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Saving and Wealth: The Adequacy of Household Saving in Canada

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^{p}	preliminary
^{r}	revised
x	suppressed to meet the confidentiality requirements of the *Statistics Act*
^{E}	use with caution
F	too unreliable to be published
*	significantly different from reference category ($p < 0.05$)

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Abstract

This paper examines one dimension of the overall state of financial preparedness for retirement in Canada by assessing the adequacy of households' private saving. The definition of *saving adequacy* followed in this study is based on the saving optimality implied by a stochastic life-cycle model. A household is said to save adequately if it accumulates more wealth than the optimal saving implied by the model. Model predictions are then compared with the observed saving behaviors in the 2005 Survey of Financial Security. Overall, the findings reveal that the median of observed wealth of the sample households that were 60 to 64 years old is higher than that of the simulated wealth from the model used in the study or 62% of the sample households have accumulated wealth exceeding the simulated median wealth implied by the life-cycle model used in this study. The overall risk of undersaving is small: only households that fall below the bottom 23rd percentile of the wealth distribution are estimated to save less than the simulated targets implied by the model. The saving adequacy and the risk of undersaving also vary with household characteristics. Households with lifetime private-pension coverage and couples fare the best: 70% of the former and 72% of the latter accumulate more wealth than the simulated medians. The risk of undersaving is also small for these groups. Households without private-pension coverage or single households are close to the simulated targets at the median level: about 54% of the former and 45% of the latter accumulate more wealth than the simulated medians. However, the risk of undersaving for these groups is higher: households without private pension coverage who fall below the bottom 40th percentile of the wealth distribution and single households who fall below the 60th percentile are likely to save less than the simulated targets implied by the model.

Executive summary

Population aging and the recent global financial crisis underscore the importance of the discussions of the adequacy of retirement preparation in Canada and the soundness of the Canadian retirement income system. The focus of this study is to examine whether the accumulated private savings of Canadian households is adequate for their retirement, given their expected entitlement to public and private pension when they retire.

The life-cycle optimal savings implied by a life-cycle consumption model provide a normative benchmark level of financial resources required to maintain similar living standards before and after retirement. In this study, the assessment of saving adequacy for Canadian households is achieved by comparing their actual wealth holdings with the optimal savings derived from a realistically calibrated life-cycle model that incorporates household demographics, mortality risk, uncertainty about future earnings and private-pension coverage, the time-varying Canadian tax and transfer system (both federal and provincial), and the public-pension system in Canada.

The life-cycle model adopted in this study is calibrated using the Longitudinal Administrative Databank which provides essential information on life-cycle earnings realization, private pension coverage, and tax and transfer systems. The 2005 Survey of Financial Security is used to document wealth accumulation by Canadian households.

The study finds that a median household aged 60 to 64 in 2005 had a saving surplus of about $73,000 (a positive difference between the observed median wealth and the simulated optimal median wealth implied by the model) and 62% of households aged 60 to 64 in 2005 had accumulated more wealth than the simulated median. This suggests that, in the context of the model, households aged 60 to 64 in 2005 saved adequately overall.

The saving surpluses for households with private pension coverage and couples are the largest, about $158,000 and $172,000, respectively. The saving surplus for the households without private pension coverage is the smallest, about $22,000.

Single households comprise the only group that incurs a saving deficit (a negative difference between the observed median wealth and the simulated median wealth). However, the deficit is not large, at about $30,000.

The same story is told by the shares of households whose wealth exceeds the simulated median wealth. The percentage of households whose wealth is higher than the simulated median wealth is highest for the households with private pension coverage (70%) and couples (72%). It is also slightly above 50% for households without private pension coverage and slightly below for singles.

This study also assesses a potential risk of undersaving for different categories of Canadian households. The adequacy threshold is defined as a point in the wealth distribution above which the actual saving exceeds the simulated target and below which saving falls short of the simulated target.

For all households aged 60 to 64 in 2005 below the bottom 10% of the wealth distribution, the actual wealth is consistently lower than the optimal simulated wealth. However, above (and including) the 25th percentile, actual wealth is consistently higher than the optimal wealth predicted by the model. This threshold appears to be the 23rd percentile in the wealth distribution, which suggests the overall risk of undersaving was low for all households aged 60 to 64 in 2005.

The adequacy thresholds for singles and households without private pension coverage are considerably higher than for all households, at the 60th percentile and 40th percentile, respectively. This suggests a higher risk of undersaving for these two types of households. Couples and households with private pension coverage fare much better, exceeding the simulated targets between the bottom 5th to 10th percentiles.

A series of robustness checks are performed to assess the sensitivity of the main findings to alternative model specifications and sample choice. The overall saving adequacy is largely robust to alternative parameter values and sample choice, although altering a number of assumptions can alter the results. The fraction of households whose wealth exceeds the simulated median wealth is always above 50% across different scenarios, ranging from 51% when only half of home equity is included into the measure of saving to 66% when a lower risk-aversion coefficient is used in the model. Meanwhile, the finding that couples and households with private pension coverage fare much better relative to singles and households without private pension coverage is also robust.

1 Introduction

Population aging and the recent global financial crisis underscore the importance of the discussions of the adequacy of retirement preparation in Canada and the soundness of the Canadian retirement income system. If Canadians are not well prepared financially, their standards of living could drop significantly in retirement. However, assessing the adequacy of retirement preparation is not an easy task. The key questions in assessing adequacy are defining what is meant by this term and how it is assessed.

One approach taken in the literature is to assess the adequacy of retirement income by focusing on the proportion of pre-retirement income that has to be replaced during retirement in order to maintain living standards at the pre-retirement level.[1] A rule of thumb is that post-retirement income should replace at least 70% of pre-retirement income. However, there is considerable heterogeneity among households in terms of family composition, number of children, home ownership, and the health condition of household members; consequently, different households will have different financial needs in retirement. A simple rule of thumb may not be optimal for households with different paths of lifetime earnings, even when their preferences are similar. A 'one-size-fits-all' replacement rate is likely to be a misleading indicator of the well-being for many retirees.

Another approach is to make inferences about the adequacy of retirement preparation by examining changes in consumption before and after retirement age. According to the standard life-cycle model of consumption, agents with rational expectations will smooth their marginal utilities of consumption over time. Consequently, predictable income changes—such as retirement—should not result in any significant consumption changes. Empirical evidence accumulated over the last two decades, however, shows that household expenditures fall after retirement and that the drop in consumption expenditures exceeds work-related expenses. This is usually referred to as the "retirement-consumption puzzle" and often interpreted as evidence that many households do not save adequately for retirement.[2] Various recent studies have offered new explanations for the puzzle (see Hurst 2008 for a survey of this literature). First, actual consumption may not be fully reflected in consumption expenditures. The former includes not only the flow of consumption generated by goods and services purchased in the current period but also the flow of consumption generated by durables purchased in previous periods. When these are included, the consumption expenditure of Canadian households changes very little in retirement (Lafrance and LaRochelle-Côté 2011). Second, the observed declines in expenditures following retirement are attributable mostly to the drop in food expenditures. It has been shown, however, that, for most retired households the actual food intake changes little compared to the pre-retirement level, even though their food expenditures drop (Aguiar and Hurst 2005, 2007; Brzozowski and Lu 2010). This is because retired households usually spend more time on shopping and food preparation at home. Third, there is substantial heterogeneity in terms of changes in consumption expenditures before and after retirement age. Many families experience only modest changes in consumption expenditures. Larger drops in spending are associated with unexpected retirement, usually related to health shocks or long-term unemployment (Haider and Stephens 2007; Smith 2006). These developments in the consumption literature underscore the difficulty in making inferences about adequacy or optimality from the changes in the patterns of consumption before and after retirement age.

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[1] See, for example, Smith (2003) and Munnell et al. (2007) for studies relating to the United States. A list of recent Canadian studies using various income measures includes LaRochelle-Côté et al. (2008), LaRochelle-Côté et al. (2010), Ostrovsky and Schellenberg (2009), Brown et al. (2010), Baldwin et al. (2011), and Moore et al. (2010).

[2] See, for example, Banks et al. (1998) and Bernheim et al. (2001).

Unlike the approach followed in the studies mentioned above, this paper assesses the adequacy of retirement preparation by examining the adequacy of private saving or wealth accumulation of Canadian households using a life-cycle stochastic model.[3] The focus is on private saving or non-pension wealth, i.e., the total wealth net of pension wealth (both public and private pensions), that is, how much a household would save privately, given its expectation of future entitlement to public and private pensions, if any. The definition of saving adequacy in this study is based on the optimality of saving implied by life-cycle models of consumption. According to consumption models, rational individuals make saving decisions to smooth their marginal utility of consumption over their lifetime subject to their lifetime budget constraints. The life-cycle optimal saving implied by such models provide a normative benchmark level of financial resources required to maintain similar living standards before and after retirement. A household is said to have saved adequately if it accumulates enough wealth to be able to smooth its marginal utility of consumption over time.[4]

This study is the first attempt to assess retirement saving adequacy for Canadian households using a stochastic life-cycle model.[5] Several U.S. studies have used a similar approach to assess the adequacy of households' retirement saving (Engen et al. 1999; Scholz et al. 2006; Gale et al. 2009). This paper differs from them in that it pays special attention to the role of private pensions in ensuring adequate retirement saving in the presence of major public-pension programs. Private-pension wealth represents a significant share of household wealth.[6] It also has an important impact on the accumulation of non-pension wealth as a result of the "crowd-out" or "offset" effect (Engelhardt and Kumar 2011; MacGee and Zhou 2010). In addition, private-pension coverage is incomplete as not all employers offer private pensions. The pension coverage may also change over a pension holder's lifetime as a result of termination of, or change in, employment. All of these factors suggest that a flexible and realistic modeling of private pensions is important in assessing retirement saving adequacy. This paper treats private-pension coverage as a stochastic process calibrated to match the age profile of private-pension coverage observed in the data. This allows for an additional source of uncertainty over the life cycle and brings the model in line with more realistic versions of the standard life-cycle model. By incorporating stochastic pension coverage into the model, the difference in the adequacy of retirement saving between households with and households without private pensions can be better assessed.

This paper develops a realistically calibrated life-cycle model that incorporates household demographics, mortality risk, uncertainty about future earnings and private-pension coverage, the time-varying Canadian tax and transfer system (both federal and provincial), and the public-pension system in Canada. An essential element of this model is information on earnings realization drawn from the Longitudinal Administrative Databank (LAD). The LAD is a longitudinal file made up of a random 20% sample of all Canadian tax filers. It is a tool designed for research. In this study, the data for a 26-year period (1982 to 2007) are used. It includes detailed information on earnings, incomes, tax and transfer, registered pension plan (RPP) coverage and contribution (RPPs are employer-sponsored private pensions in Canada), etc. This information allows us to estimate several key profiles, for example, earnings profiles and tax-and-transfer equations that are critical in the life-cycle model.

