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An evaluation of growth models for predicting 2J3KL cod stock weights-at-age

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Foreword

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ABSTRACT

Growth models are developed to estimate beginning-of-year weights-at-age for 2J3KL cod based on average weights from samples collected during the fall surveys. Beginning-of-year weights are required for computing beginning-of-year spawning stock biomass. Von Bertalanffy and Gompertz growth models were investigated. There is some evidence of changes in growth rates over time so growth models are fit by cohort. The Gompertz model did not fit the data well and a modification is proposed that results in better fits to the data. The Von Bertalanffy model provided the best fit to the data.

Évaluation des modèles de croissance servant à prédire le poids selon l'âge du stock de morue 2J3KL

RÉSUMÉ

Les modèles de croissance sont établis en vue d'estimer le poids selon l'âge des morues 2J3KL au début de l'année en fonction du poids moyen des échantillons prélevés lors des relevés d'automne. Le poids au début de l'année est nécessaire pour calculer la biomasse du stock reproducteur au début de l'année. Le modèle de croissance de Von Bertalanffy et celui de Gompertz ont été étudiés. Comme il existe des preuves que les taux de croissance changent au fil du temps, les modèles de croissance sont établis par cohorte. Le modèle de Gompertz cadrait mal avec les données, mais une modification proposée permettrait d'obtenir de meilleurs résultats. Quant au modèle de Von Bertalanffy, il s'agit du modèle qui cadrait le mieux avec les données.

INTRODUCTION

The stock assessment of cod in NAFO Div. 2J3KL uses age-based survey indices and an age-based assessment model (Cadigan 2013). One of the main indicators of stock status is spawning stock biomass (SSB) which is nominally taken as mature biomass at the beginning of the year. It is computed as age-specific survey abundance multiplied by weights and maturities, summed over all ages. Estimates of abundance may come directly from survey mean number caught per tow or swept-area total abundance estimates. However, a complication is that the DFO survey takes place in the fall each year. Ideally these estimates should represent beginning-of-year abundance which can be obtained from the assessment model. The maturities and weights should also represent stock conditions at the beginning of the year.

The fall RV survey is also the main source of biological information about maturities and weights. It is fairly straight-forward to predict beginning-of-year maturities. Samples of cod are examined each year (y) and the age (a) and maturities are determined. The proportion mature at age a and year y is considered to be a direct estimate of the maturity at the beginning of the next year, pa+1,y+1. A cohort-specific Binomial logistic regression model has been used to estimate the proportion mature as a function of age (Brattey et al. 2011) for 2J3KL cod. The rationale is that the proportion mature increases monotonically as a function of age. There are several reasons to use a modelling approach: (1) fill in values for ages and years with no data; (2) reduce sampling variability; (3) predict values over the next few years to evaluate the impact of proposed catch options on SSB.

The beginning-of-year weight-at-age (way) used in recent assessments have either been taken directly as the average weight-at-age from samples obtained in the fall survey, or these fall weights have been used for wa+1,y+1. However, modelling of the weights could provide the same benefits as outlined above for modelling of maturities, and in addition provide better estimates of beginning-of-year weights. That is, a cohort model can be used to predict weight at the beginning of the year based on fall weights. Such a model has been developed and the results are presented in this paper.

METHODS

Two growth (in weight) models were applied. There is some evidence of changes in weights at age over time. The growth models were fit by cohort to capture this variability.

VON BERTALANFFY (VONB) GROWTH MODEL

The Von Bertalanffy model is a commonly used (Quinn and Deriso 1999) growth model. It is usually applied to length data:

$$L(a) = L_\infty \left\{1 - e^{-k(a-a_o)}\right\}.$$

where *L(a)* is the length at age *a*, L_∞ is the asymptotic length as $a \rightarrow \infty$, *k* is a growth rate parameter, and a_o is an adjustment to account for non-zero length at $a = 0$. Growth in weight is approximated as a power function, $W = \alpha L^b$, and the Von Bertalanffy model for weight at age *a* is

$$W(a) = W_\infty \left\{1 - e^{-k(a-a_o)}\right\}^b$$

Often *b* is set at 3. The statistical model for estimation is

$$\log(w_{ay}) = \log(W_\infty) + b\log\left\{1 - e^{-k(a-a_o)}\right\} + \epsilon_{ay},$$

where ε_{ay} is assumed to be an independent error term with constant variance.

GOMPERTZ GROWTH MODEL

An alternative growth model is the Gompertz model (Quinn and Deriso 1999):

$$W(a) = W_\infty \exp\{\rho \exp(-ka)\},$$

Note that $\lim_{a \to \infty} W(a) = W_\infty$ and $\rho = \log\{W(0)/W_\infty\}$. The statistical model for estimation is

$$\log(w_{ay}) = \log(W_\infty) + \rho \exp(-ka) + \epsilon_{ay},$$

The Gompertz model did not fit the fall RV survey data that well (see **Results**) and weights were over-predicted at young ages. As a quick solution, the growth rate parameter *k* for the Gompertz model was estimated separately for young ages. Let k_{ca} be the growth rate parameter for age *a* and cohort *c*,

$$k_{ca} = \begin{array}{ll} k_c + \delta_1, a = 0{,}1 \\ k_c + \delta_2, a = 2, \\ k_c, \quad\quad a > 2. \end{array}$$

The δ adjustments were the same for all cohorts.

JANUARY 1 WEIGHTS

The age used when fitting the growth models was January 1 age + 0.75 to reflect the fall timing of the survey and weight data. January 1 weights were predicted using only the January 1 age, without the 0.75 added.

Only cohorts with at least five sampled average weights were used to estimate growth curves. Weights were inferred for other early and recent cohorts using the average (over age) weights of the adjacent three cohorts.

RESULTS

VON BERTALANFFY

Estimates of W_∞ (Fig. 1, Table 1) and *k* (Fig. 2, Table 1) were quite variable across cohorts and usually had wide confidence intervals. However, fits to the sampled weights from the 2J3KL fall surveys seemed reasonably accurate (Figs. 3 and 4) and overall there were no residual patterns in terms of age, year, or cohort (Fig. 5). The residual standard error was 0.16 with 267 degrees of freedom. However, there were some patterns in residuals with age and year (Fig. 6).

Beginning-of-year weights inferred using the Von Bertalanffy model were greater than incremented survey weights (i.e., previous age and fall of previous year) at younger ages (Fig. 7 and Table 2). For example, all model predicted weights at age 2 are greater than fall survey weights at age 1 in the previous year.

GOMPERTZ AND MODIFIED GOMPERTZ

Estimates of W_∞ (Fig. 8) and *k* (Fig. 9) were also quite variable across cohorts and usually had wide confidence intervals. The Gompertz model did not fit the sampled weights well (Fig. 10), especially for younger ages (Fig. 11). The residual standard error was 0.19 with 268 degrees of freedom. Although the Gompertz model has one parameter less than the Von Bertalanffy model and therefore one expects the fit to be worse, the residual pattern and lack of fit suggests the Von Bertalanffy model is the better choice.

Estimates of W_∞ (Fig. 12, Table 3) for the modified Gompertz model differed substantially from the Gompertz model results (Fig. 8), and were more broadly similar to the Von Bertalanffy results (Fig. 1). Similarly, estimates of *k* for the modified Gompertz (Fig. 13) and Von Bertalanffy (Fig. 2) models varied without trend whereas *k* for the Gompertz (Fig. 9) had an increasing trend.

The modified Gompertz model resulted in a better fit to the sampled survey weights (Figs. 14 and 15) and had less residual patterns (Figs. 16 and 17) than the Gompertz model (Figs. 10 and 11), although the Von Bertalanffy model still seems to fit better (Figs. 3-6). The residual standard error was 0.17 with 266 degrees of freedom, which was greater than the Von Bertalanffy model (residual standard error was 0.16) even though the modified Gompertz model has one additional parameter.

Beginning-of-year weights inferred using the modified Gompertz model were greater than incremented weights at younger ages (Figs. 18 and Table 4), similar to the Von Bertalanffy model (Fig. 7). However, there are differences in the modified Gompertz and Von Bertalanffy weights (Fig. 19), particularly at ages 1-5. Gompertz weights were greater than Von Bertalanffy weights for all years at ages 1-3 but less at age 5.

