3 copies

Scientific Excellence • Resource Protection & Conservation • Benefits for Canadians
Excellence scientifique • Protection et conservation des ressources • Bénéfices aux Canadiens

SAS Programs for Fitting a Seal Population Dynamics Model

N. G. Cadigan, and P. A. Shelton

Science Branch
Department of Fisheries and Oceans
P.O. Box 5667
St. John's, Newfoundland A1C 5X1

1993

Canadian Technical Report of Fisheries and Aquatic Sciences No. 1927

Fisheries and Oceans Pêches et Océans

Canada

Canadian Technical Report of Fisheries and Aquatic Sciences

Technical reports contain scientific and technical information that contributes to existing knowledge but which is not normally appropriate for primary literature. Technical reports are directed primarily toward a worldwide audience and have an international distribution. No restriction is placed on subject matter and the series reflects the broad interests and policies of the Department of Fisheries and Oceans, namely, fisheries and aquatic sciences.

Technical reports may be cited as full publications. The correct citation appears above the abstract of each report. Each report is abstracted in *Aquatic Sciences and Fisheries Abstracts* and indexed in the Department's annual index to scientific and technical publications.

Numbers 1–456 in this series were issued as Technical Reports of the Fisheries Research Board of Canada. Numbers 457–714 were issued as Department of the Environment, Fisheries and Marine Service, Research and Development Directorate Technical Reports. Numbers 715–924 were issued as Department of Fisheries and the Environment, Fisheries and Marine Service Technical Reports. The current series name was changed with report number 925.

Technical reports are produced regionally but are numbered nationally. Requests for individual reports will be filled by the issuing establishment listed on the front cover and title page. Out-of-stock reports will be supplied for a fee by commercial agents.

Rapport technique canadien des sciences halieutiques et aquatiques

Les rapports techniques contiennent des renseignements scientifiques et techniques qui constituent une contribution aux connaissances actuelles, mais qui ne sont pas normalement appropriés pour la publication dans un journal scientifique. Les rapports techniques sont destinés essentiellement à un public international et ils sont distribués à cet échelon. Il n'y a aucune restriction quant au sujet; de fait, la série reflète la vaste gamme des intérêts et des politiques du ministère des Pêches et des Océans, c'est-à-dire les sciences halieutiques et aquatiques.

Les rapports techniques peuvent être cités comme des publications complètes. Le titre exact paraît au-dessus du résumé de chaque rapport. Les rapports techniques sont résumés dans la revue *Résumés des sciences aquatiques et halieutiques*, et ils sont classés dans l'index annual des publications scientifiques et techniques du Ministère.

Les numéros 1 à 456 de cette série ont été publiés à titre de rapports techniques de l'Office des recherches sur les pêcheries du Canada. Les numéros 457 à 714 sont parus à titre de rapports techniques de la Direction générale de la recherche et du développement, Service des pêches et de la mer, ministère de l'Environnement. Les numéros 715 à 924 ont été publiés à titre de rapports techniques du Service des pêches et de la mer, ministère des Pêches et de l'Environnement. Le nom actuel de la série a été établi lors de la parution du numéro 925.

Les rapports techniques sont produits à l'échelon régional, mais numérotés à l'échelon national. Les demandes de rapports seront satisfaites par l'établissement auteur dont le nom figure sur la couverture et la page du titre. Les rapports épuisés seront fournis contre rétribution par des agents commerciaux.

Canadian Technical Report of
Fisheries and Aquatic Sciences 1927

July 1993

SAS PROGRAMS FOR FITTING A SEAL POPULATION DYNAMICS MODEL

by

Noel G. Cadigan and Peter A. Shelton

Science Branch
Department of Fisheries and Oceans
P.O Box 5667
St John's, Newfoundland A1C 5X1
Canada

©Minister of Supply and Services Canada 1993
Cat. No. Fs 97.6/1927E ISSN 0706-6457

Correct citation for this publication:

Cadigan, N. G., and P. A. Shelton. 1993. SAS programs for fitting a seal population dynamics model. Can. Tech. Rep. Fish. Aquat. Sci. 1927: iii + 34 p.

ABSTRACT

Cadigan, N. G. and P. A. Shelton. 1993. SAS programs for fitting a seal population dynamics model. Can. Tech. Rep. Fish. Aquat. Sci. 1927: iii + 34 p.

This technical report presents a description of two SAS programs (MAKED.SAS and SEAL.SAS) that fit a population dynamics model to seal data. The programs have been constructed in such a manner that special knowledge of SAS is not required to run them. The population dynamics model is transformed to a standard nonlinear regression model and parameters are estimated using SAS's PROC NLIN. Output from the programs includes estimates of intrinsic mortality, pup numbers over time and total population numbers over time. The code for the programs is presented along with two examples.

RÉSUMÉ

Cadigan, N. G. et P. A. Shelton. 1993. SAS programs for fitting a seal population dynamics model. Can. Tech. Rep. Fish. Aquat. Sci. 1927: iii + 34 p.

Ce rapport technique présente une description de deux programmes SAS (MAKED.SAS et SEAL.SAS) qui adaptent un modèle de dynamique des populations aux données sur les phoques. Ces programmes ont été conçus pour qu'on puisse les utiliser sans connaissance préalable spéciale du SAS. Le modèle de dynamique des populations est transposé en modèle de régression non linéaire standard et les paramètres sont estimés avec la fonction PROC NLIN du SAS. Ces programmes peuvent fournir des estimations du taux de mortalité intrinsèque, de la population de nouveaux-nés sur une période des temps donnée. Le codes reliés à ces programmes sont présentés, en compagnie de deux exemples.

INTRODUCTION

The programs presented in this paper are appropriate for the analysis of the population dynamics of numbers-at-age for a population in which catch-at-age (harvest), pregnancy rate-at-age and survey data for some ages exist. The analysis involves estimating the parameters of a population dynamics model (PDM) using survey data. The PDM is written in terms of two unknown parameters and known catch and pregnancy rates. A similar model was presented by Roff and Bowen (1983) however these authors did not use an efficient algorithm to obtain parameter estimates and their computer program for fitting the model is not widely available. A more efficient algorithm is presented in this report along with computer programs and instructions to implement the algorithm.

The PDM is presented in a scalar and matrix form. The PDM is then transformed to a standard nonlinear form that relates numbers-at-age in terms of 2 unknown parameters and data derived from catches and pregnancy rates. The SAS program MAKED.SAS constructs the derived data. Parameters are estimated from survey data using weighted least squares where the weights are the estimated variances of the survey estimates. The estimated parameters are then used in the transformed model to produce estimates of numbers-at-age. The SAS program SEAL.SAS produces the parameter estimates and the estimates of numbers-at-age. The programs have been constructed so that one does not need to know SAS to run them.

