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APL Programs for Stock Assessment

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coordinated by

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ABSTRACT

Rivard, D. 1980. APL programs for stock assessment. Can. Tech. Rep. Fish. Aquat. Sci. 953: 103 p.

This technical report supplies basic information on FISH, an APL-Package designed to provide assistance in stock assessment. The Package encompasses APL-functions for sequential population analysis, yield-per-recruit analysis and catch projections, and can also be used to determine the "sensitivity" of the calculated quantities with respect to certain parameters of interest in management. The calculations of the cohort analysis are extended so as to provide estimates of the surplus production and net production from a given stock. A new method is introduced for estimating survivors in the current year: the method combines catch-at-age data and the research vessel abundance index-at-age, and produces estimates of survivors in the current year, as well as estimates of the corresponding variances. Each APL-function is discussed, its use is explained and the description is accompanied by numerical examples.

Key words: stock assessment, sequential population analysis, cohort analysis, yield-per-recruit analysis, sensitivity analysis, surplus production, fisheries management.

RÉSUMÉ

Rivard, D. 1980. APL programs for stock assessment. Can. Tech. Rep. Fish. Aquat. Sci. 953: 103 p.

Nous présentons, dans ce rapport technique, la documentation de base pour FISH, une banque de programmes écrits en APL pour assister le biologiste lors de l'évaluation des stocks. Cette série de programmes permet l'analyse séquentielle des populations, l'analyse de la production par recrue et la projection des prises; elle peut aussi être utilisée pour évaluer la "sensibilité" des valeurs ainsi calculées quant à certains paramètres qui suscitent un intérêt particulier pour la gestion des pêches. L'analyse de cohorte est étendue pour produire une estimation de la production excédentaire et de la production nette pour un stock donné. Une nouvelle méthode est introduite pour évaluer le nombre de survivants à la fin de l'année courante: cette méthode utilise les données de distribution des prises par groupe d'âge, ainsi qu'un indice d'abondance par groupe d'âge établi à partir des bateaux de recherche, et évalue le nombre de survivants pour l'année courante, ainsi que la variance correspondante. Chacun des programmes est discuté; son mode d'utilisation est exposé et s'accompagne d'exemples numériques.

Mots-clés: évaluation des stocks, analyse séquentielle des populations, production par recrue, analyse de cohorte, analyse de sensibilité, production excédentaire, gestion des pêches.

Introduction.

This document gathers basic information on FISH, an APL-package designed to provide assistance in stock assessment. It describes APL-functions which can be used to perform sequential population analysis, yield-per-recruit analyses and catch projections. All functions are designed to be used interactively; the user is prompted to input data, the appropriate checks are immediately performed and error- or warning-messages are printed to assist the user in the execution of a function.

This software is based on APL-functions written in the Resource Branch, Maritimes Region, by various members of the Fisheries Systems and Data Processing Group. The original functions have been described by W.H. Marshall, in CAFSAC Research Document 78/12. Original functions have been modified by Rivard and Doubleday (1979a) so as to include a sensitivity analysis in certain population parameters. That revision included also a revision of the method which is used to determine F_{max} and $F_{0.1}$ in the yield-per-recruit analysis and the introduction of a new function for the calculation of yield-per-recruit by the method of Beverton and Holt.

In the present report, the information has been updated to accomodate the latest modifications brought to the package. The major modifications include a new function for the evaluation of total and net production from the cohort analysis, the introduction of a new method for the estimation of survivors in a given year from catch-at-age data and from age-specific research indices, as well as new functions to calculate weighted and unweighted fishing mortalities and to analyse catch rates. Since the number of new functions has increased considerably in the last year, it has been necessary to use the file system in order to avoid the multiplication of workspaces. A series of utility-functions have been written to retrieve the desired functions and to assist the user in preparing his active workspace.

Finally, we would like to thank A. Boudriault and C. Clements, who contributed to the development of the APL-functions.

General Instructions.

The workspace 2719067 FISH contains the utility-functions needed to access the APL-functions described herein. These functions are currently implemented on the I.P. Sharp Time-Sharing System in Toronto. The sign on procedure for the Sharp APL System is described in "An Introduction to Sharp APL", from I.P. Sharp Assoc. Ltd* (see also Appendix B).

In order to load the workspace FISH, type

)LOAD 2719067 FISH

You can now access any of the APL-functions described herein by typing

FILEAFETCH '2719067 *file name*'

where *file name* is one of the file names listed in Table I. Note that one file usually contains more than one APL-function and that all APL-functions stored in a given file are activated when the FILEAFETCH command is executed. Then in order to execute the desired APL-function, type in the name of the function. You will be prompted by a message which indicates the type of data to be entered. As you input data, proper checks are performed to ensure that you enter data in an acceptable format. The verifications performed by the program concern the length of input vectors and the range of acceptability of certain values. When the workspace FISH is loaded, a page width of 120 characters is automatically defined. If a smaller or larger page width is desired, simply type in

□ PW+xxxx

where xxxx is the maximum number of characters per line.

If you want to abort, for some reason, the execution of a function after you have been prompted to input characters, you have to depress the following keys:

O BACKSPACE
U BACKSPACE
T BACKSPACE

In order to escape from an input loop after you have been prompted to input numeric data, you have to type in → . Finally, you can terminate your APL session by typing

)OFF

The computer will respond by the usual sign off messages and you can hang up and turn off your terminal.

Sensitivity Analysis.

A "sensitivity analysis" represents a way to quantify the magnitude of the change in the response, say h , of a given system due to small perturbations in the values of its parameters.

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File name	APL-functions stored in this file
PFΔYIELD	YIELD YIE BEVHOLT
PFΔPROJECT	MPROJECT MPROJ MPROJOUT CALCC
PFΔPROD	COHORT VPA FFULR GRAPHΔPRODUCTION GRAPHΔNETΔPROD MEANF WEIGHTEDΔFMORT
PFΔPALOHEIM	PALOHEIMO
PFΔVONB	VONB ΔFN
PFΔSURVIVOR	SURVIVOR SURVIVORAIN T KΔCONST ESTΔSURVIVORS ESTΔVARΔSURV WEIGHTEDΔS DIAGΔSUM ANALΔVAR
Workspace FISH	DAT HEADER BELL BECOMES INPUTAMAT FILEAFETCH FETCHΔONLY OUT RESETΔINDICES AAPLUS PRANOΔSMOOTH TABLE PΔGRAPHICS

Table I. List of file names and description of file components.

In this APL-package, a sensitivity analysis can be performed on certain parameters of interest in management.

In their simplest form, sensitivity coefficients are calculated by taking the first partial derivative of a dependent variable with respect to a parameter. In other words, if a mathematical model is given by $h(b_1, b_2, \dots)$ where the b_i are the parameters, the sensitivity of h with respect to a parameter b_i is calculated as

$$x_i = \frac{\partial h}{\partial b_i} . \quad [1]$$

In practice, x_i is often approximated as

$$x_i = \frac{h(b_i + \delta) - h(b_i)}{\delta} , \quad [2]$$

where δ is a small quantity (e.g. $\delta = 0.001$) which represents a small perturbation of the parameter. For our purposes, it is convenient to define a different coefficient which is less influenced by the order of magnitude of both the dependent variable, h , and the parameters, b_i .

Relative sensitivity coefficients. Let d be a small relative change in the deterministic value b_i of one parameter, then the perturbed value of the parameter can be written as

$$b'_i = b_i + db_i = b_i(1 + d) . \quad [3]$$

Note that (db_i) represents the small perturbation which is applied to b_i , while the quantity d is really an expression which indicates the magnitude of the relative change in the parameter value ($d = (\Delta b_i)/b_i$). Also the change in the

response of the dependent variable h which is caused by the small perturbation of the parameter b_i can be expressed as

$$\Delta h = h(b_i + db_i) - h(b_i) . \quad [4]$$

The relative change in the response of the dependent variable h can simply be expressed as $\Delta h/h$. We will now write the relative sensitivity coefficients as

$$XX_i = \frac{\frac{\partial h}{\partial b_i}}{h} \approx \frac{\Delta h}{h} \times \frac{b_i}{\Delta b_i} = \frac{\Delta h}{hd} . \quad [5]$$

Therefore XX_i represents the ratio of the relative changes of the dependent variable h to a given relative change in the parameter b_i . Note that the relative sensitivities are pure numbers (i.e. no units can be attributed to them).

Analysis of relative sensitivities. The relative sensitivities can be used to analyse the response of our model to small perturbations of the parameters. A negative value for XX_i indicates that a decrease (increase) of the parameter value will give rise to an increase (decrease) of the dependent variable h . On the contrary, a positive sign indicates that an increase (decrease) of the parameter value will give rise to an increase (decrease) of the dependent variable h . A value of zero for XX_i indicates that the dependent variable is not influenced by the changes in the value of the parameter b_i . A value of one (1) for $|XX_i|$ indicates that a 1% change in the parameter value is accompanied by a 1% change in the output variable h . This correspondence is approximately true for small changes of the parameter value.

For larger changes of the parameter value, the exactitude of this correspondence depends upon the degree of nonlinearity of the model. Similarly, a value between 0 and 1 for $|XX_1|$ indicates that the relative change in the calculated variable is smaller than the relative change in the parameter. A value greater than 1 for $|XX_1|$ indicates that the relative change in the output variable is greater than the relative change in the parameter. For example, $|XX_1| = 2$ indicates that a 1% change in the parameter value gives a 2% change in the dependent variable h . Here again, this correspondence is approximately true for small changes of the parameter value.

When the relative sensitivities are calculated for the different parameters, we can draw certain conclusions regarding the relative importance of each parameter on the response or output of the model. A plot of the changes in the sensitivities, which changes can be observed when different values of the parameters are considered, would also provide a set of conditions upon which the response of the model h is highly dependent upon a given parameter.

Calculation of relative sensitivities. In our APL-programs, the relative sensitivities are approximated by finite difference (one-sided difference method). In fact, the relative sensitivities are calculated from equation [5], by using $d = 0.001$.

Sensitivity coefficients can be calculated for each parameter which is entered as input data. In many instances, however, the input information is taking the form of a vector. Partial recruitment figures, age-specific weights and natural mortalities are examples of such vectors. In those cases, sensitivities are calculated by assuming a small relative perturbation for each element of the input vector. For example, given a vector $[e_1 \ e_2]$, the relative sensitivities are calculated as

$$XX = \frac{h([e_1 + de_1 \ e_2]) - h([e_1 \ e_2])}{h \ d} . [6]$$

The extension to higher dimensions is straight forward. This procedure permits an assessment of the global effect of the input information on the response of the model and eliminates the effects of the order of magnitude of each element of the vector on the sensitivities. Even though this procedure does not permit to assess the sensitivity associated with each element of the input vector, it does provide information on the relative importance of this input vector with respect to other parameters in controlling the response of the model.

Notation.

In order to avoid confusion between time-intervals and a reference point in time, the following notation is used when specifying subscripts: greek letters are used to identify time-intervals while roman letters are used to identify a reference point in time. For example, i and t will refer to a given age and a given time, respectively, while ι and τ will refer to the intervals between age i and age $i+1$, and between time t and time $t+1$, respectively.

DESCRIPTION OF APL FUNCTIONS.

1. Utility Functions.

The following functions have been written to assist the user in preparing the data before executing certain APL-functions. For example, the functions VPA, COHORT and PALOHEIMO require that a matrix of data be created before calling these functions. The functions INPUTAMAT and BECOMES have been written to prepare and to update such matrices. Other utility functions are also available to output vectors or matrices with the appropriate heading. All utility functions described herein are kept in workspace 2719067 FISH.

Input-Output Functions.

INPUTAMAT

The function *INPUTAMAT* is used to create a matrix of data. The user is prompted to enter data column-wise. Checks are performed at each step so as to ensure that entries are non-negative and that the dimensions of each vector agree with the dimensions of the initial vector. The resulting data-matrix, which is stored in the global variable MAT, is printed in output.

In order to enter the function, type in *INPUTAMAT*. Then you will be prompted for input as follows:

ENTER FIRST YEAR AND YOUNGEST AGE-GROUP?

-enter two numbers representing the first year and the youngest age-group for your data-matrix. These values are used for headings only; dummy values, say 1 1, may be used if desired.

ERRORS: -you must enter two non-negative integers.

ENTER AGE-SPECIFIC DATA FOR EACH YEAR (ONE YEAR PER LINE), STARTING WITH YOUNGEST AGE.

TERMINATE ENTRY WITH A SCALAR AFTER NEXT PROMPT.

-enter one vector of data for each column of the matrix. Each vector entered is followed by a 'CARRIAGE RETURN'. Then wait for the prompt-signal before entering the next vector. When the last vector has been entered, depress the 'CARRIAGE RETURN' key, wait for the prompt-signal and type in a scalar (say 1), followed by a 'CARRIAGE RETURN'.

ERRORS: -dimensions must agree with the first vector entered;
-all entries must be non-negative.

ALIGN PAPER

-adjust paper for printing the data-matrix and depress the 'CARRIAGE RETURN' key to start.

The data-matrix will be printed at your terminal, with a short message indicating that the data is stored in the global variable MAT. You can now assign a name to the matrix, say NAME, by typing

NAME←MAT

You must assign a name to your newly created matrix if you are planning to create other matrices through *INPUTAMAT*. Otherwise, the information which is kept in MAT will be overwritten.

AA YY BECOMES XXXX

The diadic function BECOMES is used to correct erroneous entries of the matrix MAT. The user identifies the erroneous entry by specifying the age-group (AA) and the year (YY); he then assigns a new value (XXXX) to this location.

ERRORS: -indices must refer to an existing entry of the matrix MAT;
-the new value XXXX must be non-negative.

RESETAINDICES MMMM

The monadic function RESETAINDICES is used to reset the indices of a matrix MMMM before using the output function OUT. The user is prompted to input the following information:

ENTER FIRST YEAR AND YOUNGEST AGE-GROUP.

-enter two numbers representing the first year and the youngest age-group for your data matrix MMMM.

ERRORS: -you must enter two non-negative numbers.

The function RESETAINDICES will assign the proper values to the global variables YR and AG, which variables are used by OUT for producing the proper labels in output.

xx OUT mmmm

The diadic function OUT is used to output the matrix or the vector mmmm with xx decimal digits. Prior to using the function OUT, it may be necessary to define the variables YR and AG by using the function RESETAINDICES (YR and AG are defined during the execution of COHORT, INPUTAMAT, VPA or MPROJECT; if these functions have been executed, it is not necessary to redefine YR and AG through the use of the function RESETAINDICES). In addition, you can assign a title to the output table by typing

TIT←'any title for the table'

The matrix mmmm is printed with a format which considers the page width, the number of digits before the decimal point and the sign of any element of the matrix or vector. The current date is also printed in a standard format.

BELL

The function *BELL* is used to turn off the bell signal. But note that the bell signal is re-activated when an error is encountered. In order to turn off the bell signal again, simply type *BELL*.

(a,b) AΔPLUS mmmm

The diadic function *AΔPLUS* is used to sum the values contained in a given matrix *mmmm*, from age-group *a* to the oldest age-group represented in the matrix. The value *b* is used to indicate that the summation is to be taken *b-a+1* times, i.e. for each age-group between *a* and *b*. For example, if *a=3* and *b=5*, the function *AΔPLUS* will calculate, when applied to the matrix of population numbers, the 3⁺, 4⁺ and 5⁺ (respectively) in the population, for every year specified in the matrix of population numbers. The results are printed in output and take the form of a vector, when *a=b*, or of a matrix, when *b>a*.

The function *AΔPLUS* is often used in conjunction with the function *OUT* for printing totals on the bottom line of a given table.

A AN EXAMPLE

INPUTMAT
ENTER FIRST YEAR AND YOUNGEST AGE GROUP
□:
1965 2
ENTER AGE-SPECIFIC DATA FOR EACH YEAR(ONE YEAR PER LINE), STARTING WITH YOUNGEST AGE.
TERMINATE ENTRY WITH A SCALAR AFTER NEXT PROMPT
□:
337 144 61 26 11 5 4
□:
104 115 63 30 14 6 2
□:
69 107 73 32
MUST BE NON-NEGATIVE AND DIMENSIONS MUST AGREE WITH FIRST VECTOR
□:
69 107 73 32 14 6 3
□:
69 111 109 60 25 10 4
□:
69 111 113 89 48 18 7
□:
1
ALIGN PAPER

DATA STORED IN MAT 29/11/79

	1965	1966	1967	1968	1969
2	337	104	69	69	69
3	144	115	107	111	111
4	61	63	73	109	113
5	26	30	32	60	89
6	11	14	14	25	48
7	5	6	6	10	18
8	4	2	3	4	7

3 2100 BECOMES 104
INDICES MUST BE PRESENT IN MATRIX. FORMAT IS: AGE YEAR BECOMES NUMBER

3 1966 BECOMES 110
8 1968 BECOMES 6
2 1969 BECOMES 114

CATCHMATRIX+MAT
TIT+'CATCH MATRIX'
0 OUT CATCHMATRIX

CATCH MATRIX 29/11/79

	1965	1966	1967	1968	1969
2	337	104	69	69	114
3	144	110	107	111	111
4	61	63	73	109	113
5	26	30	32	60	89
6	11	14	14	25	48
7	5	6	6	10	18
8	4	2	3	6	7

0 OUT CATCHAMATRIX,[1] (2,4) ADPLUS CATCHAMATRIX

CATCH MATRIX 9/6/80

	1965	1966	1967	1968	1969
2	337	104	69	69	114
3	144	110	107	111	111
4	61	63	73	109	113
5	26	30	32	60	89
6	11	14	14	25	48
7	5	6	6	10	18
8	4	2	3	6	7
2+	588	329	304	390	500
3+	251	225	235	321	386
4+	107	115	128	210	275

(2,2) ADPLUS CATCHAMATRIX
588 329 304 390 500

2 3 ADPLUS CATCHAMATRIX
588 329 304 390 500
251 225 235 321 386

RESET INDICES CATCHAMATRIX
ENTER FIRST YEAR AND YOUNGEST AGE GROUP
□:
1970 6

TIT←'NEW CATCH MATRIX'

0 OUT CATCHAMATRIX

NEW CATCH MATRIX 29/11/79

	1970	1971	1972	1973	1974
6	337	104	69	69	114
7	144	110	107	111	111
8	61	63	73	109	113
9	26	30	32	60	89
10	11	14	14	25	48
11	5	6	6	10	18
12	4	2	3	6	7

2. Sequential Population Analysis by Cohort Analysis (COHORT).

This program calculates population numbers at age and the instantaneous rate of fishing mortality at age by using the method of cohort analysis described in Pope (1972). In other words, given historical information on catch-at-age and given an estimate of the instantaneous rate of natural mortality as well as an estimate of instantaneous fishing mortality for the last year of historical catch and for the last age-groups, the program reconstructs the age-composition of the stock and estimates the corresponding rates of fishing mortality for a specified number of years. In addition, the program calculates sensitivity coefficients for population numbers and for instantaneous fishing mortalities with respect to both natural mortality and initial fishing mortalities. Sensitivity coefficients are also calculated for the estimated number of recruits with respect to individual catches.

If information on weight-at-age is available, the program also calculates surplus and net production from the stock. Production is assumed to have two components: an exogenous component, recruitment, and an endogenous component, production due to somatic growth. Annual net production is calculated as the total production diminished by the losses of biomass due to fishing and natural deaths.

In addition to the above-mentioned information, the program calculates the mean age and the mean weight of individuals in the catch.

Input Information

The input information required by COHORT consists of:

t_0 : first year of prediction;

t_f : final year of prediction;

b : age of youngest age-group;

m : age of oldest age-group;

$C_{i,\tau}$: catch between time t and $t+1$ and for age i to $i+1$. Note that τ refers to the period $t, t+1$, while i refers to age-interval $i, i+1$.

F_{i,τ_f} : ($i = b, \dots, m$) : the instantaneous rate of fishing mortality for each age-category in the final year (t_f).

$F_{\mu,\tau}$: ($t = t_0, \dots, t_f$) : the instantaneous rate of fishing mortality for the oldest age-group in each year.

M_i : ($i = b, \dots, m$) : the instantaneous rate of natural mortality for age i (those are assumed to be constant for all years considered).

$W_{i+5,t+5}$: ($i = b, \dots, m ; t = t_0, \dots, t_f$) weight-at-age, expressed in kilograms. Those are required only when population biomass is to be calculated (and production). The weights are taken as mid-year estimates.

Algorithm.

The calculations are performed in the following manner:

Population numbers: $N_{i,t}$

The program calculates population numbers by using three different equations.

A) for the first ($m-b-1$) age-groups in the final year (i.e. for $i = b, \dots, m-1$ and $t = t_f$), the $N_{i,t}$ are calculated as

$$N_{i,t_f} = C_{i,\tau_f} Z_{i,\tau_f} / F_{i,\tau_f} (1 - e^{-Z_{i,\tau_f}}) \quad [2.1]$$

$$\text{where } Z_{i,\tau_f} = F_{i,\tau_f} - M_i$$

B) for the oldest age-group in each year (i.e. for $i = m$ and $t = t_0, \dots, t_f$), the population numbers are calculated as [2.1] if fishing is not complete for the oldest age-group and as

$$N_{m,t} = C_{\mu,\tau} Z_{\mu,\tau} / F_{\mu,\tau}, \quad [2.2]$$

if the fishing is complete for the oldest age-group.

Fishing is considered to be incomplete if the last row of the catch table (i.e. corresponding to the oldest age-group) includes only the catches from the oldest age-group but not the catches of older fish from the same cohort. If the catches of older fish have been added to the last row of the catch table or if there are no older fish, then fishing should be considered to be complete.

Note that μ refers to the age-interval $m, m+1$, while τ_f refers to the time-interval t_f, t_f+1 .

C) for the remainder of the table (i.e. for $i = b, \dots, m-1$ and $t = t_0, \dots, t_f-1$), population numbers are calculated by using the approximation

$$N_{i,t} = C_{i,\tau} e^{M_i/2} + N_{i+1,t+1} e^{M_i}. \quad [2.3]$$

Finally, the total population numbers are calculated for each year as

$$N_{\cdot,t} = \sum_{i=b}^m N_{i,t} \quad (t = t_0, \dots, t_f). \quad [2.4]$$

Population biomass: $B_{i,t}$.

An 'average biomass' estimate and an estimate for 'population biomass at the beginning of the year' are calculated from the estimated population numbers and from the weight-at-age data. The 'average biomass' is calculated as

$$\bar{B}_{i,\tau} = W_{i+.5,t+.5} N_{i,t} (1 - e^{-Z_{i,\tau}}) / Z_{i,\tau}.$$

Population biomass at the beginning of the year is calculated as

$$B_{i,t} = W_{i,t} N_{i,t},$$

where $W_{i,t}$ represents weight-at-age corrected to the beginning of the year. The $W_{i,t}$ are approximated by

$$W_{i,t} = e^{(\ln W_{i-.5,t-.5} + \ln W_{i+.5,t+.5})/2}.$$

For $t = t_0$ and $i = b$, the $W_{i,t}$ are approximated by the relationship

$$W_{i,t} = e^{(2 \ln W_{i+.5,t+.5} - \ln W_{i+1,t+1})}.$$

For $t = t_f+1$ and $i = m+1$, the weights are approximated by the equation

$$W_{i+1,t+1} = e^{(2 \ln W_{i+.5,t+.5} - \ln W_{i,t})}.$$

Catch biomass: $Y_{i,\tau}$

$$Y_{i,\tau} = W_{i+.5,t+.5} C_{i,\tau}.$$

Mean weight of individuals in catch: \bar{W}_{τ}

$$\bar{W}_{\tau} = \sum_i Y_{i,\tau} / \sum_i C_{i,\tau}.$$

Mean age of individuals in catch: \bar{i}_{τ}

$$\bar{i}_{\tau} = \sum_i i C_{i,\tau} / \sum_i C_{i,\tau}.$$

Production over τ .

For each age-group, total production between t and $t+1$ (or say over a period τ) is evaluated from the summation of:

- 1) the observed change in biomass between t and $t+1$:

$$\Delta B_{i,\tau} = N_{i,t} (W_{i+1,t+1} e^{-Z_{i,\tau}} - W_{i,t}) ;$$

- 2) the loss of biomass through natural mortality:

$$D_{i,\tau} = \frac{M}{Z_{i,\tau}} W_{i+.5,t+.5} N_{i,t} (1 - e^{-Z_{i,\tau}}) ;$$

- 3) the loss of biomass through fishing mortality: $Y_{i,\tau}$.

Since a number of cohorts exist simultaneously in the exploited population, total production over τ can be evaluated from

$$\begin{aligned} P_{\cdot,\tau} &= B_{b,t} + G_{\cdot,\tau} \\ &= B_{b,t} + \sum_i \Delta B_{i,\tau} + \sum_i D_{i,\tau} + \sum_i Y_{i,\tau} \end{aligned}$$

where $B_{b,t}$ is the recruitment (expressed in biomass units) entering the exploited population at the beginning of the period τ (exogenous component) and $G_{\cdot,\tau}$ is the increase of biomass due to growth only (endogenous component)*. The net production can now be calculated as

$$P_{\cdot,\tau}^* = P_{\cdot,\tau}^{**} - \sum_i Y_{i,\tau},$$

where $P_{\cdot,\tau}^{**}$, or 'surplus production', is defined as

$$P_{\cdot,\tau}^{**} = P_{\cdot,\tau} - \sum_i D_{i,\tau}.$$

Surplus production is thus defined as the excess of recruitment and growth over the loss of biomass through natural deaths.

*Note that the program also calculates the distribution of the growth component over the different age-groups from the evaluation of $(G_{i,\tau} / G_{\cdot,\tau}) \times 100$, where $G_{i,\tau} = \Delta B_{i,\tau} + D_{i,\tau} + Y_{i,\tau}$. See Rivard, 1980.

Production/Biomass ratio.

The production per unit biomass is calculated for each year as

$$P_{\cdot, \tau} / \sum_i \bar{B}_{i, \tau}$$

Fishing mortalities.

Age-specific rates of fishing mortality are calculated for each year (i.e. for $i = b, \dots, m-1$ and $t = t_0, \dots, t_f-1$) as

$$F_{i, \tau} = \ln \left(\frac{N_{i, t}}{N_{i+1, t+1}} \right) - M_i . [2.5]$$

Note that the instantaneous rates of fishing mortality are provided as input data for each age-group in the final year and for the oldest age-group in each year.

An indication of the overall fishing mortality is obtained by calculating the following weighted F:

$$\bar{F}_\tau = \sum (N_{i, t} F_{i, \tau}) / \sum N_{i, t}$$

The summation is taken over the fully recruited ages only. The weighted F is calculated for each year t of the cohort analysis and appears on the bottom line of the table for age-specific fishing mortalities.

Sensitivity coefficients.

The sensitivity coefficients are calculated for the population numbers and for the instantaneous fishing mortalities with respect to each parameter. Since the input parameters are either vectors or matrices, the sensitivities are approximated by considering the changes in the response of calculated quantities to small relative perturbations for each element of the input vectors (or matrices).

However, when we perturb simultaneously each element of the catch matrix, the relative sensitivities of $N_{i, t}$ with respect to the catch matrix take the value of 1.00. In other words, if we increase simultaneously all catch values by 1%, we also expect all $N_{i, t}$ to increase by 1%. In addition, the relative sensitivities of the instantaneous fishing mortalities with respect to the catch matrix are taking the value of zero. This null value indicates that any systematic perturbation of the catch matrix will not influence our estimation of the instantaneous fishing mortalities. Since the above-mentioned results are constant for any catch matrix, these sensitivities are not calculated by the function COHORT.

For the population numbers ($N_{i, t}$) and the fishing mortalities ($F_{i, \tau}$), the sensitivities are approximated by considering a small relative perturbation for each element of the vectors

$$[F_{i, \tau_f}] , i = b, \dots, m$$

$$[F_{\mu, \tau}] , t = t_0, \dots, t_f$$

and

$$[M_i] , i = b, \dots, m$$

Finally, we perform a sensitivity analysis for $N_{b, t}$ and N_{i, t_0} with respect to individual entries of the catch matrix. The sensitivities of the estimated population numbers at age b are particularly important for the assessment of future recruitment to the stock. Here, the matrix of sensitivities is formed in the following manner:

- 1) for each element in the top row of the matrix of population numbers, i.e. for $t = t_0, \dots, t_f$, we calculate the relative sensitivities as

$$XN_{k+1, t-t_0+k+1} = \frac{C_{b+k, \tau+k}}{N_{b, t}} \frac{\partial N_{b, t}}{\partial C_{b+k, \tau+k}}$$

where $k = 0, \dots, (t_f-t-1)$;

- 2) for the last $(m-b-1)$ elements of the first column of the matrix of population numbers, i.e. for $i = b+1, \dots, m$, we calculate the relative sensitivities as

$$XN_{i-b+k+1, k+1} = \frac{C_{i+k, \tau_0+k}}{N_{i, t_0}} \frac{\partial N_{i, t_0}}{\partial C_{i+k, \tau_0+k}}$$

where $k = 0, \dots, (m-i-1)$

Overall sensitivity of recruitment to parameters.

In order to assess the effect of simultaneous perturbations of initial parameter values on the calculation of recruitment, we define the following index, which is calculated for each year:

$$I_t^2 = \sum_\theta (XR_{t, \theta})^2$$

where $XR_{t, \theta}$ is the relative sensitivity of recruitment to parameter θ in a given year t . This index provides information on the overall sensitivity of the calculated recruitment in consecutive years with respect to initial

parameter values. The square root of this index provides an approximation of the overall relative change which can be registered for the calculated recruitment when small perturbations are applied simultaneously to each parameter (see Rivard and Doubleday, 1979b). The vector I_t ($t = t_0, \dots, t_f$) is printed in output under the heading 'Overall sensitivity of recruitment'.

We also define an index I_{sp} which gives insight on the overall importance of input parameters for the evaluation of recruitment in different species or in different stocks. This index is calculated as

$$I_{sp} = \frac{1}{\text{nb of years}} \sum_t I_t$$

The calculated value for I_{sp} is printed in output under the heading 'Overall Sensitivity Index'.

Input format.

In order to initiate the program, type COHORT. Then you will be prompted for input as follows: [note that a catch matrix ($C_{i,t}$: $i = b, \dots, m$; $t = t_0, \dots, t_f$) must be created before entering the function]

NAME OF CATCH MATRIX?...

-enter the name of the catch matrix.

ERRORS: -the name entered must represent an APL variable;
-the variable must be a matrix;
-there should not be negative values in the matrix.

SENSITIVITY ANALYSIS TO BE PERFORMED (YES OR NO)?

-enter either YES or NO, in order to indicate if you want to calculate the relative sensitivity coefficients.

ERRORS: -you must enter either YES or NO.

FIRST YEAR, YOUNGEST AGE-GROUP AND FIRST FULLY RECRUITED AGE-GROUP?

-enter three numbers representing the first year in the catch table, the youngest age-group in the catch table and the first fully recruited age-group.

ERRORS: -you must enter exactly three numbers;
-all figures must be non-negative integers.

NATURAL MORTALITY?

-enter either a single number (M) or a vector giving a separate instantaneous rate of natural mortality (M_i) for each age-group i. When a single number is entered, the M_i are set equal to M for all i.

ERRORS: -you must enter one number or a vector of numbers with the same number of elements as there are rows in the catch matrix;
-the natural mortality must be equal to or greater than zero.

WEIGHT AT AGE KNOWN (YES OR NO)?

-enter either YES or NO, in order to indicate if weight-at-age data are available.

ERRORS: -you must enter either YES or NO.

ENTER WEIGHT AT AGE DATA (KG): VECTOR OR MATRIX.

-enter either a vector or a matrix of weights at age ($W_{i,t}$: $i = b, \dots, m$; $t = t_0, \dots, t_f$). When a vector is entered, growth is assumed to be constant through time.

ERRORS: -the vector must have the same length as there are rows in the catch matrix;
or -the matrix must have the same dimension as the catch matrix;
-all entries must be positive.

ENTER 1 IF MATRIX IS TO BE PRINTED. OTHERWISE ENTER 0. ONE NUMBER IS REQUIRED FOR EACH MATRIX:

CATCH BIOMASS

MEAN WEIGHT OF INDIVIDUALS IN CATCH

WEIGHTS AT THE BEGINNING OF THE YEAR

POPULATION BIOMASS AT THE BEGINNING OF THE

YEAR

-enter a vector of 0's or 1's to indicate which matrices are to be printed.

ERRORS: -you must enter a vector of 0's and/or 1's;
-the input vector must have dimension 4.

F VALUES FOR LAST YEAR (XXXX)?

-enter the values of the instantaneous rate of fishing mortality for the last year represented in the catch matrix: (F_{i,t_f} , $i = b, \dots, m$). You can also enter a single number representing the fishing mortality to be applied to all age-groups in the final year.

ERRORS: -you must enter one number or a vector of numbers with the same number of elements as there are rows in the catch matrix;
-the fishing mortalities must be greater than zero.

F VALUES FOR OLDEST AGE GROUP (XX)?

-enter the values of the instantaneous rate of fishing mortality for the oldest age-group represented in the catch matrix: $F_{m,t}$, $t = t_0, \dots, t_f$. You can also enter a single value representing the fishing mortality to be applied to the oldest age-group in all years.

ERRORS: -you must enter one number or a vector of numbers having the same number of elements as there are columns in the catch matrix;
-the fishing mortalities must be greater than zero.

IS FISHING COMPLETE FOR THE LAST AGE GIVEN (YES OR NO)?...

-enter either YES or NO, in order to specify which equation is to be used to calculate the population numbers for the oldest age-group. Fishing is considered to be complete if the catches of older fish have been added to the last row of the catch table or if there are no older fish.

ERRORS: -you must enter either YES or NO.

Graphical output.

In addition to the above-mentioned calculations, the user can generate two graphs showing 1) the variation of the components of production (recruitment, growth) through time and 2) the fluctuations of surplus production and of the loss of biomass due to fishing mortality. The first graph is produced with the function GRAPHAPRODUCTION, while the second graph is obtained by execution of the function GRAPHANETAPROD. In order to use these plotting functions, you must copy the workspace 39 MAGIC in your active workspace and you must specify the type of terminal being used. This is achieved by the following commands:

```
)COPY 39 MAGIC
@SUPERPLOT 'TERMINAL,xxxx'
```

where xxxx identifies the type of terminal being used (the terminals which are supported by the plotting package are listed in Appendix D). Note that only terminals with graphical capabilities can be selected.

In order to initiate the first graph, type

```
GRAPHAPRODUCTION
```

The function GRAPHAPRODUCTION uses the data stored by COHORT in the variable DATA1. Paper will advance to the next page; then adjust paper and depress the 'SPACE BAR' followed by a 'CARRIAGE RETURN' to start. In order to output the second graph, type

```
GRAPHANETAPROD
```

The function GRAPHANETAPROD uses the data stored by COHORT in the variable DATA2. Paper will advance to the next page; adjust paper and depress the 'SPACE BAR' followed by a 'CARRIAGE RETURN' to start. Annual net production corresponds to the difference between the two lines plotted.

Calculating mean F-values and partial recruitment figures.

A series of functions have been written to calculate an overall rate for the age-specific fishing mortalities. These functions operate on FISHMORT, the matrix of age-specific fishing mortalities which is available as a global variable after execution of the function COHORT.

MEANF

This function calculates an unweighted mean for the annual rate of fishing mortality from the following relationship:

$$\bar{F}_\tau = (\sum F_{i,\tau}) / \text{nb of age-groups}$$

Here, the summation is over all age-groups. In order to print the vector of unweighted mean-F's, simply type in MEANF after execution of the function COHORT.

WEIGHTEDAFMORT

This function calculates a weighted mean for the annual rate of fishing mortality from the formula

$$\bar{F}_\tau = (\sum N_{i,t} F_{i,\tau}) / (\sum N_{i,t}) ,$$

where the summations are taken over all age-groups. In order to print a vector of weighted F's, type in WEIGHTEDAFMORT after execution of the function COHORT.

FFULR n (n = first fully recruited age-group)

This function calculates an overall rate of fishing mortality for the period τ from the equation

$$\bar{F}_\tau = \ln(\frac{\sum_{i=i_r}^{m-2} N_{i,t}}{\sum_{i=i_r}^{m-1} N_{i+1,t+1}}) - M ,$$

where i_r is the first fully recruited age-group. In order to print a vector representing this overall rate of fishing mortality, type in FFULR n after executing the function COHORT.

PRANOASMOOTH

This function is used to calculate partial recruitment figures from the age-specific fishing mortalities stored in the global variable FISHMORT. The partial recruitment figures are calculated as

$$r_{i,\tau} = F_{i,\tau} / FMAX_\tau ,$$

where $FMAX_\tau$ is the maximum value of $F_{i,\tau}$ which is observed over the period τ ; they are, in fact, relative measure of catchability for the different ages. In order to print the matrix of partial recruitment figures, type in PRANOASMOOTH; results are stored in the global variable SEL.

- An Example.

FILEΔFETCH '2719067 PFΔPROD'

COHORT

NAME OF CATCH MATRIX?.....

CCC

(the matrix CCC is printed on page 22)

SENSITIVITY ANALYSIS TO BE PERFORMED (YES OR NO)?

NO

FIRST YEAR, YOUNGEST AGE GROUP AND FIRST FULLY RECRUITED AGE-GROUP?

□:

1965 2 6

NATURAL MORTALITY?

□:

.2

WEIGHT AT AGE KNOWN(YES OR NO)?

YES

ENTER WEIGHT AT AGE DATA (KG): VECTOR OR MATRIX

□:

.08 .15 .24 .29 .31 .32 .33 .34 .35

ENTER 1 IF MATRIX IS TO BE PRINTED. OTHERWISE ENTER 0.

ONE NUMBER IS REQUIRED FOR EACH MATRIX

CATCH BIOMASS

MEAN WEIGHT OF INDIVIDUALS IN CATCH

WEIGHTS AT THE BEGINNING OF THE YEAR

POPULATION BIOMASS AT THE BEGINNING OF THE YEAR

□:

0 1 0 0

F VALUES FOR LAST YEAR (1977)?

□:

.12 .164 .304 .36 .4 .4 .4 .4 .4

F VALUES FOR OLDEST AGE GROUP (10)?

□:

.139 .351 .372 .525 .34 .514 .562 .773 .564 .358 .389 .352 .4

IS FISHING COMPLETE FOR LAST AGE GIVEN(YES OR NO)?.....

