On the Hydroacoustically Inferred Morphology of Arctic Ice

By
J.E. Papadakis

Institute of Ocean Sciences
Department of Fisheries and Oceans
Sidney, B.C. V8L 4B2

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ABSTRACT

Papadakis, J.E. 1987. On The Hydroacoustically Inferred Morphology of Arctic Ice. Can. Contract. Rep. Hydrogr. Ocean Sci: 28.

The inference of the underside morphology of Arctic ice by the use of hydroacoustical methodologies is investigated and an extensive reference literature is given.

Problems related to the definition of morphology are analyzed. The attention however is directed not to the study of mathematical surfaces taken in isolation, but towards their interaction (convolutions) with other idealizations representing the sound field.

A three layer model for the Arctic ice underwater sound scattering is suggested as the most appropriate. As a first step in conducting field measurements, the existence of a forwards scattered, roughness induced, boundary wave is proposed to be verified in the Arctic.

Finally, a methodological assertion is derived in promotion of a multidisciplinary approach of the problem involving acoustics, geometry, and oceanography.

RESUME

Papadakis, J.E. 1987. On The Hydroacoustically Inferred Morphology of Arctic Ice. Can. Contract. Rep. Hydrogr. Ocean Sci: 28.

L'inférence de la morphologie du dessous de la glace de l'Océan Arctique par l'usage de méthodes hydroacoustiques est recherchée et une bibliographie extensive est presentée.

Les problèmes qui ont rapport à la définition de la morphologie sont analysés. Cependant, l'attention est dirigée, non pas vers l'étude des surfaces mathématiques prises une à une, mais vers leur convolutions avec d'autres modèles de champ acoustique.

Un modèle à trois couches pour la diffusion de son dans l'eau sous la glace de l'Océan Arctique est proposé comme étant le plus approprié. Tout d'abord pour conduire les mesures sur le terrain, l'existence d'ondes diffusives limites induite par une surface rugueuse qui est proposée doit être vérifiée dans l'Océan Arctique.

Enfine, une méthode est présentée en vue de promouvoir une approche pluridisciplinaire des problèmes acoustiques, géométriques et océanographiques.

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I. INTRODUCTION

The study of Arctic ice morphological characteristics is of significance because it gives knowledge about a part of planet earth that among others plays a major role in the climate. Rapid progress has already started to occur as various political, economic, military, and scientific interests require reliable morphological knowledge of large areas of the lower surface of the ice cover.

Applications associated with transportation, pollution, construction of structures (intended to survive the Arctic conditions), military uses of underwater sound, the heat budget and other physical processes require a good knowledge of the Arctic ice morphology and its variability. In Rothrock and Thorndike, 1984, a convincing argument is given that "an understanding of the geometry of floes and how the geometry changes during the annual cycle will stimulate research on the governing physical processes."

With the aid of aircraft and satellites the upper surface of the frozen sea can be mapped and monitored in large areas through all the seasons. More problematic is the surface between ice and water. Underwater photo-cameras and upward looking sonars have been used from manned and unmanned submersibles, or suspended from holes through the ice (Lyon, 1961; Kan et al., 1974). In Sater, 1968, the history and some results from the first scientific mission on ice drifting stations are reported. In McLaren, 1985, a brief account is given of the first western attempts, using submarines, to obtain "ground truth". In addition, some of the morphological characteristics of the lower surface of the Arctic Sea cover can be inferred from photographs of the upper surface as it is practiced, for example, in Diachok, 1976 and Wadhams, 1981.

Given that the available time or expense are limiting factors, it is understandable that as an increase in the range of investigated area is attempted, the useful detailed information gathered by any sensing system will decrease. There are however certain application demands that could be fulfilled by only global morphological characteristics. The aim of the present study is to examine the use in the Arctic Sea of hydroacoustical methods in obtaining an estimate of the roughness of a large scale under-ice area. This roughness could be related to the hydrodynamic roughness needed in all the studies of flow under ice, in studying the general Arctic circulation, and as a parameter in the heat budget calculations. A similar attempt for the bottom topography of the Arctic Ocean is found in Dyer, et al., 1982.

In general, as an increase in the sensed area is attempted, one could clarify, order, and coordinate the number of useful applications with the sensed morphological characteristics only by means of a complete decision theoretical model. Even if the sensing is specialized to underwater sound and the application to hydrodynamical roughness, the encountered problems are still very difficult. If these problems are successfully defined and resolved, many efforts under the severe Arctic conditions could be saved.

The above is not an easy task. The key problems, by their nature, carry all the uncertainties connected with the methodology of inverse scattering. In addition, some deeper problems of epistemological nature have to be faced: How do we define roughness? How does the phenomenological acoustical roughness relate to the other phenomenological hydrodynamical roughnesses? How do all of these relate to a synthetic geometrical characteristic especially if the geometrical surface is of a fractal nature? (Rothrock and Thorndike, 1980, 1984; Mandelbrot, 1982).

The scientific status of hydroacoustical estimation of arctic ice morphology appears to be equivalent to the status of oceanic acoustical tomography, or to the status of sensing instruments based on fiber optics. In all these areas the difficulties to be faced and the mathematics needed are similar: Perfect boundary conditions, with a stochastic medium - Born's approximation, or vice versa, and Kirchhoff's approximation (Morse and Feshbach, 1953, p.1073); Perturbations; Green's functions; Fredholm's Integral Equations; Stochastic Integral Equations; Path Integrals, etc.

Although the last three items will not be evaluated in this work, it is clear that any advanced study of scattering, or of propagation in random media depends on these. Here, an exposition is attempted of the main factors and a presentation in a manageable form of the foregoing challenges. Mainly, multidisciplinary bases are given on which solutions could be founded, usually, after tedious elaborations. For these reasons the work is divided into 8 sections.

Section II contains general information about Arctic Hydroacoustics Section III is concerned with the existing information about the Arctic ice morphology. In Section IV various theoretical approaches to form and structure are exposed and some measures of roughness are mentioned. The scattering of sound waves from irregular surfaces is summarized in Section V. Because the flow under the ice depends on the ice morphology and generates medium inhomogeneities some related information is given in Section VI. In Section VII some conclusions are derived, and finally in Section VIII the references are contained.

II. General Arctic Hydroacoustical Characteristics.

The acoustics in the Arctic Ocean are influenced by four main factors: a) the ice cover, b) the mean sound velocity profile, c) the fluctuations from the mean profile, and d) the ambient noise. For a wide range of frequencies, the influence of bottom is not essential because the mean sound velocity profile is such as to refract the acoustic energy upwards. As a first step in the iterative cognitive process that takes place between the factors controlling sound and the sound exploring these factors, a general knowledge of the Arctic geography and oceanography is required.

The Arctic, called also a Polar Mediterranean (Rey, 1982), is divided by the Lomonosov Ridge into two main basins: the Canadian and the Eurasian. Three more ridges, the Alpha and its continuation Mendeleyev, and the eastern midoceanic Arctic Ridge subdivide further the area in the Makarov, Amundsen and Nansen basins. North of the continental Canada there are many islands, while north of Europe and Asia an extensive continental shelf is found. It has a maximum width above 800 km, being thus the widest on Earth.

As mentioned, one of the major Arctic characteristics is the ice cover. This, in brief, is strongly influenced by and influences the Earth's climate. A description of the ice cover as it seasonally varies can be found in Rey, 1982.

Many rivers discharge their waters into the Arctic. The main communication between the rest of the oceans is via the Fram Strait. Through this strait, warm Atlantic water flows in, while a surface counter current, paralleled by a deep overflow of Arctic waters over sills, bring to the Atlantic Ocean drifting ice and cold waters. It is interesting that the input from rivers equals the ice export (Aagaard and Creisman, 1975). The general pattern of water circulation is also discussed and pictured in Rey, 1982 (see however Aagaard et al., 1985).

Typical temperature and salinity profiles can be found in Lewis 1982 and Coachman and Aagaard, 1974. Both works are very informative regarding the physics of the Arctic Ocean.

Corresponding to the typical temperature and salinity profiles, a typical sound velocity profile accompanied by a ray-tracing are schematically shown in Figure 1 (for details, see Diachok, 1978). In general the propagation is governed by an upward refracting half-channel which has its axis at the surface. In practice, below a thin surface layer of about 50m, as discussed in Milne, 1967, the channel remains steady in time and horizontal distance.

The steadiness of the Arctic channel in regard to the propagation of low acoustic frequencies is taken as a fact in Kutshale, 1984, and Mellen, 1983. This does not mean, however, that for high frequencies the channel is deterministic. The high frequency acoustic variability, related to the Arctic oceanic fine structure, is the subject matter of an excellent recent study (Schulkin et al., 1985).

The Arctic acoustic channel could be considered as an extension of the deep sound channel of the non-polar oceans (SOFAR). Both the Arctic and the SOFAR waveguides are dispersive as expected. After the detailed studies in Kutschale, 1961, 1968, 1984; Yang, 1984, it has become apparent that the main difference between these two waveguides is that the long range signal content in high frequencies is less in the Arctic than in the SOFAR channel. The reason is the scattering of higher frequencies from the ice irregularities. Based on a simplified reasoning, in Kutshale, 1984, it is shown with a series of computed figures how the ice roughness influences the normal modes.

Many linear and non-linear thermal and dynamic phenomena exclusive to the Arctic cause inhomogeneities in the refractive index. However, the existence of ice cover also helps in attenuating the meteorological forcing. This, combined with the relatively small amount of marine life, makes the volume reverberation smaller in comparison to the reverberation in the non-polar oceans. As a consequence, the sound transmission in the Arctic is considered to be better, especially at low frequencies, Urick, 1982.

A picture summarizing schematically the Arctic transmission loss is presented in Figure 2. (For details, see Buck, 1968.) The line representing the spherical spreading was added for comparison. The total transmission loss N_w from the source to range R is equated to the sum of divergence, or spreading, loss N_d plus the reflection loss N_r. It has been assumed that $N_d = 20 \log \frac{r_o}{4} + 10 \log \frac{4R}{r_o} = 10 \log r_o + 10 \log R - 6$, i.e. spherical spreading up to 1/4 of the distance r_o that the deepest limiting ray travels, for first time, from the source to the surface, and cylindrical spreading thereafter as in most of the oceanic waveguides.

