HYDRAULICS RESEARCH DIVISION

Technical Note

DATE: 29 November 1977 REPORT NO: 77-15

TITLE: "Wave Information from a Floating Buoy"

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REASON FOR REPORT:

To demonstrate how a suitably instrumented floating buoy could be made to yield some useful average parameters of a wind-wave field. In particular, peak period, significant height and mean wave direction are considered. A new idea for a simple averageable magnetic compass is outlined.

CORRESPONDENCE FILE NO:

2520

1. INTRODUCTION

For many studies the wave information required is restricted to the following or some of the following:

T	=	peak period	=	$1/f$	=	$2\pi/\omega$
σ	=	rms wave height	=	(significant height, H_c)/4		
θ	=	average wave direction (or instead direction of the peak)				

Donelan (1976) demonstrated how T and σ could be measured in situ over several wave periods (10 mins $\rightarrow\infty$) and transmitted as required. T can be estimated from σ and $\dot{\sigma}$ (the rms surface vertical velocity); therefore measuring σ and $\dot{\sigma}$ yields T and σ. However, the spectral "equilibrium" parameter α is a weak function of the wave age $gT/2\pi\bar{U}$ ($\bar{U}$ is the wind speed), so that a more accurate estimate of T can be achieved if $\bar{U}$ is known (3.2% if $\bar{U}$ is known, approximately 10% if not). This arises because the relationship between T and σ and $\dot{\sigma}$ depends on the integral under the spectrum, which in turn depends on α (see Donelan, 1976 for further details).

2. DIRECTION OF A MONOCHROMATIC WAVE TRAIN θ

Let us address the question of the determination of θ. Consider a single sinusoid of amplitude a.

$$\eta(t) = a \cos \omega t \qquad 2.1$$

$$w(t) = \dot{\eta}(t) = -a\omega \sin \omega t \quad \uparrow^{+} \text{(vertical velocity)} \qquad 2.2$$

$$u(t) = a\omega \cos \omega t \quad \longrightarrow^{+} \underset{\sim}{c} \text{ (horizontal velocity)} \qquad 2.3$$

$$\underset{\sim}{c} = \text{phase velocity} \qquad 2.4$$

As indicated above u(t) is taken to be positive in the direction of $\underset{\sim}{c}$.

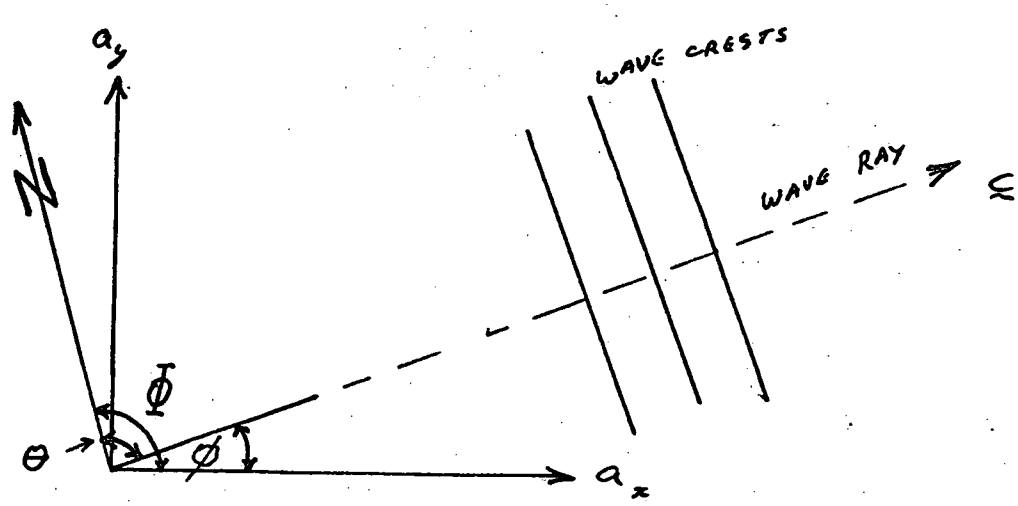
$$a_z(t) = \dot{w}(t) = -a\omega^2 \cos \omega t \quad \uparrow^{+} \text{ vertical acceleration} \qquad 2.5$$

$$a_H(t) = \dot{u}(t) = -a\omega^2 \sin \omega t \quad \rightarrow^{+} \underset{\sim}{c} \text{ horizontal acceleration} \qquad 2.6$$