---

[3] *Saving* and *wealth* have the same meaning in this paper and are used interchangeably.

[4] It should be pointed out that this definition of saving adequacy is different from that based on poverty rates among the elderly. A household in poverty during its working years and in retirement may still be considered to have saved adequately by this definition if the marginal utilities of pre-retirement and post-retirement consumption are not very different.

[5] Using a two-period model, Horner (2009) examined the saving adequacy issue for Canadian households. However, he used simplified assumptions about earnings, mortality, pension coverage, tax and transfer, etc. The saving rate is assumed to be constant over the accumulation period

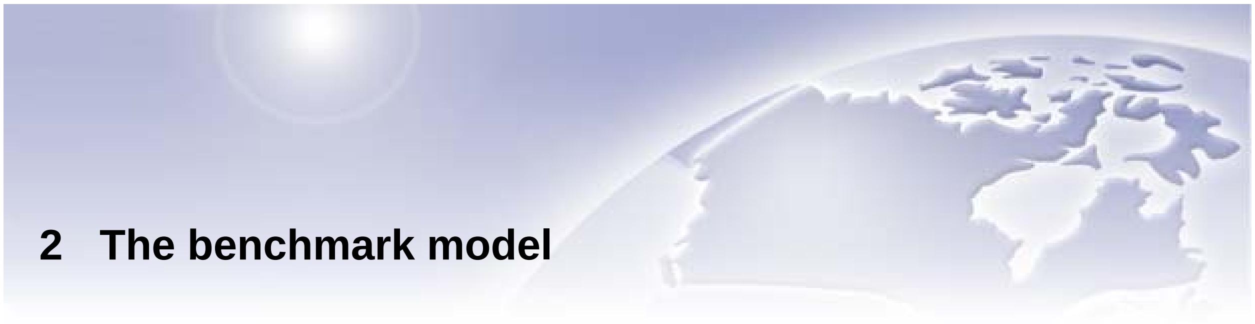
[6] According to the 2005 Survey of Financial Security (SFS), in Canada, private pensions represent 19% of total household assets, second only to the principal residence. In the U.S., private pensions account for between 20% and 40% of total retirement saving (Munnell and Perun 2006, and Gustman et al. 2010).

It is important to note that the objective of this paper is not to explain the observed saving patterns using a life-cycle model. Rather, this paper employs a theoretical model to derive optimal saving only as the normative “adequacy target,” which can be used as a benchmark for how much resource one needs in order to maintain one's living standard in retirement. The model predictions are then compared with the observed saving behaviors and any shortfall (relative to the benchmark) in observed wealth is interpreted as evidence of “undersaving.” Under this approach, a household is said to have saved adequately for retirement if it accumulates more wealth upon retirement than the simulated benchmark target based on the household's characteristics.

Because earnings histories and wealth information are not available for the same individuals in any Canadian dataset, this paper provides an overall assessment of saving adequacy rather than an assessment of exactly how many households saving adequately. That is, the simulated medians of wealth are used as the adequacy targets and the fractions of households in the Survey of Financial Security (SFS) whose wealth exceeds those targets are computed. The observed wealth distributions are also compared with the simulated wealth distributions for households that have the same characteristics, in order to identify potential risk of undersaving.

Overall, Canadian households that were 60 to 64 years old in 2005 are estimated to have saved adequately for retirement: about 62% of all households accumulated more wealth than the simulated median wealth. The overall risk of undersaving is small: only households below the 23rd percentile of the wealth distribution saved less than the simulated targets implied by the model. Households with private-pension coverage fare much better than those without coverage: 70% of households with private-pension coverage exceeded the simulated median. Couples are also estimated to have adequate retirement saving. The risk of undersaving is low for these groups, with shortfall likely happening at the bottom 5th percentile. Households without private-pension coverage or single households are found to be groups with a high risk of undersaving. Saving deficits are likely happening between the bottom 25th and 50th percentiles of the wealth distribution for households without private-pension coverage and between the 50th and 75th percentiles of the wealth distribution for singles. The results are largely robust to a series of sensitivity analyses.

The rest of the paper is organized as follows. Section 2 outlines a life-cycle model used for the analysis. Section 3 explains its parameterization. Section 4 documents empirical evidence of wealth accumulation among Canadian households. Section 5 presents simulation results from the benchmark model and compares them with observed wealth. Section 6 offers a series of robustness checks related to alternative specifications and choices of model parameters. Some limitations of this paper are also discussed in this section. Section 7 concludes.



2 The benchmark model

This section briefly lays out the life-cycle model used to derive the optimal wealth accumulation over a lifetime. In the model, households live finitely with a positive mortality risk in each period. In addition, households face uncertain labour earnings and private-pension coverage in each period. Given these uncertainties, they have to make decisions on how much to consume today and how much to save for future consumption in order to smooth out consumption over the lifetime. The resulting accumulated optimal savings from the model are then used as the "adequacy target" for comparison with actual savings to gauge overall saving adequacy.

2.1 Households

In the model, households live for *J* periods and maximize their lifetime discounted utility from consumption. Households face idiosyncratic shocks to labour earnings, longevity and private-pension coverage. The benchmark version of the model assumes no borrowing.[7]

Households maximize their discounted lifetime expected utility represented by

$$\sum_{j=1}^{J} \beta^{j-1} s_j n_j \frac{\left(\dfrac{c_j}{n_j}\right)^{1-\sigma}}{1-\sigma}, \qquad (1)$$

where: $\beta < 1$ is a discount factor; s_j is the probability of a household being alive at period *j*; $s_j = \prod_{t=0}^{j} P_t$ (where P_t denotes the conditional survival probability that a household is alive in period *t* conditional on being alive in period $t-1$); n_j adjusts consumption for the number of adults and children in the household in period *j*; σ is the coefficient of relative risk aversion; and c_j denotes consumption in period *j*.[8]

Households work in the first *R*<*J* periods. After *R*, households retire and receive retirement income. *R* and *J* are assumed to be exogenous and deterministic. The budget constraint in working periods is

$$k' = (1+r)k + (1-rpprem)y + transfer - c - tax - cppcr. \qquad (2)$$

---

[7] Because of a positive probability of death at each period, borrowing against future earnings is not allowed in the model. MacGee and Zhou (2010) found that the effect of borrowing on wealth accumulation is very small when there is a positive mortality risk. Inheritance is also incorporated in the benchmark model. However, the bequest motive is not considered in the model since the primary interest here is in saving for retirement.

[8] As in Baker et al. (2009), the first adult is counted as one, each subsequent adult counts as 0.7, and each child under age 18 counts as 0.5.

During working periods, households supply their labour to the market and receive labour income, *y*, a fraction of which is contributed to private pensions (if covered), *rpprem*. In the meantime, households receive a gross return to their financial wealth carried over from the previous period, $(1+r)k$, where r denotes a riskless interest rate. Households also receive transfer payments from government (both federal and provincial). On the expenditure side, households spend on consumption, *c*, contribute to the Canada Pension Plan (CPP/QPP)[9], *cppcr*, where this contribution is proportional to their labour earnings, and pay income tax (both federal and provincial).

Every household is assumed to begin its life with zero initial wealth. In each working period $1 \le j \le R$, labour earnings are determined by household age and a persistent productivity shock: $\ln(y_j) = h_j + e_j; e_{j+1} = \rho e_j + \varepsilon_{j+1}; and \varepsilon \sim N(0,\sigma_\varepsilon^2)$; where: $\ln(y_j)$ is the logarithm of household earnings; h_j is the deterministic age–earnings profile; e_j is the persistent productivity shock, governed by an Autoregressive (AR) (1) process; ρ is the shock persistence parameter; and ε is a zero-mean independently and identically normally distributed error term.

During retirement periods, households do not receive labour earnings any more. Instead, they continue receiving returns on their accumulated financial wealth. They receive non-contributory public-pension benefits: Old Age Security (OAS) and the Guaranteed Income Supplement (GIS), both of which are financed through general tax revenues for the purpose of providing benefits to low- and middle-income elder households. Households also receive pay-as-you-go-based public-pension benefits (CPP/QPP) and private-pension benefits (if covered). Therefore, the periodic budget constraint in retirement periods is given by

$$k' = (1+r)k + CQPP + RPP + OAS + GIS + transfer - tax - c, \tag{3}$$

where *RPP* stands for the benefits from registered pension plans (employer-sponsored private pensions in Canada).

Let V_j be the value function at period *j* for a household, which is equivalent to the maximized discounted expected utility from period *j* onward. Thus, the household's problem can be solved in the following recursive fashion:

$$V_j(k,e,\bar{y}_{cpp},pen,n_{rpp}) = \max_c \left\{ n_j \frac{(c_j/n_j)^{1-\sigma}}{1-\sigma} + \beta P_{j+1}\mathrm{E}\left[V_{j+1}(k',e',\bar{y}'_{cpp},pen',n'_{rpp})\right]\right\}, \tag{4}$$

E subject to the budget constraints provided in equations (2) or (3), according to whether the household works or is retired. $k_j, e_j, \bar{y}_{cpp}, pen_j, n_{rpp}^j$ are five state variables, where: e_j is the earnings state for period *j*; $\bar{y}_{cpp}$ is average earnings (up to Yearly Maximum Pensionable Earnings (YMPE)) prior to period *j*; pen_j is the private-pension coverage status at *j*; and n_{rpp}^j is the number of years of private-pension coverage prior to *j*. These five state variables together characterize all possible financial outcomes (including financial wealth, earnings, and future public- and private-pension benefits) that households may arrive at each period and upon which households need to make optimal decisions of consumption.

Equation (4) is numerically solved backward from period *J* (see the Appendix, Section 9.1) for technical details. Let $\{c_j^*\}_{j=1...J}$ be the stream of optimal consumption obtained from solving equation (4). Thus, the optimal accumulated saving or wealth $\{k_j^*\}_{j=1...J}$ can be derived from equations (2) and (3). These are then used as the "adequacy target" to compare with the actual saving. Meanwhile, one of important implications from stochastic life-cycle models like the one used in this paper is that households with the same age and even with the same current earnings would save differently as a result of different realization of earnings risk and of private-pension coverage. Thus, there is a distribution of optimal accumulated wealth for households with the same characteristics. This contrasts with the non-stochastic life-cycle models, where households with the same characteristics save exactly the same amount.

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[9] The QPP, the Quebec Pension Plan, is the public pension plan in Quebec and is almost identical to the CPP.

3 Calibration

This section discusses in detail the empirical approach to the calibration of lifetime earnings, the tax and transfer system, and the private- and public-pension systems.