N_r and the reflection loss per unit distance, N'_r, are frequency dependent because they include the effects of scattering from the rough ice surface. With this in mind, the relation

$$N_w = 10 \log r_o + 10 \log R - 6 + N'_r R$$

was used to construct the family of curves in Figure 2 which take into account all the available data up to 1968.

Questions have been raised regarding acoustical energy conversion to Lamb modes in the irregularly layered (with ice blocks) anisotropic ice canopy (Yang and Votaw, 1981), or, regarding the energy absorption by the overlying snow, (McCammon and McDaniel, 1984). These questions indicate the importance of frequency and grazing angle in the determination of transmission loss. However, a recent study (Diachok, et al., 1984), indicates that for grazing angles less than 10^o and for frequencies less than 100 Hz, the roughness effects predominate. Also, as it is mentioned in Urick 1982, subsequent to 1968, data from the marginal ice zone east of Greenland (Bradley, 1973), confirm the validity of Figure 2.

The ambient noise studied in Buck, 1968; Milne and Canton, 1964, and more recently in Denner, 1981; Dyer et al., 1984, is in general variable depending on the season and geographic position so that comparisons with other oceans are not easy. Broadly speaking, it could be said that the noise levels are of a comparable magnitude. Of interest, being an inverse problem eventually leading to remote sensing capabilities, is the attention paid in Buck, 1968, to the correlation between the levels of low frequency ambient noise and local wind. However, of more interest to this study is the work in Mellen and Marsh, 1963; Mellen, 1966; Diachok, 1978; Kryazhev and Kudryashov, 1985, and Mellen et al., 1985, where ice roughness is actually deduced hydroacoustically.

Ending this section it could be observed that since 1958 when the first western underwater sound propagation studies were conducted by the United States Underwater Sound Laboratory (Urick, 1982), many theoretical, numerical, and laboratory (Keller and Papadakis, 1977; De Santo, 1979) advancements have taken place. All these have influenced the formalism of current text books (Ziomek 1985) and make the field of Arctic Hydroacoustics indeed an exciting field of research.

III. Arctic Ice Morphology

The ice in the Arctic varies seasonally in extent and physical properties. Exposed to external and internal forcings it deforms and it is put in motion. It moves anticyclonically in the Canadian Basin forming the Beaufort Gyre, and drifts transpolarly from the Bering Strait to the Fram

Strait. A figure from Gordienko, 1958, representing the mean ice drift is often referenced (see for example, Hibler, 1979). Because of the gyre, multiyear ice accumulates in the Arctic's southeast arc. There, accumulations up to 6.5m could be seen in a figure composed by L.A. LeShack (Hibler, 1980, SCOR Working Group 58), representing the distribution of mean ice thickness.

Thermal effects and ice motion make the ice thickness variable. A first distinction in characterizing the ice cover morphology could be made between the marginal ice zone, where ice and open sea water meet, and the rest of the cover. As a marginal ice zone it is defined (Kozo and Tucker, 1974) to be the part of the cover, of about 200 km wide, between the edge and the point where consolidated multiyear ice predominates.

The remaining part of the ice cover could be thought as a combination of thin (up to 2m), one year ice and thick multiyear ice consisting of slowly undulating undeformed ice and deformed pressure ridged ice (Hibler, 1980; Wadhams, 1981). Divergent ice motion and melting create open areas of water (polynias) with ice floes of various sizes, and extensive cracks (leads) which when refrozen give a smooth ice surface. The polynias are of interest to the submarines for surfacing and the unfrozen leads to the navigators and to the aircraft pilots for landing.

The upper ice surface is less rough than the lower surface (Rothrock and Thorndike, 1980) because meteorological smoothing effects are stronger than oceanic. In 1971 during the AIDJEX (Arctic Ice Dynamics Joint Experiment) the highest free floating (not landed) ridge sail observed was 15m, while from submarine sonar data, the deepest keel was found to extend about 50m below the sea level (Write et al., 1978). A laser profile of the upper sea ice surface, and a portion, from the output of an upwards looking sonar, are presented in Wadhams, 1981.

After some necessary corrections on the profiles (Williams et al., 1975; Harrison, 1970) one is interested in quantization, i.e. the magnitude distributions, the spatial and temporal occurrence, and their mutual relationships, of keel drafts, of ridge sails, of ice thickness, and of width of the leads. Sonar draft distributions and number of keels versus spatial occurence are given in Wadhams, 1981. As an example of functional relations, a linear relationship between sail height and keel draft is suggested in Wadhams, 1981, after the pioneering work of Hibler, et al., 1972; Hibler, et al., 1974. Spectral analysis, of a different set of data is found in Rothrock and Thorndike, 1980.

The above quantizations depend on arbitrary but nevertheless agreeable choices of how a signal could be separated from the background noise, or simply, on how a bump is defined to be a bump and not noise. In addition, the above factors depend on the commonly used mode of describing nature and its phenomena by a mean field accompanied by noise (classical approach). The possibility that the description could be performed using mean fields which are continuous but not differentiable has been undertaken in Rothrock and Thorndike, 1980, 1984. This constitutes a different approach called, in brief, fractal.

Although their preconceptions are different, a common desire of both approaches is the discovery of cause-effect relations between the physical factors acting upon the ice cover and the various forms that the ice takes. It is thus hoped that from observations of morphology the main dynamic factors could be deduced by the use of inverse thinking. This however cannot be achieved completely but it is not outside the scientific endeavours.

Current advancements in data processing regarding the estimation of the order of a stochastic process (corresponding to the classical approach), or equivalently (corresponding to the fractal approach), in the determination of the dimensionality of the attractor corresponding to the time series, (Nicolis and Nicolis, 1984), can help us reliably to decide, at least, about the number of the essentially interplaying factors.

Regarding the classical approach and by leaving aside the ancients (Heraclitus for example), in the present century, D'Archy-Thomson and Rene Thom have also seen the form as the outcome of an optimization between antagonizing factors. Such a point of view might be valid in an ideal limit. However, it is not always possible to define, or after having defined, to resolve the required variational problems. Echoing Gestalt psychology, perhaps the recent observations by Ramon Margalef, 1982, are relevant here (see also Taylor 1972). They point to the influence of the size of the domain in which organizational structures are to be distinguished. This, in a fundamental manner, introduces relativity in the problem of deducing forces from geometry and waters down the whole exercise.

At a more practical level, the difficulties in describing the ice cover morphology in absolute coordination with the physical processes could be seen if we examine, for example, the simple case of the undeformed multiyear ice. It can be understood that the salinity has to increase with the depth of this ice cover. However, because of the various mechanisms resulting in brine drainage (thermal explusion, flushing, gravity), inversions in salinity are very common. Also, the existence of various melt ponds during the summer create horizontal inhomogeneities. Thus, the estimation of the elastic parameters for this ice cover is difficult. Even if one relates approximately the elastic parameters to the mean ice thickness, many thermal and dynamic feed-back loops still complicate the relation between the ice drift, deformation, and strength.

Inspite of all the above about the problems of description, even with the addition that the optimal descriptions of Nature could be many-valued as well as unstable (Papadakis, 1985), physical and not mathematical considerations help rescue the situation. For our purposes, it is important to see here that our problem is not to consider one mathematical surface in isolation but to consider the "physical testing" (Young, 1975), or, the interaction, or, the convolution of this surface with the plethora of light and sound waves. The mutual interaction of pairs of functions, plus smoothing and some other elements from the decision theoretical model, appear to give us entities to which, at least, a physical meaning could be attached.

An example of how physics and geometry can be combined in a simple manner is given by the so called Rayleigh roughness parameter $\mu = 2kh \cos \phi$, where k is the wavenumber of the "testing" wave, ϕ characterizes its direction with respect to the trend of the geometrical surface and h is a measure of its geometrical protuberances. Another criterion that, together with geometry and physics, combines the analytic ability for resolving the problem (i.e. decision theory elements) is the so called Brekhovskikh criterion (Brekhovskikh, 1952, 1982):

$$4\pi r_c \cos \phi >> \lambda, \ \lambda = 2\pi/k$$

Here, r_c denotes the smaller of the two radii of curvature at the point of incidence of the geometrical surface and ϕ is the angle of incidence. The last criterion is the basis of the Brekhovskikh method which is similar to the method of Kirchhoff discussed in Section V. E.1.

IV About Form and Structure

IV.A. Theoretical considerations

As we are dealing with ice morphology, bumps that we call keels, and irregularities that globally we term roughness, we find outselves surrounded by questions that relate to the very general concepts of form and structure.

The establishment, stability and variation of any kind of form, organization or event are of interest to all of us. However, the qualitative and other elements required for the study of form cover areas beyond the usual quantitative area of scientific activity which is governed by our two-valued logico-axiomatic system. As such, hard sciences avoid many related problems by postulation. Exception is found in Biology where the problem of parts and whole, or, of one and many, is of central importance (see for example the recent books by Salthe, 1985, and Brooks and Wiley, 1986).

In our times, two schools of thought predominate. One is represented by the Mathematicians, R. Thom (Theory of Catastrophes, 1975), V.I. Arnold, D. Anosov, J. Sinai, S. Smale (General Dynamical Systems, 1981). The other by I. Prigogine (1980, 1984, Order Through Fluctuations), and K. Wilson (1976, 1979, Phase Transitions, Renormalization Group). Mixing between these two has already occurred, by studying, for example, general dynamical systems in noisy environments (Synergetics - H. Haken, 1981; Form Oscillators - J. Papadakis, 1985).

