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ABSTRACT

In establishing scaling laws for physical models to simulate sediment transport rate in the vicinity of the bed in unidirectional flows, it has been a common practice to neglect the effect of the relative depth and consider only the shear Reynolds number and the mobility number. As a result, scale effects are introduced in the model because of the influence of flow depth on bed form geometry which in turn affects the friction factor and the sediment transport rate. This paper describes an experimental study to systematically measure the effect of the relative depth on sediment transport rate.

RÉSUMÉ

En établissant des lois d'échelle pour les modèles physiques qui simulent le transport des sédiments à proximité du lit en présence d'écoulements unidirectionnels, on a eu pour habitude de laisser pour compte l'effet de la profondeur relative et de ne retenir que le nombre de Reynolds pour le frottement et le nombre de la mobilité. Il en résulte donc que des effets d'échelle sont introduits dans le modèle à cause de l'influence de la profondeur du débit sur la géométrie du lit, ce qui, en retour, influe sur le facteur de friction et le taux de transport des sédiments. Le présent document décrit une étude expérimentale qui cherche à mesurer de façon systématique l'effet de la profondeur relative sur le taux de transport des sédiments.

INTRODUCTION

Design of scale models of flows transporting sediment is not straighforward because of the large number of characteristics parameters that contol the phenomenon. It often involves neglect of one or more parameters so that a small scale model is possible. When a parameter is neglected in a scale model, a certain amount of error is introduced when transfering the model results to prototype conditions. Such errors are usually called the errors due to "scale effects". It is desirable to have some quantitative knowledge of these scale effects in order to assess the reliability of a physical model. In this paper, the effect of the relative flow depth, $Z = h/D_{50}$ (h is the flow depth and D_{50} is the median size of sediment) which is often neglected in modelling the bed load transport of sediment is investigated using a laboratory sediment transport flume.

Factors Affecting Bedload Transport

Yalin (1971) has shown that when considering the motion of the bed load sediment particles en masse, the effect of the sediment density can be neglected and any property relating to bed load transport is governed by three dimensionless parameters as expressed below.

$$\Pi_A = \Phi_A(X, Y, Z) \tag{1}$$

where Π_A is a dimensionless form of a property A, X is the shear Reynolds number, Y is the mobility number and Z is the relative depth. The expansion of X, Y, and Z are shown below

$$X = \frac{v_* D_{50}}{\nu}$$

$$Y = \frac{\rho v_*^2}{\gamma_s D_{50}}$$

$$Z = \frac{h}{D_{50}}$$

where v_* is the shear velocity, ν and ρ are kinematic viscosity and density of fluid respectively, and γ_s is the submerged specific weight of sediment. Yalin (1971) further shows that the realization of a practical small scale model considering all the three parameters listed above is not possible. A compromise is to ignore the relative depth parameter Z and design the model based only on the shear Reynolds number and the Mobility number.

By neglecting the relative depth parameter, it is assumed that the effect of Z on the property under investigation is small. It can be shown (Yalin (1971)) that if the transport rate of bed load is investigated then the neglect of Z implies the assumption that the equivalent sand grain roughness of sand bed (K_s) is independent of flow depth. The above assumption is not always true. It may be true if the flow is over a plain rough bed. However, if the flow is over a dune covered bed then the size of the dune will depend on the flow depth and consequently the values of K_s and the form drag will change depending on the value of the flow depth. Since the sediment transport rate is governed by the effective skin friction shear stress which is the difference between the total shear stress and the shear stress corresponding to the form drag, one can expect an influence of the Z parameter on the bed load transport rate.

At present, the quantitative information on the effect of Z parameter on bed load transport rate is lacking. Therefore, an experimental study was initiated to systematically measure this effect. In order to isolate the effect of Z, it is necessary to conduct experiments in such a way that both the shear Reynolds number,

X, and the Mobility number, Y are held constant while the parameter Z alone is varied. This can be achieved by keeping the shear velocity, v_* constant when the flow depth, h, is varied. In order to maintain v_* constant, the slope of the channel, s, has to be adjusted in such a way that the product sh remains constant. By working with the same sediment (γ_s and D_{50} are constant) and fluid (ρ and μ constant) it is possible to achieve fixed values of X and Y while Z is varying. The description of the experimental set up, the experimental procedure and the results are given in following sections.