NO

POPULATION NUMBERS

29/ 5/80

	1965	1966	1967	1968	1969	1970	1971	1972
2	2573883	1520603	1256924	2361178	622711	789346	863222	5168989
3	961249	1916581	1205487	985697	1252997	446009	549520	576299
4	1301419	763071	1324796	925051	734694	677987	312531	292773
5	343998	855456	571734	868943	698456	493877	296884	159812
6	91885	236624	420997	368733	463037	426419	222392	140313
7	40626	65645	152581	200631	235996	278799	240340	113680
8	4460	31730	41105	72489	82269	136659	127000	112063
9	1033	3144	18991	29585	30415	46911	74556	58717
10	314	797	1045	15179	10251	19160	19161	27908
	1973	1974	1975	1976	1977			
2	799690	1261696	1683197	194992	541745			
3	3644541	627897	925947	1164915	141620			
4	406700	2474822	472818	614285	807496			
5	105319	233871	1468646	303543	384765			
6	60983	55082	143341	854384	183970			
7	46669	26799	31295	71574	500638			
8	48678	20691	14594	17156	41444			
9	47684	23925	12133	9018	9551			
10	24498	21092	9668	6784	4166			
2+	5184762	4745875	4761638	3236650	2614495			
3+	4385072	3484179	3078441	3041658	2072750			
4+	740531	2856282	2152494	1876743	1931130			
5+	333831	381460	1679676	1262458	1123634			

MEAN POPULATION BIOMASS (KG)

29/ 5/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973
2	178328	108556	89266	139917	42356	52977	56810	348997	56827
3	128753	240378	158863	128109	140437	56298	61164	72987	453307
4	255094	159073	259415	193549	145521	110761	54648	44003	74966
5	83222	177703	134223	187003	159876	98679	60584	29749	22477
6	24190	59377	92171	92208	112580	100607	59221	26371	12888
7	11518	16775	34436	42487	58185	61777	53800	24524	10220
8	1242	8190	11560	15987	20771	33822	29209	24864	11500
9	309	648	5785	6202	8281	10537	16141	13310	11084
10	93	214	278	3779	2772	4793	4693	6245	5995
2+	682749	770915	785997	809239	690780	530252	387269	591051	659264
3+	504420	662359	696731	669322	648423	477275	330459	242054	602437
4+	375668	421981	537868	541214	507986	420976	269295	169066	149130
5+	120574	262907	278453	347665	362466	310216	214647	125963	74164
	1974	1975	1976	1977					
2	86814	112656	13351	37089					
3	82004	113923	146297	17806					
4	462763	91668	117746	152229					
5	53629	328837	69093	85438					
6	13043	32034	205167	42676					
7	6426	7527	17646	120470					
8	5291	3822	4285	10284					
9	5350	3128	2136	2442					
10	5658	2557	1825	1096					
2+	720977	696152	577546	469532					
3+	634163	583497	564195	432442					
4+	552159	469574	417898	414636					
5+	89396	377906	300151	262407					

WARNING: WEIGHT AT AGE IS AVAILABLE FOR ONE YEAR ONLY.
POPULATION BIOMASS, AS CALCULATED, MAY BE BIASED.

MEAN WEIGHT OF INDIVIDUALS IN CATCH

29/ 5/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
	0.18	0.23	0.25	0.17	0.22	0.25	0.23	0.16	0.18	0.22	0.21	0.25	0.26

MEAN AGE OF INDIVIDUALS IN CATCH

29/ 5/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
	3.31	4.17	4.67	3.53	4.19	4.86	4.64	3.49	3.70	3.93	3.93	4.66	
	5.02												

DISTRIBUTION OF GROWTH OVER AGES (PER CENT)

29/ 5/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976
2	49.3	24.5	22.2	34.6	14.5	26.2	34.7	77.5	11.5	20.5	33.4	5.7
3	26.6	46.5	33.7	27.3	42.2	24.0	32.8	14.0	78.7	16.6	29.2	53.3
4	21.2	18.6	34.3	25.0	26.5	31.9	18.8	5.9	8.3	59.6	14.6	26.9
5	2.4	9.2	7.3	10.5	12.1	11.4	8.6	1.6	1.0	2.8	21.8	6.5
6	0.3	1.1	1.9	1.9	3.2	4.1	2.5	0.4	0.2	0.3	0.8	7.2
7	0.2	0.2	0.4	0.4	0.9	1.3	1.4	0.2	0.1	0.1	0.1	0.4
8	0.0	0.1	0.1	0.2	0.3	0.7	0.7	0.2	0.1	0.1	0.1	0.1
9	0.0	0.0	0.1	0.0	0.1	0.2	0.3	0.1	0.1	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1	0.0	0.0

| 1977

2	18.0
3	8.6
4	55.2
5	12.6
6	2.2
7	3.0
8	0.3
9	0.1
10	0.0

PRODUCTION

29/ 5/80

SOURCE	1965	1966	1967	1968	1969	1970
--------	------	------	------	------	------	------

RECRUITMENT BIOMASS	150376	88839	73434	137949	36381	46117
GROWTH	227589	279816	253331	252588	182936	126602
TOTAL PRODUCTION	377965	368655	326765	390537	219317	172719
LOSS THROUGH FISHING	95016	169374	172784	236722	203395	240474
SURPLUS PRODUCTION	241415	214472	169565	228689	81161	66668
NET PRODUCTION	146399	45098	3219	8033	122235	173806

SOURCE	1971	1972	1973	1974	1975	1976
--------	------	------	------	------	------	------

RECRUITMENT BIOMASS	50433	301992	46721	73713	98339	11392
GROWTH	102558	282550	311874	265783	211444	148370
TOTAL PRODUCTION	152991	584542	358595	339496	309782	159763
LOSS THROUGH FISHING	180983	189599	153089	194455	198328	155851
SURPLUS PRODUCTION	75537	466331	226742	195301	170552	44253
NET PRODUCTION	105446	276733	73653	846	27776	111598

SOURCE	1977
--------	------

RECRUITMENT BIOMASS	31651
GROWTH	129773
TOTAL PRODUCTION	161424
LOSS THROUGH FISHING	155194
SURPLUS PRODUCTION	67518
NET PRODUCTION	87676

PRODUCTION/BIOMASS RATIO

29/ 5/80

1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
------	------	------	------	------	------	------	------	------	------	------	------	------

0.55	0.48	0.42	0.48	0.32	0.33	0.40	0.99	0.54	0.47	0.44	0.28	0.34
------	------	------	------	------	------	------	------	------	------	------	------	------

FISHING MORTALITY

29/ 5/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976
2	0.095	0.032	0.043	0.434	0.134	0.162	0.204	0.149	0.042	0.109	0.168	0.120
3	0.031	0.169	0.065	0.094	0.414	0.156	0.430	0.149	0.187	0.084	0.210	0.166
4	0.220	0.089	0.222	0.081	0.197	0.626	0.471	0.822	0.353	0.322	0.243	0.268
5	0.174	0.509	0.239	0.429	0.293	0.598	0.549	0.763	0.448	0.290	0.342	0.306
6	0.136	0.239	0.541	0.246	0.307	0.373	0.471	0.901	0.622	0.365	0.494	0.334
7	0.047	0.268	0.544	0.691	0.346	0.586	0.563	0.648	0.613	0.408	0.401	0.346
8	0.150	0.313	0.129	0.669	0.362	0.406	0.571	0.654	0.510	0.334	0.281	0.386
9	0.060	0.902	0.024	0.860	0.262	0.695	0.783	0.674	0.616	0.706	0.381	0.572
10	0.139	0.351	0.372	0.525	0.340	0.514	0.562	0.773	0.564	0.358	0.389	0.352
6+	0.110	0.258	0.499	0.454	0.323	0.463	0.559	0.739	0.589	0.423	0.455	0.339

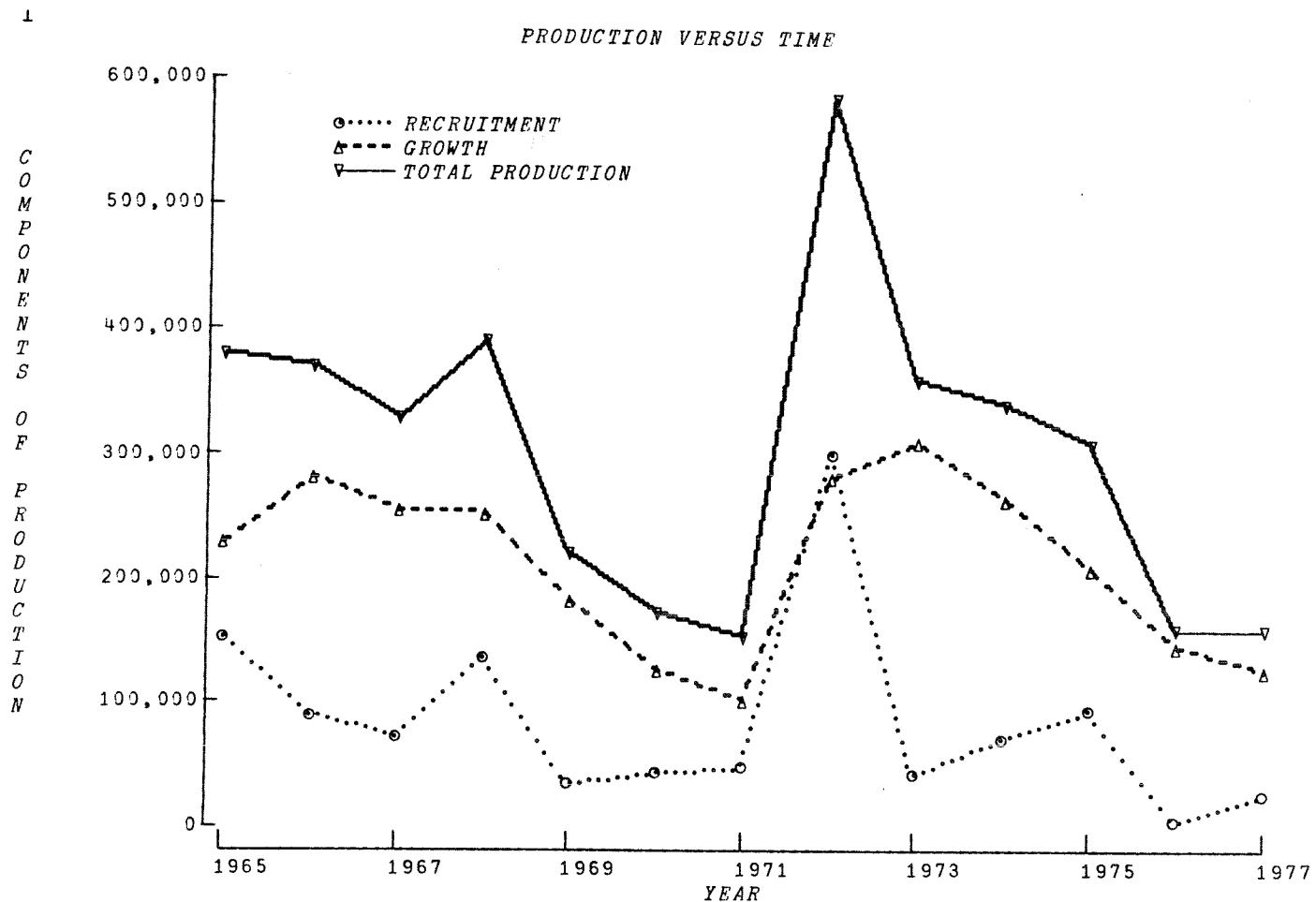
	1977
--	------

2	0.120
3	0.164
4	0.304
5	0.360
6	0.400
7	0.400
8	0.400
9	0.400
10	0.400

	1977
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)COPY 39 MAGIC
ΔSUPERPLOT 'TERMINAL,X1620'
GRAPHΔPRODUCTION

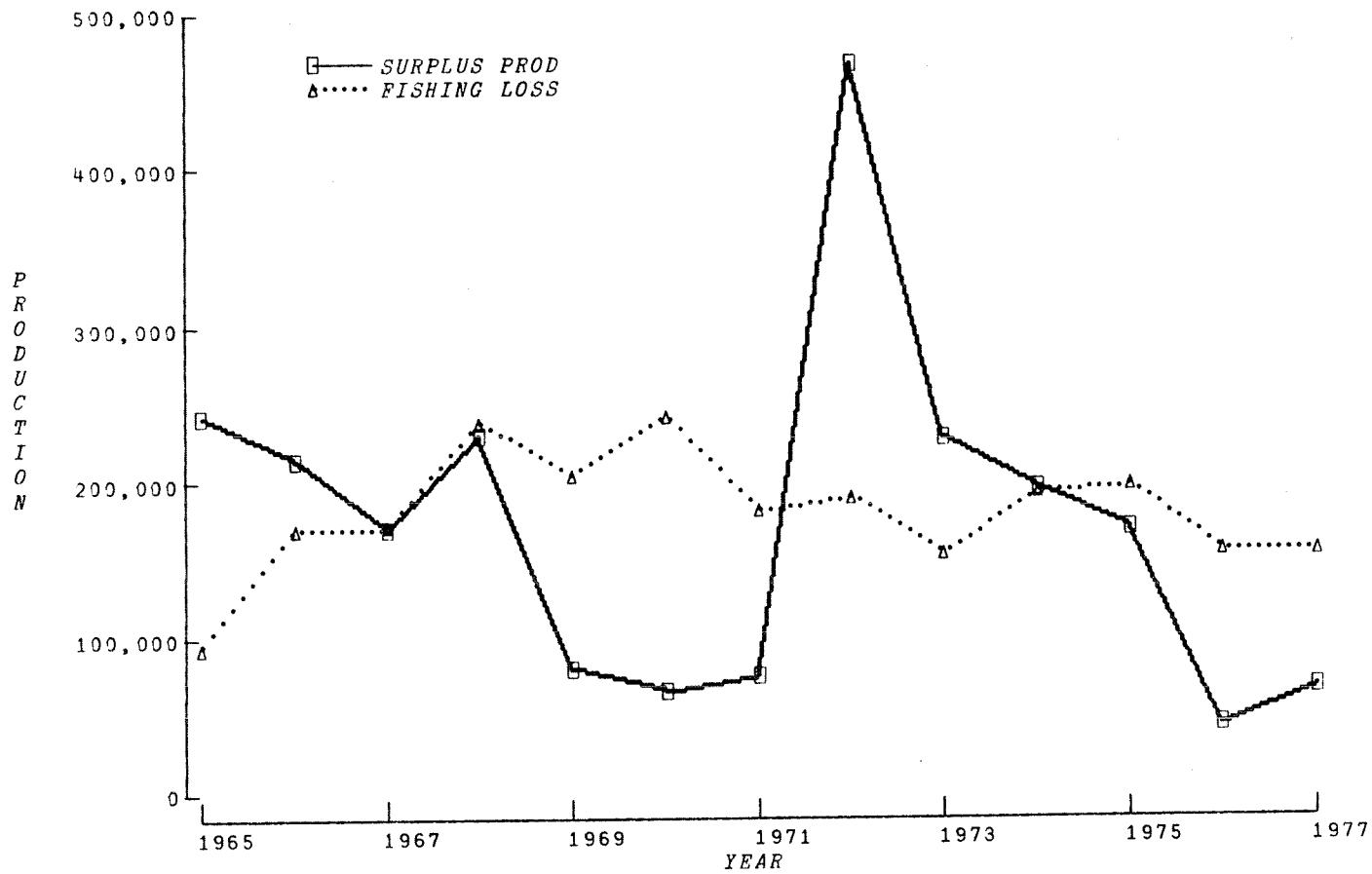
(Paper will advance to the next page)
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GRAPH&NET&PROD

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SURPLUS PRODUCTION AND YIELD VERSUS TIME



3. Sequential Population Analysis by Gulland's method (VPA).

This program calculates population numbers at age and the instantaneous rates of fishing mortality at age by using a virtual population analysis (Gulland, 1965; Pope, 1972). That is, given historical information on catch-at-age and given an estimate of the instantaneous rate of natural mortality as well as an estimate of the instantaneous rates of fishing mortality for the last year of historical catch and for the last age-groups, the program reconstructs the age-composition of the stock and estimates the corresponding rates of fishing mortality for a specified number of years. Sensitivity coefficients are calculated for population numbers and for instantaneous rates of fishing mortality with respect to both natural mortality and initial fishing mortalities, and with respect to individual entries of the catch matrix.

Input Information.*

The input information required by VPA consists of:

t_0 : first year of the catch matrix;

t_f : final year in the catch matrix;

b : age of youngest age-group in the catch matrix;

m : age of the oldest age-group in the catch matrix;

$C_{i,\tau}$ ($t = t_0, \dots, t_f$; $i = b, \dots, m$): catch-at-age, given in numbers;

F_{i,τ_f} ($i = b, \dots, m$): the instantaneous rate of fishing mortality for each age-group in the final year (t_f);

$F_{\mu,\tau}$ ($t = t_0, \dots, t_f$): the instantaneous rate of fishing mortality for the oldest age-group in each year;

M_i ($i = b, \dots, m$): the instantaneous rate of natural mortality for each age-group (those are assumed to be constant for all years considered);

$W_{i+.5,t+.5}$ ($i = b, \dots, m$; $t = t_0, \dots, t_f$): weight-at-age, expressed in kilograms. Those are required only when population biomass is to be calculated. The weights are taken as mid-year estimates.

Algorithm.

The function VPA calculates the population numbers at age and the instantaneous rate of fishing mortality at age; when information on weight-at-age is available, the function also calculates the catch biomass (yield) and the population biomass. The calculations are performed in the following manner:

Population numbers: $N_{i,t}$

The function calculates the population numbers by using three different equations:

A) for the first ($m-b-1$) age-groups in the final year (i.e. for $i = b, \dots, m-1$ and $t = t_f$), the $N_{i,t}$ are calculated from equation [2.1].

B) for the oldest age-group in each year (i.e. for $i = m$ and $t = t_0, \dots, t_f$), the $N_{i,t}$ are calculated as equation [2.1] in COHORT if fishing is not complete for the oldest age-group and as equation [2.2] in COHORT if fishing is complete for the oldest age-groups.

C) for the remainder of the table (i.e. for $i = b, \dots, m-1$ and $t = t_0, \dots, t_f-1$), the population numbers are calculated as

$$N_{i,t} = N_{i+1,t+1} e^{(F_{i,\tau} + M_i)} . \quad [3.1]$$

In order to use the preceding equation, an estimate of $F_{i,\tau}$ is necessary. That estimate can be obtained by using the catch equation

$$C_{i,\tau} = N_{i+1,t+1} F_{i,\tau} \frac{(e^{F_{i,\tau} + M_i} - 1)}{F_{i,\tau} + M_i} . \quad [3.2]$$

Equation [3.2] involves only one unknown, namely $F_{i,\tau}$, which the function calculates by using the Newton-Raphson method of successive approximations (see Seber, 1973: section 1.3.8). An initial estimate of $F_{i,\tau}$ is calculated by using equations [2.3] and [2.5]. Successive values of $F_{i,\tau}$ are then calculated by the Newton-Raphson method. We exit the iterative process when

$$\left| \frac{F_{i,\tau} (e^{F_{i,\tau} + M_i} - 1)}{F_{i,\tau} + M_i} - \frac{C_{i,\tau}}{N_{i+1,t+1}} \right| < 10^{-5} .$$

*The notation follows that of Chapter 2.

Once the final value of $F_{i,\tau}$ has been found, the $N_{i,t}$ are calculated by using equation [3.1] which is given above.

Catch Biomass: $Y_{i,\tau}$

$$Y_{i,\tau} = W_{i+5,t+5} C_{i,\tau}$$

Mean weight of individuals in catch: \bar{W}_{τ}

$$\bar{W}_{\tau} = \sum_i Y_{i,\tau} / \sum_i C_{i,\tau}$$

Mean population biomass: $\bar{B}_{i,\tau}$

$$\bar{B}_{i,\tau} = W_{i+5,t+5} N_{i,t} (1-e^{-Z_{i,\tau}}) / Z_{i,\tau}$$

Fishing mortalities: $F_{i,\tau}$

Age-specific rates of fishing mortality are determined from equation [3.2]. Thereafter, an indication of the overall fishing mortality rate is obtained by calculating the following weighted F:

$$\bar{F}_{\tau} = \sum_i (N_{i,t} F_{i,\tau}) / \sum_i N_{i,t}$$

where the summation is taken over all ages. The weighted F is calculated for each year of the cohort analysis and is printed on the bottom line of the table for age-specific rates of fishing mortality.

Sensitivity coefficients.

The sensitivity coefficients are calculated for the population numbers and for the instantaneous rates of fishing mortality with respect to each parameter. The sensitivities are approximated by considering the changes in the response of calculated quantities to small relative perturbations for each element of the input vectors (or matrices).

As in COHORT, the relative sensitivities of $N_{i,t}$ with respect to the catch matrix take the value of 1.00, while the relative sensitivities of the instantaneous rates of fishing mortality with respect to the catch matrix take the value of zero. In other words, if we increase simultaneously all catch values by 1%, we expect that all $N_{i,t}$ will increase by 1%; however, the estimated rates of fishing mortality will not be modified by such perturbation of the catch values. Since these results are constant for any catch matrix, these sensitivity coefficients are not calculated by the function VPA.

For the population numbers, $N_{i,t}$, and for the instantaneous rates of fishing mortality, $F_{i,\tau}$, the sensitivity coefficients are approximated by considering a small relative perturbation for each element of the input vectors

$$[F_{i,\tau_f}] , i = b, \dots, m ;$$

$$[F_{i,t}] , t = t_0, \dots, t_f;$$

and

$$[M_i] , i = b, \dots, m .$$

Finally, we also perform a sensitivity analysis for $N_{b,t}$ and N_{i,t_0} with respect to individual entries of the catch matrix. The calculations are performed as in COHORT. The overall sensitivity of recruitment to parameters is also estimated as in COHORT.

Input Format.

In order to initiate the program, type VPA. Then you will be prompted for input as follows: [note that a catch matrix ($C_{i,\tau}$: $i = b, \dots, m$; $t = t_0, \dots, t_f$) must be previously created before calling the function]

NAME OF CATCH MATRIX?...

-enter the name of the catch matrix.

ERRORS: -the name entered must represent an APL variable;
-the variable must be a matrix;
-there should not be negative values in the matrix.

SENSITIVITY ANALYSIS TO BE PERFORMED (YES OR NO)?

-enter either YES or NO, in order to indicate if you want to calculate the relative sensitivity coefficients.

ERRORS: -you must enter either YES or NO.

FIRST YEAR AND YOUNGEST AGE-GROUP?

-enter two numbers representing the first year in the catch matrix and the youngest age-group in the catch matrix.

ERRORS: -you must enter exactly two numbers;
-these values must be non-negative integers.

NATURAL MORTALITY?

-enter either a single number (M) or a vector giving a separate value M_i for each age-group i.

ERRORS: -you must enter one number or a vector of numbers having the same number of elements as there are rows in the catch matrix;
-the natural mortalities must be equal to or greater than zero.

WEIGHT AT AGE KNOWN (YES OR NO)?

-enter either YES or NO, in order to indicate if weight-at-age data are available.

ERRORS: -you must enter either YES or NO.

ENTER WEIGHT AT AGE DATA (KG): VECTOR OR MATRIX.

-enter either a vector or a matrix of weight-at-age ($W_{i+5,t+5}$: $i = b, \dots, m$; $t = t_0, \dots, t_f$). When a vector is entered, growth is assumed to be constant through time.

ERRORS: -the vector must have the same length as there are rows in the catch matrix;
or -the matrix must have the same dimension as the dimensions of the catch matrix;
-all entries must be positive.

STARTING F VALUES FOR LAST YEAR (XXXX)?

-enter the values of the instantaneous rates of fishing mortality for the last year represented in the catch matrix (F_{i,T_f} : $i = b, \dots, m$). You can also enter a single number representing the rate of fishing mortality to be applied to all age-groups in the final year.

ERRORS: -you must enter one number or a vector of numbers having the same number of elements as there are rows in the catch matrix;
-the values entered must be greater than zero.

STARTING F VALUES FOR OLDEST AGE-GROUP (XX)?

-enter the values of the instantaneous rate of fishing mortality for the oldest age-group represented in the catch matrix ($F_{\mu,T}$: $t = t_0, \dots, t_f$). You can also enter a single

value representing the instantaneous rate of fishing mortality to be applied to the oldest age-group in each year.

ERRORS: -you must enter one number or a vector of numbers having the same number of elements as there are columns in the catch matrix;
-the values entered must be greater than zero.

IS FISHING COMPLETE FOR THE LAST AGE GIVEN (YES OR NO)?...

-enter either YES or NO, in order to specify which equation is to be used to calculate the population numbers for the oldest age-group. Fishing is considered to be complete if the catches of older fish have been added to the last row of the catch matrix or if there are no older fish.

ERRORS: -you must enter either YES or NO.

FILE&FETCH '2719067 PFA&PROD'

ccc

210796	43630	47948	751706	70536	106916	144167	649254	29656	118301	235590	19922	55634
26450	270068	68430	79933	384467	58166	173662	71984	562616	45600	158941	161637	19468
232147	58591	238394	65107	118960	285361	106170	148516	109530	616206	92356	130597	192823
49752	308775	109814	274518	160723	201097	113561	77207	34422	53199	384646	72334	106061
10592	45479	159203	72827	110852	120223	75593	75384	25562	15254	50599	219788	50566
1693	13970	57948	90617	62506	111911	93620	49065	19361	8120	9357	18960	150588
561	7722	4497	31977	22595	41257	50022	48700	17604	5313	3239	4967	12466
54	1690	409	15441	6345	21271	36618	26055	19836	10964	3481	3556	2873
37	215	296	5668	2693	7039	7536	13792	9661	5787	2842	1835	1253

VPA

NAME OF CATCH MATRIX?.....

ccc

SENSITIVITY ANALYSIS TO BE PERFORMED (YES OR NO)?

YES

FIRST YEAR AND YOUNGEST AGE GROUP?

1

1965 2

NATURAL MORTALITY?

117

2

WEIGHT AT AGE KNOWN (YES OR NO)?

YES

ENTER WEIGHT AT AGE DATA (KG): VECTOR OR MATRIX

□ *

08 15 24 29 31 32 33 34 35

ST

STARTING F VALUES FOR BASIC ISMR (10%).

1

MEAN E VALUES FOR OLDEST AGE GROUP

31

139 .351 .372 .525 .34 .514 .562 .773 .564 .358

TG

POPULATION NUMBERS

MEAN POPULATION BIOMASS (KG)

12/ 6/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
2	177017	107732	88477	138468	41956	52449	56485	347423	56677	86623	112545	13348	37089
3	127905	238497	157618	126924	138859	55719	60501	72556	451258	81789	113718	146245	17806
4	252644	157991	257225	191960	144064	109449	54038	43588	74532	460881	91465	117656	152229
5	82595	175904	133263	185352	158480	97671	59847	29438	22339	53345	327895	69027	85438
6	24007	58898	91247	91512	111545	99667	49724	26095	12789	12978	31912	204940	42676
7	11497	16640	34171	42123	57727	61220	53303	24300	10160	6389	7500	17625	120470
8	1231	8181	11460	15915	20633	33556	28994	24683	11415	5274	3803	4279	10284
9	309	644	5785	6169	8275	10498	16068	13263	11050	5329	3124	2130	2442
10	93	214	278	3779	2772	4793	4693	6245	5995	5658	2557	1825	1096
2+	677299	764701	779525	802201	684311	525021	383652	587590	656216	718266	694519	577074	469532
3+	500282	656969	691047	663733	642356	472572	327166	240167	599539	631643	581974	563726	432442
4+	372377	418472	533429	536809	503496	416854	266666	167612	148280	549854	468257	417481	414636
5+	119732	260481	276205	344849	359432	307405	212628	124023	73748	88973	376792	299826	262407

WARNING: WEIGHT AT AGE IS AVAILABLE FOR ONE YEAR ONLY.
 POPULATION BIOMASS, AS CALCULATED MAY BE BIASED.

CATCH BIOMASS (KG)

12/ 6/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
2	16864	3490	3836	60136	5643	8553	11533	51940	2372	9464	18847	1594	4451
3	3967	40510	10264	11990	57670	8725	26049	10798	84392	6840	23841	24246	2920
4	55715	14062	57215	15626	28550	68487	25481	35644	26287	147889	22165	31343	46278
5	14428	89545	31846	79610	46610	58318	32933	22390	9982	15428	111547	20977	30758
6	3284	14098	49353	22576	34364	37269	23434	23369	7924	4729	15686	68134	17070
7	542	4470	18543	28997	20092	35812	29958	15701	6196	2598	2994	6067	48188
8	185	2548	1484	10552	7456	13615	16507	16071	5809	1753	1069	1639	4114
9	18	575	139	5250	2157	7232	12450	8859	6744	3728	1184	1209	977
10	13	75	104	1984	943	2464	2638	4827	3381	2025	995	642	439
2+	95016	169374	172784	236722	203395	240474	180983	189599	153089	194455	198328	155851	155194

MEAN WEIGHT OF INDIVIDUALS IN CATCH

12/ 6/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
	0.18	0.23	0.25	0.17	0.22	0.25	0.23	0.16	0.18	0.22	0.21	0.25	0.26

FISHING MORTALITY

12/ 6/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
2	0.095	0.032	0.043	0.434	0.134	0.163	0.204	0.150	0.042	0.109	0.167	0.119	0.120
3	0.031	0.170	0.065	0.094	0.415	0.157	0.431	0.149	0.187	0.084	0.210	0.166	0.164
4	0.221	0.089	0.222	0.081	0.198	0.626	0.472	0.818	0.353	0.321	0.242	0.266	0.304
5	0.175	0.509	0.239	0.430	0.294	0.597	0.550	0.761	0.447	0.289	0.340	0.304	0.360
6	0.137	0.239	0.541	0.247	0.308	0.374	0.471	0.896	0.620	0.364	0.492	0.332	0.400
7	0.047	0.269	0.543	0.688	0.346	0.585	0.562	0.646	0.610	0.407	0.399	0.344	0.400
8	0.150	0.311	0.129	0.663	0.361	0.406	0.569	0.651	0.509	0.332	0.281	0.383	0.400
9	0.059	0.892	0.024	0.851	0.261	0.689	0.775	0.668	0.610	0.699	0.379	0.568	0.400
10	0.139	0.351	0.372	0.525	0.340	0.514	0.362	0.773	0.564	0.358	0.389	0.352	0.400
2+	0.120	0.179	0.176	0.322	0.295	0.403	0.408	0.233	0.200	0.235	0.249	0.246	0.294

PARTIAL DERIVATIVES OF POPULATION WITH RESPECT TO PARAMETERS

0.037 0.059 0.033 0.013 0.028 0.042 0.133 0.218 0.420 0.467 0.617 0.818 0.942
 0.081 0.040 0.061 0.034 0.019 0.032 0.049 0.163 0.253 0.437 0.520 0.729 0.922
 0.038 0.084 0.048 0.065 0.037 0.029 0.038 0.075 0.189 0.305 0.476 0.641 0.860
 0.120 0.048 0.092 0.060 0.071 0.046 0.054 0.060 0.168 0.268 0.420 0.605 0.836
 0.214 0.142 0.079 0.116 0.091 0.095 0.082 0.094 0.128 0.226 0.236 0.357 0.588 0.819
 0.525 0.245 0.181 0.135 0.149 0.124 0.138 0.131 0.226 0.236 0.376 0.582 0.819
 0.297 0.550 0.321 0.309 0.266 0.210 0.221 0.240 0.457 0.414 0.353 0.560 0.819
 0.791 0.345 0.750 0.365 0.596 0.381 0.314 0.389 0.457 0.413 0.576 0.467 0.819
 0.933 0.840 0.831 0.768 0.844 0.773 0.754 0.675 0.753 0.837 0.824 0.839 0.819

SENSITIVITY TO TERMINAL F

12/ 6/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
2	-0.037	-0.059	-0.033	-0.013	-0.028	-0.042	-0.133	-0.218	-0.420	-0.467	-0.617	-0.818	-0.942
3	-0.081	-0.040	-0.061	-0.034	-0.019	-0.032	-0.049	-0.163	-0.253	-0.437	-0.520	-0.729	-0.922
4	-0.038	-0.084	-0.048	-0.065	-0.037	-0.029	-0.038	-0.075	-0.189	-0.305	-0.476	-0.641	-0.860
5	-0.120	-0.048	-0.092	-0.060	-0.071	-0.046	-0.054	-0.060	-0.168	-0.268	-0.420	-0.605	-0.836
6	-0.214	-0.142	-0.079	-0.116	-0.091	-0.095	-0.082	-0.094	-0.128	-0.226	-0.357	-0.588	-0.819
7	-0.525	-0.245	-0.181	-0.135	-0.149	-0.124	-0.138	-0.131	-0.226	-0.236	-0.376	-0.582	-0.819
8	-0.297	-0.550	-0.321	-0.309	-0.266	-0.210	-0.221	-0.240	-0.457	-0.414	-0.353	-0.560	-0.819
9	-0.791	-0.345	-0.750	-0.365	-0.596	-0.381	-0.314	-0.389	-0.457	-0.413	-0.576	-0.467	-0.819
10	-0.933	-0.840	-0.831	-0.768	-0.844	-0.773	-0.754	-0.675	-0.753	-0.837	-0.824	-0.839	-0.819

SENSITIVITY TO NATURAL MORTALITY

12/ 6/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
2	0.750	0.933	0.777	0.402	0.611	0.498	0.647	0.638	0.701	0.528	0.405	0.272	0.095
3	0.864	0.616	0.761	0.607	0.370	0.485	0.369	0.572	0.526	0.527	0.378	0.262	0.094
4	0.462	0.688	0.512	0.605	0.458	0.313	0.350	0.318	0.448	0.414	0.364	0.245	0.092
5	0.539	0.352	0.543	0.416	0.448	0.337	0.308	0.306	0.409	0.398	0.336	0.238	0.091
6	0.566	0.423	0.326	0.465	0.389	0.370	0.339	0.268	0.355	0.387	0.300	0.234	0.090
7	0.557	0.434	0.312	0.294	0.368	0.296	0.295	0.288	0.331	0.382	0.317	0.232	0.090
8	0.331	0.379	0.339	0.271	0.298	0.282	0.260	0.250	0.269	0.333	0.327	0.227	0.090
9	0.280	0.169	0.284	0.173	0.243	0.188	0.177	0.190	0.198	0.188	0.228	0.203	0.090
10	0.094	0.091	0.091	0.088	0.091	0.088	0.087	0.084	0.087	0.091	0.090	0.091	0.090

PARTIAL DERIVATIVES OF ESTIMATED F WITH RESPECT TO PARAMETERS

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
2	0.038	0.060	0.033	0.016	0.030	0.045	0.147	0.235	0.428	0.492	0.670	0.868	1.000
3	0.083	0.044	0.063	0.036	0.024	0.035	0.061	0.175	0.277	0.456	0.577	0.791	1.000
4	0.043	0.088	0.053	0.068	0.041	0.040	0.048	0.114	0.225	0.357	0.536	0.732	1.000
5	0.130	0.061	0.103	0.074	0.082	0.062	0.072	0.089	0.210	0.309	0.497	0.704	1.000
6	0.229	0.160	0.103	0.131	0.106	0.114	0.104	0.148	0.174	0.314	0.457	0.694	1.000
7	0.537	0.280	0.237	0.191	0.176	0.166	0.182	0.182	0.308	0.288	0.459	0.690	1.000
8	0.320	0.642	0.342	0.432	0.319	0.257	0.295	0.334	0.321	0.488	0.405	0.677	1.000
9	0.815	0.545	0.759	0.564	0.678	0.541	0.466	0.545	0.622	0.588	0.695	0.621	1.000
10	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

SENSITIVITY TO NATURAL MORTALITY

12/ 6/80

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
2	-0.686	-0.850	-0.695	-0.388	-0.551	-0.437	-0.611	-0.585	-0.617	-0.457	-0.338	-0.187	0.000
3	-0.779	-0.567	-0.686	-0.536	-0.345	-0.421	-0.346	-0.513	-0.473	-0.449	-0.316	-0.182	0.000
4	-0.411	-0.619	-0.467	-0.530	-0.401	-0.311	-0.331	-0.356	-0.425	-0.378	-0.306	-0.174	0.000
5	-0.484	-0.341	-0.507	-0.404	-0.412	-0.338	-0.290	-0.327	-0.399	-0.353	-0.289	-0.170	0.000
6	-0.503	-0.371	-0.312	-0.420	-0.346	-0.336	-0.316	-0.294	-0.366	-0.355	-0.270	-0.168	0.000
7	-0.471	-0.390	-0.294	-0.296	-0.329	-0.280	-0.275	-0.280	-0.332	-0.357	-0.276	-0.168	0.000
8	-0.254	-0.335	-0.260	-0.259	-0.248	-0.235	-0.229	-0.228	-0.233	-0.285	-0.270	-0.165	0.000
9	-0.189	-0.137	-0.189	-0.139	-0.172	-0.145	-0.138	-0.146	-0.152	-0.146	-0.166	-0.154	0.000
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

PARTIAL DERIVATIVES OF POPULATION NUMBERS FOR YOUNGEST AGE GROUP

	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
2	0.091	0.032	0.043	0.355	0.126	0.151	0.186	0.139	0.041	0.104	0.155	0.113	1.000
3	0.031	0.143	0.061	0.087	0.222	0.127	0.300	0.113	0.148	0.077	0.170	0.130	0.887
4	0.199	0.083	0.154	0.071	0.157	0.201	0.284	0.313	0.210	0.197	0.191	0.171	0.717
5	0.161	0.323	0.190	0.216	0.214	0.325	0.098	0.252	0.089	0.125	0.151	0.183	0.557
6	0.128	0.180	0.204	0.154	0.107	0.196	0.149	0.079	0.102	0.048	0.145	0.105	0.511
7	0.046	0.207	0.280	0.142	0.161	0.132	0.186	0.118	0.025	0.040	0.036	0.066	0.264
8	0.140	0.257	0.081	0.189	0.043	0.130	0.072	0.118	0.052	0.008	0.019	0.023	0.160
9	0.058	0.516	0.017	0.341	0.046	0.050	0.141	0.046	0.059	0.039	0.007	0.026	0.050
10	1.000	0.942	0.353	0.682	0.250	0.153	0.049	0.119	0.047	0.069	0.038	0.014	0.033

OVERALL SENSITIVITY OF RECRUITMENT

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	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	1977
	0.84	1.01	0.89	0.63	0.75	0.69	0.77	0.80	1.00	0.94	1.05	1.24	1.38

OVERALL SENSITIVITY INDEX: 0.921780309

4. Estimating the Coefficients of the Von Bertalanffy Growth Equation (VONB).

The function VONB estimates the coefficients of the Von Bertalanffy growth equation, as well as the corresponding variances, by using the Marquardt method (Ricker, 1975; Bard, 1974). Length-at-age data are the only data required for executing this function. But VONB can also be used to fit a Bertalanffy relationship to weight data.

Input Information.

The input information required by the function VONB consists of:

n : the total number of observations;

L_t : length at age t ,

Algorithm.

The Von Bertalanffy growth equation is a non-linear model of the form

$$\hat{L}_t = L_\infty (1 - e^{-K(t - t_0)}) \pm \epsilon \quad [4.1]$$

where L_∞ , K and t_0 are the unknown parameters. In general, we refer to L_∞ as the 'mean asymptotic length', to K as the 'Brody Growth Coefficient' and to t_0 as the 'hypothetical age at which the fish would have been zero length if it had always grown in the manner described by the equation' (see Ricker, 1975).