DISCUSSION

Model-predicted beginning-of-year weights were usually greater than those predicted by incrementing fall survey weights ahead one year and one age (i.e., approximating the beginning-of-year weight at age *a* and year *y* using the fall survey sampled weight at age *a-1* and year *y-1*). This is not surprising because tagging (Cadigan and Brattey 2003) and other studies (Mello and Rose 2005) indicate that most of growth occurs in the late summer and fall, and cod seem to continue to growth in length and weight between the fall survey and the beginning of the next year. A growth model incorporating seasonal variability may improve estimates of beginning-of-year weight-at-age.

The approach described here is interim and seems reasonable for estimating beginning-of-year weight-at-age for the current 2J3KL cod assessment which involves ages 1-12. However, for other purposes the approach could be problematic. For example, in yield per recruit calculations, at low fishing mortalities, one has to infer weights at ages much older than 12. The models described here may not be reliable for this purpose.

There were some patterns in residuals, where clusters (over years and ages) of residuals had the same sign indicating some model mis-specification. More flexible growth models could be explored to account for these patterns.

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Table 1. Von Bertalanffy model parameter estimates and standard errors.

	W_∞		k	
Cohort	**Estimate**	**Std. Error**	**Estimate**	**Std. Error**
1971	50.0	49.2	0.09	0.04
1972	50.0	41.4	0.08	0.03
1973	39.6	24.9	0.09	0.03
1974	50.0	30.2	0.08	0.03
1975	50.0	26.1	0.08	0.02
1976	50.0	26.7	0.08	0.02
1977	25.8	10.8	0.11	0.03
1978	10.2	2.2	0.15	0.02
1979	7.5	1.5	0.17	0.02
1980	13.5	3.3	0.14	0.02
1981	9.0	2.1	0.15	0.02
1982	9.1	2.3	0.15	0.02
1983	24.2	10.0	0.11	0.03
1984	13.3	5.7	0.14	0.03
1985	10.4	3.7	0.15	0.03
1986	8.2	2.2	0.16	0.02
1987	29.9	12.4	0.10	0.03
1988	29.5	11.4	0.11	0.03
1989	41.1	15.9	0.10	0.03
1990	42.5	17.5	0.10	0.03
1991	37.8	21.4	0.10	0.03
1992	32.6	14.9	0.11	0.03
1993	20.9	9.5	0.12	0.03
1994	10.9	2.6	0.15	0.02
1995	24.9	6.7	0.12	0.02
1996	25.5	8.1	0.12	0.02
1997	40.7	17.1	0.10	0.03
1998	50.0	24.0	0.10	0.03
1999	27.5	10.1	0.11	0.03
2000	21.1	6.2	0.12	0.02
2001	18.0	4.9	0.13	0.02
2002	21.3	7.6	0.12	0.03
2003	16.6	5.6	0.13	0.03
2004	20.1	8.9	0.12	0.03
2005	30.8	20.5	0.11	0.03
2006	19.9	12.3	0.13	0.03
2007	42.9	43.1	0.10	0.04
2008	50.0	73.6	0.10	0.04
	b		t_o	
	5.00	1.19	-1.37	0.63

Table 2. Beginning-of-year predicted weights at age from the Von Bertalanffy model.

	Age											
Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1983	0.02	0.09	0.26	0.61	0.97	1.28	1.64	2.05	3.06	3.78	5.66	7.54
1984	0.02	0.09	0.23	0.54	1.00	1.46	1.91	2.39	2.85	4.07	4.81	7.03
1985	0.02	0.08	0.24	0.47	0.95	1.47	2.03	2.68	3.32	3.81	5.22	5.95
1986	0.02	0.09	0.22	0.48	0.80	1.46	1.98	2.64	3.56	4.39	4.90	6.51
1987	0.02	0.09	0.24	0.47	0.82	1.21	2.08	2.51	3.28	4.53	5.62	6.12
1988	0.02	0.10	0.25	0.49	0.85	1.24	1.69	2.77	3.04	3.92	5.58	6.97
1989	0.02	0.07	0.25	0.50	0.85	1.38	1.72	2.21	3.51	3.55	4.55	6.68
1990	0.02	0.08	0.20	0.49	0.85	1.33	2.05	2.26	2.76	4.27	4.04	5.14
1991	0.02	0.08	0.22	0.44	0.83	1.30	1.90	2.85	2.81	3.31	5.04	4.48
1992	0.02	0.08	0.24	0.49	0.81	1.25	1.83	2.55	3.76	3.37	3.86	5.80
1993	0.02	0.08	0.24	0.53	0.90	1.34	1.72	2.41	3.24	4.76	3.93	4.38
1994	0.02	0.08	0.22	0.54	1.00	1.47	2.02	2.24	3.02	3.97	5.82	4.46
1995	0.03	0.09	0.23	0.49	1.01	1.65	2.20	2.84	2.76	3.64	4.70	6.91
1996	0.02	0.10	0.24	0.50	0.92	1.67	2.51	3.08	3.79	3.29	4.25	5.43
1997	0.02	0.09	0.27	0.52	0.93	1.52	2.54	3.55	4.09	4.86	3.80	4.85
1998	0.02	0.10	0.26	0.55	0.93	1.52	2.31	3.60	4.77	5.21	6.01	4.29
1999	0.02	0.09	0.28	0.56	0.94	1.48	2.28	3.27	4.84	6.14	6.42	7.23
2000	0.02	0.09	0.25	0.59	1.01	1.43	2.17	3.21	4.40	6.24	7.64	7.68
2001	0.02	0.09	0.26	0.55	1.07	1.63	2.00	2.97	4.28	5.66	7.77	9.23
2002	0.03	0.09	0.26	0.57	1.03	1.72	2.40	2.62	3.87	5.47	7.04	9.39
2003	0.02	0.11	0.25	0.55	1.08	1.70	2.52	3.30	3.28	4.84	6.75	8.50
2004	0.02	0.10	0.30	0.53	1.01	1.80	2.57	3.47	4.32	3.94	5.85	8.10
2005	0.02	0.11	0.28	0.62	0.96	1.63	2.75	3.64	4.53	5.43	4.60	6.87
2006	0.02	0.09	0.29	0.58	1.09	1.53	2.41	3.92	4.88	5.68	6.59	5.23
2007	0.02	0.08	0.26	0.59	1.04	1.71	2.24	3.34	5.29	6.27	6.88	7.78
2008	0.02	0.10	0.23	0.55	1.04	1.66	2.45	3.07	4.40	6.85	7.78	8.11
2009	0.02	0.10	0.27	0.50	0.98	1.62	2.41	3.30	3.99	5.55	8.56	9.39
2010	0.02	0.10	0.28	0.57	0.93	1.56	2.32	3.28	4.21	4.97	6.77	10.39
2011	0.02	0.10	0.28	0.61	1.02	1.52	2.26	3.11	4.25	5.17	6.01	8.04
2012	0.02	0.10	0.28	0.61	1.14	1.62	2.28	3.09	3.97	5.29	6.15	7.05
2013	0.02	0.10	0.28	0.60	1.15	1.88	2.35	3.20	4.00	4.87	6.36	7.12
2014	0.02	0.10	0.28	0.60	1.11	1.92	2.84	3.19	4.25	4.97	5.78	7.44
2015	0.02	0.10	0.28	0.60	1.11	1.81	2.91	4.00	4.12	5.41	5.98	6.68

Table 3. Modified Gompertz model parameter estimates and standard errors.