A note of caution: PROC NLIN in SEAL.SAS automatically produces confidence intervals for parameter estimates and can produce confidence intervals for model output and derived parameters. For the model in this paper the assumptions underlying the construction of the above confidence intervals are unrealistic.

PROGRAM SUMMARY

POPULATION DYNAMICS MODEL

The PDM is:

$$n_{a,t} = \begin{cases} (n_{a-1,t-1}e^{-\frac{m}{2}} - c_{a-1,t-1})e^{-\frac{m}{2}} & 0 < a < A, \\ \sum_i n_{i,t} f_{i,t} & a = 0, \end{cases} \tag{1}$$

where $n_{a,t}$ = the population number at age a in year t,

$c_{a,t}$ = the number caught at age a in year t,

$f_{a,t}$ = the per capita pregnancy (fecundity) rate of age a parents in year t assuming a 1:1 sex ratio,

m = instantaneous natural mortality rate.

A plus age class A, i.e. ages A and greater, is included in the model so that

$$n_{A,t} = (n_{A-1,t-1}e^{-\frac{m}{2}} - c_{A-1,t-1})e^{-\frac{m}{2}}.$$

$A-1$ is taken as ages $A-1$ and greater.

It is necessary to construct a different model for numbers-at-age in the year prior to the first year for which reliable pregnancy rate data exists. The year for which reliable pregnancy rate data exists is designated the initial year, t_0, in the model. The model is modified by removing the plus age class and by assuming seals do not live longer than $\mathring{A}$ years. This procedure is similar to that used by Cooke et al (1985). Assuming that pup production prior to t_0 is equal to a hunting selection parameter (s) times the pup catch and using (1) recursively, it follows that

$$n_{a,t_0-1} = se^{-ma}c_{0,t_0-a-1} - \sum_{i=1}^{a} e^{-m(i-\frac{1}{2})}c_{a-i,t_0-i-1}, a = 1, ...\mathring{A}. \qquad (2)$$

If, in the above equation, catches are required for which no data exist it can be assumed that those catches are equal to the earliest catch available, although alternative assumptions can be made (the average of the first 5 catches available etc.).

To summarize, population numbers-at-age are obtained from (1) in the initial year and onwards because of the availability of $f_{i,t}$'s. For the years prior to the initial year, ie $t_0 - 1$ and earlier, (2) is used because it requires only catch numbers. Note that the PDM makes several simplifying assumptions. For example, s and m are assumed to be constant, independent ot t, etc. Changes to the assumptions in the model will have an impact on the estimates.

MATRIX FORMULATION

In matrix notation the model is:

$$\begin{aligned}
\boldsymbol{n}_t &= e^{-m}\boldsymbol{P}\boldsymbol{n}_{t-1} - e^{-\frac{m}{2}}\boldsymbol{P}\boldsymbol{c}_{t-1} + \boldsymbol{F}_t\boldsymbol{n}_t, \\
\text{where } \boldsymbol{n}_t' &= (n_{0,t}, n_{1,t}, ...n_{A,t}), \\
\boldsymbol{c}_t' &= (c_{0,t}, c_{1,t}, ...c_{A,t}), \\
\boldsymbol{P} &= \begin{bmatrix} 0' & 0 \\ \mathbf{I} & 1 \end{bmatrix}, \\
\boldsymbol{F}_t &= \begin{bmatrix} \boldsymbol{f}_t' \\ 0 \end{bmatrix}, \\
\boldsymbol{f}_t' &= (f_{0,t}, f_{1,t}, ...f_{A,t}).
\end{aligned}$$

The model may equivalently be written as:

$$\begin{aligned} \boldsymbol{n}_t &= e^{-m}\boldsymbol{\Lambda}_t \mathbf{n}_{t-1} - e^{-\frac{m}{2}}\boldsymbol{\Lambda}_t \mathbf{c}_{t-1}, \qquad 0 \le m < \infty, \\ \text{where } \boldsymbol{\Lambda}_t &= (\boldsymbol{I} - \boldsymbol{F}_t)^{-1}\boldsymbol{P}, \\ &= (\boldsymbol{I} + \boldsymbol{F}_t)\boldsymbol{P}, \qquad \text{when } f_{0,t} = 0, t = 1, 2, \ldots. \end{aligned}$$

The population vector in the year prior to the initial year of the model is constructed as follows. Denote the vector of numbers-at-age in year t without the plus age class as $\boldsymbol{n}_t^*$. $\boldsymbol{n}_t^*$ is an $\mathring{A}\times 1$ vector. Let

$$\boldsymbol{D}_{\mathring{A}\times\mathring{A}} = \begin{bmatrix} \mathbf{0}' & 0 \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$$

and

$$\boldsymbol{D}_{1\,\mathring{A}\times\mathring{A}} = \begin{bmatrix} 1 & \mathbf{0}' \\ \mathbf{0} & 0 \end{bmatrix}.$$

Then $\boldsymbol{n}_{t_0-1}^* = e^{-m}\boldsymbol{D}\boldsymbol{n}_{t_0-2}^* - e^{-\frac{m}{2}}\boldsymbol{D}\boldsymbol{c}_{t_0-2}^* + s\boldsymbol{D}_1\boldsymbol{c}_{t_0-1}^*$. Using this formula recursively $\mathring{A}$ times yields:

$$\boldsymbol{n}_{t_0-1}^* = e^{-\mathring{A}m}\boldsymbol{D}^{\mathring{A}}\boldsymbol{n}_{t_0-1-\mathring{A}}^* - \sum_{i=1}^{\mathring{A}} e^{-m(i-\frac{1}{2})}\boldsymbol{D}^i\boldsymbol{c}_{t_0-i-1}^* + s\sum_{i=1}^{\mathring{A}} e^{-m(i-1)}\boldsymbol{D}^{i-1}\boldsymbol{D}_1\boldsymbol{c}_{t_0-i}^*,$$

where $\boldsymbol{D}^0 = \boldsymbol{I}$.

Note that $\boldsymbol{D}^{\mathring{A}} = \mathbf{0}$ so that

$$\boldsymbol{n}_{t_0-1}^* = s\sum_{i=1}^{\mathring{A}} e^{-m(i-1)}\boldsymbol{D}^{i-1}\boldsymbol{D}_1\boldsymbol{c}_{t_0-i}^* - \sum_{i=1}^{\mathring{A}} e^{-m(i-\frac{1}{2})}\boldsymbol{D}^i\boldsymbol{c}_{t_0-i-1}^*, \qquad 0 \le s < \infty.$$

Let

$$\boldsymbol{B}_{(A+1)\times\mathring{A}} = \begin{bmatrix} \mathbf{I}_{A\times A} & \mathbf{0}_{A\times(\mathring{A}-A)} \\ \mathbf{0}' & \mathbf{1}' \end{bmatrix}.$$

Then $\boldsymbol{n}_{t_0-1} = \boldsymbol{B}\boldsymbol{n}_{t_0-1}^*$.

