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Impact of stock-recruit and natural mortality process errors on MSY reference points

Incidence des erreurs dues au processus d'évaluation des stocks-recrues et de la mortalité naturelle sur les points de référence en matière de rendement maximal soutemu (RMS)

Noel Cadigan

Science Branch
Fisheries and Oceans Canada
PO Box 5667
St. John's NL Canada A1C 5X1

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ABSTRACT

The impact of stock-recruit and natural mortality process errors on MSY reference points (RPs) is investigated using simulations of stochastic projections. Process errors are the main source of variability in MSY projections. If the stochastic population projections achieve a stationary (i.e. steady-state) distribution then the mean of the stationary distribution may be used for RP's for fisheries management, although the variance of the stationary distribution of population size should be accounted for in management decisions.

Two important characteristics of the process errors, namely their variance and auto-correlation, are varied to examine how these factors affect MSY RP's. Results suggest that when the process error variance or auto-correlation is large then values for MSY RP's may be lower. However, in this situation the stochastic projections may not lead to a stationary distribution depending on how the process errors are incorporated into the population dynamics model.

RÉSUMÉ

On étudie l'incidence des erreurs dues au processus d'évaluation des stocks-recrues et de la mortalité naturelle sur les points de référence du RMS à l'aide de simulations des projections stochastiques. Les erreurs dues au processus sont la principale source de variabilité des projections du RMS. Si les projections stochastiques de la population donnent à une répartition stationnaire (c'est-à-dire un état stable), on peut utiliser la moyenne de la répartition stationnaire comme points de référence pour la gestion des pêches, même s'il faut tenir compte de la variance de la répartition stationnaire de la taille de la population dans les décisions de gestion.

On fait varier deux facteurs importants des erreurs dues au processus, la variance et l'autocorrélation, pour examiner leur influence sur les points de référence du RMS. Les résultats permettent de penser que les valeurs des points de référence du RMS peuvent diminuer lorsque la variance ou l'autocorrélation des erreurs dues au processus est élevée. Cependant, dans ce cas, les projections stochastiques ne donneront peut-être pas une répartition stationnaire selon la manière dont les erreurs dues au processus sont intégrées dans le modèle de dynamique de la population.

1. INTRODUCTION

Calculation of MSY reference points (RPs) conceptually involves evaluating long-term stock projections in which fishing mortality is varied to find the level (Fmsy) that maximizes long term yield. MSY is the maximized yield and Bmsy is the equilibrium stock size that gives MSY. If the projection is deterministic then the calculation of MSY RPs is also deterministic. In this context some theory has been developed related to MSY calculations (e.g. Sissenwine and Shepherd, 1987). In the traditional age-based MSY calculations, all population processes are assumed to be constant; that is, the age-based values of natural mortality, maturity, weight, and fishery selectivity are held constant in the long-term stock projections, as is the recruit-per-spawner functional relationship with SSB. Estimation error in these population processes contributes to uncertainty, and some bias, in MSY RPs.

If the population processes themselves are variable, then MSY RPs will also vary. For example, if natural mortality (M) changes in the future, as a function of predators or other factors, then this will affect MSY RPs. If the population processes that vary have, or are expected to achieve, a stationary (i.e. steady-state) distribution then there may also be stationary distributions for MSY RPs that are useful for fisheries management, although the RPs will be random and this should be accounted for in management decisions.

In a stochastic environment, harvesting according to the deterministic MSY rule is an under-optimized strategy and can lead to strong decreases in stock size (e.g. Bousquet et al., 2008). These authors provided analytical descriptions of the impacts of process error for the Schaefer surplus production model with a particular type of bounded process error. They showed that the stochastic mean values for MSY, Bmsy, and Fmsy were less than the deterministic results. They concluded that the deterministic Fmsy is incompatible with the assumption of equilibrium: on average, one cannot hope to harvest more than the stochastic MSY. Constant harvesting at the deterministic Fmsy would eventually lead to stock extinction.

Some preliminary investigations are presented in this paper on the impact on MSY RP's of process error in the stock-recruit relationship and natural mortality. Analyses are based on an artificial example that is loosely based on 3Ps American plaice. Stochastic simulations are used to find the fishing mortality rate (Fmsy) that optimizes long term equilibrium expected catch and produce equilibrium distributions for biomass and catch at F = Fmsy. The impacts of the magnitude (i.e. variance) of the process errors and the amount of auto-correlation in the errors are explored, to illustrate when process errors matter in terms of MSY RP's.

2. METHODS

2.1 Deterministic reference points

The purpose of this section is to provide a simple review of traditional age-based MSY calculations for those less familiar with the subject.

Let No denote the initial age distribution (numbers-at-age) of the population. If the realistic maximum age is A, then No is an A×1 vector. The stock projection is based on the standard cohort model,

$$N_{a+1,y+1} = N_{a,y} e^{-Z_{ay}}, \qquad (1)$$

where $Z_{a,y} = F_{a,y} + M_{a,y}$ is the total mortality rate which is split into a component due to fishing (F) and a component due to other sources (M). The latter component is usually referred to as the natural mortality rate, and is assumed to be known. I will assume M=0.2 for all projection ages and years. Equation (1) can be used to project the age distribution in year y+1 based on the age distribution in year y, except for the number at the first age (a_o) in the model. Note that a_o is often greater than one because for some stocks the fishery does not exploit younger ages and information about M for these ages is too speculative to make it worthwhile to extend the cohort model to these ages.

I assume $N_{A+1} = 0$ for all years in Equation (1), and it important for A to be large enough so that this assumption is reasonable. Otherwise, a plus group could be used,
$N_{a,y} = N_{a-1,y-1}e^{-Z_{a-1,y-1}} + N_{a,y-1}e^{-Z_{a,y-1}}$.

For MSY calculations it is necessary to use some type of model to determine recruitment (i.e. stock numbers at age a_o) in the projection. Stock-recruit models provide a functional relationship between recruitment and parental stock size. Let S_y denote the parental stock size in year y. Typically parental stock size is expressed as spawning stock biomass (SSB) which I define more explicitly later. A stock-recruit model (SR) predicts recruitment, $N_{a_o,y} = SR(S_{y-a_o})$. In this paper I focus on the Beverton-Holt (BH) SR model,

$$N_{a_o,y} = BH(S_{y-a_o}) = \frac{\alpha S_{y-a_o}}{\beta + S_{y-a_o}} \quad (2)$$

For this model, recruitment is a monotone increasing function of S, and α is the maximum recruitment: $\lim_{S\to\infty} BH(S)$. β is the stock size that produces 50% of maximum recruitment.

Equations (2) and (1) can be used to project future stock size if total mortality rates are known. For illustration purposes I set α = 20 000 and β = 10 000. This BH SR curve is shown in Figure 1.