	W_∞		k	
Cohort	**Estimate**	**Std. Error**	**Estimate**	**Std. Error**
1971	50.0	75.2	0.11	0.07
1972	50.0	68.3	0.10	0.06
1973	34.2	29.1	0.11	0.04
1974	50.0	61.5	0.10	0.05
1975	50.0	59.4	0.10	0.05
1976	50.0	68.1	0.10	0.06
1977	50.0	121.2	0.10	0.10
1978	50.0	64.4	0.10	0.05
1979	50.0	37.9	0.10	0.03
1980	50.0	26.7	0.10	0.02
1981	50.0	18.4	0.09	0.02
1982	50.0	14.8	0.10	0.01
1983	25.3	11.8	0.14	0.03
1984	10.0	8.9	0.21	0.08
1985	43.3	15.0	0.11	0.02
1986	50.0	15.3	0.10	0.01
1987	22.9	10.6	0.14	0.03
1988	15.9	5.1	0.18	0.03
1989	31.9	12.3	0.14	0.02
1990	33.1	13.0	0.14	0.02
1991	19.1	11.6	0.16	0.04
1992	23.0	10.9	0.15	0.03
1993	33.3	12.8	0.13	0.02
1994	36.1	13.6	0.12	0.02
1995	42.4	15.7	0.12	0.02
1996	50.0	16.3	0.11	0.02
1997	36.2	15.5	0.13	0.02
1998	37.4	17.8	0.14	0.03
1999	34.7	13.6	0.13	0.02
2000	40.0	14.5	0.12	0.02
2001	50.0	15.5	0.11	0.02
2002	23.7	12.2	0.16	0.04
2003	49.6	16.5	0.11	0.02
2004	35.7	13.9	0.13	0.02
2005	18.5	11.4	0.17	0.04
2006	37.5	15.9	0.13	0.03
2007	30.2	15.8	0.14	0.04
2008	21.1	15.8	0.17	0.06
	δ_1		δ_1	
	-0.08	0.01	-0.03	0.00
	ρ			
	-6.79	0.10		

Table 4. Beginning-of-year predicted weights at age from the modified Gompertz model.

	Age											
Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1983	0.06	0.06	0.23	0.53	0.81	1.28	1.82	2.21	3.16	3.81	5.61	7.41
1984	0.04	0.07	0.20	0.54	0.81	1.19	1.82	2.51	2.95	4.12	4.81	6.95
1985	0.02	0.06	0.21	0.48	0.84	1.20	1.70	2.52	3.36	3.84	5.22	5.93
1986	0.06	0.05	0.20	0.51	0.73	1.24	1.71	2.34	3.37	4.37	4.87	6.47
1987	0.06	0.07	0.20	0.52	0.77	1.07	1.78	2.36	3.13	4.38	5.54	6.04
1988	0.04	0.07	0.22	0.52	0.85	1.14	1.52	2.45	3.15	4.07	5.56	6.86
1989	0.03	0.06	0.21	0.53	0.90	1.33	1.62	2.08	3.28	4.10	5.16	6.89
1990	0.05	0.06	0.20	0.51	0.83	1.42	1.94	2.23	2.78	4.26	5.20	6.40
1991	0.05	0.07	0.22	0.50	0.78	1.24	2.05	2.71	2.98	3.61	5.41	6.44
1992	0.04	0.07	0.25	0.57	0.84	1.16	1.78	2.76	3.62	3.87	4.57	6.70
1993	0.04	0.06	0.26	0.64	0.97	1.31	1.64	2.47	3.51	4.66	4.91	5.68
1994	0.05	0.06	0.22	0.65	1.05	1.53	1.92	2.26	3.31	4.27	5.81	6.09
1995	0.05	0.07	0.22	0.57	1.07	1.63	2.25	2.67	3.02	4.31	5.01	7.03
1996	0.06	0.07	0.23	0.58	0.97	1.65	2.39	3.09	3.56	3.93	5.45	5.70
1997	0.07	0.07	0.22	0.57	0.97	1.53	2.42	3.34	4.03	4.57	4.98	6.74
1998	0.06	0.08	0.24	0.55	0.93	1.52	2.25	3.38	4.46	5.04	5.67	6.18
1999	0.06	0.07	0.26	0.59	0.89	1.43	2.23	3.11	4.52	5.75	6.08	6.84
2000	0.05	0.08	0.26	0.64	0.94	1.35	2.09	3.10	4.10	5.83	7.16	7.11
2001	0.06	0.07	0.29	0.65	1.02	1.43	1.97	2.92	4.12	5.18	7.27	8.68
2002	0.07	0.07	0.24	0.72	1.06	1.53	2.07	2.74	3.91	5.26	6.31	8.82
2003	0.04	0.08	0.23	0.61	1.19	1.64	2.21	2.88	3.68	5.06	6.48	7.47
2004	0.07	0.07	0.27	0.57	1.01	1.85	2.39	3.07	3.86	4.77	6.35	7.75
2005	0.05	0.08	0.24	0.65	0.91	1.55	2.70	3.34	4.11	5.02	6.01	7.75
2006	0.04	0.07	0.26	0.62	1.04	1.39	2.26	3.77	4.47	5.35	6.34	7.37
2007	0.06	0.06	0.24	0.64	1.05	1.56	2.01	3.16	5.04	5.78	6.77	7.80
2008	0.05	0.07	0.21	0.60	1.01	1.64	2.26	2.80	4.23	6.50	7.23	8.35
2009	0.04	0.07	0.25	0.56	0.97	1.52	2.41	3.13	3.76	5.47	8.12	8.81
2010	0.05	0.07	0.26	0.62	0.95	1.49	2.20	3.35	4.20	4.88	6.85	9.85
2011	0.05	0.07	0.25	0.67	1.02	1.49	2.17	3.05	4.44	5.46	6.16	8.35
2012	0.05	0.07	0.25	0.66	1.11	1.56	2.19	3.03	4.09	5.66	6.91	7.58
2013	0.05	0.07	0.25	0.65	1.14	1.73	2.27	3.04	4.06	5.31	6.95	8.52
2014	0.05	0.07	0.25	0.65	1.09	1.79	2.54	3.17	4.00	5.25	6.72	8.30
2015	0.05	0.07	0.25	0.65	1.09	1.69	2.62	3.53	4.25	5.05	6.60	8.29

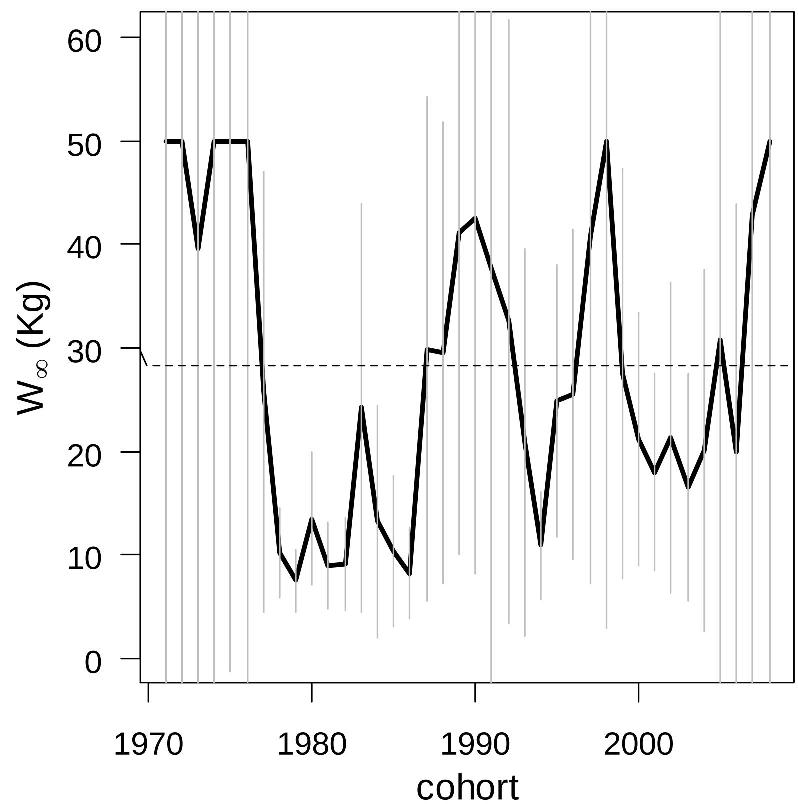

Figure 1. Solid lines connect estimates of the Von Bertalanffy W_∞ parameter for each cohort. The horizontal dashed line indicates the series average. Vertical lines indicate 95% confidence intervals.

