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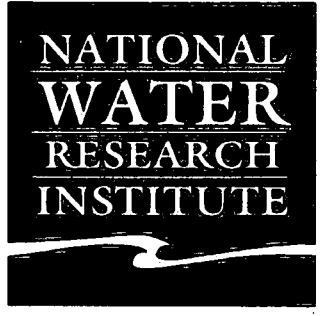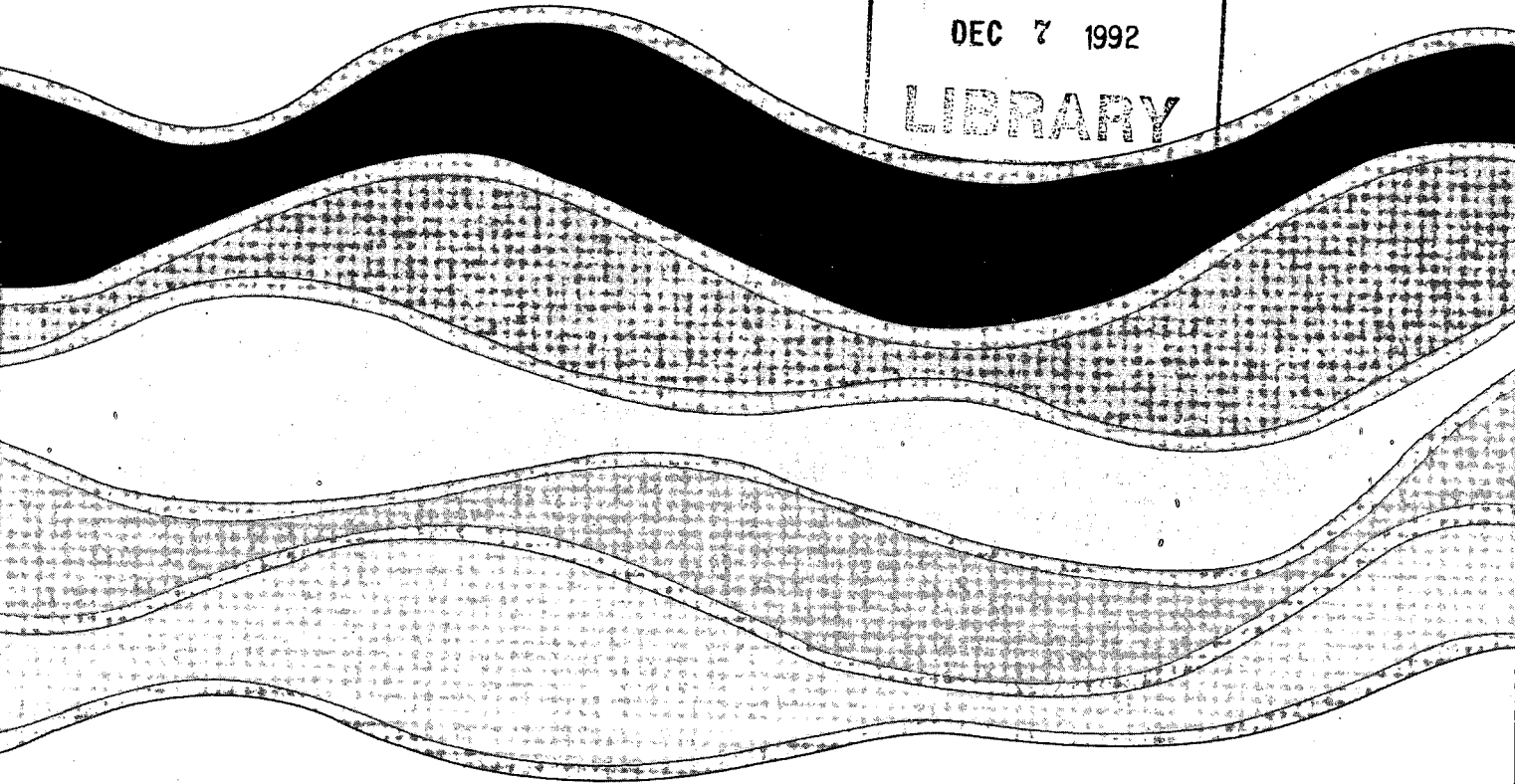EAUX

**UNCERTAINTY IN THE CALIBRATION
EQUATION FOR ROD SUSPENDED
PRICE METERS**

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UNCERTAINTY IN THE CALIBRATION EQUATION FOR ROD SUSPENDED PRICE METERS

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MANAGEMENT PERSPECTIVE

Increased awareness of river pollution and the importance of water quality monitoring has made it necessary to improve the accuracy of discharge measurements. One of the factors contributing to the error in a flow velocity measurement is the uncertainty in the current meter calibration itself. This uncertainty must be determined experimentally. In this report, repeated calibrations of five Price winter current meters, obtained in the towing tank of the Hydraulics Laboratory (HL) at the National Water Research Institute (NWRI), are examined to determine the uncertainty in a new form of calibration equation at the 95% confidence level. The results provide important information for the development of data quality control standards and development of an updated calibration strategy by the Surveys and Information Systems Branch (SISB) for measurement of flow in rivers with solid ice cover.

SOMMAIRE À L'INTENTION DE LA DIRECTION

En raison d'une sensibilisation accrue à la pollution des cours d'eau et à l'importance de la surveillance de la qualité de l'eau, il a été nécessaire d'améliorer la précision des mesures du débit. L'un des facteurs responsables de l'erreur au niveau de la mesure de la vitesse du débit est l'incertitude au niveau de l'étalonnage du courantomètre lui-même. Cette incertitude doit être établie expérimentalement. Dans le cadre du présent rapport, des étalonnages répétés de cinq courantomètres d'hiver de marque Price, effectués dans le canal à chariot mobile du laboratoire d'hydraulique de l'Institut national de recherche sur les eaux, sont étudiés afin de déterminer l'incertitude sous une nouvelle forme d'équation d'étalonnage à un seuil de confiance de 95%. Les résultats fournissent des informations importantes pour l'établissement de normes pour le contrôle de la qualité des données et l'élaboration par la Direction des relevés et des systèmes d'information d'une stratégie d'étalonnage modifiée pour mesurer le débit dans les rivières entièrement recouvertes de glace.

ABSTRACT

Five Price winter current meters were calibrated separately, each ten times, for a total of fifty calibrations. A new form of calibration equation fitted to the data by least squares methods gave excellent results. Analysis showed that, even at velocities of 10 cm/s, the uncertainty due to the calibration equation was 2% or less at the 95% confidence level for the five meters tested.

RÉSUMÉ

Cinq courantomètres d'hiver de marque Price ont été étalonnés séparément, dix fois chacun, pour un total de cinquante étalonnages. Une nouvelle forme d'équation d'étalonnage ajustée aux données par la méthode des moindres carrés a donné d'excellents résultats. L'analyse a montré que, même pour des vitesses de 10 cm/s, l'incertitude due à l'équation d'étalonnage était de 2% ou moins à un seuil de confiance de 95% pour les cinq appareils testés.

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1. INTRODUCTION

Increased awareness of river pollution and the importance of water quality monitoring has made it necessary to improve the accuracy of discharge measurements. The determination of river discharge requires the measurement of the flow velocity. The velocity is measured by placing a meter into the flow and recording the rate of rotation of the rotor, usually in revolutions per second. The relationship between the linear velocity of the flow and the revolutions per second is determined by calibrating the meter in a towing tank. The current meter calibrations are normally expressed by some form of equation from which calibration tables are prepared for use in the field. One of the factors contributing to the error in a flow velocity measurement is the uncertainty in the current meter calibration itself (Smoot and Carter 1968). This uncertainty in the calibration is due to two reasons. Firstly, there is the uncertainty in the calibration data and secondly, there is the uncertainty due to the fit of the calibration equation to the calibration data.

In this report, repeated calibrations of five rod suspended Price current meters, obtained in the towing tank of the Hydraulics Laboratory (HL) at the National Water Research Institute (NWRI), are examined to determine the uncertainty in the calibration equation developed by Engel (1989) at the 95% confidence level. The work was done by the Research and Applications Branch (RAB), NWRI, for the Operational Technology Section (OTS), the Monitoring and Surveys Division (MSD), Ottawa, in accordance with the R&D plan of the Committee for the Measurement of Flow Under Ice (MFUI).