Fishing mortalities and M are held constant for all years in projections for MSY calculations. However, the projections are performed for a range of fishing mortalities to find the ones that maximize yield. Typically the age-pattern in F's is also treated as fixed and the same in each year; that is, $F_{ay} = F \times S_a$, where S_a is the fixed fishery age-selectivity. The projected catch (C) is obtained using the Baranov catch equation,

$$C_{ay} = N_{ay}\left(1 - e^{-Z_a}\right)\frac{F_a}{Z_a}. \quad (3)$$

In equation (3), $N_{ay}\{1 - \exp(-Z_a)\}$ is the total number of deaths in year y from the fish at age a alive at the beginning of the year, and the fraction F_a / Z_a is assigned to fishing. The yield or landed weight in projection year y is $L_y = \Sigma_a W_{ay}^c C_{ay}$, where W_{ay}^c's are the projected average catch weights-at-age, which are assumed to be known and usually the same from year to year; that is, $W_{ay}^c = W_a^c$ for all projection years. If the projections are run long enough for a specific F then eventually the population reaches an equilibrium with a constant age-distribution and landings. Let L denote the equilibrium landings, $L = \lim_{y\to\infty} L_y$, and note that L is a function of F.

Definition: Fmsy is the value of F that maximizes L(F).

The corresponding stock biomass is called Bmsy, but often Bmsy is expressed in terms of parental stock size.

$SSB_y = \Sigma_a W^s_{ay} Mat_{ay} N_{ay}$ where Mat_{ay} is the proportion of fish mature at age a in year y, and W^s_{ay}'s are the average beginning of year stock weights-at-age, as opposed to the catch weight-at-age W^c_{ay}. Usually $W^c_{ay} > W^s_{ay}$ because of fish growth. Both maturities and stock weights are assumed to be known and usually the same from year to year, but different by age.

Definition: If SSB(F) is the equilibrium SSB for some value of F, then Bmsy is often taken to be SSB(Fmsy). To avoid confusion, I refer to this as SSBmsy.

SSB is often used to represent parental stock size in equation (2).

The values of W^s_a's, Mat_a's, and S_a's that I use to illustrate MSY results are shown in Figure 2. For simplicity I assume $W^s_a = W^c_a$. These values are loosely based on American plaice (*Hippoglossoides platessoides*) in NAFO Subdivision 3Ps. The fishery selectivities (S_a's) were derived from a survey-based cohort model (SURBA) but this is not the focus of this paper and is not described further. Although these biological and fishery characteristics can have important effects on MSY RP's, they are not the focus of this paper and are merely assumed to be known without error.

2.2 Stochastic reference points when there is process error

When there is process error in some aspects of population dynamics in the projections then the projections will usually not converge to fixed values. The best one can hope for is that the statistical distribution of projection values (SSB, etc) eventually achieves a stationary distribution that does not change from year to year. A particular set of errors will produce a deterministic projection, and the statistical distribution of process errors generates a distribution of projected population values. When there is a stationary distribution, then:

Definition: Fmsy is taken to be the value of F that maximizes the mean of the stationary distribution for landings (i.e. yield).

In this case there is also a stationary distribution of SSB's, and a value for Bmsy could be the mean of this distribution, although the uncertainty in the value of Bmsy should be taken into account when evaluating stock status relative to Bmsy. I visually check that stochastic projections result in a stationary distribution by plotting quantiles of the projection distributions each year. To aid in interpreting these quantiles, I also run a smoother through them. I used the loess function in the R software package (R 2011) for this.

Two situations are investigated. The first is when there is process error in the SR model (equation 2). Stock-recruit relationships are usually poorly defined because of highly variable pre-recruitment life history processes (Quinn and Dersio, 1999; Ch. 3). We account for this uncertainty using lognormal multiplicative process errors, standardized to have mean one;

$$N_{a_o,y} = BH(S_{y-a_o})\varepsilon_y = \frac{\alpha S_{y-a_o}}{\beta + S_{y-a_o}}\varepsilon_y; \varepsilon_y = \exp\left(\sigma_R Z_y - \sigma_R^2/2\right),\ Z_y \overset{iid}{\sim} N(0,1), \quad (4)$$

where iid is an acronym for independent and identically distributed. Note that the $-\sigma_R^2/2$ term in (4) is required so that $E(\varepsilon_y) = 1$. A range of values for σ_R is investigated to see what impact these errors have on MSY reference points.

Another possibly important feature of SR process error is auto-correlation. It is quite common when examining residuals obtained from fitting a SR model to see strong evidence of auto-correlation. Extended periods of above- or below-average recruitment productivity could affect the value of Fmsy and Bmsy. To examine the potential impact of auto-correlated SR errors I performed stochastic projections in which the Z_y's in equation (4) are from an AR(1) process; that is, $Z_y = \varphi Z_{y-1} + \delta_y$, where $\delta_y \sim N(0,1)$. If $|\varphi|<1$ then the AR(1) process achieves a stationary distribution, and $\lim_{y\to\infty} Var(Z_y) = 1/(1-\varphi^2)$. Hence, to make the stationary variance comparable to the iid case I generated process errors using $\varepsilon_y = \exp(\sigma_P Z_y - \sigma_R^2/2)$ where $\sigma_p = \sigma_R(1-\varphi^2)^{1/2}$. In this case, $\lim_{y\to\infty} Var(\sigma_p Z_y) = \sigma_R^2$. The lognormal AR(1) errors when $\varphi = 0.95$ are illustrated in Figure 3 for a 250 year projection. With this amount of auto-correlation there can be extended periods of time, longer than the monitoring periods for most stocks, with above- or below-average recruitment productivity.

Although the auto-correlated process errors are standardized so that $\lim_{y\to\infty} E(\varepsilon_y) = 1$, the average errors for a 250 year projection realization can be quite different from one when φ is close to one. In this case recruitment generated by equation (4) will differ systematically and substantially from the deterministic case in the particular projection. To reduce this problem, but still have positive errors in equation (4), as a second approach I standardized the Z_y 's by subtracting their mean for all years in each projection.

The second type of process error I examine is error in M. In the stochastic projections, $M_{ay} = M\,\varepsilon_y$, where M is the nominal value (i.e. M = 0.2) and ε_y's were generated the same as with the SR process errors, either lognormal iid or AR(1), and mean standardized or not.

A large number of projections were conducted, each for 250 years. Projections were simulated at least 2000 times, and even higher when σ_R or φ were large. The projections were summarized in terms on means or quantiles, and MSY RP's were derived by averaging results for the final 50 years; that is, Fmsy was the value of F that maximized the average of the projected mean landings for the final 50 projection years. Note that the annual projected mean landings are approximated by averaging across simulations.

3 RESULTS

3.1 Stock-recruit process error

The mean yield curve for iid ε's when $\sigma_R = 0.3$ (Figure 4) had a maximum very close to the deterministic result. Means of the equilibrium distributions for Bmsy and MSY were also similar to deterministic results (Figure 5). However, differences were observed for larger values of σ_R. When $\sigma_R = 1.0$ the the mean yield curve (Figure 6) had a somewhat lower maximum and Fmsy was slightly less than the deterministic result. Means of the equilibrium distributions for Bmsy and MSY were somewhat lower than the deterministic results (Figure 7).