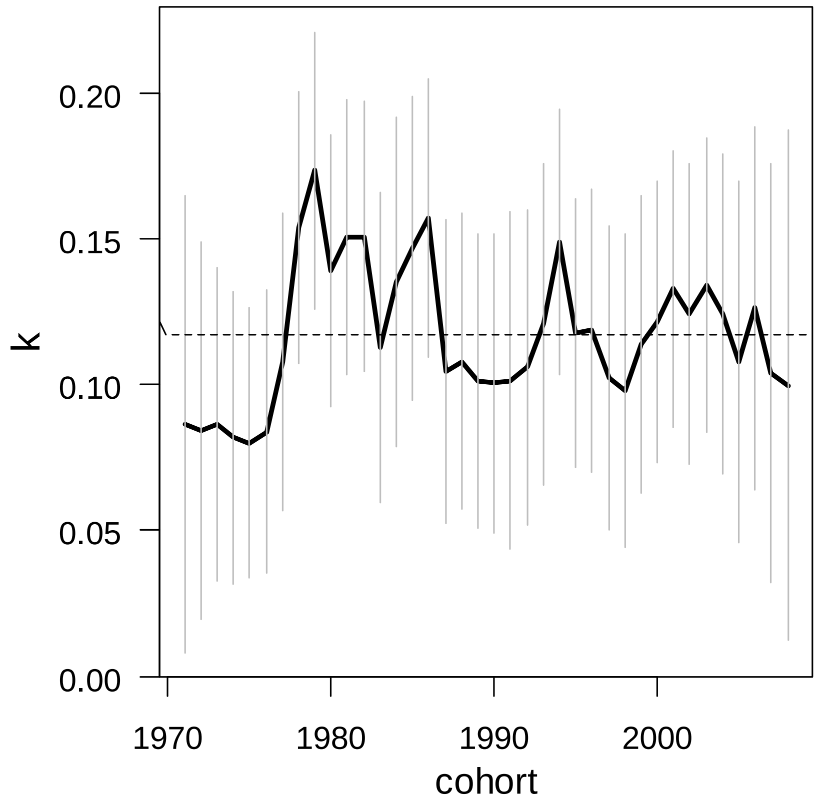

Figure 2. Solid lines connect estimates of the Von Bertalanffy k parameter for each cohort. The horizontal dashed line indicates the series average. Vertical lines indicate 95% confidence intervals.

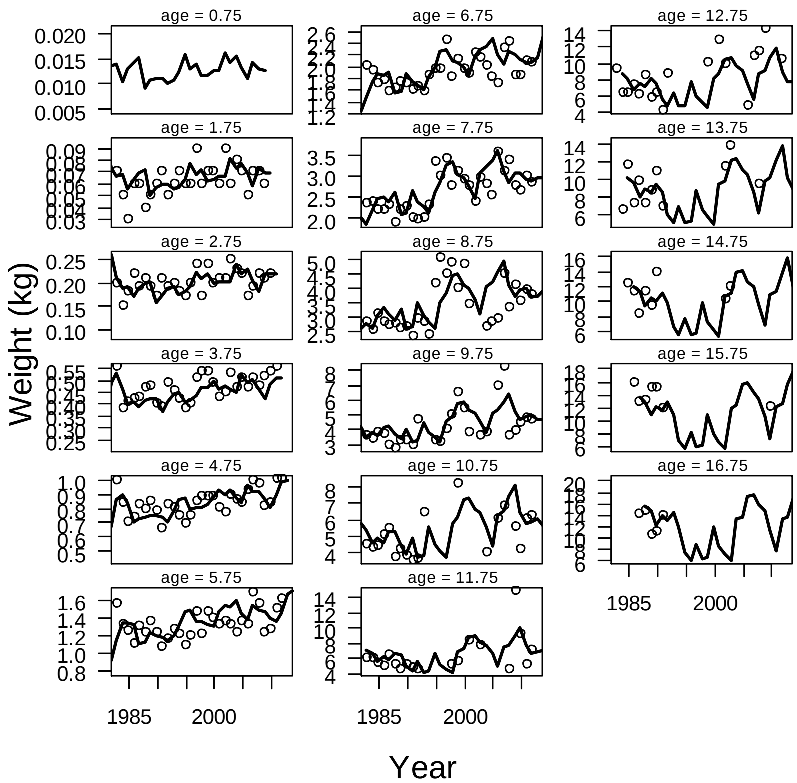

Figure 3. Von Bertalanffy model predicted versus observed survey weights. Each panel shows the results for an age class. The survey occurs during the fall and the age at the top of each panel includes the fraction of year.

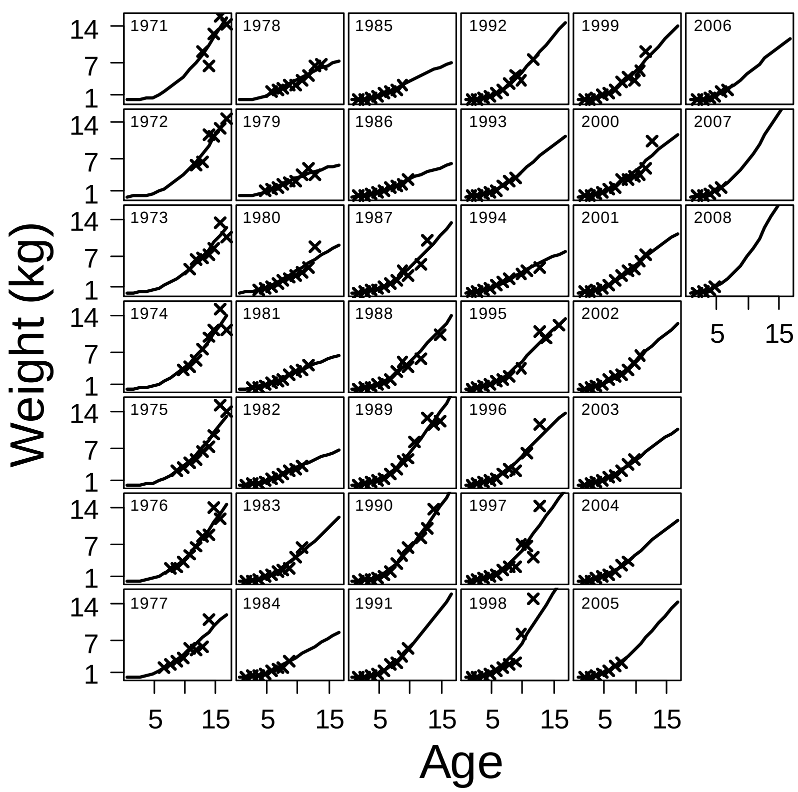

Figure 4. Von Bertalanffy model predicted versus observed survey weights, by cohort.

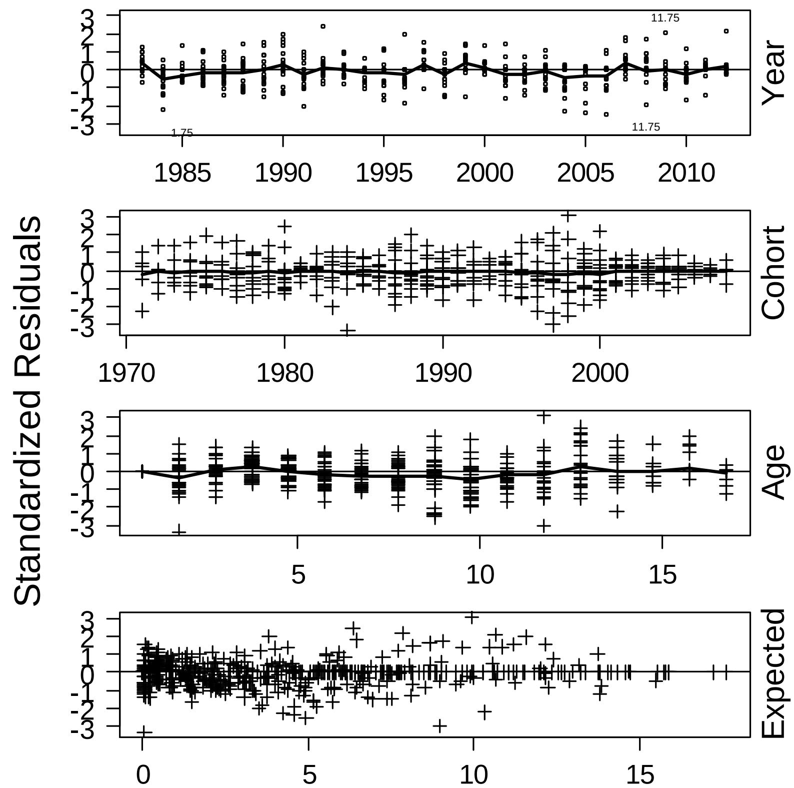

Figure 5. Standardized residuals from fitting the Von Bertalanffy model, versus year, age, cohort, and predicted value. Solid lines in the top three panels indicates the average residual each year, cohort, and age, respectively. Text in the top panel indicates ages that have residual absolute values greater than three.