TRANSFORMED MODEL

It is useful to rewrite the model in the following form. Standardize time so that $t_0 = 0$. At time t, using the model recursively, it follows that

$$\begin{aligned}
\boldsymbol{n}_t &= \sum_{i=1}^{\overset{\circ}{A}} \left[s e^{-m(t+i-1)} \mathbf{w}_t^i - e^{-m(t+i-\frac{1}{2})} \mathbf{u}_t^i \right] - \sum_{i=0}^{t} e^{-m(t-i+\frac{1}{2})} \mathbf{v}_t^i, \qquad t \geq 0, \\
\text{where } \mathbf{u}_t^i &= \left[\prod_{k=0}^{t} \boldsymbol{\Lambda}_{t-k} \right] \boldsymbol{B} \boldsymbol{D}^i \boldsymbol{c}_{t_0-i-1}^*, \\
\mathbf{w}_t^i &= \left[\prod_{k=0}^{t} \boldsymbol{\Lambda}_{t-k} \right] \boldsymbol{B} \boldsymbol{D}^{i-1} \boldsymbol{D}_1 \boldsymbol{c}_{t_0-i}^*, \\
\text{and } \mathbf{v}_t^i &= \left[\prod_{j=i}^{t} \boldsymbol{\Lambda}_{t-i+j} \right] \mathbf{c}_{i-1}.
\end{aligned}$$

For pups

$$n_{0,t} = \sum_{i=1}^{\overset{\circ}{A}} \left[s e^{-m(t+i-1)} \mathbf{w}_t^i(1) - e^{m(t+i-\frac{1}{2})} \mathbf{u}_t^i(1) \right] - \sum_{i=0}^{t} e^{-m(t-i+\frac{1}{2})} \mathbf{v}_t^i(1),$$

and for total population numbers

$$n_{.,t} = \sum_{i=1}^{\overset{\circ}{A}} \left[s e^{-m(t+i-1)} \mathbf{w}_t^i(.) - e^{m(t+i-\frac{1}{2})} \mathbf{u}_t^i(.) \right] - \sum_{i=0}^{t} e^{-m(t-i+\frac{1}{2})} \mathbf{v}_t^i(.),$$

where $\mathbf{w}_t^i(1)$ is the first element in the vector $\mathbf{w}_t^i$, etc. The notation (.) denotes summation over all ages.

The model for pup numbers, total population numbers, or numbers at any specific age is now in a standard nonlinear from. It relates numbers in terms of unknown parameters and known data.

A statistical model is assumed that describes the distribution of survey estimates of $n_{a,t}$'s in terms of the dynamic models $n_{a,t}$'s. Let $\tilde{n}_{0,t}$ denote a survey estimate of $n_{0,t}$ and $\tilde{\sigma}_t^2$ denote the estimated variance of $\tilde{n}_{0,t}$. The statistical model is:

$$\tilde{n}_{0,t_i} \sim \mathrm{N}(n_{0,t_i}, \tilde{\sigma}_{t_i}^2), \qquad i = 1, ..., l.$$

Maximum likelihood (or equivalently least-squares) parameter estimates are obtained using PROC NLIN of SAS (1989).

IMPLEMENTATION

Two SAS programs were written to estimate the parameters of the model. The programs are separate because the transformation of the data need only be performed once although model parameters may be estimated many times based on different survey data. MAKED.SAS is the program that computes the transformed data from the catch and pregnancy rate data. This program requires BASE SAS (1988), SAS/IML (1989) and SAS/MACRO (1987) software. SEAL.SAS reads the output from MAKED.SAS as well as an ascii file that contains the survey data and computes the parameter estimates as well as the number-at-age for pups and total population size. This program requires BASE SAS (1988), SAS/IML (1989), SAS/STAT (1989) and SAS/MACRO (1987) software.

The programs were developed on a SUN UNIX workstation running SAS V6.07.02 and tested successfully (see Example 1). The SAS programs were used in extensive simulations. SEAL.SAS would not converge in at most .5% of the simulations. The solutions have always converged when applied to real data.

INSTRUCTIONS FOR RUNNING MAKED.SAS

Copy MAKED.SAS to a working directory. Ensure the following files are in the working directory,

- catch.dat. This ascii file contains a column of years for which catch data exists as well as columns of catches for each age up to the maximum age for which catch data exists.

- preg.dat. This ascii file contains a column of years for which pregnancy rate data exists as well as columns of pregnancy rates for each age upto the plus age class. The program infers the initial year the model starts in from this file.

- info.dat. This ascii file contains 1 record with three columns. The first column contains the working directory name in parentheses, the second contains the maximum age (numeric) and the third contains the plus age class (numeric).

All columns should be separated by blank spaces. Examples of these files are given in Appendix 3 and 4. To run the program type **sas maked**. After the program completes two new files should exist whose first name extensions are pupData and popData. Second extensions depend on the operating system, for example in SAS V6.07.02 running on a UNIX computer the extensions are .ssd01 for both files. These files are required by SEAL.SAS

INSTRUCTIONS FOR RUNNING SEAL.SAS

Copy SEAL.SAS to a working directory. Ensure the following files are in the working directory,

- survey.dat. This ascii file contains a column of years for which survey data exists as well as columns of survey numbers and standard errors.
- info.dat. same as above.

Again all columns should be separated by blank spaces. Examples of these files are given in Appendix 3 and 4. To run the program type **sas seal**. As a result of executing the code two new files should be created by the program. pup.dat is an ascii file containing years and pup estimates. pop.dat is an ascii file containing years and total population estimates. Parameter estimates and other information are obtained from the SEAL list file. PROC NLIN in SEAL.SAS automatically produces confidence intervals for parameter estimates however the assumptions underlying the construction of these confidence intervals are unrealistic for the model considered here. One should have to modify this program only if the range of starting values in PROC NLIN is too far away from the parameter estimates. The range (found in the section of code marked by %'s) has been deliberately made wide and should work for many types of data although it is conceivable that they will not work for some data.

Acknowledgements

We thank Don Bowen for information on the model, Garry Stenson for assistance in understanding the data sets and harp seal biology and Beth Perry for compiling catch-at-age and pregnancy data.

REFERENCES

Bowen, W. D. 1982. Age structure of Northwest Atlantic Harp Seal catches, 1952-1980. NAFO Sci. Coun. Stud. No. 3: 53-65.

Cooke, J. A., A. W. Trites and P. A. Larkin (1985). A view of the population dynamics of the Northwest Atlantic Harp al (*Phoca groenlandica*). Final report submitted to the Royal Commission on Seals and ng Industry in Canada, June 30, 1985, 139p.

Roff, D. A., W. D. Bowen. 1983. Population dynamics and management of the Northwest Atlantic Harp Seal (*Phoca groenlandica*). Can. J. Fish. Aquat. Sci. 40: 919-932.