Auto-correlation "boosts" the impact of process errors. When $\sigma_R = 1.0$ and $\varphi = 0.9$, Fmsy is lower than when $\varphi = 0$ (compare Figures 8 and 6). Similarly, Bmsy and MSY were lower (compare Figures 9 and 7). The quantiles in Figure 9 are wider than in Figure 7. I summarize results for a range of σ_R and φ in Tables 1-4. There is a trend to get a lower Fmsy, Bmsy, and MSY catch when σ_R increases or when φ increases; however, this trend to does hold the same when φ = 0.99 (e.g. see Table 1). Fishing at the deterministic Fmsy leads to a lower equilibrium mean Bmsy (Table 5).

Some results are substantially different when the auto-correlated errors are mean-standardized for the 250 year projection period. The standardization is described in Section 2.2. Fmsy is lower (compare Tables 6 and 1) when σ_R and φ are large. Similarly, SSBmsy (compare Tables 7 and 2) , Bmsy (compare Tables 8 and 3), and particularly MSY (compare Tables 9 and 4) are lower when errors are mean-standardized. In addition, percentiles are not as wide when σ_R and φ are large. However, there is evidence of non-stationarity when projection errors are mean-standardization. For example, the 95^{th} quantile starts to increase (Figure 10) towards the end of the projection period when σ_R = 0.5 and φ = 0.95, and in the worst case (σ_R = 1.0 and φ = 0.99; Figure 11) there is clear evidence of non-stationarity because the upper quantiles and the mean are increasing, and the lower quantiles are decreasing, in the latter part of the projection. Non-stationarity does not seem to be a problem when process errors are not mean standardized.

3.2 M process error

I do not present results for Bmsy, which are similar in pattern to SSBmsy results.

The stochastic Fmsy's that maximized the means of the equilibrium distributions for yield (Table 10) are similar to the deterministic value, although somewhat greater when σ_R and φ are large. The means of the equilibrium distributions for SSBmsy (Table 11) and MSY catch (Table 12) also increased substantially with σ_R and φ, as did the range of percentiles.

Non-stationarity was evident when the auto-correlated errors are mean-standardized for the 250 year projection period (Tables 13-15). I illustrate this σ_R = 0.3 and φ = 0.95 (Figure 12). The 95^{th} percentile increases towards the end of the projection period, and the mean is slowly increasing as well. This is more evident when φ = 0.99 (i.e. Figure 13). Non-stationarity does not seem to be a problem when process errors are not mean standardized.

4 DISCUSSION

Process error introduces uncertainty into MSY reference points (RPs). If the magnitude and auto-correlation in the errors is large enough then process errors can affect the mean values for RPs.

I only investigated a single BH stock-recruit curve. It would be desirable to better understand how the form the stock-recruit curve and other aspects of stock productivity interact with process errors in terms of MSY RP's.

The M process errors demonstrated that the process error variance and the way the errors are incorporated in the population dynamics model can have a large impact. The way I added process errors to M actually lead to increases in Bmsy and MSY catch. This may not be the best approach; a better approach may be to incorporate process error directly in the cohort model, $N_{a+1,y+1} = N_{a,y} e^{-Z_{ay}} \varepsilon_y$.

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6. TABLES AND FIGURES

Table 1. *Stochastic mean Fmsy with recruitment auto-correlated process errors. The deterministic Fmsy is 0.128.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	0.129	0.128	0.128	0.129
0.30	0.128	0.127	0.127	0.129
0.50	0.128	0.122	0.126	0.127
0.75	0.128	0.111	0.113	0.127
1.00	0.124	0.105	0.105	0.129

Table 2. *Stochastic SSBmsy with auto-correlated recruitment process errors. 5th and 95th quantiles are in parentheses and grey. The deterministic SSBmsy is 16.273 Kt.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	16.2 (15.4,17.1)	16.2 (13.6,19.2)	16.2 (13.1,19.7)	16.3 (12.5,20.4)
0.30	16.2 (13.7,19.1)	16.3 (8.83,26.3)	16.3 (7.90,27.6)	16.2 (6.17,30.4)
0.50	16.1 (12.1,21.1)	16.1 (5.59,33.2)	15.8 (3.96,36.0)	16.4 (1.77,42.0)
0.75	15.9 (9.93,23.9)	16.8 (2.62,45.9)	16.6 (1.15,51.1)	15.8 (0.19,53.6)
1.00	16.0 (8.04,28.1)	15.4 (0.78,52.6)	16.0 (0.18,60.4)	15.8 (0.04,67.8)

Table 3. *Stochastic Bmsy with auto-correlated recruitment process errors. 5th and 95th quantiles are in parentheses and grey. The deterministic Bmsy is 23.335 Kt.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	23.3 (22.2,24.5)	23.3 (19.5,27.4)	23.3 (18.8,28.2)	23.4 (17.9,29.3)
0.30	23.3 (19.9,27.1)	23.3 (12.8,37.4)	23.3 (11.4,39.4)	23.3 (8.84,43.7)
0.50	23.1 (17.6,29.7)	23.0 (8.06,46.7)	22.6 (5.68,51.1)	23.5 (2.52,60.1)
0.75	22.7 (14.6,33.6)	23.6 (3.74,63.7)	23.4 (1.62,71.3)	22.5 (0.26,76.6)
1.00	22.8 (11.9,38.9)	21.5 (1.10,72.5)	22.3 (0.25,84.1)	22.6 (0.04,97.5)

Table 4. *Stochastic Cmsy with auto-correlated recruitment process errors. 5th and 95th quantiles are in parentheses and grey. The deterministic Cmsy is 1.955 Kt.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	1.96 (1.86,2.07)	1.96 (1.64,2.31)	1.96 (1.58,2.37)	1.97 (1.51,2.48)
0.30	1.96 (1.66,2.30)	1.94 (1.06,3.13)	1.95 (0.95,3.30)	1.96 (0.75,3.68)
0.50	1.95 (1.46,2.53)	1.85 (0.64,3.79)	1.87 (0.47,4.25)	1.95 (0.21,5.00)
0.75	1.90 (1.20,2.86)	1.74 (0.27,4.73)	1.75 (0.12,5.35)	1.87 (0.02,6.37)
1.00	1.86 (0.95,3.25)	1.50 (0.07,5.09)	1.56 (0.01,5.90)	1.92 (0.00,8.25)

Table 5. *Difference between SSBmsy and equilibrium SSB at deterministic Fmsy, with auto-correlated recruitment process errors.*

Recruitment process error variance (σ_R)	Auto-correlation (φ) 0.0	0.9	0.95	0.99
0.10	0.2	0.2	0.2	0.1
0.30	0.1	0.2	0.4	0.1
0.50	0.0	0.7	0.1	0.3
0.75	0.0	2.6	2.4	0.1
1.00	0.5	3.5	3.6	0.2

Table 6. *Stochastic mean Fmsy with projection mean-standardized recruitment auto-correlated process errors. The deterministic Fmsy is 0.128. Values in red indicate cases where there is evidence of non-stationarity.*

Recruitment process error variance (σ_R)	Auto-correlation (φ) 0.0	0.9	0.95	0.99
0.10	0.129	0.129	**0.128**	**0.128**
0.30	0.128	0.127	**0.128**	**0.128**
0.50	0.128	0.120	**0.123**	**0.126**
0.75	0.128	0.111	**0.110**	**0.114**
1.00	0.124	**0.101**	**0.100**	**0.101**