The importance of light scattering and of the geographical-climatological conditions for the genesis of geometry in Greece have been stressed by S.P. Zervos, 1972. It is interesting to note that in the Greek language "logical term", "definition" and "mountain" all have the same root, "o os", (orography for example). It thus becomes apparent that the concept of a bump (a keel!) and the basis of Chrysippian logic (not Aristotelian, for in Aristotel are also found elements of three-valued logic) are of equivalent depth.

The notion then of a clear cut bump is found in the roots of any entity that is acceptable to the two-valued Chrysippian system. To be avoided, if possible, are the many-valued logico-mathematical systems without the Eudoxus-Dedekind cuts of analysis, or, fuzzy sets such as the "blobs" (Batchelor and Townsend, 1949), or the "scales" for example that the Swiftian rather than the Newtonian (discrete and atomistic) physicists use. This avoidance has been noticed by K. Wilson, 1976, to be followed especially in Quantum Physics.

The notion "form oscillator" (Papadakis, 1985) exemplifies the stability problems connected with a single two-valued system, the statistical outlyiers, and the empirical establishment of a simple bump. However, as it has happened before (Logic of Quantum Mechanics - Birkhoff and J. von Neumann, 1936; Jammer, 1974), the three-valued system can technically be avoided (see Popper, 1978) by a couple of two-valued systems. Thus, it is hoped that the metaphysical questions may be circumvented and the Chrysippian tradition preserved. A unifying perspective of the above is found in D. Bolm's work (1979).

The logical forcing on the data that the Dedekind cuts perform is but an aspect of the interaction between the form establishing mechanisms (FEM) and the data source (DS). The primordial motivation of using the extracted form for survival, or for any other reasons, defines a certain distance between FEM and DS and simultaneously dictates the size of the data domain. The same motivation requires verification of the established form. All these depend on the richness of preexisting forms in the FEM and its abilities for coordination between the available and the desirable. We thus advocate that the whole process is iterative in the Ionian sense, i.e. physical and not mathematical. It is modelled not by a simple information theoretical model but by a more complete, decision theoretical model described among others and in Papadakis, 1973.

Examples for the above are found in G.K. Batchelor (1967, pp 4-6) where he describes the variations of density as the volume tends to zero, B. Mandelbrot (1983, pp 6-9) where he extensively quotes J. Perin and describes the changes in dimensionality of an object as the observing distance changes, and finally in E. Schrodinger (1951, pp 26-39) who discusses continuity, discontinuity and distinguishability and who, in Broda (1983, p 77), attributes these ideas to L. Boltzmann.

Since, however, our main problem is the successful simulation of the physical environment it is pertinent to review some modes of describing form and structure, and, with a specific example, uncover their main points and inter-relationships. We call these modes a) entropic b) Newtonian or, analytic c) parametric, and d) Weierstrassian.

Consider a salinity (S) profile of multiyear ice of a depth (D) equal to 10m. Suppose we can measure salinity every e_d = 10 cm and that the salinometer has an accuracy of e_s = 0.1 and range from 0 to 40. Already, we can define the numbers d = 10:0.1 = 100 and s = 40:0.1 = 400. The defined grid is relatively coarse and the cartesian product is (d*s) = 100*400 = 40,000. The set of all possible discrete single-valued functions S = f(D) has A = 400^{100} members. This finite number of functions (remember that only 10^{80} is the number of atoms in the known universe) can produce B = $\binom{499}{100}$ discrete histograms. Note that B is less than A.

The number of functions (realizations or complexions) for each one of the B histograms is given by the well known partitioning formula used by Jacob Bernulli, Laplace, Boltzmann, etc. A further reduction of B to C groups is possible if the order of the bins (columns) in the histogram is irrelevant.

C represents all the possible entropies one could compute from the data. Thus, with the Theory of Information we could recognize C essential structures in the data. Although the reduction A B C is remarkable it has to be remembered that C represents histograms and not functions. Also it has to be observed that $C \to \infty$ as s and $d \to \infty$.

Jaynes, 1982, has shown that the C histograms concentrate at the histogram of maximum entropy and that this is valid for any dictated constraint. Any given a priori information about the data could pass as a constraint and help define the histogram of maximum entropy. One could thus define a ΔH of information tolerance around H_{max} and ignore the realizations which correspond to H values outside the interval ($H_{max} + \Delta H$). The new C', however, is not very much smaller than C. Also, the complexions behind each C' are still vast. Reduction will occur if one considers the joint probability distributions of D and S. But for these the Gibbsian notion of the ensembles is needed and thus past experience (time evolution) of many realizations.

Before we consider the problem of defining structure (and thus data reduction) from the Newtonian or from the parametric point of view, whereby the histograms are of the noise and not of the noiseless signals (functions), we find it appropriate to introduce the following quotation from Jaynes 1978, about the role of the physicist's intuition.

> "In other words, merely knowing the physical meaning of our parameters, already constitutes highly relevant prior information which our intuition is able to use at once; in favorable cases its effect is to give us an inner conviction that there is no ambiguity after all in applying the Principle of Indifference. Can we analyze how our intuition does this, extract the essence, and express it as a formal mathematical principle that might apply in cases where our intuition fails us? This problem is not completely solved today, although I believe we have made a good start on it in the principle of transformation groups ([Jaynes, 1968, 1973, 1979]). Perhaps these remarks will encourage others to try their hand at resolving these puzzles; this is an area where important new results might turn up with comparatively little effort, given the right inspiration on how to approach them."

In the above information theoretic considerations, or entropic mode, it was not taken into account that the data represent Arctic ice characteristics. Thus, the concepts of physical continuity, inertia, diffusion, cohesive forces, and, in general, the concepts of known dynamic laws were ignored.

The dynamical laws when introduced, will lead to classes of structure that the data globally must follow, while any local details may be taken as noise. At this point one could observe that Newton's greatest discovery was not the gravity but the incorporation of the derivatives to the description of Nature. The existence of well behaved derivatives imposes tremendous constraints on the data and structures them, in what otherwise is called a physical law (Newtonian mode). In a similar manner, one could explain the vast power of reduction found in the parametric or regression analysis.

The problem left open in the parametric mode is what these structure classes are, given that they are dictated from the data, as the various physical laws have acted upon them. This is expressed in Statistics as the problem of ignorance about the real model. A big chapter is devoted to the so called "aptness of the model". Because our main problem here is not to discover physical laws but to effectively codify the data, the F statistic for example could serve us well. Briefly, we are looking at the data space and not at their histograms. We try to discover inductively relations among data successions which are, in general, of variable but finitely describable step length.

Thus far the entropic, the Newtonian (or analytic) and the parametric approaches to form were discussed. In view of the high reductionistic power of the parametric approach it is suggested that at least the parametric fitting has to be used in order to separate the sonar under-ice profile into areas, or domains, of stationarity and establish a representative form for a keel and for a correlation function (see, for example, Papadakis, 1981 for the required numerical procedures).

K. Weierstrass and B. Mandelbrot are mostly responsible for the modern fractal geometry of form which is an extension of the analytic representation, as it relaxes the condition of differentiability. As far as we know, D.A. Rothrock and A.S. Thorndike (1980) introduced for the first time the fractal approach to the description of Arctic ice morphology. They present spectra of ice profiles whereby self-similarity is suggested. This characteristic of self similarity is a sufficient only condition and it helps in computing the dimensionality of the fractal curve.

Weierstrass has also influenced the current theory of General Dynamical Systems (GDS) in two ways: a) he promoted and directed the work of the first Russian woman to become a professional mathematician, S. Kovalevskaya, who successfully studied and opened new roads in the problem of integrability of systems, and b) by extending the calculus of variations, he influenced the American school of variations and especially the teachers of S. Smale who in turn inspired M. Feigenbaum to his theory of universality via the iterative maps. Interestingly enough, Feigenbaum's philosophical idea that, in simulating nature, complexity is the reality and not randomness could be attributed to the colleague of Weierstrass, L. Kronecker.

Our mentioning in Section III that the estimation of the dynamic system's attractor dimensionality, from a time or space series is equivalent to clarifying the number of essentially participating physical factors that produce the series, constitutes an example of how the concept of strange attractors arising in GDS could give flesh to the geometry of fractals. With this way of attaching physical meaning to the ideal geometrical entities one achieves at least a partial answer to the expectation quoted in the introduction to Rothrock and Thorndike, 1984.

IV.B. Descriptions of Roughness

Having in mind that the reasons for describing the irregular ice surface is to study first the sound scattering and then to infer from the received sound the elements of this description, we direct our attention to how irregularities are practically introduced in the models of scattering.

We recognize four general approaches to study scattering: a) Rayleigh b) Twersky-Biot, c) Analytic Approximative, and d) Lebesgue-Berry. Although this distinction is not clear-cut (for example the basic ideas of Rayleigh are found in all of them) it will be followed in the next section.

To simplify matters, only functions of one variable will be considered. One of the simplest descriptions of irregularity of the underside of the ice would be the detection from the sonar profile of the predominant harmonic (Lanczos, 1961; Bloomfield, 1976; see also Thomson and Papadakis, 1986 for an application of the method) and the subsequent arbitrary representation of the profile as a periodic function having a period equal to that of the detected predominant harmonic. Extending to general periodic functions, other shapes could be used as periodic elements. These could be the various shapes that the ice keels have been thought to have, for example, squares, trapezoids, triangles, and ellipses Furthermore, one could consider these protuberances distributed randomly (Twersky, 1949, 1983a,b).

The perturbational (included in the analytic approximative approach) and Lebesgue-Berry modelling require a statistical description of the irregularities. Various measures of roughness have been suggested in various fields (see for example, Nye, 1969). Below we mainly follow Fox and Hayes, 1985, and Rothrock and Thorndike, 1980.

Given a set of points X_i, $i = 1,2,\ldots,N$ we define the mean value $\bar{x} = \frac{1}{N}\sum_{1}^{N} X_i$. In underwater acoustics, as in many other areas, the quantity

$$A = \frac{\sum_{i=1}^{N} (X_i - c)^2}{N - 1}$$

where $c = \bar{x}$, is customarily called the variance. If $c \neq \bar{x}$, the square root of A is called the root mean square. Here surfaces the problem of what is the trend or, in other contexts, the mean field, c. The previous quantities change according to the trend considered. Further, by interchanging the exponent 2 with any other positive integer, various informative measures could be produced. These, under the title "norms", could be found, in Rice, 1963, together with other norms.