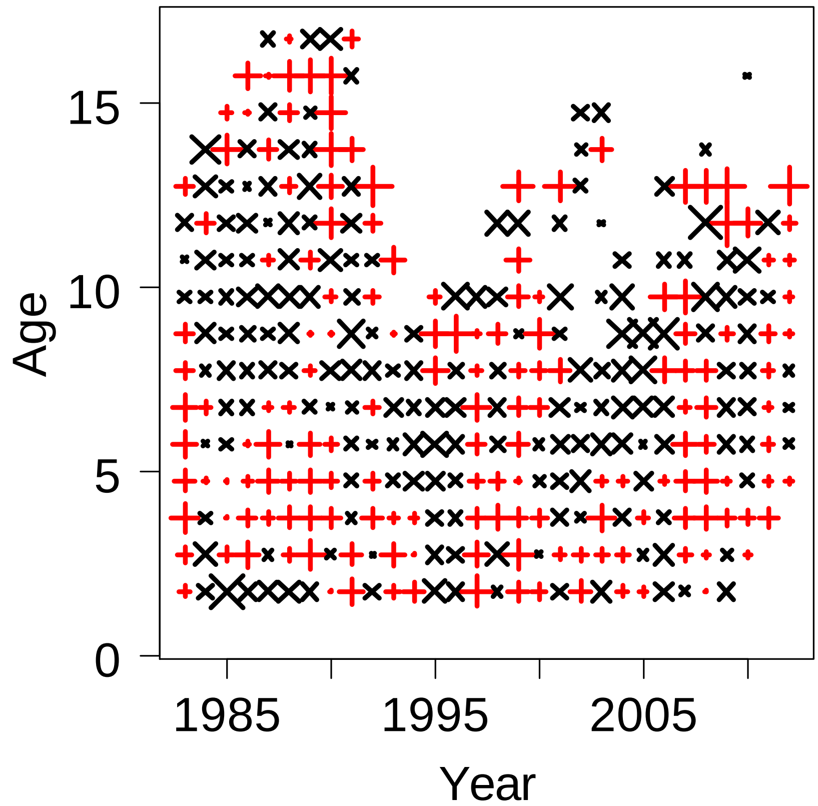

Figure 6. Matrix plot of residuals from fitting the Von Bertalanffy model. Red +'s are positive and black ×'s are negative. The sizes of plotting symbols are proportional to the absolute value of the residuals. Blanks indicate ages and years with no sampled weights or too few (in a cohort) to fit the model.

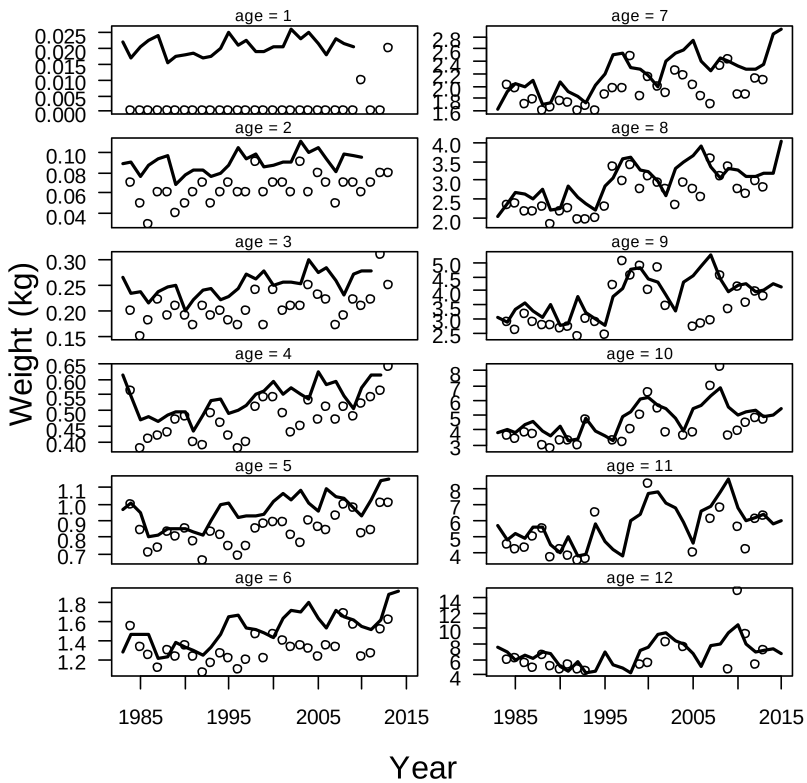

Figure 7. Von Bertalanffy model predicted beginning-of-year weights versus the sampled weights in the fall surveys, incremented by one age and year. Each panel shows the results for an age class.

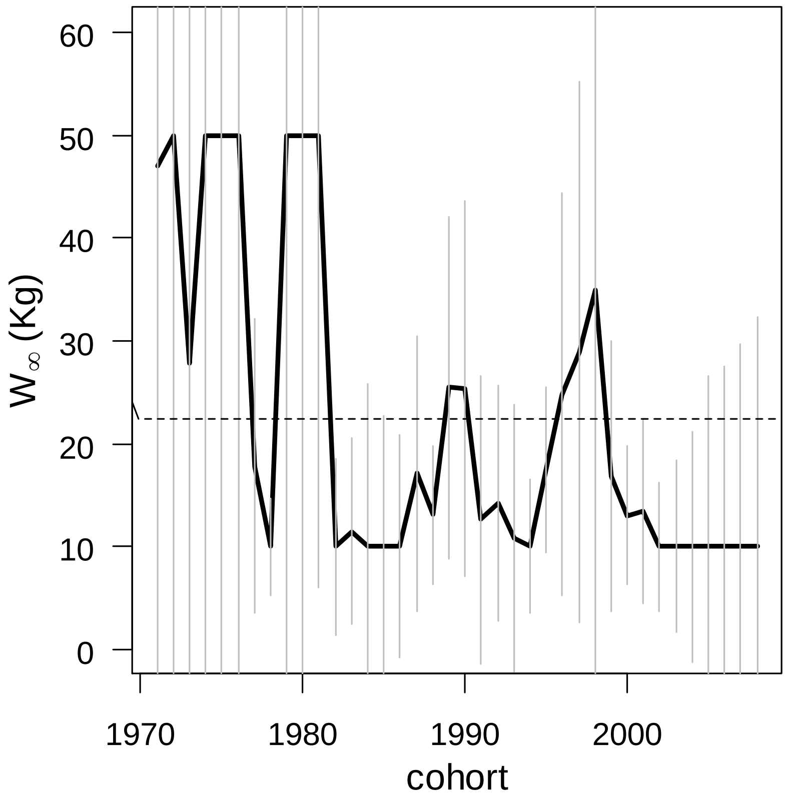

Figure 8. Solid lines connect estimates of the Gompertz W_{∞} parameter for each cohort. The horizontal dashed line indicates the series average. Vertical lines indicate 95% confidence intervals.