SAS Institute Inc., 1987. SAS Guide to Macro Processing, Version 6, First Edition, Cary, NC. 233p.

SAS Institute Inc., 1989. SAS/IML Software: Usage and Reference, Version 6, First Edition, Cary, NC. 501p.

SAS Institute Inc., 1988. SAS Language Guide, Release 6.03 Edition. Cary, NC. 530p.

SAS Institute Inc., 1989. SAS/STAT User's Guide, Version 6, Fourth Edition, Volume2, Cary, NC. 846p.

Shelton, P. A., N. G. Cadigan and G. B. Shelton. 1992. Model estimates of harp seal population trajectories in the Northwest Atlantic. CAFSAC Res. Doc 92/ , 15p.

APPENDIX 1 - MAKED.SAS

```
* This program computes the input data for seal.sas - a program;
* that estimates the parameters of a population dynamics model ;
* for numbers-at-age. The model was developed by Noel Cadigan and;
* Peter Shelton, Dept. of Fisheries and Oceans, Government of  ;
* Canada, St. John's, Newfoundland, Canada. The program was    ;
* written in the winter of 1992.                               ;

* The program is written using SAS V6.07.02 on a SUN UNIX      ;
* workstation. This program requires BASE SAS, SAS/IML and SAS/;
* MACRO SOFTWARE. The program was successfully executed on a    ;
* VAX-VMS mainframe running SAS V6.06.01. The user is advised  ;
* to refer to the associated technical report.                 ;

* The program requires three ascii data files to be located in  ;
* the working directory. These are 1. catch.dat. This file      ;
* contains a year column and the catch-at-age matrix where rows;
* correspond to years and columns to ages (with no plus age    ;
* class). All catch data should be in this file. 2. preg.dat.  ;
* This file contains a year column and the pregnancy rates-at- ;
* age matrix where rows correspond to years and columns to ages;
* (with plus age class). The program infers the initial        ;
* year from this file. 3. info.dat. This file contains one row ;
* and 3 columns (separated by blanks). The first column contains;
* the working directory name in parentheses. The maximum age is;
* in the second column (numeric) and the plus age class is     ;
* the third column (numeric).                                  ;

* The program produces two SAS data files named pupData and    ;
* popData. Both these files are required by seal.sas           ;

* This section creates SAS macro variables from info.dat       ;
data info;
infile 'info.dat';
input current :$100. Adot A;
call symput('current',current);
call symput('Adot',Adot);
call symput('A',A);
run;
```

```
libname current &current;

%let Adot=&Adot;
%let A=&A;

* This section creates a SAS data set for catch data          ;
data catch;
infile 'catch.dat';
input year c0-c&Adot;
run;

* This section creates a SAS data set for pregnancy data      ;
data preg;
infile 'preg.dat';
input year p0-p&A;
run;

options ls=75 ps=64;

proc iml;

* SAS/IML interactive matrix language is used to modify the   ;
* catch and pregnancy data for the nonlinear model (see Tech  ;
* Rep)                                                        ;

* create a temporary vector to read catch file into           ;
use catch;
read all into temp;

yearc = temp[,1];
catch = temp[,2:ncol(temp)];

* create a temporary vector to read pregnancy file into       ;
use preg;
read all into temp;

yearp = temp[,1];
preg = temp[,2:ncol(temp)];
```

```
* a is the plus age class plus 1                                  ;
a = ncol(preg);

* compute catch matrix (C) with year as columns and with          ;
* plus age class. Put maximum age + 1 of catch in Adot            ;
C = t(catch[,1:a]);
Adot = ncol(catch);
C[a,] = C[a,] + t(catch[,(a+1):Adot][,+]);

* create catch matrix (Cl) for generating initial numbers-at-age;
ta = yearp[1]-yearc[1];
Cl = repeat(t(catch[1,]),1,Adot-ta-1);
if ta > 0 then Cl = Cl||t(catch[1:ta,]);

* create matrices involved in generating initial numbers-at-age;
D = repeat(0,Adot,Adot);
D[2:Adot,1:(Adot-1)] = diag(repeat(1,Adot-1,1));
D1 = repeat(0,Adot,Adot);D1[1,1]=1;
Am = repeat(0,a,Adot);
Am[1:a,1:a] = Diag(repeat(1,a,1));
Am[a,(a+1):Adot] = repeat(1,1,Adot-a);

U = repeat(0,a,Adot-1);
W=U;
Di = D;

Adot = Adot - 1;
W[,1] = Am*D1*Cl[,Adot];
do i = 1 to Adot-1;
  U[,i] = Am*Di*Cl[,Adot-i];
  W[,i+1] = Am*Di*D1*Cl[,Adot-i];
  Di = D*Di;
end;
U[,Adot] = Am*Di*Cl[,1];

* create matrices of modified data for nonlinear model. These ;
* are Datapup for pup numbers and Datapop for total population;
* numbers. T is the numbers of years used in fitting the model;
T=nrow(yearp);
```

```
D = repeat(0,a,a);
D[2:a,1:(a-1)] = diag(repeat(1,a-1,1));
D[a,a] = 1;
Ft = repeat(0,a,a);

Datapup = repeat(0,t,t+2*Adot+1);
Datapop = repeat(0,t,t+2*Adot+1);

Datapup[,1] = yearp;
Datapop[,1] = yearp;
one = repeat(1,a,1);
ID = diag(repeat(1,a,1));

do it = 1 to t;
  Gu = ID;
  do i = 1 to it;
    Ft[1,] = 0.5*preg[it-i+1,];
    Gt = (ID + Ft)*D;
    Gu = Gu*Gt;
  end;
  do i = 1 to Adot;
    tu = Gu*U[,i];
    tw = Gu*W[,i];
    Datapup[it,i+1] = tu[1];
    Datapop[it,i+1] = sum(tu);
    Datapup[it,i+Adot+1] = tw[1];
    Datapop[it,i+Adot+1] = sum(tw);
  end;
end;

do it = 1 to t;
  do i = 1 to it;
    Gv = diag(repeat(1,a,1));
    do j = i to it;
      Ft[1,] = 0.5*preg[it+i-j,];
      Gt = (ID + Ft)*D;
      Gv = Gv*Gt;
    end;
  if (ta=0&i=1) then v = Gv*C[,1];
  else v = Gv*C[,ta+i-1];
```

```
  Datapup[it,i+2*Adot+1] = v[1];
  Datapop[it,i+2*Adot+1] = sum(v);
  end;
end;

* create names for modified data assuming less than 1000 years;
* data used in fitting the model                                ;
names = repeat('    ',1,t+2*Adot+1);
names[1] = 'year';
un = t(concat('u',char(1:Adot,3,0)));
un = t(rowcatc(un));
wn = t(concat('w',char(1:Adot,3,0)));
wn = t(rowcatc(wn));
vn = t(concat('v',char(1:t,3,0)));
vn = t(rowcatc(vn));
names[2:(Adot+1)] = un;
names[(Adot+2):(2*Adot+1)]=wn;
names[(2*Adot+2):(t+2*Adot+1)]=vn;


create current.pupData from Datapup[colname=names];
append from Datapup;
close current.pupData;

create current.popData from Datapop[colname=names];
append from Datapop;
close current.popData;

quit;
run;

endsas;
```