By using the Marquardt algorithm, we find the values of L_∞ , K and t_0 which minimize the sum of squares of the residuals

$$S^2 = \sum_{t=1}^n (\hat{L}_t - L_t)^2 \quad .$$

The Marquardt algorithm is an iterative process which requires initial estimates of the coefficients. These are calculated from the regression of L_{t+1} on L_t (slope b , constant a) as follows:

$$L_\infty = a / (1 - b) \quad , \quad [4.2]$$

$$K = -\ln(b) \quad , \quad [4.3]$$

$$t_0 = \bar{t} + \frac{1}{K} \ln \left(1 - \frac{\bar{L}}{L_\infty} \right) \quad . \quad [4.4]$$

Variance estimates are calculated as

$$V_\theta = S^2 \left[[\partial \hat{L}_t / \partial \theta] [\partial \hat{L}_t / \partial \theta]^T \right]^{-1} \quad ,$$

where

- θ is the vector of parameters: $[L_\infty \ K \ t_0]$;
- $[\partial \hat{L}_t / \partial \theta]$ is a 3 by n matrix representing the partial derivative of \hat{L}_t ($t=1, \dots, n$) with respect to each parameter;
- V_θ is a 3 by 3 matrix referred to as the variance-covariance matrix.

The correlation matrix of the estimated parameters is then calculated from the variance-covariance matrix.

An alternative formulation.

Equation [4.1] can be rewritten as

$$\hat{L}_t = \frac{\omega}{K} (1 - e^{-K(t-t_0)}) \pm \epsilon \quad , \quad [4.5]$$

where ω , K and t_0 are the three unknowns to be estimated by using the Marquardt algorithm. The estimates of ω and K can be used to estimate L_∞ :

$$\hat{L}_\infty = \hat{\omega} / \hat{K} \quad .$$

Gallucci and Quinn (1979) suggest that the new parameter ω is more robust than either K or L_∞ . They recommend the use of parameter ω as a means to compare statistically growth between regional stocks (see also Kingsley et al., 1980).

Output.

In order to monitor the convergence of the Marquardt algorithm, the following information is printed: (the user is referred to the Sharp APL reference manual for statistical functions if additional information is necessary)

- A) the sum of squares and the eigenvalues of the moment matrix, as calculated by using the initial parameter estimates;
- B) a message indicating which convergence criterion has been met: namely, the iteration is stopped if
 - the relative change in each parameter is less than 0.00001,
 - the relative change in the sum of squares is less than 0.00001,
 - the total number of iterations exceeds 20;

- C) the statistics of the final iteration:
 - the determinant of the matrix which must be inverted in order to calculate the correction vector;
 - the angle in degrees between the correction vector of Gauss linearization and the correction vector of steepest descent;
 - the new parameter estimates and the final sum of squares;
 - the value of lambda (λ), a coefficient used in the Marquardt algorithm for interpolating between the correction vectors of Gauss linearization and of steepest descent;
- D) a summary table giving, for each parameter, the final estimated value, the standard error and the computed t-value (order of the parameters is L_∞ , K and t_0);
- E) the correlation matrix of the estimated parameters;
- F) the variance of the residuals, R-squared, R-bar-squared (i.e. R adjusted for the number of degrees of freedom), the Durbin-Watson statistic (for testing the significance of autocorrelation in the residuals) and a table of residuals;
- G) a graph showing the residuals as a function of age and a graph showing the observed and the estimated length at age.

Input format.

In order to initiate the function, type in VONB. Then you will be prompted for input as follows:

AGES?

-enter a vector whose values represent the ages corresponding to the length data.

ERRORS: -all entries must be positive;
-there must be at least 4 values in the vector entered.

LENGTH AT AGE?

-enter a vector whose values represent the length data corresponding to the ages provided in the first vector entered.

ERRORS: -all entries must be positive;
-the vector entered must have the same length as the vector of ages.

THE FORMS OF THE EQUATION FOR FITTING ARE:

1. $L(T)=LINF(1-EXP(-K(T-T(0))))$
2. $L(T)=(\omega/T)(1-EXP(-K(T-T(0))))$

WHICH FORM IS TO BE FITTED(1 OR 2)?

-enter either 1 or 2 , in order to indicate which form of the equation is to be fitted.

ERRORS: -you must enter only one number;
-this number must be either 1 or 2.

YOUR OWN INITIAL VALUES (YES OR NO)?

-enter either YES or NO. If you enter YES, then the program will ask for the values of the initial estimates for the coefficients to be estimated.

ERRORS: -you must enter either YES or NO.

INITIAL VALUES (ASYMP. LENGTH, K, T(0))?

-enter three values which represent your initial estimates for L_∞ , K and t_0 .

ERRORS: -you must enter only three numbers;
-the first two values have to be positive.

AN EXAMPLE.

FILEΔFETCH '2719067 PFΔVONB'

AGES

2 2 3 4 4 4 5 6 6 7 8 9 10 10 11 12 13 14 14 15 16 16

LENGTHS

0.77 0.81 1.52 2.3 2.36 2.32 3.01 3.65 3.81 4.51 5.37 6.65 7.19 7.3 7.75 7.13 8.12 7.82 7.9 9.42
9.82 9.7

VONB

AGES?

AGES

LENGTH AT AGE?

LENGTHS

THE FORMS OF THE EQUATION FOR FITTING ARE:

1. $L(T) = \text{LINE}(1 - \text{EXP}(-K(T-T(0))))$
2. $L(T) = (\omega/T)(1 - \text{EXP}(-K(T-T(0))))$

WHICH FORM IS TO BE FITTED(1 OR 2)?

2

YOUR OWN INITIAL VALUES (YES OR NO) ?

NO

VON BERTALANFFY PARAMETERS

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ORDER OF PARAMETERS IS AS FOLLOWS:

1. ω
2. BRODY GROWTH COEFFICIENT
3. $T(0)$

INITIAL PARAMETER VALUES

1.000923
0.063947
2.035363

PRELIMINARY ANALYSIS

INITIAL SUM OF SQUARES 8.2619663E0

EIGENVALUES OF MOMENT MATRIX

5.2788667E1
3.0846627E0
1.8029452E4

ITERATION STOPS. RELATIVE CHANGE IN EACH PARAMETER LESS THAN 0.00001.

ITERATION NUMBER 2 (FINAL ITERATION)

DETERMINANT 1.3125709E3
ANGLE IN SCALED COORDINATES 2.92 DEGREES.
NEW PARAMETER ESTIMATES

1.012151
0.065994
1.400601

NEW SUM OF SQUARES 3.6909289E0
LAMBDA 1.00000E1

APPROXIMATE STATISTICS FROM LINEAR THEORY

EST. PAR.	STD. ERR.	T-VALUE
1.012151	0.106946	9.464095
0.065994	0.017923	3.682011
1.400601	0.299104	4.682659

CORRELATION MATRIX OF THE ESTIMATED PARAMETERS

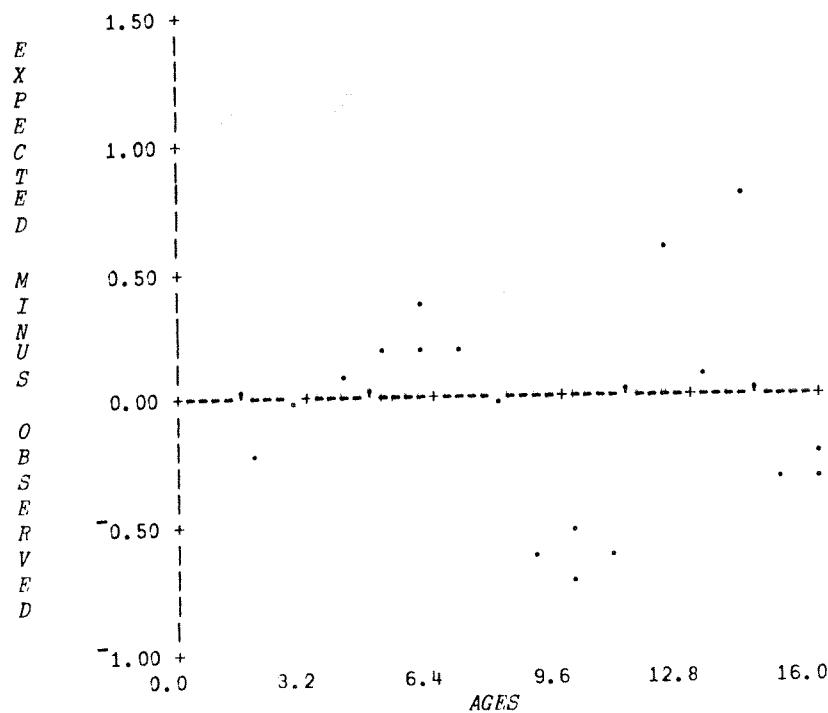
1.000000	0.978405	0.834029
0.978405	1.000000	0.753207
0.834029	0.753207	1.000000

VARIANCE OF RESIDUALS	0.194259
R SQUARED	0.980427
R-BAR SQUARED	0.978366
DURBIN-WATSON STATISTIC	1.070183

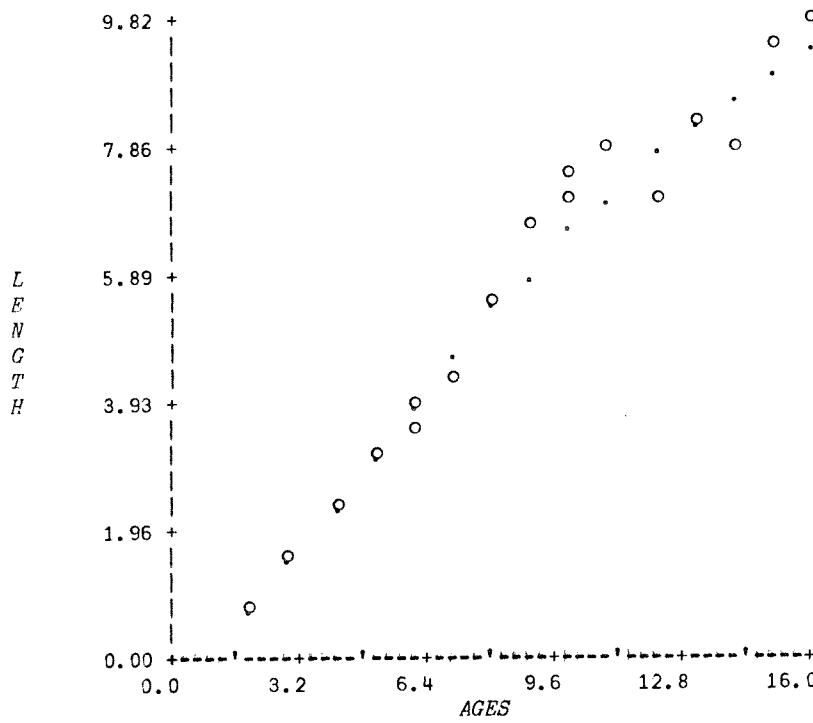
OBSERVED Y	ESTIMATED Y	RESIDUALS
0.770000	0.594840	0.175160
0.810000	0.594840	0.215160
1.520000	1.536327	-0.016327
2.300000	2.417687	-0.117687
2.360000	2.417687	-0.057687
2.320000	2.417687	-0.097687
3.010000	3.242761	-0.232761
3.650000	4.015143	-0.365143
3.810000	4.015143	-0.205143
4.510000	4.738197	-0.228197
5.370000	5.415075	-0.045075
6.650000	6.048725	0.601275
7.190000	6.641908	0.548092
7.300000	6.641908	0.658092
7.750000	7.197209	0.552791
7.130000	7.717045	-0.587045
8.120000	8.203684	-0.083684
7.820000	8.659243	-0.839243
7.900000	8.659243	-0.759243
9.420000	9.085709	0.334291
9.820000	9.484940	0.335060
9.700000	9.484940	0.215060

ESTIMATE OF THE ASYMPTOTIC LENGTH IS: 15.337018

RESIDUALS



OBSERVED AND EXPECTED LENGTH AT AGE



5. Computation of Yield from a Given Recruitment (BEVHOLT)

This program estimates the equilibrium yield by the method of Beverton and Holt. In other words, the program calculates population numbers and biomass, as well as catch numbers and weight from a given recruitment. The control parameters of the model are 1) the growth characteristics of the stock, 2) age of the recruits, 3) age at first capture and 4) maximum age attained. The program generates a table showing numbers and biomass of exploited population, catch numbers, catch biomass (yield) and the average weight for predetermined values of the instantaneous rate of fishing mortality. The program also calculates relative sensitivity coefficients of computed quantities with respect to certain parameters.

Input information.

The input information required by the program consists of (the notation follows Gulland, 1969):

- M : the instantaneous rate of natural mortality;
- t_R : age of the recruits (i.e. average age at which fish enter the "target" population);
- t_c : age at first capture ($t_c > t_R$);
- t₀ : the hypothetical age at which the fish would have been zero weight if it had always grown according to the assumed Brody's relationship;
- t_L : maximum age attained;
- W_∞ : the average asymptotic weight of a fish;
- K : the Brody growth coefficient.

Algorithm.

For predetermined values of the instantaneous rate of fishing mortality (F), the following calculations are performed by the method of Beverton and Holt (1957, see page 310):

Average numbers per recruit in the exploited phase :

$$\left(\frac{\bar{N}}{R} \right) = \frac{e^{-M(t_c-t_R)}}{F+M} (1 - e^{-(F+M)(t_L-t_c)}).$$

Average biomass per recruit for fish in the exploited phase:

$$\left(\frac{\bar{B}}{R} \right) = e^{-M(t_c-t_R)} W_{\infty} \times \frac{\sum_{n=0}^3 U_n e^{-nK(t_c-t_0)} (1 - e^{-(F+M+nK)(t_L-t_c)})}{F + M + nK}$$

$$\text{where } U_0 = 1, U_1 = -3, U_2 = 3, U_3 = -1.$$

Catch per recruit :

$$\left(\frac{C}{R} \right) = F \frac{\bar{N}}{R} .$$

Average weight of fish caught:

$$\bar{W} = (Y/R) / (C/R) .$$

Yield per recruit:

$$\left(\frac{Y}{R} \right) = F \frac{\bar{B}}{R} .$$

Instantaneous fishing mortality at maximum yield per recruit (F_{max}):

$$\begin{aligned} \text{Let } A &= t_c - t_R & U_0 &= 1 \\ B &= t_c - t_0 & U_1 &= -3 \\ C &= t_L - t_c & U_2 &= 3 \\ D &= F + M + nK & U_3 &= -1 \end{aligned}$$

Then the value of F_{max} can be determined from the slope

$$X = \partial \left(\frac{Y}{R} \right) / \partial F =$$

$$W_{\infty} e^{-MA} \sum_{n=0}^3 \left[U_n e^{-nKB} \left(\frac{CF e^{-DC}}{D} + (1 - e^{-DC}) \left(\frac{D-F}{D^2} \right) \right) \right].$$

F_{\max} is the value of F in X which ensures that $X = 0$. In practice, F_{\max} is found by the method of "false position" (Seber, 1973):

- 1) Find an initial value F_{\max}^0 and calculate X with $F = F_{\max}^0$. In order to determine F_{\max}^0 , the vector of calculated yields per recruit is searched for a maximum. The value of F which corresponds to the maximum yield per recruit is taken as the starting value for the iterative process.

2) Let $F_{\max}^1 = F_{\max}^0 - 0.005$ and calculate $X(F_{\max}^1)$.

- 3) For all $i > 1$, calculate

$$F_{\max}^i = F_{\max}^{i-1} - H^{i-1},$$

where

$$H^{i-1} = \frac{X(F_{\max}^{i-1}) (F_{\max}^{i-1} - F_{\max}^{i-2})}{X(F_{\max}^{i-1}) - X(F_{\max}^{i-2})}.$$

Exit the iterative process when

$$|X(F_{\max}^i)| < 1.0 \times 10^{-6}.$$

Convergence is usually rapid and direct. However, the value of F_{\max} may become excessively large when the yield-per-recruit curve is nearly asymptotic. In that case, the iterative procedure will produce values of F_{\max} which exceed the computing capacity of the computer; in addition, convergence then becomes much slower. In order to avoid these problems, the iteration is aborted and a warning message is issued when F_{\max} is seen to take a value greater than 10 or when the number of iterations exceeds 30.

Determination of $F_{0.1}$:

Similarly, $F_{0.1}$ is the value of F which ensures that $X' = 0.1 \times SL$, where SL is the value of the slope X at $F = 0$. The iterative procedure for the determination of $F_{0.1}$ appears as:

- 1) Find an initial value $F_{0.1}^0 = 0.7 F_{\max}$ and calculate

$$Q(F_{0.1}^0) = X(F_{0.1}^0) - (0.1 \times SL).$$

If convergence is not achieved during the estimation of F_{\max} , $F_{0.1}^0$ is given the arbitrary value 0.1.

- 2) Let $F_{0.1}^1 = F_{0.1}^0 - 0.005$ and calculate

$$Q(F_{0.1}^1) = X(F_{0.1}^1) - (0.1 \times SL).$$

- 3) For all $i > 1$, calculate

$$F_{0.1}^i = F_{0.1}^{i-1} - H^{i-1}$$

where

$$H^{i-1} = \frac{Q(F_{0.1}^{i-1}) (F_{0.1}^{i-1} - F_{0.1}^{i-2})}{Q(F_{0.1}^{i-1}) - Q(F_{0.1}^{i-2})}.$$

- 4) Exit the iterative process when

$$|Q(F_{0.1}^i)| < 1.0 \times 10^{-6}.$$

Sensitivity coefficients.

For each quantity computed in the yield per recruit analysis, sensitivity coefficients are calculated with respect to M, K, t_0 , t_R and t_c . All sensitivity coefficients are approximated by finite difference. For a given F-value, the sensitivity coefficients are evaluated by taking the partial derivative of (N/R), (B/R), W, (C/R) and (Y/R) with respect to a chosen parameter, say θ . Therefore, the relative sensitivity coefficients are calculated as (assuming R = 1):

$$\bar{XN}_\theta = \theta \frac{\partial \bar{N}}{\partial \theta} / \bar{N} \frac{\partial \theta}{\partial \theta},$$

$$\bar{XB}_\theta = \theta \frac{\partial \bar{B}}{\partial \theta} / \bar{B} \frac{\partial \theta}{\partial \theta},$$

$$\bar{XC}_\theta = \theta \frac{\partial \bar{C}}{\partial \theta} / \bar{C} \frac{\partial \theta}{\partial \theta},$$

$$\bar{XW}_\theta = \theta \frac{\partial \bar{W}}{\partial \theta} / \bar{W} \frac{\partial \theta}{\partial \theta},$$

$$\bar{XY}_\theta = \theta \frac{\partial \bar{Y}}{\partial \theta} / \bar{Y} \frac{\partial \theta}{\partial \theta}.$$

For each parameter θ considered, a table of sensitivity coefficients is constructed. Each row of that table contains the value of \bar{XN}_θ , \bar{XB}_θ , \bar{XC}_θ , \bar{XW}_θ and \bar{XY}_θ for a given F.

The program also calculates the sensitivity of F_{\max} and $F_{0.1}$ with respect to M, K, t_0 , t_R and t_c . That is,

$$\bar{XF}_\theta = (\partial F_{\max} / \partial \theta) \times (\theta / F_{\max}),$$

$$\bar{XXF}_\theta = (\partial F_{0.1} / \partial \theta) \times (\theta / F_{0.1}),$$

where θ is the chosen parameter.

Input format.

In order to initiate the program, type BEVHOLT. Then you will be prompted for input as follows:

NATURAL MORTALITY?

-enter a number representing the instantaneous rate of natural mortality (M).

ERRORS: -you must enter a single value;
-the entry must be greater than zero.

ASYMPTOTIC WEIGHT? (GR)

-enter a number representing the asymptotic weight of a fish.

ERRORS: -you must enter a single value;
-the entry must be greater than zero.

BRODY COEFFICIENT (PARAMETER K OF BRODY'S EQUATION?)

-enter a value representing the Brody growth coefficient.

ERRORS: -you must enter a single value;
-the entry must be a value between 0 and 1.

HYPOTHETICAL AGE AT WHICH THE FISH WOULD HAVE ZERO WEIGHT ACCORDING TO BRODY'S EQUATION?

-enter a value representing t_0 .

ERRORS: -you must enter a single value.

AGE OF RECRUITMENT TO STOCK?

-enter a value representing the average age at which fish enter the "target" population.

ERRORS: -you must enter a single value;
-the entry must be greater than zero.

AGE AT FIRST CAPTURE?

-enter a value representing the age at first capture (t_c).

ERRORS: -you must enter a single value;
-the entry must be greater than zero.

MAXIMUM AGE ATTAINED?

-enter a value representing the maximum age (t_L).

ERRORS: -you must enter a single value;
-the entry must be greater than zero.

ENTER A VECTOR OF ZEROS AND/OR ONES TO INDICATE WITH RESPECT TO WHICH PARAMETERS THE SENSITIVITY COEFFICIENTS ARE TO BE CALCULATED:

NATURAL MORTALITY
BRODY COEFFICIENT
AGE AT ZERO WEIGHT
AGE AT FIRST CAPTURE
AGE OF RECRUITMENT

-enter a value of 1 to indicate that sensitivity coefficients are to be calculated with respect to this parameter. Enter a value of 0 to delete calculation of sensitivity coefficients.

ERRORS: -the entry must be a vector of dimension 5;
-all entries must be 0 or 1.

MAXIMUM FISHING MORTALITY IN OUTPUT TABLE?

-enter a positive value which indicates the maximum fishing mortality for which we want to perform the calculations. This value is used to control the amount of printing. Reasonable values for this entry range between 0.5 and 2.0. A value smaller than 0.1 forces the program to find the value of $F_{0.1}$ and F_{\max} and to calculate the corresponding yield per recruit; this will yield the smallest amount of output. A value greater than 2.0 may generate a large amount of output when sensitivities are requested for all parameters.

ERRORS: -you must enter a single value;
-the entry must be greater than zero (the program will reset this entry to a value of 5.0 if a value greater than 5.0 is entered).

AN EXAMPLE.

FILE@FETCH '2719067 PF@YIELD'

BEVHOLT
NATURAL MORTALITY?
□:
 .1
ASYMPTOTIC WEIGHT?(GR)
□:
 2867
BRODY COEFFICIENT (PARAMETER K OF BRODY'S EQUATION)?
□:
 .095
HYPOTHETICAL AGE AT WHICH THE FISH WOULD HAVE ZERO WEIGHT ACCORDING TO BRODY'S EQUATION?
□:
 -.815
AGE OF RECRUITMENT TO STOCK?
□:
 3.72
AGE AT FIRST CAPTURE?
□:
 3.72
MAXIMUM AGE ATTAINED?
□:
 15

ENTER A VECTOR OF ZEROS AND\OR ONES TO INDICATE WITH RESPECT TO WHICH PARAMETERS THE SENSITIVITY COEFFICIENTS ARE TO BE CALCULATED:

NATURAL MORTALITY
BRODY COEFFICIENT
AGE AT ZERO WEIGHT
AGE AT FIRST CAPTURE
AGE OF RECRUITMENT

□:
 1 1 1 1 1
MAXIMUM FISHING MORTALITY IN OUTPUT TABLE
□:
 .2

FISHING MORTALITY	EXP-POP NUMBERS	EXP-POP BIOMASS	CATCH NUMBERS	YIELD (GR)	AVG. WEIGHT (GR)
0.000	6.763	3935	0.000	0	0
0.100	4.476	2149	0.448	215	480
F0.1---	0.148	3.783	0.561	246	438
	0.200	3.220	0.644	257	399
FMAX---	0.223	3.016	0.672	258	384

SENSITIVITY TO NATURAL MORTALITY

FISHING MORTALITY	EXP-POP NUMBERS	EXP-POP BIOMASS	CATCH NUMBERS	YIELD (GR)	AVG. WEIGHT (GR)
0.000	-0.460	-0.650	0.000	0.000	0.000
0.100	-0.368	-0.559	-0.368	-0.559	-0.191
F0.1---	0.148	-0.330	-0.514	-0.330	-0.185
	0.200	-0.294	-0.468	-0.294	-0.175
FMAX---	0.223	-0.279	-0.449	-0.279	-0.169

SENSITIVITY TO BRODY COEFFICIENT(K)

FISHING MORTALITY	EXP-POP NUMBERS	EXP-POP BIOMASS	CATCH NUMBERS	YIELD (GR)	AVG. WEIGHT (GR)
0.000	0.000	1.715	0.000	0.000	0.000
0.100	0.000	1.804	0.000	1.804	1.804
F0.1---	0.148	0.000	1.848	0.000	1.848
	0.200	0.000	1.894	0.000	1.894
FMAX---	0.223	0.000	1.914	0.000	1.914

SENSITIVITY TO AGE AT ZERO WEIGHT

FISHING MORTALITY	EXP-POP NUMBERS	EXP-POP BIOMASS	CATCH NUMBERS	YIELD (GR)	AVG. WEIGHT (GR)
F0.1---	0.000	0.000	0.000	0.000	0.000
	0.100	0.000	0.000	0.170	0.170
	0.148	0.000	0.000	0.182	0.182
	0.200	0.000	0.000	0.196	0.196
	FMAX---	0.223	0.000	0.201	0.201

SENSITIVITY TO AGE AT FIRST CAPTURE

FISHING MORTALITY	EXP-POP NUMBERS	EXP-POP BIOMASS	CATCH NUMBERS	YIELD (GR)	AVG. WEIGHT (GR)
F0.1---	0.000	0.550	0.000	0.000	0.000
	0.100	0.459	0.000	0.159	0.618
	0.148	0.432	0.000	0.276	0.708
	0.200	0.411	0.000	0.388	0.800
	FMAX---	0.404	0.000	0.434	0.839

SENSITIVITY TO AGE OF RECRUITMENT

FISHING MORTALITY	EXP-POP NUMBERS	EXP-POP BIOMASS	CATCH NUMBERS	YIELD (GR)	AVG. WEIGHT (GR)
F0.1---	0.000	0.372	0.000	0.000	0.000
	0.100	0.372	0.372	0.372	0.000
	0.148	0.372	0.372	0.372	0.000
	0.200	0.372	0.372	0.372	0.000
	FMAX---	0.372	0.372	0.372	0.000

SENSITIVITY OF FMAX AND F0.1 TO PARAMETERS

PARAMETERS	FMAX	F0.1
SENSITIVITY TO NATURAL MORTALITY	0.3265	0.0173
SENSITIVITY TO BRODY COEFFICIENT(K)	0.3275	0.7684
SENSITIVITY TO AGE AT ZERO WEIGHT	0.0993	0.1094
SENSITIVITY TO AGE AT FIRST CAPTURE	0.7503	0.5468
SENSITIVITY TO AGE OF RECRUITMENT	0.0000	0.1186

6. Computation of Yield from a Given Recruitment by the Method of Thompson and Bell (YIELD).

This program uses the Thompson and Bell's method to determine the yield for a single recruit at various levels of fishing mortality (ref. Ricker, 1975). The program generates a table showing catch numbers, yield (kg), average weight and a relative index of the yield per unit-effort. For these calculated quantities, the relative sensitivities are calculated with respect to input parameters.

Input information.

The input information required by the program consists of:

b : age of youngest age-group;

m : age of oldest age-group;

W_i ($i = b, \dots, m$) : weight at age i , in kilograms;

r_i ($i = b, \dots, m$) : a value, between 0 and 1, which indicates the proportion of fishing mortality which can be allocated to age group i (often referred to as "partial recruitment" to fishing mortality);

M_i ($i = b, \dots, m$) : instantaneous rate of natural mortality for age i .

Algorithm.

For predetermined values of instantaneous rate of fishing mortality (F_*), the following calculations are performed by the method of Thompson and Bell:

Catch per recruit (in numbers):

$$C_* = \sum_{i=b}^m N_i \frac{F_i}{Z_i} (1 - e^{-Z_i}) ,$$

where $F_i = r_i F_*$.

and $Z_i = F_i + M_i$;

also, the N_i are calculated from the recursive relationship (by assuming that $N_b = 1$):

$$N_i = N_{i-1} e^{-Z_{i-1}} .$$

Yield per recruit (kg):

$$Y_* = \sum_{i=b}^m W_i N_i \frac{F_i}{Z_i} (1 - e^{-Z_i}) ,$$

where F_i , Z_i and N_i are defined as in C. .

Average weight (kg) of fish caught:

$$\bar{W} = Y_* / C_* .$$

Relative index of yield per unit of effort:

$$I = \left[\frac{Y_*}{F_*} \right] / \max_F \left[\frac{Y_*(F)}{F} \right] ,$$

where F takes the values of fishing mortality which appear in the output table.

Determination of F_{\max} :

F_{\max} can be determined from the slope $X = \frac{\partial(Y_*)}{\partial F_*}$. So F_{\max} is the value of F_* in X which ensures that $X = 0$. In practice, X is found by finite difference and F_{\max} is evaluated by the method of "false position" (Seber, 1973):

- 1) Find an initial value, F_{\max}^0 , and calculate X with $F_* = F_{\max}^0$. In order to determine F_{\max}^0 , the vector of calculated yields per recruit is searched for a maximum. The value of F which corresponds to the maximum yield per recruit is taken as the starting value for the iterative process.
- 2) Let $F_{\max}^1 = F_{\max}^0 - 0.005$ and calculate $X(F_{\max}^1)$.
- 3) For all $i > 1$, calculate

$$F_{\max}^i = F_{\max}^{i-1} - H^{i-1} ,$$

where

$$H^{i-1} = \frac{X(F_{\max}^{i-1}) (F_{\max}^{i-1} - F_{\max}^{i-2})}{X(F_{\max}^{i-1}) - X(F_{\max}^{i-2})} .$$

Exit the iterative process when

$$|X(F_{\max}^i)| \leq 1.0 \times 10^{-9} .$$

Then the final value F_{\max}^i is taken as the true value, F_{\max} .

When the yield-per-recruit curve is nearly asymptotic, the value of F_{\max} may become excessively large and convergence becomes much slower. In order to avoid memory overflow, the iterative process is aborted and a warning message is issued when F_{\max} is seen to take a value greater than 10 or when the number of iterations exceeds 30.

Determination of $F_{0.1}$:

Similarly, $F_{0.1}^0$ is the value of F in X which ensures that $\bar{X} = 0.1 \times SL$, where SL is the slope $\partial(Y)/\partial F$ at $F = 0$. The iterative procedure for the determination of $F_{0.1}^0$ appears as:

- 1) find an initial value, $F_{0.1}^0 = 0.7 F_{\max}$, and calculate

$$Q(F_{0.1}^0) = X(F_{0.1}^0) - (0.1 \times SL)$$

If convergence to a final value is not achieved during the estimation of F_{\max}^0 , $F_{0.1}^0$ is given the arbitrary value 0.1.

- 2) let $F_{0.1}^1 = F_{0.1}^0 - 0.005$ and calculate

$$Q(F_{0.1}^1) = X(F_{0.1}^1) - (0.1 \times SL)$$

- 3) For all $i > 1$, calculate

$$F_{0.1}^i = F_{0.1}^{i-1} - H^{i-1},$$

where

$$H^{i-1} = \frac{Q(F_{0.1}^{i-1}) (F_{0.1}^{i-1} - F_{0.1}^{i-2})}{Q(F_{0.1}^{i-1}) - Q(F_{0.1}^{i-2})}.$$

Exit the iterative process when

$$|Q(F_{0.1}^i)| \leq 1.0 \times 10^{-9}.$$

Then the final value $F_{0.1}^i$ is taken as the true value, $F_{0.1}$.

Sensitivity coefficients.

Since input parameters (mortality, weight-at-age, partial recruitment) are expressed as vectors, the relative sensitivities of calculated quantities are approximated from equation [6]. In other words, the sensitivities

are calculated from the response of the calculated quantities to small relative perturbations in each element of the input vector under consideration. This can be shown to be equivalent to the response obtained from a small relative perturbation of the mean of the input vector.

Input format.

In order to initiate the program, you have to type YIELD. Then you will be prompted for input as follows:

WEIGHT AT AGE? (KG)

-enter a vector of weights at age (W_i , $i = b, \dots, m$).

ERRORS: -the values entered must be positive.

PARTIAL RECRUITMENT?

-enter the values r_i ($i = b, \dots, m$) which indicate the proportion of fishing mortality which can be allocated to age-group i .

ERRORS: -the vector must have the same length as the vector of weights at age;
-the values entered must all be greater than or equal to zero and less than or equal to one.

NATURAL MORTALITY?

-enter either a single number (M) or a vector which assigns a separate instantaneous rate of natural mortality (M_i , $i = b, \dots, m$) for each age-group i . When a single number is entered, the M_i are set equal to M for all i .

ERRORS: -the entry must either be a single number or a vector which has the same length as the vector of weights at age;
-all entries must be greater than zero.

SENSITIVITY COEFFICIENTS FOR THE FOLLOWING PARAMETERS:

NATURAL MORTALITY
PARTIAL RECRUITMENT
WEIGHT AT AGE

-enter a vector of zeros and/or ones, to indicate your choice of parameters for which the sensitivity coefficients are to be calculated.

ERRORS: -the vector must have exactly three elements;
-the values entered must be zeros or ones.

MAXIMUM FISHING MORTALITY IN OUTPUT TABLE?

-enter a positive value which indicates the maximum fishing mortality for which we want to perform the calculations. This value is used to control the amount of printing. Reasonable

values for this entry range between 0.5 and 2.0. A value smaller than 0.1 forces the program to find the value of $F_{0,1}$ and F_{\max} , and to calculate the corresponding yield per recruit; this will yield the smallest amount of output. A value greater than 2.0 may generate a large amount of output when sensitivities are requested for all parameters.

ERRORS: -you must enter a single value;
-the entry must be greater than zero (the program will reset this entry to a value of 5.0 if a value greater than 5.0 is entered).

A EXAMPLE.

WG
0.766 1.017 1.727 2.512 3.287 3.994 4.806 5.607 6.481 8.046

PR
0.474 0.632 0.737 0.895 0.947 0.947 1 1 1 1

FILEΔFETCH '2719067 PFΔYIELD'

YIELD
WEIGHT AT AGE? (KG)

WG
PARTIAL RECRUITMENT?

PR
NATURAL MORTALITY?

.2

SENSITIVITY COEFFICIENTS FOR THE FOLLOWING PARAMETERS:
(ENTER A VECTOR OF ZEROS AND/OR ONES IN THIS ORDER)

NATURAL MORTALITY

PARTIAL RECRUITMENT

WEIGHT AT AGE

0 1 0

MAXIMUM FISHING MORTALITY IN OUTPUT TABLE

0.8

YIELD PER RECRUIT ANALYSIS

FISHING MORTALITY	CATCH (NUMBER)	YIELD (KG)	AVG. WEIGHT (KG)	YIELD PER UNIT EFFORT
0.1000	0.253	0.666	2.638	1.000
0.2000	0.402	0.920	2.289	0.690
F0.1---	0.2285	0.433	0.954	2.203
	0.3000	0.497	1.001	2.013
FMAX---	0.3751	0.549	1.013	1.846
	0.4000	0.563	1.012	1.797
	0.5000	0.611	0.995	1.629
	0.6000	0.648	0.970	1.497
	0.7000	0.678	0.944	1.392
	0.8000	0.703	0.919	1.308

SENSITIVITY TO PARTIAL RECRUITMENT

FISHING MORTALITY	CATCH (NUMBER)	YIELD (KG)	AVG. WEIGHT (KG)	YIELD PER UNIT EFFORT
0.1000	0.750	0.603	0.147	0.000
0.2000	0.581	0.310	0.271	0.293
F0.1---	0.2285	0.545	0.243	0.301
	0.3000	0.469	0.106	0.363
FMAX---	0.3751	0.410	0.000	0.410
	0.4000	0.394	0.028	0.422
	0.5000	0.342	0.113	0.455
	0.6000	0.305	0.164	0.469
	0.7000	0.277	0.194	0.471
	0.8000	0.255	0.209	0.464

SENSITIVITY OF FMAX AND F0.1 TO PARAMETERS.

PARAMETERS	FMAX	F0.1
SENSITIVITY TO PARTIAL RECRUITMENT	-0.9990	-0.9988

7. Calculation of total mortality by Paloheimo's method (PALOHEIMO).

The function PALOHEIMO provides estimates of the instantaneous rate of total mortality (Z) by using the method of Paloheimo (Ricker, 1975). An estimate of the catchability coefficient (q) and an estimate of the instantaneous rate of natural mortality (M) are calculated by regressing Z against f , the fishing effort.

Input Information.

The input information required by the function PALOHEIMO consists of:

t_0 : the first year represented in the CPUE matrix;

t_f : final year represented in the CPUE matrix;

b : age of the youngest age-group in the CPUE matrix;

m : age of the oldest age-group in the CPUE matrix;

$U_{i,t}$: ($i = b, \dots, m$; $t = t_0, \dots, t_f$) a matrix (time- and age-specific) of catch per unit of effort (CPUE);

f_t ($t = t_0, \dots, t_f$): a vector whose values represent annual estimates of fishing effort for the period covered by the CPUE matrix.

Algorithm.

This function approximates the mean of the instantaneous mortality rates for a cohort in two consecutive years from the relationship

$$\bar{Z}_{i,t} = -\ln(U_{i+1,t+1} / U_{i,t}) .$$

An overall estimate of mortality for the period $\tau, \tau+1$ is calculated as

$$\bar{Z}_{\tau, \tau+1} = (\sum_i \bar{Z}_{i,\tau}) / (b - m + 1) .$$

Finally, the mean effort for the period $[\tau, \tau+1]$ is given by

$$\bar{f}_{\tau, \tau+1} = (f_{\tau} + f_{\tau+1}) / 2 ,$$

and the regression of $\bar{Z}_{\tau, \tau+1}$ against $\bar{f}_{\tau, \tau+1}$ will

provide an estimate of the instantaneous rate of natural mortality (M) and an estimate of the catchability coefficient (q). The regression model appears as

$$\bar{Z}_{\tau, \tau+1} = M + q \bar{f}_{\tau, \tau+1} + \epsilon ,$$

where M is the regression constant and where q appears as the slope of the regression line. Note that M and q are assumed to be constant over the whole period covered by the data.

Input Format.

In order to initiate the function, type in PALOHEIMO. Then you will be prompted for input as follows: [note that the matrix of age- and time-specific CPUE must be previously created before entering the function]

ENTER NAME OF CPUE MATRIX FOR RESEARCH VESSELS (NUM/TOW)?

-enter the name of the CPUE matrix.

ERRORS: -the name entered must represent an APL variable.

ENTER FIRST YEAR AND YOUNGEST AGE-GROUP?

-enter two numbers representing the first year and the youngest age-group represented in the CPUE matrix.

ERRORS: -you must enter exactly two numbers;
-the two values entered must be non-negative integers.

REGRESSION WITH AN INDEX OF FISHING EFFORT DESIRED?(YES OR NO)

-enter either YES or NO in order to indicate if the regression of Z against fishing effort is to be performed.

ERRORS: -you must enter either YES or NO.