Table 7. *Stochastic SSBmsy with projection mean-standardized auto-correlated recruitment process errors. 5th and 95th quantiles are in parentheses and grey. The deterministic SSBmsy is 16.273 Kt. Values in red indicate cases where there is evidence of non-stationarity.*

Recruitment process error variance (σ_R)	Auto-correlation (φ) 0.0	0.9	0.95	0.99
0.10	16.2 (15.4,17.1)	16.2 (13.8,18.8)	**16.3** (13.4,19.5)	**16.2** (12.1,19.5)
0.30	16.2 (13.7,19.1)	16.0 (9.48,24.6)	**16.1** (8.49,26.6)	**16.1** (8.14,26.8)
0.50	16.1 (12.1,21.1)	16.0 (6.19,31.5)	**15.9** (4.88,34.1)	15.4 (4.07,33.2)
0.75	15.9 (9.93,23.9)	16.0 (3.27,40.3)	**16.3** (2.25,45.5)	**15.8** (1.56,45.8)
1.00	16.0 (8.04,28.1)	**15.4** (1.40,48.6)	**14.9** (0.61,51.7)	**15.0** (0.42,53.6)

Table 8. *Stochastic Bmsy with projection mean-standardized auto-correlated recruitment process errors. 5th and 95th quantiles are in parentheses and grey. The deterministic Bmsy is 23.335 Kt. Values in red indicate cases where there is evidence of non-stationarity.*

Recruitment process error variance (σ_R)	Auto-correlation (φ) 0.0	0.9	0.95	0.99
0.10	23.3 (22.2,24.5)	23.2 (19.8,27.0)	**23.4** (19.3,28.0)	**23.2** (18.9,28.0)
0.30	23.3 (19.9,27.1)	22.9 (13.7,35.0)	**23.1** (12.2,38.0)	**23.1** (11.6,38.6)
0.50	23.1 (17.6,29.7)	23.0 (8.06,46.7)	**22.6** (5.68,51.1)	**23.5** (2.52,60.1)
0.75	22.7 (14.6,33.6)	22.5 (4.71,56.3)	**22.9** (3.18,63.6)	**22.4** (2.11,65.1)
1.00	22.8 (11.9,38.9)	**21.4** (1.99,66.9)	**20.8** (0.84,71.8)	**21.0** (0.54,75.5)

Table 9. *Stochastic Cmsy with projection mean-standardized auto-correlated recruitment process errors. 5th and 95th quantiles are in parentheses and grey. The deterministic Cmsy is 1.955 Kt. Values in red indicate cases where there is evidence of non-stationarity.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	1.96 (1.86,2.07)	1.96 (1.67,2.28)	**1.97** (1.62,2.36)	**1.96** (1.59,2.35)
0.30	1.96 (1.66,2.30)	1.91 (1.13,2.93)	**1.94** (1.02,3.19)	**1.93** (0.97,3.23)
0.50	1.95 (1.46,2.53)	1.81 (0.70,3.54)	**1.83** (0.56,3.92)	**1.82** (0.47,3.94)
0.75	1.91 (1.20,2.86)	1.66 (0.34,4.18)	**1.67** (0.23,4.64)	**1.68** (0.16,4.90)
1.00	1.86 (0.95,3.25)	**1.44** (0.13,4.52)	**1.39** (0.06,4.80)	**1.41** (0.04,5.08)

Table 10. *Stochastic mean Fmsy with M auto-correlated process errors. The deterministic Fmsy is 0.128.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	0.129	0.129	0.128	0.128
0.30	0.128	0.126	0.127	0.128
0.50	0.128	0.126	0.126	0.130
0.75	0.129	0.125	0.126	0.136
1.00	0.130	0.127	0.126	0.133

Table 11. *Stochastic SSBmsy with auto-correlated M process errors. 5th and 95th quantiles are in parentheses and grey. The deterministic SSBmsy is 16.273 Kt.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	16.3 (14.9,17.7)	16.5 (12.3,21.1)	16.4 (11.3,22.1)	16.6 (10.6,23.1)
0.30	16.5 (12.3,21.0)	18.1 (6.28,33.5)	18.3 (4.72,36.3)	19.3 (2.73,41.1)
0.50	16.9 (9.96,24.9)	20.6 (2.42,48.3)	21.8 (1.02,55.0)	23.4 (0.17,61.1)
0.75	18.0 (7.39,30.9)	25.2 (0.47,68.8)	27.2 (0.08,78.2)	29.0 (0.03,83.4)
1.00	19.5 (5.10,37.7)	30.2 (0.10,88.5)	33.7 (0.02,101)	39.1 (0.00,108)

Table 12. *Stochastic Cmsy with auto-correlated M process errors. 5th and 95th quantiles are in parentheses and grey. The deterministic Cmsy is 1.955 Kt.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	1.97 (1.81,2.13)	1.99 (1.51,2.52)	1.98 (1.38,2.62)	2.00 (1.31,2.74)
0.30	1.98 (1.52,2.48)	2.13 (0.78,3.84)	2.16 (0.60,4.18)	2.29 (0.35,4.75)
0.50	2.03 (1.25,2.92)	2.39 (0.32,5.44)	2.50 (0.13,6.14)	2.77 (0.02,7.03)
0.75	2.16 (0.96,3.59)	2.85 (0.06,7.58)	3.09 (0.01,8.65)	3.53 (0.00,9.92)
1.00	2.34 (0.70,4.39)	3.42 (0.01,9.75)	3.78 (0.00,11.1)	4.58 (0.00,12.4)

Table 13. *Stochastic mean Fmsy with projection mean-standardized M auto-correlated process errors. The deterministic Fmsy is 0.128. Values in red indicate cases where there is evidence of non-stationarity.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	0.129	0.129	**0.129**	**0.129**
0.30	0.128	**0.127**	**0.128**	**0.131**
0.50	0.128	**0.126**	**0.127**	**0.130**
0.75	0.129	0.126	**0.127**	**0.130**
1.00	0.130	0.117	**0.126**	**0.132**

Table 14. *Stochastic SSBmsy with projection mean-standardized auto-correlated M process errors. 5th and 95th quantiles are in parentheses and grey. The deterministic SSBmsy is 16.273 Kt. Values in red indicate cases where there is evidence of non-stationarity.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	16.3 (14.9,17.7)	16.4 (12.5,20.7)	**16.5** (11.9,21.4)	**16.5** (11.6,21.8)
0.30	16.5 (12.3,21.0)	**17.9** (6.91,32.2)	**18.5** (5.80,35.1)	**18.6** (5.17,36.1)
0.50	16.9 (9.96,24.9)	**20.7** (3.37,45.9)	**21.9** (2.24,52.0)	**23.4** (2.06,54.7)
0.75	18.0 (7.39,30.9)	24.8 (1.04,64.2)	**26.9** (0.41,72.4)	**31.5** (0.58,78.3)
1.00	19.5 (5.10,37.7)	33.8 (0.45,90.8)	**34.6** (0.10,94.7)	**40.6** (0.02,99.6)