2.   PRELIMINARY CONSIDERATIONS

2.1   Calibration Equation

In developing a new calibration equation for rod suspended Price meters, it was shown by Engel (1989), that for a frictionless current meter, the dimensionless rotor response could be expressed as

$$\frac{ND}{V} = \frac{1}{\pi}\left[\frac{K-1}{K+1}\right]$$ (1)

where N = the rate of rotation of the rotor, D = the effective diameter of the rotor, V = the average flow velocity or towing speed, $K = \frac{C_{D1}}{C_{D2}}$, C_{D1} = the drag coefficient of the conical elements on the stoss-side and C_{D2} = the drag coefficient of the conical elements on the lee-side. The value of K must be determined experimentally.

Equation (1) reflects the typical response characteristics of the Price current meter in a two dimensional flow field if there is no frictional resistance in the bearings and other contact surfaces. ND/V is dependent only on the value of K which reflects primarily the shape and orientation of the conical elements of the rotor. The sensitivity of the meter is dependent on both D and K. The sensitivity can be increased by reducing D and increasing K because the rate of rotation of the rotor will be increased for a given value of the flow velocity. For a given meter the value of K and D are constant and a practical calibration equation is normally expressed in a form of V as a function of N. Therefore, equation (1) may be rearranged to give

$$V = \frac{D}{\pi}\left[\frac{K+1}{K-1}\right]N = AN$$ (2)

where A = the meter constant. Equation (2) is linear, with slope A and passes through the origin of a $V vs. N$ plot. Such a behaviour would be ideal for a current meter. It is known, however, that calibration curves are nonlinear, particularly in the region of lower velocities. This effect can best be illustrated with the plot of $ND/V vs. V$ in

Figure 1. The average curve fitted to the data shows that the meter response is very nonlinear for velocities less than about 30 cm/s. For velocities greater than 30 cm/s the values of ND/V are approximately constant, indicating that the rotor response in this range tends to be linear. The non-linearity of the rotor response manifests itself in the standard $Vvs.N$ format of the calibration plot by its departure from the curve for the frictionless meter as shown schematically in Figure 2.

The nonlinearity in the calibration equation is the result of frictional resistance due to the bearings and electrical contact brushes in the meter head, density of the fluid as well as possible effects of the meter yoke on the local flow field. The nonlinearity is not observable in a standard $Vvs.N$ plot because of the scale that is normally adopted. However, the magnitude of the nonlinearity increases as the density of the fluid decreases. This was demonstrated by Engel (1976) who calibrated Price type current meters in both water and air, for which data are plotted in Figure 3. Curves fitted to the data show virtually no discernible nonlinearity when the fluid is water, whereas in the case of air, the nonlinearity is very pronounced. It is also interesting to note in Figure 3 that both curves merge into a single curve indicating that the meter behaves similarly in all Newtonian fluids in the range where the factors contributing to the nonlinearity become insignificantly small. A single, continuous calibration equation which combines the linear and the frictional components of the rotor response, was developed by Engel (1989) and is given as

$$V = AN + Be^{-kN} \qquad (3)$$

where A, B and k are coefficients to be determined by calibration in a towing tank. The coefficient A accounts for the hydro-dynamic characteristics of the rotor as shown in equation (2), B accounts for the static friction of the rotor assembly and k accounts for the rate of decline from static friction to dynamic friction in the rotor assembly. Five rod suspended Price meters were calibrated individually ten times to determine the uncertainty in the fit to the test data obtained with equation (3). This information will

provide necessary input for the development of an updated current meter calibration strategy.

3.2   Underlying Equation

It can been shown that for equation (3) the error in the computed velocity may be expressed as

$$\delta V = \left\{ \left[\frac{\partial V}{\partial A} \delta A \right]^2 + \left[\frac{\partial V}{\partial N} \delta N \right]^2 + \left[\frac{\partial V}{\partial B} \delta B \right]^2 + \left[\frac{\partial V}{\partial k} \delta k \right]^2 \right\}^{\frac{1}{2}} \qquad (4)$$

Equation (4) states that the error in V, δV, is the square root of the sum of the squares of the errors due to an error in A, N, B and k. The error in N can be considered to be very small because the measurement of time and rotor revolutions are very precise. Therefore the quantity $[\frac{\partial V}{\partial N} \delta N]$ can be eliminated from equation (4) and the error in V becomes

$$\delta V = \left\{ \left[\frac{\partial V}{\partial A} \delta A \right]^2 + \left[\frac{\partial V}{\partial B} \delta B \right]^2 + \left[\frac{\partial V}{\partial k} \delta k \right]^2 \right\}^{\frac{1}{2}} \qquad (5)$$

The partial derivatives are obtained by differentiating equation (3) resulting in

$$\frac{\partial V}{\partial A} = N; \quad \frac{\partial V}{\partial B} = e^{-kN}; \quad \frac{\partial V}{\partial k} = -BNe^{-kN}$$

and substituting into equation (5) gives

$$\delta V = \left\{ \left[N \delta A \right]^2 + \left[e^{-kN} \delta B \right]^2 + \left[-BNe^{-kN} \delta k \right]^2 \right\}^{\frac{1}{2}} \qquad (6)$$

After some rearranging, the relative error in the velocity can be written in terms of the relative errors in A, B and k as

$$\frac{\delta V}{V} = \frac{\left\{ (AN)^2 \left(\frac{\delta A}{A} \right)^2 + B^2 e^{-2kN} \left(\frac{\delta B}{B} \right)^2 + (BNk)^2 e^{-2kN} \left(\frac{\delta k}{k} \right)^2 \right\}^{\frac{1}{2}}}{V} \qquad (7)$$

The relative error ratios $(\frac{\delta V}{V})$, $(\frac{\delta A}{A})$, $(\frac{\delta B}{B})$ and $(\frac{\delta k}{k})$ can be expressed as ratios of the standard deviation to the corresponding mean and as such become coefficients of variation (Herschy, 1978). The coefficient of variation is a basic measure of the relative

uncertainty in the mean value of the variable it represents. Normally, the uncertainty is expressed at the 95% confidence level which, in the case of the velocity V, has been expressed by Engel (1991) as

$$E_V = \pm \frac{100 t_{0.975} C_V}{\sqrt{n-1}} \tag{8}$$

where E_V = the relative uncertainty in the computed velocity in percent at the 95% confidence level, $t_{0.975}$ = the confidence coefficient at the 95% confidence level from Student's t distribution for $(n-1)$ degrees of freedom (Spiegel, 1961) and C_V is the coefficient of variation for the velocity V. The same reasoning applies to A, B and k for which the relative uncertainties can be given as E_A, E_B and E_k. Replacing the relative error ratios in equation (7) with the percent uncertainties, substituting equation (3) for V in the denominator and after further algebraic manipulation, the relative uncertainty in the velocity, computed with equation (3), becomes

$$E_V = \frac{\left\{ A^2 N^2 E_A^2 + B^2 e^{-2kN} \left[E_B^2 + k^2 N^2 E_k^2 \right] \right\}^{\frac{1}{2}}}{AN + Be^{-kN}} \tag{9}$$

Uncertainties in hydrometry are generally expressed as percentages. This practice has been recommended by the International Standards Organization (ISO 5168) and experience in the field has proved this approach to be convenient both in statistical analysis of the data and in the use to which data is put (Herschy 1978). Equation (9) will be used to examine the effectiveness of the meter calibrations obtained when equation (3) is fitted to the data by least squares methods.

3. EXPERIMENTAL METHOD AND PROCEDURE

3.1 Towing Tank

The towing tank is constructed of reinforced concrete, is founded on piles and is 122 metres long and 5 metres wide. The full depth of the tank is 3 metres, of

which 1.5 metres is below ground level. Normally the water depth is maintained at 2.7 metres. Concrete was chosen for its stability, vibration reduction and to minimize possible convection currents.

At one end of the tank is an overflow weir. Waves arising from towed current meters and their suspensions are washed over the crest, reducing wave reflections. Parallel to the sides of the tank perforated beaches serve to dampen lateral surface wave disturbances.