EXPERIMENTAL EQUIPMENT

The experiments were conducted in the sediment transport flume at the National water Resarch Institute, Burlington, Ontario, Canada. It is a tilting flume, rectangular in cross section, 2m wide with glass sidewalls 3/4 m high and having an overall length of about 22 m. The flume could be tilted to slopes up to ±1%.

Water was fed from a large constant head tank through a 16 in (406 mm) I.D. pipe, which was terminated by a diffuser in the head box of the flume. In addition, baffles were placed in the head box to improve the entrance condition. Sediment was introduced into the flow from a gravity feed hopper located above the entrance section of the flume. The feed rate could be accurately controlled by using a rotating grooved shaft installed at the bottom of the hopper, which was driven by a variable speed motor. The flume channel floor was recessed 20 cm below the lip of the head box floor to permit placement of a sediment bed. The water level in the flume was controlled by a set of vertical louvres at the downstream end of the flume. The flow leaving the flume was split into three streams of equal width, each of which flowed into a separate sediment trap. The two outside traps were used to collect sediment during the setup period. The centre trap was kept closed by means of a pneumatically controlled gate. Once the right flow conditions were achieved, the centre trap was also opened to begin collection of sediment for weighing. With this procedure sediment transport from the centre 1/3 of the flume, where the flow was close to being two dimensional, was collected. As a result the effects of side walls could be more or less neglected.

The sediment flume is equipped with a self propelled instrument carriage which can travel along the length of the flume on rails fastened to the laboratory floor. This carriage was used to make profile traverses of the mobile bed and water surface, as well as to level the sand bed. To measure the profiles, an adjustable instrument rack was used which was mounted on the upstream face of the carriage. The bed level and water level probes were mounted on this instrument rack. The bed level probe was an electro-optical sensor that follows the sediment bed and is able to pick up changes in bed elevation as small as 1.5 mm. The water level probe measured water surface elevations by maintaining contact with the water surface using the water as a conductor and keeping a constant resistance through a feed back system. Displacement of the carriage along the length of the flume was measured by an aluminum wheel whose circumference was 1/10 of the overall length of the flume. It was attached to the instrument carriage and travelled on the same rails. This wheel ensured precise measurement of profile lengths. Each of the profilers and the measuring wheel provided a signal between 0 and 10 volts. These signals were fed through an A/D converter into an ARISIA minicomputer which is run on a CPM operating system. The profiling information was stored on 8 in (20 cm) double sided, double density discs having a storage capacity of 981 K bytes.

The river wash sand used for the experiments was fairly uniform in size with a median sieve diameter of 1.2 mm (Figure 1). Most of the grains were rounded with specific gravity of about 2.65 and a porosity of 0.45.

EXPERIMENTAL PROCEDURE

Prior to each test, the granular bed was carefully levelled while the flume was set at zero slope. To begin a given test, water was passed very slowly over

