A study of some relevance to the problem of extreme wave slopes is that of Burcharth (1981) who, in addition to his investigation of groupiness, also examined the same wave records, off Cornwall and Norway, for the frequency of occurrence of extremely rapid jumps in wave height. Burcharth (1981) defined a "jump-parameter"  $C$  in terms of a change in wave-height between successive waves as shown in Figure 27. In such a jump, a wave of height  $C\bar{H}$  ( $\bar{H}$  is the mean wave height) is followed by a larger one of height greater than the significant wave height  $H_g$ . Absolute probabilities of such jumps are shown in Figure 28, from Burcharth's data. Again, the passage from the temporal (jumps in a Waverider buoy trace) to the space domain (slopes) depends on which wave speed is chosen.

One must note also that for extreme wave slopes (when waves become steep enough to break) as well as for extreme wave heights, Waverider buoy measurements as well as those of larger, slope-sensing, buoys become subject to possibly severe instrumental errors.

#### Summary of Observations

Mostly because of instrumental problems (wave buoys in deep-water having difficulties conforming to very steep water surfaces) measurements of extreme wave

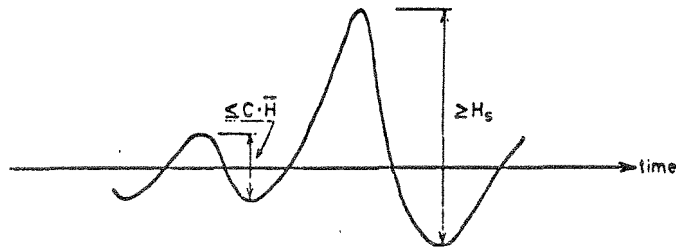


Figure 27. Definition of a jump in wave height and of the jump parameter C used by Burcharth (1981).

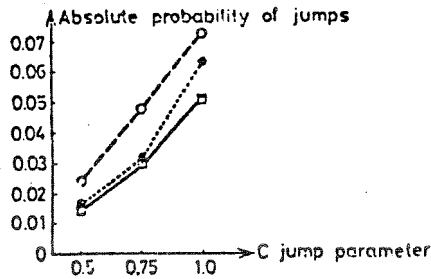


Figure 28. Absolute probability of jump measured by Burcharth (1981) at Hansthalm (squares) and Perran Bay (black circles) as a function of the jump parameter C defined in Figure 27. The open circles show the results expected from Rayleigh statistics as per equation (12).

slopes in deep water lack adequate instrumental documentation. Indirect high slope estimates may also be inferred from high observed values of vertical velocity,  $w$ , using the relation

$$\text{slope} = \frac{kH}{2} = \frac{|w|}{c}$$

(LeBlond and Mysak, 1978, p.85).

## 5.2 Review of Theory

The theory of extreme wave slopes is the same as that developed to examine waves of extreme heights and incipient breaking, mostly by Longuet-Higgins and his students or collaborators, as referred to in the previous chapter. Much more is known about laboratory experiments, than about actual oceanic conditions. Huang et al. (1981) have emphasized the importance of "significant slopes" in empirical wind-wave studies; Cummins and Bales (1980) also note their significance in naval architectural studies. Since field measurements are rather inadequate in terms of specification of extreme values or frequencies of high wave slopes, one can only rely on theoretical and laboratory estimations.

One thus expects because of wave breaking, usually of spilling type, but sometimes of plunging type, wave slopes defined in terms of the forward steepness reaching up to 0.78 empirically or even higher values theoretically. How frequently such shockwave-like wave forms do actually occur in real sea-states is not properly documented. Burcharth (1981) has presented an expression for  $P(C)$ , the absolute probability of occurrence of a jump from a wave height  $H \leq C\bar{H}$  to a wave height  $H > H_s$  as

$$P(C) = [1 - P(H > C\bar{H})] P(H > H_s) \quad (12)$$

Assuming Rayleigh statistics, values of  $P(C)$  can be readily obtained. Values presented by Burcharth (1981) for comparison with observed data are seen in Figure 28. Jumps are seen to be rarer than expected from Rayleigh statistics.

### 5.3 Relevance to Offshore Design

Inasmuch as extreme rare slopes are associated with wave breaking, they are just as important as breaking in setting extreme loads on offshore structures. There seems to be little probability of gathering useful knowledge about extreme slopes from extrapolation of the relatively small slopes measureable with available instruments.

Research on this topic will have to proceed hand-in-hand with work on wave breaking, its frequency of occurrence and the conditions (slope, speeds, accelerations, shock pressures) which accompany it.

6. RECOMMENDATIONS FOR USE OF MEDS DATA

Of the points discussed in this report all but the first, on wave groupiness, are concerned with strong nonlinearities of surface gravity waves on the ocean. As Arhan and Plaisted (1981) have pointed out, Waverider buoy measurements show little nonlinearity, although experiments carried out in wave tanks lead to highly nonlinear wave profiles. As a result one suspects an inadequacy of Waverider buoys to resolve the strongest asymmetries or steepnesses of nonlinear sea-states.

The following recommendations can be made as a result of this review:

- 1) The performance of Waverider buoys should be studied further to assess their response in extreme wave slopes and breaking waves.
- 2) Data from Waverider buoys should be examined in some storm events to ascertain if upper bounds appear to exist in the values of acceleration and of vertical velocity. These upper bounds could then be compared with values expected in breaking waves.
- 3) The phase relation between signals at different frequencies should be examined, using the bispectral method discussed by Arhan and Plaisted (1981) and Masuda and Kuo (1981a,b) to detect the type of asymmetry associated with steep forward faces which precede breaking, as well as possible evidence of phase-locking between different frequencies.
4. Wave slopes could be calculated using time-series of vertical velocity  $w$  (obtained by differentiation of the sea-level displacement) and the wave speed of the spectral peak frequency  $c(T_p)$

as  $w/c(T_p)$ . These results could be compared to those obtained by the method used by Kjeldsen and Myrhaug (1979) to achieve a better understanding of the significance of wave slopes inferred from Waverider time-series.

5. Jump occurrences and the relation of their frequency with that predicted by Rayleigh statistics would be worth estimating, to continue and broaden the preliminary work of Burcharth (1981) in this direction.
6. The detection of wave groups is not related to nonlinearity. In order to compare the SIWEH method to other methods already used on natural wave data, sufficiently long time-series of Waverider buoy sea-level displacement data should be analyzed for groupiness via wave-counting, envelope fitting and SIWEH calculations.



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