APPENDIX 2 - SEAL.SAS

```
* This program computes the estimates of the parameters of a   ;
* population dynamics model for numbers-at-age. The model was  ;
* developed by Noel Cadigan and Peter Shelton, Dept. of        ;
* Fisheries and Oceans, Government of Canada, St. John's,      ;
* Newfoundland, Canada. It is based in part by work by Roff and;
* Bowen (1983) and Cooke et al (1985). The program was written ;
* in the winter of 1992.                                       ;

* The program is written using SAS V6.07.02 on a SUN UNIX      ;
* workstation. This program requires BASE SAS, SAS/IML, SAS/   ;
* MACRO and SAS/STAT SOFTWARE. The program was successfully    ;
* executed on a VAX-VMS mainframe running SAS V6.06.01. The    ;
* user is advised to refer to the associated Technical report. ;

* The program requires an ascii data file (survey.dat) to be    ;
* located in the working directory. This file contains a year  ;
* column, a survey column and a standard error column. The     ;
* program also requires the ascii file info.dat (located in    ;
* the working directory). This file contains one row  and 3    ;
* columns (separated by blanks). The first column contains the ;
* working directory name in parentheses. The maximum age is in ;
* the second column (numeric) and the plus age class is the;
* third column (numeric). The program uses the SAS data files  ;
* pupdata and popdata as inputs in the estimation procedure.   ;

* The program computes two ascii data files named pup.dat and  ;
* pop.dat. These files contain time series of pup and total    ;
* population numbers. Parameter estimates and other information;
* are contained in the list file produced by seal.sas.         ;

options ls=64 ps=75;

* This section creates SAS macro variables from info.dat       ;
data info;
infile 'info.dat';
input current :$100. Adot A;
call symput('current',current);
call symput('Adot',Adot);
```

```
call symput('A',A);
run;

libname current &current;

%let Adot=&Adot;
%let A=&A;

* This section creates a SAS data set from survey.dat. t is years;
* n is surveys and w is survey standard errors                   ;
data surveys;
  infile 'survey.dat';
  input t n w;
  year=t;

* This section creates a SAS macro variable for the number of   ;
* years of catch data                                           ;

data _null_;
  set current.pupdata;
  call symput('n_obs',_n_);
  run;
%let n_obs=&n_obs;

* This section creates a SAS data set for parameter estimation ;

data min;
  merge current.pupdata surveys;
  by year;
  tt = t;t = _N_;
  if year eq tt;
  drop year tt;

* This section computes the parameter estimates                  ;

proc nlin method=newton data=min best=10 save;

*%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%;
* initial values;
```

```
  parms m = 0 to 1 by 0.02
        s = 0 to 20 by 1;

*%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%;

* weights;

  _weight_= 1/(w**2);

* arrays for nonlinear model;

  array um{&Adot} u1-u&Adot;
  array wm{&Adot} w1-w&Adot;
  array vm{&n_obs} v1-v&n_obs;

* temporary computations;

  tu = 0;
  tv = 0;
  tw = 0;
  do i = 1 to t;
    tv = tv + exp(-m*(t-i+0.5))*vm{i};
  end;
  do i = 1 to &Adot;
    tu = tu + exp(-m*(t+i-0.5))*um{i};
    tw = tw + exp(-m*(t+i-1))*wm{i};
  end;

* nonlinear model;

  model n = s*tw - tu - tv;

* derivatives;

  der.s = tw;
  der.s.s = 0;
  dmu = 0;
  dmv = 0;
  dmw = 0;
  do i = 1 to t;
```

```
    dmv = dmv - (t-i+0.5)*exp(-m*(t-i+0.5))*vm{i};
  end;
  do i = 1 to &Adot;
    dmu = dmu - (t+i-0.5)*exp(-m*(t+i-0.5))*um{i};
    dmw = dmw - (t+i-1)*exp(-m*(t+i-1))*wm{i};
  end;
  d2mu = 0;
  d2mv = 0;
  d2mw = 0;
  do i = 1 to t;
    d2mv = d2mv + ((t-i+0.5)**2)*exp(-m*(t-i+0.5))*vm{i};
  end;
  do i = 1 to &Adot;
    d2mu = d2mu + ((t+i-0.5)**2)*exp(-m*(t+i-0.5))*um{i};
    d2mw = d2mw + ((t+i-1)**2)*exp(-m*(t+i-1))*wm{i};
  end;
  der.m = s*dmw - dmu - dmv;
  der.m.m = s*d2mw - d2mu - d2mv;
  der.m.s = dmw;

  output out=out p=pred parms=m s sse=sse;
run;

* print some output from NLIN;

data out1;
merge out surveys;
rename t=year n=survey w=std;
keep t n w pred m s sse;
proc print data=out1 round;

* SAS data set of parameter estimates;

data params;
set out;
if _N_ = 1;
keep m s;

* produce estimates of numbers-at-age for both pups and total ;
* population                                                  ;
```

```
proc iml;

* numb is an IML function that computes the nonlinear model for ;
* numbers at age. b is a 2x1 vector of parameters, u v and w are;
* data derived from catch and pregnancy rates for a particular ;
* age i.e. pups, total numbers etc. tti is a vector of julian  ;
* years and to is the initial julian year for the model.       ;

 start numb(b,u,v,w,tti,to);
  m = b[1];
  s = b[2];
  t = nrow(tti);
  ti = tti - to + 1;
  ret = repeat(0,t,1);
  do it = 1 to t;
    tt=ti[it];
    td = t((1:&Adot)+tt);
    tu = u[it,];
    tw = w[it,];
    ret[it] = s*tw*exp(-m*(td-1))-tu*exp(-m*(td-0.5));
    td = tt-t(1:tt);
    tv = v[it,];tv=t(tv[1:tt]);
    ret[it] = ret[it] - tv*exp(-m*(td+0.5));
  end;
return(ret);
finish;

* input parameter estimates;

use params;
read all var{m s};
close params;

* input pup and total population data and create u,w and v   ;
* matrices                                                   ;

use current.pupData;
read all into temp;
close current.pupData;
```

```
year = temp[,1];
ufpup = temp[,2:(&Adot+1)];
wfpup = temp[,(&Adot+2):(2*&Adot+1)];
vfpup = temp[,(2*&Adot+2):(2*&Adot+1+&n_obs)];

use current.popData;
read all into temp;
close current.popData;

ufpop = temp[,2:(&Adot+1)];
wfpop = temp[,(&Adot+2):(2*&Adot+1)];
vfpop = temp[,(2*&Adot+2):(2*&Adot+1+&n_obs)];

* compute population estimates;

b = m\\s;
pup = year||numb(b,ufpup,vfpup,wfpup,year,min(year));
pop = numb(b,ufpop,vfpop,wfpop,year,min(year));
reset noname;
print pup[colname={'YEAR','PUP #S'}] pop[colname={'TOTAL POP #S'}];
pop = year||pop;

* write data to ascii files;

file 'pup.dat';
do i = 1 to &n_obs;
  put (pup[i,1]) +1 (pup[i,2]);
end;
file 'pop.dat';
do i = 1 to &n_obs;
  put (pop[i,1]) +1 (pop[i,2]);
end;

quit;
run;endsas;
```