A simple step, in considering the distribution of variance over the wavelengths that we suppose (by following Fourier and not Walsh, for example) could, by superposition reconstruct the profile, is the quantity (John von Neumann)

$$B = \frac{\sum_{i=1}^{N-1} (X_{i+1} - X_i)^2}{\sum_{i=1}^{N} (X_i - \bar{x})^2}$$

This is a measure of the ratio of high wavenumber variability over low wavenumber (trend) variability.

A more complete description of roughness is given by the autocorrelation, or equivalently, by its Fourier transform (Bendat and Piersol, 1971; Papoulis, 1965). The fractal approach is, in practice, concerned with spectra described by power laws as $S(k) = \beta k^p$, k is the wavenumber, and β is a proportionality constant named "topothesy" (Sayles and Thomas, 1978). In logarithmic plots they appear as lines. This demonstrates that a special scale cannot be defined as no wave number or bands are characteristic. In this case, the slope of the line, i.e. p, will characterize the roughness but without any reference to any particular scale. In practice, however, the largest wavenumber (inner scale) and the smallest (outer scale) are present.

Going back, from the wavenumber domain to the space domain, a correspondence can be established between p and α, the order of the Lipschitz-Holder (LH) condition that a function (profile) f(x) may satisfy. This condition is satisfied at x, when

$$|f(\xi) - f(x)| \leq E|\xi - x|^{\alpha}, \text{ for all } |\xi - x| < \delta$$

where $\alpha > 0$, and E, a proportionality constant, are independent of ξ.

While a function satisfying the (LH) condition is continuous it need not be differentiable or have bounded variation (Jeffreys, 1956). For example, for $\alpha = 1$ the function $f(o) = 0$, $f(x) = x \sin \frac{1}{x}$ is not differentiable at $x = 0$. Interesting functions, for reasons of calculational simplicity, are the ones that satisfy the LH condition over all the domain of their definition with the same α. An example of a function which in addition does not present a smallest or a largest scale is a modification by Mandelbrot (1977) of the Weierstrass function called by Berry and Lewis (1980) the Weierstrass-Mandelbrot function:

$$W(x) = \sum_{n=-\infty}^{\infty} \gamma^{-n(2-D)} [1 - \exp(i\gamma^n x) \exp(i\phi_n)]$$

(See also Ausloos and Berman, 1985) with γ^n as wavenumbers and amplitude following a power law. The phases ϕ_n are random and $\gamma > 1$, $1 < D < 2$ hold.

The above correspondence between p and α, for a restricted but very wide class of functions is, according to Rothrock and Thorndike (1980), given as $-p = 2\alpha + 1$. The parameter α that characterizes the differentiability of a function could serve in the space domain as a measure of roughness, as p serves in the wavenumber domain.

Finally, probability density functions of various quantities measurable in the ice profile play an important role in the description of roughness as well as in the scattering of sound. The differentiability of their Fourier transforms, called characteristic functions, Papoulis, 1965, and their connection with the variability moments are of great use and make the description more comprehensive.

V Scattering of Sound

V.A. A brief scanning of the literature

The scattering of electromagnetic (EM) waves is in general more complicated than the scattering of sound waves. Historically, sound waves were studied first. However in the present century the attention paid to EM waves is responsible for the production of many valuable articles and books about scattering that are useful in many other fields. Such a book is the one by Beckmann and Spizzichino, 1963. Together with the original work of the authors, the main contributions to scattering of other workers until 1961 are also treated.

Another well quoted book is written by Bass and Fuks, 1979. It gives an original and detailed presentation of the method of small perturbations and also contains a study of waveguides with rough walls. The book by Lighthill, 1980, is especially mentioned here because of its theoretical insights, its intuitive background and its strength in unifying the various waves observed in fluids.

Many standard books in Hydroacoustics contain chapters, or articles, about underwater sound scattering from the free ocean surface or from the ocean bottom: Tolstoy and Clay (1966), Urick (1967 and 1982), Clay and Medwin (1977), De Santo (1979) and Brekhovskikh and Lysanov (1982). The propagation of sound in an inhomogeneous ocean is the theme of the book by Flatté et al. (1979). Since the methodologies in Flatté's book have direct applications to the scattering of sound from irregular boundaries it is also cited here.

Recent books, theoretical in nature, are by Ishimaru (1978), Colton and Kress (1983) and Tsang et al. (1985). From other related fields, the book in Geophysics by Tolstoy, 1973, is cited as well as the optics book by Born and Wolf (1975), the book by Tatarski (1971) on atmospheric propagation, and, from the field of Acoustic Imaging, an article by Stone (1982). Also, of interest is an article on the topological approach to remote sensing by Dangelmayr and Guttinger, 1982.

Finally, the classic by Rayleigh, "Theory of Sound", and his article (Rayleigh, 1907) which contains the "image method" which was rediscovered by Twersky, 1949, and used by many others. In underwater acoustics the article by Eckart, 1953, has been truly influential. The best reviews and introduction are by Fortuin, 1969; Horton, 1971, and Tolstoy, 1984. A series of papers by Middleton, 1967a,b, represent a phenomenological approach to scattering. Although they do not relate explicitly to the geometrical characteristics of the surface, they have to be seriously taken into account for the simple reason that phenomenological methods play a role even in cases where the approach is considered as pure physical or pure geometrical.

V.B. Modelling approaches to scattering

In Ishimaru, 1978, three types of scatterers are considered: 1) discrete, tenuous or dense, 2) continuous random media, and 3) rough surfaces. The scattering is taken to be either simple, whereby few scatterers participate, or multiple, where scattering of the already scattered energy has occurred in various possible orders and combinations. In modelling the scattering, two approaches are recognized in Ishimaru (1978): a) the analytic approach based on the wave equation, the boundary conditions, the wave amplitudes and the phases, and b) the transport or radiative transfer theory, which is based on the geometric propagation of intensities, and in some heuristic phenomenological interpretations of reflection and absorption.

Although there are many correspondences between the two approaches, the geometrical characteristics of the scattering surface are not present in (b) as pointed out in Watson and Keller, 1983. Three of our four approaches mentioned in the previous section, Rayleigh, Analytic Approximative, and Lebesgue-Berry, belong to Ishimaru's analytic approach, while in Twersky-Biot, elements from the radiative transfer theory are present. It has to be noted however that when in Ishimaru's book the rough surfaces are studied, he distinguishes between the method of small perturbations and the method based on the Kirchhoff approximation.

The deviations from a flat surface may be simple deterministic, complicated deterministic, or stochastic. As a representative of simple deterministic, the periodic surface, studied by Rayleigh in 1896, gives us a good start for the understanding of scattering from irregular surfaces.

Feigenbaum's opinion that, in reality, randomness does not exist, as the ideal Euclidean line does not exist, must be taken into account whenever approximative models with exact solutions are weighted against approximative solutions based on exact models. In practice, any random surface is effectively truncated in frequency by the Nyquist sampling interval. The remainder is considered to be noise. However, even this not-random surface is not easily handled. Imitations of randomness with simple deterministic iterative algorithms that describe evolutions in the theory of GDS have not yet been widely applied to scattering. Their natural position is in the type of modelling we call the Lebesgue-Berry (L-B).

Usually, even the deterministic (but still complicated) surface is further simplified by defining a trend, and considering the deviations from this trend as another form of noise. If these deviations are sufficiently gentle, they are approximated by their local tangent planes (Brekhovskikh and Lysanov, 1982). But now the model is evidently not exact, and an error, different than the original noise, must be accepted.

In most of the cases, as a trend is taken to be, simply, the mean value plane. In order to be able to simplify the resulting algebraic expressions from the application of the wave equation in accord with the boundary conditions, the deviations (heights), and the slopes must be classified as small while the typical surface wavelengths must be classified either as small or large in comparison with the incident sound wavelengths. This classification, however, depends on the angle of incidence because the normal to the mean plane sound wavelength projection depends on this angle In addition, the shadowing of the irregularities is also changing which gives different multiscattering effects and different close range interferences.

The previous concerns relate to what we call Analytic Approximative (AA) modelling. Two main methods belong to this approach. The method of small perturbations and the method of Kirchhoff. In the following, both methods will be examined together with the three other approaches. We will start with the L-B approach. Already it has been indicated that whenever notions from the theory of GDS, like strange attractors, and for the same reason, fractals, are involved, this type of description will be classified as a member of the L-B approach. Because this approach is still in its infancy, only a few will be mentioned while the reasoning behind its name is explained.

V.C. The Lebesgue-Berry (L-B) approach

Waves have been, justifiably, called retarded potentials for the reason that the state of the field at the point P at time t is determined by states at another point Q at a previous time $t - {}^{r}/c$, where r is the distance between P and Q and c the phase velocity (Phillips, 1933). Lebesgue, who is one of the protagonists of fractals, in 1913 produced an example of a region with a spine for which a Green's function does not exist. In other words, the associated Dirichlet problem is impossible to solve (Kellogg, 1953).

Recently, M.V. Berry, 1979, studied monochromatic waves that encounter fractal surfaces. He calls "diffractals" the new wave regime which in the short wave limit has such a rich fine structure that cannot be described by geometrical optics. This is to be expected because it is well known that geometrical optics is based on functions having at least first and second derivatives in order for the WKBJ approximation to be valid. See, for example, what discontinuities of first and second derivatives can do to ray tracing (Pedersen, 1961) and to normal modes (Maslowe, 1982).

We thus conclude that the first case started with Lebesgue, while with Berry new frontiers are to be opened. It is noted here that if one takes into account the footnote on page 29 of Beckmann and Spizzichino, 1963, he will reject the Lebesgue-Berry approach as "inconsiderate". We think however that the main problem is the intractable algebra and that the computers will play a major role in establishing this approach.