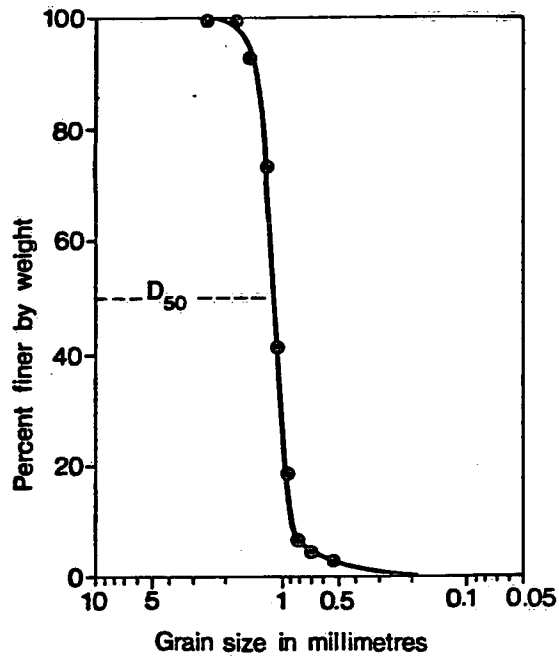

Fig.1 GRAINSIZE DISTRIBUTION OF TEST SAND

the levelled bed until a depth of several cm had been obtained. At this point the flume was set at the desired slope and the point gauge, placed at the beginning of the working section, was set at the desired depth. The instrument carriage was then placed in position so that the bed and water surface profiles were at the same location. The water was allowed to continue rising slowly until the water surface contacted the point gauge. At this instant, the computer was triggered to take a 10 second sample from the two profilers to obtain the reference voltages that are needed for the computation of the average flow depth from the traces of bed and water levels. Once this initializing was completed, a profile of the plane bed was taken, at the same time obtaining reference voltages at the beginning and end of the 11 m long working section for the distance measuring wheel. The initial bed slope was then computed and compared with the pre-set slope of the flume. Normally, the agreement between these slopes was very good and the bed profile was adopted as the reference profile for monitoring the flow conditions during the test.

Once the reference values for the instruments had been obtained, the flow was gradually increased, allowing the depth to rise to almost the full height of the flume walls. The flow was then kept at this depth as the discharge was increased to the pre-determined required value to ensure that there was no bed material movement during this phase. When the desired discharge had been obtained the louvres at the end of the flume were opened to reduce the flow depth slightly below the desired value to allow for subsequent increase due to the dune development. With the onset of sediment transport the sediment feeder was turned on.

As the flow transported sediment, dune patterns began to form. The bed levels and water levels along the centre line of the flume were then monitored at regular intervals of time by traversing the probes along the length of the working section. After each traverse, the slope of the water surface was computed by performing a linear regression on the measured water levels. If the computed slope is different from the desired slope adjustments to the louver-gate are made until the two slopes become equal.