3.2   Towing Carriage

The carriage is 3 metres long, 5 metres wide, weighs 6 tonnes and travels on four precision machined steel wheels. The carriage is operated in three overlapping speed ranges:

$$0.005 \text{ m/s} - 0.06 \text{ m/s}$$
$$0.05 \text{ m/s} - 0.60 \text{ m/s}$$
$$0.50 \text{ m/s} - 6.00 \text{ m/s}$$

The maximum speed of 6.00 m/s can be maintained for 12 seconds. Tachometer generators connected to the drive shafts emit a voltage signal proportional to the speed of the carriage. A feedback control system uses these signals as input to maintain constant speed during tests. The average speed data for the towing carriage is obtained by recording the voltage pulses emitted from a measuring wheel. This wheel is attached to the frame of the towing carriage and travels on one of the towing tank rails, emitting a pulse for each millimeter of travel. The pulses and measured time are collected and processed to produce an average towing speed with a micro computer data acquisition system. Analysis of the towing speed variability by Engel (1989), showed that for speeds between 0.2 m/s and 3.00 m/s, the error in the mean speed was less than 0.15

percent at the 99 percent confidence level. Occasionally, these tolerances are exceeded as a result of irregular occurrences such as "spikes" in the data transmission system of the towing carriage. Tests with such anomalies are recognized by the computer and are automatically abandoned.

3.3   Meter Suspension

The calibration tests were conducted using five Price type winter meters, each fastened to a standard 20 mm diameter solid steel suspension rod. The meters were secured to the rods in accordance with standards used by the WSC for meters with rod suspensions. All meters were suspended 30 cm below the water surface. This depth was chosen to avoid surface effects and to create a minimum of drag on the suspension rods, thereby reducing undesirable vibrations. In all cases great care was taken that the meters were always aligned so that their longitudinal axis was parallel to the direction of travel of the towing carriage. Small deviations from true alignment, especially for velocities less than 30 cm/s do not affect the meter (Engel and Dezeeuw 1978) and therefore any uncertainty due to meter alignment can be considered to be insignificant.

3.4   Test Procedure

A run of the towing carriage, with a meter mounted on the centre-line of the towing tank at a particular speed was defined as a test. To begin a set of tests the meter was carefully aligned in its specified position at the back of the towing carriage. The meter was then towed at preselected speeds. Tests were conducted, beginning at velocities of 6 cm/s up to a maximum of 300 cm/s, for a total of 20 tests per calibration. After each set of 20 tests, the meter was thoroughly inspected before the next set of tests was begun. Each time a meter was towed, care was taken that steady state conditions prevailed when measurements were recorded. The lengths of the waiting

times between successive tests were in accordance with routine procedures used by the National Calibration Service. For each test, the towing speed, revolutions of the meter rotors and the measuring time were recorded. Water temperatures were not noted because temperature changes during the tests were small and do not affect the performance of the meters (Engel, 1976). A total of 5 meters were calibrated. Each meter was calibrated 10 times, resulting in a total of 50 calibrations.

4. DATA ANALYSIS

4.1 Least Squares Fit

An optimized fit of equation (3) to the calibration data is obtained by ensuring that the sum of the squared deviations between the observed values and their estimated values are as small as possible (Stanton, 1961). Mathematically, this is expressed as

$$S = \sum_{i=1}^{n} \left(V_{ci} - AN_i - Be^{-kN_i} \right)^2 \tag{10}$$

where S = the sum of the squared deviations, n = the total number of data pairs, i = the ith data pair in the range from 1 to n, V_c = the towing carriage velocity and all other variables are already defined. For the sake of simplicity the subscripts i are dropped and their presence is taken for granted. The sum S is a minimum for the conditions

$$\frac{\partial S}{\partial A} = \frac{\partial S}{\partial B} = \frac{\partial S}{\partial k} = 0 \tag{11}$$

Equation (11) results in a set of linear equations from which A and B can be expressed in terms of the third coefficient k. The values of A and B are given by

$$A = \frac{(\sum V_c e^{-kN}) - B(\sum V_c e^{-2kN})}{(\sum N e^{-kN})} \tag{12}$$

and

$$B = \frac{(\sum N V_c)(\sum N e^{-kN}) - (\sum N^2)(\sum e^{-kN})}{(\sum N e^{-kN})(\sum N e^{-kN}) - (\sum N^2)(\sum e^{-2kN})} \tag{13}$$

The solution of equations (12) and (13) requires a trial and error procedure. A value of k is initially assumed and values of A and B are computed. These initial values of A and B and the assumed value of k are then used to solve for the sum S in equation (10). Additional values of k are chosen and the process is repeated until the value of k which gives the minimum value of S has been found. Substitution of this optimum value of k into equations (12) and (13) ensures optimum values of A and B thereby providing the best fit of equation (3) to the calibration data. Values of A, B and k are given in Tables 1 through 5 for the five meters calibrated.

4.2   The Effect of A

The coefficient A, as shown in equation (2) can be expressed as

$$A = \frac{D}{\pi}\left[\frac{K+1}{K-1}\right] \tag{14}$$

which shows that it depends on the shape and orientation of the conical rotor cups and the size of the rotor. For a given meter type, the rotor geometry is the same with minor variances due to fabrication tolerances. Therefore, one should expect very little variation in A from one meter to another. Mean values of A and the uncertainty E_A at the 95% confidence level for each of the five meters tested were computed from the data in Tables 1 to 5 and these are given in Table 6. Examination of the data show that indeed the variation in A from meter to meter is trivial. The uncertainty E_A is more pronounced, but its magnitude is very small and therefore does not have a great impact on the effect of A for the Price meters.

4.3   The Effect of B

The coefficient B represents the threshold velocity of the meter. Theoretically, the threshold velocity is the maximum velocity for which the rotor will remain stationary. In other words, it is the flow velocity at which the rotor is on the verge of the

beginning of rotation. Using dimensional analysis, it was shown by Engel (1989) that B can be expressed as

$$B = \frac{bT_o}{D^{\frac{7}{2}}\sqrt{\rho\gamma}}$$ (15)

where T_o = the resistance in the meter at the point of beginning of rotation which occurs at the threshold velocity (i.e. when $N = 0$) and b = a coefficient, ρ = density of the fluid and γ = the unit weight of the fluid. One can expect that the threshold velocity increases as T_o increases. Clearly, for best performance, T_o should be kept as small as possible. In the case of the Price meter, the dependence of the threshold velocity on T_o has significant implications. The Price meter has "cat-whisker" electrical contact brushes which form part of the pulse signal circuit. The overall resistance torque T_o is strongly dependent on how snugly these contact brushes are set. It is therefore important that these adjustments and settings made at the time of meter calibration are maintained during use in the field. Equation (15) also shows that the threshold velocity is inversely proportional to the rotor diameter. Therefore, for a given static resistance T_o, the threshold velocity can be significantly decreased by increasing the rotor diameter. Finally, the effect of fluid density on B can be seen in Figure 5 in which data for the average calibrations of three Price meters in both air and water are plotted as V vs. N. The curves clearly show that the threshold velocity, when the fluid is air is much larger than when the fluid is water. Fortunately, changes in density of the water, as a result of temperature changes are small and therefore, the density of the water does not affect the response of the meter rotor significantly for standard calibrations.

Examination of Table 6 shows that average values of B vary considerably from meter to meter. Although the magnitudes of B are small their relative effects at low velocities is significant. The uncertainty in determining B is much higher than that for A. Values of E_B are between about 12% and 25% for the five meters tested. This uncertainty is largely due to towing tank environment. At very low towing velocities,

residual velocities in the tank due to the disturbance from previous meter tows (Kamphuis 1971) and density currents can cause significant variability in the meter response. This effect can be reduced by increasing the waiting times between tows, but cannot be completely eliminated.

4.4   The Effect of k

The exponent kN in equation (3) is dimensionless and therefore, k has the units of s/rev. Physically, k is a decay constant, the magnitude of which dictates the rate at which the non-linear component of equation (3) approaches the linear component. The rate of change in the non-linear component reflects the rate of change of the resistance in the meter. Since the threshold velocity is directly proportional to T_o, then k should be directly related to B. Examination of the average values of B and k in Table 6, shows that B decreases as k decreases. Physically, one would expect that $B \rightarrow 0$ as $k \rightarrow 0$, implying that when $k = 0$ the meter operates as an ideal frictionless meter.

The uncertainties E_k in Table 6 are considerably larger than E_B varying between 38% and 79% for the five meters tested. This reflects the high sensitivity of k in equation (3). Considering the dependence of k on B, the high uncertainty must be largely a magnification of the uncertainty in determining B.