ENTER VALUES OF FISHING EFFORT?

-if the answer to the preceding prompt is YES, enter a vector whose values represent annual estimates of fishing effort for the period covered by the CPUE matrix.

ERRORS: -the dimensions of the vector must be equal to the dimensions of each row of the CPUE matrix;
-each entry must be greater than zero.

Graphical Output.

The function PALOHEIMO generates a graph showing the observed points $(z_{\tau, \tau+1}, f_{\tau, \tau+1})$ and two points of the regression line (the regression line is obtained by tracing a straight line through these two points).

AN EXAMPLE

FILEAFETCH '2719067 PFΔPALOHEIM'

MAT

0.45	0.513	0.59	0.563	0.546
0.22	0.27	0.326	0.363	0.342
0.125	0.125	0.127	0.146	0.159
0.05	0.057	0.052	0.051	0.057
0.027	0.024	0.012	0.019	0.017
0.009	0.011	0.009	0.008	0.006

PALOHEIMO

ENTER NAME OF CPU MATRIX FOR RESEARCH VESSELS(NUM/TOW)

MAT

ENTER FIRST YEAR AND YOUNGEST AGE GROUP

[]:

1975 3

ESTIMATES OF TOTAL MORTALITIES(Z) 13 / 6/80

	1975	1976	1977	1978
3	0.511	0.453	0.486	0.498
4	0.565	0.754	0.803	0.825
5	0.785	0.877	0.912	0.941
6	0.734	1.558	1.007	1.099
7	0.898	0.981	0.405	1.153
3+	0.699	0.925	0.723	0.903

REGRESSION WITH AN INDEX OF FISHING EFFORT DESIRED(YES OR NO)?

YES

ENTER VALUES OF FISHING EFFORT

[]:

350 432 440 560

THERE MUST BE 5 VALUES ALL NON-NEGATIVE. RE-ENTER

[]:

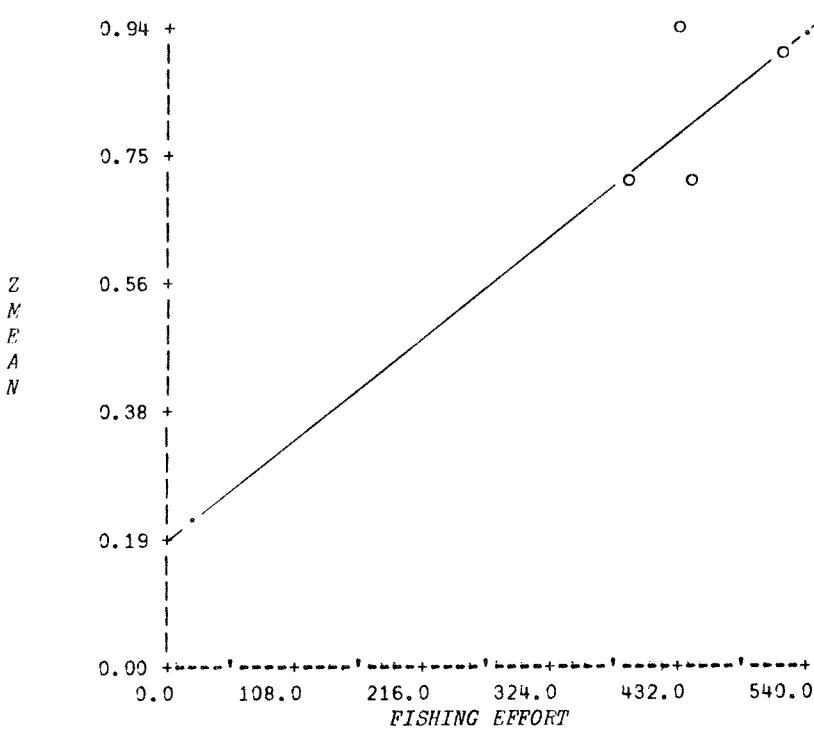
350 432 440 450 590

WAS ANOVA

WAS CORRELATION

ALIGN PAPER

MEAN OF DEPENDENT VARIABLE	0.81232			
VARIABLE	MEAN	ESTIMATED COEFFICIENT	STD. ERROR	T-VALUE
CONSTANT TERM		0.18937	0.54595	0.34686
1	448.00000	0.00139	0.00121	1.14713
MULTIPLE CORRELATION COEFFICIENT (R^2).....	0.3968456421			
CORRECTED R^2 (R^2).....	0.0952684631			
F-STATISTIC FOR SIGNIFICANCE OF REGRESSION(1, 2)	1.3159007701			
STANDARD ERROR OF THE ESTIMATE.....	0.1123210365			
DURBIN-WATSON STATISTIC.....	3.1129800356			
COEFFICIENT OF VARIATION (AT THE MEAN OF Y)...(%)	13.8271684155			



8. Estimating Survivors in the current year (SURVIVOR)

In this section, we introduce a new method for estimating survivors in the current year (W.G. Doubleday, pers. comm.). The method combines catch-at-age information and a research vessel abundance index-at-age, and produces estimates of survivors-at-age in the current year, as well as variance estimates. In input, this iterative method requires an initial estimate of survivors for the last year and for the oldest age-group, an estimate of natural mortality and a matrix of catch-at-age for all years considered. Convergence to final estimates of survivors is generally direct and rapid.

In practice, our estimation is derived from a comparison between mid-year estimates obtained from research vessels and mid-year population numbers calculated from a VPA formula. Underlying assumptions specify that 1) catch is taken uniformly during the year, 2) that the research vessel abundance index is a mid-year estimate and that 3) the instantaneous rate of natural mortality is a 'known' constant which can be applied to all years and all age-groups of the catch matrix. The survivor estimates are end-of-year estimates. In order to 'tune' the research vessel abundance index with the VPA population numbers, we need to define calibration constants which have to be estimated within a pre-defined calibration block.

Input Information.

The input information required by the function SURVIVOR consists of

- t_0 : first year represented in the catch matrix;
- t_f : final year represented in the catch matrix;
- t_1 : final year of the calibration block;
- b : age of the youngest age-group in the catch matrix;
- m : age of the oldest age-group in the catch matrix;
- i_1 : age at which fish are fully recruited to both commercial exploitation and research surveys;
- i_2 : age of the oldest age-group in the calibration block (note that $i_1 \leq i_2$);
- $C_{i,\tau}$: ($i = b, \dots, m$; $t = t_0, \dots, t_f$) catch between time t and $t+1$ and for age-group $i, i+1$. Note that τ refers to the period $t, t+1$, while i refers to age-group $i, i+1$.
- M : instantaneous rate of natural mortality.

$A_{i+5,t+5}$: ($i = b, \dots, m$; $t = t_0, \dots, t_f$) research vessel abundance index-at-age;

$S_{m,t}^0$: ($t = t_0, \dots, t_f$) an initial estimate of survivors in population for the oldest age-groups;

S_{i,t_f}^0 : ($i = b, \dots, m$) an initial estimate of survivors in population for each age-groups in the final year.

Algorithm.

From the initial estimates of survivors in the population, we calculate a series of calibration constants which are used to generate successive estimates of survivors in the population. For each iteration, the following calculations are performed: (superscripts refer to the ℓ th iterative step)

1) Calibration of the survey index:

A mid-year population estimate can be derived from the survey index as follows:

$$N_{i+5,t+5}^\ell = k_{i+5}^\ell A_{i+5,t+5} + \epsilon . \quad [8.1]$$

The k_{i+5} are calibration constants (one value for each age-group between b and i_1). From [8.1], we derive the following expression for the estimation of the calibration constant k_{i+5}^ℓ :

$$\ln k_{i+5}^\ell = \frac{\sum_{t=t_0}^{t_1} \ln(N_{i+5,t+5}^\ell) - \ln(A_{i+5,t+5})}{(t_1 - t_0 + 1)}, \quad [8.2]$$

where the summation is taken from $t = t_0$ to $t = t_1$. For $i_1 \leq i \leq i_2$, we calculate a single calibration constant, k^ℓ , from the formula

$$\ln k^\ell = \frac{\sum_{i_1}^{i_2} \sum_{t_0}^{t_1} \ln(N_{i+5,t+5}^\ell) - \ln(A_{i+5,t+5})}{(t_1 - t_0 + 1) (i_2 - i_1 + 1)}. \quad [8.3]$$

The logarithmic transformation is necessary, both for [8.2] and [8.3], in order to eliminate the influence of the order of magnitude for the survey index-at-age.

In order to estimate k_{i+5}^ℓ and k , we need mid-year estimates of the population numbers-at-age. These are derived from the iterative relationship

$$N_{i+5,t+5}^\ell = CINT_{i+5,t+5} + SINT_{i+5,t+5}^\ell \quad [8.4]$$

where CINT and SINT are calculated as

$$\text{CINT}_{i+.5, t+.5} = \sum_{j=1}^{t_f-t} C_{i+j, t+j} \frac{(e^{M(j+.5)} - e^{M(j-.5)})}{M} + C_{i, t} \frac{(e^{M/2} - 1)}{M}$$

or

$$\text{CINT}_{i+.5, t+.5} = \sum_{j=1}^{m-i} C_{i+j, t+j} \frac{(e^{M(j+.5)} - e^{M(j-.5)})}{M} + C_{i, t} \frac{(e^{M/2} - 1)}{M}$$

and

$$\text{SINT}_{i+.5, t+.5}^{\ell} = S_{i+t_f-t, t_f}^{\ell} e^{(t_f-t+.5)M}$$

or

$$\text{SINT}_{i+.5, t+.5}^{\ell} = S_{m, t+m-1}^{\ell} e^{(m-i+.5)M}$$

2) Estimating survivors in the final year and for the oldest age-groups in each year:

From the estimated $k_{i+.5}$ and k , we estimate a weighted average of the survivors in the final year from the relationship

$$\hat{S}_{i, t_f}^{\ell} = \sum_{j \leq i} w_{i, t_f, j}^{\ell} \times \hat{S}_{i, t_f, j}^{\ell},$$

where (let $j \leq i$)

$$\begin{aligned} \hat{S}_{i, t_f, j}^{\ell} &= (k_{j+.5}^{\ell} A_{j+.5, t_f-i+j+.5} \\ &\quad - \text{CINT}_{j+.5, t_f-i+j+.5}) e^{-M(i-j+.5)} \end{aligned}$$

and

$$w_{i, t_f, j}^{\ell} = (\text{Var } S_{i, t_f, j}^{\ell})^{-1} / \sum_{g=b}^i (\text{Var } S_{i, t_f, g}^{\ell})^{-1}.$$

The $S_{i, t_f, j}$ are independent estimates of the survivors in the final year, for age-group i . The number of estimates which can be evaluated is equal to the length of the cohort. Variances for successive estimates of survivors are estimated as

$$\begin{aligned} \text{Var } S_{i, t_f, j}^{\ell} &= (k_{j+.5}^{\ell} A_{j+.5, t_f-i+j+.5})^2 \text{RMS}^{\ell} \\ &\quad \times \left(1 + \frac{1}{t_1 - t_0 + 1} \right) e^{-2M(i-j+.5)} \end{aligned}$$

when $j < i_1$ and as

$$\begin{aligned} \text{Var } S_{i, t_f, j}^{\ell} &= (k_{j+.5}^{\ell} A_{j+.5, t_f-i+j+.5})^2 \text{RMS}^{\ell} \\ &\quad \times \left(\frac{1}{1 + \frac{1}{(i_2 - i_1 - 1)(t_1 - t_0 + 1)}} \right) e^{-2M(i-j+.5)} \end{aligned}$$

when $j \geq i_1$. The quantity RMS is evaluated as

$$\text{RMS}^{\ell} = \frac{\sum_{i=b}^{i_2} \sum_{t=t_0}^{t_1} \left[\frac{N_{i+.5, t+.5}^{\ell} k_{i+.5, t+.5}^{\ell} \times A_{i+.5, t+.5}}{N_{i+.5, t+.5}^{\ell}} \right]^2}{(t_1 - t_0 + 1) (i_2 - b + 1)}$$

Finally, the pooled variances of the estimated survivors for the current year are calculated as

$$\text{Var } \hat{S}_{i, t_f}^{\ell} = \left(\sum_j 1/\text{Var } S_{i, t_f, j}^{\ell} \right)^{-1}.$$

Similar formulae have been developed in order to estimate the survivors for the oldest age-groups. That is

$$\hat{S}_{m, t}^{\ell} = \sum_{j \leq i} w_{m, t, j}^{\ell} \times \hat{S}_{m, t, j}^{\ell},$$

where

$$\begin{aligned} \hat{S}_{m, t, j}^{\ell} &= (k_{j+.5}^{\ell} A_{j+.5, t-m+j+.5} - \text{CINT}_{j+.5, t-m+j+.5} \\ &\quad \times e^{-M(m-j+.5)}) \quad (j \leq m) \end{aligned}$$

and

$$w_{m, t, j}^{\ell} = (\text{Var } S_{m, t, j}^{\ell})^{-1} / \sum_{g=b}^m (\text{Var } S_{m, t, g}^{\ell})^{-1}.$$

The $S_{m, t, j}$ are independent estimates of the survivors for the oldest age-group in each year t . Variances for successive estimates of survivors are estimated as

$$\text{Var } \hat{s}_{m,t,j}^{\ell} = (k_{j+0.5}^{\ell} A_{j+0.5, t-m+j+0.5})^2 \text{RMS}^{\ell} \\ \times \left(1 + \frac{1}{t_1 - t_0 + 1} \right) e^{-2M(m-j+0.5)}$$

when $j < i_1$ and as

$$\text{Var } \hat{s}_{m,t,j}^{\ell} = (k_{j+0.5}^{\ell} A_{j+0.5, t-m+j+0.5})^2 \text{RMS}^{\ell} \\ \times \left[1 + \frac{1}{(i_2 - i_1 - 1)(t_1 - t_0 + 1)} \right] e^{-2M(m-j+0.5)}$$

when $j \geq i_1$. Finally, the pooled variances of the estimated survivors for the oldest age-groups are calculated as

$$\text{Var } \hat{s}_{m,t}^{\ell} = \left(\sum_{j} \frac{1}{\text{Var } \hat{s}_{m,t,j}^{\ell}} \right)^{-1}$$

3) Iterations ($\ell = 0, 1, \dots$):

Steps 1) and 2) are repeated in an iterative manner in order to produce successive estimates of $k_{i+0.5}$, k , S_{i,t_f} and $S_{m,t}$, as well as estimates of the corresponding variances. Convergence is considered to be achieved when

$$\left| \frac{s_{i,t}^{\ell} - s_{i,t}^{\ell-1}}{s_{i,t}^{\ell}} \right| \leq 0.001$$

for each age-group in the final year (t_f) and for each age-group m in the different years t considered.

The iterative process can also be stopped when the total number of iterations exceeds a given value (provided in input).

4) Final iteration.

At the end of the iterative process, the following information can be printed in a matrix form:

the integrated survivors: $SINT_{i+0.5, t_f}$
 the population numbers: $N_{i+0.5, t_f}$
 the estimated survivors: $S_{i,t_f,j}$ and $S_{m,t}$
 the estimated variance of
 survivors: $\text{Var } S_{i,t_f,j}$ and $\text{Var } S_{m,t}$.

The weighted estimates (time-specific) of survivors for the last age-groups and the weighted estimates (age-specific) of survivors for the last year are also printed, as well as their corresponding variance. The final estimates of $K_{i+0.5}$ (age-specific) are finally printed,

along with their corresponding variances, standard errors and degrees of freedom. In addition, the matrix of residuals ($e_{i,t}$: $i = b, \dots, m$; $t = t_0, \dots, t_f$) is calculated as

$$e_{i,t} = \ln (K_{i+0.5} A_{i+0.5, t_f}) - \ln N_{i+0.5, t_f}$$

Analysis of variance.

In order to identify the presence of systematic errors in the residuals (like the errors related to age-effects, year-effects and year-class effects), an analysis of variance is performed in the following manner:

- 1) Calculation of the sum-of-squares of residuals:

$$SSS = \sum_i \sum_t e_{i,t}^2$$

- 2) Calculation of the sum-of-squares of residuals (RES) for a regression which considers that the $e_{i,t}$ come from a model showing a year-effect, an age-effect and a year-class effect. The regression model has the form

$$e_{i,t} = \mu + a_1 \text{year.1} + \dots + a_x \text{year.x} \\ + b_1 \text{age.1} + \dots + b_y \text{age.y} \\ + c_1 \text{year-class.1} + \dots + c_z \text{year-class.z} + \epsilon$$

where $\text{year.1}, \dots, \text{year.x}$, $\text{age.1}, \dots, \text{age.y}$, $\text{year-class.1}, \dots, \text{year-class.z}$ are dummy variables taking the value of 0 or 1. Note that this model has more parameters to estimate than there are independent normal equations. This model is modified, in practice, so as to remove the singularity problems which will arise in an attempt to estimate the coefficients of the above-mentioned model. The sum-of-squares due to this regression is referred to as SS in the following text.

- 3) Calculation of the sum-of-squares (SS2) due to regression for a regression model which considers that the $e_{i,t}$ come from a model showing only a year-effect and a year-class effect.

- 4) Calculation of the sum-of-squares (SS1) due to regression for a regression model which considers that the $e_{i,t}$ come from a model showing only a year-effect. The mean-square error is calculated as

$$MS1 = SS1 / (x-1)$$

Source of variation	Sum of squares	degrees of freedom	Mean square	F value
Constant	$(\sum e_{i,t})^2 / r$	1	-	-
Age	SS4	x - 1	MS4	MS4/MSR
Year	SS1	y - 1	MS1	MS1/MSR
Year-class	SS3	z - 1	MS3	MS3/MSR
Residuals	RES	s**	MSR = RES/s	-
Total	SSS	r*	--	-

* number of residuals in the initial matrix

** $r = x + y + z + 2$

Table 5.1 . Analysis of variance for testing the presence of systematic errors in the residuals.

5) The sum-of-squares due to a year-class effect is calculated as

$$SS3 = SS2 - SS1$$

The corresponding mean-square error is then calculated as

$$MS3 = SS3 / (z-1)$$

6) The sum-of-squares due to an age-effect is calculated as

$$SS4 = SS - SS2$$

while the corresponding mean-square error has the form

$$MS4 = SS4 / (y-1)$$

Then the table of analysis of variance is constructed as shown in Table 5.1 .

Input Format.

In order to initiate the function, type in SURVIVOR. Then you will be prompted for input as follows: [note that a catch matrix ($C_{i,t}$: $i = b, \dots, m$; $t = t_0, \dots, t_f$) and a matrix of 'research vessel abundance index' ($A_{i+.5,t+.5}$: $i = b, \dots, m$; $t = t_0, \dots, t_f$) must be previously created before starting the program]

FIRST YEAR AND YOUNGEST AGE-GROUP?

-enter two numbers representing the first year and the youngest age-group in the catch matrix.

ERRORS: -you must enter exactly two numbers; -these values must be non-negative integers.

FINAL YEAR AND OLDEST AGE-GROUP IN CALIBRATION BLOCK??

-enter two numbers representing the final year and the oldest age-group of the calibration block.

ERRORS: -you must enter exactly two numbers; -these two values must be, respectively, greater than the two values entered in the preceding prompt.

AGE AT WHICH K SHOULD LEVEL OFF?

-enter a single value representing the age at which fish are fully recruited to both commercial exploitation and research surveys.

ERRORS: -you must enter one number which is greater than the value representing the youngest age-group, but smaller than or equal to the value of the oldest age-group in the calibration block.

NAME OF RESEARCH VESSEL ABUNDANCE MATRIX?

-enter the name of the matrix tabulating the research vessel abundance-index.

ERRORS: -the name entered must correspond to an APL variable;
-the variable must be a matrix;
-there should not be negative values in the matrix.

NAME OF CATCH MATRIX?

-enter the name of the catch matrix.

ERRORS: -the name entered must correspond to an APL variable;
-the variable must be a matrix;
-there should not be negative values in the matrix;
-the dimensions of this matrix must be equal to the dimensions of the 'research vessel abundance' matrix.

SURVIVORS IN POPULATION FOR LAST YEAR?

-enter a vector representing initial estimates of survivors in population for each age-groups in the last year.

ERRORS: -the vector should have the same dimensions as the dimensions of each column of the catch matrix;
-each value entered must be positive.

SURVIVORS IN POPULATION FOR OLDEST AGE-GROUPS?

-enter a vector representing initial estimates of survivors in population for the oldest age-group in each year.

ERRORS: -the vector should have the same dimension as the dimension of each row of the catch matrix;
-each value entered must be positive.

NATURAL MORTALITY?

-enter a single value representing the instantaneous rate of natural mortality for each age-group i.

ERRORS: -you must enter only one number;
-the natural mortality must be equal to or greater than zero.

MAXIMUM NUMBER OF ITERATIONS?

-enter a number representing the maximum number of iterations to be allowed in the current run.

ERRORS: -you must enter only one number;
-the value entered must be greater than zero.

ENTER 1 IF MATRIX IS TO BE PRINTED. OTHERWISE ENTER 0. ONE NUMBER IS REQUIRED IN VECTOR FOR EACH MATRIX:

INTEGRATED CATCH
INTEGRATED SURVIVORS
ESTIMATED SURVIVORS
ESTIMATED VARIANCE OF SURVIVORS
WEIGHTED SURVIVORS
OUTLIERS OF RESIDUALS

-enter a vector of 0's and/or 1's in order to indicate which matrices are to be printed.

ERRORS: -you must enter a vector containing only 0's and/or 1's;
-the input vector must have six entries.

ANALYSIS OF VARIANCE TO BE PERFORMED (YES OR NO)?

-enter either 'YES' or 'NO', in order to indicate if the analysis of variance is to be performed on the residuals.

ERRORS: -you must enter either 'YES' or 'NO'.

An Example.

FILE@FETCH '2719067 PF@SURVIVOR'

	RESEARCH VESSEL ABUNDANCE INDEX												8 / 1/80
	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	
2	47188	27436	20061	35263	9443	10968	13220	93968	14709	18790	30595	3450	
3	21477	28398	19193	14992	11476	5945	8530	9107	54558	8503	12318	17053	
4	15110	12437	17190	11707	9562	7072	3122	2537	4211	27082	6145	6985	
5	4122	9073	6438	10160	7088	4664	2918	1440	1203	2534	13822	3351	
6	765	2055	3359	4268	4087	3682	2097	1122	419	435	1252	6592	
7	278	637	1171	2008	2617	2418	2093	924	566	181	248	630	
8	94	152	379	644	1402	1382	1085	889	369	302	116	128	
9	9	60	106	234	362	774	742	507	339	228	192	58	
10	0	6	44	60	120	228	408	515	287	190	95	122	

	CATCH MATRIX												8 / 1/80
	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976	
2	210796	43630	47948	751706	70536	106916	144167	649254	29656	118301	235590	19922	
3	26450	270068	68430	79933	384467	58166	173662	71984	562616	45600	158941	161637	
4	232147	58591	238394	65107	118960	285361	106170	148516	109530	616206	92356	130597	
5	49752	308775	109814	274518	160723	201097	113561	77207	34422	53199	384646	72334	
6	10592	45479	159203	72827	110852	120223	75593	75384	25562	15254	50599	219788	
7	1693	13970	57948	90617	62506	111911	93620	49065	19361	8120	9357	18960	
8	561	7722	4497	31977	22595	41257	50022	48700	17604	5313	3238	4967	
9	54	1690	409	15441	6345	21271	36618	26055	19836	10964	3481	3556	
10	37	215	296	5668	2693	7039	7536	13792	9661	5787	2842	1835	

SURVIVOR
FIRST YEAR AND YOUNGEST AGE GROUP?
 1965 2
FINAL YEAR AND OLDEST AGE GROUP IN CALIBRATION BLOCK
 1970 8
AGE AT WHICH K SHOULD LEVEL OFF
 6
NAME OF RESEARCH VESSEL ABUNDANCE MATRIX
RV@MAT
NAME OF CATCH MATRIX?
CATM
SURVIVORS IN POPULATION FOR LAST YEAR?
 70000 400000 150000 50000 200000 20000 4000 1500 5000
SURVIVORS IN POPULATION FOR OLDEST AGE GROUP?
 1 100 1500 1000 4000 7000 15000 15000 10000 7000 3000 5000
NATURAL MORTALITY
 0.2
MAXIMUM NUMBER OF ITERATIONS?
 10
ENTER 1 IF MATRIX IS TO BE PRINTED. OTHERWISE ENTER 0.
ONE NUMBER IS REQUIRED IN VECTOR FOR EACH MATRIX
INTEGRATED CATCH
INTEGRATED SURVIVORS
ESTIMATED SURVIVORS
ESTIMATED VARIANCE OF SURVIVORS
WEIGHTED SURVIVORS
OUTLIERS OF RESIDUALS
 0 0 1 1 1 1
ANALYSIS OF VARIANCE TO BE PERFORMED(YES OR NO)?
YES

ITERATION 1 RMS=0.02475074238
O/O CHANGE IN SURVIVORS=110.5401764

<u>AGE</u>	<u>SURVIVORS</u>	<u>K</u>
2	152952	52.04
3	874298	59.06
4	343942	64.79
5	183048	71.45
6	400943	80.73
7	36735	80.73
8	6799	80.73
9	2151	80.73
10	8243	80.73

ITERATION 2 RMS=0.01432995738
O/O CHANGE IN SURVIVORS=10.54317433

<u>AGE</u>	<u>SURVIVORS</u>	<u>K</u>
2	154728	52.61
3	888078	59.96
4	352774	66.45
5	189644	74.07
6	442160	89.07
7	41781	89.07
8	8065	89.07
9	2751	89.07
10	9515	89.07

ITERATION 3 RMS=0.01308929534
O/O CHANGE IN SURVIVORS=4.625389576

<u>AGE</u>	<u>SURVIVORS</u>	<u>K</u>
2	155752	52.93
3	896123	60.50
4	357508	67.28
5	193144	75.43
6	462675	93.30
7	44322	93.30
8	8708	93.30
9	3064	93.30
10	10164	93.30

ITERATION 4 RMS=0.01264046776
O/O CHANGE IN SURVIVORS=2.25145645

<u>AGE</u>	<u>SURVIVORS</u>	<u>K</u>
2	156278	53.10
3	900255	60.77
4	359943	67.71
5	194943	76.14
6	473222	95.52
7	45640	95.52
8	9047	95.52
9	3228	95.52
10	10505	95.52

ITERATION 5 RMS=0.01244700991
O/O CHANGE IN SURVIVORS=1.140071357

<u>AGE</u>	<u>SURVIVORS</u>	<u>K</u>
2	156555	53.19
3	902424	60.91
4	361220	67.94
5	195885	76.50
6	478702	96.68
7	46327	96.68
8	9227	96.68
9	3314	96.68
10	10683	96.68

ITERATION 6 RMS=0.0123564317
O/O CHANGE IN SURVIVORS=0.5852887504

<u>AGE</u>	<u>SURVIVORS</u>	<u>K</u>
2	156700	53.24
3	903560	60.99
4	361888	68.05
5	196378	76.70
6	481557	97.28
7	46686	97.28
8	9320	97.28
9	3359	97.28
10	10776	97.28

ITERATION 7 RMS=0.01231187254
O/O CHANGE IN SURVIVORS=0.3028787829

<u>AGE</u>	<u>SURVIVORS</u>	<u>K</u>
2	156775	53.26
3	904154	61.03
4	362238	68.12
5	196636	76.80
6	483045	97.60
7	46873	97.60
8	9369	97.60
9	3383	97.60
10	10824	97.60

ITERATION 8 RMS=0.01228934311
O/O CHANGE IN SURVIVORS=0.1573808265

<u>AGE</u>	<u>SURVIVORS</u>	<u>K</u>
2	156815	53.27
3	904464	61.05
4	362420	68.15
5	196770	76.85
6	483822	97.77
7	46971	97.77
8	9395	97.77
9	3395	97.77
10	10850	97.77

ITERATION 9 RMS=0.01227778083
O/O CHANGE IN SURVIVORS=0.08195092202

<u>AGE</u>	<u>SURVIVORS</u>	<u>K</u>
2	156836	53.28
3	904626	61.06
4	362515	68.16
5	196840	76.88
6	484227	97.85
7	47021	97.85
8	9408	97.85
9	3401	97.85
10	10863	97.85

FINAL ITERATION (10)

	POPULATION NUMBERS											8 / 1/80
	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	
2	2235287	1338938	1114582	1811383	527370	663964	730330	4333552	737801	1049503	1542756	
3	970945	1650084	1068392	865410	983920	373730	414946	501624	3000787	569595	732702	
4	1169136	755676	1121271	814285	617521	504261	230425	194296	327588	1921413	402769	
5	309929	710342	541219	684763	562176	358683	235963	106210	78756	195737	1124803	
6	84913	210689	372890	361182	389703	333870	171118	108430	41554	42402	113276	
7	25975	58313	125345	193637	234615	218087	177044	84208	47107	19153	23697	
8	8857	16863	39589	62460	108772	145545	106572	81473	39443	27704	10645	
9	884	6206	10287	23129	34352	69206	83977	53320	36299	19498	18739	
10	24	598	4212	5549	11007	22044	43918	46427	27841	18426	9891	

| 1976

2	183806
3	1084763
4	469316
5	255579
6	650730
7	61937
8	13009
9	5629
10	12970

ESTIMATED SURVIVORS

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	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976
2	71581	36384	0	23210	0	0	38389	758027	219664	333214	969406	156847
3	111424	39338	37036	18768	0	485	44669	69169	648476	166307	376951	865329
4	0	60576	34399	9509	16967	4826	0	724	26899	438664	208830	368740
5	18945	31939	20086	52718	8215	7590	7004	5242	16239	46503	438300	198707
6	4597	12735	18235	58465	24819	24713	21500	11454	3152	9499	53936	479316
7	2921	10687	11284	37617	46174	29821	27750	10700	14984	2511	9843	46800
8	3857	1123	7158	16968	53438	29216	20400	17278	5591	11978	3957	8935
9	416	3434	2380	8508	17369	41045	27069	17855	11643	9713	10927	3429
10	4	458	3773	2629	9307	16867	32537	39088	20869	14052	7080	9920

ESTIMATED VARIANCE OF SURVIVORS

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	1965	1966	1967	1968	1969	1970	1971	1972
2	3020772392	1021166892	545958948	1686921587	180468596	363226539	712157419	53680044158
3	1225968495	2143447607	979091684	597396136	350039790	140142453	389413753	662158425
4	1128392751	764475858	1460455165	677345810	451909311	247161249	65037050	64036516
5	159397663	772190316	388821885	968292693	471262781	204067837	72255258	26257187
6	13267068	95793880	255883707	413106900	378832201	307425810	90258169	25854417
7	2359026	12420897	41985244	123447424	209700039	178918236	134105621	26137061
8	400176	1055526	6552649	18956621	89795150	87200349	53797061	36078961
9	5178	246356	763338	3727915	8906074	40857175	37540647	17505321
10	6	4060	198458	367479	1452328	5294114	16902449	26994293

	1973	1974	1975	1976
2	1962213234	4776738566	18892737260	358350382
3	35454243852	1284859624	4022219746	11500324627
4	263232084	16244108001	1247791836	2404930209
5	27338340	180984929	8030131194	703988788
6	5383383	8628920	106872776	4416285111
7	9806030	1490353	4192268	40363865
8	6222561	4159071	922962	1653490
9	7848861	3552356	2519339	339650
10	8400684	3658600	920952	1508915

WEIGHTED SURVIVORS

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	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976
2	115	80	0	9	0	0	1162	30741	31895	79834	366810	156847
3	1044	89	85	20	0	1	113	2253	39817	36878	107255	537900
4	0	910	115	32	24	13	0	11	2204	58787	47683	175476
5	301	380	594	265	39	23	66	40	584	5541	118822	80420
6	274	337	654	1626	319	181	150	303	118	1083	10884	236271
7	229	680	681	2796	2530	811	465	258	1045	340	2309	25004
8	831	197	863	2269	5461	3849	1845	1077	566	1970	866	5315
9	183	1202	577	1804	4944	9219	8285	4963	3335	1722	2966	2039
10	4	256	1640	1324	5065	8077	17666	16637	12089	8636	4841	4496

ESTIMATED SURVIVORS FOR AGE 10 (WEIGHTED)

<u>YEAR</u>	<u>SURVIVORS</u>	<u>VARIANCE</u>	<u>STANDARD ERROR</u>	<u>C.V. (%)</u>
1965	4	6	2	65.31
1966	439	2276	48	10.86
1967	3673	86230	294	8.00
1968	2328	185097	430	18.48
1969	8686	790333	889	10.24
1970	16610	2535146	1592	9.59
1971	36177	9177270	3029	8.37
1972	35475	11489559	3390	9.55
1973	20612	4866283	2206	10.70
1974	13930	2248398	1499	10.76
1975	7603	629723	794	10.44
1976	10870	683954	827	7.61

ESTIMATED SURVIVORS FOR 1976 (WEIGHTED)

<u>AGE</u>	<u>SURVIVORS</u>	<u>VARIANCE</u>	<u>STANDARD ERROR</u>	<u>C.V. (o/o)</u>
2	156847	358350382	18930	12.07
3	904710	7148756923	84550	9.35
4	362565	1144453779	33830	9.33
5	196877	284915169	16879	8.57
6	484438	2176938657	46658	9.63
7	47048	21565574	4644	9.87
8	9415	983554	992	10.53
9	3405	201998	449	13.20
10	10870	683954	827	7.61

FINAL ESTIMATION FOR K

<u>AGE</u>	<u>K</u>	<u>LN(K)</u>	<u>VAR(LN(K))</u>	<u>STANDARD_ERROR</u>	<u>D.F.</u>
2	53.28	3.9713	0.0086	0.0379	6
3	61.06	4.0901	0.0435	0.0852	6
4	68.17	4.2184	0.0073	0.0349	6
5	76.89	4.3397	0.0054	0.0300	6
6	97.90	4.5744	0.0191	0.0326	17
7	97.90	4.5744	0.0191	0.0326	17
8	97.90	4.5744	0.0191	0.0326	17
9	97.90	4.5744	0.0191	0.0326	17
10	97.90	4.5744	0.0191	0.0326	17

RESIDUALS

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	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976
2	0.1177	0.0879	0.0418	0.0366	0.0470	0.1276	0.0362	0.1444	0.0604	0.0471	0.0551	0.0001
3	0.3006	0.0497	0.0926	0.0562	0.3394	0.0291	0.2274	0.1031	0.1046	0.0925	0.0262	0.0409
4	-0.1266	0.1151	0.0442	0.0201	0.0542	0.0449	0.0793	0.1165	0.1320	0.0399	0.0394	0.0146
5	0.0225	-0.0180	0.0891	0.1318	0.0310	0.0001	0.0504	0.0417	0.1610	0.0044	0.0567	0.0080
6	-0.1258	-0.0461	0.1258	0.1456	0.0263	0.0765	0.1822	0.0133	0.0122	0.0035	0.0792	0.0084
7	0.0452	0.0671	-0.0892	0.0151	0.0881	0.0818	0.1461	0.0716	0.1623	0.0798	0.0245	0.0040
8	-0.0341	-0.1249	0.0654	0.0097	0.2327	-0.0732	-0.0030	0.0658	-0.0876	0.0642	0.0681	0.0410
9	-0.0354	-0.0528	0.0073	-0.0100	0.0299	0.0911	-0.1446	-0.0719	0.0884	0.1367	0.0046	0.0054
10	0.0005	0.0334	0.0266	0.0590	0.0612	0.0135	-0.0954	0.0831	0.0108	0.0080	0.0596	-0.0838

MEAN OF RESIDUALS=0.01150081995

STANDARD DEVIATION OF RESIDUALS = 0.09162356142

OUTLIERS OF RESIDUALS

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ANALYSIS OF VARIANCE

<u>SOURCE</u>	<u>B</u>	<u>STAND ERROR OF B</u>
<i>CONSTANT</i>	0.011	
<i>AGE</i>	3 0.017	0.042
	4 -0.047	0.044
	5 -0.013	0.047
	6 -0.005	0.051
	7 0.023	0.056
	8 -0.018	0.062
	9 -0.035	0.068
	10 -0.021	0.069
	1966 -0.017	0.049
	1967 -0.058	0.050
<i>YEAR</i>	1968 0.023	0.053
	1969 -0.013	0.056
	1970 -0.025	0.060
	1971 -0.007	0.065
	1972 0.013	0.070
	1973 0.001	0.076
	1974 -0.019	0.083
	1975 0.013	0.089
	1976 -0.019	0.086
	1956 0.024	0.100
	1957 0.041	0.089
	1958 0.011	0.080
	1959 -0.002	0.074
	1960 0.005	0.068
<i>YRCLASS</i>	1961 0.015	0.064
	1962 0.065	0.060
	1963 0.052	0.057
	1964 0.048	0.059
	1965 0.042	0.060
	1966 -0.023	0.062
	1967 -0.011	0.066
	1968 0.019	0.071
	1969 -0.004	0.077
	1970 0.029	0.084
	1971 0.009	0.091
	1972 0.004	0.100
	1973 -0.010	0.112

SUMMARY OF ANALYSIS OF VARIANCE

<u>SOURCE</u>	<u>SS</u>	<u>DF</u>	<u>MS</u>	<u>F</u>
<i>CONSTANT</i>	0.0140	1		
<i>AGE</i>	0.04596	8	0.0057	0.549
<i>YEAR</i>	0.04338	11	0.0039	0.377
<i>YRCLASS</i>	0.06556	19	0.0035	0.330
<i>RESIDUALS</i>	0.74335	69	0.0108	
<i>TOTAL</i>	0.91227	108		

9. Catch Projections (MPROJECT).

This program simulates a population over a number of years by the combination of 1) a simple catch equation, 2) age-specific information on weight and 3) a discrete function representing age-specific allocation of fishing mortality (partial recruitment). It generates tables (both age- and time-specific) showing population numbers and biomass, numbers and biomass of mature fish, catch numbers and biomass, as well as fishing mortalities. The program also calculates surplus and net production from the stock. Annual net production is calculated as the total production diminished by the loss of biomass due to fishing and to natural deaths. The program also calculates the mean age and the mean weight of individuals in the catch.

The relative sensitivity coefficients are calculated with respect to certain parameters (ref. Arnold and Beck, 1977). The parameters of interest here are initial population numbers, recruitment, partial recruitment, weight-at-age, and annual rates of fishing and natural mortality.

Input Information.

In this section, greek letters are used to identify time-intervals. For example, ι refers to age-interval $i, i+1$, while τ refers to the time-interval $t, t+1$.