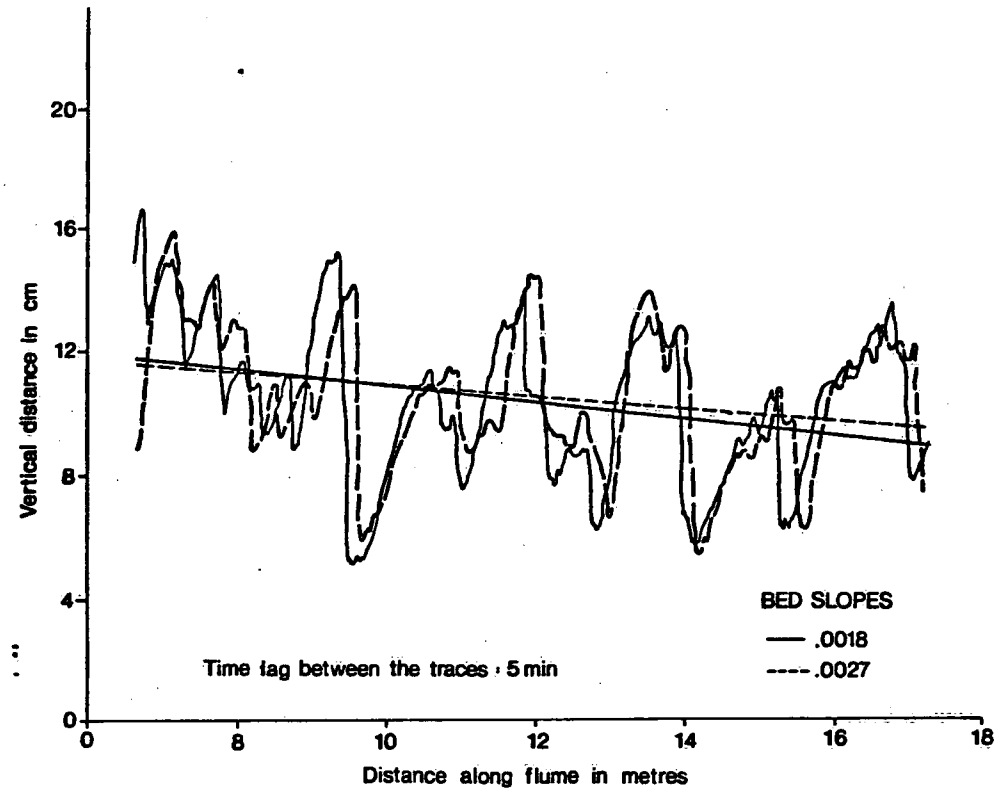

Fig. 2 VARIATION OF THE COMPUTED BED SLOPE DEPENDING ON THE DUNE PROFILE WITHIN THE WORKING SECTION.

As the transport rate is not known appriori, the setting for the feed rate is established by a trial and error procedure. At the beginning of the test, a particular feed rate was set and the bed levels were monitored. Calculation of the bed slope by performing linear regression analysis on the measured bed levels was found to be an unsatisfactory approach as the slope of the bed depended on the positions of the crest and trough of the sand waves at the beginning and at the end of the traverse.

To illustrate this problem, two bed profiles taken at 5 min interval are shown in Fig. 2. The slopes computed using linear regression for these profiles are also given. It can be seen that the computed slopes depend on the position of dune patterns within the working section. The solid trace begins with a crest of a dune while the dotted trace begins with a trough. Notice that the solid trace gives a much higher slope than the dotted one. Since the changes in bed slope can occur only over a longer time duration, the observed difference in slope over a 5 minute duration has to do with the position of the dune patterns and it does not reflect the true change of bed slope.

To overcome this problem, an alternate approach was devised which invovles comparing the bed profile with the original plain bed profile and computing the net volume of sediment that is eroded or deposited within the working section. This net volume was not as sensitive to the location of the crests and troughs with respect to the length of record as the slope and also it gave an indication as to whether the bed is aggrading or degrading. If the net volume is negative then the bed is degrading and the amount of sediment feed is not enough to compensate the transport rate and if the net volume is positive then the reverse is true.

Therefore, by monitoring the bed levels at regular intervals of time and computing the net volume of sediment deposited or eroded from the bed with respect to the original bed level, it was possible to adjust the feed rate to match the transport rate and to obtain an equilibrium flow condition with the desired slopes. When the feed rate is adjusted, it takes a considerably longer time for conditions to stabilize and often it may take one or two days before the equilibrium conditions can be established. Because of the limited capacity of the sand hopper, sometimes the tests had to be terminated before the equilibrium conditions are attained. In this case, the bed was releveled and the test restarted.

When equilibrium condition was achieved, the centre trap is opened for a known duration of time (usually a few hours) and the sediment is collected. At the end of the sand collection period, the sediment trap was closed and a sample of the sediment feed at the hopper collected to verify the feed rate. When this was done, water and sediment feed were shut off and the test considered completed. Photographs of the dune bed were taken, the working section was re-levelled and the net volume of deposition or scour determined by removing or adding material to return the bed to its original plane bed level. The net volume was compared with the computed net volume of scour or deposition. For successful tests there was always good agreement between these values. The sediment in the center trap was removed and its submerged weight determined by standard methods. The transport rate was then compared with the feed rate.

Due to time constraints, only four different values of Z were tested for constant values of X and Y. The hydraulic characteristics of the tests are summarized in Table 1.