4.5   Effect of Uncertainty in A, B and k

Examination of equation (9) shows that the uncertainty in the velocity E_V varies with the flow velocity reflected by the rate of rotation of the meter rotor N. Values of E_V were computed for different values of N over the range of towing velocities used for the tests using the values of E_A, E_B and E_k in Table 6. The results were plotted as E_V vs. N for each of the five meters in Figures 6 through 10. Smooth

curves were drawn through the plotted points to facilitate the analysis. The curves clearly show that the greatest uncertainty in the velocity occurs at low values of N. Although uncertainties in B and k are very large, the effect of this variability is small because the product $B^2 e^{-2kN}$ in equation (9) is small and decreases as N increases. As N increases, E_V decreases, initially rapidly, with the rate of change decreasing as N increases. Even at the low values of N (i.e. low velocities), the uncertainties E_V are quite low. For $N = 0.15$, representing a velocity of about 0.10 m/s, the largest value of E_V is about 2.0%. Such low values indicate that the calibrations in the towing tank are very consistent and that equation (3) is a good representation of the response characteristics of the Price meters when they are mounted on a rod suspension. The goodness of the fit of equation (3) can be seen in Figures 11 through 15 in which curves of equation (3) are superimposed on the plotted data for one of the ten calibrations conducted for each of the five meters. The data are plotted as $\frac{N}{V_c}$ vs. V_c. The ratio $\frac{N}{V_c}$ was used because of its high sensitivity to changes in V_c. It represents the rate of rotation of the meter rotor for each meter of towing distance along the tank and can therefore be considered to be a form of meter rotor efficiency. The curves fit the data extremely well over the full range of velocities tested. This accuracy cannot be obtained with linear calibration equations presently used for calibrations of Price meters.

5.   CONCLUSIONS

Using theoretical analysis, an equation for the uncertainty in the calibration equation for rod suspended Price current meters, developed by Engel (1989), was obtained.

Calibrations of five rod suspended Price current meters, each calibrated ten times, were successfully conducted in a towing tank to provide the necessary data for the determination of calibration uncertainty. The equation that should be used for calibration of

rod suspended Price meters is given by

$$V = AN + Be^{-kN} \tag{3}$$

Analysis of the data showed that the uncertainty, at the 95% confidence level, of A was very low and that values of A varied only slightly from meter to meter. This is primarily due to the high precision in the fabrication of the rotor elements of the meters. The uncertainty E_A is mainly due to normally expected experimental error.

Analysis of the data showed that the uncertainty, at the 95% confidence level, of B was quite high and much higher than that of A. The reason for this is mainly due to residual currents in the towing tank as a result of disturbances created by towing the current meters. This uncertainty can be reduced by increasing the time interval between successive meter tows.

Analysis of the data showed that the uncertainty, at the 95% confidence level, of k was considerably higher than that of B. The primary reason for this is that k is dependent on B, with k decreasing as B decreases. Therefore, the factors that affect B can be expected to affect k with the uncertainties being amplified.

The effect of uncertainties in the coefficients A, B and k is greatest at the lowest velocities and decreases as velocities increase. This shows that the greatest error in meter calibrations are likely to occur at the low velocities. The results confirm that equation (3) provides the most accurate calibration for rod suspended Price winter current meters. Considerations should be given to adopting this equation to replace the linear equation format presently used.

ACKNOWLEDGEMENT

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TABLE 1   Test Data for Meter No. 6-226

i	A [m/rev]	B [m/s]	k
01	0.6789	0.012848	7.00
02	-	-	-
03	0.6784	0.007490	2.30
04	0.6771	0.009844	1.56
05	0.6767	0.010322	2.00
06	0.6783	0.013220	5.28
07	0.6785	0.016758	4.56
08	0.6781	0.012318	3.40
09	0.6794	0.012652	4.68
10	0.6792	0.014091	2.71

TABLE 2 Test Data for Meter No. 6-273

i	A [m/rev]	B [m/s]	k
01	0.6794	0.009858	1.56
02	0.6783	0.006840	2.56
03	0.6782	0.011043	1.97
04	0.6777	0.009495	2.20
05	0.6803	0.005852	2.10
06	0.6794	0.010138	1.28
07	0.6787	0.009577	5.43
08	0.6783	0.009655	6.30
09	0.6786	0.009415	4.14
10	0.6787	0.011075	6.21

TABLE 3   Test Data for Meter No. 6-322

i	A [m/rev]	B [m/s]	k
01	0.6790	0.007292	2.06
02	0.6787	0.009273	1.44
03	0.6791	0.006766	1.19
04	0.6790	0.003567	0.72
05	0.6795	0.005858	2.30
06	0.6791	0.011226	9.28
07	0.6786	0.006351	0.24
08	0.6792	0.008138	1.01
09	0.6792	0.009746	2.55
10	0.6796	0.005794	4.18

TABLE 4   Test Data for Meter No. 6-449

i	A $[m/rev]$	B $[m/s]$	k
01	0.6829	0.006821	1.06
02	0.6817	0.007587	0.71
03	0.6822	0.005696	2.70
04	0.6820	0.007698	1.04
05	0.6812	0.007786	1.94
06	0.6820	0.005206	1.59
07	0.6801	0.006800	1.03
08	0.6807	0.008448	1.86
09	0.6823	0.006563	3.02
10	0.6811	0.005601	0.88

TABLE 5   Test Data for Meter No. 6-487

i	A [m/rev]	B [m/s]	k
01	0.6832	0.004944	0.95
02	0.6821	0.003787	0.00
03	0.6813	0.006031	0.00
04	0.6831	0.003485	1.47
05	0.6824	0.003277	3.18
06	0.6828	0.003408	0.41
07	0.6833	0.007299	2.80
08	0.6833	0.003135	0.54
09	0.6833	0.006687	2.14
10	0.6838	0.005900	1.49