Suppose we know the three components of acceleration of a particle on the surface.

$$\vec{k}\, a_z + \vec{j}\, a_y + \vec{i}\, a_x = \vec{k}\, a_z + \vec{h}\, a_H = \vec{a} \qquad 2.7$$

and let the angle of approach of the wave ray to a_x be ϕ.



$$a_x(t) = A \sin \omega t \cos \phi \qquad 2.8$$

$$a_y(t) = A \sin \omega t \sin \phi \qquad 2.9$$

$$a\ (t) = A \cos \omega t \qquad 2.10$$

$$w(t) = \frac{A}{\omega} \sin \omega t \qquad 2.11$$

$$\eta\ (t) = \frac{-A}{\omega^2} \cos \omega t \qquad 2.12$$

where $A = -a\omega^2$.

We note that

$$\overline{a_x(t).w(t)} = \frac{A^2}{2\omega} \cos \phi \qquad 2.13$$

$$\overline{a_y(t).w(t)} = \frac{A^2}{2\omega} \sin \phi \qquad 2.14$$

$$\overline{(w(t))^2} = \frac{A^2}{2\omega^2} \qquad 2.15$$

$$\overline{(\eta(t))^2} = \frac{A^2}{2\omega^4} \qquad 2.16$$

$$\overline{(a_{z(t)})^2} = \frac{A^2}{2} \qquad 2.17$$

where ——— denotes a time average over several cycles.

Clearly if we compute the parameters 2.13, 2.14 and any one of 2.15, 2.16 and 2.17 in situ on the floating particle (accelerometer buoy) and transmit these, we can readily solve for ϕ, ω, and a.

The relationship between 2.15 and 2.16 and the peak period, for typical wave spectra, is discussed at length in Donelan (1976) and will not be repeated here. It should be clear that 2.13 and 2.14 will together yield the value of ϕ for a spectrum of waves provided that all frequency components have the same average direction. For another method of computation of ϕ see Appendix A.

Evidently the minimum in situ computation is required if 2.13, 2.14 and 2.15 or 2.17 are used. Only one integration $\int a_{z(t)} dt = w(t)$ is needed.

This yields ϕ, T and a. To obtain Θ, we need the mean orientation of a_x .r.t.compass North, Φ. This is, in effect, the orientation of the buoy.

3. DETERMINATION OF MEAN BUOY ORIENTATION, Φ

Several methods of establishing the mean compass direction Φ are available. Most of these methods consist of a measurement immediately followed by a recording of it. This is not practicable for our case because recordings (transmissions) are to be made at relatively long intervals compared to the possible rotational rate of a small spherical buoy. The following ideas are offered to reduce the error in Φ.

(a) Add a small fixed vane to one side of the buoy so that it is steadied by the wind.

(b) Measure the instantaneous Φ' frequently (once per second perhaps). At the end of the interval, transmit two bits of information: $\overline{\Phi}$ and Φ'_m.

$\overline{\Phi}$ = average over entire interval

Φ'_m = a single sample preferably the middle sample.

Φ'_m is simply a "dead-band" indicator, so that if the averaging has included the dead band, one can make allowances for it.

(c) A better method would be to use a compass with two dead bands 180° apart and transmit both average values, $\overline{\Phi}_1$, $\overline{\Phi}_2$. The choice of which is the correct direction would depend on the past history of the directions $\overline{\Phi}_1$, $\overline{\Phi}_2$. A suggested compass design is outlined in the next section.