TABLE 1. SUMMARY OF EXPERIMENTAL DATA

Test No.	Slope s	Flow Depth h in cm	Flow Rate Q in l/s	Shear Velocity v_* in cm/s	Sediment Trans. Rate q_s in gm/s/cm	Shear Reynolds Number $v_* D_{50}/\nu$	Mobility Number $\rho v_*^2/\gamma_s D_{50}$	Relative Depth h/D_{50}	$(\rho^{1/2} q_s)/(\gamma_s D_{50})^{3/2}$
1	0.0030	9.94	119.3	5.409	0.481	66.53	0.147	80.8	0.146
2	0.0020	15.40	186.6	5.497	0.367	67.61	0.152	125.2	0.128
3	0.0015	19.86	240.3	5.406	0.357	66.49	0.147	161.5	0.125
4	0.0012	24.78	300.8	5.401	0.334	66.43	0.147	201.5	0.117

RESULTS AND DISCUSSION

For the present case X and Y are both held constant and thus equation (1) is reduced to

$$q_* = f_z[Z] \qquad (2)$$

where $q_* = \rho^{1/2} q_s/(\gamma_s D_{50})^{3/2}$ (q_s is the submerged weight of transported material material per unit time and unit width). Data from Table 1 were plotted as q_* vs Z in Figure 3, with X = 67 and Y = 0.15. A smooth average curve was drawn through the four data points, showing that there is a non-linear decrease in q_* as Z increases from about 80 to 200. The rate of decrease becomes less as Z increases indicating that q_* approaches a constant value of about 1.15 when Z > 200.

A possible explanation for the observed effect of Z on the sediment transport rate can be offered as follows: In a study on geometry of dunes, Yalin and Karahan (1981) had observed that the dune steepness Δ/Λ (Δ is the dune-height and Λ is the dune-length) is a function of both the mobility number, Y and the relative depth, Z