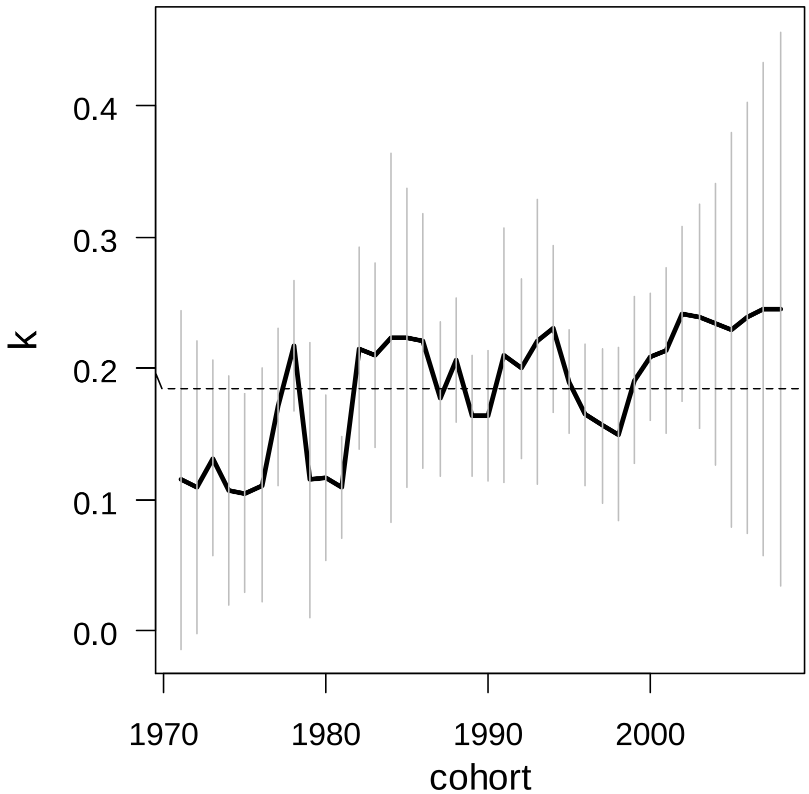

Figure 9. Solid lines connect estimates of the Gompertz k parameter for each cohort. The horizontal dashed line indicates the series average. Vertical lines indicate 95% confidence intervals.

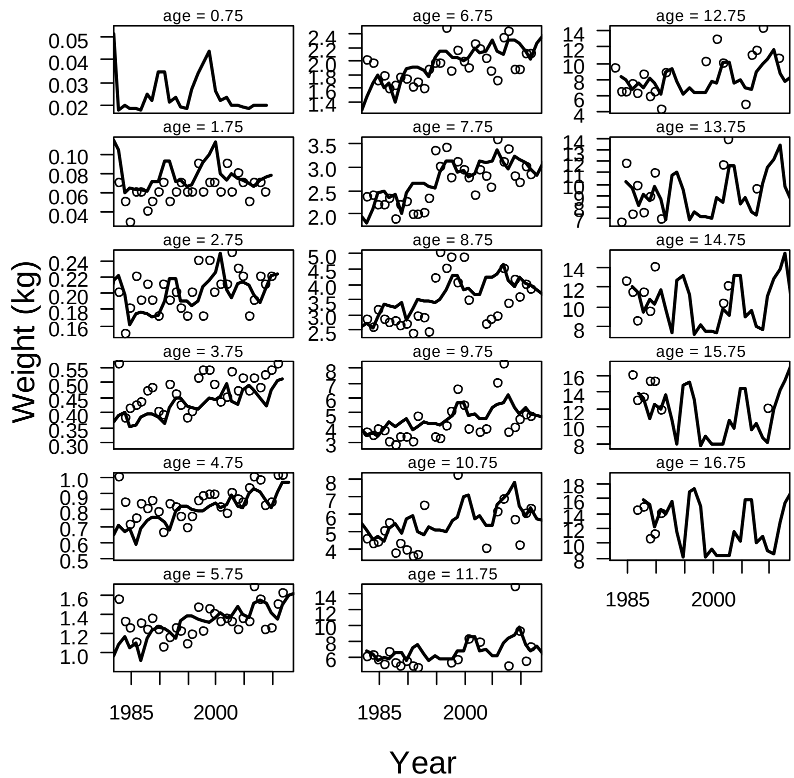

Figure 10. Gompertz model predicted versus observed survey weights. Each panel shows the results for an age class. The survey occurs during the fall and the age at the top of each panel includes the fraction of year.

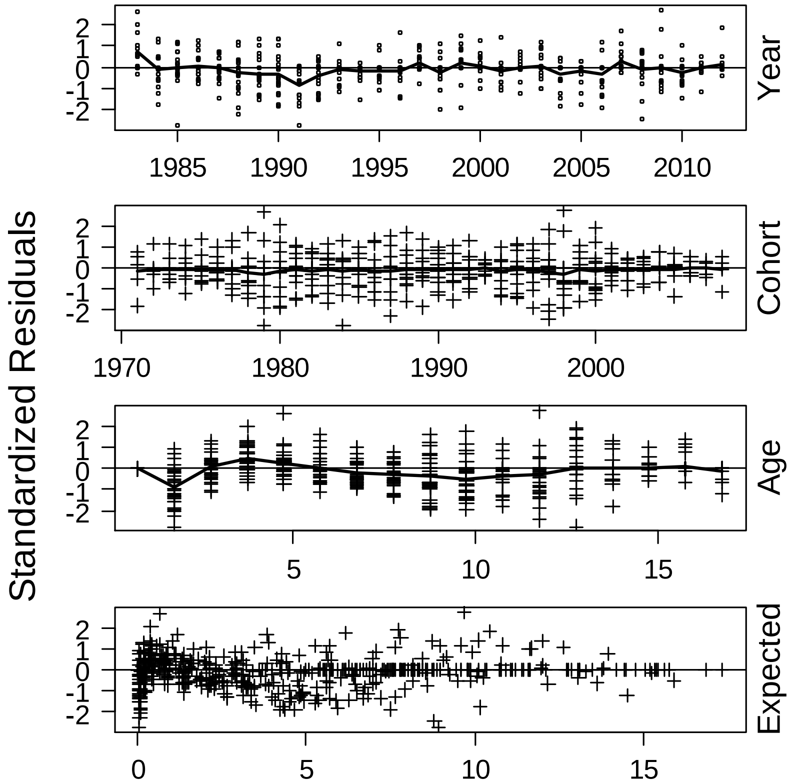

Figure 11. Standardized residuals from fitting the Gompertz model, versus year, age, cohort, and predicted value. Solid lines in the top three panels indicates the average residual each year, cohort, and age, respectively.

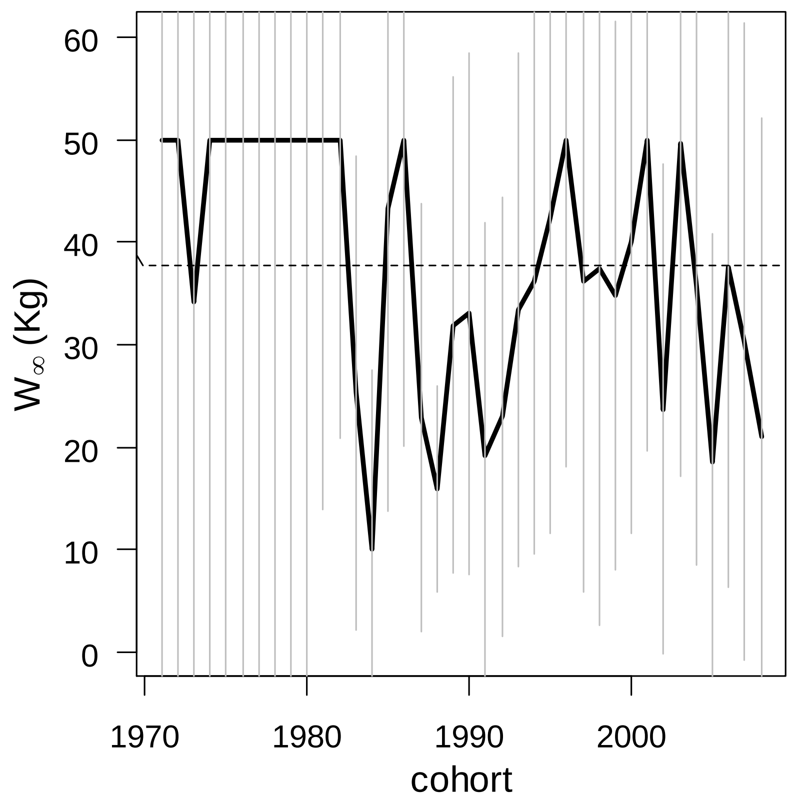

Figure 12. Solid lines connect estimates of the modified Gompertz W_∞ parameter for each cohort. The horizontal dashed line indicates the series average. Vertical lines indicate 95% confidence intervals.