APPENDIX 3 - EXAMPLE 1

The following example has been artificially constructed and should be easy to implement. The survey data was generated by fixing $m = 0.1$ and $s = 5$ in the model. Note in the list file that is created after executing **SEAL.SAS** that the parameters are reproduced exactly. Info.dat is constructed for a UNIX machine.

In this example the initial year, t_0, is 1969. The program infers this from preg.dat. 1969 is also the first year for which catch data exists so the program uses the 1969 catch as the catch for all years prior to 1969 that is required by the PDM. The plus age class (A) is 5, remember pups have age 0, and the maximum age class ($\mathring{A}$) is 19. Survey data exists for 4 years (1973-1976); the standard errors were arbitrarily chosen as 1's - this has no influence on the estimates because an exact solution exists for all years.

Data

catch.dat

```
1969 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1970 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1971 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1972 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1973 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1974 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1975 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1976 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1977 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1978 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1979 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
1980 90 30 20 10 10 10 10 9 7 5 3 1 1 0 1 0 0 1 0 1
```

preg.dat

```
1969 0.0 0.0 0.0 0.0 0.4 0.8
1970 0.0 0.0 0.0 0.0 0.4 0.8
1971 0.0 0.0 0.0 0.0 0.4 0.8
1972 0.0 0.0 0.0 0.0 0.4 0.8
1973 0.0 0.0 0.0 0.0 0.4 0.8
1974 0.0 0.0 0.0 0.0 0.4 0.8
1975 0.0 0.0 0.0 0.0 0.4 0.8
1976 0.0 0.0 0.0 0.0 0.4 0.8
1977 0.0 0.0 0.0 0.0 0.4 0.8
1978 0.0 0.0 0.0 0.0 0.4 0.8
1979 0.0 0.0 0.0 0.0 0.4 0.8
1980 0.0 0.0 0.0 0.0 0.4 0.8
```

info.dat

```
'/home/cadigan/shelton/seal/models/randb/newmodel/techm' 19 5
```

survey.dat

```
1973 445.45270 1
1974 445.08454 1
1975 447.07098 1
1976 450.96723 1
```

List File for Example 1

```
                        The SAS System                          1
                              11:38 Thursday, November 26, 1992

Non-Linear Least Squares Grid Search     Dependent Variable N
                        M              S    Weighted SS
                 0.100000       5.000000  4.4867293E-11
                 0.160000       8.000000    7708.123699
                 0.180000       9.000000    7842.628919
                 0.120000       6.000000   10118.478139
                 0.140000       7.000000   12607.949610
                 0.200000      11.000000   15970.630796
                 0.060000       4.000000   17100.380965
                 0.220000      12.000000   20203.693349
                 0.200000      10.000000   20559.705815
                 0.220000      13.000000   22970.260413

            Non-Linear Least Squares Iterative Phase
             Dependent Variable N   Method: Newton
       Iter              M              S    Weighted SS
          0       0.100000       5.000000  4.4867293E-11
          1       0.100000       5.000000  1.4762294E-11
          2       0.100000       5.000000  1.4762292E-11
NOTE: Convergence criterion met.
```

Non-Linear Least Squares Summary Statistics
Dependent Variable N

Source	DF	Weighted SS	Weighted MS
Regression	2	799772.25938	399886.12969
Residual	2	0.00000	0.00000
Uncorrected Total	4	799772.25938	
(Corrected Total)	3	21.72429	

Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval Lower	Upper
M	0.099999995	2.57104754E-9	0.0999999838	0.1000000059
S	4.999999825	8.83579694E-8	4.9999994445	5.0000002049

Asymptotic Correlation Matrix

Corr	M	S
M	1	0.9982931045
S	0.9982931045	1

The SAS System 2
11:38 Thursday, November 26, 1992

OBS	YEAR	SURVEY	STD	PRED	M	S	SSE
1	1973	445.45	1	445.45	0.10	5.00	0
2	1974	445.08	1	445.08	0.10	5.00	0
3	1975	447.07	1	447.07	0.10	5.00	0
4	1976	450.97	1	450.97	0.10	5.00	0

```
The SAS System                                    3
      11:38 Thursday, November 26, 1992

YEAR    PUP #S  TOTAL POP #S

1969 416.78345     2355.2326
1970 426.34428     2358.6399
1971 434.99527      2370.374
1972 442.82301     2388.8191
1973  445.4527     2408.1387
1974 445.08454     2425.2516
1975 447.07098     2442.7224
1976 450.96723      2462.427
1977  455.7948     2485.0839
1978  460.4326     2510.2226
1979 464.85072     2537.3871
1980  469.6117     2566.7275
```

The following ascii output files are produced for Example 1. These file are made available for plotting etc.

pup.dat

```
1969 416.78345
1970 426.34428
1971 434.99527
1972 442.82301
1973 445.45270
1974 445.08454
1975 447.07098
1976 450.96723
1977 455.79480
1978 460.43260
1979 464.85072
1980 469.61170
```

pop.dat

```
1969 2355.2326
1970 2358.6399
1971 2370.3740
1972 2388.8191
1973 2408.1387
1974 2425.2516
1975 2442.7224
1976 2462.4270
1977 2485.0839
1978 2510.2226
1979 2537.3871
1980 2566.7275
```

APPENDIX 4 - EXAMPLE 2

The following example is based on data for the Northwest Atlantic harp seal (*Phoca groenlandica*). Unpublished catch and pregnancy rate data have been made available by Dr. G. Stenson of the Department of Fisheries and Oceans, Northwest Atlantic Fisheries Centre, St. John's, Newfoundland, Canada. Catch-at-age data for the period prior to 1980 appear in Bowen (1980) and for the period 1965-1990 in Shelton et al (1992). Note that the number of columns in the catch file is too large to fit on one line so the columns are broken over three lines. In the ascii file the data for a year should be on a single line with no carriage returns. Info.dat is constructed for a VAX/VMS machine.
In this example the initial year, t_0, is 1965. Again the program infers this from preg.dat. The plus age class (A) is 7 and the maximum age class ($\mathring{A}$) is 25. 1952 is the first year for which catch data exists so the program uses the 1952 catch as the catch for all years prior to 1952 (1939-1951) that is required by the PDM. Survey data exists for 6 years - standard errors are important because an exact fit to all data is not produced.