(d) The safest method would be to rotate the measured horizontal accelerations into north (a_N) and east (a_E) components and apply these directly in 2.13 and 2.14 yielding θ instead of ϕ.

$$a_N = a_x \cos \Phi + a_z \sin \Phi$$

$$a_E = a_x \sin \Phi - a_y \cos \Phi$$

This could be done either by (analog or digital) computation of $\cos \Phi$ and $\sin \Phi$ from Φ or by designing the compass to yield outputs proportional to $\cos \Phi$ and $\sin \Phi$.

4. A LOW POWERED UNAMBIGUOUS ANALOG COMPASS

As indicated above, there is a need for a magnetic compass having the following characteristics:

(a) Low power requirements.

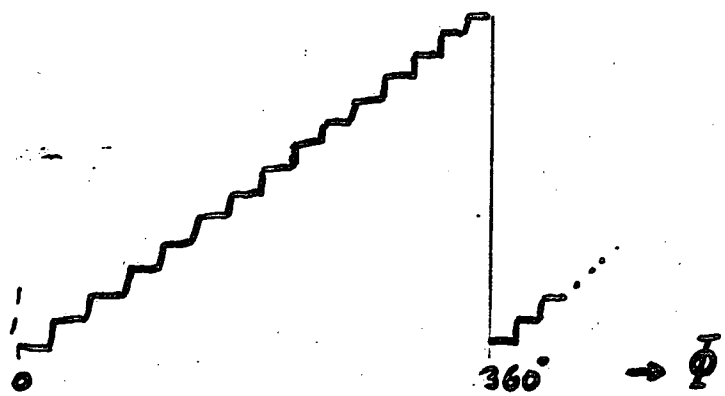
(b) Electrical analog or digital read-out for easy integration.

(c) No circular ("dead-band") discontinuity.

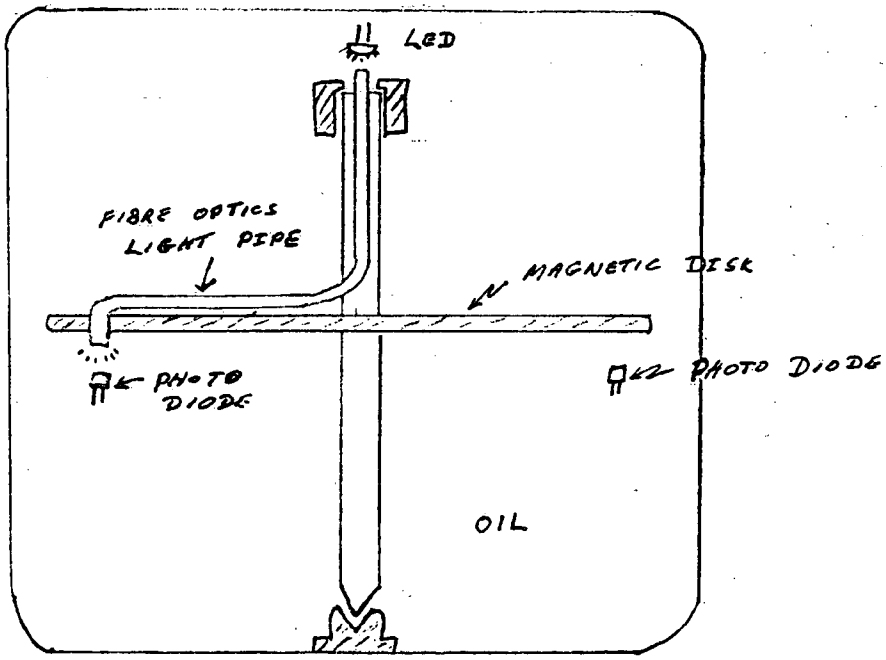
The following design has these characteristics. Consider a North seeking disk, which is mounted on bearings and which holds a light source (LED). The light shines on a ring of photo diodes which are connected so as to make a stepped potentiometer out of a network of resistances. (See Figure 2 on following page).

The light source could be an LED (light emitting diode) on the magnetic disk, but this entails some sort of slip-ring arrangement to get power to the rotating light. A better method (F. M. Boyce, personal communication over coffee) would be to use fibre optics as shown to conduct the light from a source external to the rotating disk or to use annulli of mercury into which the metallic contacts trail to serve as nearly frictionless slip-rings.