Table 15. *Stochastic Cmsy with projection mean-standardized auto-correlated M process errors. 5th and 95th quantiles are in parentheses and grey. The deterministic Cmsy is 1.955 Kt. Values in red indicate cases where there is evidence of non-stationarity.*

	Auto-correlation (φ)			
Recruitment process error variance (σ_R)	0.0	0.9	0.95	0.99
0.10	1.97 (1.81,2.13)	1.99 (1.54,2.48)	**1.99** (1.46,2.56)	**2.00** (1.43,2.61)
0.30	1.98 (1.52,2.48)	**2.12** (0.87,3.72)	**2.20** (0.74,4.06)	**2.26** (0.67,4.28)
0.50	2.03 (1.25,2.92)	**2.40** (0.44,5.18)	**2.55** (0.29,5.88)	**2.78** (0.27,6.35)
0.75	2.16 (0.96,3.59)	2.82 (0.14,7.10)	**3.10** (0.06,8.10)	**3.69** (0.08,8.97)
1.00	2.34 (0.70,4.39)	3.52 (0.06,9.23)	**3.89** (0.01,10.4)	**4.75** (0.00,11.4)

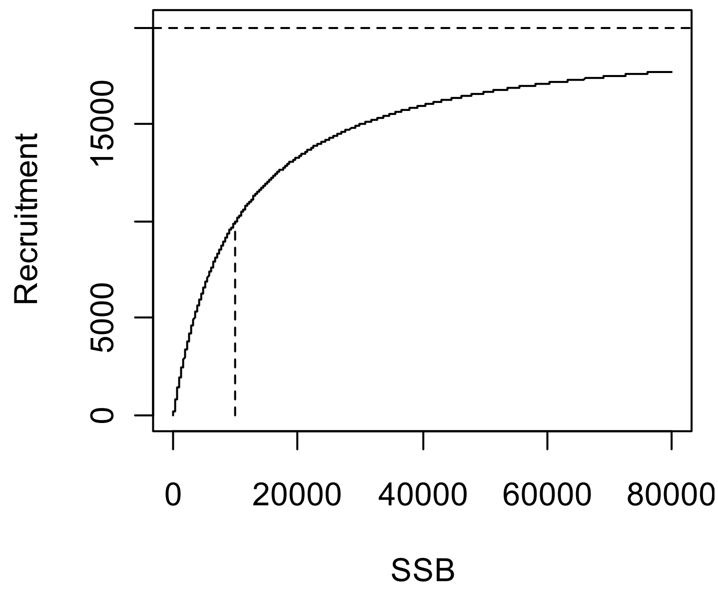

Figure 1. The Beverton-Holt stock-recruit function used to illustrate the impact of process error on MSY reference points. The horizontal dashed line indicates the maximum recruitment (Rmax), and the vertical dashed line indicates the stock size that produces 50% of Rmax.

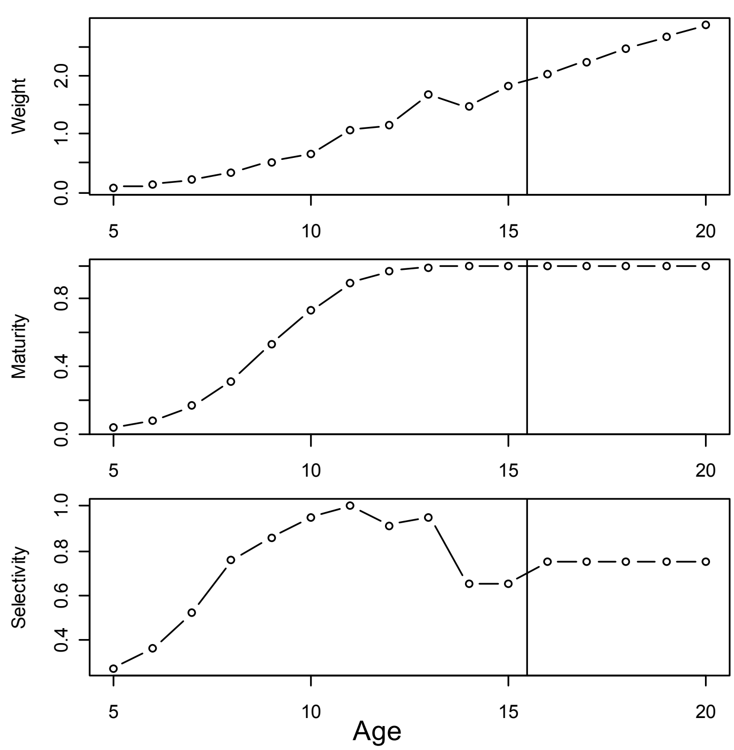

Figure 2. *Stock weights-at-age (top panel), maturity-at-age (middle panel), and fishery selectivity (bottom panel). Note that values for ages greater than 15 are extrapolations.*

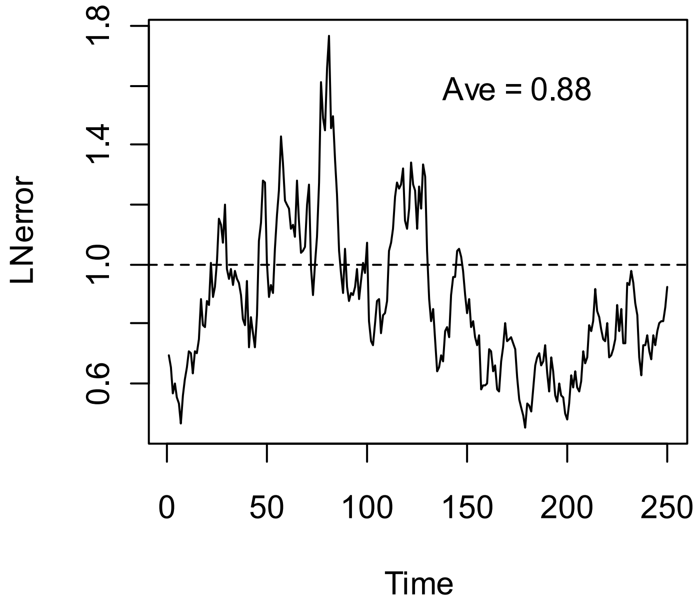

Figure 3. *A simulated realization of lognormal auto-correlated errors when φ = 0.95. The average error for the 250 years is indicated in the figure. A horizontal line (dashed) at one is shown for reference.*

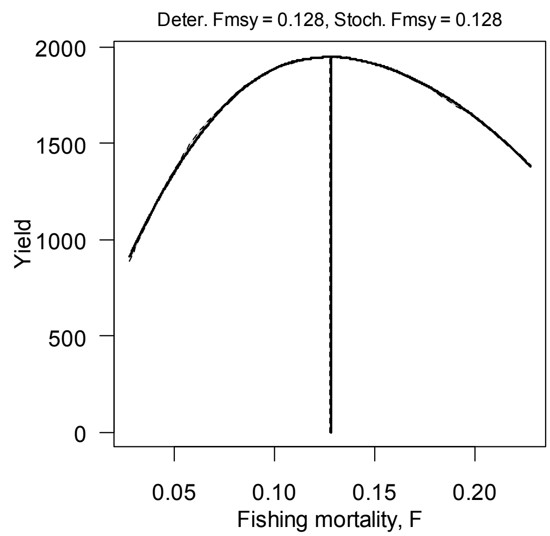

Figure 4. *Equilibrium mean yield versus fishing mortality (solid line). Process errors (σ_R = 0.3, φ=0) are in the stock-recruit relationship. The value of F that maximized mean yield (i.e. stoch. Fmsy) is indicated at the top of the panel, and by the solid vertical line.*