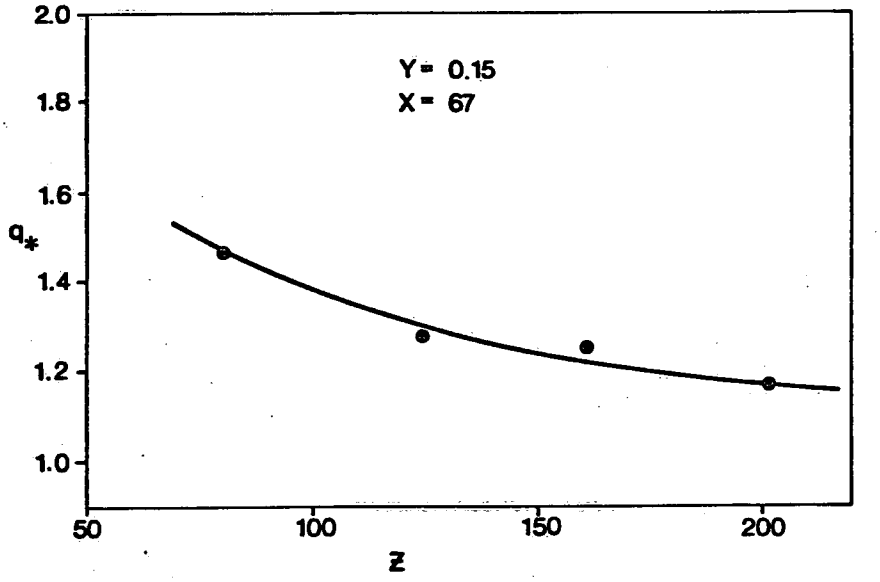

Fig. 3 EFFECT OF RELATIVE DEPTH ON BEDLOAD TRANSPORT RATE

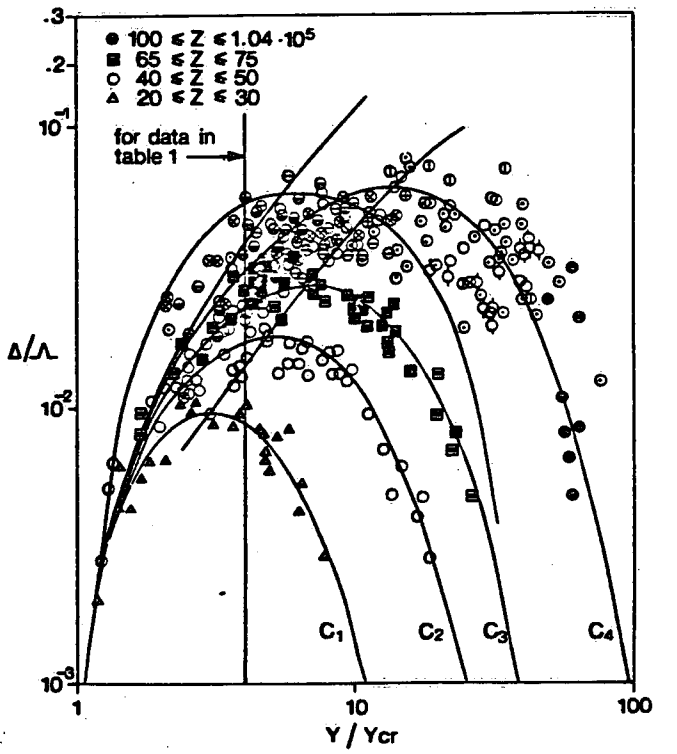

Fig.4 VARIATION OF DUNE STEEPNESS (taken from Yalin et al 1981)

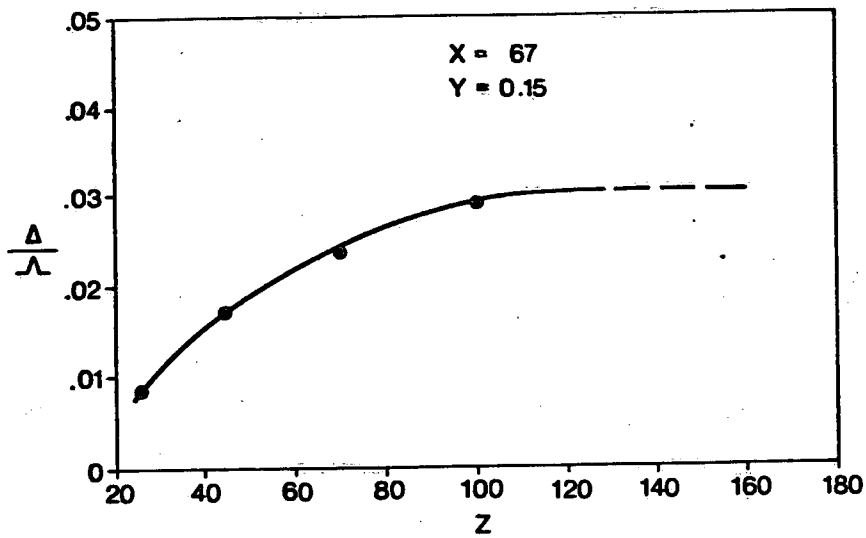

Fig.5 VARIATION OF DUNE STEEPNESS WITH RELATIVE DEPTH
(data from Yalin, Karahan, 1981)

and plotted a family of curves giving the dune steepness in terms of Y/Y_{cr}(Y_{cr} is the critical mobility number for the initiation of sediment transport) and Z. These curves are reproduced in Fig. 4. For the present runs the ratio Y/Y_{cr} takes a constant value of 3.95 and hence the variation of the dune-steepness with respect to Z can be obtained by picking off the intersection points between the vertical line corresponding to Y/Y_{cr} = 3.95 and the curves representing different Z values in Fig. 4. A graph so constructed is shown in Fig. 5. It shows that the dune-steepness increases as Z increases, but the rate of increase becomes less and less and it tends to approach a constant value of about .035 for values of Z > 200.

As the dune-steepness increases, the form drag increases thereby reducing the effective shear stress responsible for bed load transport. Therefore, the reduction of bed load transport rate for increased values of Z in the range between 80 and 200 can be attributed to the increase in the bed form steepness for the same range of Z parameter.

The present results indicate that if the model and prototype values of Z are greater than about 200, then there may not be scale effect even if the Z parameter is not taken into account in designing the model. But, if the prototype Z value is larger than 200, and if the model Z value is less than 200 then, the scale effects will then begin to show. This effect will increase as Z value of the model decreases (i.e., the size of the model decreases). The transport rate measured in the model will be an overestimate of the actual transport rate in the prototype. The percent error in the transport rate obtained for the data in Fig. 3 for different values of Z are given in Table 2. These results clearly show that for small models scale effect errors can be large, exceeding values of 30% for models for which Z < 75 and the mobility number is 0.15.