3.1 Demographics and preferences

Households are assumed in the model to start working from age 22 until age 64, and retire at age 65. Households can live up to a maximum age of 95, subject to mortality risk, in every period for which the female mortality rates from the 2000−2002 Life Tables are used. The coefficient of relative risk aversion σ represents households' tolerance to risk. The higher σ is, the less tolerant households are to risk and the more precautionary saving is made. The value of σ is set to 2.0. The real riskless interest rate r is set to 4%.The annual discount factor β, set to equal to $1/(1+r)$, thus is equal to 0.96.[10] The discount factor represents how patient households are in delaying consumption to the future. The higher the discount factor is, the more patient households are and the more savings are made for future consumption.

3.2 Longitudinal Administrative Databank

The main dataset used to calibrate the model is the LAD. The LAD is a random 20% sample of the T1 Family File, a yearly cross-sectional file of all tax filers.[11] Individuals selected into the LAD can be linked across years. Consequently, it is possible to create a longitudinal profile of each individual. The LAD contains basic demographics (year of birth, sex, and marital status) and detailed income and tax information for the period from 1982 to 2007 (at the time of writing). The LAD sample includes only those who lived in the ten Canadian provinces; it excludes those who lived in the territories and on Indian reserves. The sample is further restricted to the individuals who filed tax returns in all years starting from 1992 and were 22 to 64 years old in all years. The filing restriction for the years before 1992 was not imposed. This is done mainly because, before the introduction of the Goods and Services Tax rebate (in 1986) and, particularly, before the introduction of the Canada Child Tax Benefit (in 1992), low-income families had fewer incentives to file tax returns. The subsample of low-income families can be retained by relaxing the filing restriction prior to 1992.[12]

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[10] The coefficient of relative risk aversion of 2.0 falls in the range of 1−3, which is commonly used in the literature. The real rate of return of 4% is also used in the *25th Actuarial Report on the Canada Pension Plan* (Office of the Superintendent of Financial Institutions 2009). Sensitivity analysis is conducted on these parameters and the results are presented in Section 6.

[11] A smaller, 10%, random LAD sample was used for calibration.

[12] The age restriction is conditional on the filer's age. For married or common-law couples, the filer's age is not necessarily the age of the husband. This age restriction may result in a loss of households in which the wife is the only filer, and the wife is aged 65 or over but the husband is younger than 65.

3.3 Life-cycle earnings process

The LAD sample is divided into three types according to marital status and the number of earners: singles and lone parents (hereafter, *S*); one-earner couples (married or common-law) (1EC); and two-earner couples (2EC).[13]

The age–earnings profiles and persistent productivity shocks are estimated for each of the three household types defined above. Household earnings are total pre-tax household earnings that include the household head’s and the spouse's (if any) earnings from T4 slips, other employment income, and net self-employment income (net business income, net professional income, net commission income, net farming income, and net fishing income). All household earnings are converted to 2005 dollars using the consumer price index (CPI).

All households of the same type are pooled together in all years and the logarithm of household earnings of the same type are then regressed against household age, household age squared, and year dummies as

$$\ln(y_{it}) = \alpha^{\tau} + \beta_1^{\tau} age_{it} + \beta_2^{\tau}(age_{it})^2 + \sum_{k=1983}^{2007} \gamma_{k-1982}^{\tau} year_{k-1982} + e_{it}, \tag{5}$$

where: τ is the household-type index; *i* is the individual; *t* is the year; age_{it} is the household's age (husband's age in the case of couples); and $year_{k-1982}$ denotes the year dummies for the years from 1983 to 2007. The residuals from the earnings regression (5) are then used to estimate the parameter of the persistence shock ρ for each type:

$$e_{it} = \rho^{\tau} e_{i(t-1)} + \varepsilon_{it}. \tag{6}$$

To be consistent with the model’s assumption that all individuals retire at 65, the earnings sample includes only households aged between 22 and 64 with positive earnings. Table 1 shows the coefficient estimates for equations (5) and (6). Panel A in Table 1 lists the estimated age–earnings profiles for all three household types.[14] The implied lifetime earnings profiles of singles (*S*) lie below the earnings profiles of couples (1EC or 2EC).[15] Couples with one earner also have a lower lifetime earnings profile than couples with two earners.

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[13] Because the LAD does not include information on education, education cannot be used as a grouping criterion. Moreover, this study does not model marriage, divorce, fertility, or labour supply in this paper. Each type of household is assumed to make its optimal decisions as though its type were maintained through the life cycle.

[14] To take into account life-cycle changes in the labour force participation and the fact that the earnings regressions are based only on observed market earnings, this study adjusts the age-earnings profile by multiplying the coefficients from the earnings regressions by the labour force participation rates among Canadian males during the period from 1980 to 2005. Hendricks (2007) and MacGee and Zhou (2010) used a similar adjustment.

[15] For simplicity, the word ‘singles’ is used hereafter in this paper to replace singles and lone parents.

Table 1
Coefficient estimates for labour income processes

	Singles[1]	One-earner couples	Two-earner couples
		coefficient	
Panel A			
Constant	6.965	6.165	8.490
Age	0.159	0.218	0.122
Age squared divided by 100	-0.186	-0.255	-0.139
Panel B			
Earnings shock persistence	0.739	0.822	0.855
Standard deviation of earnings shock	0.764	0.941	0.551
Standard deviation of earnings shock at age 22	0.964	0.979	0.566

1. Singles and lone parents.
Source: Statistics Canada, authors' estimations using the Longitudinal Administrative Databank.

Panel B in Table 1 lists the estimates for the stochastic part of the labour income process, governed by an AR (1) process. The estimates of the persistence parameter ρ range from 0.74 for singles to 0.86 for two-earner couples. The persistence parameter ρ measures the persistence of earnings shocks. For example, the estimate of ρ for singles is 0.74, which implies that, for single households, less than 10% of a given earnings shock (good or bad) will remain after eight years.[16] In general, the earnings shocks are more persistent for couples than for singles. A similar relationship has been found in other studies, such as Scholz et al (2006). The standard deviation of earnings shocks, σ_ε, ranges from 0.55 for two-earner couples to 0.94 for one-earner couples. These estimates are much larger than those found in other studies, e.g., Scholz et al. (2006). This difference may be attributed to the following: (1) single cohorts are used in Scholz et al. (2006), while multiple cohorts are used in the current study; and (2) education is controlled for in the Scholz et al. earnings regression, while in this study it effectively enters into the error terms of the earnings regressions because the LAD does not contain any information about households' education.

The remaining parameters of the earnings process are the mean earnings and standard deviation, σ_{e1}, for the households in the first period (age 22). Households draw their labour endowments from a normal distribution with a mean earnings and standard deviation σ_{e1}. The estimates of σ_{e1} are based on the earnings of households aged 22 in 1982 (see Table 1).[17] Finally, the AR (1) process is discretized as a seven-state Markov process according to the Tauchen method (Tauchen 1986).

3.4 Children

The presence of children plays an important role in determining the optimal retirement wealth of a household (Scholz and Seshadri 2009). Households with children would have higher consumption expenditure and thus accumulate less wealth than households without children, all else being equal. Children also affect household earnings profiles, taxes paid by households, and transfer income received by households.

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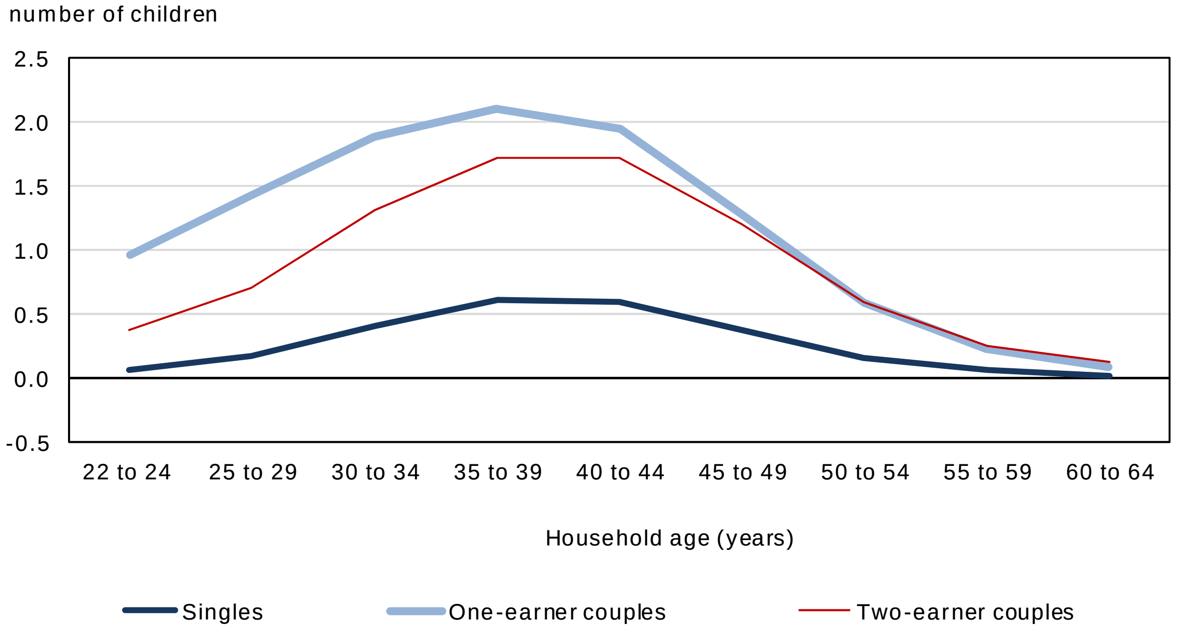
[16] Based on the derivation of equation (6).

[17] In the model, however, the household's earnings process starts at the age of 22, in 1962, and continues until the age of 65, in 2005. To take into account the earnings growth before 1982—the first year in the LAD—the mean earnings of 22-year-old households in 1982 is deflated by an annual growth rate of 1.5% during that period to obtain the mean earnings in the first period.

Children are incorporated in the model and are defined as dependents under 18 years of age.[18] They do not contribute to household income but affect household consumption through the household size in the household's utility function (equivalence scale n_j in equation (1)). The number of children within each of the three types of households is allowed to vary with age, but exogenously. To do so, households of the same type (across years) in the LAD are pooled together to compute the average number of children for each household type over various age ranges. Chart 1 plots the number of children for various age groups for the three household types. The number of children is hump-shaped, increasing toward the middle age and then decreasing toward zero. Singles (or lone parents) have the lowest number of children over the lifetime while one-earner couples have the highest number of children over the lifetime. The number of children is assumed to be zero in the model for those 65 and over for all three household types.

Chart 1

Average number of children by household age group and type in the Longitudinal Administrative Databank



Note: 'Singles' comprises singles and lone parents.
Source: Statistics Canada, authors' calculations using the Longitudinal Administrative Databank.