TABLE 6   Means and Uncertainties at 95% level for A, B and k

Meter	$\overline{A}$ [m/rev]	E_A [%]	$\overline{B}$ [m/s]	E_B [%]	$\overline{k}$	E_k [%]
6-226	0.6783	0.1006	0.012172	16.562	3.721	36.202
6-273	0.6788	0.0821	0.009295	13.616	3.375	43.847
6-322	0.6791	0.0366	0.007401	22.851	2.497	79.415
6-449	0.6817	0.0906	0.006832	11.732	1.583	37.695
6-487	0.6829	0.0805	0.004795	24.760	1.298	65.397
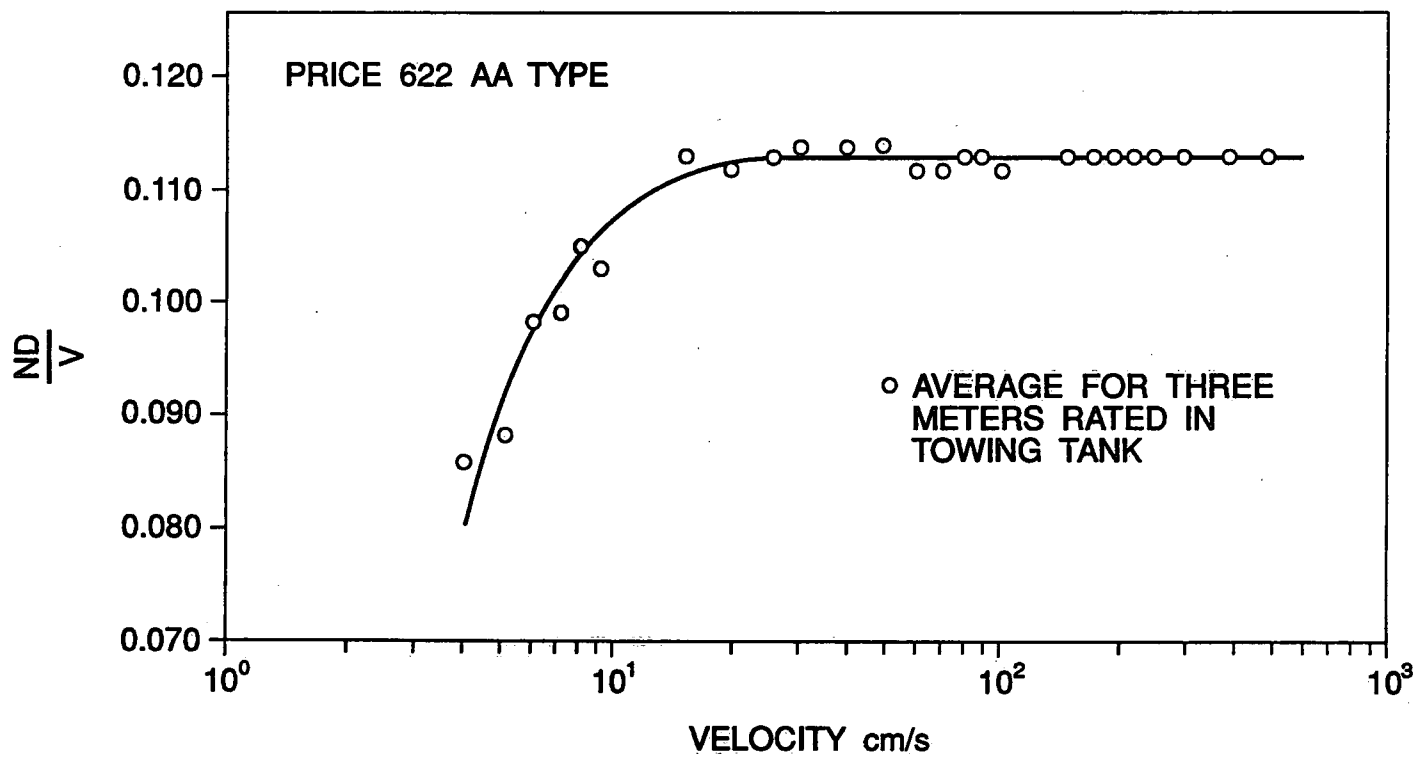
FIGURE 1 TYPICAL PRICE METER PERFORMANCE CURVE
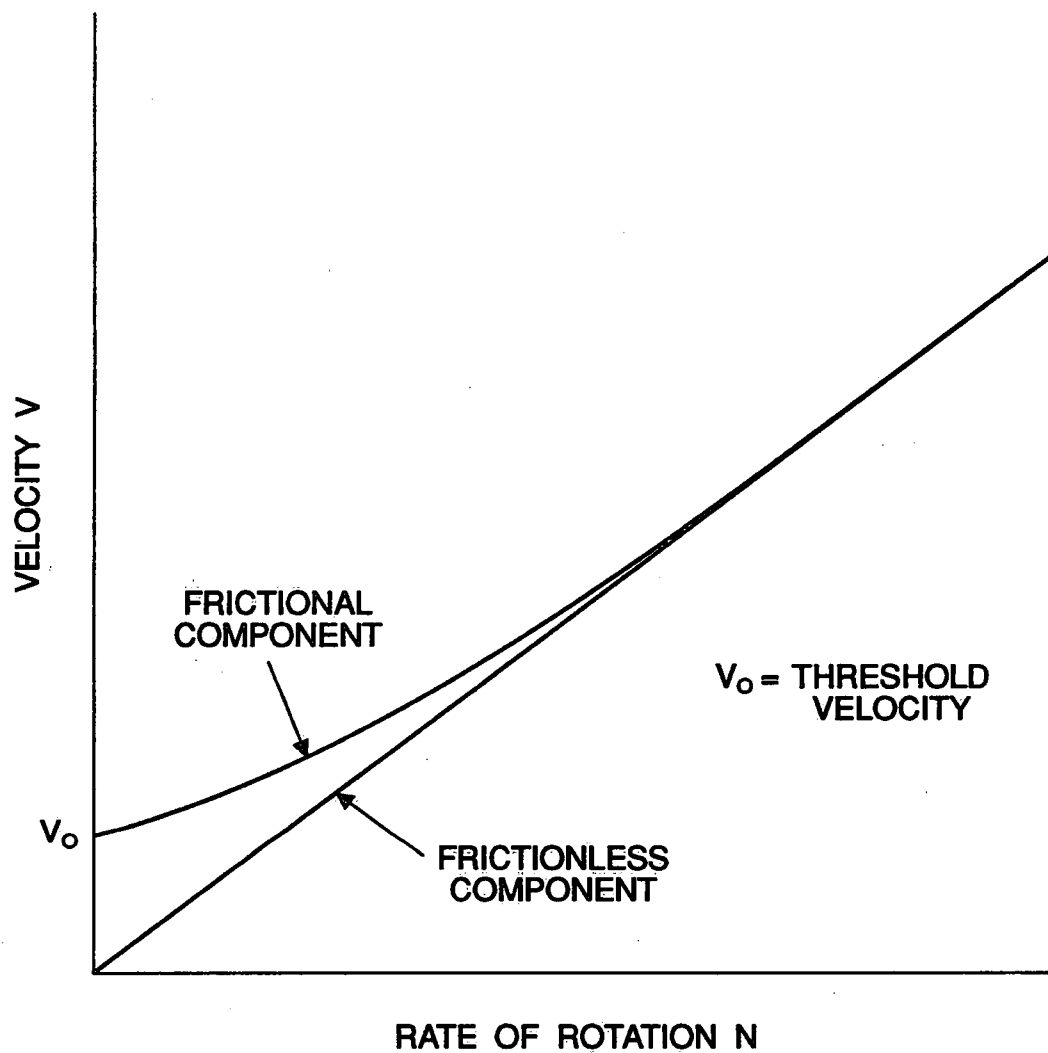
FIGURE 2 COMPONENTS OF CALIBRATION EQUATION
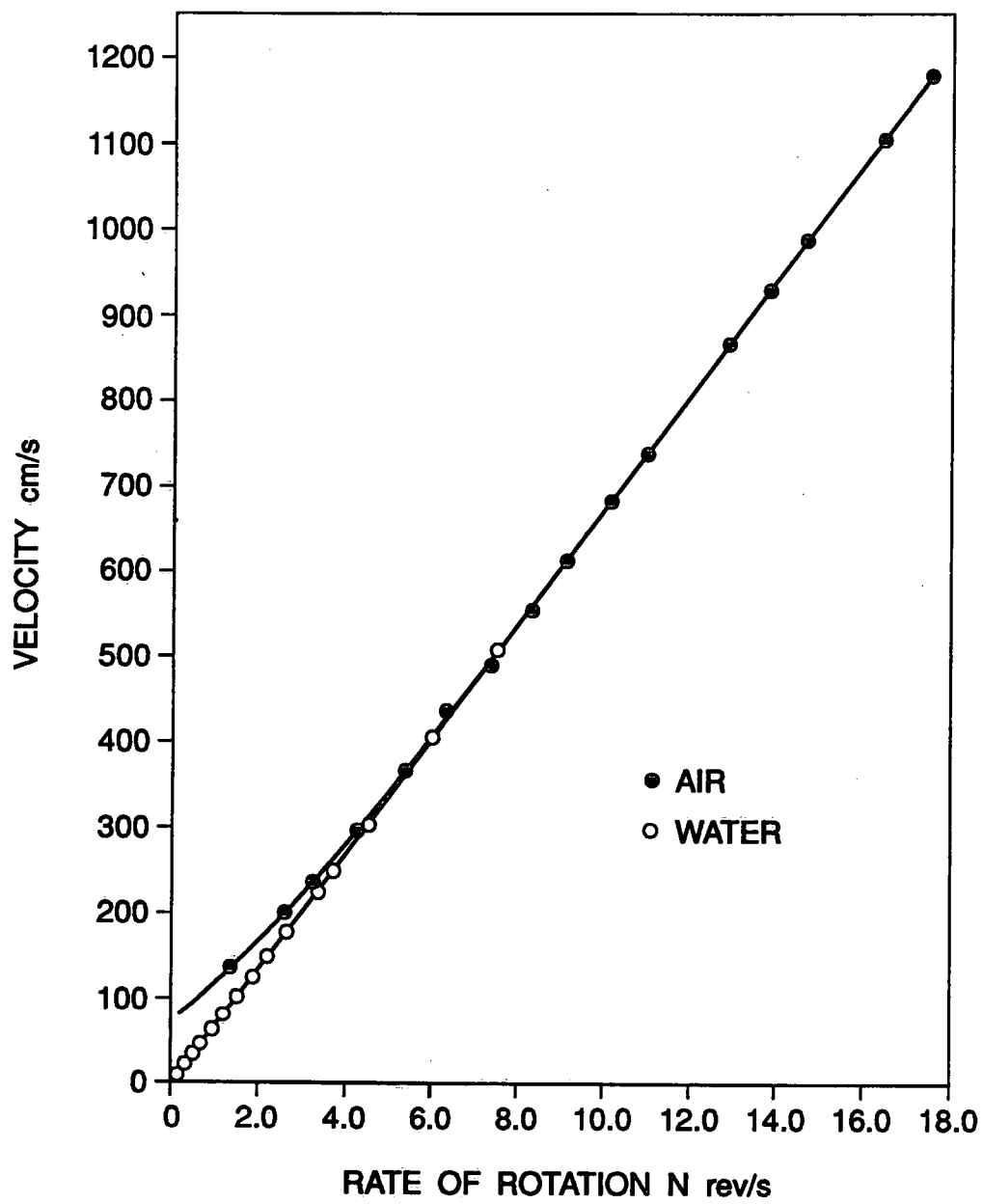
FIGURE 3 CALIBRATION CURVES FOR PRICE METER IN AIR
AND WATER
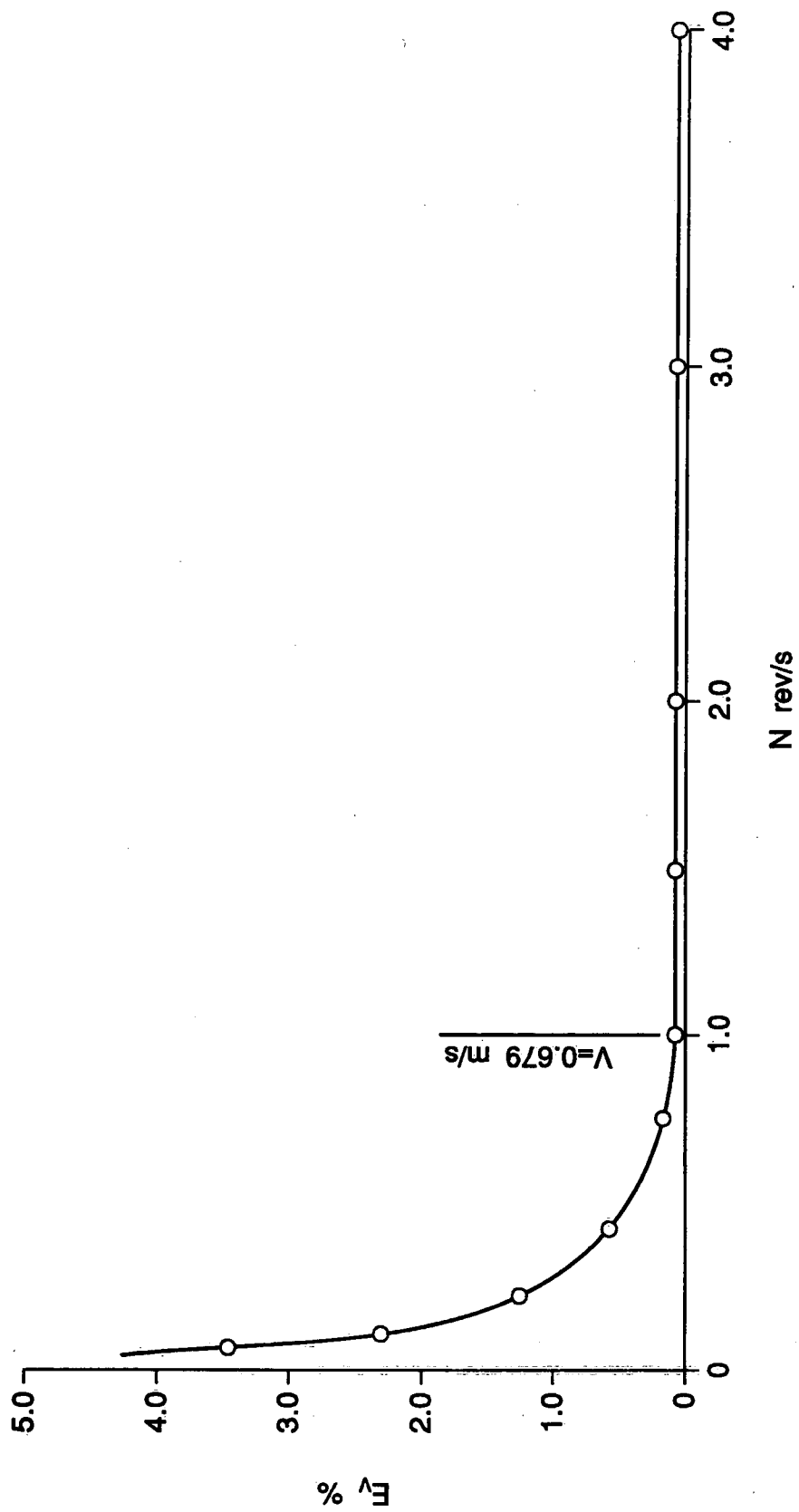