V.D. The Rayleigh approach

Rayleigh studied, at first, the normal incidence of plane monochromatic waves, of wavelength , on a sinusoidal boundary and obtained an approximation for the first few modes. Later, Rayleigh (1907), and La Gasce and Tamarkin (1956), who also performed experiments in water tanks, considered any angle of incidence. In 1951 S.O. Rice, known also for his theoretical work on noise, developed a generalization of Rayleigh's approach by applying it to irregular surfaces which could be decomposed into a spectrum of spatial harmonics.

Specifying the periodic boundary by $z = \xi(x) = \xi(x + \Lambda) = h \cos (Kx)$, $\Lambda = 2\pi/K$, the incident plane waves will be scattered in m discrete directions forming angles θm with the normal to the mean level. This is due to the constructive interference of the scattered waves from each individual harmonic bump. The directions of scattering are given by the so called grating formula:

$$\sin\theta m = \sin\theta + m\lambda/\Lambda \tag{1}$$

For m = 0 we have the specular mode. There is a cut-off defining a maximum M given by the condition $|\sin\theta M| \leq 1$.

A variation of the same ideas is used by Marsh (1961a,b, 1963) where he considers a rough boundary as a diffraction grating. The backscattered energy in the direction of incidence, is predominantly produced by that spectral component of the rough boundary for which phase reinforcement occurs.

In the Rayleigh approach it is assumed that the total pressure field p, can be represented by an infinite sum of plane waves

$$P(x,y) = \exp[ik(x\sin\theta - 2\cos\theta)] + \\ + \sum_{m=-\infty}^{\infty} A_m \exp[ik(x\sin\theta_m + z\cos\theta_m)] \tag{2}$$

and that at the boundary p = 0. Note that (2) is a complete representation in the sense that it is valid everywhere, on and away from the boundary; $k = 2\pi/\lambda$.

All the above assumptions have been criticized. For example, Heaps (1955), Mecham (1956) and by Uretsky (1963, 1965) concluded that the amplitude coefficients Am in equation (2) are only valid when the boundary undulations are gentle, i.e. small h.K. For more clarification, see Milar (1969), De Santo (1979) or McCammon and McDaniel (1985).

Having p = 0 at the boundary, the two remaining terms in (2) can be expanded in a Fourier series with respect to x. A set of infinite linear equations in the coefficients Am results. By approximation, or by recursion, the amplitude coefficients can be obtained. It can be seen that the surface Rayleigh waves are represented in (2) whenever cos θm is imaginary. This happens when m > M. A complete investigation of (1) and (2), with informative graphs is given in Beckmann and Spizzichino, 1963. It has to be observed that due to the finiteness of the illuminated periodic surface by actual sources, the energy is found not in m lines but in m lobes. Their width becomes larger as the illuminated area becomes smaller.

V.E. The Analytic Approximative Approach

The method of small perturbations (P) and the method based on the Kirchhoff approximation (K) are included in the Analytic Approximative approach. A detailed exposition of the (P) method is found in Bass and Fuks (1979) and also Wenzel (1973). The Kirchhoff method could be found in Eckart (1953), Clay and Medwin (1977) and Brekhovskikh and Lysanov (1982). Enlightening comparisons between (P) and (K) are given in Kuperman (1975) and in Labianca and Harper (1977). Although in the last reference it is demonstrated that the (P) method is a generalization of the Rayleigh-Rice approach and that it incorporates the (K) method, the (K) method, in many simple cases, has an intuitive appeal that will be exposed in brief below.

V.E.1. The Kirchhoff method

In underwater acoustics the (K) method is based on a generalization of the Helmholtz integral representation devised by Kirchhoff and is called the Helmholtz-Kirchhoff theorem or integral representation. This representation could be considered as the analytic expression of Huygen's principle (Baker and Copson, 1969) although in Lamb (1945) reservations are presented. One could arrive at this theorem by applying Green's second identity to a pair of arbitrary functions which subsequently are supposed to be harmonic as they are required to obey Helmholtz wave-type equations.

One of these functions is the Green's function g associated with the linear differential operator $(\nabla^2 + k^2)$ and the other could be the excess pressure P above the ambient pressure, or the displacement potential, ϕ. The function g is the wavefield at r generated by a point source function at r_o. For example, a delta pulse, $\delta(r - r_o)$, concentrated at the point r_o together with the Green's function

$$g = g(^r/r_o) = g(r - r_o) \quad \text{satisfy}$$
$$(\nabla^2 + k^2)g = \delta(r - r_o) \qquad (1)$$

In other words, the linear operator $(\nabla^2 + k^2)$ maps g into a delta pulse. The symmetry of g (Phillips, 1946), is guaranteed by reciprocity and, hence, the above equation will also be valid for $\delta = \delta(r_o - r)$.

For a homogeneous infinite medium and a harmonically time varying source, or forcing function, with frequency w, the Green's function is given (Tolstoy, 1973, Roos, 1967) by:

$$g(r - r_o) = \frac{1}{4\pi R} e^{i(kR - wt)} ,$$

$$R = |\vec{r} - \vec{r}_o| \text{ and } kR = \vec{k} \cdot (\vec{r} - \vec{r}_o)$$

It is well known that P and g are assumed to satisfy the linear inhomogeneous wave equation with constant coefficients:

$$\nabla^2 P = \frac{1}{c^2} \cdot \frac{\partial^2 P}{\partial t^2} = f \qquad (2)$$

where c is the adiabatic sound speed and where the source f is assumed to be at $r = r_s$ and is given by:

$$f = A\, e^{-iwt}\, \delta(r - r_s), \text{ where A is a constant}$$

After applying Green's second identity, the pressure field at r is given by the Helmholtz-Kirchhoff theorem:

$$P(r) = \frac{A}{4\pi R_s} e^{ikR_s} - \iint_S (g\nabla P_o - P_o \nabla g) \cdot ds \qquad (3)$$

where $R_s = |r_s - r|$.

The first term on the right of equation (3), is the contribution to the pressure field from the source. In problems dealing with scattering from surfaces it represents the incident field. The surface integral is over a surface S enclosing the medium in which Eq. (2) holds. P_o is the value of P on S. Also, g must be taken on the same surface. This surface may include not only the walls (boundaries) but also the surfaces enclosing inclusions, suspended fluid particles, for example.

The expression (3) is an integral equation because P is included in both parts. If P_o is known at S then (3) is an exact equation. In order to resolve this equation various numerical schemes (Lin, 1985) are applied. The usual classic approach is to use an iterative scheme whereby an initial guess for P_o is made and then by iteration better estimates, $P_{o,1}$, are obtained. If the problem refers to a refraction by obstacles, the surface integral is replaced by a volume integral enclosing the obstacle and the iteration is named the "Born approximation" (Morse and Feshbach, 1953).

For problems of diffraction from surfaces or apertures, the first iterative step applied in (3) is equivalent to the Kirchhoff approximation. This approximation is formed by assuming that the surface $\delta(x,y)$ is gentle, i.e. locally flat.

Let us suppose that at R_1 there is a source with a directivity pattern D and that the point of incidence 0 of the beam's axis is taken as the origin. Let θ_1 be the angle of incidence and consider at a distance r from 0 an element ds while the receiver is at R_2. The incident field, P_{in}, on ds is approximately

$$P_{in} = \frac{D}{R_1} e^{i\vec{k}_1 \cdot (\vec{R}_1+\vec{r})} ,$$ where $\vec{k}$ is the incident wavenumber.

The Green's function for the element ds is

$$g = \frac{\exp[i\vec{k}_2 \cdot (\vec{R}_2 - \vec{r})]}{4\pi R_2}$$

The Kirchhoff approximation assumes that the scattered pressure field P_c could be given by

$$P_c = \mathbf{R} P_{in}$$

R is the Rayleigh relection coefficient of an infinite plane wave on an infinite plane surface. Thus this assumption imposes smoothness conditions for the surface. From these conditions it also must follow that:

$$\nabla P_c \cdot ds = \mathbf{R}\nabla P_{in} \cdot ds$$

Introducing now in (3) $P_o = P_{in} + P_c = P_{in} + \mathbf{R}P_{in}$ we have

$$P_c = - \iint_S (g\nabla P_o - P_o\nabla g) \cdot ds = \iint_S \mathbf{R}\nabla(P_{in}g) \cdot ds$$

or, taking into account g and P_{in} as defined above, we have in a first approximation:

$$P_c \approx \frac{i \exp\ [ik(R_1 + R_2)]}{4\pi R_1 R_2} \iint_{\hat{S}} D\mathbf{R}\vec{V} e^{i\vec{V}} \cdot ds \tag{4}$$

where $\vec{V} = \vec{k}_1 - \vec{k}_2$

and

$$ds = \frac{\nabla \perp \xi}{\sqrt{1 + \xi_x{}^2 + \xi_y{}^2}}\, dxdy$$

$\Delta\perp$ is the operator $\vec{1}_{x'} \frac{\partial}{\partial x'} + \vec{1}_{y'} \frac{\partial}{y'}$ taken in the plane (x',y') which is perpendicular to the beam axis. ξ_x and ξ_y are the derivatives of the rough surface. If these derivatives are small and if we denote the differences between the x,y,z, components of the wavenumber vectors k_1 and k_2 by 2α, 2β, 2γ, respectively, the scattered field is given by (Tolstoy, 1973):

$$P_c \approx \frac{i \exp\ [ik(R_1 - R_2)]}{2\pi R_1 R_2} \iint_S D\,\mathbf{R}\, [\alpha\xi_x + \beta\xi_y - \gamma] \cdot$$
$$\cdot \exp\ [2i(\alpha_x + \beta_y + \gamma\xi)]\ dx\ dy \tag{5}$$

For a small spread of angles near the specular and for constant R one could obtain (Tolstoy, 1973):

$$P_c \approx \mathbf{R} \frac{ik \cos\theta_1}{2\pi R_1 R_2}\ e^{ik(R_1+R_2)} \iint_S D\ e^{2i(\alpha_x+\beta_y+\gamma\xi)}\ dxdy \tag{6}$$

From this equation many results for various surfaces can be obtained For example for a smooth surface $\xi = 0$ we have:

$$P_c = R \frac{ik \cos\theta_1}{2\pi R_1 R_2} e^{ik(R_1+R_2)} \iint_S D\, e^{2i(\alpha_x+\beta_y)} \, dxdy$$

Also, from (6) the scattering by sinusoidal surfaces can be calculated. The same equation can also be used when the rough surface is represented by a Fourier series. Finally, averages and variances of the field could be obtained from (6) as a function of the irregular surface statistics.