To compare the present measurements of bed load transport rate with the existing data, the dimensionless transport rate q_* was plotted against the mobility number Y as shown in Fig. 6. In this figure the present measurements are shown as triangles and the existing data are plotted as filled and open circles. It can be seen from this figure that the present data show a general agreement with the existing data. The variation with respect to Z that is apparent in Fig. 3 shows up as experiment scatter in this figure. The range of variations is also suppressed because of the logarithmic scale used in this figure.

TABLE 2. % ERRORS DUE TO SCALE EFFECTS FOR Y = 0.15, X = 67

$Z = h/D_{50}$	$q_* = \frac{\rho^{1/2} q_s}{(\gamma_s D_{50})^{3/2}}$	% Error
75	1.495	30.0
100	1.378	19.8
125	1.297	12.8
150	1.240	7.8
175	1.200	4.4
200	1.170	1.7
>200	1.150	0

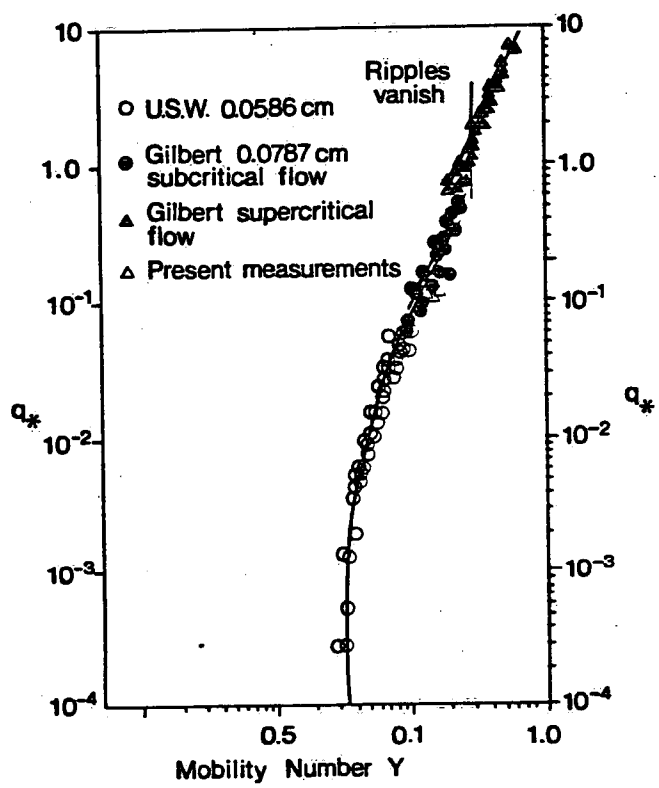

Fig.6 Comparison between present measurements and existing data.

The present measurements were also compared with the existing bedload transport rate equations and the comparison is shown in Fig. 7. Five equations were selected. These are:

1) Bagnold's bed load equation (1956)
2) Einstein's bed load function (1942)
3) Engulund & Hansen (1967)
4) Ackers and White (1973)
5) Van Rijn (1984).

The details of these equations can be found in the original references. From Fig. 7, it can be seen that all of the equations underpredict the transport rate. Predictions of Ackers and White appears to be the closest to the measurement.