The transfer function of such a device would have the following stepped ramp form. It is, of course, necessary to ensure that at least one diode is illuminated at all times.



This may be integrated by analog methods or A to D converted and summed. In either case cognizance needs to be taken of the fact that there is a discontinuity in the ramp at some spot ("dead band"). The method of producing an unambiguous reading of direction differs between the analog and the digital.



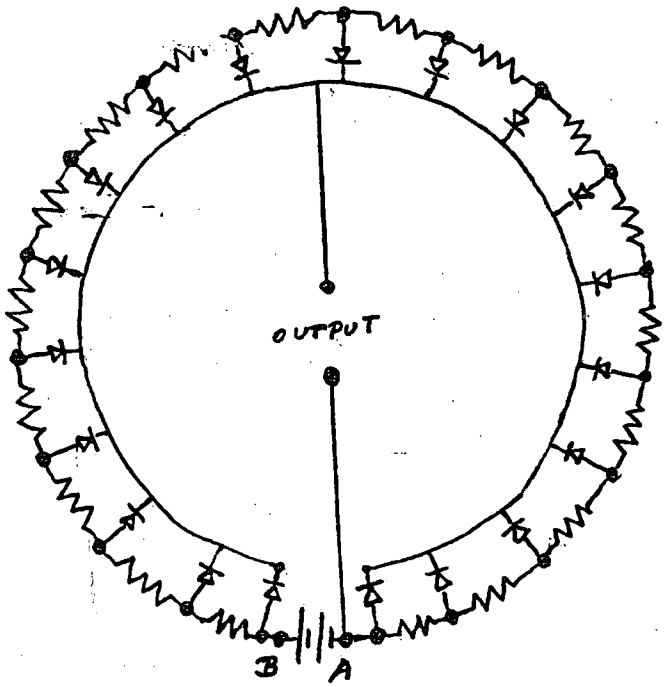


FIG. 2. LOW POWERED COMPASS.

(a) Analog: The photo-diodes are replaced by dual photo-diodes (or two strings of single photo-diodes). One set of diodes is hooked up to a resistance network as shown before. The other set is hooked up to a resistive network with the "dead band" displaced 180°. This produces two indicators of direction which are separately integrated and averaged.

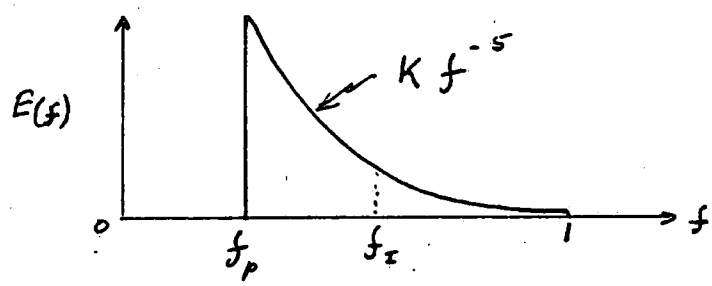
(b) Digital: Only one string of diodes and resistors is required. An A to D convertor* having N binary bits is applied to the output of the string. This yields numbers from 0 to 2^N-1. These can be summed to yield average direction. The output of the convertor is applied to a binary adder having N+1 bits. The other input to the adder is the binary number equal to 2^{N-1}. Only the lowest N bits of the adder are accumulated and averaged. This has the effect of shifting the "dead band" to 2^{N-1} i.e. 180° away from 2^N.

In both cases the results of averaging - one reading having been shifted by 180° - yields two directions which will imply the same compass direction except when one of them was obtained from instantaneous readings which were traversing the dead band. The choice of which direction suffers from this fault will usually be obvious from readings before and after; i.e. the faulty indication of direction will have a much higher variability than the other.

* An alternate method would be to use the switching of the photodiodes to provide digital inputs to the adder directly, thereby avoiding the use of an A to D convertor.