V.E.2. The method of small perturbations

Whenever the rough surface has a small root mean square height $a \ll \lambda$, where λ is the sound wavelength and has gentle slopes, i.e. $a \ll \Lambda$, where is a typical surface wavelength, the perturbational method can be applied. Since the (K) method is valid for $\lambda \ll \Lambda$ (physical optics) the domain of validity of (P) (also known as the method of small-wave height) incorporates that of (K). A more detailed discussion and clarifications are given in Labianca and Harper, 1977.

For any realization of the random surface, a perturbational expansion of the solution according to the small parameter "a" is carried out to terms of second order. Then, by taking ensemble averages, the first and the second order statistics of the field can be obtained as a function of "a" and the correlation function of the surface. The computed average field is coherent. There are similarities with the Rayleigh-Rice method in that the far field is represented as a superposition of plane waves, while the expansion up to the second order of the boundary conditions about the mean plane is equivalent to the inclusion of second order scattering effects.

Representing the surface as $z_s = a\xi(x,y)$, where ξ is a homogeneous zero mean random process and "a" is small, and where the source is on the z axis at $r' = (0,0,z')$, the pressure field at the point r is expanded as

$$P(r,r',a) = P_o(r,r') + aP_1(r,r') + a^2P_2(r,r') + 0(a^3) \qquad (7)$$

On the surface z_s the pressure field $P(x,y,z_s,r',a)$ is expressed by a second order Taylor series about the trend plane $z = 0$ as:

$$P(x,y,z_s,r',a) \equiv 0 = (P)_{z=0} + z_s\left(\frac{\partial P}{\partial z}\right)_{z=0} + \frac{z^2{}_s}{2}\left(\frac{\partial^2 P}{\partial z^2}\right)_{z=0} + 0(z^3{}_s) \tag{8}$$

In (8) we assume that at $z = z_s$ there is a pressure release surface. The use of Taylor series implies the analytic continuation of the boundary condition from the free surface to the plane $z = 0$. Equating the corresponding coefficients between (7) and (8) on the plane $z = 0$ yields the recursion

$$P_o = 0, \; P_1 = -\xi\frac{\partial P_o}{\partial z}, \; P_2 = -\xi\,\frac{\partial P_1}{\partial z} - \xi^2\,\frac{\partial^2 P_o}{2\partial z^2} \tag{9}$$

For a rigid rough wall the assumption is that the normal derivative of P must vanish, i.e. on z_s:

$$\frac{\partial P}{\partial n} = 0 \tag{10}$$

For this case, the relations corresponding to (9) are given in Wenzel, 1974, by the recursive formula

$$\frac{\partial P_\mu}{\partial z} = \frac{\partial \xi}{\partial x} \cdot \frac{\partial P_{\mu-1}}{\partial x} + \frac{\partial \xi}{\partial y} \cdot \frac{\partial P_{\mu-1}}{\partial y} - \xi\frac{\partial^2 P_{\mu-1}}{\partial z^2} \tag{11}$$

Where $P_{-1} = 0$. These are obtained by combining (9) and (10) and accepting the small slope assumption. Observe here, as in Brekhovskikh and Lysanov (1982) that the terms in (11) for $\mu > 0$ depend on the slope of the random surface, while in (9) they depend only on ξ.

In both cases, pressure release and rigid wall, the fields P_μ, $\mu = 0,1,2$ have to satisfy the radiation condition

$$P\mu(r,r',a) = 0 \text{ for } |r| \to \infty$$

Also, these fields must satisfy the equations

$$(\nabla^2 + k_o{}^2)P_\mu = \begin{cases} f \text{ for } \mu = 0 \\ 0 \text{ for } \mu \neq 0 \end{cases} \tag{12}$$

After the successive introduction of P in Eq.(3) the field can be computed. Of interest is the quantity P_o (r,r') = g_o(r,r') and P (r,r') = P(r',r), (i.e. reciprocal for $\mu < 3$). From (7) it can be inferred that $\overline{P} = \overline{P}_o + a\overline{P}_1 + a^2\overline{P}_2$. Because P_o is not random and $P_1 = 0$ (Bekhovskikh and Lysanov, 1983), for the coherent field:

$$\overline{P} = P_o + a^2\overline{P}_2 + O(a^3) \tag{13}$$

The coherent field P on the mean plane z = 0 satisfies a "generalized smooth boundary condition" (Wenzel, 1974; Tolstoy, 1984), in the sense that a linear relationship can be established between the total field and P The value of P_2 is given in the same references and depends on the correlation function of the rough surface.

V.F. The Twersky-Biot Approach

In his 1949 Ph.D. Thesis "On the Theory of the Non-Specular Reflections of Plane Waves of Sound", Victor Twersky established a method variations of which have also been evaluated by Biot (1957, 1958), Tolstoy (1979, 1984), Medwin and Novarini (1981), and which has been applied to the inference of Arctic ice morphological characeristics by Diachok (1976). In general this approach consists of studying first the scattered field from a model protuberance on a plane which is then used to approximate the combined result of many periodic or randomly located protuberances on a plane.

The protuberances (bumps or bosses) which have been investigated are half circular or elliptic cylinders and hemispheres. In Medwin and Novarini (1981, 1985) a conjoint assemblage of crests and troughs or facets and wadges is used. Although in the literature (Beckmann and Spizzichino, 1963, for example) many works could be found that describe rough surfaces by protuberances, the subsequent treatment of the problem is different. The same could be said for the remarkable computer dependent work of Dozier (1984) who uses trapezoids to express the irregularities. Subsequently he uses conformal mappings to locally flatten the segments of the surface and then the parabolic equation is employed; finally, after transforming back to the real space, transmission losses are computed.

It appears that Twersky, independent of Rayleigh, used the "image method" whereby the plane, on which the half cylinder is located is substituted by the rest of the cylinder and by an additional incident wave which is the image of the original wave according to the replaced plane. This method is, in principle, identical to the image method used by Kelvin in Electricity and by Helmholtz and Stokes in Fluid Dynamics (Granger, 1985).

An additional characteristic of the Twersky-Biot approach is the observation that the scattered field is composed of three components. One corresponds to the specular reflection, the other two are due to the presence of the obstacle. From these two, one is equivalent to a simple source of sound and the other to a double source. The simple source is explained as a reaction of the bump to the periodic condensations and rarefractions of the fluid. The double source is the result of the immobility of the boss. If the bump was absent, in its place the fluid would sway back and forth as a dipole. More details on this physical interpretation are to be found in Twersky, 1949; Lamb, 1945, and Lighthill, 1980.

The previous interpretations, in theoretical terms, are included in Equation (3). One could state that the Helmholtz theorem in a finite region expresses the possibility of resolving a piece wise differentiable vector field F in two components; one is an irrotational component, $\nabla\phi$, and the other solenoidal component, $\nabla x A$, where ϕ (a scalar) and A (a vector) are properly chosen.

From this point on, the works based on Biot and on Twersky differ. In scope, the work of Twersky is more general while the work based on Biot is specialized for $a < \Lambda << \lambda$, where 'a' is a typical dimension of the bumps and Λ their spacing, i.e. conditions encountered in low frequency propagation. In the case of $\Lambda << \lambda$, the protuberances are replaced by a continuous distribution of monopoles and dipoles. Biot showed that the effect of roughness is equivalent to a linear boundary condition obeyed by the total acoustical pressure (or displacement) ϕ and the coherent scattered field ϕ_s on a plane other than the zero plane that is a smooth representative of the corrugated surface. Thus, if $\phi = \phi_o + \phi_s$, where ϕ_o is the field in the absence of roughness the condition is:

$$\frac{\partial \phi_s}{\partial z} = \eta\phi \qquad (14)$$

The above condition is the result of an analysis that includes multiple scattering effects. The factor η has various forms depending on the modelling of the bumps; it depends on the roughness volume per unit area, on the typical shape chosen for the bosses and on the spacing between them. In a sense, (14) has absorbed in the geometrical characteristics of the bumps substituting them by their result (i.e. a properly chosen distribution of monopoles and dipoles that after some simplifications and assumptions is condensed in η).

By comparison with the (P) method, Biot's method gives a better approximation. That is, it is a first order approximation in ka, where k is the acoustic wavenumber, while (P) is only an approximation of the second order, k^2a^2. For $a/\Lambda \rightarrow 0$ (small slopes) both methods give the same result. It has to be noticed here that both the Twersky and the Biot methods are not limited by the small slope condition. The bumps could be finite needles protuding normally outwards from the surface.

Biot's method is less sophisticated than Twersky's, however, it easily predicts the existence of a boundary coherent wave when the source is close to the surface. This wave decays exponentially from the surface, but propagates parallel to it and in the absence of attenuation it decreases as the $-\frac{1}{2}$ power of the range. Therefore, for sufficiently large range it dominates the direct wave which decays as a -1 power. As it has been generalized by Tolstoy, 1980, Biot's method presents an advantage over Twersky's in that constraints for the scatterers can be incorporated in the formulation. Thus, the scattering from rough fluid interfaces between media with the same density but different sound speeds can be studied. Similarly, the efforts by Menke and Richards, 1982, who present a generalization of Biot's method, reflect the physical circumstances better.