The input information required by the program consists of:

t_0 : initial year;

t_f : final year of projection;

b : age of youngest age-group;

m : age of oldest age-group;

$N_{i,0}$ ($i = b, \dots, m$) : population numbers at age i , at the beginning of the initial year t_0 ;

$C_{i,0}$ ($i = b, \dots, m$) : catch (in numbers), if known, for the initial time period;

$w_{i+0.5}$ ($i = b, \dots, m$) : weight-at-age $i+0.5$, in kilograms. These are taken as mid-year estimates;

r_i ($i = b, \dots, m$) : partial recruitment. These are values, between 0 and 1, which indicate the proportion of fishing mortality which can be allocated to age-group $i, i+1$. The r_i are constant for all years of prediction considered);

M_i ($i = b, \dots, m$): instantaneous rate of natural mortality for age $i, i+1$ (assumed to be constant for all years of prediction considered);

$R_{\cdot, t}$ ($t = t_0, \dots, t_f$) : estimated recruitment (in numbers) to the youngest age-group at the beginning of year t ;

$F_{\cdot, \tau}$ ($t = t_0, \dots, t_f$) : an estimate of instantaneous fishing mortality for each year considered in the projection;

or

$Q_{\cdot, \tau}$ ($t = t_0, \dots, t_f$) : an estimate of total catch or quota for each year considered in the projection;

d_i ($i = b, \dots, m$) : a value, between 0 and 1, indicating the proportion of fish in age-group i which are assumed to have attained maturity (those values are assumed to be constant for all years of prediction considered).

Algorithm.

For a given time horizon, the population numbers, the population biomass, the catch (both in numbers and in biomass units) and the instantaneous fishing mortalities are calculated from the following relationships:

Age-specific fishing mortalities: $F_{\cdot, \tau}$

The calculation of the instantaneous fishing mortalities is governed by the information provided in input:

A) Estimates of population and catch, both in numbers, may be available for the initial year t_0 . Then age-specific fishing mortalities are calculated by the following iterative procedure:

$$\text{STEP 1. } F_{\cdot, 0}^0 = r_i . \quad [9.1].$$

STEP 2. Then we calculate the catch as follows (let $j = 0$):

$$C_{\cdot, 0}^j = \frac{F_{\cdot, 0}^j}{Z_{\cdot, 0}^j} N_{\cdot, 0} (1 - e^{-Z_{\cdot, 0}^j}) \quad [9.2]$$

$$\text{where } Z_{\cdot, 0}^j = F_{\cdot, 0}^j + M_i .$$

STEP 3. If $|C_{\cdot, 0}^j - C_{\cdot, 0}| < 0.01$ for all i , then $F_{\cdot, 0}^j$ is accepted as the final value for the instantaneous fishing mortality. If not, calculate a $F_{\cdot, 0}^{j+1}$ from the relationship

$$F_{\cdot, 0}^{j+1} = F_{\cdot, 0}^j - \frac{(C_{\cdot, 0}^j - C_{\cdot, 0})}{C_1 + C_2} ,$$

$$\text{where } C_1 = N_{i,0} \frac{F_{i,0}}{Z_{i,0}^j} e^{-Z_{i,0}^j}$$

$$C_2 = N_{i,0} \frac{M_{i,0}}{(Z_{i,0}^j)^2} (1-e^{-Z_{i,0}^j}) .$$

Then steps 2 and 3 are repeated in an iterative manner in order to find the final values for the instantaneous fishing mortalities.

If catches are not available for the first year, an estimate of annual fishing mortality is provided in input. Then,

$$F_{i,0} = r_i F_{*,0} .$$

and the $C_{i,0}$ are calculated as in [9.2].

B) For the years of prediction ($t > t_0$), the fishing mortalities can be evaluated in two ways:

1) When an estimate of the annual fishing mortality, $F_{*,\tau}$, is given, the age-specific fishing mortalities are calculated as

$$F_{i,\tau} = r_i F_{*,\tau} .$$

2) When quotas, say $Q_{*,\tau}$, are entered as input data in a given year t , the instantaneous fishing mortalities are calculated according to the following iterative procedure:

$$\text{STEP 1: } F_{i,\tau}^0 = r_i .$$

$$\text{STEP 2: } C_{i,\tau}^j = \frac{F_{i,\tau}^j}{Z_{i,\tau}^j} N_{i,t} (1-e^{-Z_{i,\tau}^j}) ,$$

$$\text{where } Z_{i,\tau}^j = F_{i,\tau}^j + M_i .$$

$$\text{STEP 3: If } |Q_{*,\tau} - \sum (C_{i,\tau} W_{i+5})| \leq 0.1 ,$$

then the $F_{i,\tau}^j$ are accepted as the final values for the instantaneous fishing mortality. If not, calculate the next F -value from the recursive equation

$$F_{i,\tau}^{j+1} = F_{i,\tau}^j \frac{Q_{*,\tau}}{\sum C_{i,\tau} W_{i+5}} .$$

Then steps 2 and 3 are repeated in an iterative manner in order to find the final values for the instantaneous fishing mortalities.

Weighted fishing mortalities (\bar{F}_τ):

An indication of the overall fishing mortality for a given stock can be obtained through the calculation of the following index weighted F :

$$\bar{F}_\tau = \sum (N_{i,t} F_{i,t}) / \sum N_{i,t} ,$$

where the summation is taken over all age-groups . The weighted F is calculated for each year t of the projection and appears on the bottom line of the table for age-specific fishing mortalities.

Population numbers ($N_{i,t}$):

$$N_{i,t} = N_{i-1,t-1} e^{-Z_{i-1,t-1}} ,$$

where

$$Z_{i,\tau} = F_{i,\tau} + M_i .$$

Fish are assumed to leave completely the exploited stock when $i>m$. In other words, $N_{m,t}$ is the population for the last age-group. For each year of prediction, the numbers in the first age-group considered are set equal to the recruits, $R_{*,t}$.

Population biomass (average biomass for the current year):

$$\bar{B}_{i,\tau} = W_{i+5} N_{i,t} (1-e^{-Z_{i,\tau}}) / Z_{i,\tau} .$$

Catch, in numbers ($C_{i,\tau}$):

$$C_{i,\tau} = F_{i,\tau} N_{i,t} (1-e^{-Z_{i,\tau}}) / Z_{i,\tau} .$$

Yield ($Y_{i,\tau}$):

$$Y_{i,\tau} = W_{i+5} C_{i,\tau} .$$

Number of mature fish ($N_{\cdot,t}$):

$$N_{\cdot,t} = \sum_{i=b}^m d_i N_{i,t}$$

Biomass of mature fish ($B_{\cdot,t}$):

$$B_{\cdot,t} = \sum_{i=b}^m d_i W_i N_{i,t}$$

Components of production:

The components of production are calculated as in COHORT (see page 10).

Mean weight of individuals in the catch (\bar{W}_{τ}):

$$\bar{W}_{\tau} = \frac{\sum_i Y_{i,\tau}}{\sum_i C_{i,\tau}}$$

Sensitivity coefficients.

Since input parameters (population numbers in the initial year, recruitment in years of prediction, partial recruitment, weight-at-age, annual fishing and natural mortalities) are expressed as vectors, the relative sensitivities are calculated from the response of the predicted catch to small relative perturbations in each element of the input vector under consideration. In fact, the relative sensitivities are approximated from equation [6].

Input format.

In order to initiate the function, type in MPROJECT. Thereafter, you will be prompted for input as follows:

FIRST YEAR AND YOUNGEST AGE-GROUP?

-enter two numbers representing respectively the year on which the projection is to start and the youngest age (b) to be included in the projection.

ERRORS: -you must enter 2 positive integers.

NUMBERS AT AGE FOR XXXX?

-enter the vector of population numbers (N_{i,t_0} , $i = b, \dots, m$) for the year on which the projection is to start (the first number in the vector should be for the youngest age-group as entered above).

ERRORS: -the values entered must be non-negative.

CATCH AT AGE KNOWN FOR XXXX? (YES OR NO)

-enter either YES or NO. If you enter YES, then the program will ask for the values of the catch at age.

CATCH AT AGE FOR XXXX?

-enter the vector of catch at age (C_{i,t_0} , $i = b, \dots, m$) for the first year of the projection.

ERRORS: -the catch vector must have the same length as the vector of numbers at age;
-the values entered must be non-negative;
-the catch numbers must be less than or equal to the population numbers.

WEIGHT AT AGE? (KG)

-enter a vector of weights at age (W_i , $i = b, \dots, m$).

ERRORS: -the vector must have the same length as the vector of numbers at age;
-the weights must be positive.

PARTIAL RECRUITMENT TO FISHING? (VALUES BETWEEN 0 AND 1)

-enter the values (r_i , $i = b, \dots, m$) which indicate the proportion of fishing mortality which can be allocated to age-group i.

ERRORS: -the vector must have the same length as the vector of numbers at age;
-the values entered must be greater than or equal to zero and less than or equal to one. But at least one value should be different than zero.

NATURAL MORTALITY?

-enter either a single number (M) or a vector giving separate instantaneous rate of natural mortality (M_i , $i = b, \dots, m$) for each age-group i. When a single number is entered, the M_i are set equal to M for all i.

ERRORS: -the entry must either be a single number or a vector which has the same number of elements as the vector of numbers at age;
-all values must be greater than zero.

NUMBER OF YEARS TO BE PROJECTED?

-enter a value representing the number of years for which the projection is to be carried forward (the initial year, t_0 , counts as one year).

ERRORS: -the entry must be a single integer greater than one.

RECRUITMENT FOR XXXX TO YYYY?

-enter a vector containing the recruitment values ($R_{\cdot,t}$) for the range of years requested.

ERRORS: -the entry must be a vector of dimension one less than the number of years for which the projection is to be done;
-all entries must be greater than or equal to zero.

QUOTA (IN BIOMASS) OR F VALUES FOR XXXX TO YYYY?

-enter a vector of quota ($Q_{\cdot,t}$) and/or F-values ($F_{\cdot,t}$) for the predetermined years of prediction. Any number which is less than 10 will be assumed to be an F-value and a number greater than or equal to 10 will be interpreted as a quota. Note that quotas and F-values may be intermixed in the vector.

ERRORS: -the entry must be a vector of dimension one less than the number of years for which the projection is to be done if the catches (in numbers) were input for the initial year. If catches were not provided for the initial year, the entry must be a vector of the same dimension as the number of years for which the projection is to be made.

MATURITY FIGURES TO BE CALCULATED?

-enter either YES or NO in order to indicate whether or not you wish to get printouts of mature biomass and mature numbers. If you answer YES, the program will next ask for:

MATURITY AT AGE?

-enter a vector indicating the proportion of mature fish (d_i) in each age-group i .

ERRORS: -the maturity vector must have the same length as the vector of numbers at age;
-the values must be between 0 and 1, or equal to 0, or 1 .

SENSITIVITY ANALYSIS TO BE PERFORMED (YES OR NO)?

-enter either YES or NO in order to indicate if you want to calculate the relative sensitivity coefficients.

ERRORS: -you must enter either YES or NO.

After the first set of results have been printed, you will be asked:

MORE PROJECTIONS WITH THIS DATA?

-enter either YES or NO. If you do wish to perform additional projections, the program will return to the point at which partial recruitments are requested and continue from there.

An example.

FILEΔFETCH '2719067 PFAPROJECT'

MPROJECT
FIRST YEAR AND YOUNGEST AGE GROUP?
□:
1978 3
NUMBERS AT AGE FOR 1978?
□:
1000 427 182 78 33 14 6
CATCH AT AGE KNOWN FOR 1978?(YES OR NO)
YES
CATCH AT AGE FOR 1978?
□:
337 144 61 26 11 5 4
WEIGHT AT AGE? (KG) :VECTOR OR MATRIX
□:
1.86 5.53 8.8 10.96 12.28 13.6 14.5
PARTIAL RECRUITMENT TO FISHING?(VALUES BETWEEN 0 AND 1)
□:
.3 .6 .8 .9 1 1 1
NATURAL MORTALITY?
□:
.1
NUMBER OF YEARS TO BE PROJECTED?
□:
5
RECRUITMENT FOR 1979 TO 1982?
□:
1000 1000 1000 1000
QUOTA (IN BIOMASS) OR F VALUES FOR 1979 TO 1982?
□:
3952 4410 1 1

MATURITY FIGURES TO BE CALCULATED?(YES OR NO)
NO

SENSITIVITY ANALYSIS TO BE PERFORMED (YES OR NO)?
YES

POPULATION NUMBERS 19/ 6/80

	1978	1979	1980	1981	1982
3	1000	1000	1000	1000	1000
4	427	586	691	691	670
5	182	250	309	364	343
6	78	107	110	136	148
7	33	46	43	44	50
8	14	19	17	16	15
9	6	8	7	6	5
-----+-----					
3+	1740	2016	2177	2257	2231
4+	740	1016	1177	1257	1231
5+	313	430	486	567	561
6+	131	180	177	202	218

POPULATION BIOMASS (AVERAGE) 19/ 6/80

	1978	1979	1980	1981	1982
3	1440.36	1554.67	1554.69	1533.01	1533.01
4	1828.25	2391.68	2821.26	2747.06	2665.85
5	1241.93	1500.88	1854.10	2113.35	1990.30
6	663.79	769.14	792.11	942.12	1025.90
7	314.68	356.63	333.96	330.00	372.58
8	145.26	167.12	145.30	130.55	121.65
9	49.00	72.71	65.55	54.68	46.33
-----+-----					
3+	5683.25	6812.84	7566.98	7850.76	7755.63
4+	4242.90	5258.17	6012.29	6317.75	6222.61
5+	2414.65	2866.49	3191.02	3570.69	3556.77
6+	1172.73	1365.60	1336.92	1457.34	1566.47

CATCH NUMBERS 19/ 6/80

	1978	1979	1980	1981	1982
3	337	226	226	247	247
4	144	234	275	298	289
5	61	123	152	192	181
6	26	57	59	77	84
7	11	26	24	27	30
8	5	11	10	10	9
9	4	5	4	4	3
3+	588	681	750	855	844
4+	251	455	524	608	597
5+	107	221	248	310	308
6+	46	99	97	118	127

CATCH BIOMASS 19/ 6/80

	1978	1979	1980	1981	1982
3	627	420	420	460	460
4	796	1292	1523	1648	1600
5	537	1081	1335	1691	1592
6	285	623	642	848	923
7	135	321	301	330	373
8	68	150	131	131	122
9	58	65	59	55	46
3+	2506	3952	4410	5162	5116
4+	1879	3532	3990	4702	4656
5+	1083	2241	2467	3054	3056
6+	546	1160	1132	1363	1464

MEAN WEIGHT OF INDIVIDUALS IN CATCH 19/ 6/80

	1978	1979	1980	1981	1982
	4.3	5.8	5.9	6.0	6.1

FISHING MORTALITY 19/ 6/80

	1978	1979	1980	1981	1982
3	0.435	0.270	0.270	0.300	0.300
4	0.436	0.540	0.540	0.600	0.600
5	0.432	0.720	0.720	0.800	0.800
6	0.429	0.810	0.810	0.900	0.900
7	0.429	0.900	0.900	1.000	1.000
8	0.468	0.900	0.900	1.000	1.000
9	1.184	0.900	0.900	1.000	1.000
3+	0.437	0.456	0.466	0.529	0.529

DISTRIBUTION OF GROWTH OVER AGES (PERCENT) 19/ 6/80

	1978	1979	1980	1981	1982
3	44.4	39.8	36.0	34.8	35.4
4	38.3	42.0	44.8	43.4	42.9
5	12.5	13.2	14.8	16.8	16.2
6	3.3	3.5	3.2	3.9	4.3
7	1.0	1.0	0.8	0.8	0.9
8	0.4	0.4	0.3	0.3	0.3
9	0.1	0.1	0.1	0.1	0.1

PRODUCTION 19/ 6/80

SOURCE	1978	1979	1980	1981	1982
RECRUITMENT BIOMASS	1079	1079	1079	1079	1079
GROWTH	3534	4299	4757	4844	4754
TOTAL PRODUCTION	4613	5378	5836	5922	5833
LOSS THROUGH FISHING	2506	3952	4410	5162	5116
SURPLUS PRODUCTION	4945	4697	5079	5137	5058
NET PRODUCTION	1539	745	669	25	58

PRODUCTION/BIOMASS RATIO 19/ 6/80

	1978	1979	1980	1981	1982
	0.81	0.79	0.77	0.75	0.75

SENSITIVITY OF CATCH BIOMASS WITH RESPECT TO PARAMETERS

SENSITIVITY TO NATURAL MORTALITY 19/ 6/80

	1978	1979	1980	1981	1982
3	0.000	0.142	0.321	0.047	0.047
4	0.000	0.005	0.116	0.258	0.144
5	0.000	0.019	0.095	0.528	0.356
6	0.000	0.025	0.147	0.787	0.628
7	0.000	0.031	0.179	0.863	0.886
8	0.000	0.031	0.198	0.920	0.963
9	0.000	0.034	0.198	0.939	1.020

SENSITIVITY TO PARTIAL RECRUITMENT 19/ 6/80

	1978	1979	1980	1981	1982
3	0.000	0.000	0.000	0.860	0.860
4	0.000	0.000	0.000	0.735	0.434
5	0.000	0.000	0.000	0.659	0.059
6	0.000	0.000	0.000	0.624	0.177
7	0.000	0.000	0.000	0.590	0.310
8	0.000	0.002	0.001	0.590	0.410
9	0.000	0.001	0.002	0.590	0.410

SENSITIVITY TO WEIGHT 19/ 6/80

	1978	1979	1980	1981	1982
3	1.000	0.218	0.986	1.000	1.000
4	1.000	0.058	0.350	1.614	1.000
5	1.000	0.040	0.185	2.605	1.614
6	1.000	0.086	0.511	3.392	2.605
7	1.000	0.130	0.709	3.849	3.392
8	1.000	0.130	0.834	4.180	3.849
9	1.000	0.130	0.834	4.306	4.180

SENSITIVITY TO POPULATION NUMBERS IN 1978? 19/ 6/80

	1978	1979	1980	1981	1982
3	0.000	-1.676	-2.108	0.000	0.000
4	0.000	0.085	-1.315	0.652	0.000
5	0.000	0.219	0.913	1.823	0.652
6	0.000	0.278	1.340	4.324	1.823
7	0.000	0.335	1.584	4.890	4.324
8	0.000	0.336	1.753	5.277	4.890
9	0.000	0.395	1.754	5.446	5.277

SENSITIVITY TO RECRUITMENT 19/ 6/80

	1978	1979	1980	1981	1982
3	0.000	0.871	0.382	1.000	1.000
4	0.000	0.112	0.503	1.191	1.000
5	0.000	0.102	0.407	1.422	1.191
6	0.000	0.097	0.357	0.589	1.422
7	0.000	0.092	0.321	0.679	0.589
8	0.000	0.092	0.308	0.756	0.679
9	0.000	0.092	0.308	0.770	0.756

MORE PROJECTIONS WITH THIS DATA?(YES OR NO)
YES

PARTIAL RECRUITMENT TO FISHING? (VALUES BETWEEN 0 AND 1)

.3 .65

.8 .9 1 1 1

NATURAL MORTALITY?

.1

NUMBER OF YEARS TO BE PROJECTED?

5

RECRUITMENT FOR 1979 TO 1982?

1000 1000 1000 1000

QUOTA (IN BIOMASS) OR F VALUES FOR 1979 TO 1982?

3952 4410 5162 5116

MATURITY FIGURES TO BE CALCULATED? (YES OR NO)

NO

SENSITIVITY ANALYSIS TO BE PERFORMED (YES OR NO)?

NO

POPULATION NUMBERS 19/ 6/80

	1978	1979	1980	1981	1982
3	1000	1000	1000	1000	1000
4	427	586	691	691	670
5	182	250	309	364	343
6	78	107	110	136	148
7	33	46	43	44	50
8	14	19	17	16	15
9	6	8	7	6	5
3+	1740	2016	2177	2257	2231
4+	740	1016	1177	1257	1231
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7	314.68	356.63	333.96	330.00	372.55
8	145.26	167.12	145.30	130.55	121.65
9	49.00	72.71	65.55	54.68	46.33
3+	5683.25	6812.84	7566.98	7850.73	7755.27
4+	4242.90	5258.17	6012.29	6317.72	6222.29
5+	2414.65	2866.49	3191.02	3570.68	3556.55
6+	1172.73	1365.60	1336.92	1457.34	1566.36

CATCH NUMBERS 19/ 6/80

	1978	1979	1980	1981	1982
3	337	226	226	247	247
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9	4	5	4	4	3
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7	135	321	301	330	373
8	68	150	131	131	122
9	58	65	59	55	46
3+	2506	3952	4410	5162	5116
4+	1879	3532	3990	4702	4656
5+	1083	2241	2467	3054	3056
6+	546	1160	1132	1363	1464

MEAN WEIGHT OF INDIVIDUALS IN CATCH 19/ 6/80

	1978	1979	1980	1981	1982
	4.3	5.8	5.9	6.0	6.1

FISHING MORTALITY 19/ 6/80

	1978	1979	1980	1981	1982
3	0.435	0.270	0.270	0.300	0.300
4	0.436	0.540	0.540	0.600	0.600
5	0.432	0.720	0.720	0.800	0.800
6	0.429	0.810	0.810	0.900	0.900
7	0.429	0.900	0.900	1.000	1.000
8	0.468	0.900	0.900	1.000	1.000
9	1.184	0.900	0.900	1.000	1.000
3+	0.437	0.456	0.466	0.529	0.529

DISTRIBUTION OF GROWTH OVER AGES (PERCENT) 19/ 6/80

	1978	1979	1980	1981	1982
3	44.4	39.8	36.0	34.8	35.4
4	38.3	42.0	44.8	43.4	42.9
5	12.5	13.2	14.8	16.8	16.2
6	3.3	3.5	3.2	3.9	4.3
7	1.0	1.0	0.8	0.8	0.9
8	0.4	0.4	0.3	0.3	0.3
9	0.1	0.1	0.1	0.1	0.1

PRODUCTION 19/ 6/80

SOURCE	1978	1979	1980	1981	1982
RECRUITMENT BIOMASS	1079	1079	1079	1079	1079
GROWTH	3534	4299	4757	4844	4754
TOTAL PRODUCTION	4613	5378	5836	5922	5833
LOSS THROUGH FISHING	2506	3952	4410	5162	5116
SURPLUS PRODUCTION	4045	4697	5079	5137	5057
NET PRODUCTION	1539	745	669	725	59

PRODUCTION/BIOMASS RATIO 19/ 6/80

	1978	1979	1980	1981	1982
	0.81	0.79	0.77	0.75	0.75

MORE PROJECTIONS WITH THIS DATA?(YES OR NO)
NO

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APPENDIX A

LISTING OF APL FUNCTIONS

UTILITY FUNCTIONS

DAT

The function DAT prints the current date in a standard format.

```
    V X<-DAT
[1]   X<-1↓ 3 0 ¶100TΦ3+¶TS
[2]   X[3 6]<-'/'
    V
```

HEADER

(calling form: HEADER SUB)

The function HEADER prints, on the OUTPUT file, the characters sent in the right argument, SUB, and underlines them.

```
    V HEADER SUB
[1]   '
[2]   SUB
[3]   (pSUB)p'+''
    V
```

AΔPLUS

(calling form: AGES AΔPLUS MATRIX)

The diadic function AΔPLUS calculates the sum of the values contained in MATRIX for the range of age-groups which is specified by the values of the vector AGES.

```
    V VECT+AGES AΔPLUS MATRIX
[1]   CNT<-FIRSTAGE+1+AGES
[2]   LASTAGE<+1+AGES
[3]   +(v/(2=pAGES,AGES),(LASTAGE<FIRSTAGE),(LASTAGE>AG[pAG]),FIRSTAGE<AG[1])/
      ERROR
[4]   +(^(0≤LASTAGE-FIRSTAGE),(2=pAGES))/OK
[5]   ERROR:+0,0+¶+^INVALID. FORMAT IS: FIRSTAGE LASTAGE PLUS MATRIX'
[6]   OK:VECT<+/[1] MATRIX<((FIRSTAGE-AG[1]),0)+MATRIX
[7]   CHECK:>(CNT>LASTAGE)/SHAPE
[8]   VECT<-VECT,[1]+/[1] MATRIX+ 1 0 +MATRIX
[9]   CNT<-CNT+1 ^ +CHECK
[10]  SHAPE:VECT<((1+LASTAGE-FIRSTAGE),1+pMATRIX)pVECT
    V
```

INPUTAMAT

INPUTAMAT is an utility function used to create a matrix of data. The matrix which is created by INPUTAMAT is called MAT.

```
V INPUTAMAT;TEMP;CHECK;SAVE;SIZE
[1] RET1:' ENTER FIRST YEAR AND YOUNGEST AGE GROUP '
[2] -(^/(STORE=[STORE),(0≤2↑STORE),2=pSTORE-,□)/OK1
[3] ->RET1,04[]+"MUST BE TWO NON-NEGATIVE NUMBERS"
[4] OK1:YEAR<1↑STORE ◇ AGE<1↑STORE
[5] ' ENTER AGE-SPECIFIC DATA FOR EACH YEAR(ONE YEAR PER LINE), STARTING WITH
   YOUNGEST AGE.
[6] ' TERMINATE ENTRY WITH A SCALAR AFTER NEXT PROMPT'
[7] RET2:SIZE<pTEMP-,□
[8] -(^/(0≤TEMP<(SIZE,1)pTEMP))/OK2
[9] ->RET2,04[]+"MUST BE NON-NEGATIVE NUMBERS"
[10] OK2:CHECK<10
[11] SAVE+TEMP
[12] MAT+TEMP
[13] NEXT:SIZE<pTEMP-,□
[14] -(^/SIZE=1)/LABEL
[15] TEMP+(SIZE,1)pTEMP
[16] -(^/(^/0≤TEMP),(^/(pSAVE)=pTEMP))/OK3
[17] ->NEXT,04[]+"MUST BE NON-NEGATIVE AND DIMENSIONS MUST AGREE WITH FIRST VECTO
   R"
[18] OK3:CHECK+MAT
[19] MAT+MAT,TEMP
[20] ->NEXT
[21] LABEL:YR<~1+YEAR+1pMAT[1;]
[22] AG<~1+AGE+1pMAT[;1]
[23] TIT<"DATA STORED IN MAT"
[24] 'ALIGN PAPER'
[25] R+□DL 5
[26] O OUT MAT
V
```

BECOMES (calling form: EDIT BECOMES VALUE)

The function BECOMES is used to correct erroneous entries of the matrix MAT. VALUE is the corrected value to be inserted in the matrix MAT, at the location identified by EDIT.

```
V EDIT BECOMES VALUE;EDITAGE;EDITYEAR;ROW;COL
[1] EDITAGE<1↑EDIT
[2] EDITYEAR<1↑EDIT
[3] -(2=pEDIT,EDIT)/ERROR
[4] -(^/(EDITYEAR≥YEAR),(EDITAGE≥AGE),(2=pEDIT),(1=pVALUE),(^/VALUE≥0),^/EDIT
   >0)/OK1
[5] ERROR:+0,04[]+"INDICES MUST BE PRESENT IN MATRIX. FORMAT IS: AGE YEAR BECOM
   ES NUMBER"
[6] OK1:ROW<1+EDITAGE-AGE
[7] COL<1+EDITYEAR-YEAR
[8] -(^/(ROW≤1↑pMAT),(COL≤1↑pMAT))/OK2
[9] ->ERROR
[10] OK2:MAT[ROW;COL]←VALUE
V
```

RESETAINDEXES (calling form: RESETAINDEXES NAME)

This function is used to reset the indices of a matrix NAME before using the output function OUT.

```
VRESETAINDEXES[]V
V RESETAINDEXES NAME;TEMP
[1] RET:'ENTER FIRST YEAR AND YOUNGEST AGE GROUP'
[2] -(^/(TEMP=[TEMP),(0≤2↑TEMP),2=pTEMP-,□)/OK
[3] ->RET,04[]+"MUST BE TWO NON-NEGATIVE NUMBERS"
[4] OK:YEAR<1↑TEMP ◇ AGE<1↑TEMP
[5] YR<~1+YEAR+1pNAME[1;]
[6] AG<~1+AGE+1pNAME[;1]
V
```

MEANF

The function MEANF calculates an unweighted mean for the annual rates of fishing mortality which are stored in the matrix FISHMORT.

```
V MEANF
[1] FUW←(+/[1] FISHMORT)+pFISHMORT[,1]
[2] TIT←'MEAN FISHING MORTALITY (UNWEIGHTED)'
[3] YR←_1+YEAR+pFISHMORT[1,]
[4] 4 OUT FUW
V
```

WEIGHTEDAFMORT

The function WEIGHTEDAFMORT calculates a weighted mean for the annual rates of fishing mortality which are stored in the matrix FISHMORT.

```
V WEIGHTEDAFMORT
[1] FWG←(+/[1](POPNB×FISHMORT))+/[1] POPNB
[2] YR←_1+YEAR+pFISHMORT[1,]
[3] TIT←'WEIGHTED FISHING MORTALITIES'
[4] 4 OUT FWG
V
```

OUT (calling form: A OUT B)

The function OUT prints, with A decimal places, the matrix B, sent in the right argument. The matrix B is printed with an adequate format which considers the number of digits before the decimal point and the sign of any element of the matrix. The function OUT calls the function DAT and outputs the current date.

```
V A OUT B;C;D;W;Y;PW;TEST
[1] →(1×pB)/CHECK1
[2] B←(1,pB)pB
[3] CHECK1:A←,¶(2,_1+pB)ρ(5↑2+(A≠0)+A+(0>(L/[1] B))+Γ1001ΓΓ/[1] B),(~1+pB)ρA
[4] PW←(20+pTIT)Γ([PW]4++/((ρA)ρ 1 0)/A
[5] Y←YR
[6] '
[7] ((~8+PW)↑((L0.5×PW+pTIT)ρ^ 1),TIT),DAT
[8] SK1:_^
[9] C←_1+([PW]4++\A[_1+2×10.5×ρA])_1
[10] D←(2×C1ρY)↑A
[11] D[2×1C1ρY]=0
[12] ' | ',D¶(C1ρY)↑Y
[13] '---+' ,(+/A[_1+2×1C])ρ^ 1
[14] →(1×1+pB)/MAT
[15] (1 4 ρ^ 1),((2×C)↑A)¶(1,C)↑B
[16] →CHECK
[17] MAT:(2 0 ¶((ρAG),1)ρAG),((ρAG),2)ρ^ 1,((2×C)↑A)¶((ρAG),C)↑B
[18] →((ρAG)=1+pB)/CHECK
[19] '---+' ,(+/A[_1+2×1C])ρ^ 1
[20] (2 0 ¶((TEST,1)ρAG)),((TEST←(1+pB)-ρAG),2)ρ^ 1,((2×C)↑A)¶(((ρAG)-1+pB)
,C)↑B
[21] CHECK:A←(2×C)↑A
[22] B←(0,C)↑B
[23] Y←C↑Y
[24] →(0×ρA)/SK1
V
```

FETCHAONLY (calling form: `FETCHAONLY NAME`)

The function `FETCHAONLY` is used to 'define' the pack specified by the value of the variable `NAME`. This function is used by the function `FILEAFETCH`.

```
▽ FETCHAONLY PACKNAME
[1]  □PDEFPACKNAME
    ▽
```

FILEAFETCH (calling form: `FILEAFETCH FILENAME`)

The function `FILEAFETCH` is a utility function which is used to fetch the APL-functions which have been stored in permanent files. `FILENAME` specifies the name of the permanent file to be fetched.

```
▽ FILEAFETCH FILENAME
[1]  □UNTIE □WUMS
[2]  FILENAME □TIE FN+1+⌈/0,□WUMS
[3]  PACK+□READ FN,1
[4]  FETCHAONLY 'PACK'
    ▽
```

FFULR (calling form: `FFULR x`)

The function `FFULR` calculates an overall rate of fishing mortality for the fully-recruited age-groups. The variable `x` is a value representing the first fully-recruited age-group.

```
▽ FFULR X
[1]  →(ʌ/(X>0),((pAG)≥X-AG[1]))/OK
[2]  →0,0†'AGE NOT PRESENT'
[3]  OK:NO←((X-AG[1]),0)+POPNB
[4]  FULR←(⊖(+/[1](~1~1+NO))++[1](1~1+NO))-K
[5]  TIT←'FULLY RECRUITED FISHMORTALITY'
[6]  4 OUT FULR
    ▽
```

TABLE (Calling form: A TABLE B)

The function TABLE prints, with A decimal places, the matrix B, sent in the right argument. This function is used to tabulate, with appropriate headings, the information provided in the matrix B. The function TABLE also calls the function DAT.

```

▽ A TABLE B
[1] A←,Q(2,(ρB)[2])ρ(5[2+((A≠0)+0>([1] B))+1001[1|B],((ρB)[2])ρA
[2] +(ρTIT1)>(ρLB)[2])/CHANGELB
[3] NEXT:PW←(20+ρTIT)⌈(PW)⌈(ρLB)[2]++/((ρA)ρ 1 0)/A
[4] Y←YR ⋄ '
[5] ((-8+PW)↑((L0.5×PW=ρTIT)ρ' '),TIT),DAT
[6] +(ρTIT1)>(ρLB)[2])/CHANGELB
[7] SK1:' ⋄ C←-1+([PW<(ρLB)[2]+4++\A[-1+2×10.5×ρA])1
[8] D←(2×CLρY)↑A ⋄ D[2×1CLρY]←0
[9] (WTIT1),((2+(ρLB)[2]-(ρTIT1))ρ' ','|',D,CLρY)↑Y
[10] ((2+(ρLB)[2])ρ' ','+',(+/A[-1+2×1C])ρ' '-'
[11] (W(-SECT[2]),0)+LB),((SECT[2],3)ρ' '|'),((2×C)+A)W(SECT[1],C)+B
[12] ((2+(ρLB)[2])ρ' ','+',(+/A[-1+2×1C])ρ' '-'
[13] (W(SECT[1]),0)+LB),((SECT[2],3)ρ' '|'),((2×C)+A)W((-SECT[2]),C)+B
[14] A←(2×C)+A
[15] B←(0,C)+B
[16] Y←C↓Y
[17] +(0≠ρA)/SK1
[18] →0
[19] CHANGELB:LB←LB,((ρLB)[1],((ρTIT1)-(ρLB)[2]))ρ' '
[20] →NEXT
▽

```

OUTAF (calling form: A OUTAF B)

The function OUTAF is used to output a matrix of fishing mortalities when a vector of weighted fishing mortalities is printed at the bottom of the table. The quantity A represents the first fully recruited age-group.

```

▽ FA OUTAF B;C;D;W;Y;PW;TEST
[1] +(1≠ρρB)/CHECK1
[2] B←(1,ρB)ρB
[3] CHECK1:A←,Q(2,-1+ρB)ρ(5[2+4+(0>([1] B))+1001[1|B],(-1+ρB)ρ3
[4] PW←(20+ρTIT)⌈(PW)⌈4++/((ρA)ρ 1 0)/A
[5] Y←YR
[6] '
[7] ((-8+PW)↑((L0.5×PW=ρTIT)ρ' '),TIT),DAT
[8] SK1:' '
[9] C←-1+([PW<4++\A[-1+2×10.5×ρA])1
[10] D←(2×CLρY)↑A
[11] D[2×1CLρY]←0
[12] '|',D,CLρY)↑Y
[13] '---+' ,(+/A[-1+2×1C])ρ' '-'
[14] +(1≠1+ρB)/MAT
[15] (1 4 ρ' '|'),((2×C)+A)W(1,C)+B
[16] +CHECK
[17] MAT:(2 0 W((ρAG),1)ρAG),(((ρAG),2)ρ' '|'),((2×C)+A)W((ρAG),C)+B
[18] +(ρAG)=1+ρB)/CHECK
[19] '---+' ,(+/A[-1+2×1C])ρ' '-'
[20] (2 0 W((TEST,1)ρ' -1+FA+1TEST)),(((TEST+(1+ρB)-ρAG),2)ρ' '+|'),((2×C)+A)W((ρAG)-1+ρB),C)+B
[21] CHECK:A←(2×C)+A
[22] B←(0,C)+B
[23] Y←C↓Y
[24] +(0≠ρA)/SK1
▽