5. EFFECT OF SKEWNESS IN THE DIRECTIONAL SPECTRUM ON THE ESTIMATION OF θ

Consider the simple spectrum represented by a "-5" equilibrium region and a sharp low frequency cut-off.



$$\left.\begin{aligned} E(f) &= 0 && f < f_p \\ E(f) &= Kf^{-5} && f \geq f_p \end{aligned}\right\} \qquad 5.1$$

Further assume that the direction of travel of the waves θ(f) is given by

$$\left.\begin{aligned} \theta(f) &= \theta_L && f < f_I \\ \theta(f) &= \theta_H && f \geq f_I \end{aligned}\right\} \qquad 5.2$$

From 5.1 we note that the acceleration spectrum A(f) and velocity spectrum V(f) are given by:

$$\mathcal{A}(f) = \left.\begin{aligned} &0 && f < f_p \\ &Kf^{-1} && f \geq f_p \end{aligned}\right\} \qquad 5.3$$

$$\mathcal{V}(f) = \left.\begin{aligned} &0 && f < f_p \\ &Kf^{-3} && f \geq f_p \end{aligned}\right\} \qquad 5.4$$

by definition:

$$\int E(f)\, d_f = \sigma_\eta^2 \qquad 5.5$$

where σ_η is the standard deviation of the surface elevation.

Similarly

$$\int V(f)\, d_f = \sigma_w^2 = \sigma_{u_H}^2 \qquad 5.6$$

and

$$\int \hat{A}(f)\, d_f = \sigma_{a_H}^2 = \sigma_{a_z}^2 \qquad 5.7$$

The equivalence of vertical and horizontal velocity and acceleration components is, of course, only exact for small amplitude (slope) deep water waves. In fact, it is a reasonable approximation for waves of average steepness. Clearly:

$$\overline{a(t) \cdot w(t)} = \sigma_a \sigma_w \qquad 5.8$$

integrating 5.3

$$\sigma_a^2 = K \ln \frac{f_T}{f_B} \qquad 5.9$$

where f_T = top frequency of the range, which is limited by the floating response of the buoy to about 1 H_z.

f_B = bottom frequency of the range - limited to f_p.

integrating 5.4

$$\sigma_w^2 = +\frac{K}{2} \left[\frac{1}{f_B^2} - \frac{1}{f_T^2}\right] \qquad 5.10$$

Taking $\phi_L = 0^o$ and $\phi_H = 90^o$, 2.13 and 2.14 are evaluated for ϕ using various values of f_p. f_T is set equal to the arithmetic mean of f_p and 1 H_z. The results are shown in Table 1.

For comparison the average direction of momentum flux of the wave field is also shown. This direction or the direction of travel of the peak ϕ_L are probably the two most useful directions for most studies of waves or their effects on coasts and coastal structures.

Table I Various Estimates of Wave Direction with a "Split Directional Spectrum"

f_p	f_I	Computed ϕ^o		Diff.	Computed ϕ^o	Diff.
H_z	H_z	5.11 (1)	5.9, 5.10 (2)	(2)-(1)	5.13, 5.14 (3)	(3)-(1)
0.05	0.525	0.04	2.40	2.4	0.04	0.00
0.10	0.55	0.29	5.22	4.9	0.28	-0.01
0.15	0.575	0.84	8.08	7.2	0.82	-0.02
0.20	0.60	1.73	10.92	9.2	1.69	-0.04
0.25	0.625	2.96	13.71	10.8	2.91	-0.05
0.30	0.65	4.52	16.44	11.9	4.47	-0.05
0.35	0.675	6.40	19.09	12.7	6.34	-0.06
0.40	0.70	8.57	21.65	13.1	8.52	-0.05
0.45	0.725	11.01	24.12	13.1	10.96	-0.05
0.50	0.75	13.70	26.5	12.8	13.64	-0.06

The average direction of wave momentum is given by:

$$\tan^{-1}\phi = \frac{\left[\frac{1}{f_I^3} - \frac{1}{f_T^3}\right]}{\left[\frac{1}{f_B^3} - \frac{1}{f_I^3}\right]} \qquad 5.11$$

It is clear that the directional estimate obtained using 5.9 and 5.10 (i.e. an automatic buoy) is close to the average direction of the momentum even in this hypothetical case of a split spectrum. The difference in direction varies from 2.4^o for a very low frequency peak to 13.1^o for a peak frequency of 0.5 H_z.