Twersky's method is related to the theory of radiative transfer as it aims to obtain results consistent with the preservation of energy. His fundamental approximations and results in the study of multiple scattering effects relate to the diagram methods (Feynman); the Dyson equation for the mean field and the Bethe-Salpeter equation for the correlation function (Tatarski, 1971; Ishimaru, 1978). Although all the possible multi-scattering interactions are not included the most important interactions are included. The approximation named the Foldy-Twersky integral equation is equivalent to the so called "first order smoothing approximation" (Frish, 1968).

A summary of Twersky's method is given in Tolstoy, 1984, while in Ishimaru, 1978, a more detailed exposition is found. The latest work on the subject is found in Twersky, 1983a,b. The following considerations include only the basic elements. More details about the elliptic protuberances which have been applied to the Arctic by Diachock, 1974, are to be found in Burke and Twersky, 1966.

In a medium containing M scatterers, at r_m positions, the total field $\psi(r)$ at position r is given as a sum of the incident $\phi_i(r)$ in the absence of the scatterers plus the contribution of the scattered field U_m from the M scattering points, i.e.

$$\psi(r) = \phi_i(r) + \sum_{m=1}^{M} u_m(r)U_m \qquad (15)$$

$u_m(r)$ is called the single scattering operator or scattering characteristic of the m particle as it is observed at position r. Consider now that each of U_m is the result of incidence $\phi_i(r_m)$ and scattering from the rest i scatterers where $i \neq m$. An iteration could be started if in (15) is substituted:

$$u_m = \phi_i(r_m) + \sum_{i=1, i\neq m}^{M} u_i U_i \qquad (16)$$

Evaluation of the above iteration gives an infinity of sums Twersky's interpretation consists of eliminating sums that contain scattering combinations representing a scatterer more than once. Included are the incident field $\phi_i(r)$ plus all the combinations of sums representing scattering paths through different scatterers.

By taking the average of (15) the coherent field at the point r could be obtained. For $M \to \infty$, and with $\rho(r)$ representing the density of scatterers one obtains the following expression named the Foldy-Twersky integral representation

$$\bar{\psi}(r) = \phi_{in}(r) + \int \rho(r')u_{r'}(r)\bar{\psi}(r')dr' \qquad (17)$$

For the computation of the correlation function of the field the same rules and approximations are followed as in the case of the average (coherent) field.

V.G. Scattering in waveguides with rough walls

Any review of the Arctic Hydroacoustics would be incomplete without a reference to the effects of rough walls on the far field represented by normal modes. We have already mentioned in Sec.II, the work of Kutschale, 1984, and in Sec.V.A., we mentioned the book by Bass and Fuks which contains an elegant treatment of the problem with the method of small perturbation.

Since 1948 when C.L. Pekeris laid the foundations by studying successfully simple two liquid layer waveguides (Pekeris, 1948, see also Papadakis, 1973) the theory has been developed further and more realistic models have been considered (Labianca, 1977; Tolstoy, 1973; Boyles, 1984).

In Kryazhev et al., 1976, it is written that "The most conspicuous example of an underwater acoustic waveguide in which the scattering of sound by the boundary is significant is the Arctic sound channel ...." To date, more effort has been put in the propagation of sound in shallow water (Clay et al., 1985; McDaniel, 1981; Kuperman and Ingenito, 1977; Tolstoy and Clay, 1966) and in the surface duct (Bucker, 1980).

Generally, all the above cases are examples where the effect of the roughness is to transfer energy from the coherent field to the incoherent. The imagined mechanism is that energy is transmitted from the trapped modes to the other trapped modes so that the randomness of the field increases, and/or, from the trapped modes to the untrapped so that the attenuation increases.

In well posed circumstances, the sound field can be computed at a given point in the waveguide, as well as the correlation function of the sound field at two points. In a recent study (Kryazhev and Mudryashov, 1985, it is concluded that "the form of the array response in the sound field generated in a waveguide with statistically rough boundary differs from the array response in a regular waveguide, and this fact can be utilized to discern the nature of the stochasticity of a sound field and to determine the parameters of a rough boundary".

VI Flow Under Ice

The reasons for including this section in the present work are mainly twofold. First, to reiterate that flows in the Arctic depend on the physical characteristics of ice and water, as well as, on the underside of ice morphology. Second, to note that for certain dynamical conditions, expressed by the Froude number, medium inhomogeneities are possible and that these inhomogeneities may influence the scattering. Indeed, recently in the Arctic, Topham (1985, Figure 3) observed substantial inhomogeneities in the vicinity of an ice keel extending downwards to about 40m.

These observations immediately indicate, at least for a high frequency band of sound waves, that computations from a simple acoustical model of "ice roughness" and "roughness" which influences the flow are not simply related because a strong scattering will result not only from the ice surface but also from the inhomogeneities. This however is not discouraging; the history of open sea surface reverberation (Medwin, 1970) teaches that after two scales of roughness were tried for the sea surface, a new layer was introduced that represents the air bubbles in the vicinity of the sea surface. This is suggesting that a three layered model for the scattering of sound in the Arctic is the most appropriate.

Except for the above interaction between flow and sound, the hydrodynamical interaction between flow and ice is not a simple matter. Starting with Nansen (1902), works followed by Sverdup (1928), Rossby and Montgomery (1935), Felzenbaum (1958), Reed and Campbell (1960), Campbell (1965), Galt (1973), and Rothrock (1975), studying the dynamics of ice drift, while in Hibler (1979) dynamics and thermodynamics are combined.

The general equation for the ice drift equilibrium is thought to include five terms as given in Campbell, 1960 and Hibler, 1979:

$$T_a + T_i + T_w + C + G = 0$$

where:

T_a the wind stress at the air-ice interface

T_i the internal stress of ice

T_w the water stress at the water-ice interface

C the Coriolis force

G the pressure gradient force due to the tilting of the sea surface on which the ice floats.

Detailed examination, especially of the first three terms, which are complicated and temperature dependent, has been undertaken in the last fifteen years. For example, in the Arctic, Arya (1973) has studied the wind stress while McPhee and Smith (1976) have studied the water-ice stress. Also, work in experimental tanks and computer simulations have proved helpful. A detailed review of the works related to the dynamics of sea and ice cover is contained in Chung and Rowe, 1984.

Specializing now in the interaction between water and ice, it has been found that the flow under ice is influenced by thermodynamical processes. The small scale ice roughness is connected with the friction or skin drag r_s; the shape, orientation and spacing of the large scale protuberances (keels, for example) are connected with the form drag r_f; the density and velocity profile of the water are connected with the wave drag r_v.

The various kinds of drag mentioned constitute the total water-ice drag. They are, however, interdependent. For example, the distribution of small scale roughness on an ice keel controls the separation of flow Similarly, the internal waves that are supported by the water stratification change the pressure distribution in the proximity of the rough surface This gives a different regime of flow separation so that the emerging eddies (corresponding to r_f) and the wetting area (corresponding to r_s) are different than without the existence of the gravity waves.

From Chung and Rowe (1984) it could be inferred that given the seasonal variability in the Arctic, the three models that they suggest may give, when studied, a satisfactory description of most of the phenomena of ice-water interaction underneath an isolated Arctic ice keel. The problem however of the interaction of many keels still remains open. It is hoped that phenomena analogous to multiscattering mentioned in Section V take place and that the emerging picture would be of a wave governed layered structure, interrupted by areas of eddies.

The previous description is corroborated by the data of Topham, 1985. In order to have an evaluation of the acoustical influence of the inhomogenous fluid mass, an accurate statistical description of the inhomogeneities is needed together with the dynamic conditions that generate them. As a theoretical guide for the fluid dynamic-acoustical interaction, the works by Lighthill, 1980; Tolstoy, 1982 and Menke and Richards, 1982, are suggested.

VII CONCLUSIONS

The analytic approximative approach represented by Mellen et al. 1985a,b, and called by them statistical, is inadequate in explaining the available data. In the first of these works, leaking of energy from the sound channel is suggested as a reason for the discrepancy. In the second, it is indicated that even when a second scale of roughness for the ice is introduced (see, for example, Kurianov, 1962), satisfactory results are still not obtained.

Apart from the fact that correlations and power spectra are reductions of the actual surface forms so that many physically different surfaces may be represented by them (Kinney and Clay, 1985), other reasons to explain the inadequacies could be that the multiple scaling (Lovejoy and Schertzer, 1985, Schertzer and Lovejoy, 1986) fractal nature of the ice surface has not been taken into account, or perhaps, that improper quantities have been introduced. For example, asymptotic forms of correlation functions and spectra are used, reflecting infinite number of interactions between acoustic waves and the ice surface, while, from the general propagation characteristics (Figure 1), it is evident that only few in number interactions are taking place. Another reason for the discrepancies could be that the inhomogeneous layer adjacent to the ice cover which was presented in the previous section was not included in these studies. The acoustical significance of the observations by Topham, 1985, remains to be evaluated, especially at the low acoustic frequencies. The work of Menke, and Richards, 1982 which is based on the work of Tolstoy, 1982 and Biot, 1968 appears to be the best suited to the incorporation of a layer of inhomogeneities. In relation to these last three works, it would be also of interest to verify the existence in the Arctic of the boundary wave that they predict (see equation 14). Although experimental work in the laboratory has verified this wave (Medwin et al., 1979), as far as we know from the available literature, verification has not yet been conducted in the Arctic. For an analogy in oceanography see Mysak and Howe, 1978.

Calculations based on the method of Twersky are more promising as it can be seen in Diachock, 1976, and Tolstoy, et al. 1985. This however must not be taken as an absolute and objective determination of the surface geometrical characteristics. Twersky (1983a,b) already felt the need to introduce new factors that compensate for the distortion of the shape of the bumps as they are deduced by the inverse scattering methodology.

The possible complications introduced by the flow, as well as the thermal effects - two factors that have only been mentioned in this study but not evaluated - make the effort of deducing the ice morphology from hydroacoustical measurements an inter-disciplinary affair. It is fortunate that we have as a guide a recent article by Thom, 1985, about the dangers and the benefits arising from any interdisciplinary quest.