```

MAIN FUNCTIONS

BEVHOLT

This function computes the yield from a given recruitment by the method of Beverton and Holt.

```
▽ BEVHOLT;MES;M;L;W;K;TI;TR;TC;U;RP;RFW;T1;T2;T3;T4;FLG;FLG1;FLG2;SL;P1;F;NO;TL;B;C;Y;
WB;SV;IN;A;CHECK;TIT;U;TEMP;VEC;INC;LIM;Z;KFP;IND;FO1;FMAX;FLG9;TCI;TCR;E;V;MA;MAX;D
;H;TOT;FT;FLAG3;I;DEB;EX;FP;N;OUTM;P2;TLC;IND;KKK;OUTF
[1] MES←'ONE POSITIVE VALUE. RE-ENTER '
[2] ER1:'NATURAL MORTALITY?'
[3] +(Λ/(M>0),1=pM←,□)/ER2
[4] +ER1,0+□+MES
[5] ER2:'ASYMPTOTIC WEIGHT?(GR)'
[6] +(Λ/(W>0),1=pW←,□)/ER3
[7] +ER2,0+□+MES
[8] ER3:'BRODY COEFFICIENT (PARAMETER K OF BRODY''S EQUATION)?'
[9] +(Λ/(K≤1),(K≥0),1=pK←,□)/ER4
[10] +ER3,0+□+MUST BE BETWEEN 0 AND 1. RE-ENTER '
[11] ER4:'HYPOTHETICAL AGE AT WHICH THE FISH WOULD HAVE ZERO WEIGHT ACCORDING TO BRODY''S
EQUATION?'
[12] +(1=pTI←,□)/ER5
[13] +ER4,0+□+ONE VALUE ONLY. RE-ENTER '
[14] ER5:'AGE OF RECRUITMENT TO STOCK?'
[15] +(Λ/(TR>0),1=pTR←,□)/ER6
[16] +ER5,0+□+MES
[17] ER6:'AGE AT FIRST CAPTURE?'
[18] +(Λ/(TC>0),1=pTC←,□)/ER7
[19] +ER6,0+□+MES
[20] ER7:'MAXIMUM AGE ATTAINED?'
[21] +(Λ/(TL>0),1=pTL←,□)/ER8
[22] +ER7,0+□+MES
[23] ER8:'ENTER A VECTOR OF ZEROS AND\OR ONES TO INDICATE WITH RESPECT TO'
[24] 'WHICH PARAMETERS THE SENSITIVITY COEFFICIENTS ARE TO BE CALCULATED:'
[25] ' NATURAL MORTALITY' ^ ' BRODY COEFFICIENT'
[26] ' AGE AT ZERO WEIGHT' ^ ' AGE AT FIRST CAPTURE'
[27] ' AGE OF RECRUITMENT'
[28] +(Λ/((SV=0)∨SV=1),5=pSV←,□)/ER9
[29] +ER8,0+□+MUST BE A VECTOR OF 5 ELEMENTS WITH ZEROS AND\OR ONES'
[30] ER9:'MAXIMUM FISHING MORTALITY IN OUTPUT TABLE'
[31] +(Λ/(KKK>0),1=pKKK←,□)/ES1
[32] +ER9,0+□+MES
[33] ES1:+(KKK<5)/ES2 ^ KKK+5
[34] ES2:KKK+5+KKK+⌈(KKK×10)
[35] IN←(SV=1)/1pSV
[36] A←I←IND←1 ^ FMAX←FO1+6p0 ^ FLG←FLG9+0 ^ TIT← 6 35 p'
[37] U← 1 ^ 3 3 ^ 1 ^ TEMP←VEC←M,K,TI,TC,TR
[38] F←NO←B+C+Y+WB←((1++/SV),KKK)p0 ^ FT←TOT←H←30p0
[39] LP1:RP←*-VEC[1]×TCR+VEC[4]-VEC[5]
[40] T1←VEC[2]×TCI+VEC[4]-VEC[3]
[41] T2←RP×TLC+TL-VEC[4]
[42] I←1 ^ V←2 ^ FLG2←F[A;I]+0 ^ →DD
[43] AA:I←2 ^ F[A;I]←0.0001 ^ FLG+1 ^ →DD
[44] BB:SL←1000×Y[A;2] ^ I←1 ^ FLG+2
[45] CC:I←I+1
[46] +( ((KKK-5)÷10) < F[A;I] + F[A;I+1] + 0.1 ) / FF
[47] DD:P1←0 ^ N←1 ^ Z←F[A;I]+VEC[1]
[48] EE:T3←Z+VEC[2]×N←1
[49] P1←P1+(1-*T3×TLC)×(U[N]×*-T1×N-1)+T3
[50] +(5>N←N+1)/EE
[51] NO[A;I]←RP×(1-*Z×TLC)+Z
[52] BL[A;I]←RP×W×P1
[53] CL[A;I]←F[A;I]×NO[A;I]
[54] Y[A;I]←F[A;I]×BL[A;I]
[55] WB[A;I]←Y[A;I]×C[A;I]
[56] +( (FLG=0),FLG=1 ) / AA,BB
[57] +( (V=0),V=1 ) / HH,00
```

```

[58] →CC
[59] FF:I←I=1
[60] →(A≠1)/FF1
[61] MAX←Y[1;],Y[1;]
[62] FT[1]←F[1;MAX] ⪻ L←E←V←0 ⪻ →II
[63] FF1:FT[1]←FMAX[1] ⪻ L←E←V←0 ⪻ →II
[64] GG:MA←I←I+1
[65] FMAX[A]←F[A;I]+FT[L] ⪻ F[A;I]+F[1;I] ⪻ →DD
[66] HH:L←E←0 ⪻ V←1
[67] →(A=1)/HH1
[68] FT[1]←FO1[1] ⪻ →II
[69] HH1:FT[1]←0.7×FMAX[1]
[70] →(FLG9≠1)/II ⪻ FT←(ρFT)ρ0 ⪻ FT[1]←0.1
[71] II:→(∨/(30≤L≤L+1),(∨/(|FT|>10))/VV
[72] P2←0 ⪻ N←1
[73] JJ:D←FT[L]+VEC[1]+VEC[2]×N←1
[74] →(100<|D|)/NN
[75] P2←P2+U[N]×(*=(N=1)×VEC[2]×TCI)×((TLC×FT[L]×EX)÷D)+(1-EX+*-*D×TLC)×(D-FT[L])÷D*2
[76] →(5>N+N+1)/JJ
[77] TOT[L]←W×P2×**VEC[1]×TCR
[78] →(V=1)/LL
[79] KK:→(E=1)/MM
[80] FT[L+1]←FT[L]-0.005 ⪻ E←1 ⪻ →II
[81] LL:TOT[L]←TOT[L]-SL ⪻ →KK
[82] MM:→(0.000000001≥|TOT[L]|)/NN
[83] H[L]←(TOT[L]×FT[L]-FT[L-1])÷TOT[L]-TOT[L-1]
[84] FT[L+1]←FT[L]-H[L] ⪻ →II
[85] NN:→(V=0)/GG
[86] FP←I←I+1
[87] FO1[A]←F[A;I]+FT[L] ⪻ F[A;I]+F[1;I] ⪻ →DD
[88] OO:DEB←((ρI←I),7)ρ'
[89] I←ΔF[1;I]
[90] DFB[I]MA;]←'FMAX---'
[91] DEB[I]FP;]←'FO.1---'
[92] →(A≠1)/PP1
[93] →QQ
[94] PP1:PTIT[A;]
[95] NO[A;]←(1000×NO[A;]-NO[1;])÷NO[1;]
[96] B[A;]←(1000×B[A;]-B[1;])÷B[1;]
[97] C[A;]←(1000×C[A;]-C[1;])÷C[1;]
[98] Y[A;]←(1000×Y[A;]-Y[1;])÷Y[1;]
[99] WB[A;]←(1000×WB[A;]-WB[1;])÷WB[1;]
[100] QQ:*
[101] *      FISHING    EXP-POP    EXP-POP    CATCH     YIELD    AVG. WEIGHT'
[102] *      MORTALITY   NUMBERS   BIOMASS   NUMBERS   (GR)     (GR)'
[103] *      -----      -----      -----      -----      -----      -----
[104] C[A;1]←Y[A;1]←
[105] OUTM←NO[A;I],B[A;I],C[A;I],Y[A;I],WB[A;I]
[106] OUTM[1+4×KKK-2]←0
[107] QQ:→(A>1)/QQR
[108] (*DEB), 7 3 12 3 10 9 10 3 8 0 11 0 ρ(6,KKK-2)ρ(F[1;I],OUTM)
[109] FISHMORT←F[1;I] ⪻ YIELDPERRECRUIT←Y[1;I] ⪻ MEANWEIGHT←WB[1;I]
[110] QQR:(A>1)/(*DEB), 7 3 12 3 10 9 10 3 8 3 11 3 ρ(6,KKK-2)ρ(F[1;I],OUTM)
[111] →((ρIN)<1)/WW
[112] VEC←TEMP
[113] VEC[IND]←VEC[IND]+|(0.001×VEC[IND+1+IN])|
[114] A←A+1 ⪻ *
[115] →((IND=2),(IND=3),(IND=4),(IND=5))/RR,SS,TT,TT1
[116] TIT[A;]←"SENSITIVITY TO NATURAL MORTALITY" ⪻ →UU
[117] RR:TIT[A;]←"SENSITIVITY TO BRODY COEFFICIENT(K)" ⪻ →UU
[118] SS:TIT[A;]←"SENSITIVITY TO AGE AT ZERO WEIGHT" ⪻ →UU
[119] TT:TIT[A;]←"SENSITIVITY TO AGE AT FIRST CAPTURE" ⪻ →UU
[120] TT1:TIT[A;]←"SENSITIVITY TO AGE OF RECRUITMENT" ⪻
[121] UU:IN+1+IN ⪻ →LP1
[122] VV:→(FLG9=1)/NN ⪻ *
[123] *WARNING: CONVERGENCE CRITERION NOT SATISFIED AFTER 30 ITERATIONS.*
```

```

[124] ' THE VALUE OF FMAX AND THE CORRESPONDING DERIVATIVES MAY
[125] ' BE INACCURATE.' < ' ' < FLC9+1 < +NN
[126] WW:FLAG3+A
[127] +(FLAG3=1)/0
[128] A+1 < ' '
[129] 'SENSITIVITY OF FMAX AND FO.1 TO PARAMETERS' < ' '
[130] '           PARAMETERS          FMAX          FO.1'
[131] '-----'-----'-----'
[132] IN+(SV=1)/1oSV
[133] WW1:A+A+1 < IND+1+IN
[134] +(A>FLAG3)/0
[135] FMAX[A]<-(1000×FMAX[A]-FMAX[1])×FMAX[1]
[136] FO1[A]<-(1000×FO1[A]-FO1[1])×FO1[1]
[137] OUTF<(FMAX[A],FO1[A])
[138] WWW:(#TIT[A;]), 15 4 15 4 #OUTF
[139] IN+1+IN < WWW1

```

COHORT

The function COHORT calculates population numbers at age and the instantaneous rate of fishing mortality at age by using the method of cohort analysis described in Pope (1972). If information on weight is also provided in input, the function calculates population biomass at age, as well as total and net production.

The function COHORT calls the utility functions OUT, HEADER, OUTAF, TABLE and DAT.

```
V COHORT;J;I;MORT;FI;FC;X;YR;Y;TIT;NAM;CATCH;A;CHECK;CHECK1;CHECK2;ZZ;YY;POP
;CTEMP;FIT;FCT;F;SUB;SENSM;SENSMF;SENSMF1;XX;K;L;IN1;XX;IK;JK;M1;M2;Z
[1] A←CHECK+ZZ+YY←1
[2] NAM←NL 2 ⌈ FLAG←0
[3] ERO: 'NAME OF CATCH MATRIX?....'
[4] +(∨/NAM) =X-(1+pNAM)+( ' ' X)/X←,M)/OK1
[5] →ERO, 04M←'NAMED MATRIX DOES NOT EXIST. RE-ENTER '
[6] OK1:+(2=pCATCH←,X)/OK2
[7] →ERO, 04M←(( ' ' X)/X), ' IS NOT A MATRIX.'
[8] OK2:+(A/0≤,CATCH)/ERO5
[9] →0, 04M←'NEGATIVE VALUES IN CATCH MATRIX.'
[10] ER05: ' ⌈ ' SENSITIVITY ANALYSIS TO BE PERFORMED (YES OR NO)?'
[11] +(A/' YES'=3←CHECK1),~A/' NO'=2←CHECK1+( ' ' CHECK1)/CHECK1←,M)/ER1,ER05
[12] ER1:'FIRST YEAR, YOUNGEST AGE GROUP AND FIRST FULLY RECRUITED AGE-GROUP?'
[13] +(A/(X=[X], 0≤3*X), 3=pX←,M)/OK3
[14] →ER1, 04M←'MUST BE 2 NON-NEGATIVE INTEGERS. RE-ENTER '
[15] OK3:YR←(1+X[1])+I,J←(pCATCH)[2]
[16] AG←(1+X[2])+I,J←(pCATCH)[1]
[17] POP←F←MORT←SENSM←(4,I,J)p0
[18] ER2:'NATURAL MORTALITY?'
[19] +(∨/(1,I)=pM←,M)/OK4
[20] →ER2, 04M←'EITHER 1 OR ',(V,I), ' NUMBERS. RE-ENTER '
[21] OK4:+(A/M≥0)/OK5
[22] →ER2, 04M←'MORTALITIES MUST BE POSITIVE. RE-ENTER '
[23] OK5:'WEIGHT AT AGE KNOWN(YES OR NO)?'
[24] +(A/' NO'=2←CHFC2),~A/' YES'=3←CHECK2+( ' ' CHECK2)/CHECK2←,M)/ER3,OK5
[25] BMASS:'ENTER WEIGHT AT AGE DATA (KG): VECTOR OR MATRIX'
[26] +(A/(A/0≤WGT), (2=ppWGT←,M))/MAT
[27] +(1=ppWGT)/VECTOR
[28] ERROR:→BMASS, 04M←' WEIGHTS MUST BE POSITIVE '
[29] VECTOR:+((pWGT)=pCATCH[,1])/RESHAPE
[30] +BMASS, 04M←'DIMENSION MUST AGREE. '
[31] RESHAPE:WGT←((pWGT),1)pWGT
[32] FLAG←1 ⌈ →PRINTOPTION
[33] MAT:+((pWGT)≠pCATCH)/BMASS
[34] PRINTOPTION: ' ⌈ ' ENTER 1 IF MATRIX IS TO BE PRINTED. OTHERWISE ENTER 0. '
[35] 'ONE NUMBER IS REQUIRED FOR EACH MATRIX'
[36] 'CATCH BIOMASS' ⌈ 'MEAN WEIGHT OF INDIVIDUALS IN CATCH'
[37] 'WEIGHTS AT THE BEGINNING OF THE YEAR'
[38] 'POPULATION BIOMASS AT THE BEGINNING OF THE YEAR'
[39] +(A/(4=+/1=PRINTMAT),(0=PRINTMAT)),4=pPRINTMAT←,M)/ER3
[40] →PRINTOPTION, 04M←'RE-ENTER '
[41] ER3:'F VALUES FOR LAST YEAR (',(V-1)YR),')?'
[42] +(∨/(1,I)=pFI←,M)/OK6
[43] →ER3, 04M←'EITHER 1 OR ',(V,I), ' NUMBERS. RE-ENTER '
[44] OK6:+(A/FT>0)/ER4
[45] →ER3, 04M←'MORTALITIES MUST BE POSITIVE. RE-ENTER '
[46] ER4:'F VALUES FOR OLDEST AGE GROUP (',(V-1)AG),')?'
[47] +(∨/(1,J)=pFC←,M)/OK8
[48] →ER4, 04M←'EITHER 1 OR ',(V,J), ' NUMBERS. RE-ENTER '
[49] OK8:+(A/FC>0)/OK9
[50] →ER4, 04M←'MORTALITIES MUST BE POSITIVE. RE-ENTER '
[51] OK9:V←X[3]-AG[1]
[52] RET:F[A;I;]←FC+JpFC
[53] F[A;:J]←FI+IpFI
[54] MORT[A;:]←((J,I)pM)
[55] POP[A;:J]←(CATCH[,J]*FI+MORT[A;:J])*FI×1←*FI+MORT[A;:J]
[56] POP[A;I;]←(CATCH[I;]*FC+MORT[A;I;])*FC
```

```

[57] +(A=1)/ERS
[58] +(CHECK=0),CHECK#0)/DD,SK1
[59] ER5: 'IS FISHING COMPLETE FOR LAST AGE GIVEN(YES OR NO)?.....'
[60] +(A/'YES'=3+XX),~A/'NO'=2+XX+( ' 'XX,.)/SK1,ERS
[61] CHECK#0
[62] DD:POP[A;I;]+POP[A;I;]*1**FC+MORT[A;I;]
[63] SK1:Y<J-1
[64] AA:X<MORT[A;I-1;Y]
[65] POP[A;I-1;Y]+(CATCH[I-1;Y]**X*2)+(POP[A;I-1;Y+1]**X)
[66] +(1≤Y-Y+1)/AA
[67] F[A;I-1;J-1]+(POP[A;I-1;J-1]+POP[A;I-1;J-1])-MORT[A;I-1;J-1]
[68] +(NO=2+CHECK1)/EE4
[69] +(A=2),(A=3))/BB,EE
[70] FI+FIT+(0.001*FIT-FI) ◇ FC+FC+(0.001*FCT-FC) ◇ A+2
[71] +RET
[72] BB:M=M+(0.001*M)
[73] FI+FIT ◇ FC+FCT ◇ A+3
[74] +RET
[75] EE:SENSM[2;;]←((POP[2;;]-POP[1;;])*POP[1;;])*1000
[76] SENS[3;;]←((F[2;;]-FL1;;)*F[1;;])*1000
[77] SENS[4;;]←((POP[3;;]-POP[1;;])*1000)*POP[1;;]
[78] SENSMF←((F[3;;]-F[1;;])*1000)*F[1;;]
[79] FE4:TIT+'POPULATION NUMBERS'
[80] POPNB+POP[1;;]
[81] 0 OUT POP[1;;],[1] NEWPOPNB←(AG[1],AG[4]) AAPLUS POPNB
[82] +(NO=2+CHECK2)/MORTALITY
[83] WG←WGT
[84] NEXT: +(A/(pWGT)=pPOPNB)/PRINT
[85] WGT←WGT,WG
[86] +NEXT
[87] PRINT:MEANPOPNB←(POPNB*(1**F[1;;]+MORT[1;;]))*F[1;;]+MORT[1;;]
[88] POPBIOMASSΔMIDYR←WGT*MEANPOPNB
[89] TIT+'MEAN POPULATION BIOMASS (KG)'
[90] 0 OUT POPBIOMASSΔMIDYR,[1] NEWPOPBIOMASS←(AG[1],AG[4]) AAPLUS
    POPBIOMASSΔMIDYR
[91] +(FLAG=0)/CATBIOMASS
[92] ' ' 'WARNING: WEIGHT AT AGE IS AVAILABLE FOR ONE YEAR ONLY.'
[93] ' ' 'POPULATION BIOMASS, AS CALCULATED, MAY BE BIASED.' ' '
[94] CATBIOMASS:CATCHBIOMASS←WGT*CATCH
[95] SUMΔCATBIOMASS←(AG[1],AG[4]) AAPLUS CATCHBIOMASS
[96] +(PRINTMAT[1]=0)/MEANCATCH
[97] TIT+'CATCH BIOMASS'
[98] 0 OUT CATCHBIOMASS,[1] SUMΔCATBIOMASS
[99] MEANCATCH: +(PRINTMAT[2]=0)/WTΔBEGAYR
[100] TIT+'MEAN WEIGHT OF INDIVIDUALS IN CATCH'
[101] 2 OUT (+/[1] CATCHBIOMASS)+/[1] CATCH
[102] TIT+'MEAN AGE OF INDIVIDUALS IN CATCH'
[103] 2 OUT (+/[1](CATCH*(@(pCATCH)pAG)))+(+/[1] CATCH)
[104] WTΔBEGAYR:WT←*((@(-1 1+WGT))+* 1 1+WGT)*2
[105] FIRSTAGWT←*2*(@-1+WGT)+(*WT[1;])+2
[106] FIRSTYRWWT←*2*(@-1+WGT)+(*WT[1;])+2
[107] FIRSTYRLASTAGWT←*((@WT[1;])+(*WT[1;])+2)
[108] WTΔBEG←((1,J)p0),[1]((I-1,1)p0),WT
[109] WTΔBEG[1;1+J]←FIRSTAGWT
[110] WTΔBEG[1;1]←FIRSTYRWWT,FIRSTYRLASTAGWT
[111] LSTYR←*(2*WT[1;J])→WTΔBEG[1;J]
[112] LSTYR←0,(-1+LSTYR)
[113] WTΔBEG←WTΔBEG,[2] LSTYR
[114] LSTAGE←*(2*WT[1;])→1+WTΔBEG[1;]
[115] LSTAGE←0,LSTAGE
[116] WTΔBEG←WTΔBEG,[1] LSTAGE
[117] +(PRINTMAT[3]=0)/POPBMASS
[118] YR←YR,(YR[pYR]+1) ◇ AG←AG,(AG[pAG]+1)
[119] TIT+'WEIGHTS AT THE BEGINNING OF THE YEAR' ◇ 2 OUT WTΔBEG
[120] YR←-1+YR ◇ AG←-1+AG
[121] POPBMASS:POPBMASSΔBEG←POPNB* -1 -1 +WTΔBEG
[122] +(PRINTMAT[4]=0)/RECBMASS
[123] TIT+' POPULATION BIOMASS AT BEGINNING OF YEAR'
[124] 0 OUT POBPBIOMASSΔBEG,[1]+/[1] POBPBIOMASSΔBEG
[125] FECΔBMASS:RECRUITΔBIOMASS←POPBMASSΔBEG[1;]
[126] ΔB←(POPNB)*(1 1+WGTΔBEG)**-(F[1;])+MORT[1;]) - -1 -1 +WTΔBEG

```

```

[127] NETPROD<(+/[1] ΔB)+RECRUITΔBIOMASS
[128] FISHINGBIOMASSLOSS<CATCH×WGT
[129] DT<MORT[1;;]*POPBIOMASSAMIDYR
[130] BG<ΔB+DT+FISHINGBIOMASSLOSS
[131] TIT<`DISTRIBUTION OF GROWTH OVER AGES (PER CENT)' ◁ 1 OUT(BG*((pBG)p(+/[1]
    BG)))×100
[132] PRODUCTION<(+/[1] BG)+RECRUITΔBIOMASS
[133] LB<`RECRUITMENT BIOMASS GROWTH' TOTAL PRODUCTION LOSS THROU
    GH FISHINGSURPLUS PRODUCTION NET PRODUCTION
[134] LB< 6 20 pLB
[135] TIT1<`SOURCE' ◁ TIT<`PRODUCTION'
[136] OUTM<RECRUITΔBIOMASS,(+/[1] BG),PRODUCTION,(+/[1] FISHINGBIOMASSLOSS),(
    PRODUCTION+/[1] DT),NETPROD
[137] OUTM<(6,pYR)pOUTM
[138] SECT< 3 3
[139] 0 TABLE OUTM
[140] DATA1<(`3 0 '+OUTM)
[141] DATAΔYR<100*(1,pYR)pYR
[142] DATA2<(2,pYR)p(OUTM[5;],OUTM[4;])
[143] TIT<`PRODUCTION/BIOMASS RATIO'
[144] 2 OUT(PRODUCTION+(/([1] POPBIOMASSAMIDYR))
[145] MORTALITY:FWGT<(+/[1](((V,0)+POPNB)×((V,0)+FISHMORT+F[1;])))+/[1]((V,0)+
    POPNB)
[146] TIT<`FISHING MORTALITY' ◁ (V+AG[1]) OUTΔF FISHMORT,[1] FWGT
[147] +(‘NO’=2+CHECK1)/0
[148] SUB<`PARTIAL DERIVATIVES OF POPULATION WITH RESPECT TO PARAMETERS' ◁
    HEADER SUB
[149] TIT<`SENSITIVITY TO TERMINAL F' ◁ 3 OUT SENS[2;;]
[150] TIT<`SENSITIVITY TO NATURAL MORTALITY' ◁ 3 OUT SENS[4;;]
[151] OVERSENS<(SENS[2;1;]*2)+SENS[4;1;]*2
[152] SUB<`PARTIAL DERIVATIVES OF ESTIMATED F WITH RESPECT TO PARAMETERS' ◁
    HEADER SUB
[153] TIT<`SENSITIVITY TO TERMINAL F' ◁ 3 OUT SENS[3;;]
[154] TIT<`SENSITIVITY TO NATURAL MORTALITY' ◁ 3 OUT SENSMF
[155] SUB<`PARTIAL DERIVATIVES OF POPULATION NUMBERS FOR YOUNGEST AGE GROUP'
[156] HEADER SUB
[157] XK<(pMORT[1;])p0 ◁ J+I+IK+JK+0
[158] EEO:K<(-0.5)
[159] EE1:I+I+1 ◁ J+J+1 ◁ K+K+1
[160] XK[I;J]<(CATCH[I;J]×(*((MORT[1;I;J]×K))×POP[1;(I-K-0.5);(J-K-0.5)])
[161] +(I=pFI)∨(J=pFC))/EE2
[162] +FE1
[163] EE2:>(IK=((pFI)×1))/EE3
[164] I+IK+IK+1 ◁ J+0 ◁ +EEO
[165] EE3:J-JK-JK+1 ◁ I+0 ◁ +(JK×pFC)/EEO
[166] M1<(pFI)pMORT[1;;pFC]
[167] M2<(pFC)pMORT[1;pFI;]
[168] FIT<XK[;pFC]×FIT<(pXK[;pFC])pFIT+((*(-0.5×M1))×Z+(FI×1+*(-Z+FI+M1)))
[169] FCT<XK[pFI;]×FCT<(pXK[pFI;])pFCT+(((*(-0.5×M2))×(FC))×(Z+FC+M2))
[170] +(‘YES’=3×XX)/EE6
[171] FCT<FCT×IN1<(pFCT)pIN1<(1+*Z)
[172] EE6:XK[pFI;]×FCT ◁ XK[;pFC]×FIT
[173] TIT<`SENSITIVITY TO INDIVIDUAL CATCH VALUES' ◁ 3 OUT XK
[174] OVERSENS<OVERSENS+XK[1;]*2 ◁ I<1
[175] SUM:OVERSENS<OVERSENS+((I+XK[I+1;]),I×0)*2
[176] I+I+1
[177] +((1+pXK)-I)>0)/SUM
[178] OVERSENS<OVERSENS*0.5
[179] TIT<`OVERALL SENSITIVITY OF RECRUITMENT'
[180] 2 OUT OVERSENS
[181] ISP<(+/OVERSENS)*pOVERSENS
[182] ' ' ◁ `OVERALL SENSITIVITY INDEX: ',ISP

```

GRAPHAPRODUCTION

The function GRAPHAPRODUCTION can be used after a call to the function COHORT in order to generate a graph showing the variations of the components of production (recruitment, growth) through time.

```
▽ GRAPHAPRODUCTION
[1]  ΔSUPERPLOT 'RESET'
[2]  PER←1
[3]  NEWTITLE 'PRODUCTION VERSUS TIME'
[4]  NEWLABEL 'RECRUITMENT,GROWTH,TOTAL PRODUCTION'
[5]  ΔSUPERPLOT 'XLABEL,YEAR/'
[6]  ΔSUPERPLOT 'YLABEL,COMPONENTS OF PRODUCTION/'
[7]  ΔSUPERPLOT 'LINE,1,STR,BLACK,DOT,○,1/'
[8]  ΔSUPERPLOT 'LINE,2,STR,BLACK,SDASH,△,1/'
[9]  ΔSUPERPLOT 'LINE,3,STR,BLACK,SOLID,▽,1/'
[10] DATA←DATAΔYR
[11] PUT DATA1
[12] PLOT ABOVE
▽
```

GRAPHNETAPROD

The function GRAPHNETAPROD can be used after a call to the function COHORT in order to generate a graph showing the fluctuations of annual net production (the area between the two lines plotted) through time.

```
▽ GRAPHNETAPROD
[1]  ΔSUPERPLOT 'RESET'
[2]  PER←1
[3]  NEWTITLE 'SURPLUS PRODUCTION AND YIELD VERSUS TIME'
[4]  NEWLABEL 'SURPLUS PROD,FISHING LOSS'
[5]  ΔSUPERPLOT 'YLABEL,PRODUCTION'
[6]  ΔSUPERPLOT 'LINE,1,STR,BLACK,SOLID,□,1/'
[7]  ΔSUPERPLOT 'LINE,2,STR,BLACK,DOT,△,1/'
[8]  DATA←DATAΔYR
[9]  PUT DATA2
[10] PLOT ABOVE
▽
```

PRANOASMOOTH

The function PRANOASMOOTH calculates 'partial recruitment' figures and store the results in the global variable SEL.

```
▽ PRANOASMOOTH;INTERSEL
[1]  FMAX←⌈FISHMORT ⌈ I←1
[2]  SEL←((pINTERSEL),1)ρINTERSEL+FISHMORT[;I]+FMAX[I]
[3]  NEXT:→((pFMAX)<I←I+1)/PRINT
[4]  SEL←SEL,((pINTERSEL),1)ρINTERSEL+FISHMORT[;I]+FMAX[I]
[5]  →NEXT
[6]  PRINT:TIT+'SELECTIVITY COEFFICIENTS CALCULATED WITHOUT SMOOTHING'
[7]  YR←-1+YEAR+1ρFISHMORT[1;]
[8]  AG←-1+AGE+1ρFISHMORT[;1]
[9]  2 OUT SEL
▽
```

MPROJECT

The function MPROJECT prepares input data required by MPROJ. The function MPROJECT also calls MPROJOUT, which function prints the output tables evaluated in MPROJ. The function MPROJECT provides also for more projections from the same data when desired. MPROJECT may be seen as the "main" program, which calls the different subroutines necessary to perform the catch projections and to output the results.

```
v MPROJECT;YR;AC;X;M;ANS;Y;POP;CAT;SEL;MAT;REC;CURR;F;SENSM;A;D;I;ONES;SUB;TIT;REC1;OUTM;OUTM1;
CHECK1;CHECK2
[1] ER1:'FIRST YEAR AND YOUNGEST AGE GROUP?'
[2] +(Λ/(X=1), (0≤2+X), 2=pX+,[]) /ER2
[3] +ER1, 0+[]-'MUST BE 2 NON-NEGATIVE INTEGERS. RE-ENTER '
[4] FR2:'NUMBERS AT AGE FOR ',(vX[1]),'?
[5] +(Λ/0≤POP+,[]) /FR3
[6] +ER2, 0+[]-'POPULATION NUMBERS MUST BE POSITIVE. RE-ENTER '
[7] ER3:AC←1+X[2]+pPOP
[8] CAT←(pAG)p-1
[9] ER4:'CATCH AT AGE KNOWN FOR ',(vX[1]),'?(YES OR NO)'
[10] +((Λ/'NO'=2+CHECK), ~Λ/'YES'=3+CHECK←(' '≠CHECK)/CHECK←,[]) /ER6,ER4
[11] ER5:'CATCH AT AGE FOR ',(vX[1]),'?
[12] +(Λ/(CAT≥0),(pAG)=pCAT+,[]) /SK1
[13] +ER5, 0+[]-'ONE NON-NEGATIVE CATCH FOR EACH OF ',(vPAG), ' AGES. RE-ENTER '
[14] SK1:+(Λ/CAT≤POP) /FR6
[15] +FR5, 0+[]-'CATCHES MUST BE LESS THAN POPULATION NUMBERS. RE-ENTER '
[16] FR6:'WEIGHT AT AGE? (KG) :VECTOR OR MATRIX'
[17] +(Λ/(v/((pCAT)=pWG), ((pWG)[1]=pCAT)), 0≤WG+,[]) /ER7
[18] +ER6, 0+[]-'ONE NON-NEGATIVE WEIGHT FOR EACH OF ',(vPAG), ' AGES. RE-ENTER '
[19] ER7:'PARTIAL RECRUITMENT TO FISHING?(VALUES BETWEEN 0 AND 1)'
[20] +((pAG)=pSEL+,[]) /SK2
[21] +ER7, 0+[]-'ONE VALUE FOR EACH OF ',(vPAG), ' AGES. RE-ENTER '
[22] SK2:+(Λ/(v/SFL≥0),(SFL≥0),SEL≤1) /ER8
[23] +FR7, 0+[]-'PARTIAL RECRUITMENTS MUST BE BETWEEN 0 AND 1 AND AT LEAST ONE VALUE MUST DIFFER FROM
0. RE-ENTER '
[24] FR8:'NATURAL MORTALITY?'
[25] +(v/(1,pAG)=pM+,[]) /SK3
[26] +FR8, 0+[]-'EITHER 1 OR ',(vPAG), ' VALUES. RE-ENTER '
[27] SK3:+(Λ/M>0) /FR9
[28] +FR8, 0+[]-'MORTALITIES MUST BE POSITIVE. RE-ENTER '
[29] ER9:'NUMBER OF YEARS TO BE PROJECTED?'
[30] +(Λ/(1=pYR), (YR=YR), 1<YR+,[]) /SK4
[31] +ER9, 0+[]-'A POSITIVE INTEGER GREATER THAN ONE. RE-ENTER '
[32] SK4:YR←1+X[1]+1YR
[33] FR10:'RECRUITMENT FOR ',(vYR[2]), ' TO ',(v-1+pYR), '?
[34] +(Λ/(REC≥0), (-1+pYR)=pREC+,[]) /SK6
[35] +ER10, 0+[]-(v-1+pYR), ' POSITIVE VALUES. RE-ENTER '
[36] SK6:Y←YR[1]+v/-1≠,CAT
[37] +(Y=YR[1]) /SK5
[38] D←(pYR)-1
[39] +ER13
[40] SK5:D+(pYR)
[41] ER13:'QUOTA (IN BIOMASS) OR F VALUES FOR ',(vY), ' TO ',(v-1+pYR), '?
[42] +(Λ/(ANS≥0),(pYR)=pANS←((Y≠YR[1])/0),FVAL&QU+,[]) /ER14
[43] +ER13, 0+[]-(vD), ' NON-NEGATIVE NUMBERS. RE-ENTER '
[44] ER14:' 'Λ ' MATURITY FIGURES TO BE CALCULATED?(YES OR NO)'
[45] MAT←(pAG)p0
[46] +((Λ/'NO'=2+Y), ~Λ/'YES'=3+Y←(' '≠Y)/Y←[]) /ER16,ER14
[47] ER15:'MATURITY AT AGE?'
[48] +((pAG)=pMAT+,[]) /SK7
[49] +ER15, 0+[]-'ONE VALUE FOR EACH OF ',(vPAG), ' AGES. RE-ENTER '
[50] SK7:+(Λ/(MAT≥0),MAT≤1) /ER16
[51] +ER15, 0+[]-'MATURITIES MUST BE BETWEEN 0 AND 1. RE-ENTER '
[52] ER16:' 'Λ ' SENSITIVITY ANALYSIS TO BE PERFORMED (YES OR NO)?'
[53] +((Λ/'YES'=3+CHECK1), ~Λ/'NO'=2+CHECK1←(CHECK1≠' ')/CHECK1+,[]) /ER17,ER16
[54] ER17:REC1+REC
[55] SK8:+(1=pWG) /VECTOR
[56] WGT←WG
[57] +SK9
```

```
[58] VECTOR:WGT←Q((2+ρYR),ρAG)ρWG
[59] SK9:MPROJ
[60] MPROJOUT
[61] +( 'NO'=2↑CHECK1)/SK10
[62] +(CHECK2=1)/SK10
[63] SUB←'SENSITIVITY OF CATCH BIOMASS WITH RESPECT TO PARAMETERS'
[64] HEADER SUB
[65] OUTM1←OUTM
[66] TIT←'SENSITIVITY TO NATURAL MORTALITY'
[67] M←M×1.001 ← MPROJ ← M+M+1.001
[68] 3 OUT SENSM←((OUTM-OUTM1)×1000)÷OUTM1
[69] SEL←SEL×1.001
[70] TIT←'SENSITIVITY TO PARTIAL RECRUITMENT' ← MPROJ ← SEL+SEL+1.001
[71] 3 OUT SENSM←((OUTM-OUTM1)×1000)÷OUTM1
[72] WGT←WGT×1.001
[73] TIT←'SENSITIVITY TO WEIGHT' ← MPROJ ← WGT←WGT+1.001
[74] 3 OUT SENSM←((OUTM-OUTM1)×1000)÷OUTM1
[75] POP←POP×1.001
[76] TIT←'SENSITIVITY TO POPULATION NUMBERS IN ',(↙X[1]),'?' ← MPROJ
[77] POP←POP+1.001
[78] 3 OUT SENSM←((OUTM-OUTM1)×1000)÷OUTM1
[79] REC←REC1←REC1×1.001 ← MPROJ
[80] TIT←'SENSITIVITY TO RECRUITMENT' ← REC1←REC1+1.001
[81] 3 OUT SENSM←((OUTM-OUTM1)×1000)÷OUTM1
[82] +( (ρFVALΔQU)≠ρQUOTA←(ANS>10)/ANS )/SK10
[83] ANS←ANS×1.001
[84] TIT←'SENSITIVITY OF F TO QUOTA' ← MPROJ
[85] ANS←ANS+1.001
[86] 3 OUT SENSM←(((~1 0 +CURR[3::])-FISHMORT)×1000)÷FISHMORT
[87] SK10:' ' ← 'MORE PROJECTIONS WITH THIS DATA?(YES OR NO)'
[88] +( (~/'YES'=3↑Y),~~/'NO'=2↑Y+(Y↑' ') / Y←,[] )/ER7,SK10
      ▽
```

MPRC

The function MPROJ calculates population numbers and biomass, catch (both in numbers and biomass), age-specific fishing mortalities and sensitivity coefficients with respect to certain parameters. The function MPROJ performs all the calculations by using the data entered as input information in MPROJECT.

```

    ▽ MPROJ;C;C1;C2;X;Z;ZS;S
[1] INIT:CURR←(3,(ρAC),ρYR)ρ⁻¹
[2] CURR[1;;1]←POP
[3] CURR[2;;1]←CAT ∧ REC←REC1 ∧ CHECK2←0
[4] X*1
[5] FINP:→((∨/CURR[2;;X]≠-1),ANS[X]≥10)/CHECK,YICALC
[6] CURR[3;;X]←SEL×ANS[X]
[7] CURR[2;;X]←CURR[3;;X] CALCC CURR[1;;X]
[8] +UPDATE
[9] CHECK:CURR[3;;X]←SEL
[10] →(∧/CURR[1;;X]>CURR[2;;X])/ADJUST
[11] 'CATCH GREATER THAN POPULATION'
[12] CURR[2;;X]←CURR[2;;X]LCURR[1;;X]
[13] ADJUST:C←CURR[3;;X] CALCC CURR[1;;X]
[14] +(0.01>/|C-CURR[2;;X])/UPDATE
[15] C1←-(CURR[3;;X]×CURR[1;;X]+CURR[3;;X]+M)×**-CURR[3;;X]+M
[16] C2←-(CURR[1;;X]×M+(CURR[3;;X]+M)*2)×1**-CURR[3;;X]+M
[17] CURR[3;;X]←CURR[3;;X]-(CURR[2;;X]-C)÷C1+C2
[18] +ADJUST
[19] YICALC:CURR[3;;X]←SEL
[20] +(ANS[X]<+/WGT[;X]×CURR[1;;X])/ADJTOQUO
[21] 'QUOTA GREATER THAN POPULATION BIOMASS FOR YEAR ',(¬YR[1]+X-1)
[22] CURR[3;;X]←SEL ∧ CHECK2←1
[23] CURR[2;;X]←CURR[3;;X] CALCC CURR[1;;X]
[24] +UPDATE
[25] ADJTOQUO:CURR[2;;X]←CURR[3;;X] CALCC CURR[1;;X]
[26] +(0.01>|ANS[X]-+/WGT[;X]×CURR[2;;X])/UPDATE
[27] CURR[3;;X]←CURR[3;;X]×ANS[X]÷+/WGT[;X]×CURR[2;;X]
[28] +ADJTOQUO
[29] UPDATE:→((ρYR)<X+X+1)/Fxit
[30] CURR[1;;X]←(1+REC),-1+CURR[1;;X]←CURR[1;;X-1]×**-CURR[3;;X-1]+M
[31] REC←1+REC
[32] +FINP
[33] EXIT:CURR←0[CURR,[2]+/[2] 0[CURR
[34] OUTM←(-1 0 +CURR[2;;])×WGT[,1ρYR]

```

The function CALCC calculates the catch in numbers by using the formula

$$C = \frac{F}{Z} N (1 - e^{-Z})$$

The parameters F and N must be sent as left and right argument respectively.
This function is used by MPROJ.

```
[1] CATCH←((F×N)+F+M)×1★-F+I
```