FIGURE 4   UNCERTAINTY E_v AS A FUNCTION OF N FOR METER No. 6-226
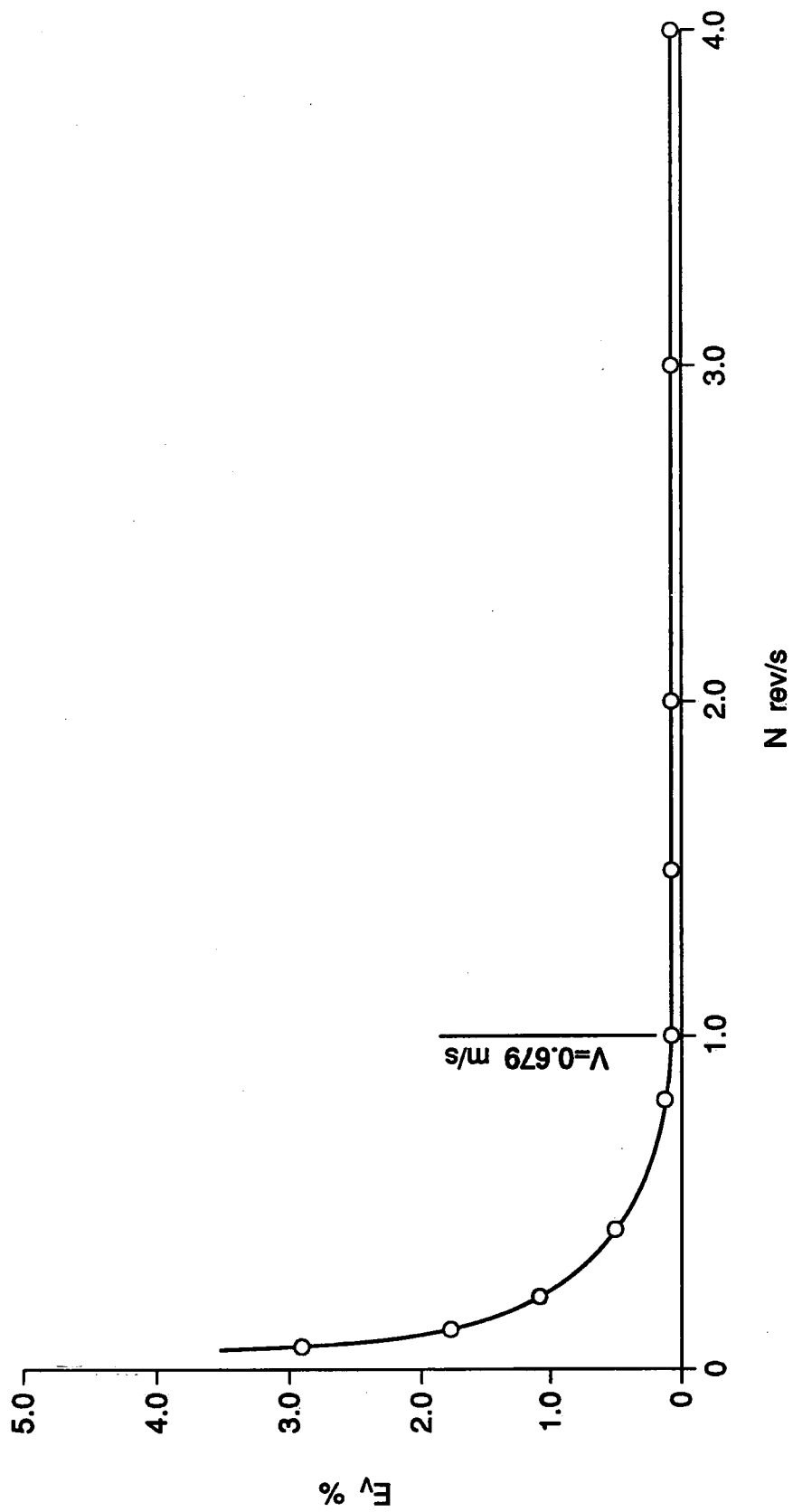
FIGURE 5   UNCERTAINTY E_V AS A FUNCTION OF N FOR METER No. 6-273
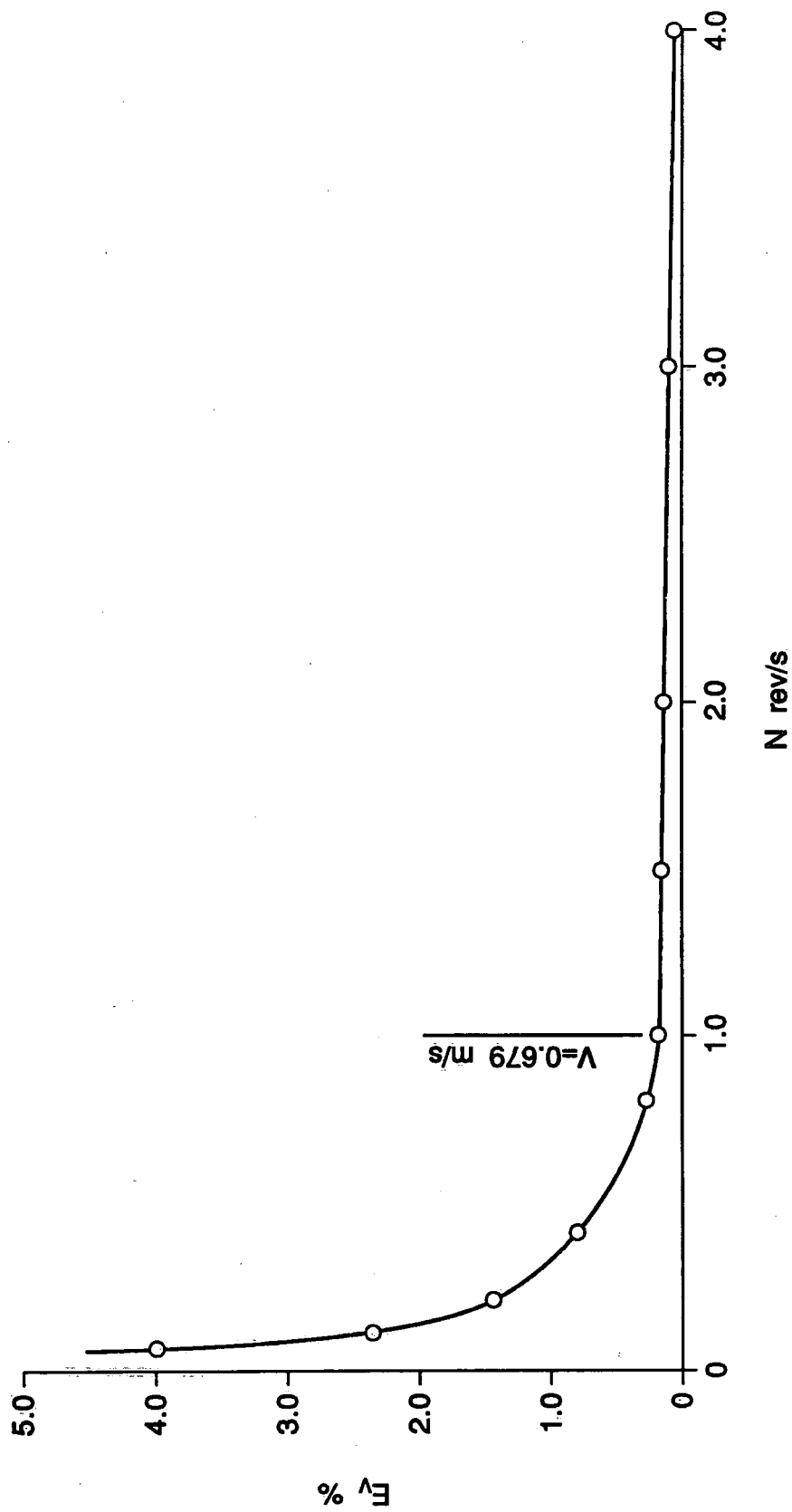
FIGURE 6   UNCERTAINTY E_v AS A FUNCTION OF N FOR METER No. 6-322
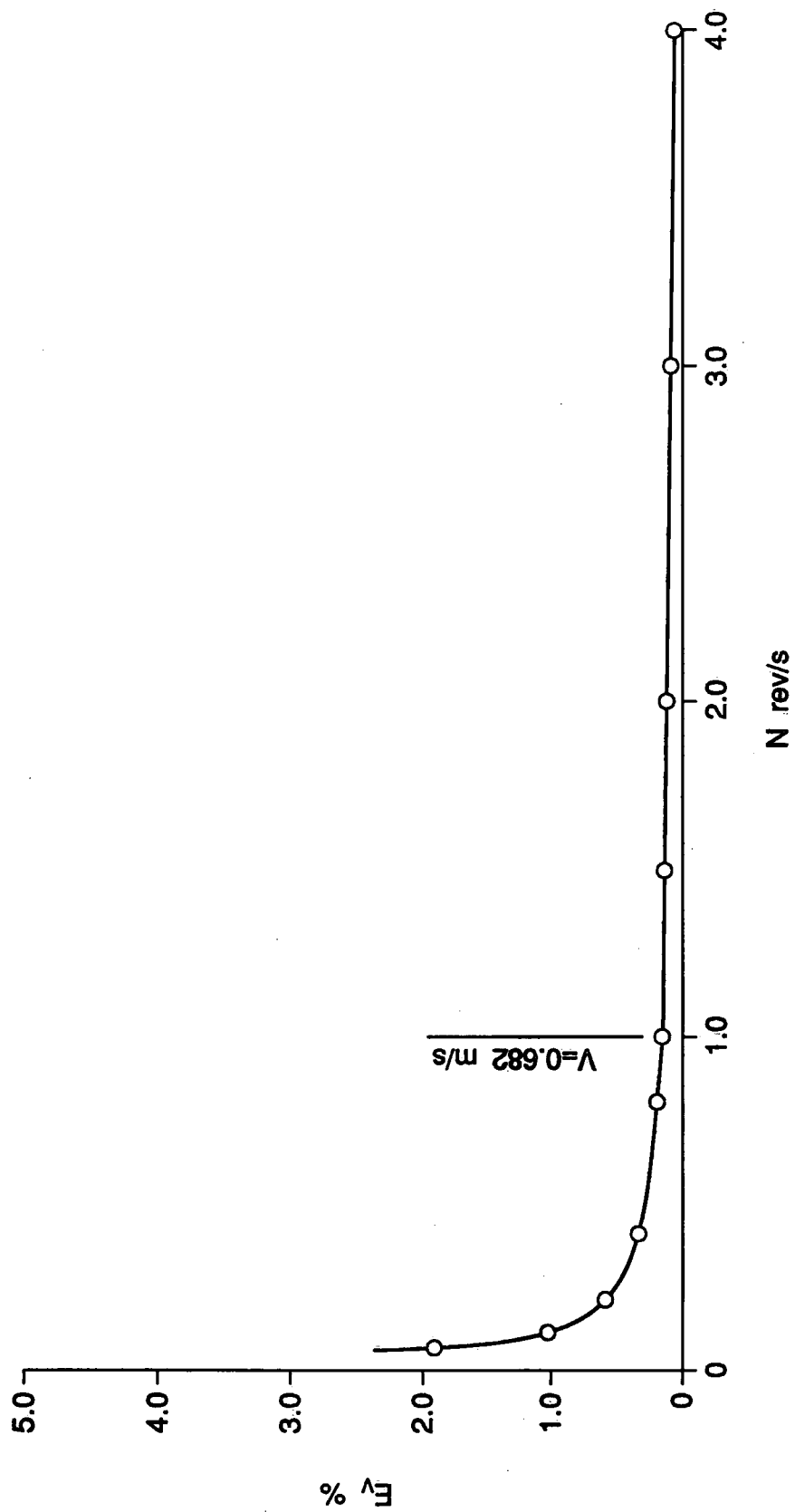