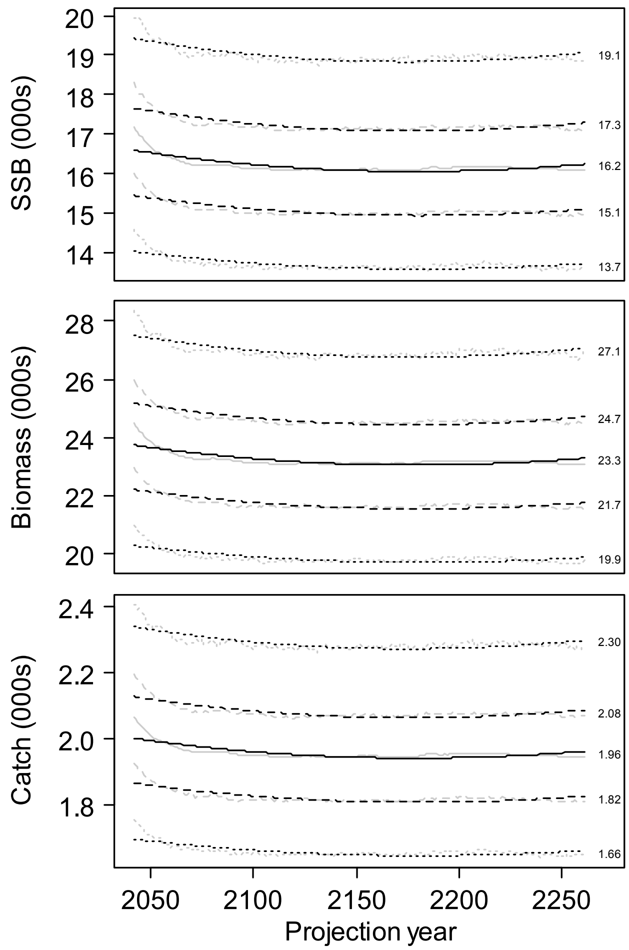

***Figure 5**. Equilibrium distribution results (grey lines) when process errors (σ_R = 0.3, φ=0) are in the stock-recruit relationship; for SSB (top panel), total biomass (middle panel), and catch weight (bottom panel; i.e. yield). Solid lines are for means, dashed lines are for 25th and 75th percentiles, and dotted lines are for 5th and 95th percentiles. Black lines show loess smoothes of the corresponding results. Numbers at the right in each panel indicate averages for the last 50 years in the projections. The deterministic SSBmsy is 16.27, total Bmsy is 23.33, and MSY catch is 1.95, in 000s.*

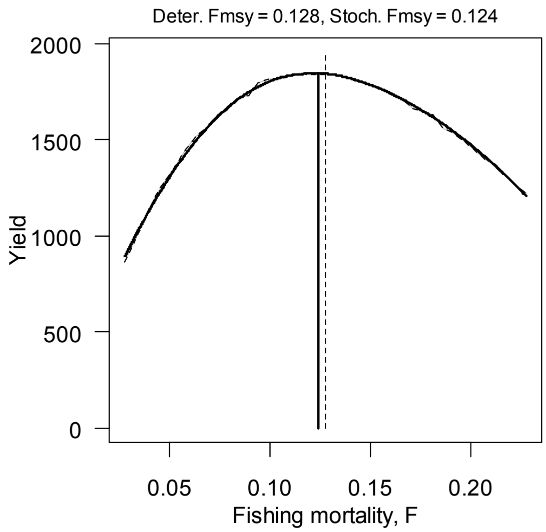

***Figure 6**. Equilibrium mean yield versus fishing mortality (solid line). Process errors (σ_R = 1.0, φ=0) are in the stock-recruit relationship. The value of F that maximized mean yield (i.e. stoch. Fmsy) is indicated at the top of the panel, and by the solid vertical line. The dashed line indicates the deterministic Fmsy, and the height of this line indicates the deterministic MSY.*

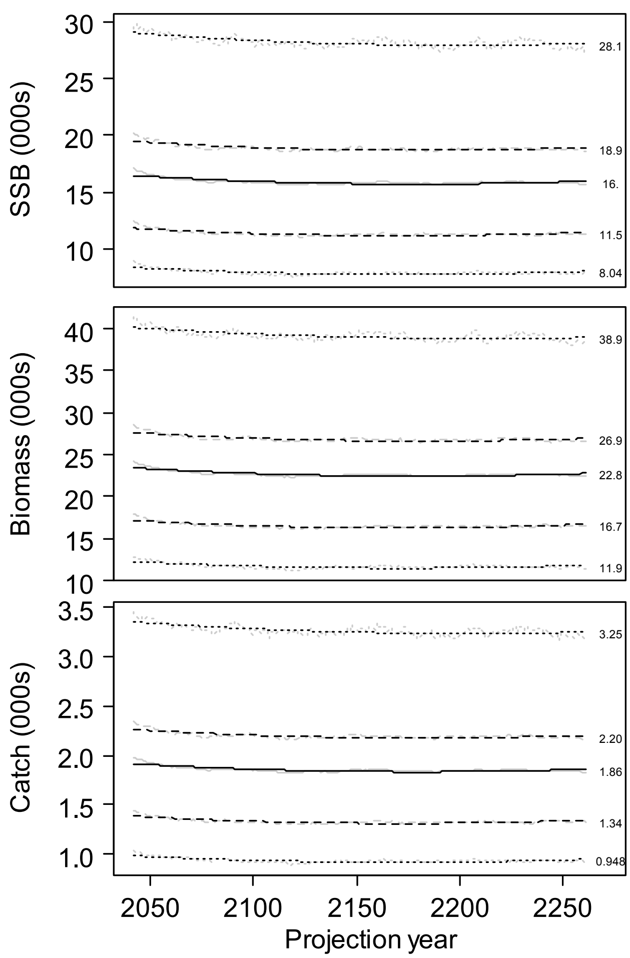

***Figure 7**. Equilibrium distribution results (grey lines) when process errors (σ_R = 1.0, φ=0) are in the stock-recruit relationship; for SSB (top panel), total biomass (middle panel), and catch weight (bottom panel; i.e.*

yield). Solid lines are for means, dashed lines are for 25th and 75th percentiles, and dotted lines are for 5th and 95th percentiles. Black lines show loess smoothes of the corresponding results. Numbers at the right in each panel indicate averages for the last 50 years in the projections. The deterministic SSBmsy is 16.27, total Bmsy is 23.33, and MSY catch is 1.95, in 000s.