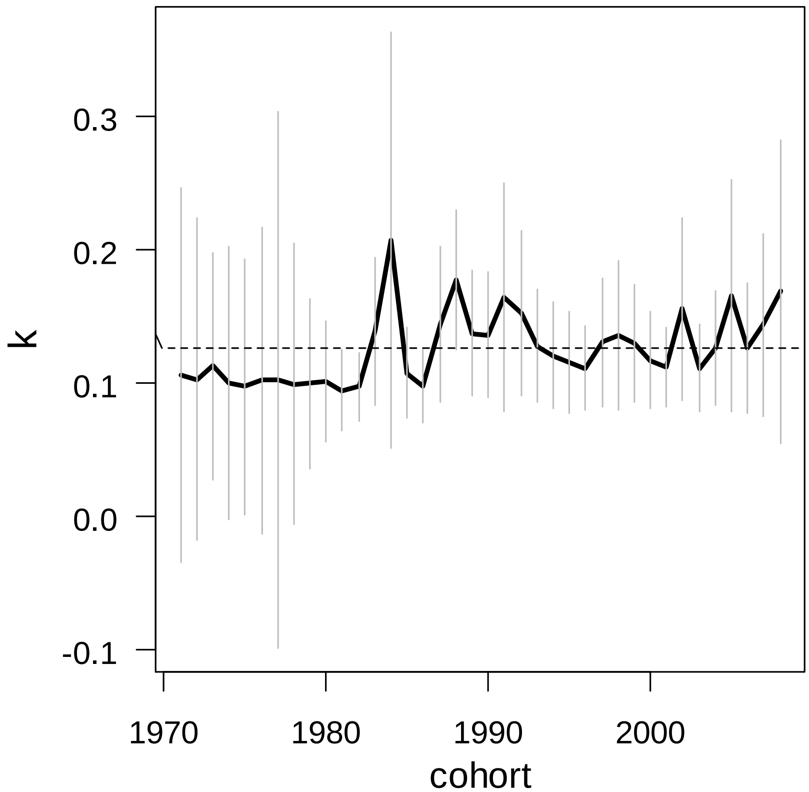

Figure 13. Solid lines connect estimates of the modified Gompertz k parameter for each cohort. The horizontal dashed line indicates the series average. Vertical lines indicate 95% confidence intervals.

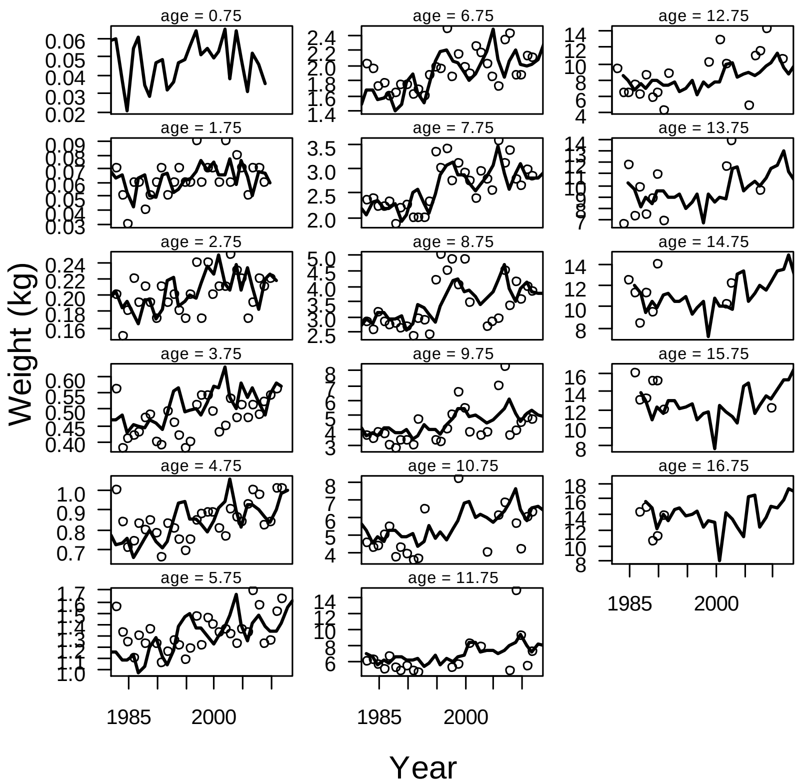

Figure 14. Modified Gompertz model predicted versus observed survey weights. Each panel shows the results for an age class. The survey occurs during the fall and the age at the top of each panel includes the fraction of year.

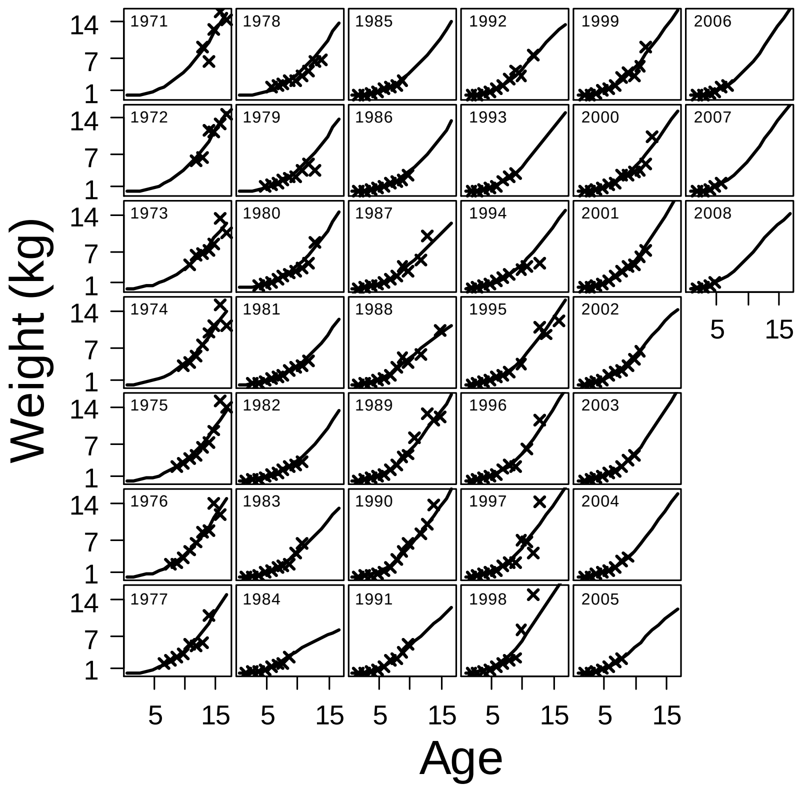

Figure 15. Modified Gompertz model predicted versus observed survey weights, by cohort.