3.5 Tax and transfer system

Since disposable income is particularly important to households' consumption and saving decisions, it is important to incorporate the effects of the tax and transfer system into the model. Following an approach in Frenette et al. (2007), the LAD is used to estimate the tax and transfer equations for each province and each year from 1982 to 2007. The total tax paid by a household is regressed against a number of household income brackets, the interaction terms between household income and income brackets, the number of children under 18, a senior-status (65 and over) dummy, household-type dummies, and provincial dummies (for purposes of federal tax estimation only).

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18. Dependents aged 18 or over with no income are treated in the same way as those under 18. If they do file a tax return (having income), they are treated as singles (unattached individuals).

$$\begin{aligned} Tax_{it} &= \alpha_t + \sum_{m=1}^{10} \beta_{mt} IR_{imt} + \sum_{m=1}^{10} \gamma_{mt} IR_{imt} I_{it} + \lambda_t KID_{it} + \delta_t (KID_{it})^2 + \sum_{q=1}^{2} \phi_{qt} TYPE_{iqt} \\ &+ \theta_t SENIOR_{it} + \sum_{s=1}^{9} \varphi_{st} PROV_{ist} + u_{it}, \end{aligned} \tag{7}$$

where: Tax_{it} is the total tax paid by each household (either federal or provincial tax); IR_{imt} denotes 10 income bracket dummies ($5,000 or less (omitted); $5,000 to $10,000; $10,000 to $20,000; $20,000 to $30,000; $30,000 to $40,000; $40,000 to $50,000; $50,000 to $60,000; $60,000 to $100,000; $100,000 to $150,000; $150,000 to $250,000; and $250,000 and more); *m* is the bracket index; and I_{it} is the household taxable income that includes both employment and investment incomes, net of private-pension contributions and other similar deductions. The income brackets and the interaction terms with household incomes capture the progressive nature of the personal income tax system. Two household-type dummies, $TYPE_{iqt}$, are also included (the "singles" as the reference type) with *q* as the type index. The household-type dummies, the number of children, KID_{it}, senior status (65 and over), $SENIOR_{it}$, and provincial dummies $PROV_{ist}$ capture differences in tax exemption amounts, age credit, provincial tax credits, and other non-refundable tax credits. The regressions are run on all households aged 22 and over, separately for federal and provincial taxes. The tax regressions show very good fit with R^2 above 0.8 or 0.9 in all years and for all provinces.

The transfer incomes are defined as total transfers except CPP/QPP, Unemployment Insurance/Employment Insurance (UI/EI) and OAS/GIS benefits.[19] Hence, transfer incomes include family benefits, child care benefits, Goods and Services Tax/Federal Sales Tax (GST/FST) credits, social assistance, and workers' compensation. Total household transfer incomes are regressed on the same set of explanatory variables as in the provincial tax regression, for all households aged 22 and over with positive transfer incomes, by province and year. The transfer regressions also work well: the estimated mean transfer income received by a household is identical to the observed mean in the LAD sample: $1,860. The estimated median is slightly higher than the observed median transfer income: $1,170 compared to $830 in the LAD. Also, the transfer regression predicts a slightly higher fraction of households with positive transfer incomes, 68% as compared to 63% in the LAD.

3.6 Registered pension plans

RPPs are employer-sponsored private-pension plans registered through the Canada Revenue Agency. The assets held in RPPs are among the most important assets held by Canadians. According to the 2005 SFS, assets held in RPPs represented 19% of total family assets, second only to the principal residence. Recent studies also show that private pensions play an important role in wealth accumulation. For example, MacGee and Zhou (2010) found that a life-cycle model with private pensions can lead to greater wealth dispersion for households with similar lifetime earnings and the resulting wealth dispersion is closer to those observed in data. Hence, it is important to incorporate private pensions into the life-cycle model.

There are two types of RPPs in Canada: defined benefit (DB) and defined contribution (DC) pension plans. Under DB pension plans, benefits are generally determined according to a formula based on the earnings history and years of coverage. DB pension plans are managed by employers, and employees typically do not make active investment decisions in this type of plan. By contrast, participation in DC pension plans often requires active decisions by eligible employees, who are given investment options and can choose the amount of their contributions (subject to plan and legislative provisions). Under DC pension plans, employers often match employees' contributions. Although participation in DB pension plans have been steadily declining since early 1980, about 75% of all RPP members in 2008 were still DB plan members. Unfortunately, it is not possible to distinguish DB pension plans from DC pension plans in the LAD. As DB pension plans still represent the majority of RPP membership, all RPP members in the sample are assumed to be members of DB pension plans.

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[19] CPP/QPP and OAS/GIS are modeled explicitly. UI/EI is excluded from the transfer system in the model because it depends on additional variables such as weeks worked and hours worked, which are not available in the LAD.

Coverage

Not every Canadian is covered by an RPP. According to *Pension Plans in Canada,*[20] about 38% of all employed workers were covered by RPPs in 2008.[21] As well, RPP participation changes with age. RPP participation rates are generally lower for younger workers and increase with age. There is also a great deal of uncertainty with regard to the RPP coverage over time. Individuals, especially under DB pension plans, may lose their coverage as a result of being laid off, as a result of quitting, or as a result of a firm closure. To take this uncertainty into account, RPP coverage is modeled as a stochastic process that matches the lifetime RPP coverage rate in data. This approach to modeling the lifetime pension coverage as a stochastic process constitutes one of the main differences between this study and similar U.S. studies on saving adequacy (Engen et al. 1999; Scholz et al. 2006; and Gale et al. 2009).[22]

The LAD contains a variable measuring employees' annual contributions to RPPs (since 1986) and a variable measuring total annual contribution to RPPs by employers and employees, i.e., pension adjustment. The former captures only contributory plans and may lead to an underestimating of the total RPP coverage rate. The latter captures both contributory and non-contributory (requiring only employers' contributions) plans, although it also covers credits from deferred profit-sharing pension plans, which account for only a small portion of pension adjustment. The "pension adjustment" variable in the LAD is then used to proxy RPP coverage.[23] A household is said to be covered by an RPP plan if it reports a positive amount of pension adjustment.[24]

The initial household RPP coverage rate and transition matrices that are conditional on the initial coverage are necessary in order to generate a stochastic lifetime RPP coverage process. The initial RPP coverage in the model is set to 13%, the average RPP coverage rate of 22-year-old households in the LAD. The initial RPP coverage is also allowed to vary by household earnings level and household type. In general, high earners have a higher RPP coverage rate than low earners, and couples with two earners have a higher coverage rate than other types of households. Given the initial coverage rate, the transition matrices containing the probabilities of having and not having an RPP conditional on prior RPP coverage are then calibrated to match the lifetime pension coverage rate observed in the SFS, which was 57.9% for households aged 60 to 64 in 2005. The transition matrices that contain the probabilities of having and not having RPP coverage conditional on prior coverage are also allowed to differ by household type and earnings level. Together with the initial RPP coverage, the transition matrices predict a lifetime private-pension coverage rate of 56.5% compared to 57.9% in the 2005 SFS.[25]

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[20] See Statistics Canada 2000.
[21] *The Daily*, Statistics Canada, May 25, 2010.
[22] The private-pension coverage is deterministic in those U.S. studies.
[23] Morissette and Ostrovsky (2006) did the same to proxy the RPP coverage.
[24] In the case of a couple, either the husband or wife, or both, may report positive pension adjustment.
[25] The RPP coverage rate in the LAD for households 55 to 57 years of age is 52%.

Contributions

Employees' RPP contributions need to be computed in order to derive a measure of taxable income in the model as the contributions are tax-deductable.[26] For this purpose, the distributions of RPP contributions as percentages of household earnings are examined for all households under age 64 covered by an RPP in the LAD. The distributions are fairly similar across different household types. Therefore, in the benchmark model, the RPP contribution rate is set to 3.5% of household earnings, which is the median contribution rate as a percentage of household earnings for all households with positive RPP contributions in the LAD.

Benefits

DB pension benefits are determined by various formulas that are usually tied to employees' earnings and length of pensionable service. In some cases, benefits are computed according to the average earnings in the last few years of work, often in the final five years. In other cases, benefits are computed on the basis of the average earnings over the whole career. Under the flat benefit plans, each pensionable year earns a fixed amount towards pension benefits. In this paper, the following formula is used to compute the amount of household private-pension benefits[27]:

$$rpp = \alpha(n_{rpp})n_{rpp}\tilde{y}_R, \tag{8}$$

where α is an accrual rate that depends on number of years of pensionable service, n_{rpp}. Let $\alpha = 0$ if $n_{rpp} < 5$, the vesting period in the benchmark model, and $\alpha = 1.5\%$ if $n_{rpp} \geq 5$.[28] n_{rpp} is the number of years of pensionable service over the lifetime. In other words, multiple incidences of RPP coverage over the lifetime are treated as continuous coverage by adding up the number of years of pensionable service. The adjusted earnings in the last working year, $\tilde{y}_R$, are used to calculate pension benefits.[29] The definition of adjusted earnings is $\tilde{y}_R = \max\left\{y_R, \bar{y}_{cpp}\right\}$. That is, if the last year's earnings, y_R, are higher than $\bar{y}_{cpp}$, the lifetime average earnings up to YMPE used to compute CPP/QPP benefits, the adjusted last year's earnings, $\tilde{y}_R$, are equal to y_R. Otherwise, $\tilde{y}_R = \bar{y}_{cpp}$.

3.7 Public pensions

The CPP/QPP is the second of the two pillars (CPP/QPP and OAS/GIS) of the Canadian public-pension system. Employed individuals aged 18 and over are required to make CPP/QPP contributions up to YMPE until they retire. In the model, households pay CPP/QPP contributions from age 22 to age 64 according to the historical CPP/QPP contribution rate from 1966 to 2005. The historical YMPE are also converted to 2005 dollars. CPP/QPP benefits are calculated as 25% of the average earnings-to-YMPE ratio (capped at 1) over the last 35 working years, adjusted by the average YMPE in the last five years prior to retirement (which takes place at age 65 in the model). Currently, the CPP/QPP regulation also applies actuarial factors to the beneficiaries who do not claim the benefits at age 65, reducing (increasing) actual benefits for those retiring before (after) the age of 65. The actuarial factors are not applicable in the model since every household is assumed to retire at 65 and is thus entitled to full benefits. The CPP/QPP benefits are fully indexed to the CPI and subject to income tax.

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[26] Note that some plans are not contributory (that is, they do not require employee contributions). More than 70% of RPP members belonged to contributory plans in 2000.

[27] The integration of the CPP/QPP and RPPs is not considered in this paper. The impact is discussed in Section 7.

[28] According to *Pension Plans in Canada* (Statistics Canada 2000), the vesting periods range from two to five years. A vesting period shorter than five years would yield a much higher lifetime RPP coverage rate in the model.