Kantian, i.e. well posed questions are always possible to be formulated, if in advance a task is set with precision. However, as Karl Popper has written many times, the answer to one question generates other questions. This, as we understand it, is due to the forcing that the clear cut, well posed, compartmentalized question requires in order to be formulated (see section IV.A).

It appears, then, that it is not sufficient to study only specific cases, or conduct specific experiments, but in addition strong efforts for coordination of the whole affair are needed. A path guided by the decision theoretical model has been put forward in Papadakis, 1973, whereby the main characteristics resulting from each specific study are weighted and carefully combined in a new synthesis.

Despite the thermal and dynamic complications, our quest of ice morphology has an advantage over the acoustical tomography, or more generally, the propagation in inhomogeneous media. Oceanographers and acousticians, by being polarized, put unreasonable demands to each other, forgetting their common roots. It would have been an extremity to demand an effective resolution of the Navier-Stokes equations from the knowledge only of some partial solutions of a wave propagation problem. Intuitively, there are many reasons to seek inference of flows from the behaviour of acoustic waves. A very elementary case of this, is presented in Lighthill, 1982, page 18. The idea, however, that it could be possible to pass with the aid of mathematical transformations from the behaviour of one form of material organization to the behaviour of the other, escapes the abilities of the present generation.

A third element, the geometry and morphology of the ice surface, and a different tactic, to pass from waves to geometry and from there to flows, is the advantage of the effort described in this work over the propagation in random media that makes it feasible in our generation. Of course, in the study of random media, the same elements exist there too, but in the Arctic the geometrical element is predominant.

The general conclusion then of this study is that in the quest of deducing hydroacoustically the Arctic ice morphology there is a better chance for a harmonious and mutually beneficial cooperation between oceanographers and acousticians. Critical to this quest is the role that geometry, structure, and form play. It is hoped that what is described in section IV and in Papadakis, 1985, will prove to be helpful.

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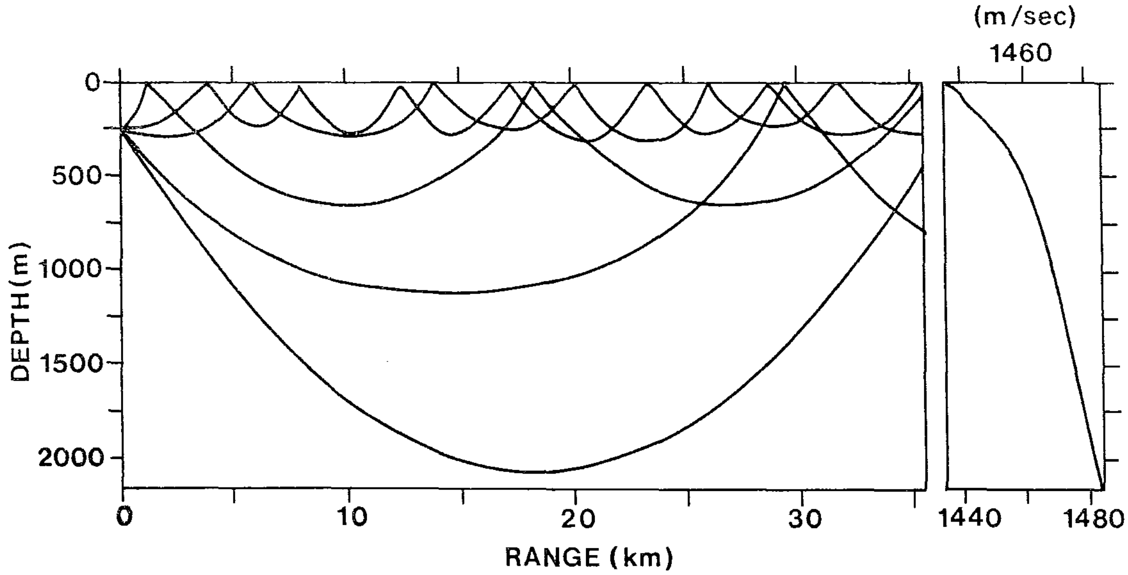

Figure 1. A Typical sound velocity profile and a corresponding ray tracing sketch (modified from O.I. Diachok, 1978)

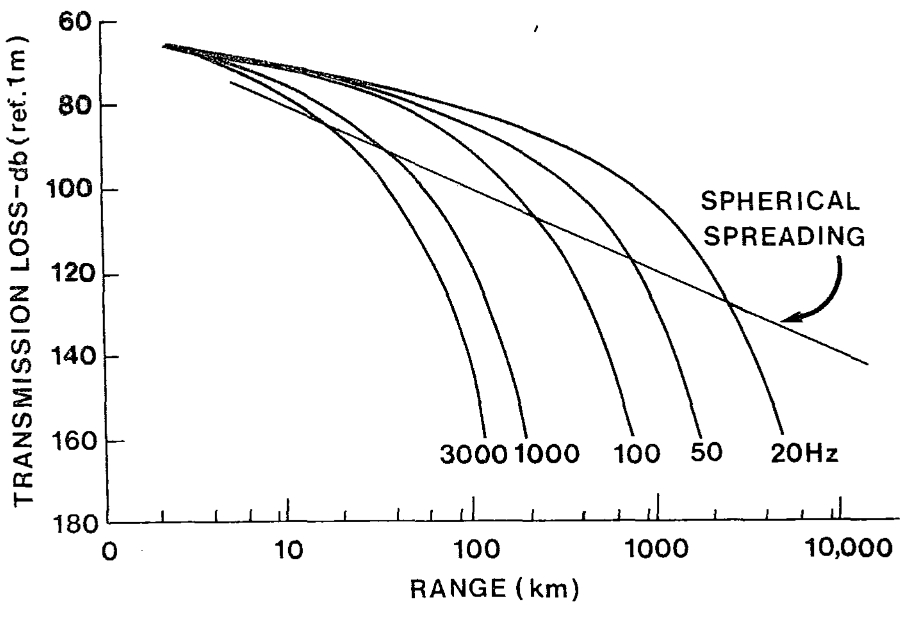

Figure 2. Transmission loss in the Arctic (modified from B.M. Buck, 1968)

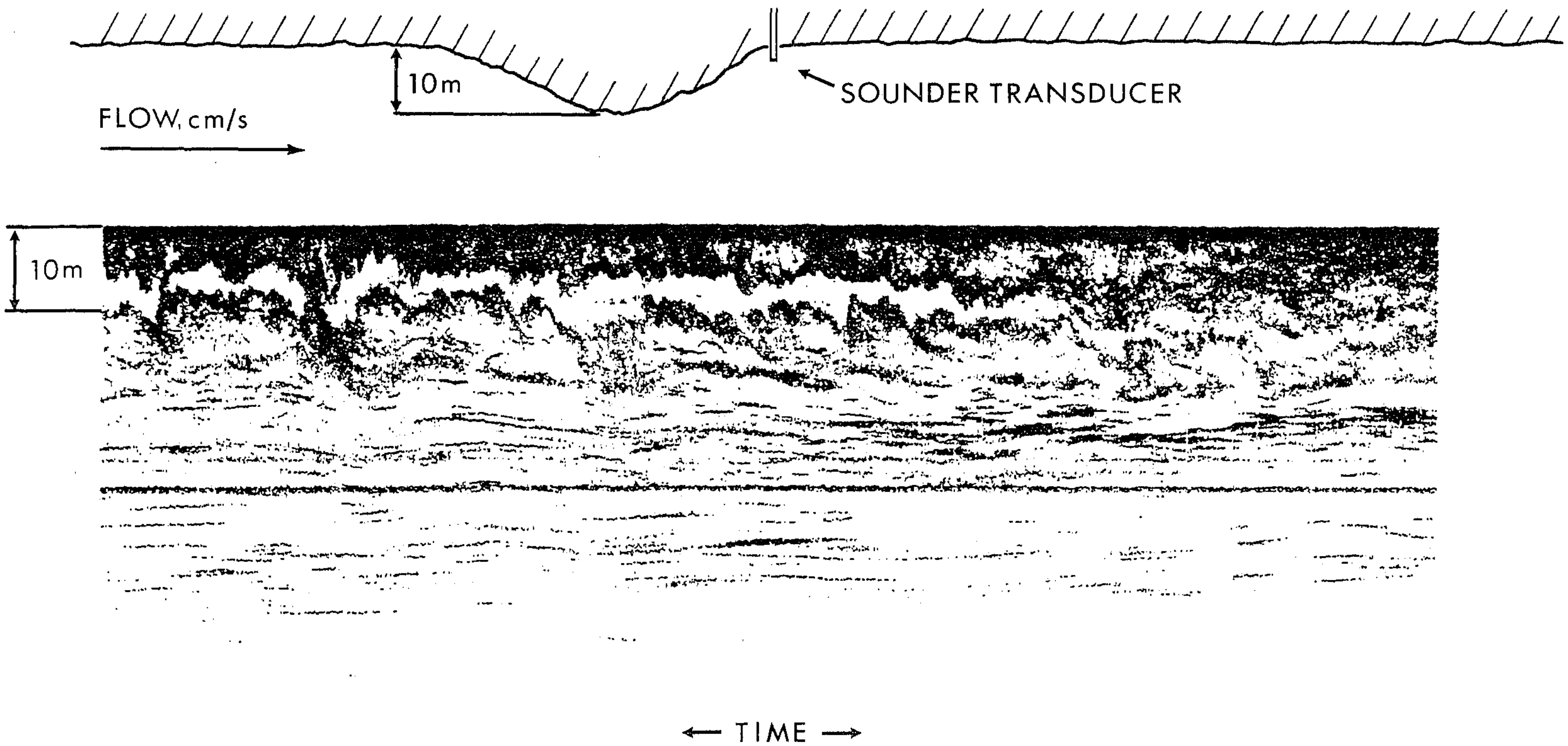

Figure 3. A 200 kHz sounder record taken in the lee of an ice keel, April 1985, N.W.P. by Dr. D.R. Topham of IOS.