Data

catch.dat

```
1952 207800  7939 12105  8279  6321 7143 12212 8400
8177   5910  6770  6457  1743  1638 2411  4205 1544
2349   1892   988  5363   992    74  487  1336 1873
1953 207712 23415  7530  6418  4396 3960  3235 2855
2915   2766  2358  3539  1617  1211  972  1901 1899
1380    880   656  2789  1627   915  642   467  374
1954 186253 35151 14146  5539  6033 3529  4080 3440
3377   2125  3121  2211  2619  2479 1300  1736 2245
1019    340  1124   626   323   309  707   152 1102
1955 261471 23964  9134  6327  4948 4070  3956 3377
3510   3020  3292  3503  2206  1707 1378  2357 2278
1491    910   932  2570  1317   808  669   589  857
1956 347879 13991  5305  3777  3024 2393  2303 2020
2027   1736  1928  2246  1284  1057  855  1407 1307
 864    497   518  1420   753   475  357   323  524
```

1957	173121	23875	8853	6680	5888	4793	4679	3658
3737	3109	3468	3620	2276	1871	1391	2377	2180
1471	804	838	2475	1306	796	671	582	1033
1958	150916	27050	10962	11685	11365	9828	6378	6086
4999	4887	9733	6376	6958	4001	3104	8904	5905
2998	545	1530	5377	2921	2430	994	1940	3420
1959	244112	23615	9549	7664	5052	4176	4223	3439
3256	2781	3198	3167	2124	1670	1273	2339	2209
1429	769	803	2467	1321	801	634	544	813
1960	165659	35044	13243	9350	7714	5882	5860	4887
4801	4191	4693	4763	3133	2522	1801	3486	3220
2082	1101	1175	3640	1946	1163	951	806	1250
1961	175893	7092	2641	2370	2632	1622	1137	1252
971	789	1127	1224	323	355	433	435	381
294	218	201	243	44	179	85	40	195
1962	212095	31029	34823	10145	8941	6208	3092	2461
2565	2554	1233	1762	1959	958	1418	1856	734
1747	757	651	1338	223	617	98	195	714
1963	276283	10361	8751	7326	4274	3489	3869	3873
3590	3277	3769	4191	2615	2855	2802	2103	2635
1807	1179	961	930	850	606	463	397	589
1964	271745	6500	5836	6067	7317	4912	6995	3765
3075	2742	4172	2770	2174	1710	1639	2658	2092
2520	3892	2035	88	1979	989	983	1471	2477
1965	188184	12952	6501	5317	5139	6248	5921	2471
984	867	1435	1131	1612	193	1208	1046	622
336	658	699	287	455	259	33	18	728
1966	255874	14385	11278	5189	4849	5206	5133	4934
3384	1783	1987	2793	1745	1505	1745	1484	1382
1021	1613	996	1088	787	317	638	402	1050
1967	280257	14683	6826	2992	2452	2931	3784	3232
2438	1553	1465	2108	1334	1002	1438	1541	1064
1307	1362	1405	906	585	462	504	283	943
1968	160595	7530	4865	3590	2371	2225	1766	2576
2566	1818	1874	1898	1025	1001	1201	1092	980
818	1151	982	738	464	567	316	245	567
1969	237103	21346	3905	3422	2722	3099	2200	2241
2980	2397	2117	2107	1181	1286	1300	1419	944
1402	911	1055	958	605	394	475	243	829

1970	221075	9399	7603	2865	2345	2204	1352	1394
1309	1074	1566	1392	994	716	630	721	810
399	453	444	383	305	231	172	111	203
1971	212854	8281	3098	2068	1328	1011	745	608
485	648	648	897	521	315	268	281	218
184	225	172	231	152	131	90	51	305
1972	120263	4862	2798	1745	1475	746	657	664
417	364	402	814	457	329	313	352	276
190	165	103	151	63	176	16	18	44
1973	103435	7060	4875	3264	2575	3583	1845	1129
1460	782	819	1306	574	570	400	481	452
285	179	222	225	103	85	71	54	271
1974	119413	13192	7783	3370	2556	2407	2771	1358
1051	1002	755	881	680	700	524	421	382
316	307	193	116	85	94	79	130	432
1975	144449	14183	6247	3276	1886	1371	1282	1104
674	580	925	558	456	375	265	389	301
259	161	218	158	114	92	94	96	87
1976	136974	15565	7691	4166	2563	743	395	410
419	182	159	400	232	124	172	144	118
79	75	49	78	34	47	48	33	35
1977	134893	9222	6831	6580	5066	3075	1702	1081
681	407	507	537	291	233	448	463	271
120	81	83	84	45	36	41	38	98
1978	121058	18409	11010	5958	3938	2532	1846	663
823	290	815	224	243	279	122	286	107
143	182	98	152	123	118	47	41	93
1979	139200	16161	7580	4345	2691	2009	1459	1058
729	515	796	268	260	179	294	391	208
195	72	84	200	120	110	44	47	625
1980	136182	18205	9770	6269	4249	3305	2243	1690
1117	756	1053	595	634	433	534	586	357
342	153	222	248	109	85	79	81	104
1981	184593	9164	5038	3830	2409	1887	1748	1027
845	482	1071	505	332	231	239	311	267
335	234	211	159	211	198	92	68	200

1982	153096	14996	7195	3444	1727	1307	715	733
425	351	886	244	172	135	138	278	124
174	89	154	194	102	68	90	55	152
1983	58544	7608	4576	2714	1416	1150	943	679
535	350	1050	220	230	127	174	306	146
146	146	123	165	85	53	80	72	121
1984	31850	5906	5315	2806	1729	921	722	582
420	336	955	203	198	112	118	213	98
112	86	95	139	61	80	45	51	62
1985	21690	6725	4913	2517	1222	747	591	536
388	281	935	149	155	113	187	295	132
105	115	121	146	61	50	63	50	74
1986	28240	4747	3366	2412	1210	662	562	419
328	263	759	178	184	110	139	215	106
111	99	99	149	86	62	60	66	115
1987	40951	5686	4139	3369	2234	1171	1012	806
591	536	985	369	254	215	220	353	215
226	154	150	259	138	72	132	138	594
1988	75108	11191	9867	5613	3553	1970	1716	819
790	546	968	236	288	357	286	419	357
281	340	433	477	148	93	131	270	670
1989	62037	7228	5394	3501	2220	1547	1095	495
366	298	967	235	241	180	139	285	180
134	222	112	213	119	252	130	136	229
1990	41346	9374	6217	5265	4181	3548	1876	1074
446	656	1159	772	641	650	318	615	232
383	285	83	475	61	188	252	50	956