[29] The main reason for using the adjusted earnings is to reduce the state-space. Using the average earnings for the last five years of work or the career average earnings will add an additional state variable to the model.

The OAS is a monthly benefit available to most Canadians aged 65 and over who meet legal and residence requirements.[30] The maximum monthly OAS benefit in 2005 was $471.76. OAS benefits were fully taxable and subject to a 15% clawback for a net income of more than $60,806 in 2005. The GIS is a monthly benefit paid to eligible Canadian residents who receive OAS benefits and who have little or no other income.[31] The maximum monthly benefits in 2005 were $561 for single seniors and $365 for married seniors. GIS benefits are not taxable; however, for each dollar of net family income (excluding OAS), GIS benefits are reduced by 50 cents for single seniors and by 25 cents for married seniors. Both OAS and GIS benefits are indexed to CPI, but the former are fully taxable while the latter are not.

3.8 Survey of Financial Security

To assess the extent to which Canadians save adequately, their actual wealth holdings are compared with the predictions of the life-cycle model. The 2005 SFS is used to document wealth accumulation by Canadian households, as information on wealth is not available in the LAD. The SFS covers the population of all ten provinces; it excludes the territories. It provides a comprehensive picture of the net worth of Canadians. Information about the value of all major financial and non-financial household assets and debts was collected for more than 5200 families in Canada. The 2005 SFS is also used to calibrate household-type composition and the distribution of inheritance in the model.

Household composition

As the primary focus is the adequacy of household saving for retirement, the SFS sample is restricted to those households with heads aged 60 to 64 in 2005 who were near retirement or who had recently retired.[32] This selection of a sample that is seemingly inconsistent with the model where individuals work until age 65 is the result of several considerations. First, the 2005 SFS has a relatively small sample size. The current sample section yields only 393 observations.[33] If to select only those non-retired households, the model would yield even fewer observations. Second, the model already takes into account the choice of labour supply, although in a somewhat *ad hoc* fashion, by multiplying the age–earnings profiles for each type of household by the labour force participation rate. Hence, the earnings in the model are a weighted average of individuals who are still in the labour force and individuals who have retired.

As well, the SFS sample is also divided into several subgroups according to marital status and the number of earners, as in the LAD sample.[34] Table 2 shows the composition of the households in the 2005 SFS that are also used in the benchmark model. The composition across provinces in the 2005 SFS varies little. Thus, the composition is assumed to be the same across provinces in the simulation. The Appendix provides more details on how the simulation is conducted.

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[30] A minimum of 10 years of residence in Canada after reaching age 18 is required in order to receive OAS benefits in Canada and a minimum of 20 years is required in order to receive OAS benefits outside of Canada.

[31] The survivor and children's benefits are not considered in the benchmark model.

[32] The retirement of a household in the SFS is defined according to the retirement status of the major income earner within the household.

[33] The robustness of the model's implications is tested using a much larger sample combining both the 1999 SFS and the 2005 SFS in Section 6.

[34] Information used to define the number of earners for each household in the 2005 SFS includes that on whether respondents worked full-time or part-time in the last year, the earnings of household members, the amount of CPP benefits received, and the RPP status. Seven households in the sample cannot be identified in the end and are dropped from the sample.

Table 2
Household composition

Household type	2005 Survey of Financial Security	Benchmark model
	percent	
Singles[1]	36.55	36.55
One-earner couples	30.67	30.67
Two-earner couples	32.78	32.78

1. Singles and lone parents.
Source: Statistics Canada, 2005 Survey of Financial Security.

Inheritances

The 2005 SFS also collects information on the date and value of any inheritance (up to five times) received by household members.[35] The proportion of households with heads aged 55 to 64 in 2005 that reported receiving an inheritance is about 30%. The mean and median age of receiving the first inheritance among those households is 47. The median value of all inheritances received by those households is about $42,000 in 2005 dollars.[36] The inheritances are incorporated into the model and assumed to be unanticipated, random, and not correlated with household earnings. In the model, 30% of households receive an inheritance randomly at age 47. The value of inheritances is based on the corresponding distribution that was observed in the 2005 SFS.

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[35] For those households that received at least one inheritance, the majority (more than 70%) received only one. Thus, the first inheritance is used to compute the mean age at which the inheritance is received. However, all inheritances received (up to five times) are used to calculate the inheritance amount.

[36] The average inheritance for the top 10% of these households is $639,000 in 2005 dollars.

4 Retirement wealth in the Survey of Financial Security

Households’ private saving in the model are defined as total non-pension wealth.[37] In the SFS, this corresponds to net worth minus private-pension wealth. The net worth in the SFS is equal to total assets, which exclude public pensions but include private pensions, net of total debts.[38] The private-pension assets that were subtracted from the net worth consist of the pension value (termination) and other retirement funds (deferred profit sharing pension plans, executive and foreign pension plans, and annuities).[39]

Table 3 summarizes the main sample statistics by marital status, singles and couples (1EC and 2EC), and by RPP coverage status. On average, singles have much lower average annual earnings than couples: $11,600 versus $45,400, respectively. The fraction of retirees is also higher for singles than for couples: 60% versus 39%, respectively. Not only do singles have a lower RPP coverage rate than couples, 42% versus 67%, but they also accumulate less RPP wealth.[40] Overall, the total amount of private saving (non-pension wealth) of singles, at about $215,300, is much lower than that of couples ($585,700). Households with RPP coverage have higher household earnings than those without RPP coverage. The fraction of retired households is also higher for households with RPP coverage than for those without it. Overall, households with RPP coverage also accumulate more non-pension wealth at both the median level and the mean level.

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[37] Focusing on household private saving is a common practice in the literature. See Engen et al. (1999), Scholz et al. (2006), and Gale et al. (2009), for example.

[38] Total assets in the SFS include all financial (deposits, stocks, mutual funds, bonds, registered retirement saving plans (RRSPs), locked-in retirement accounts, registered retirement income funds, and RPPs) and non-financial (principal residence, other real estate, vehicles, and business equity) assets. Total debts include any mortgage on the principal residence, any mortgage on other real estate, any line of credit, credit card and instalment debt, student loans, vehicle loans, and other debts.

[39] The public- and private-pension wealth is excluded from the non-pension wealth, but RRSPs are included.

[40] The RPP coverage in the SFS is defined as either household head or spouse (if any) being covered by deferred RPPs (offered by previous employers), current RPPs, or RPPs in pay. Hence, it represents the lifetime RPP coverage status as of the interview date.

Table 3
Sample statistics for households aged 60 to 64

	Singles[1]			Couples[2]			Singles and couples not covered by registered pensions plans			Singles and couples covered by registered pension plans		
	mean	median	standard deviation	mean	median	standard deviation	mean	median	standard deviation	mean	median	standard deviation
Age (years)	61.90	62.00	1.36	62.05	62.00	1.44	61.94	62.00	1.35	62.01	62.00	1.46
Household current earnings (thousands of dollars)	11.64	0.00	21.91	45.43	33.07	57.50	24.06	12.40	45.34	39.63	25.84	52.85
Registered pension plan wealth (thousands of dollars)	104.16	0.00	171.01	265.94	97.42	343.43	...	...	...	356.50	279.44	323.45
Non-pension wealth (thousands of dollars)	215.31	90.23	353.85	585.73	361.80	1,142.86	405.78	183.75	1,117.57	482.67	313.00	810.72
Percentage who are retired (percent)	60.00	...	...	39.00	...	...	38.00	...	...	53.00	...	...
Registered pension plan coverage rate (percent)	42.00	...	...	67.00	...	...	...	...	...	...	...	...

	Singles	Couples	Singles and couples not covered by registered pensions plans	Singles and couples covered by registered pension plans
Number of observations	124	269	187	206

1. Singles and lone parents.
2. One-earner couples and two-earner couples.

Source: Statistics Canada, 2005 Survey of Financial Security.

Table 4 illustrates the distributions of non-pension wealth for households with heads aged 60 to 64 by household type.[41] Overall, households with heads aged 60 to 64 accumulate a median wealth of $231,000 and a mean wealth of $450,300 in 2005. Couples hold more wealth than singles throughout the wealth distribution, and their median wealth is more than twice the median wealth of singles. Households covered by RPP have accumulated more wealth almost throughout the wealth distribution than households with no RPP coverage.

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[41] The detailed results by the distribution of household current earnings are not presented here, since the current earnings are not sufficient to capture lifetime earnings that are central to wealth accumulation. It should be noted that current earnings are subject to selection bias, especially around retirement age. That is, more wealthy households can afford to retire earlier. Results are available upon request.

Table 4
Distribution of non-pension wealth among households aged 60 to 64

	Distribution of non-pension wealth							Mean
	5th percentile	10th percentile	25th percentile	50th percentile	75th percentile	90th percentile	95th percentile	
	thousands of dollars							
All households	1.00	4.20	68.90	231.00	545.00	871.50	1,244.00	450.33
Singles[1]	0.25	0.65	4.70	90.23	218.00	710.70	1,074.10	215.31
Couples[2]	29.20	46.00	172.50	361.80	609.80	1,042.00	1,525.90	585.73
Covered by registered pension plan	4.00	24.50	117.26	313.00	564.10	977.10	1,244.00	482.67
Not covered by registered pension plan	0.25	0.65	10.00	183.75	488.10	829.70	1,252.35	405.78

1. Singles and lone parents.
2. One-earner couples and two-earner couples.

Source: Statistics Canada, 2005 Survey of Financial Security.

5 Simulation results

The model is simulated after being calibrated and solved to generate the retirement wealth for 20,000 households. The simulated results are used as benchmarks to evaluate the actual wealth of households in the SFS. The Appendix (Section 9.1) provides technical details of the simulation. The predicted life-cycle profiles of optimal consumption and wealth from the model are consistent with those in the literature on stochastic life-cycle model. These are discussed in Section 9.2 of the Appendix.

5.1 Saving adequacy

According to the definition of adequacy in this paper, a household is said to save adequately if it accumulates more wealth than the optimal wealth implied by the benchmark model. However, as a result of data limitations, this comparison cannot be performed for every household in the sample in order to identify which particular households save adequately and how many of these households there are. Instead, the median of observed wealth is compared with the median of the simulated wealth for households with similar characteristics to gauge overall saving adequacy.[42] If the median of observed wealth is higher than the median of the simulated wealth for households with the same characteristics, it is concluded that, overall, those households save adequately. Following the same logic, the share of the households in the SFS whose wealth exceeds the simulated median wealth for households with the same characteristics is also computed. If more than 50% of households in the SFS have accumulated wealth in excess of the simulated median wealth from the benchmark model, it is concluded that households in the sample save adequately.