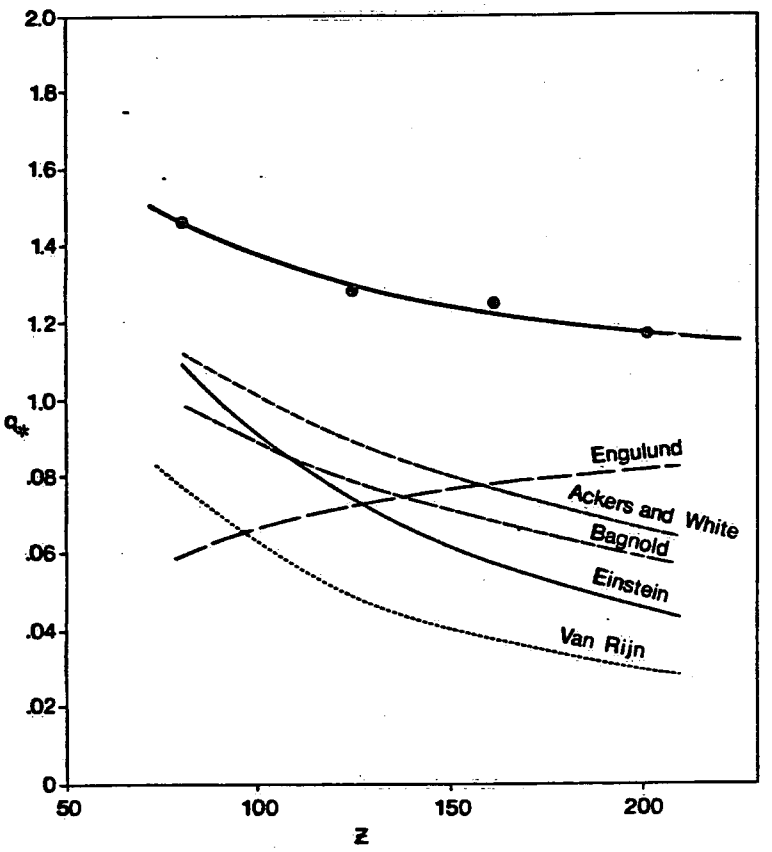

Fig.7 COMPARISON OF MEASURED BED LOAD TRANSPORT RATES WITH THOSE PREDICTED BY THE EXISTING EQUATIONS.

None of the above equations contains the relative depth parameter Z explicitly. The variation with respect to Z predicted by these equations is due to the use of the effective shear stress which is evaluated independently of the equations. Therefore, the predicted variation does not reflect the equations' ability, but it is a measure of the ability of the method used to evaluate the effective shear stress. The equation of Engulund and Hansen predicts a reverse trend with respect to Z. This is due to the fact that the Engulund and Hansen's equation uses the inverse of the total friction factor which shows a decreasing trend with Z, even though the form drag component increases with Z.

SUMMARY AND CONCLUSION

The effect of the relative depth parameter, Z, on the bed load transport rate of mobile bed channel flows was investigated experimentally. Experimental results show that the effect is noticeable when Z is in the range of 80 to 200. It diminishes for increasing values of Z. The implication of this result for the evaluation of the scale effects in physical models of mobile boundary flows is discussed. From this study one can conclude that for the flows with Y/Y_{cr} of about 4.0, and the shear Reynolds number of about 67, the error due to the omission of Z from scaling considerations will be negligible only if the Z values of both model and prototype are greater than 200. If the prototype value is greater than 20 and if the model value is less than 200, then the scale effects will begin to show and will increase for decreasing value of Z of the model. An error as high as 30% can be expected if the model value of Z goes below 75. The error is in the positive side, i.e., the model will overestimate the sediment transport rate. Corrections can be applied to the model results if the variation of the sediment transport rate with respect to Z can be completely determined.

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LIST OF SYMBOLS

D_{50}	median size of sediment particles
g	acceleration due to gravity
h	flow depth
K_s	equivalent sand grain roughness height
Q	flow rate
q_s	bed load transport rate per unit width
q_*	dimensionless transport rate = $q_s \rho^{1/2} (\nu_s D_{50})^{3/2}$
s	slope of uniform flow
v_*	shear velocity
X	shear Reynolds number = $(v_* D_{50}/\nu$
Y	mobility number = $(\rho v_*^2/\nu_s D_{50})$
Y_{cr}	critical mobility number for initiation of sediment motion
Z	relative depth parameter = h/D_{50}
Δ	dune height
Λ	dune length
γ_s	submerged specific weight of sediment
ν	kinematic viscosity of fluid
ρ	density of fluid