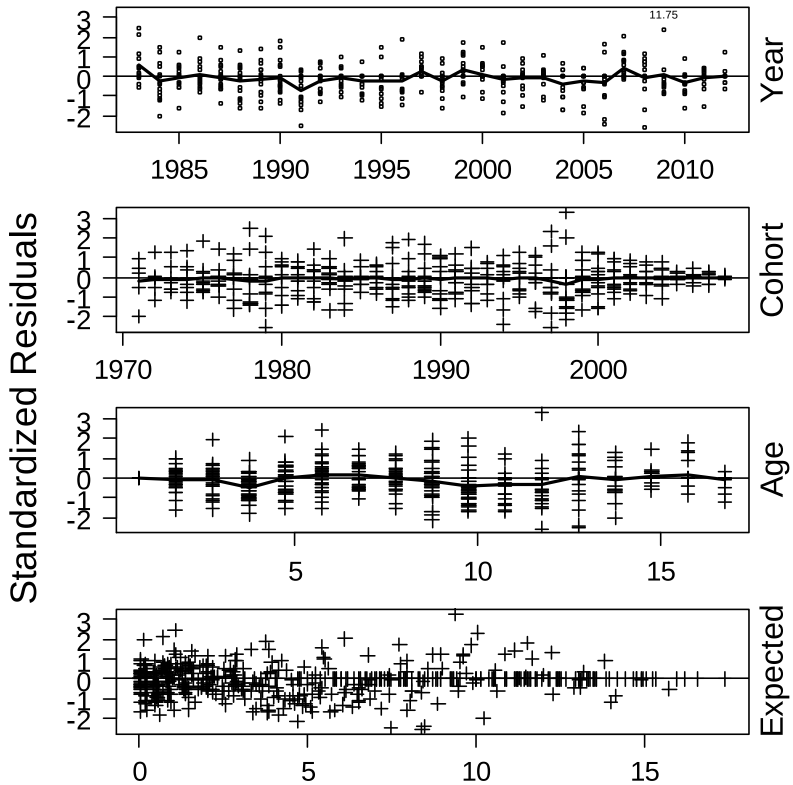

Figure 16. Standardized residuals from fitting the modified Gompertz model, versus year, age, cohort, and predicted value. Solid lines in the top three panels indicates the average residual each year, cohort, and age, respectively.

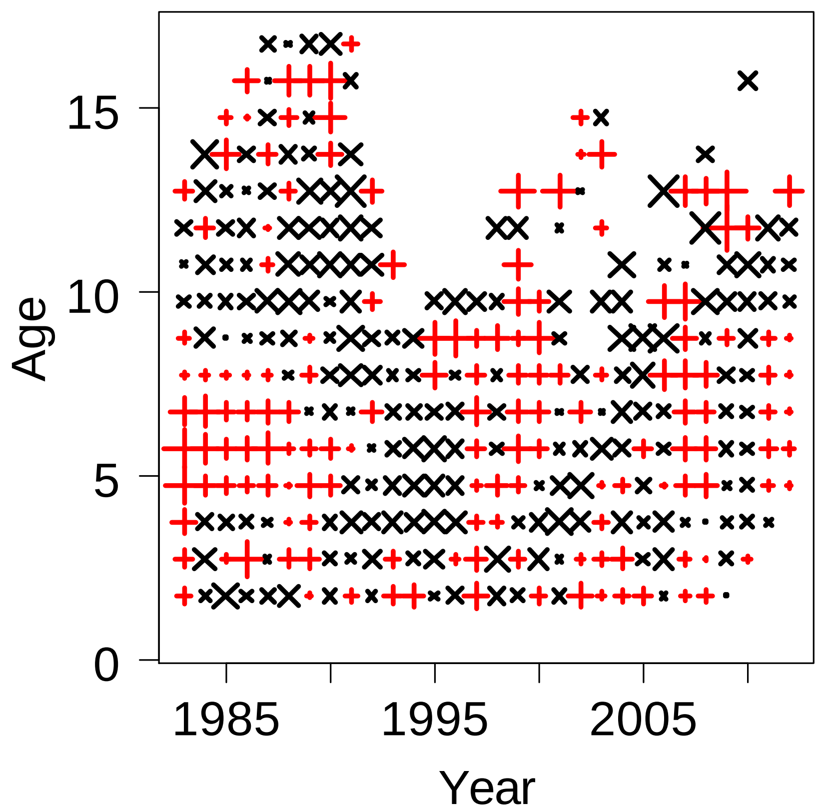

Figure 17. Matrix plot of residuals from fitting the modified Gompertz model. Red +'s are positive and black ×'s are negative. The sizes of plotting symbols are proportional to the absolute value of the residuals. Blanks indicate ages and years with no sampled weights or too few (in a cohort) to fit the model.

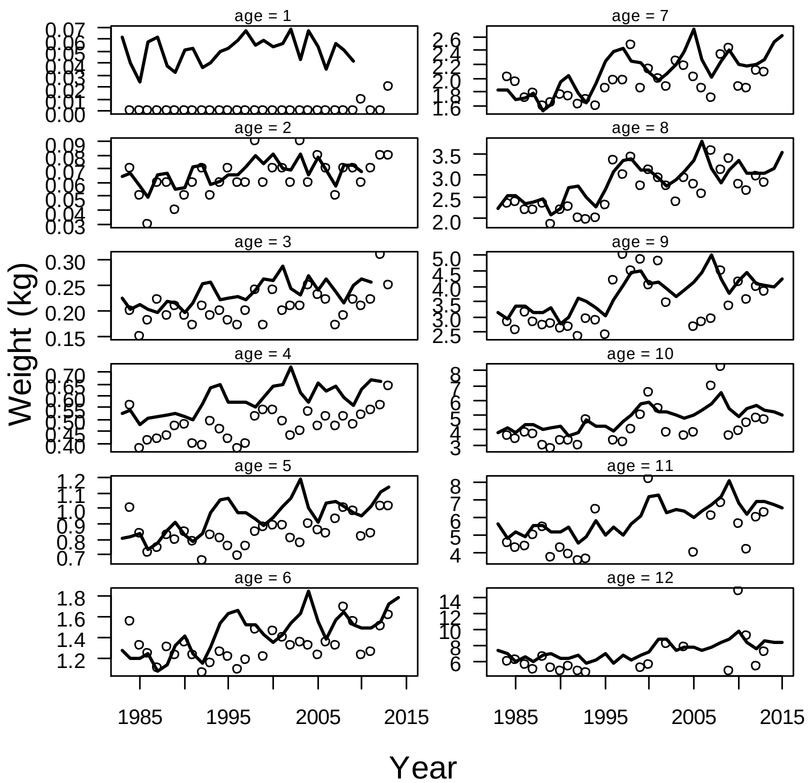

Figure 18. Modified Gompertz model predicted beginning-of-year weights versus the sampled weights in the fall surveys, incremented by one age and year. Each panel shows the results for an age class.

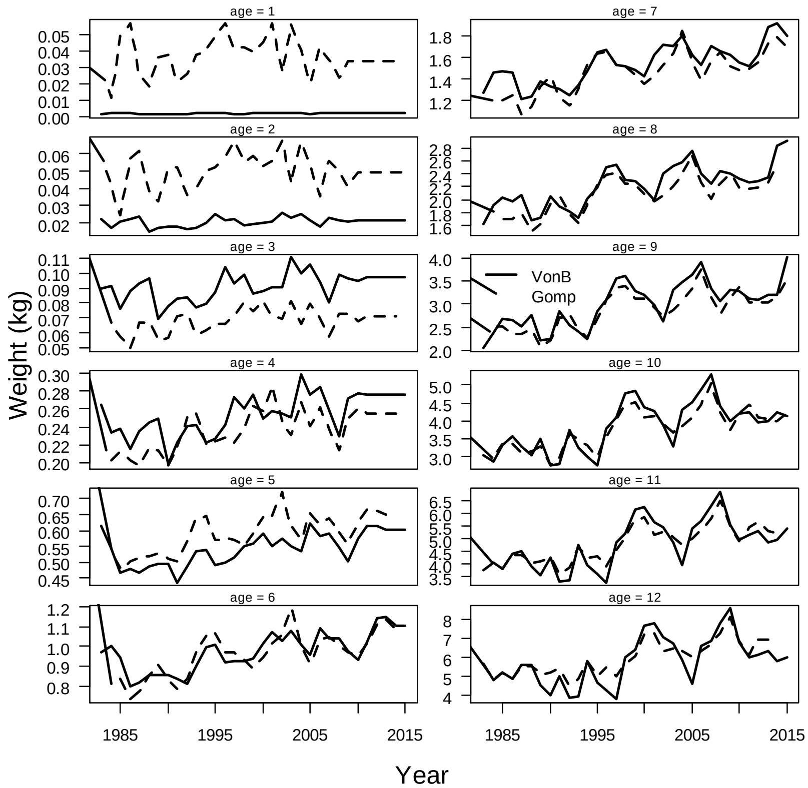

Figure 19. A comparison of modified Gompertz and Von Bertalanffy model predicted beginning-of-year weights. Each panel shows the results for an age class.