preg.dat

1965	0.000	0.000	0.000	0.000	0.033	0.114	0.541	0.837
1966	0.000	0.000	0.000	0.000	0.000	0.111	0.353	0.850
1967	0.000	0.000	0.000	0.000	0.000	0.211	0.606	0.901
1968	0.000	0.000	0.000	0.000	0.000	0.316	0.700	0.881
1969	0.000	0.000	0.000	0.000	0.004	0.160	0.438	0.880
1970	0.000	0.000	0.000	0.000	0.000	0.231	0.506	0.863
1971	0.000	0.000	0.000	0.000	0.000	0.269	0.573	0.858
1972	0.000	0.000	0.000	0.000	0.000	0.311	0.638	0.853
1973	0.000	0.000	0.000	0.000	0.000	0.356	0.698	0.848
1974	0.000	0.000	0.000	0.000	0.002	0.404	0.752	0.843
1975	0.000	0.000	0.000	0.000	0.003	0.454	0.799	0.837
1976	0.000	0.000	0.000	0.000	0.006	0.505	0.839	0.832
1977	0.000	0.000	0.000	0.000	0.013	0.555	0.873	0.826
1978	0.000	0.000	0.000	0.000	0.025	0.605	0.900	0.820
1979	0.000	0.000	0.000	0.000	0.071	0.661	0.896	0.827
1980	0.000	0.000	0.000	0.000	0.184	0.712	0.893	0.833
1981	0.000	0.000	0.000	0.001	0.400	0.759	0.889	0.737
1982	0.000	0.000	0.000	0.003	0.200	0.800	1.000	0.923
1983	0.000	0.000	0.000	0.008	0.166	0.620	1.000	0.904
1984	0.000	0.000	0.000	0.019	0.137	0.400	1.000	0.880
1985	0.000	0.000	0.000	0.044	0.112	0.536	1.000	0.852
1986	0.000	0.000	0.000	0.100	0.091	0.667	1.000	0.818
1987	0.000	0.000	0.000	0.167	0.375	0.551	1.000	0.787
1988	0.000	0.000	0.000	0.000	0.000	0.429	1.000	0.955
1989	0.000	0.000	0.000	0.000	0.000	0.534	1.000	0.714
1990	0.000	0.000	0.000	0.077	0.250	0.636	0.667	0.878

info.dat

'[cadigan.shelton]' 25 7

survey.dat

```
1977 318000 48000
1978 497000 34000
1979 478000 35000
1980 475000 47000
1983 534000 33000
1990 577900 38800
```

List File for Example 2

```
                    The SAS System                          1
                          15:16 Thursday, November 26, 1992

Non-Linear Least Squares Grid Search     Dependent Variable N
                     M              S    Weighted SS
              0.120000       3.000000      31.316913
              0.180000       6.000000      58.351430
              0.200000       8.000000      95.855267
              0.160000       5.000000     116.772752
              0.200000       7.000000     137.871088
              0.220000      10.000000     138.530395
              0.220000      11.000000     143.219833
              0.160000       4.000000     173.136062
              0.240000      14.000000     174.866874
              0.240000      15.000000     186.987171

          Non-Linear Least Squares Iterative Phase
           Dependent Variable N   Method: Newton
     Iter              M              S    Weighted SS
        0       0.120000       3.000000      31.316913
        1       0.122259       3.000000      10.438637
        2       0.129866       3.223491       9.848276
        3       0.128051       3.168963       9.673490
        4       0.127466       3.150434       9.666161
        5       0.127448       3.149961       9.666136
        6       0.127448       3.149958       9.666136
NOTE: Convergence criterion met.
```

Non-Linear Least Squares Summary Statistics
Dependent Variable N

Source	DF	Weighted SS	Weighted MS
Regression	2	1020.2489250	510.1244625
Residual	4	9.6661363	2.4165341
Uncorrected Total	6	1029.9150612	
(Corrected Total)	5	19.9675590	

Parameter	Estimate	Asymptotic Std. Error	Asymptotic 95 % Confidence Interval Lower	Upper
M	0.127448125	0.01125965083	0.0961867452	0.1587095041
S	3.149958419	0.36210653207	2.1446031139	4.1553137246

Asymptotic Correlation Matrix

Corr	M	S
M	1	0.9963459775
S	0.9963459775	1

OBS	YEAR	SURVEY	STD	PRED	M	S	SSE
1	1977	318000	48000	447435.34	0.13	3.15	9.67
2	1978	497000	34000	459317.34	0.13	3.15	9.67
3	1979	478000	35000	470075.55	0.13	3.15	9.67
4	1980	475000	47000	477890.63	0.13	3.15	9.67
5	1983	534000	33000	502988.66	0.13	3.15	9.67
6	1990	577900	38800	596432.35	0.13	3.15	9.67

YEAR	PUP #S	TOTAL POP #S
1965	438603.83	3081778.3
1966	429795.99	2912646
1967	468118.18	2720197.7
1968	499283.05	2576042.2
1969	481661.5	2557275.9
1970	507100.15	2478166.1
1971	499959.75	2437496.8
1972	468589.32	2393156.4
1973	453099.57	2430539.6
1974	443923.09	2455919.7
1975	436397.04	2447380.5
1976	436078.68	2422091.2
1977	447435.34	2419314.7
1978	459317.34	2426895.7
1979	470075.55	2447437.1
1980	477890.63	2463915.5
1981	448929.82	2440302.8
1982	521463.82	2467386.4
1983	502988.66	2499628.7
1984	481105.57	2604914.9
1985	485312.55	2728591.2
1986	486356.2	2848695.9
1987	518806.59	2984640.4
1988	534798.38	3101335.9
1989	471472.73	3091991.6
1990	596432.35	3235908.4

Output files for Example 2

pup.dat

```
1965 438603.83
1966 429795.99
1967 468118.18
1968 499283.05
1969 481661.50
1970 507100.15
1971 499959.75
1972 468589.32
1973 453099.57
1974 443923.09
1975 436397.04
1976 436078.68
1977 447435.34
1978 459317.34
1979 470075.55
1980 477890.63
1981 448929.82
1982 521463.82
1983 502988.66
1984 481105.57
1985 485312.55
1986 486356.20
1987 518806.59
1988 534798.38
1989 471472.73
1990 596432.35
```

pop.dat

```
1965 3081778.3
1966 2912646.0
1967 2720197.7
1968 2576042.2
1969 2557275.9
1970 2478166.1
1971 2437496.8
1972 2393156.4
1973 2430539.6
1974 2455919.7
1975 2447380.5
1976 2422091.2
1977 2419314.7
1978 2426895.7
1979 2447437.1
1980 2463915.5
1981 2440302.8
1982 2467386.4
1983 2499628.7
1984 2604914.9
1985 2728591.2
1986 2848695.9
1987 2984640.4
1988 3101335.9
1989 3091991.6
1990 3235908.4
```