Table 5 reports the measures of saving adequacy defined above for all households in the SFS sample and for subgroups by marital status and RPP coverage status. Overall, at the median level, households aged 60 to 64 have a saving surplus of about $73,000 (a positive difference between the observed median wealth and the simulated median wealth), 62% of whom have accumulated more wealth than the simulated median. This suggests that, overall, households aged 60 to 64 save adequately. However, it is important to note that it is possible that some households among the 62% of households whose wealth exceeds the simulated median wealth may not save enough relative to their own optimal wealth targets. Similarly, some of those 38% whose wealth falls below the simulated median wealth may in fact save enough relative to their own optimal wealth targets.

The median wealth of all subgroups in the SFS, except singles, exceeds the median of simulated wealth from the benchmark model for the same category. The saving surpluses for households with RPP coverage and couples are the largest, about $158,000 and $172,000, respectively.[43] The saving surplus for the households without RPP coverage is the smallest, about $22,000. Single households comprise the only group that incurs a saving deficit (a negative difference between the observed median wealth and the simulated median wealth).[44] However, the deficit is not large, at about $30,000. The same story is told by the shares of households whose wealth exceeds the simulated median wealth. The percentage of households whose wealth is higher than the simulated median wealth is highest for the households with RPP coverage and couples, at 70% and 72%, respectively. It is also slightly above 50% for households without RPP coverage and slightly below for singles. All in all, a majority of households aged 60 to 64 in the SFS appear to have saved adequately, according to the definition adopted here.

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[42] The simulated median wealth is based on households in the model economy with the exact same age, marital status, and RPP coverage status as corresponding households in the SFS sample.

[43] Engen et al. (1999) also found that households with private pensions appear to save significantly more adequately than households without private pensions.

Table 5

Household wealth, 2005 Survey of Financial Security (SFS) versus the benchmark

Sample	Median non-pension household wealth		Mean non-pension household wealth		Households exceeding the benchmark median in 2005 SFS
	2005 SFS	Benchmark	2005 SFS	Benchmark	
	thousands of dollars				percent
All households	231.00	158.09	450.33	197.16	62.47
Singles[1]	90.23	120.14	215.31	146.54	44.65
Couples[2]	361.80	189.44	585.73	226.32	72.48
Covered by registered pension plan	313.00	155.44	482.67	194.44	69.52
Not covered by registered pension plan	183.75	161.28	405.78	200.67	53.60

1. Singles and lone parents.
2. One-earner couples and two-earner couples.

Sources: Statistics Canada, 2005 Survey of Financial Security and authors' simulations.

5.2 Risk of undersaving

Although it is not possible to assess saving adequacy for each household in the SFS given data constraints, it is possible to compare the observed and simulated wealth distributions side by side to see whether there is a potential risk of undersaving at certain points of the wealth distribution.

Table 6 compares the simulated and actual wealth distributions at various percentiles for all households aged 60 to 64 in the sample. The benchmark model underestimates wealth at the high end of the distribution. It is likely that there is a significant amount of wealth accumulation among households in this category for which the model does not account.[45] For all households aged 60 to 64 below (and including) the 10th percentile, the actual wealth is consistently lower than the simulated wealth while, above (and including) the 25th percentile, actual wealth is consistently higher than predicted wealth. The adequacy threshold is defined as a point in the wealth distribution above which the actual saving exceeds the simulated target and below which saving falls short of the simulated target. This threshold appears to be the 23rd percentile in the wealth distribution for all households aged 60 to 64 in the SFS. This may suggest that the overall saving deficit is small and that the undersaving likely happens only at the low end of the wealth distribution. The adequacy threshold for singles is considerably higher, at the 60th percentile. This suggests a higher risk of undersaving. Couples fare much better than singles, exceeding the simulated targets even at the 5th percentile.

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[44] Scholz et al. (2006) found that being single, rather than lifetime earnings, is the only factor that strongly correlates with having a saving deficit

[45] In the model, households save only for retirement and to protect against uncertainty in earnings, RPP coverage, and life span. Any other potential saving motives are ignored.

A similar comparison by RPP coverage is given in Table 7. Households with lifetime RPP coverage appear to save adequately according to the criteria used in this study. All households above the 10th percentile of the wealth distribution accumulate more wealth than what is implied by the benchmark model. Even at the 5th percentile, the deficit is only about $1,400. Households without lifetime RPP coverage accumulate more wealth than what is implied by the benchmark model at the median level. However, the adequacy threshold for households without RPP coverage locates in the neighborhood of the 40th percentile. This also suggests a high risk of undersaving.

Table 6
Distribution of households' non-pension wealth by marital status

	Households' non-pension wealth							Mean
	5th percentile	10th percentile	25th percentile	50th percentile	75th percentile	90th percentile	95th percentile	
	thousands of dollars							
All households								
2005 SFS	1.00	4.20	68.90	231.00	545.00	871.50	1,244.00	450.33
Benchmark	6.14	17.92	60.28	158.09	287.54	436.20	534.57	197.16
Singles[1]								
2005 SFS	0.25	0.65	4.70	90.23	218.00	710.70	1,074.10	215.31
Benchmark	4.70	13.52	46.04	120.14	211.36	315.49	390.60	146.54
Couples[2]								
2005 SFS	29.20	46.00	172.50	361.80	609.80	1,042.00	1,525.90	585.73
Benchmark	7.68	21.38	74.02	189.44	336.70	486.84	579.98	226.32

1. Singles and lone parents.
2. One-earner couples and two-earner couples.

Note: SFS stands for Survey of Financial Security.
Sources: Statistics Canada, 2005 Survey of Financial Security and authors' simulations.

Table 7
Distribution of households' non-pension wealth by registered pension plan coverage

	Households' non-pension wealth							Mean
	5th percentile	10th percentile	25th percentile	50th percentile	75th percentile	90th percentile	95th percentile	
	thousands of dollars							
Covered by registered pension plan								
2005 SFS	4.00	24.50	117.26	313.00	564.10	977.10	1,244.00	482.67
Benchmark	5.40	16.56	57.28	155.44	287.44	430.14	530.55	194.44
Not covered by registered pension plan								
2005 SFS	0.25	0.65	10.00	183.75	488.10	829.70	1,252.35	405.78
Benchmark	7.08	20.15	64.09	161.28	287.85	444.63	540.29	200.67

Note: SFS stands for Survey of Financial Security.
Sources: Statistics Canada, 2005 Survey of Financial Security and authors' simulations.

5.3 Comparison with other studies

This section compares the benchmark results with those in several other similar studies that use optimal wealth accumulation as the saving adequacy benchmark. By and large, the results in this paper are consistent with the results from those studies, although some differences need to be recognized.

Engen et al. (1999) examined the adequacy of household saving by comparing the median of observed wealth to that of simulated wealth. Their study was based on a sample of married couples aged 51 to 61 in the 1992 Health and Retirement Study (HRS) , which is slightly younger than the sample used in this study. They find that the saving of roughly 60% of households in their sample exceed the simulated median of non-pension wealth. It should be noted that Engen et al. set their discount factor parameter, β , to 0.97 and the relative risk-aversion parameter, σ , to 3: both are higher than the corresponding parameters in the benchmark model. A higher discount factor leads to more saving because agents value future utility at a higher rate. The higher is the relative risk aversion—the more risk-averse agents are—the more precautionary saving are accumulated. Therefore, their model implies higher saving targets than the model used in this study, everything else being equal. Assuming a similar wealth distribution in both samples, one can expect that fewer households would save adequately by the Engen et al. model—since the requirements are more stringent—than by this study.

Scholz et al. (2006) also used a sample from 1992 HRS although their sample includes both married couples and singles. Unlike Engen et al. (1999), Scholz et al. derived optimal saving targets for each household in their sample and found that about 84% of households in their sample accumulated more wealth than their optimal targets. The relative risk aversion is also set to 3 in their paper. This means that the saving adequacy requirements followed by Scholz et al. are more stringent. A more recent paper, by Gale et al. (2009), presents an updated version of Scholz et al. (2006) using a 2004 HRS sample. Gale et al. found that 74% of households in their sample exceeded optimal retirement wealth targets, a lower rate of saving adequacy than in Scholz et al. (2006). It should be remembered, however, that the results of this paper pertain only to the overall adequacy of household saving, not to the adequacy of the household saving of any particular households in the sample. It is possible that some households among the 62% of households whose wealth exceeds the simulated median wealth do not save enough relative to their specific optimal targets.

Horner (2009) is the only other Canadian study to have examined saving adequacy based on the life-cycle theory of consumption. However, there are several fundamental differences between Horner's work and this paper. For example, the life-cycle model used in his paper has only two periods and no uncertainties. Saving in his model is assumed to be constant over time. Yet, some of his results are quite close. For instance, Horner found that, overall, 69% of families aged 30 to 64 in 2006 met their saving target to equalize pre- and post-retirement consumptions. About 60% of modest-earner families and 63% of middle-earner families met the consumption-continuity saving target. The share of families who met the saving target is lowest for singles among the different family types in the middle of the earnings distribution. RPP members are more likely than those without RPP coverage to achieve consumption continuity.

However, this similarity should be interpreted with caution. A life-cycle model with certainty, like the one used in Horner, implies that all households with the same characteristics (age, earnings, pension status) save equally. Hence, Horner's finding that 69% of households aged 30 to 64 in 2006 met their saving targets would also imply that the saving of exactly 31% of households in that group fall short. This would (erroneously) suggest that a large proportion of the population is undersaving. In contrast, in a stochastic life-cycle model, households with the same characteristics can still save differently in response to the different realization of shocks. Thus, in this paper, a distribution of optimal saving targets is used, instead of a single target. Therefore, the finding that the observed wealth of 62% of households in the sample exceeds the simulated median does not necessarily imply that the other 38% undersave relative to their optimal targets. Instead, it suggests adequate amounts of wealth accumulation relative to the median of the simulated wealth.

6 Sensitivity analysis

This section conducts a number of sensitivity analyses to test the robustness of the implications from the benchmark model. Table 8 provides the main set of results for the sensitivity analyses.

Pooled Survey of Financial Security sample

The 2005 SFS sample consists of only 393 observations. The number of observations becomes even smaller when the sample is broken down by marital status and RPP coverage status. It is possible that some of the results may be driven by the relatively small sample size. To investigate this possibility, households aged 60 to 64 from the 1999 SFS are pooled together with those in the 2005 SFS in order to construct a larger sample. The resulting sample has 1397 observations, more than three times the number in the original sample. The new sample, however, may suffer from inconsistency as a result of the difference in economic conditions between 1999 and 2005. For instance, the booming housing market of the early 2000s may have had an effect on the wealth of the households in the 2005 SFS but not on that of the households in the 1999 SFS. To control for this type of year effect, household non-pension wealth in the pooled sample is regressed on age, age squared, education, and a dummy variable for the year 1999, and construct new household wealth with the year effect removed. The resulting wealth in the pooled sample is then compared with the simulated wealth from the benchmark model, and the results are presented in Column 2 of Table 8.