MPROJOUT

The function MPROJOUT outputs all the information which has been stored in tables by MPROJ. The calculations of the numbers and biomass of mature fish in the population is also performed within this function. The function MPROJOUT calls the utility functions OUT and HEADER.

```

▼ MPROJOUT;X;TIT;I
[1] TIT←'POPULATION NUMBERS'
[2] POPNB← 1 0 +CURR[1;;]
[3] 0 OUT POPNB,[1](AG[1],AG[4]) APLUS POPNB
[4] TIT←'POPULATION BIOMASS (AVERAGE)'
[5] I←(ρPOPNB)[1] ⌈ J←(ρPOPNB)[2]
[6] FISHMORT← 1 0 +CURR[3;;]
[7] MORT←Q((J,I)ρM)
[8] MEANPOP←(POPNB×(1+*FISHMORT+MORT))÷FISHMORT+MORT
[9] POPBIOMASSΔMIDYR←OUTM×MFANPOPKWT[,1+J+1]
[10] 2 OUT OUTM,[1](AG[1],AG[4]) APLUS OUTM
[11] TIT←'MATURE NUMBERS'
[12] +(^/0=MAT)/N2
[13] OUTM←(1 0 +CURR[1;;])×Q((ρYR),ρAG)ρMAT
[14] 0 OUT OUTM,[1]+/[1] OUTM
[15] TIT←'MATURE BIOMASS'
[16] OUTM←(1 0 +CURR[1;;])×WGT[,1+J+1]×Q((ρYR),ρAG)ρMAT
[17] 2 OUT OUTM,[1]+/[1] OUTM
[18] N2:TIT←'CATCH NUMBERS'
[19] 0 OUT CATCH,[1](AG[1],AG[4]) APLUS CATCH← 1 0 +CURR[2;;]
[20] TIT←'CATCH BIOMASS'
[21] OUTM←(1 0 +CURR[2;;])× 0 -1 + 0 1 +WGT
[22] 0 OUT OUTM,[1](AG[1],AG[4]) APLUS OUTM
[23] TIT←'MEAN WEIGHT OF INDIVIDUALS IN CATCH'
[24] 1 OUT (+/[1] OUTM)+/[1] 1 0 +CURR[2;;]
[25] TIT←'FISHING MORTALITY'
[26] FWGT←(+/[1](POPNB×FISHMORT))+/[1] POPNB
[27] 3 OUT FISHMORT,[1] FWGT
[28] WTΔBEGΔYR:WT←*((@(-1 -1 +WGT))+@ 1 1 +WGT)+2
[29] FIRSTAGWT←*2×(@-1+WGT[1;1])-@WT[1;1]+2
[30] FIRSTYRWT←*2×(@-1+WGT[,1])-@WT[,1]+2
[31] FIRSTYRLASTAGWT←*((@WT[1;1])+(@WT[I-1;2]))+2
[32] WGTΔBEG←((1,J+2)ρ0),[1](((I-1),1)ρ0),WT
[33] WGTΔBEG[1;1+J+2]←FIRSTAGWT
[34] WGTΔBEG[,1]←FIRSTYRWT,FIRSTYRLASTAGWT
[35] LSTAGE←*(2×WGT[,1])-@WGTΔBEG[,1]
[36] WGTΔBEG←WGTΔBEG,[1] LSTAGE
[37] +(PRINTΔPROD[1]=0)/POPBIOMASS
[38] YR←(YR[1]-1),YR,(YR[ρYR]+1) ⌈ AG←AG,AC[ρAG]+1
[39] TIT←'WEIGHTS AT THE BEGINNING OF THE YEAR' ⌈ 2 OUT WGTΔBEG
[40] YR←-1+1+YR ⌈ AG←-1+AG
[41] POPBIOMASS:POPBIOMASSΔBEG←POPNB× 1 1 + 0 -1 +WGTΔBEG
[42] +(PRINTΔPROD[2]=0)/RECΔBIOMASS
[43] TIT←'POPULATION BIOMASS AT BEGINNING OF YEAR'
[44] 0 OUT POPBIOMASSΔBEG,[1]+/[1] POPBIOMASSΔBEG
[45] RECΔBIOMASS:RECRUITΔBIOMASS←POPBIOMASSΔBEG[1;]
[46] ΔB←POPNB×((1 2 +WGTΔBEG)*-(FISHMORT)+MORT)- 1 -1 + 0 1 +WGTΔBEG
[47] NETPROD←(+/[1] ΔB)+RECRUITΔBIOMASS
[48] FISHINGBIOMASSLOSS←CATCH×WGT[,1+J+1]
[49] DT←MORT×POPBIOMASSΔMIDYR
[50] BG←ΔB+DT+FISHINGBIOMASSLOSS
[51] TIT←'DISTRIBUTION OF GROWTH OVER AGES (PERCENT)'
[52] 1 OUT(BG×((ρBG)ρ(+/[1] BG)))×100
[53] PRODUCTION←(+/[1] BG)+RECRUITΔBIOMASS
[54] LB←'RECRUITMENT BIOMASS GROWTH' TOTAL PRODUCTION LOSS THROUGH FISHINGSURPLUS PR
ODUCTION NET PRODUCTION
[55] LB← 6 20 ρLB
[56] TIT1←'SOURCE' ⌈ TIT←'PRODUCTION'
[57] PROD←RECRUITΔBIOMASS,(+/[1] BG),PRODUCTION,(+/[1] FISHINGBIOMASSLOSS),(PRODUCTION+/[1] DT),
NETPROD
[58] PROD←(6,ρYR)ρPROD
[59] SECT← 3 3
[60] 0 TABLE PROD
[61] TIT←'PRODUCTION/BIOMASS RATIO'
[62] 2 OUT PRODUCTION+/[1] POPBIOMASSΔMIDYR

```

PALOHEIMO

The function PALOHEIMO estimates the annual rates of total mortality (Z), the catchability coefficient (q) and the annual rate of natural mortality (M). PALOHEIMO calls the library functions REGR and PLOT, as well as the utility function FILEAFETCH.

```
v PALOHEIMO
[1]  NAM←NL 2
[2]  ENTRY:'ENTER NAME OF CPU MATRIX FOR RESEARCH VESSELS(NUM/TOW)'
[3]  REENTRY1:→(v/NAM^.=CHECK1←(`1↑pNAM)↑(' '≠CHECK1)/CHECK1←,[])/RET
[4]  →REENTRY,0↑[]←'NAMED MATRIX DOES NOT EXIST. RE-ENTER.'
[5]  RET:'ENTER FIRST YEAR AND YOUNGEST AGE GROUP'
[6]  →(^/(TEMP=[TEMP],(0≤2↑TEMP),2=pTEMP←,[])/OK1
[7]  →RET,0↑[]←'MUST BE TWO NON-NEGATIVE NUMBERS '
[8]  OK1:YEAR←1↑TEMP ◁ AGE←1↑TEMP
[9]  YR←(YEAR-1)+`2+YEAR+1↑p←CHECK1
[10] AG←(AGE-1)+`2+AGE+1↑p←CHECK1
[11] CPUE←p←CHECK1
[12] CPUE←CPUE+(CPUE=0)×9.9999×10*7
[13] Z←(`1 `1 +CPUE)‡(1 1 +CPUE)
[14] INSERT←((Z>1000)+(Z<10*`3))
[15] Z←Z+INSERT+Z×~INSERT
[16] TIT←'ESTIMATES OF TOTAL MORTALITIES(Z)'
[17] 3 OUT Z,[1] ZMEAN←(+/[1] Z)+(pZ[,1])→+/[1] INSERT
[18] TEST:' ' ◁ 'REGRESSION WITH AN INDEX OF FISHING EFFORT DESIRED(YES OR NO)?'
[19] →((^/'NO'=2↑CHECK2),~^/'YES'=3↑CHECK2←(' '≠CHECK2)/CHECK2←,[])/0,TEST
[20] 'ENTER VALUES OF FISHING EFFORT'
[21] REENTRY2:→(^/(FVAL>0),(1+pZMEAN)=pFVAL+[],)/CALC
[22] →REENTRY2,0↑[]←' THERE MUST BE ',(1+pZMEAN), ' VALUES ALL NON-NEGATIVE. RE-ENTER '
[23] CALC:F←((`1+FVAL)+(1+FVAL))÷2
[24] F←((pF),1)pF
[25] FILEAFETCH '2719067 PFΔREGRESS'
[26] NOANOVA ◁ NOCORRELATION ◁ ZMEAN REGR F
[27] DIST←`(`F)‡30
[28] Z←B[,]+(B[,]*FF+DIST× 1 30)
[29] ZMEAN←((pZMEAN),1)pZMEAN
[30] Z←((pZ),1)pZ
[31] FF←((pFF),1)pFF
[32] Z←Z,[1] ZMEAN
[33] FF←FF,[1] F
[34] DATA←FF,Z,((2,1)p1),[1]((pF[,1]),1)p2
[35] LPDEF PAGRAPHICS
[36] ABSISSA LOW LIMIT 0 ◁ ORDINATE LOW LIMIT 0
[37] ABSISSA HIGH LIMIT`FF ◁ ORDINATE HIGH LIMIT`Z
[38] ABSISSACOL 1 ◁ SYMBOLCOL 3 ◁ PLOTCHARS '.o'
[39] AXES FLOATING ◁ NORMAL
[40] SET ABSISSA TOTAL 6 SPACED 10 NUMBERED 1
[41] SET ORDINATE TOTAL 6 SPACED 5 NUMBERED 1
[42] ABSISSA TEXT 'FISHING EFFORT'
[43] ORDINATE TEXT 'ZMEAN'
[44] PLOT DATA
```

v

SURVIVOR

The function SURVIVOR is used to input data which are required for estimating the survivors in the current year. This function may be seen as the 'main' program, which calls the subroutines necessary to perform the calculations and to output the results. Namely, SURVIVORS calls the functions SURVIVORINT, KCONST, ESTASURVIVORS, WEIGHTEDAS and ANALVAR.

```
v SURVIVOR;NAM;X;P1;P2;ITER;NUMALTER;RVABUND;CATCH;SYR;SAG;I;T;YR;AG;YRPT;
AGPT;BLKSIZEAG;BLKSIZEYR;CALBLK;CHECK;CHECK1;CHECK2;K;KT;KT2;LNDIF;LNK;
MVAR;MVAR2;OUTM;CNT;MM;PRINTMAT;KK;SHAT;SUM;I1
[1] NAM<NL 2 & ITER<1
[2] ER1:'FIRST YEAR AND YOUNGEST AGE GROUP?'
[3] +(^(P1=P1),(0≤2↑P1),2=pP1<,□)/ER2
[4] +ER1,0↑□<" MUST BE 2 POSITIVE INTEGERS. RE-ENTER "
[5] ER2:'FINAL YEAR AND OLDEST AGE GROUP IN CALIBRATION BLOCK'
[6] +(^(0<P2-P1),(P2=P2),(0≤2↑P2),2=pP2<,□)/ER3
[7] +ER2,0↑□<" MUST BE 2 POSITIVE INTEGERS LARGER THAN INITIAL VALUES. RE-ENT
ER "
[8] ER3:'AGE AT WHICH K SHOULD LEVEL OFF'
[9] +(^(I1≤P2[2]),P1[2]<I1<,□)/ER4
[10] +ER3,0↑□<" MUST BE SINGLE VALUE BETWEEN ",(▼P1[2])," AND ",▼P2[2]
[11] ER4:'NAME OF RESEARCH VESSEL ABUNDANCE MATRIX'
[12] +(▼/NAM=.=X<(^1+pNAM)↑(^ 'zX)/X<,□)/OK1
[13] +ER4,0↑□<" NAMED MATRIX DOES NOT EXIST. RE-ENTER "
[14] OK1:+(2=pP1RVABUND+□)/OK2
[15] +ER4,0↑□<(^ 'zX)/X)," IS NOT A MATRIX. RE-ENTER "
[16] OK2:+(^(^/0<,RVABUND)/ER5
[17] +0,0↑□<" NEGATIVE VALUES IN RESEARCH VESSEL ABUNDANCE MATRIX"
[18] ER5:'NAME OF CATCH MATRIX?'
[19] +(▼/NAM=.=X<(^1+pNAM)↑(^ 'zX)/X<,□)/OK3
[20] +ER5,0↑□<" NAMED MATRIX DOES NOT EXIST. RE-ENTER "
[21] OK3:+(2=pP1CATCH+□)/OK4
[22] +ER5,0↑□<(^ 'zX)/X)," IS NOT A MATRIX. RE-ENTER "
[23] OK4:+(^(^/pCATCH)=pP1RVABUND)/OK5
[24] +ER5,0↑□<" DIMENSIONS MUST BE THE SAME AS THOSE OF THE CATCH MATRIX"
[25] OK5:+(^(^/0≤,CATCH)/OK6
[26] +0,0↑□<" NEGATIVE VALUE IN CATCH MATRIX"
[27] OK6:YR<^1+P1[1]+,T<(pCATCH)[2]
[28] AG<^1+P1[2]+,I<(pCATCH)[1]
[29] ER6:'SURVIVORS IN POPULATION FOR LAST YEAR?'
[30] +(1,I)=pSYR<,□)/OK7
[31] +ER6,0↑□<" ENTER 1 VALUE FOR EACH ROW OF CATCH MATRIX. RE-ENTER "
[32] OK7:+(^(^/0≤SYR)/ER7
[33] +ER6,0↑□<" VALUES MUST BE POSITIVE. RE-ENTER "
[34] ER7:'SURVIVORS IN POPULATION FOR OLDEST AGE GROUP?'
[35] +(1,T)=pSAG<,□)/OK8
[36] +ER7,0↑□<" ENTER ONE VALUE FOR EACH COLUMN OF CATCH MATRIX. RE-ENTER "
[37] OK8:+(^(^/0≤SAG)/ER8
[38] +ER7,0↑□<" VALUES MUST BE POSITIVE. RE-ENTER "
[39] ER8:'NATURAL MORTALITY'
[40] +(^(^/M≥0),0=pPM<,□)/ER9
[41] +ER8,0↑□<" NATURAL MORTALITY MUST BE 1 POSITIVE NUMBER. RE-ENTER "
[42] ER9:'MAXIMUM NUMBER OF ITERATIONS?'
[43] +(^(^/NUMALTER≥1),0=pP1NUMALTER<,□)/ER10
[44] +ER9,0↑□<" MUST BE ONE NUMBER≥1. RE-ENTER "
[45] ER10:'ENTER 1 IF MATRIX IS TO BE PRINTED. OTHERWISE ENTER 0.'
[46] +'ONE NUMBER IS REQUIRED IN VECTOR FOR EACH MATRIX'
[47] +'INTEGRATED CATCH' & 'INTEGRATED SURVIVORS' & 'ESTIMATED SURVIVORS'
[48] +'ESTIMATED VARIANCE OF SURVIVORS' & 'WEIGHTED SURVIVORS' & 'OUTLIERS OF R
ESIDUALS'
[49] +(^(6=+(1=PRINTMAT),(0=PRINTMAT),6=pP1PRINTMAT<,□)/ER11
[50] +ER10,0↑□<" RE-ENTER "
[51] ER11:'ANALYSIS OF VARIANCE TO BE PERFORMED(YES OR NO)?'
[52] +(^(^/YES)=3↑CHECK),~^(^/NO)=2↑CHECK<(^ 'zCHECK)/CHECK<,□)/OK9,ER11
[53] OK9:YRPT<P2[1] & AGPT<P2[2]
[54] MM<AG[pAG] & CNT<0
[55] SUM<CATCH<(^1+*M*2)*M
[56] SUMMATION:+(▼/(CHECK2<=T≤CNT+1),CHECK1<I≤1+CNT<=CNT+1)/LAST
```

```
[57] +(^(1=CHECK1),CHECK1=CHECK2)/LAST
[58] SUM←SUM+(((M×CNT+0.5)-M×CNT-0.5)×M)×((CNT,CNT)+CATCH),((-CNT)+I),CNT)
    p0,[1](CNT,T)p0
[59] →SUMMATION
[60] LAST:SUM[I;]←CATCH[I;]×(1+M×2)×M
[61] SUM[;T]←CATCH[;T]×(1+M×2)×M
[62] CINT←SUM
[63] +(PRINTMAT[1]=0)/SURVEY
[64] TIT←'INTEGRATED CATCH' ◊ 0 OUT CINT
[65] SURVEY:SURVIVORINT
[66] POPNB←CINT+SINT
[67] +(ITER=NUMAITER)/CONSTANT
[68] TIT←'POPULATION NUMBERS'
[69] 0 OUT POPNB
[70] CONSTANT:KΔCONST
[71] ESTΔSURVIVORS
[72] ESTΔVARΔSURV
[73] WEIGHTEDΔS
[74] +(ITER=NUMAITER)/VARIANCEANALYSIS
[75] ITER←ITER+1 ◆ SYR←SURVΔLASTΔYR ◊ SAG←SURVΔOLDESTΔAG
[76] →SURVEY
[77] VARIANCEANALYSIS:+('NO'=2+CHECK)/0
[78] ANALΔVAR
    ▽
```

SURVIVORINT

This function is used to calculate the integrated survivors ($SINT_{i+.5, t+.5}$) and to print the calculated values after the last iteration.

```
▽ SURVIVORINT;UPPERΔ;LOWERΔ;ROW;COL;CNT1;CNT2;TIT
[1]   UPPERΔ←LOWERΔ+(pCATCH)p0
[2]   COL+T ◆ ROW+I ◆ CNT1+CNT2+1
[3]   UPPERΔ[;T]←SYR ◆ LOWERΔ[I;]←SAG
[4]   UPPER:+(1≥COL+COL+1)/LOWER
[5]   UPPERΔ[;COL]←(CNT1+SYR),[1] CNT1p0
[6]   +(T≥CNT1+1)/UPPER
[7]   LOWER:+(1>ROW+ROW-1)/JOIN
[8]   LOWERΔ[ROW;]←(CNT2+SAG),CNT2p0
[9]   +(T≥CNT2+1)/LOWER
[10]  JOIN:UPPERΔ←UPPERΔ×(1,T)pF1+M×YR[T]+0.5-((P1[1]-1)+1YR[T])
[11]  LOWERΔ←LOWERΔ×(T,I)pF2+M×AG[I]+0.5-((P1[2]-1)+1AG[I])
[12]  UPPERΔ←UPPERΔ×UPPERΔ*LOWERΔ
[13]  SINT←UPPERΔ+LOWERΔ
[14]  +(ITER=NUMAITER)/0
[15]  ' ' ◆ 'FINAL ITERATION (',(ITER),')'
[16]  +(PRINTMAT[2]=0)/0
[17]  TIT←'INTEGRATED SURVIVORS'
[18]  0 OUT SINT
    ▽
```

ESTAVARASURV

The function ESTAVARASURV calculates the variance for the successive estimates of survivors which are calculated by the function ESTASURVIVORS. The function ESTAVARASURV is called by the 'main' function SURVIVOR.

```
▽ ESTAVARASURV;FACTOR;OUTERFACT
[1] RMS←(+/[1]+/((BLKSIZEAG,BLKSIZEYR)+(POPNB-CORAR)+POPNB)*2)*BLKSIZEAG×
    BLKSIZEYR
[2] ESTAVARAS←(EE*2)×RMS×CORAR*2
[3] FACTOR←(I,T)ρ1+1×BLKSIZEYR
[4] FACTOR[(1+BLKSIZEAG×CALBLK)+I;]->OUTERFACT←1+1×BLKSIZEYR×CALBLK
[5] FACTOR[;BLKSIZEYR+I,T]->OUTERFACT
[6] ESTAVARAS←ESTAVARAS×FACTOR
[7] +(ITER*NUMΔITER)/0
[8] +(PRINTMAT[4]=0)/0
[9] TIT←'ESTIMATED VARIANCE OF SURVIVORS'
[10] 0 OUT ESTAVARAS
    ▽
```

WEIGHTEDAS

The function WEIGHTEDAS calculates the weighted average of the survivors for each age-group in the final year (S_{i,t_f}) and of the oldest age-group in each year ($S_{m,t}$). This function is also used to output the statistics which are necessary to monitor the convergence of the iterative process and to print the results after the final iteration (estimated survivors, estimated K, residuals). The function WEIGHTEDAS is called by the 'main' function SURVIVOR.

```
▽ WEIGHTEDAS;POOL;POOLAG;POOLYR;INVΔVAR;CNT;POOLEDΔVAR;W;WΔSURV;CRITΔYR;
    PERCΔSURV;INVΔSURVΔSUM;VARΔSΔLASTΔYR;VARΔSΔOLDESTΔAG;STANDΔERΔLASTΔYR;
    STANDΔERΔOLDESTΔAG;CΔVΔLASTΔYR;CΔVΔOLDESTΔAG;OUTM1;OUTM2;OUTM3;STANDΔERΔK
    ;DF;SDARESID;MEANΔRESID;UPPERBND;LOWERBND;OUTER
[1] POOL←INVΔVAR←1×ESTAVARAS ◇ DIAGΔSUM INVΔVAR
[2] POOLAG←POOLYR←(I,T)ρ0
[3] POOLAG[1;]←POOL[1;] ◇ POOLYR[;1]←POOL[;1]
[4] CNT←0
[5] UPPER:=(√/(I<1+CNT),T<1+CNT+CNT+1)/COLUMN
[6] POOLAG[CNT+1;]←(CNTρ0),(-CNT)×POOL[1;]
[7] →UPPER
[8] COLUMN:CNT←0
[9] LOWER:=(√/(I<1+CNT),T<1+CNT+CNT+1)/CALC
[10] POOLYR[;CNT+1]←(CNTρ0),(-CNT)×POOL[;1]
[11] →LOWER
[12] CALC:POOLEDΔVAR←POOLYR+POOLAG×POOLAG×POOLYR
[13] W←1×ESTAVARAS×POOLEDΔVAR
[14] WΔSURV←W×SHAT
[15] +(ITER*NUMΔITER)/SUM
[16] +(PRINTMAT[5]=0)/SUM
[17] TIT←'WEIGHTED SURVIVORS' ◇ 0 OUT WΔSURV
[18] SUM:WΔSURV←WΔSURV+0.00001×WΔSURV=0
[19] SURVΔYR←SURVΔAG++/ 1 1 ×WΔSURV
[20] CNT←0
[21] NEXTCOL:=(T≤CNT+CNT+1)/OLDESTAGE
[22] SURVΔYR←SURVΔYR,+/ 1 1 ×(0,CNT)×WΔSURV
[23] →NEXTCOL
[24] OLDESTAGE:CNT←0
[25] NEXTROW:=(I≤CNT+CNT+1)/WEIGHTEDS
[26] SURVΔAG←SURVΔAG,+/ 1 1 ×(CNT,0)×WΔSURV
[27] →NEXTROW
[28] WEIGHTEDS:SURVIVORS←(Φ1+SURVΔAG),SURVΔYR
[29] SURVΔOLDESTΔAG←T+SURVIVORS
[30] SURVΔLASTΔYR←Φ(-I)×SURVIVORS
```

```
[31] →(ITER=NUMAITER)/FINALITERATION
[32] CRITΔYR←(|SYR-SURVALASTΔYR)*SYR
[33] +(Δ/(Δ/0.001≥CRITΔYR))/CONVERGENCEMET
[34] PERCΔSURV←(+/CRITΔYR*100)‡I
[35] ' ' ◇ ' ITERATION ', (‐ITER), ' RMS=' , (‐RMS)
[36] ' ' ◇ ' O/O CHANGE IN SURVIVORS=' , (‐PERCΔSURV)
[37] ' ' ◇ ' AGE SURVIVORS K'
[38] OUTM←(((ρAG),1)ρAG,(((ρAG),1)ρSURVALASTΔYR),((ρAG),1)ρK
[39] 4 0 12 0 12 2 ‐OUTM
[40] →0
[41] FINALITERATION:NUMAITER+ITER
[42] INVΔSURVΔSUM←(Φ1+POOL[;1]),POOL[1;]
[43] VARΔSALASTΔYR←1‡Φ(‐I)↑INVΔSURVΔSUM
[44] VARΔSALDESTΔAG←1‡T↑INVΔSURVΔSUM
[45] STANDΔERΔLASTΔYR←VARΔSALASTΔYR*0.5
[46] STANDΔERΔOLDESTΔAG←VARΔSALDESTΔAG*0.5
[47] CΔVΔLASTΔYR←100*STANDΔERΔLASTΔYR+SURVΔLASTΔYR
[48] CΔVΔOLDESTΔAG←100*STANDΔERΔOLDESTΔAG+SURVΔOLDESTΔAG
[49] ' ' ◇ ' RMS=' , (‐RMS)
[50] ' ' ◇ ' ESTIMATED SURVIVORS FOR AGE ', (‐AG[I]), ' (WEIGHTED)' ◇ ' '
[51] ' ' ◇ ' YEAR SURVIVORS VARIANCE STANDARD ERROR C.V.(O/O)'
[52] OUTM1←(((ρYR),1)ρYR,(((ρYR),1)ρSURVΔOLDESTΔAG)
[53] OUTM1←OUTM1,(((ρYR),1)ρVARΔSALDESTΔAG),(((ρYR),1)ρSTANDΔERΔOLDESTΔAG)
[54] OUTM1←OUTM1,(((ρYR),1)ρCΔVΔOLDESTΔAG)
[55] 8 0 15 0 15 0 12 0 12 2 ‐OUTM1
[56] ' ' ◇ ' ESTIMATED SURVIVORS FOR ', (‐YR[T]), ' (WEIGHTED)' ◇ ' '
[57] ' ' ◇ ' AGE SURVIVORS VARIANCE STANDARD ERROR C.V.(O/O)'
[58] OUTM2←(((ρAG),1)ρAG,((ρAG),1)ρSURVALASTΔYR),((ρAG),1)ρVARΔSALASTΔYR
[59] OUTM2←OUTM2,((ρAG),1)ρSTANDΔERΔLASTΔYR),((ρAG),1)ρCΔVΔLASTΔYR
[60] 8 0 15 0 15 0 12 0 12 2 ‐OUTM2
[61] STANDΔERΔK←MVAR*0.5
[62] ' ' ◇ ' FINAL ESTIMATION FOR K'
[63] ' ' ◇ ' AGE K LN(K) VAR(LN(K)) STANDARD ERROR D.F'
[64] OUTM3←(((ρAG),1)ρAG,(((ρAG),1)ρK),((ρAG),1)ρLNK
[65] DF←(((I1-AG[1]),1)YR[T]-YRPT),[1](I1+AG[1]-I1),1)ρ⁻¹+CALBLK×BLKSIZEYR
[66] OUTM3←OUTM3,((ρAG),1)ρVARΔLNK),(((ρAG),1)ρSTANDΔERΔK),DF
[67] 4 0 10 2 10 4 10 4 15 4 8 0 ‐OUTM3
[68] RESIDUAL←(OKK×KVABUND)-POPNB
[69] TIT←'RESIDUALS'
[70] 4 OUT RESIDUAL
[71] MEANΔRESID←(+/+/[1] RESIDUAL)‡⁻¹+T×I
[72] SDΔRESID←((+/+[1](RESIDUAL-MEANΔRESID)*2)*(I×T)-1)*0.5
[73] UPPERBND←MEANΔRESID+2×SDΔRESID
[74] LOWERBND←MEANΔRESID-2×SDΔRESID
[75] OUTFR←RESIDUAL×(RESIDUAL<LOWERBND)+(RESIDUAL>UPPERBND)
[76] +(PRINTMAT[6]=0)/0
[77] ' ' ◇ ' MEAN OF RESIDUALS=' , (‐MEANΔRESID)
[78] ' ' ◇ ' STANDARD DEVIATION OF RESIDUALS=' , (‐SDΔRESID)
[79] TIT←'OUTLIERS OF RESIDUALS' ◇ 2 OUT OUTER
[80] →0
[81] CONVERGENCEMET:' ' ◇ ' FINAL ITERATION (' , (‐ITER) , ')'
[82] +(PRINTMAT[2]=0)/POP
[83] TIT←'INTEGRATED SURVIVORS' ◇ 0 OUT SINT
[84] POP:TIT←'POPULATION NUMBERS' ◇ 0 OUT POPNB
[85] +(PRINTMAT[3]=0)/NEXT
[86] TIT←'ESTIMATED SURVIVORS' ◇ 0 OUT SHAT
[87] NEXT: +(PRINTMAT[4]=0)/NEXT1
[88] TIT←'ESTIMATED VARIANCE OF SURVIVORS' ◇ 0 OUT ESTΔVARΔS
[89] NEXT1: +(PRINTMAT[5]=0)/FINALITERATION
[90] TIT←'WEIGHTED SURVIVORS' ◇ 0 OUT WASURV
[91] →FINALITERATION
```

KACONST

The function KACONST is used to estimate the calibration constants, K_{i+5} , and their variances. The function KACONST is called by the 'main' function SURVIVOR.

```
▽ KACONST
[1] BLKSIZEYR+YRPT+1-YR[1]
[2] BLKSIZEAG+AGPT+1-AG[1]
[3] LNDIF+POPNB*RVABUND
[4] LNK<-(KT+/(BLKSIZEAG,BLKSIZEYR)+LNDIF)*BLKSIZEYR
[5] LNK<((~CALBLK)+LNK),(1+AG[I]-I1)*KT2+(/[1]((I1+AG[1])+KT))+BLKSIZEYR*
CALBLK+1+AGPT-I1
[6] LNKK+Q(T,I)*LNK
[7] VARΔLNKDUM<+/(I1-AG[1]),BLKSIZEYR)+(LNKK+(RVABUND)-POPNB)*2
[8] MVAR<(VARΔLNK+VARΔLNKDUM+BLKSIZEYR-1)*BLKSIZEYR
[9] CALBLK+1+AGPT-I1
[10] MVAR2<(VARΔLNKT2+(/[1] VARΔLNKDUM)+1+BLKSIZEYR*CALBLK)+BLKSIZEYR*
CALBLK
[11] VARΔLNK<VARΔLNK,(1+AG[I]-I1)*VARΔLNKT2
[12] MVAR<MVAR,(1+AG[I]-I1)*MVAR2
[13] K+*LNK+VARΔLNK*2
▽
```

ESTΔSURVIVORS

The function ESTΔSURVIVORS provide estimates of survivors for all age-groups in the last year ($S_{i,t_f,j}$) and for the oldest age-group in each year ($S_{m,t,j}$). This function is called by the 'main' function SURVIVOR,

```
▽ ESTΔSURVIVORS;CNT;ROW;COL;TIT;E
[1] E+(I,T)*0.5
[2] CNT+0 < ROW+I-1
[3] INCREASE1:+(v/(T≤1+CNT+CNT+1),ROW≤0)/INCREASE
[4] E[ROW;1T-CNT]+E[ROW;1T-CNT]+CNT
[5] ROW<ROW-1 < +INCREASE1
[6] INCREASE:CNT+0 ◇ COL+T-1
[7] INCREASE2:+(v/(I≤1+CNT+CNT+1),COL≤0)/MORTADJUST
[8] E[1I-CNT+1;COL]+E[1I-CNT+1;COL]+CNT
[9] COL<COL-1 < +INCREASE2
[10] MORTADJUST:EE+*MxE
[11] KK+Q(T,I)*K
[12] SHAT<EE*(CORAR+KK*RVABUND)-CINT
[13] SHAT<SHAT*0≤SHAT
[14] +(ITER≠NUMAITER)/0
[15] +(PRINTMAT[3]=0)/0
[16] TIT<ESTIMATED SURVIVORS'
[17] 0 OUT SHAT
▽
```

DIAGASUM (calling form: DIAGASUM MATRIX)

DIAGASUM is an utility function which is used by the function WEIGHTEDAS in order to perform summations along the diagonals of a matrix (global variable called MATRIX). The results are stored in the global variable (vector) POOL.

▽ DIAGASUM MATRIX

```
[1] CNT←0
[2] SUMMATION:→(v/(CHECK2+T<1+CNT),CHECK1←I<1+CNT+CNT+1)/0
[3] →(A/(1=CHECK1),CHECK1=CHECK2)/0
[4] POOL←POOL+(((CNT,CNT)↑MATRIX),(((-CNT)+I),CNT)ρ0),[1](CNT,T)ρ0
[5] →SUMMATION
```

▽

ANALAVAR

The function ANALAVAR performs an analysis of variance on the residuals in order to identify the presence of systematic errors. ANALAVAR is called only once by the 'main' function SURVIVOR, at the end of the iterative process.

▽ ANALAVAR;CNT;YY;AGES;YEARS;YEARCLASS;AGE;DIMAYR;CLASS;YRCLASS;YRAYRCL;

DATA;TEMP1;TEMP2;B;VARAB;SSQAB;STANDAERAB;CONST;SSQ;TOTASSQ;RESIDASSQ;SS;
SS1;SS2;LABEL1;LABEL2;OUTM;OUTM1;TYEARS;TYRAYRCL;MSARESID;DFAG;DFAYR;

DFAVRCL;MSDAG;MSAYR;MSAYRCL;FAAG;FAYR;FAYRCL;RESIDADF

```
[1] YY←Y←,RESIDUAL ◇ CONST+(+/Y)*ρY ◇ Y-Y-CONST
[2] AGES←(T,I)ρ0
[3] CNT←1
[4] AGES[;T;CNT]←1
[5] YEARS←DIMAYR+(,T)◦.=,T
[6] CLASS←(,T)◦.=,I+T-AG[1]
[7] YEARCLASS←((-I)+CNT)↓CLASS
[8] YEARCLASS[;I-CNT]←0
[9] NEXTAGE:→(I<CNT+CNT+1)/REGRESSION
[10] AGE←(T,I)ρ0
[11] AGE[,T;CNT]←1
[12] AGES←AGES,[1] AGE
[13] YEARS←YEARS,[1] DIMAYR
[14] YRCLASS←((-I)+CNT)↓CLASS
[15] YRCLASS[;I-CNT]←0
[16] YEARCLASS←YEARCLASS,[1] YRCLASS
[17] →NEXTAGE
[18] REGRESSION:AGES← 0 1 ↑AGES
[19] YEARS← 0 1 ↑YEARS
[20] YEARCLASS← 0 1 ↑YEARCLASS
[21] DATA←AGES,YRAYRCL←YEARS,YEARCLASS
[22] TEMP1←B(TDATA+QDATA)+.×DATA
[23] TEMP2←TDATA+.×Y
[24] B←TEMP1+.×TEMP2
[25] MSARESID←S2←(((QY)+.×Y)+(QB)+.×TEMP2)+(ρY)-ρB
[26] VARAB←TEMP1×S2
[27] SS←B+.×TEMP2
[28] ' ' ◇ ' ANALYSIS OF VARIANCE' ◇ ' '
[29] ' SOURCE      B      STAND ERROR OF B '
[30] SSQAB←(ρY)×CONST*2
[31] 'CONSTANT      ', 10 3 *CONST
[32] STANDAERAB←(1 1 QVARAB)*0.5
[33] LABEL←'AGE      ',((7×I-2)ρ','),'YEAR      ',(7×T-2)ρ','
[34] LABEL←LABEL,'YRCLASS',(7×I+T-AG[1])ρ','
[35] LABEL←((ρB),7)ρLABEL
[36] OUTM←((ρOUTM),1)ρOUTM←(1+AG),(1+YR),(YR[1]=AG[I])+,I+T-AG[1]+1
```

```
[37] OUTM $\leftarrow$ OUTM, $((\rho_B,1)\rho_B),((\rho_{STANDAERAB},1)\rho_{STANDAERAB}$ 
[38] '7A1,I5,2F10.3' FMT(LABEL;OUTM)
[39] TOTASSQ $\leftarrow$ (QYY)+.XY
[40] RESIDASSQ $\leftarrow$ TOTASSQ-SSQAB+SS
[41] ' ' C ' SUMMARY OF ANALYSIS OF VARIANCE' C '
[42] ' SOURCE SS DF MS F '
[43] 'CONSTANT', 12 4 8 0 PSSQAB,1
[44] TYEARS $\leftarrow$ YEARS
[45] SS1 $\leftarrow$ ((BYEARS+.XYEARS)+.XYEARS+.XY)+.XYEARS+.XY
[46] TYRAYRCL $\leftarrow$ QRAYRCL
[47] SS2 $\leftarrow$ ((BTYRAYRCL+.XYRAYRCL)+.XYRAYRCL+.XY)+.XYRAYRCL+.XY
[48] SSAAG $\leftarrow$ SS-SS2 C SSAAYRCL-SS2-SS1
[49] DFAG $\leftarrow$ I-1 C DFAYR $\leftarrow$ T-1 C DFAYRCL-I+T-AG[1]
[50] MSAG $\leftarrow$ SSAAG+DFAG C MSAYR $\leftarrow$ SS1+DFAYR
[51] MSAYRCL $\leftarrow$ SSAYRCL+DFAYRCL
[52] FAAG $\leftarrow$ MSAG+MSARESID C FAAYR $\leftarrow$ MSAYR+MSARESID
[53] FAAYRCL $\leftarrow$ MSAYRCL+MSARESID
[54] OUTM1 $\leftarrow$ SSAAG,DFAG,MSAG,FAAG,SS1,DFAYR,MSAYR,FAAYR,SSAYRCL,DFAYRCL,MSAYRCL
,FAAYRCL
[55] OUTM1 $\leftarrow$ (3,4)pOUTM1
[56] LABEL1 $\leftarrow$ (3,7)p'AGE ', 'YEAR ', 'YRCLASS'
[57] '7A1,F12.5,I8,F12.4,F10.3' FMT(LABEL1;OUTM1)
[58] RESIDADF $\leftarrow$ (pY)-1+DFAG+DFAYR+DFAYRCL
[59] 'RESIDUALS', 12 5 8 0 12 4 PRESIDASSQ,RESIDADF,RESIDASSQ+RESIDADF
[60] 'TOTAL ', 12 5 8 0 PTOTASSQ,pY
    ▽
```

VONB

The function VONB is used to estimate the parameters of the Bertalanffy equation by using the Marquardt algorithm. VONB calls the library functions MARQUARDTP and the utility function FILEAFETCH. Graphical output is obtained through the use of the function PLOT.

```

▽ VONB;L;AG;X;Y;P;PL;B;OUTM
[1]  FILEAFETCH '2719067 PFΔMARQ'
[2]  ER1:' ^ C 'AGES?'
[3]  +(^/0<AC-,[],)/SK1
[4]  +ER1,0+M+^'AGES MUST BE POSITIVE. RE-ENTER '
[5]  SK1:AC<-AG[P-&AG]
[6]  +(3<+/PL+AG*1#AG)/ER2
[7]  +ER2,0+M+^'MUST BE MORE THAN 3 DISTINCT AGES. RE-ENTER '
[8]  ER2:' ^ C 'LENGTH AT AGE?'
[9]  +(pAG)=pL-,[],OK1
[10] +ER2,0+M+^'ONE LENGTH FOR EACH OF ',(pAG),', AGES. RE-ENTER '
[11] OK1:+(A/L>0)/OK2
[12] +ER2,0+M+^'LENGTHS MUST BE POSITIVE. RE-ENTER '
[13] OK2:' ^ C 'THE FORMS OF THE EQUATION FOR FITTING ARE:'
[14]   1. L(T)=LINF(1-EXP(-K(T-T(0))))
[15]   2. L(T)=(ω/T)(1-EXP(-K(T-T(0))))
[16] FR3:' ^ C 'WHICH FORM IS TO BE FITTED (1 OR 2)?'
[17] +(A/(V/(2=FORM),1=FORM),1=pFORM-,[],)/ER4
[18] +ER3,0+M+^'FORM MUST BE 1 OR 2. RE-ENTER '
[19] ER4:' ^ C 'YOUR OWN INITIAL VALUES (YES OR NO) ?'
[20] +(A/'NO'=2#CHECK),~A/'YES'=3#CHECK+( ' *CHECK)/CHECK-,[],INITVALUES,ER4
[21] OK3:+(FORM=2)/FORM2
[22] FORM1:' ^ C 'INITIAL VALUES FOR ASYMPTOTIC LENGTH, K, AND T(0)?'
[23] +(A/(A/0≤2#INITIAL),3=pINITIAL-,[],)/OK4
[24] +FORM1,0+M+^'MUST ENTER 3 NUMBERS THE FIRST 2 OF WHICH ARE POSITIVE. RE-ENTER '
[25] FORM2:' ^ C 'INITIAL VALUES FOR ω, K, AND T(0)?'
[26] +(A/(A/0≤2#INITIAL),3=pINITIAL-,[],)/OK4
[27] +FORM2,0+M+^'MUST ENTER 3 NUMBERS THE FIRST 2 OF WHICH ARE POSITIVE. RE-ENTER '
[28] INITVALUES:L+L[P]
[29] X<-(1++/PL)p1,[1.5] -1#P+PL/L
[30] Y<1#P
[31] B+(B(X)+,X)+,×(X)+,×Y
[32] +(FORM=2)/SK2
[33] P<(B[1]+1-B[2]),-#B[2]
[34] +SK3
[35] SK2:P+(B[1]+1-B[2])×-#B[2],-#B[2]
[36] INITIAL+P,((+/#AG)+pAG)+(#1+(P[2]×(+/#L)+pL)+P[1])+P[2]
[37] +OK4
[38] SK3:INITIAL+P,((+/#AG)+pAG)+(#1+(+/#L)+pL)+P[1])+P[2]
[39] OK4:LIMIT<20
[40] PRINT EVERY 20
[41] ' ^ C '(62#(22p)), ' VON BERTALANFFY PARAMETERS ',DAT C '
[42] ' ORDER OF PARAMETERS IS AS FOLLOWS:'
[43] +(FORM=2)/ALTERNAT
[44]   1. MEAN ASYMPTOTIC LENGTH
[45] +NEXT
[46] ALTERNAT:' 1. ω'
[47] NEXT:' 2. BRODY GROWTH COEFFICIENT'
[48]   3. T(0)'
[49] ' ^ C 'INITIAL PARAMETER VALUES' C 14 6 #P(3,1)pINITIAL
[50] L MARQUARDTP AG
[51] +(FORM=1)/SK4
[52] ' ^ C 'ESTIMATE OF THE ASYMPTOTIC LENGTH IS: ', 14 6 #P(B[1]+B[2])
[53] SK4:OUTM<#(5,pAG)pAG,M[1],M[2],(M[2]-M[1]),(M[2]-M[1])-M[1]
[54] DATA<OUTM[1],[1.5] OUTM[4]
[55] □PDEF PAGRAPHICS
[56] TITLE TEXT 'RESIDUALS'
[57] NORMAL C ' C '
[58] ABCISSACOL 1 C SYMBOLCOL 0
[59] ABCISSA TEXT 'AGES'
[60] ORDINATE TEXT 'EXPECTED MINUS OBSERVED'
[61] AXES FLOATING

```

```
[62] ABSCISSA HIGH LIMIT[/OUTM[;1]
[63] ORDINATE HIGH LIMIT[/OUTM[;4]
[64] PLOT DATA
[65] DATA<@(3,pAG)pOUTM[;1],M[;2],M[;1]
[66] TITLE TEXT 'OBSERVED AND EXPECTED LENGTH AT AGE'
[67] ORDINATE TEXT 'LENGTH'
[68] AXES FIXED ◊ ABSCISSA HIGH LIMIT[/DATA[;1]
[69] ORDINATE HIGH LIMIT([/DATA[;2])(([/DATA[;3]))
[70] ABSCISSA LOW LIMIT 0 ◊ ORDINATE LOW LIMIT 0
[71] ' ' ◊ ' ' ◊ MULTIPLE ◊ PLOT DATA
▽
```

ΔFN

The function ΔFN is used by the library function MARQUARDTP to estimate the parameters of the Bertalanffy equation.

```
▽ R+A ΔFN X
[1] →(FORM=2)/OTHER
[2] R+A[1]×1-★-A[2]×X-A[3]
[3] →0
[4] OTHER:R+(A[1]+A[2])×1-★-A[2]×X-A[3]
▽
```

VPA

The function VPA calculates population numbers at age and the instantaneous rates of fishing mortality at age by using a virtual population analysis (Gulland, 1965; Pope, 1972).