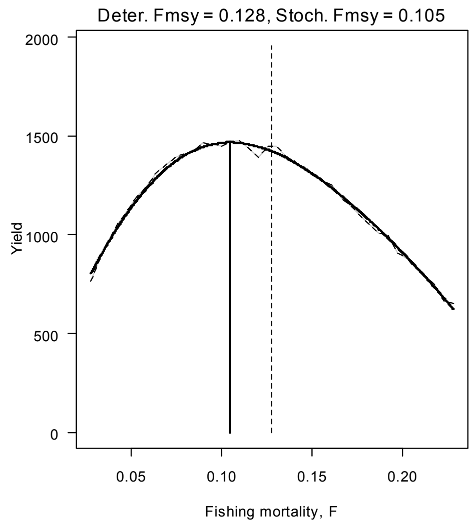

***Figure 8**. Equilibrium mean yield versus fishing mortality (solid line). Process errors (σ_R = 1.0, φ=0.9) are in the stock-recruit relationship. The value of F that maximized mean yield (i.e. stoch. Fmsy) is indicated at the top of the panel, and by the solid vertical line. The dashed line indicates the deterministic Fmsy, and the height of this line indicates the deterministic MSY.*

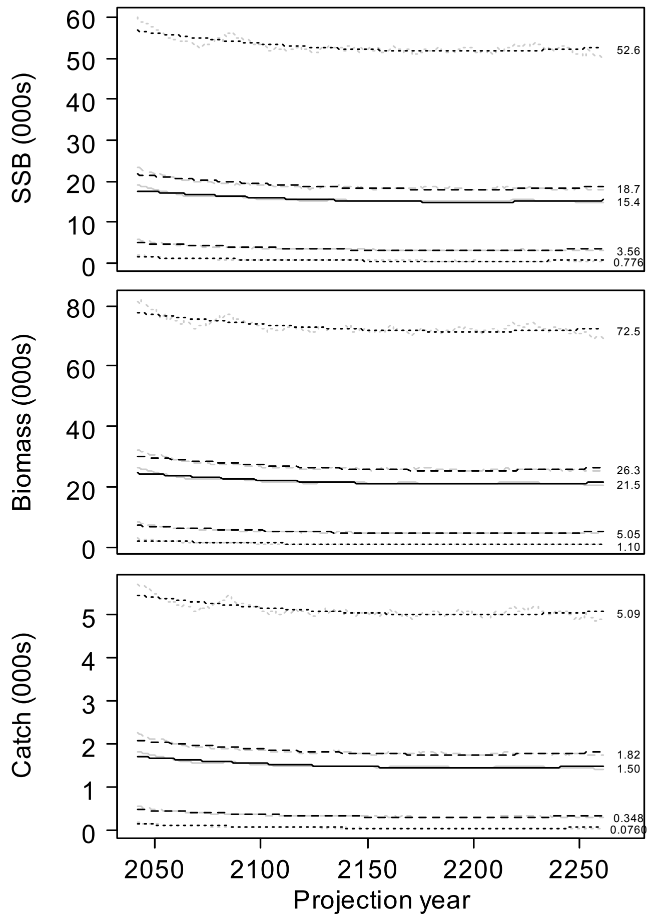

Figure 9*. Equilibrium distribution results (grey lines) when process errors (σ_R = 1.0, φ=0.9) are in the stock-recruit relationship; for SSB (top panel), total biomass (middle panel), and catch weight (bottom panel; i.e. yield). Solid lines are for means, dashed lines are for 25th and 75th percentiles, and dotted lines are for 5th and 95th percentiles. Black lines show loess smoothes of the corresponding results. Numbers at the right in each panel indicate averages for the last 50 years in the projections. The deterministic SSBmsy is 16.27, total Bmsy is 23.33, and MSY catch is 1.95, in 000s.*

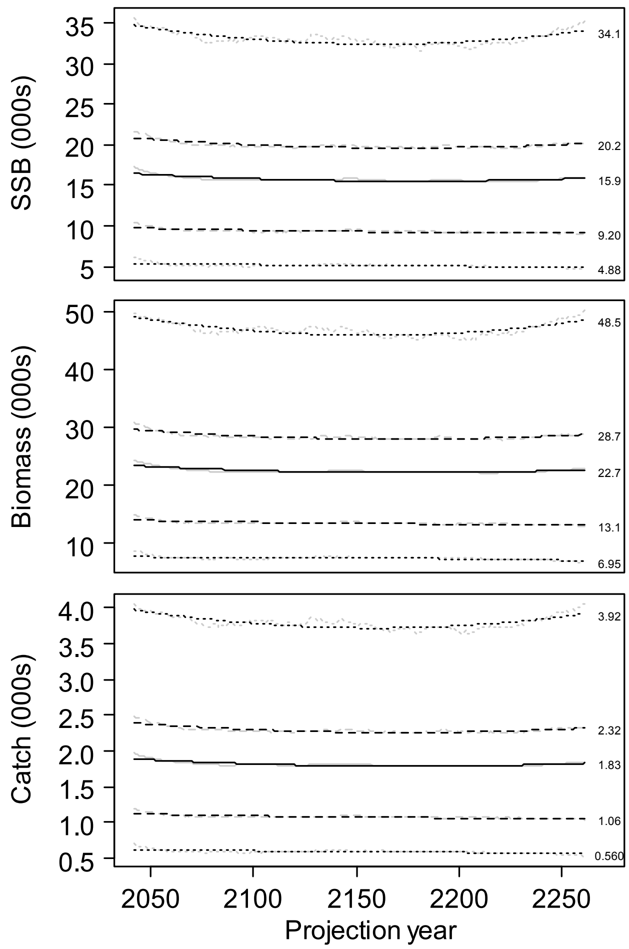

***Figure 10**. Long-term distribution results (grey lines) when projection mean-standardized process errors (σ_R = 0.5, φ=0.05) are in the stock-recruit relationship; for SSB (top panel), total biomass (middle panel), and catch weight (bottom panel; i.e. yield). Solid lines are for means, dashed lines are for 25th and 75th percentiles, and dotted lines are for 5th and 95th percentiles. Black lines show loess smoothes of the corresponding results. Numbers at the right in each panel indicate averages.*