The results are largely similar to those based on the 2005 SFS sample (Column 1 in Table 8). The main reason appears to be the similarity between the wealth distributions in the pooled sample and the 2005 sample. For example, after controlling for year effects, the median of the non-pension wealth of all households between 60 and 64 years old in the pooled sample is about $237,000, compared to $231,000 in the 2005 sample alone. The mean wealth is about $432,000 in the pooled sample, compared to $450,000 in the 2005 sample. As a result, about 63% of households in the pooled sample accumulated more wealth than the simulated median, compared to 62% that did so in the 2005 sample. The number is 74% for households with RPP coverage in the pooled sample as opposed to 70% in the 2005 sample, and 50% for households without RPP coverage in the pooled sample as opposed to 54% in the 2005 sample. The numbers for singles and couples are very close to those for their counterparts in the 2005 sample.

Home equity

The benchmark model includes all housing equity into the measure of private saving in the 2005 SFS. There have been debates in the literature about the degree to which people are willing to reduce housing equity in order to sustain consumption in retirement. To explore the impact of altering the treatment of housing in the analysis, only half of the housing equity is included in the resources available for the sample households in the 2005 SFS for this sensitivity analysis. The results (Column 3 of Table 8) show that the percentage of households whose wealth exceeds the simulated median wealth drops to about 51% for all households when half of housing equity is excluded. The proportions are more than 10 percentage points lower for all subgroups when compared to those in the benchmark. This suggests a significant role of home equity in measuring the adequacy of retirement saving.

Risk aversion

The impact of the relative risk aversion on saving adequacy is tested by considering a lower risk-aversion parameter of 1.5 and a higher risk-aversion parameter of 3.0.[46] The results are shown in columns 4 and 5 of Table 8. As expected, households in the model save less (more) to hedge against future risk when they are less (more) risk-averse. Therefore, the indicators of saving adequacy are much higher (lower) than those in the benchmark case when the risk-aversion parameter is lower (higher). The share of all households aged 60 to 64 whose wealth exceeds the simulated median increases to 66% when the coefficient of risk aversion is lower (1.5) while it drops to 55% with a higher risk aversion (3.0). Similar patterns apply to other subgroups.

The accrual rate of registered pension plans

The generosity of private pensions also influences saving decisions. Households with more generous private pensions would reduce other private saving. In the next analysis, the accrual rate of private pensions is increased to 2% from 1.5% that is used in the benchmark model. This results a slightly higher share of households whose wealth exceeds the simulated target than in the benchmark model, 64% as opposed to 62% in the benchmark case (Column 6 of Table 8). This is because more generous private pensions reduce the median non-pension wealth, allowing more households to exceed the simulated saving targets implied by the model. About 72% of households with RPP coverage accumulate more wealth than the simulated median, as opposed to 70% in the benchmark case. Singles and couples also experience slight increases in their saving adequacy figures. Not surprisingly, the generosity of private pensions has no impact on the saving adequacy of households without RPP coverage.

Discount factor

Saving decisions are also affected by the values of the discount factor, β. A higher discount factor increases the incentive to save, as agents value the future more. This raises the optimal level of wealth in the life-cycle models. It is possible that households have different discount factors (i.e., they discount the future at different rates). The next sensitivity analysis examines the effect of discount-factor heterogeneity. $\beta = 0.96$ *is randomly assinged* to 60% of the population, a lower β (0.94) to 20% of the population, and a higher β (0.98) to the remaining 20%. The results are shown in Column 7 of Table 8. The overall figures for saving adequacy are almost the same as those in the benchmark case. This is due to the impact of more saving with a higher β being nearly cancelled out as a result of the impact of less saving with a lower β. Instead, when all households in the model have a lower discount factor (0.94), the target is much lower and 71% of all sample households meet the median wealth target; when all households in the model have a higher discount factor (0.98), only 49% meet the median target.

Rate of return

Another parameter that plays an important role in saving decisions is the rate of return on investment. In life-cycle models, rational agents make consumption decisions by comparing the marginal utility of consumption made today and the marginal utility of consumption enjoyed tomorrow. A higher rate of return makes saving for tomorrow easier and thus makes delaying consumption more beneficial. This leads to more wealth accumulation in the model and thus to the fraction of households meeting simulated saving targets becoming smaller. Also, the rates of return are likely to be different for different households for a number of reasons, including differences in portfolio holdings and local financial institutions. The next analysis introduces a simple form of heterogeneity with respect to the rate of return.[47] Each household randomly draws a rate of return on saving in the first period and keeps it over the entire life-cycle. More specifically, 60% of households receive a 4% return, 20% of households receive a 2% return, and 20% of households receive a 6% return on their saving. The results are shown in Column 8 of Table 8. The results are very similar to those in the benchmark case. This is due largely to the same proportion of households with a high rate of return as with a low rate of return, and this in turn cancels out the impacts of different rates of return on saving. However, when all households in the model have a lower rate of return (2%), 71% of all sample households meet the median wealth target; when all households in the model have a higher rate of return (6%), only 53% meet the median target.

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[46] These numbers fall in the range of 1 to 3, which is widely used in the literature.

In summary, the overall saving adequacy is largely robust to alternative parameter values and sample choice. The fraction of households whose wealth exceeds the simulated median wealth is always above 50% across different scenarios, ranging from 51% when only half of home equity is included into the measure of saving to 66% when a lower risk-aversion coefficient is used in the model. Meanwhile, the finding that couples and households with RPP coverage fare much better relative to singles and households without RPP coverage is also robust.

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[47] This study does not consider the investment risks or stochastic returns to investment in the model. However, adding investment risks in the model, as long as the expected rate of return is similar and given that the risks are highly non-persistent, would have a larger impact on the wealth dispersion in the model than on the median wealth.

Table 8

Sensitivity analysis of percentage of households with wealth higher than simulated median wealth

Sample	Benchmark	Pooled SFS	Half housing equity	Lower risk aversion[1]	Higher risk aversion[2]	Higher RPP accrual rate[3]	Discounter factor heterogeneity	Rate-of-return heterogeneity
	Column 1	Column 2	Column 3	Column 4	Column 5	Column 6	Column 7	Column 8
				percent				
All households	62.47	63.22	50.92	65.57	55.27	63.97	62.36	62.52
Singles	44.65	44.93	33.48	49.98	39.42	49.98	44.65	44.65
Couples	72.48	72.18	59.97	76.32	65.73	75.63	72.48	72.48
Covered by RPP	69.52	73.62	57.55	73.67	61.19	71.72	69.00	69.52
Not covered by RPP	53.60	50.32	41.51	56.93	47.76	53.60	53.60	53.60

1. In this sensitivity analysis, the relative risk-aversion parameter is set to 1.5.
2. In this sensitivity analysis, the relative risk-aversion parameter is set to 3.0.
3. In this sensitivity analysis, the RPP accrual rate is set to 2% per year.

Note: SFS stands for Survey of Financial Security. RPP stands for registered pension plan. ‘Singles’ comprises singles and lone parents. ‘Couples’ comprises one-earner couples and two-earner couples.

Sources: Statistics Canada, 1999 and 2005 Survey of Financial Security, and authors’ simulations**.**

7 Discussion

This section discusses some limitations of the analysis and their impact on the results.

First, the benchmark model does not consider the integration between the CPP/QPP and RPPs. Since the introduction of the CPP/QPP, many RPPs have coordinated with the CPP/QPP in both contributions and benefits. As a result of the integration, the benefits from RPPs at 65 are reduced by roughly the same amount one would expect to receive at age 65 from the CPP/QPP.[48] Therefore, the benchmark model will overestimate the RPP benefits for those households with RPP coverage. This in turn will result in a lower saving target in the model for those households with RPP coverage, thus overestimating their saving adequacy. [49]

Second, the benchmark model produces a larger non-pension wealth offset effect for households with RPP coverage than that observed in the data. This may be partly due to the fact that inflation expectations are not considered in the model.[50] Private-pension benefits often take the form of fixed nominal annuities (i.e., they are not protected against inflation by formal indexation). According to Statistics Canada,[51] at the beginning of 2000, only 33% of all DB pension plans provided for the automatic indexing of pension benefits. Those plans covered about half of all members of DB pension plans, the majority of those working in the public sector. It is likely that the inflation-free benchmark model underestimates wealth accumulation that would be made if these expectations were incorporated for households with RPP coverage. This is because households in the model economy do not worry about inflation, while households in reality may worry and decide to save more in order to insure against the loss of their private-pension benefits resulting from inflation. It is also acknowledged that the higher saving for households with RPP coverage in the SFS may be due to some unobserved characteristics associated with RPP coverage (for instance, preference for saving) that is not captured by the model.

Finally, the focus of this paper is on preparedness for retirement among current retirees, specifically, households between 60 and 64 years of age in 2005. This paper does not address the retirement prospects of future retirees. This would require modeling of future earnings processes and making assumptions about future economic conditions. A recent study (Moore et al. 2010) that examined the prospects of future retirees argued that Canadians may find it more difficult to maintain the level of their working-life consumption in retirement given declining RPP coverage, among other reasons.[52]

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[48] In 2005, about 77% of RPPs members belonged to plans that integrated with the CPP/QPP (CANSIM table 280-0019).

[49] The income replacement rate for RPP in the benchmark model is about 52%, assuming maximum 35 years of pensionable service and the annual accrual rate to be 1.5%. Together with CPP/QPP, this results in a total income replacement rate of about 77%, which is slightly higher than the 70% that is often the case when RPP is integrated with CPP/QPP. Therefore, the impact of overestimating RPP benefits is not expected to be large.

[50] MacGee and Zhou (2010) showed that a model with private pensions, but without inflation, generates a larger pension offset effect than that observed in the data.

[51] See Statistics Canada 2000.

[52] Wolfson (2011) also argued that a significant portion of young Canadians with middle earnings may, under some circumstances, experience at least a 25% decrease in their living standards when they retire.

8 Conclusion

This paper assesses the adequacy of retirement saving in Canada by comparing observed retirement wealth with a simulated benchmark derived from a life-cycle stochastic model. In such models, agents reach optimal paths of their lifetime wealth accumulation and retirement saving by smoothing their marginal utilities of consumption over a lifetime, subject to budget constraints and given the uncertainties that they face. In this paper, households are said to save adequately when they accumulate more wealth than the optimal saving target implied by the model.