▼ VPA;CATCH;J;I;M;MORT;FI;FC;X;YR;AG;Y;TIT;FT;TMP;EXP;FFM;DIF;NAM;CHECK;
CHECK1;SENSM;SENSMF;SENSMF1;F;POP;TEMP;A;CTEMP;FIT;FCT;SUB;IN1;K;L
[1] A<CHECK>+1
[2] NAM<NL 2 ◇ FLAG<0
[3] ERO:'NAME OF CATCH MATRIX?.....'
[4] →(v/NAM.=X<(~1+pNAM)†(' 'zX)/X<,□)/OK1
[5] →ERO,04□~"NAMED MATRIX DOES NOT EXIST. RE-ENTER '
[6] OK1:→(2=pCATCH<□)/OK2
[7] →ERO,04□~((' 'zX)/X), ' IS NOT A MATRIX.'
[8] OK2:→(A/0≤,CATCH)/ERO5
[9] →0,04□~"NEGATIVE VALUES IN CATCH MATRIX.'
[10] ERO5: ' ◇ 'SENSITIVITY ANALYSIS TO BE PERFORMED (YES OR NO)?'
[11] CHECK1<□
[12] ER1:'FIRST YEAR AND YOUNGEST AGE GROUP?'
[13] →(A/(X=[X],(0≤2*X),2=pX<,□)/OK3
[14] →ER1,04□~"MUST BE 2 NON-NEGATIVE INTEGERS. RE-ENTER '
[15] OK3:YR<(~1+X[1])+1,I<(pCATCH)[2]
[16] AG<(~1+X[2])+1,I<(pCATCH)[1]
[17] POP<F<MORT<SENSM<(4,I,J)p0
[18] TEMP<(I,J)p0
[19] ER2:'NATURAL MORTALITY?'
[20] →(v/(1,I)=pM<,□)/OK4
[21] →ER2,04□~"EITHER 1 OR ,(VI), ' NUMBERS. RE-ENTER '
[22] OK4:→(A/M>0)/OK5
[23] →ER2,04□~"MORTALITIES MUST BE POSITIVE. RE-ENTER '
[24] OK5:'WEIGHT AT AGE KNOWN (YES OR NO)?'
[25] →((A/'NO'=2+CHECK2),~A/'YES'=3+CHECK2<(~'zCHECK2)/CHECK2<,□)/ER3,OK5
[26] BMASS:'ENTER WEIGHT AT AGE DATA (KG): VECTOR OR MATRIX'
[27] →(A/(A/0<WGT),(2=pWGT<□))/MAT
[28] →(1=pWGT)/VECTOR
[29] ERROR:→BMASS,04□~" WEIGHTS MUST BE POSITIVE '
[30] VECTOR:→((pWGT)=pCATCH[,1])/RESHAPE
[31] →BMASS,04□~" DIMENSION MUST AGREE , '
[32] RESHAPE:WGT<((pWGT),1)pWGT
[33] FLAG<1 ◇ →ER3
[34] MAT:→((pWGT)≠pCATCH)/BMASS
[35] ER3:'STARTING F VALUES FOR LAST YEAR (',(~1+YR),')?'
[36] →(v/(1,I)=pFI<,□)/OK6
[37] →ER3,04□~"EITHER 1 OR ,(VI), ' NUMBERS. RE-ENTER '
[38] OK6:→(A/FI>0)/ER4
[39] →ER3,04□~"MORTALITIES MUST BE POSITIVE. RE-ENTER '
[40] ER4:'STARTING F VALUES FOR OLDEST AGE GROUP (',(~1+AG),')?'
[41] →(v/(1,J)=pFC<,□)/OK8
[42] →ER4,04□~"EITHER 1 OR ,(VJ), ' NUMBERS. RE-ENTER '
[43] OK8:→(A/FC>0)/OK9
[44] →ER4,04□~"MORTALITIES MUST BE POSITIVE. RE-ENTER '
[45] OK9:F[A;I]<JpFC
[46] F[A;;J]<IpFI
[47] MORT[A;;]=Q((J,I)pM)
[48] POP[A;;J]←(CATCH[,J]×FI+MORT[A;;J])×FI×1-*×FI+MORT[A;;J]
[49] POP[A;;I]←(CATCH[I;]×FC+MORT[A;I;])×FC
[50] →(A=1)/ER5
[51] →((CHECK=0),CHECK≠0)/EE,SK1
[52] ER5:'IS FISHING COMPLETE FOR LAST AGE GIVEN(YES OR NO)?.....'
[53] →((A/'YES'=3+XX),~A/'NO'=2+XX<(~'zXX)/XX<,□)/SK1,ER5
[54] CHECK<0
[55] EE:POP[A;I;]+POP[A;I;]*1-*FC+MORT[A;I;]
[56] SK1:Y<J<1
[57] AA:X<MORT[A;;I-1;Y]
[58] POP[A;;I-1;Y]←(CATCH[,I-1;Y]×X×2)+(POP[A;1+I;Y+1]×X)
[59] F[A;;I-1;Y]←FT←(POP[A;;I-1;Y]+POP[A;1+I;Y+1])-X
[60] BB:TMP<(EXP-*FT+X)×FFM-*FT+FT+X

```

[61] →(Λ/0.00001>|DIF*TMP*FFM+CATCH[1,I=1;Y]*POP[A;1+1,I;Y+1])/CC
[62] F[A;1,I=1;Y]←FT*FT*DIF*TMP*X*(EXP*1)/(FT+X)*2
[63] →BB
[64] CC:POP[A;1,I=1;Y]←POP[A;1+1,I;Y+1]*X*FT
[65] →(1≤Y<Y=1)/AA
[66] F[A;1,I=1;1,J=1]←(⊖(1-1+POP[A;;])×1 1+POP[A;;])-1-1+MORT[A;;]
[67] →(NO!=2+CHECK1)/EE4
[68] →((A=2),(A=3))/FF,GG
[69] FI←FI+(0.001*FIT←FI) ◇ FC←FC+(0.001*FCT←FC) ◇ A←2
[70] →OK9
[71] FF:M=M+(0.001*M) ◇ FI←FIT ◇ FC←FCT ◇ A←3
[72] →OK9
[73] GG:SENSM[2;;]←((POP[2;;]-POP[1;;])*POP[1;;])×1000
[74] SENS[3;;]←((F[2;;]-F[1;;])*F[1;;])×1000
[75] SENS[4;;]←((POP[3;;]-POP[1;;])×1000)*POP[1;;]
[76] SENSMF←((F[3;;]-F[1;;])×1000)*F[1;;]
[77] EE4:TIT←"POPULATION NUMBERS"
[78] POPNB←POP[1;;]
[79] 0 OUT POPNB,[1](AC[1],AG[4]) AAPLUS POPNB
[80] +(NO!=2+CHECK2)/MORTALITY
[81] WG←WGT
[82] NEXT: +(Λ/(ρWGT)=ρPOPNB)/PRINT
[83] WGT←WGT,WG ◇ →NEXT
[84] PRINT:POPBIOMASS←WGT*((POPNB×(1+*F[1;;]+MORT[1;;]))*F[1;;]+MORT[1;;])
[85] TIT←"MEAN POPULATION BIOMASS (KG)"
[86] 0 OUT POPBIOMASS,[1] NEWPOPBIOMASS+(AG[1],AG[4]) AAPLUS POPBIOMASS
[87] +(FLAG=0)/CATBIOMASS
[88] ' ' ◇ "WARNING: WEIGHT AT AGE IS AVAILABLE FOR ONE YEAR ONLY."
[89] ' POPULATION BIOMASS, AS CALCULATED MAY BE BIASED." ◇ "
[90] CATBIOMASS:CATCHBIOMASS←WGT*CATCH
[91] TIT←"CATCH BIOMASS (KG)"
[92] 0 OUT CATCHBIOMASS,[1]+/[1] CATCHBIOMASS
[93] TIT←"MEAN WEIGHT OF INDIVIDUALS IN CATCH"
[94] 2 OUT(+/[1] CATCHBIOMASS)+/[1] CATCH
[95] MORTALITY:FWGT←(+/[1](POPNB×FISHMORT*F[1;;]))+/[1] POPNB
[96] TIT←"FISHING MORTALITY" ◇ 3 OUT FISHMORT,[1] FWGT
[97] +(NO!=2+CHECK1)/0
[98] SUB←"PARTIAL DERIVATIVES OF POPULATION WITH RESPECT TO PARAMETERS" ◇ HEADER SUB
[99] TIT←"SENSITIVITY TO TERMINAL F" ◇ 3 OUT SENS[2;;]
[100] TIT←"SENSITIVITY TO NATURAL MORTALITY" ◇ 3 OUT SENS[4;;]
[101] OVERSENS←(SENSM[2;1;]*2)+SENSM[4;1;]*2
[102] SUB←"PARTIAL DERIVATIVES OF ESTIMATED F WITH RESPECT TO PARAMETERS" ◇ HEADER SUB
[103] TIT←"SENSITIVITY TO TERMINAL F" ◇ 3 OUT SENS[3;;]
[104] TIT←"SENSITIVITY TO NATURAL MORTALITY" ◇ 3 OUT SENSMF
[105] SUB←"PARTIAL DERIVATIVES OF POPULATION NUMBERS FOR YOUNGEST AGE GROUP"
[106] HEADER SUB
[107] XK←(ρMORT[1;;])ρ0 ◇ J+I+IK+JK+0
[108] EEO:K←0.5
[109] FE1:I+I+1 ◇ J+J+1 ◇ K+K+1
[110] XK[I;J]←(CATCH[I;J]×(*(MORT[1;I;J]×K)))*POP[1;(I-K=0.5);(J-K=0.5)]
[111] +(I=ρFI)∨(J=ρFC))/EE2
[112] →FE1
[113] EE2:→(IK=((ρFI)=1))/EE3
[114] I+IK+JK+K+1 ◇ J+0 ◇ →EEO
[115] EE3:J+JK+JK+1 ◇ I+0 ◇ +(JK+ρFC)/EE0
[116] M1←(ρFI)ρMORT[1;;ρFC]
[117] M2←(ρFC)ρMORT[1;ρFI;]
[118] FIT←XK[;ρFC]*FIT←(ρXK[;ρFC])ρFIT←(((*(+0.5×M1))×Z)*(FI×1+*(+Z+FI+M1)))
[119] FCT←XK[ρFI;]*FCT←(ρXK[ρFI;])ρFCT←(((*(+0.5×M2))×FC)×(Z+FC+M2))
[120] +(YES=3+XX)/EE6
[121] FCT←FCT*IN1←(ρFCT)ρIN1←(1+*Z)
[122] EE6:XK[ρFI;]*FCT ◇ XK[;ρFC]←FIT
[123] TIT←"SENSITIVITY TO INDIVIDUAL CATCH VALUES" ◇ 3 OUT XK
[124] OVERSENS←OVERSENS+XK[1;]*2 ◇ I+1
[125] SUM:OVERSENS←OVERSENS+((I+XK[I+1;]),Iρ0)*2
[126] I+I+1
[127] +(((1+ρXK)=I)>0)/SUM
[128] OVERSENS←OVERSENS*0.5
[129] TIT←"OVERALL SENSITIVITY OF RECRUITMENT"
[130] 2 OUT OVERSENS
[131] ISP←(+/OVERSENS)*ρOVERSENS
[132] ' ' ◇ "OVERALL SENSITIVITY INDEX: ",ISP
    ▽

```

YIELD

The function YIELD calculates the yield for a single recruit at various levels of fishing mortality by the method of Thompson and Bell (ref. Ricker, 1975).

```
v YIELD;M;OUTM;OUTM1;OUTM2;FMX;MAX;SL;ROT;WGT;SEL;FLG;FLG1;WGT1;L;H;A;IN;IND;M1;SEL1;
  FLG9;FLAG3;FT;TOT;TIT;FMAX;FT;FO1;MSL;XSL;I;SV;KKK;OUTF
[1] ERO:'WEIGHT AT AGE? (KG)'
[2] +(^/0<WGT<,0)/ER1
[3] +ERO,0+^"WEIGHTS MUST BE POSITIVE. RE-ENTER "
[4] FR1:'PARTIAL RECRUITMENT?'
[5] +((pWGT)=pSEL<,0)/OK1
[6] +ER1,0+^"ONE VALUE FOR EACH OF ",(^pWGT)," AGES. RE-ENTER "
[7] OK1:+(^/(SEL>0),SEL<1)/ER2
[8] +ER1,0+^"PARTIAL RECRUITMENTS MUST BE BETWEEN 0 AND 1. RE-ENTER "
[9] ER2:'NATURAL MORTALITY?'
[10] +(v/(1,pWGT)=pM<,0)/OK2
[11] +ER2,0+^"EITHER 1 OR ",(^pWGT)," NUMBERS. RE-ENTER "
[12] OK2:+(^/M>0)/OK3
[13] +ER2,0+^"MORTALITIES MUST BE POSITIVE. RE-ENTER "
[14] OK3:M+(pWGT)pM
[15] ER3:' ^ C "SENSITIVITY COEFFICIENTS FOR THE FOLLOWING PARAMETERS:'
[16] '(ENTER A VECTOR OF ZEROS AND/OR ONES IN THIS ORDER)'
[17] ' NATURAL MORTALITY' ^ ' PARTIAL RECRUITMENT' ^ ' WEIGHT AT AGE'
[18] +(^/((SV=0)&SV=1),3=pSV<,0)/ER4
[19] +ER3,0+^"MUST BE A VECTOR OF 3 ELEMENTS WITH ZEROS AND/OR ONES"
[20] ER4:'MAXIMUM FISHING MORTALITY IN OUTPUT TABLE'
[21] +(^/(KKK>0),1=pKKK<,0)/ES1
[22] +ER4,0+^"ONE POSITIVE VALUE, RE-ENTER"
[23] ES1:+(KKK<5)/ER5 ◇ KKK<5
[24] ER5:KKK<2+KKK+1(KKK×10)
[25] IN+(SV=1)/1pSV ◇ '
[26] ' YIELD PER RECRUIT ANALYSIS' ◇ '
[27] M1+M ◇ SEL1+SEL ◇ WGT1+WGT ◇ FLG9+0
[28] FT+TOT+H+30p0 ◇ A+1 ◇ TIT+4 35 p' ◇ FMAX+FO1+4p0
[29] AA:MSL+YIE 0.0001 ◇ SL+1000×MSL[3]
[30] OUTM+YIE 0.1×FMX+1(KKK-2) ◇ +(A×1)/CC
[31] BB:MAX+OUTM[ ;3],1/OUTM[ ;3]
[32] FT[1]+OUTM[MAX;1] ◇ FLG1+0
[33] CC:L+0
[34] II:+(v/(30<L+L+1),(v/(|FT|>10))/VV
[35] XSL+YIE(FT[L]-0.001),(FT[L]+0.001)
[36] TOT[L]+500×XSL[2;3]-XSL[1;3] ◇ +(FLG1=1)/LL
[37] KK:+(L>2)/MM
[38] FT[L+1]+FT[L]-0.005 ◇ +II
[39] LL:TOT[L]+TOT[L]-SL ◇ +KK
[40] MM:+(0.0000000001>|TOT[L]|)/NN
[41] H[L]+(TOT[L]×FT[L]-FT[L-1])×TOT[L]-TOT[L-1]
[42] FT[L+1]+FT[L]-H[L] ◇ +II
[43] NN:+(FLG1=1)/NN1
[44] FMAX[A]+FT[L]
[45] FT[1]+0.7×FMAX[A] ◇ FLG1+1
[46] +(FLG9#1)/NM ◇ FT+(pFT)p0 ◇ FT[1]+0.1
[47] NM:+(A=1)/CC
[48] FT[1]+FO1[1] ◇ +CC
[49] NN1:FO1[A]+FT[L]
[50] NN05:OUTM+OUTM,[1] YIE FMAX[1],FO1[1]
[51] OUTM+OUTM[I+1;OUTM[ ;1];]
[52] OUTM+OUTM,[2](OUTM[ ;3]+OUTM[ ;2]),[1.5] OUTM[ ;3]+OUTM[ ;1]
[53] OUTM[ ;5]+OUTM[ ;5]+1/OUTM[ ;5]
[54] +(A=1)/NN2
[55] OUTM[ ;2]+((1000×OUTM[ ;2]-OUTM1[ ;2])×OUTM1[ ;2])
[56] OUTM[ ;3]+((1000×OUTM[ ;3]-OUTM1[ ;3])×OUTM1[ ;3])
[57] OUTM[ ;4]+((1000×OUTM[ ;4]-OUTM1[ ;4])×OUTM1[ ;4])
[58] OUTM[ ;5]+((1000×OUTM[ ;5]-OUTM1[ ;5])×OUTM1[ ;5])
```

```

[59] NN2: '
[60]   '      FISHING      CATCH      YIELD      AVG. WEIGHT      YIELD PER'
[61]   '      MORTALITY    (NUMBER)    (KG)        (KG)        UNIT EFFORT'
[62]   '
[63] ROT<- (KKK,7) p' '
[64] ROT[I;KKK;] <- F0.1---'
[65] ROT[I;(KKK=1);] <- "FMAX"---'
[66] (ROT), 7 4 10 3 9 3 10 3 14 3  OUTM
[67] M<-M1 ^ SEL<-SEL1 ◇ WGT<-WGT1
[68] +(pIN)<1)/WW
[69] +(2#A+A+1)/OO
[70] OUTM1<-OUTM
[71] FISHMORT<-OUTM[;1] ◇ YIELDPERRECruit<-OUTM[;3] ◇ MEANWEIGHT<-OUTM[;4]
[72] OO:IND<-1+IN ◇ '
[73] +(IND=2),IND=3)/RR,SS
[74] #TIT[A;] <- "SENSITIVITY TO NATURAL MORTALITY" '
[75] M<-M+(0.001#M) ◇ →UU
[76] RR:#TIT[A;] <- "SENSITIVITY TO PARTIAL RECRUITMENT" '
[77] SEL<-SEL+(0.001#SEL) ◇ →UU
[78] SS:#TIT[A;] <- "SENSITIVITY TO WEIGHT AT AGE" '
[79] WGT<-WGT+(0.001#WGT)
[80] UU:IN<-1+IN
[81] FLC1<-0 ◇ FT[1]<-FMAX[1] ◇ →AA
[82] VV: +(FLG9)/NN ◇ '
[83] 'WARNING: CONVERGENCE CRITERION NOT SATISFIED.'
[84] '          THE VALUE OF FMAX AND THE CORRESPONDING DERIVATIVES MAY'
[85] '          MAY BE INACCURATE.' ◇ ' ' ◇ FLG9<-1 ◇ +NN
[86] WW:FLAG3<-A
[87] +(FLAG3=1)/0
[88] A<-1 ◇ ' ' ◇ 'SENSITIVITY OF FMAX AND F0.1 TO PARAMETERS.' ◇ ' '
[89] '          PARAMETERS                      FMAX                  F0.1
[90] '          -----'
[91] IN<-(SV=1)/1pSV
[92] WW1:A<-A+1 ◇ IND<-1+IN
[93] +(A>FLAG3)/0
[94] FMAX[A]<-(1000×FMAX[A]-FMAX[1])+FMAX[1]
[95] F01[A]<-(1000×F01[A]-F01[1])+F01[1]
[96] OUTF<-(FMAX[A],F01[A])
[97] WWW:(#TIT[A;]), 15 4 15 4  OUTF
[98] IN<-1+IN ◇ →WW1

```

YIE (Calling form: YIE X)

The function YIE is used by the function YIELD to calculate the number of fish caught from each recruit and the total weight of fish caught from each recruit.

```

     $\forall YIE[\square] \forall$ 
 $\forall Y \leftarrow YIE \ X; NUM; CAT; YLD; I; FT; TMP$ 
[1]  $NUM \leftarrow (\rho X) \rho 1$ 
[2]  $CAT \leftarrow YLD \leftarrow (\rho X) \rho 0$ 
[3]  $I \leftarrow 1$ 
[4]  $AA: FT \leftarrow X \times SEL[I]$ 
[5]  $CAT \leftarrow CAT + TMP \times NUM \times FT \times (1 \leftarrow \star \leftarrow FT + M[I]) \div FT + M[I]$ 
[6]  $YLD \leftarrow YLD + TMP \times WGT[I]$ 
[7]  $NUM \leftarrow NUM \times \star \leftarrow FT + M[I]$ 
[8]  $\rightarrow ((\rho WGT) \geq I \leftarrow I + 1) / AA$ 
[9]  $Y \leftarrow Q(3, \rho X) \rho X, CAT, YLD$ 

```

▼

APPENDIX B

NAMES OF APL VECTORS AND MATRICES

APL FUNCTION: BEVHOLT

SYMBOL	Description	APL variable local	APL variable global	Vec- tor	Matrix
M	Instantaneous rate of natural mortality	M	-	-	-
t _R	Age of the recruits	TR	-	-	-
t _c	Age at first capture	TC	-	-	-
t _o	Age at zero weight	TI	-	-	-
t _L	Maximum age attained	TL	-	-	-
W _∞	Average asymptotic weight	W	-	-	-
K	Brody growth coefficient	K	-	-	-
F	Instantaneous rate of fishing mortality	F[1;1]	FISHMORT	X	-
N/R	Number per recruit in population	NO[1;1]	-	X	-
B/R	Biomass per recruit in population	BL[1;1]	-	X	-
C/R	Catch per recruit	CL[1;1]	-	X	-
Y/R	Yield per recruit	Y[1;1]	YIELDPERRECRUIT	X	-
W̄	Mean weight of fish in population	WB[1;1]	MEANWEIGHT	X	-
F _{0.1}	F _{0.1}	F0[1;1]	-	-	-
F _{max}	F at maximum yield per recruit	FMAX[1]	-	-	-
X̄N	Sensitivities of N/R to parameters	NO[A;1] ¹	-	X	-
X̄B	Sensitivities of B/R to parameters	BL[A;1] ¹	-	X	-
X̄C	Sensitivities of C/R to parameters	CL[A;1] ¹	-	X	-
X̄W	Sensitivities of W̄ to parameters	WB[A;1] ¹	-	X	-
XY	Sensitivities of Y/R to parameters	Y[I,A;1] ¹	-	X	-
XF	Sensitivities of F _{max} to parameters	OUTFI[1;1]	-	X	-
XXF	Sensitivities of F _{0.1} to parameters	OUTFI[2;1]	-	X	-
SL	Slope at origin	SL	-	-	-
F(i)	Consecutive values of F. ²	FT	-	X	-
Q or X	Consecutive values of Q or X. ²	TOT	-	X	-
H(i)	Consecutive values of H. ²	H	-	X	-
-	Headings for output matrices	TIT	-	-	X
-	Flag: controls calculation of SL	FLG	-	-	-
-	Flag: controls printing of warning message	FLG9	-	-	-
-	Flag: to exit function	FLAG3	-	-	-
-	Maximum F in output tables	KKK	-	-	-
-	Controls calculation of sensitivities	SV,IN	-	X	-
-	Indices for calculation of sensitivities	IND	-	-	-
-	Branching and loop indices	A,E,I,L,N,V	-	-	-

¹The value of A determines with respect to which parameter the sensitivities are calculated.

²As calculated by the method of false position.

APL FUNCTIONS: COHORT, GRAPHAPRODUCTION, GRAPHANETAPROD

Symbol	Description	APL variable local	APL variable global	Vector	Matrix
$C_{i,\tau}$	Catch matrix	CATCH	as given	-	X
$F_{i,\tau}^f$	Initial F-values for the last year	FI	-	X	-
$F_{\mu,\tau}$	Initial F-values for the oldest age-groups	FC	-	X	-
M_i	Natural mortalities	MORT	-	-	X
$W_{i+.5,t+.5}$	Weight-at-age	-	WGT	X	-
$N_{i,t}$	Calculated population numbers	POP[1;;]	POPNB	-	X
$B_{i+.5,t+.5}$	Calculated population biomass	-	POPBIOMASSAMIDYR	X	-
$B_{i,t}$	Calculated pop. biomass at beginning of year	-	POPBIOMASSABEG	-	X
$Y_{i,\tau}$	Catch biomass (yield)	-	CATCHBIOMASS FISHINGBIOMASSLOSS	-	X X
\bar{W}_{τ}	Mean weight of individuals in catch	-	-	X	-
\bar{i}_{τ}	Mean age of individuals in catch	-	-	X	-
$D_{i,\tau}$	Loss of biomass through natural mortality	-	DT	-	X
$P_{i,\tau}^*$	Net production (annual)	-	NETPROD	X	-
$B_{b,t}$	Recruitment biomass	-	RECRUITABIOMASS	X	-
$P_{i,\tau}$	Total production	-	PRODUCTION	X	-
$G_{i,\tau}$	Growth component of production	-	BG	-	X
$\Delta B_{i,\tau}$	Observed change of biomass over time	-	ΔB	-	X
$F_{i,\tau}$	Calculated fishing mortalities	F[1;;]	FISHMORT	-	X
\bar{F}_{τ}	Weighted F	-	-	X	-
-	Sensitivities of $N_{i,t}$ to parameters	SENSM[2;;] SENSM[4;;],XK	- -	-	X X
-	Sensitivities of $F_{i,\tau}$ to parameters	SENSM[3;;] SENSMF	- -	-	X X
-	Headings for output tables	TIT	-	X	-
I_t	Overall sensitivity of recruitment	-	OVERSENS	X	-
I_{sp}	Overall sensitivity index	-	ISP	-	-
-	Components of production (for graph)	-	DATA1	-	X
-	Production - deaths ; fishing loss	-	DATA2	-	X

APL FUNCTIONS: MPROJECT, MPROJ, MPROJOUT

Symbol	Description	APL variable	Vector	Matrix
		local	global	
$N_{i,0}$	Population numbers for the initial year	POP	-	X
$C_{i,0}$	Catch vector for the initial year	CAT	-	X
$W_{i+0.5}$	Weight-at-age	WGT	-	X
r_i	Partial recruitment	SEL	-	X
M_i	Natural mortality	M	-	X
$R_{\cdot,t}$	Estimated recruitment	REC	-	X
$F_{\cdot,t}$	Initial F-values for years of projection	ANS	-	X
$Q_{\cdot,t}$	Quota for years of projection	ANS	-	X
d_i	Maturity at age	MAT	-	X
$F_{i,\tau}$	Calculated fishing mortalities	CURR[3;;] ¹	FISHMORT	- X
$C_{i,\tau}$	Projected catch	CURR[2;;] ¹	CATCHNUM	- X
$N_{i,t}$	Projected population numbers	CURR[1;;] ¹	POPNB	- X
$B_{i+0.5,t+0.5}$	Projected population biomass	OUTM ¹	POPBIOMASS	- X
$Y_{i,\tau}$	Projected yield (catch biomass)	OUTM1	-	- X
$N_{\cdot,t}$	Projected number of mature fish in population	OUTM ¹	-	- X
$B_{\cdot,t}$	Projected biomass of mature fish in pop.	OUTM ¹	-	- X
-	Headings for output tables	TIT	-	X
-	Sensitivities of yield to parameters	SENSM ²	-	- X
\bar{W}_{τ}	Mean weight of individuals in the catch	OUTM ¹	-	X

¹ Overwritten during the calculation of sensitivities

² The same matrix is used for each parameter

APL FUNCTIONS: SURVIVOR and related functions.

Symbol	Description	APL variable local	Vector global	Matrix
t_0	First year in catch matrix	P1[1]	-	-
t_1	Final year of calibration block	P2[1]	-	-
i_1	First fully recruited age	I1	-	-
i_2	Oldest age-group in calibration block	P2[2]	-	-
$C_{i,t}$	Catch matrix	CATCH	as given	- X
$A_{i+5,t+5}$	Matrix of research vessel abundance index	RVABUND	as given	- X
S_m^0, t	Initial estimate of survivors for the oldest age-groups	SAG	as given	X -
S_i^0, t_f	Initial estimate of survivors for the final year (all age-groups)	SYR	as given	X -
$N_{i+5,t+5}$	Population numbers, mid-year	-	POPNB	- X
k_{i+5}	Calibration constants	K	-	X -
$\ln k_{i+5}$	Natural logarithm of calibration constants	LNK	-	X -
CINT	Integrated catch	-	CINT	- X
SINT	Integrated survivors	-	SINT	- X
$S_{i,t_f,j}$	Estimated survivors for final year	SHAT	-	- X
$S_{m,t,j}$	Estimated survivors for oldest age-groups	SHAT	-	- X
Var $S_{i,t_f,j}$	Variance of estimated survivors	-	ESTAVARS	- X
Var $S_{m,t,j}$	Variance of estimated survivors	-	ESTAVARS	- X
-	Maximum number of iterations	NUMITER	-	-
S_{i,t_f} and $S_{m,t}$	Weighted average of survivors	WASURV	-	- X
$e_{i,t}$	Matrix of residuals	-	RESIDUAL	- X
-	Matrix of outliers of residuals	OUTER	-	- X
SS4	Sum of squares due to an age-effect	-	SSΔAG	-
MS4	Mean square due to an age-effect	MSΔAG	-	-
SS1	Sum of squares due to a year-effect	SSL	-	-
MS1	Mean square due to a year-effect	MSΔYR	-	-
SS3	Sum of squares due to a yearclass-effect	-	SSΔYRCL	-
MS3	Mean square due to a yearclass-effect	MSΔYRCL	-	-

APL FUNCTION: PALOHEIMO

Symbol	Description		APL variable local	Vector global	Matrix
t_0	First year in the CPUE matrix	-	TEMP[1]	-	-
b	Youngest age-group in the CPUE matrix	-	TEMP[2]	-	-
$U_{i,\tau}$	Matrix of catch per unit of effort (time- and age-specific)	-	as given CPUE	-	X X
f_τ	Annual estimates of fishing effort	-	FVAL	X	-
$\bar{Z}_{i,\tau}$	Mean instantaneous rate of mortality	-	Z	-	X
$\bar{Z}_{\tau,\tau+1}$	Overall mortality rate for the period $\tau, \tau+1$	-	-	X	-
$\bar{f}_{\tau,\tau+1}$	Mean effort for the period $\tau, \tau+1$	-	F	X	-
q	Catchability coefficient (estimated)	-	B[2;]	-	-
M	Instantaneous rate of natural mortality (estimated)	-	B[1;]	-	-

APL FUNCTION: VONB

Symbol	Description		APL variable local	Vector global	Matrix
n	Total number of observations	ρL	-	-	-
L_t or W_t	Length-at-age or weight-at-age	L	as given	X	-
L_∞ or W_∞	Mean asymptotic length or weight	-	B[1]	-	-
K or K'	Brody Growth Coefficient	-	B[2]	-	-
t_0 or t'_0	Hypothetical age for zero length or zero weight	-	B[3]	-	-

APL FUNCTION: VPA

SYMBOL	Description	APL Variable		Vector	Matrix
		local	global		
$C_{i,\tau}$	Catch matrix	CATCH	as given	-	X
F_{i,t_f}	Initial F values for the last year	FI	-	X	-
$F_{m,\tau}$	Initial F's for oldest age-groups	FC	-	X	-
M	Natural mortality	MORT	-	-	X
$N_{i,t}$	Calculated population numbers	POP[1;:]	POPNB	-	X
$F_{i,\tau}$	Calculated F values	F[1;:]	FISHMORT	-	X
-	Sensitivities of $N_{i,t}$ to parameters	SENSM[2;:]	-	-	X
-	Sensitivities of $F_{i,\tau}$ to parameters	SENSM[4;:], XK SESSM[3;:] SENSMF	- -	-	X
-	Headings for output tables	TIT	-	X	-
I_t	Overall sensitivity of recruitment	-	OVERSENS	X	-
I_{sp}	Overall sensitivity index	-	ISP	-	-
$B_{i,t}$	Calculated population biomass	-	POPBIOMASS	-	X
$Y_{i,\tau}$	Catch biomass (yield)	-	CATCHBIOMASS-		X

APL FUNCTIONS: YIELD, YIE

SYMBOL	Description	APL variable local	APL variable global	Vec- tor	Matrix
W_i	Weight at age	WGT	-	X	-
r_i	Partial recruitment	SEL	-	X	-
M_i	Instantaneous rates of natural mortality	M	-	X	-
$F.$	Instantaneous rate of fishing mortality	OUTM1[;1]	FISHMORT	X	-
$C.$	Catch per recruit	OUTM1[;2]	-	X	-
$Y.$	Yield per recruit	OUTM1[;3]	YIELDPERRE-	X	-
			CRUIT		
\bar{W}	Average weight of fish caught	OUTM1[;4]	MEANWEIGHT	X	-
I	Relative index of yield per unit-effort	OUTM1[;5]	-	X	-
$F_{0.1}$	$F_{0.1}$	F01[1]	-	-	-
F_{max}	F at maximum yield per recruit	FMAX[1]	-	-	-
SL	Slope at origin	SL	-	-	-
$F(i)$	Consecutive values of $F.$ ¹	FT	-	X	-
Q or X	Consecutive values of Q or X. ¹	TOT	-	X	-
$H(i)$	Consecutive values of H. ¹	H	-	X	-
-	Sensitivities of C. to parameters	OUTM1;2	-	X	-
-	Sensitivities of Y. to parameters	OUTM1;3	-	X	-
-	Sensitivities of \bar{W} to parameters	OUTM1;4	-	X	-
-	Sensitivities of I to parameters	OUTM1;5	-	X	-
-	Headings for output matrices	TIT	-	-	X
-	Flag: controls calc. of $F_{0.1}$ and F_{max}	FLG1	-	-	-
-	Flag: controls printing of warning message	FLG9	-	-	-
-	Flag: to exit function	FLAG3	-	-	-
-	Maximum F in output tables	KKK	-	-	-
-	Controls calculation of sensitivities	SV,IN	-	X	-
-	Indices for calc. of sensitivities	IND	-	-	-
-	Branching and loop indices	A,I,L	-	-	-

¹As calculated by the method of false position.

²The matrix OUTM is overwritten after the calculation of sensitivities for each parameter.

APPENDIX C

SIGN ON and SIGN OFF

This appendix briefly describes the sign on and sign off procedure for I.P. Sharp APL, in Ottawa.

SIGN ON:

- 1) Turn on your terminal and acoustic coupler
(on terminal, LCL/COM switch set to COM)
- 2) Dial 9-563-9458
- 3) Wait for high-pitched tone and place the phone into the acoustic coupler.
- 4) Type: 0) and press the RETURN key.
- 5) Type:)Account.no:password and press the RETURN key.
- 6) Then the Computer will respond and you can proceed to)LOAD 2719067 FISH.

SIGN OFF:

- 1) Type:)OFF
- 2) Wait for the response of the Computer.
- 3) Hang up the phone.
- 4) Turn off the terminal and the acoustic coupler.

APPENDIX D

Terminals supported by the plotting routines.

Description	Identification
Xerox Diablo 1550	X1550
Xerox Diablo 1620	X1620
Xerox Diablo 1641	X1641
Xerox Diablo 1700	X1700
Trendata 4000	T4000
Trendata 4000A	T4000A
Anderson Jacobsen 832	AJ832
Anderson Jacobsen 832U	AJ832U
Tektronix	TEK
Hewlett Packard	HP
DTC 300	DTC300
DTC 302	DTC302
ASL 3	ASL3
ASL 2	ASL2