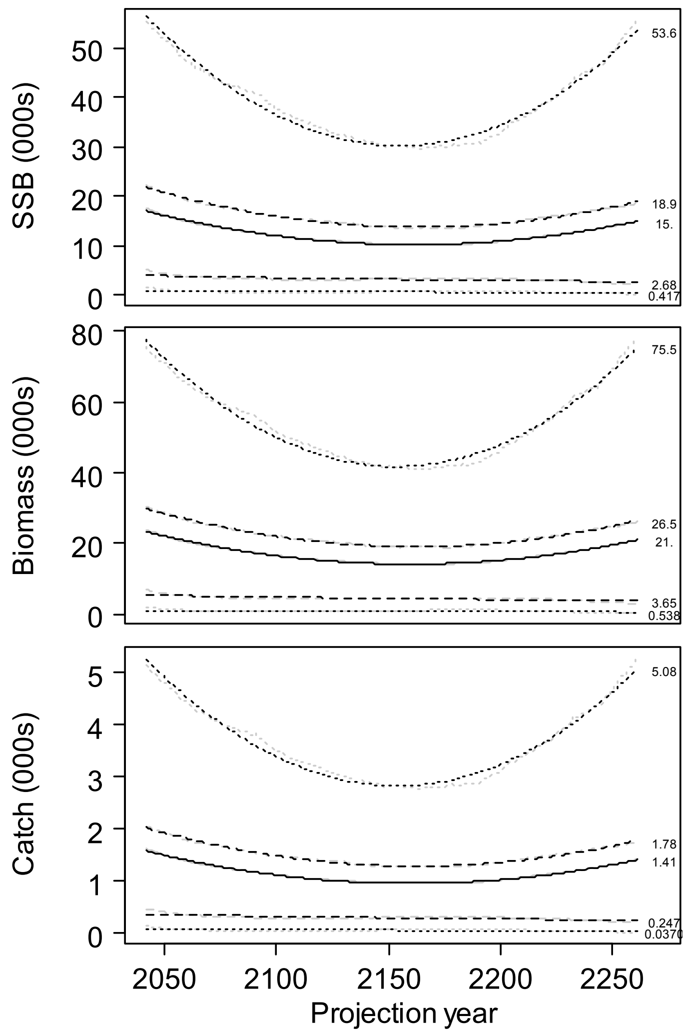

***Figure 11**. Long-term distribution results (grey lines) when projection mean-standardized process errors (σ_R = 1.0, φ=0.99) are in the stock-recruit relationship; for SSB (top panel), total biomass (middle panel), and catch weight (bottom panel; i.e. yield). Solid lines are for means, dashed lines are for 25th and 75th percentiles, and dotted lines are for 5th and 95th percentiles. Black lines show loess smoothes of the corresponding results. Numbers at the right in each panel indicate averages.*

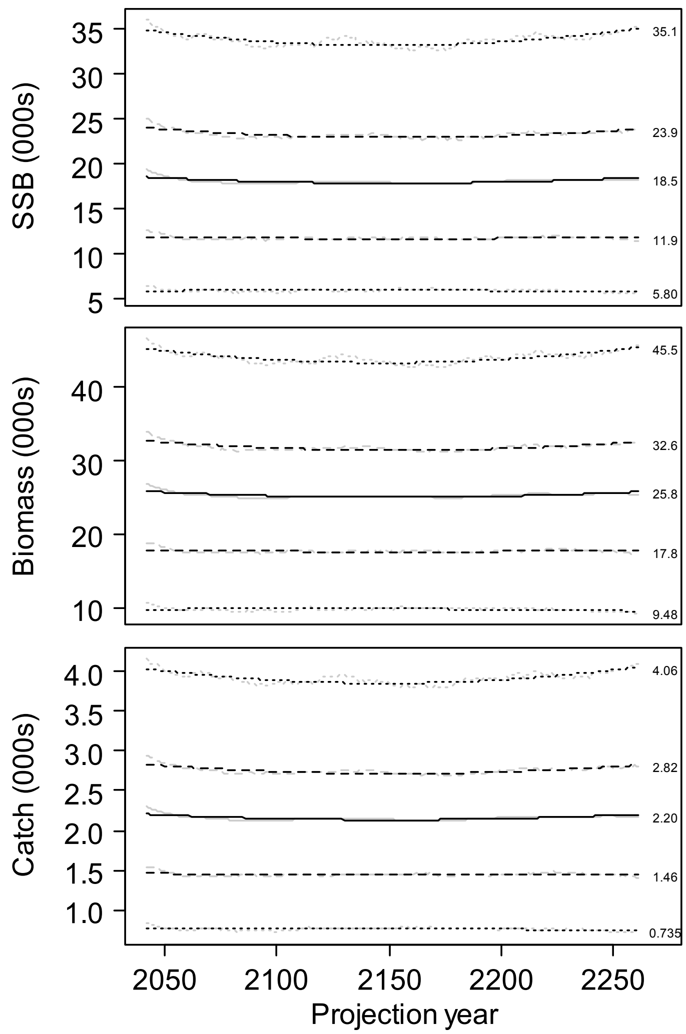

***Figure 12**. Long-term distribution results (grey lines) when projection mean-standardized process errors (σ_R = 0.3, φ=0.99) are in M; for SSB (top panel), total biomass (middle panel), and catch weight (bottom panel; i.e. yield). Solid lines are for means, dashed lines are for 25th and 75th percentiles, and dotted lines are for 5th and 95th percentiles. Black lines show loess smoothes of the corresponding results. Numbers at the right in each panel indicate averages.*

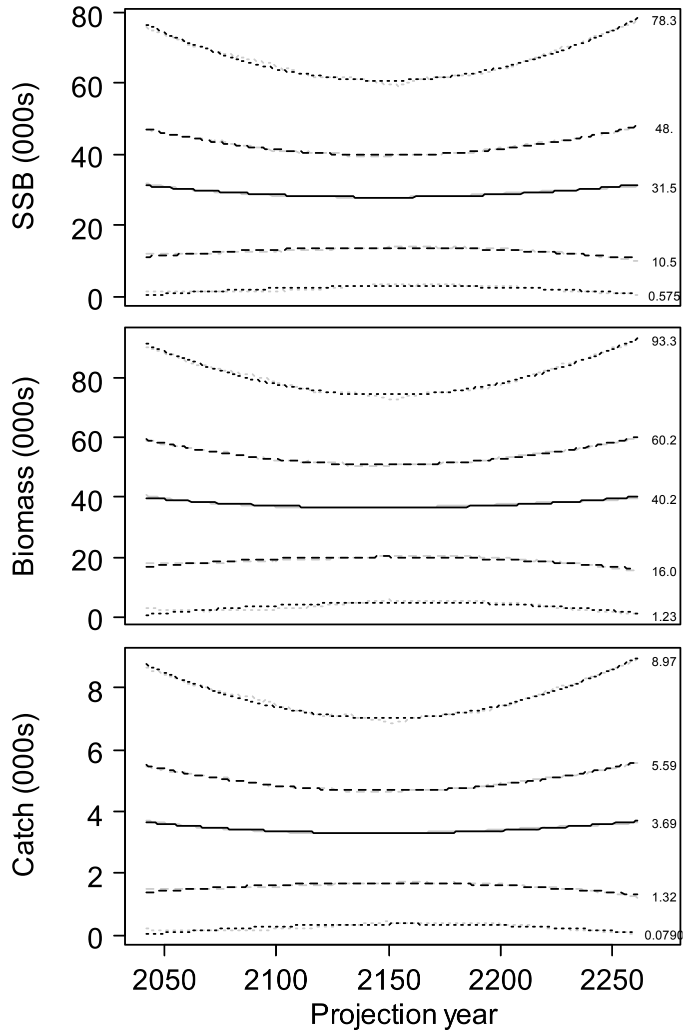

***Figure 13**. Long-term distribution results (grey lines) when projection mean-standardized process errors (σ_R = 0.75, φ=0.99) are in M; for SSB (top panel), total biomass (middle panel), and catch weight (bottom panel; i.e. yield). Solid lines are for means, dashed lines are for 25th and 75th percentiles, and dotted lines are for 5th and 95th percentiles. Black lines show loess smoothes of the corresponding results. Numbers at the right in each panel indicate averages.*