The observed wealth of households aged 60 to 64 in the 2005 SFS sample is compared with the simulated wealth implied by the life-cycle model for households with the same characteristics. The objective is to examine overall saving adequacy in Canada rather than to investigate exactly how many households are saving adequately given the data limitation. Using the definition of adequacy, it is concluded that a majority of Canadian households save adequately for retirement: about 62% of all households accumulate more wealth than the simulated wealth median. Households with lifetime RPP coverage and couples appear to be in a better financial position for retirement than other households: 70% of the former and 72% of the latter have accumulated more wealth than the simulated wealth medians. The estimated risk of undersaving is also small for these groups. Households without RPP coverage and singles do reasonably well at the median: about 54% of households without RPPs and 45% of singles have accumulated more wealth than the simulated wealth median. However, the estimated risk of undersaving by these groups is large, given that the adequacy threshold is located at the 40th and 60th percentiles of the wealth distribution for households without RPP coverage and for singles, respectively. These findings are also largely robust to various alternative parameter specifications and sample choice.

The assessment of the current state of saving adequacy in Canada is broadly consistent with results from other Canadian studies that assess the adequacy of retirement saving by focusing on retirement income and consumption. The results of this paper are also largely consistent with the results from U.S. studies that use a similar approach. However, this paper also points to the need to better understand more vulnerable groups, such as households with no RPP coverage and singles, who are considerably more likely than other groups to accumulate less retirement wealth than the simulated saving targets. It is also acknowledged that the main analysis is based on 2005 wealth records, a year characterized by rapid growth in the financial and housing markets that resulted in a high level of household net worth. The conclusions might have been different had post-2008 wealth data been examined. This underscores the importance of re-examining the issue as more recent wealth data become available.

9 Appendix

9.1 Numerical solution

Numerical dynamic programming techniques are used to approximate the decision rules as well as the value function. The dynamic program has five state variables in addition to period *j*: non-pension wealth *k*; earnings state *e*; the average ratio of earnings to YMPE until current period $\bar{y}_{cpp}$; RPP coverage status *pen*; and years of RPP coverage until current period n_{rpp}.

The state-space along two continuous state variables, *k* and $\bar{y}_{cpp}$ is discretized. The model is solved using backward induction. In the last period ($j = J$), the policy functions are trivial. In periods prior to *J*, optimal decision rules are computed for each possible combination of nodes, using stored information about the subsequent period's decision rules and value function. The approach by Tauchen (1986) is used to approximate the distributions of the innovations to the labour earnings process. For points that do not lie on the state-space grids, the value functions are evaluated using a bi-cubic spline interpolation along the two dimensions. The decision rules for the current period are obtained by picking the maximum from all alternatives.

Once the optimal decision rules are determined for all possible nodes in each period, the wealth accumulation is simulated for 20,000 households. To do so, these households are first assigned to each of the 10 provinces according to the geographic distribution of households aged 60 to 64 derived from the SFS sample.[53] Within each province, households are sorted into different types according to the household composition as shown in Table 2, assuming the same household composition for every province.[54] The simulation starts from age 22. The initial earnings are first simulated on the basis of the log earnings regression (equation 5) and a random draw from the initial distribution of earnings residual, which is assumed to be normally distributed, $N(0, \sigma_{e1})$, for each household. Along with a random draw of initial RPP coverage, the optimal saving is derived for each household according to the model solution. Then the earnings and RPP coverage are updated in each period according to equations (5) and (6) and the transition matrix for RPP coverage. The optimal saving for each household is then updated according to the updated earnings and RPP status at each period. In the end, the accumulated saving or wealth for households of different types between 60 and 64 years of age is used as the benchmark and to compare with the wealth level in the SFS for households with similar characteristics. All programs are parallelized and run on SHARCNET.[55]

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[53] Prince Edward Island and Newfoundland and Labrador are joined together in the simulation because of their relatively small populations.

[54] The household composition across provinces is examined in the 2005 SFS. The composition varies, but the differences are not large. Therefore, for simplicity, the same composition across provinces is assumed.

[55] SHARCNET is a multi-institutional high-performance computing network spanning 17 academic institutions in Ontario, Canada.

9.2 Optimal life-cycle consumption and wealth profiles

The optimal life cycle profiles of consumption and wealth predicted by the benchmark model are plotted in Chart 2 and Chart 3, respectively. They match very well with those predicted by other models used in the literature.[56]

The consumption in Chart 2 refers to optimal median consumption per adult equivalent over the life cycle by marital status and private-pension status. It is also normalized so that one unit is equal to $20,000 in 2005 dollars. Consumption is hump-shaped over the life cycle regardless of household type. When households are young, consumption is low as a result of low incomes and households' desire to build up their precautionary saving. As households age, income and saving rise and the precautionary-saving motive edges off. This leads to rising consumption during working years and peaking before retirement. Consumption then falls throughout old age as an increasing mortality risk makes households less patient and less willing to defer consumption to future. The consumption profile also differs by household type. Singles have a higher consumption level per adult equivalent than couples before age 52, and a lower consumption level than couples after age 52. This is due mostly to the fact that couples have more children on average than singles (including lone parents—see Chart 1), a fact that drives down consumption per adult equivalent for couples. Households with private pension have much higher consumption than those without a private pension throughout the life cycle.[57] This is due to the positive correlation between RPP coverage and earnings. Households with RPP are more likely to have higher earnings thus a higher consumption level.

Wealth in Chart 3 refers to optimal median total household wealth (non-pension) and is also normalized the same way as consumption. Wealth is also hump-shaped over the life cycle, regardless of household marital status and private-pension status. Wealth rises and peaks before retirement, reflecting a period of saving for retirement and precautionary motives. It then declines in old age, a period of dissaving. Couples have a much higher non-pension wealth profile than singles throughout the life cycle as a result of a higher life-cycle earnings profile and more persistent earnings shocks (see Table 1). Households with RPP coverage have a much higher non-pension wealth profile than households without RPP coverage before retirement and then a lower profile afterward. This is due to the former being more likely to have a higher earnings profile than the latter, a fact that encourages more saving among households with RPP coverage. This higher saving dominates the offset effect that the former tend to save less than the latter, conditional on same earnings level. However, in retirement, households with RPP deplete their non-pension wealth more quickly than do households without RPP because the former have access to additional wealth, namely, RPP wealth*,* while the latter do not.

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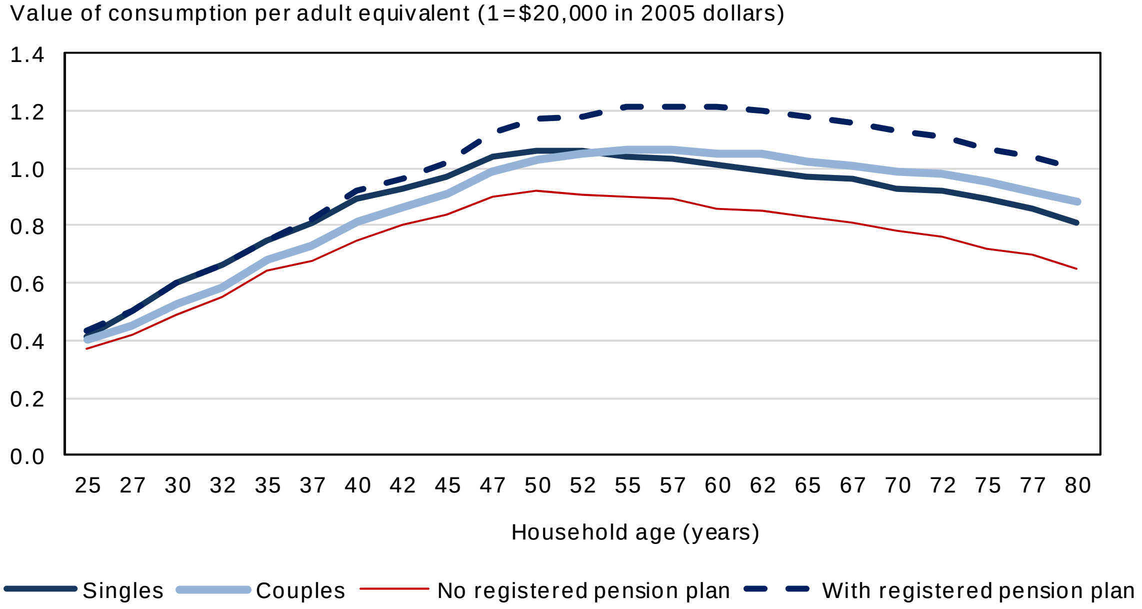
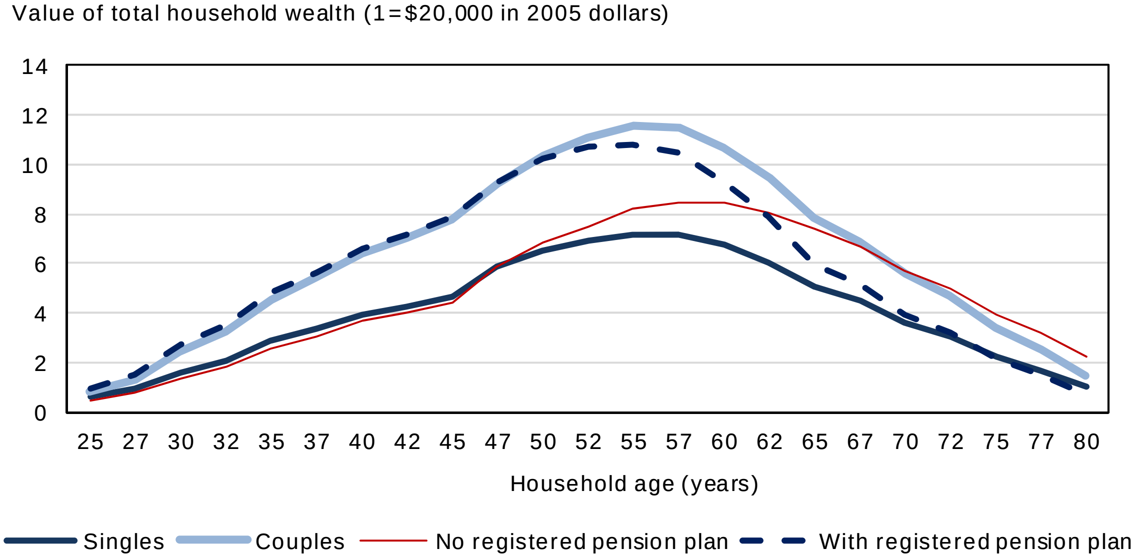
56. See, for example, Hubbard et al. (1995), Huggett (1996), Engen et al. (1999), De Nardi (2004), and Cocco et al. (2005) for comparison.
57. Engen et al. (1999) also found that optimal consumption is higher for households with private pension than for households without private pension.

Chart 2
Optimal life-cycle profiles of median consumption by household type



Note: 'Singles' comprises singles and lone parents. 'Couples' comprises one-earner couples and two-earner couples.
Source: Statistics Canada, authors' simulation based on the benchmark model.

Chart 3
Optimal life-cycle profiles of median wealth by household type



Note: 'Singles' comprises singles and lone parents. 'Couples' comprises one-earner couples and two-earner couples.
Source: Statistics Canada, authors' simulation based on the benchmark model.

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