Generally, all the wave components will have much the same average direction and the automatic buoy will yield an average direction which is much closer to the truth.

Some improvement in the average direction estimate may be

achieved by integrating the horizontal acceleration components so that instead of 5.8 we have

$$\overline{u_H(t) \cdot \eta(t)} = \sigma_{u_H} \sigma_\eta \qquad 5.12$$

integrating 5.4:

$$\sigma^2_{u_H} = \frac{K}{2}\left[\frac{1}{f_B^2} - \frac{1}{f_T^2}\right] \qquad 5.13$$

integrating 5.1:

$$\sigma^2_\eta = \frac{K}{4}\left[\frac{1}{f_B^4} - \frac{1}{f_T^4}\right] \qquad 5.14$$

The results are shown in Table 1. The improvement is dramatic because the desired behaviour of the frequency weighting goes as f^{-3} (c.f. 5.11) and this is closely approximated by the square root of the product of 5.13 and 5.14.

Clearly then, by integrating the horizontal acceleration components once and the vertical twice, an excellent estimate of the mean direction of wave momentum is obtained. Unfortunately, integration of the horizontal accelerations is subject to more serious "tilt" induced errors than the vertical acceleration since the spurious "g signal" due to a tilt ε appears as $g \sin \varepsilon$ in the horizontal acceleration but only as $g(1-\cos \varepsilon)$ in the vertical.

However, suitable band-pass filtering together with appropriate accelerometer damping should reduce this problem to a manageable level. A further factor in our favour is the fact that we require only the mean product of horizontal acceleration (or velocity) and the vertical velocity (or displacement). Therefore, any tilts which do not have frequency counterparts in the corresponding vertical velocity (or displacement) are averaged to zero in the long run.

6. PROPOSAL FOR WAVE BUOY

It is proposed, therefore, that a surface buoy be designed with the following characteristics:

(1) Spherical with its centre at the mean water level under still conditions. (See Appendix B.)

(2) Contains three accelerometers orthogonally placed so that one is vertical and the other two referenced to fixed axes in the buoy.

(3) Contains a magnetic north seeking compass capable of averaging direction such as the one described in section 4. The compass is referenced to the same axes as the horizontal accelerometers.

(4) All three accelerometers and the compass (if necessary and possible) mounted on a damped gimballed support to maintain the vertical accelerometer as vertical as possible and the other accelerometers and compass horizontal.

(5) Computational circuitry (including: band-pass filters, integrators, multipliers, averagers) to obtain the following instantaneous and average values:

(a) The instantaneous vertical buoy displacement.

(b) The root-mean-square of the vertical buoy displacement.

(c) The root-mean square of the vertical buoy velocity.

(d) (i) The average product of each component of horizontal buoy velocity with the vertical buoy displacement.

or

(ii) The average product of each component of horizontal buoy acceleration with the vertical buoy velocity.

(e) The average direction of the buoy relative to magnetic north and the average direction of the buoy relative to magnetic north $+180^{\circ}$.

From intermittent transmission or recordings of the six averages outlined in (b), (c), (d) and (e) one can readily and easily determine the peak period, significant height and average direction of a uni-modal spectrum of surface gravity waves. However, many users of remote wave measuring devices require knowledge of the complete spectrum (Personal communication, Dr. J. R. Wilson, Marine Data Centre, Ottawa), and this may be obtained from (a). On enclosed water bodies the spectrum is usually uni-modal and the average information (b) to (e) may be adequate.

A buoy having this kind of internal computational ability admits of

an entirely different approach to the handling of remote wave data. The following possibilities seem to me to be the most likely to find application:

(i) Transmit continuous information on the surface displacement. Considerable power would be consumed in transmission, thus transmission distances would necessarily be short.

(ii) Transmit or record the six averages ((b) to (e) above) at the end of the averaging period. Typically a second or so of transmission whenever wave information is required - usually hourly or three-hourly . This would allow the use of a relatively powerful transmitter or compact recorder and, in the case of transmission, a two-way system could be used to allow interrogation and repeated transmission with resetting only after shore acknowledged receipt.

(iii) A combination of (i) and (ii) with continuous transmission either on a preset cycle or, preferably, by request from shore. Since the average information is available, the continuous record need not be very long and could be scaled after-the-fact by the average values. Typically, averages over 30 minutes with continuous records for ten minutes should be adequate in most circumstances.

It is worth noting that the additional computation required to determine the rms values of all acceleration, velocity, displacement and compass direction signals treated would be well worth the diagnostic value of these averages.

It is not difficult to show that the method of estimating wave direction works in water of any depth, since the estimate of ϕ does not depend on the ratio of horizontal to vertical acceleration - only that there be some of each.

ACKNOWLEDGEMENTS

These thoughts, which stemmed initially from the author's past and current attempts to be able to provide other CCIW studies with average wave information, were crystallized in conversation on 23 November 1977 wth Mr. Bill McGregor of McGregor and Associates, Carlton Place, Ontario. I thank T. M. Dick, M. Skafel, J. Hamilton and F. M. Boyce for their helpful comments. The ideas of the latter two are incorporated in Appendices A and B respectively.

REFERENCES

Donelan, M., 1976. "A Method for the Automatic Measurement of Wave Frequency". J. Fish. Res. Board Can., V 33, pp 2318-2322.

APPENDIX A

An Alternate Method of Computing ϕ

Using the same notation as in section 2.

$$\overline{4(a_x(t).a_y(t))} = A^2 \sin 2\phi \qquad \text{A.1}$$

$$\overline{2\left(a_x^2(t) - a_y^2(t)\right)} = A^2 \cos 2\phi \qquad \text{A.2}$$

This yields ϕ with an ambiguity of π, which however may be resolved by determining the sign of $\overline{\left(a_x(t) + a_y(t)\right) . w(t)}$

It is interesting to note that the two methods (section 2 and this one) yield different sensitivities to the frequencies of rotation of the buoy. To see this we write the instantaneous products corresponding to 2.13, 2.14:

$$\left.\begin{aligned} a_x(t).w_{(t)} &= \frac{A^2}{2\omega} \cos\phi\,(1 - \cos 2\omega t) \\ a_y(t).w_{(t)} &= \frac{A^2}{2\omega} \sin\phi\,(1 - \cos 2\omega t) \end{aligned}\right\} \qquad \text{A.3}$$

and to A.1 and A.2

$$\left.\begin{aligned} a_x(t).a_y(t) &= \frac{A^2}{4} \sin 2\phi\,(1 - \cos 2\omega t) \\ a_x^2(t) - a_y^2(t) &= \frac{A^2}{2} \cos 2\phi\,(1 - \cos 2\omega t) \end{aligned}\right\} \qquad \text{A.4}$$

Clearly, errors in the time averages arise if ϕ has frequency components near 2ω in the first case (A.3) and near ω in the second (A.4).

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APPENDIX B

A Cylindrical Buoy

A spherical buoy, having no preferred horizontal orientation, may be very susceptible to rotation caused, for instance, by variations in torsion in the mooring line as it strained and relaxed. Perhaps buoy geometrics with large rotational inertia around a vertical axis would avoid this problem. Specifically, a horizontal cylinder moored by means of "Y" shaped yoke attached, through bearings, to the ends of the cylinder's axis would tend to orient itself normal to the surface current. If the current is entirely due to the waves and if the buoy has sufficient rotational inertia around the vertical, the buoy will lie normal to the waves and, in fact, its orientation could be used to yield the wave direction with a 180° ambiguity.

However, in the presence of surface currents not lined up with the wave direction, the buoy would be forced to rotate to some degree. The question of buoy geometry can best be resolved by laboratory tests or careful analytical work outside the scope of this work. It is, nonetheless, apparent that if a sphere is used the mass should be concentrated away from its centre.