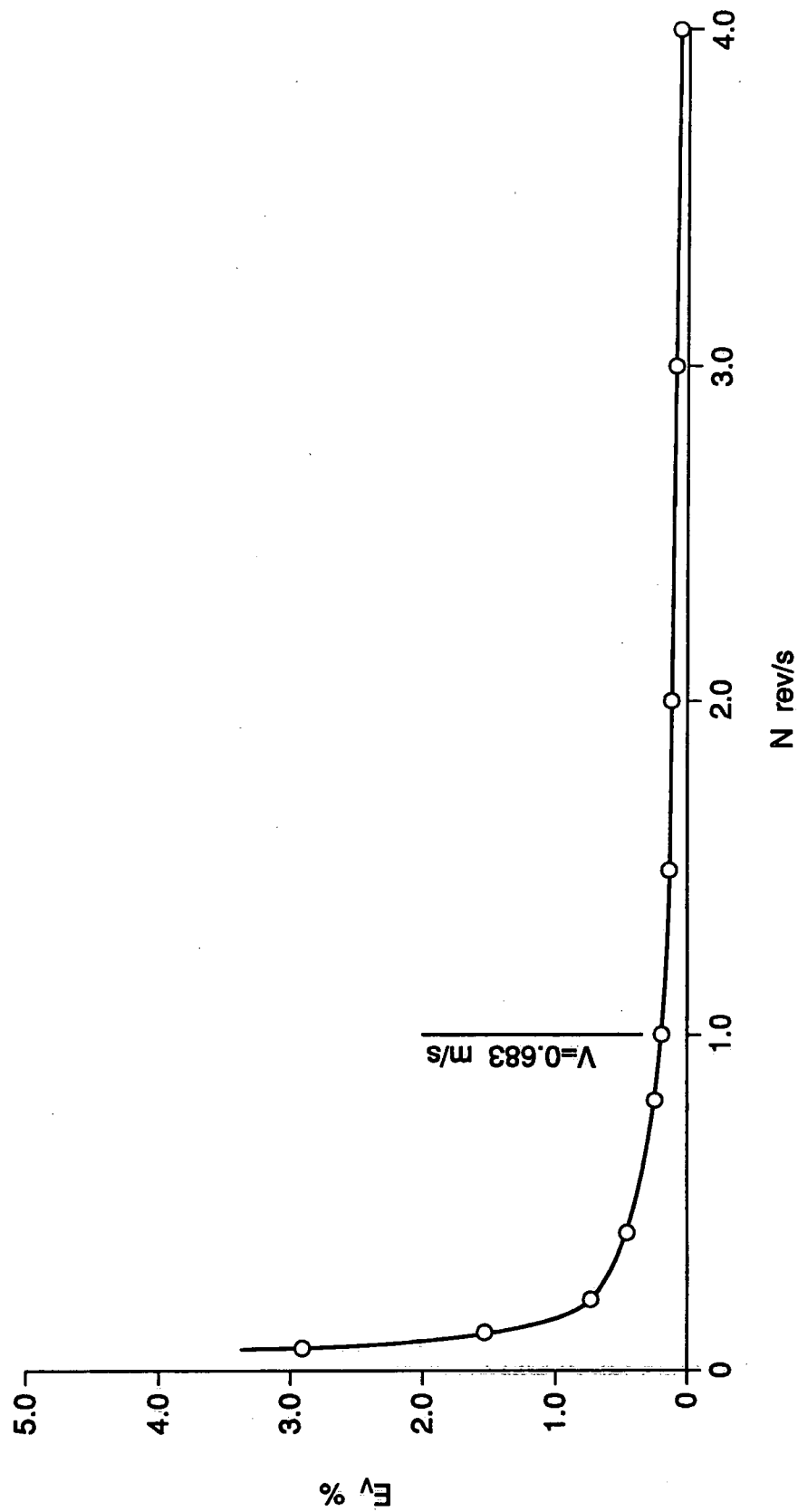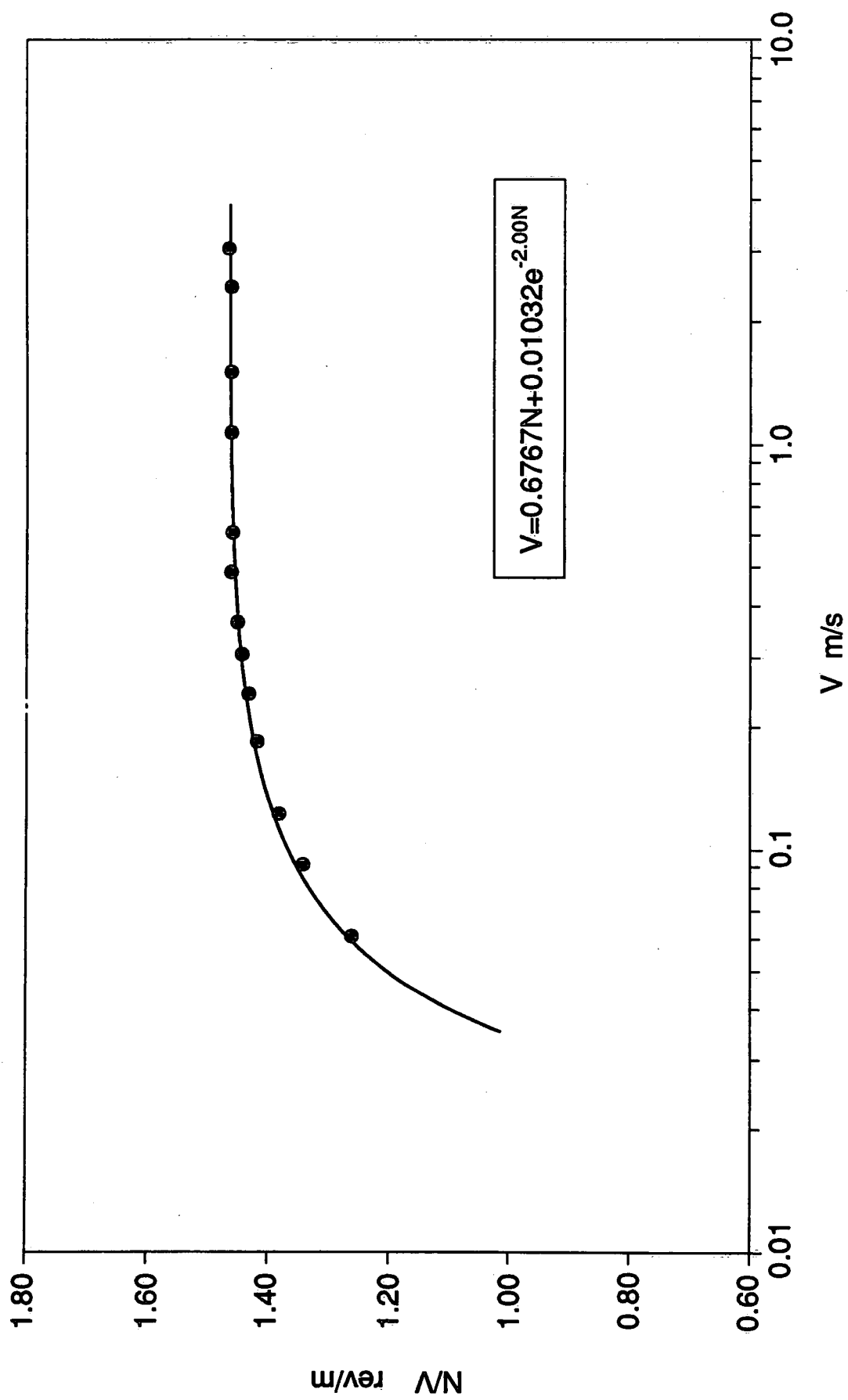FIGURE 7   UNCERTAINTY E_V AS A FUNCTION OF N FOR METER No. 6-449

N rev/s

E_V %

V=0.682 m/s

5.0 4.0 3.0 2.0 1.0 0

0 1.0 2.0 3.0 4.0

FIGURE 8 · UNCERTAINTY E_v AS A FUNCTION OF N FOR METER No. 6-487

$$V = 0.6767N + 0.010032e^{-2.00N}$$

N/V rev/m

V m/s

FIGURE 9 TYPICAL CALIBRATION FOR METER No. 6-226
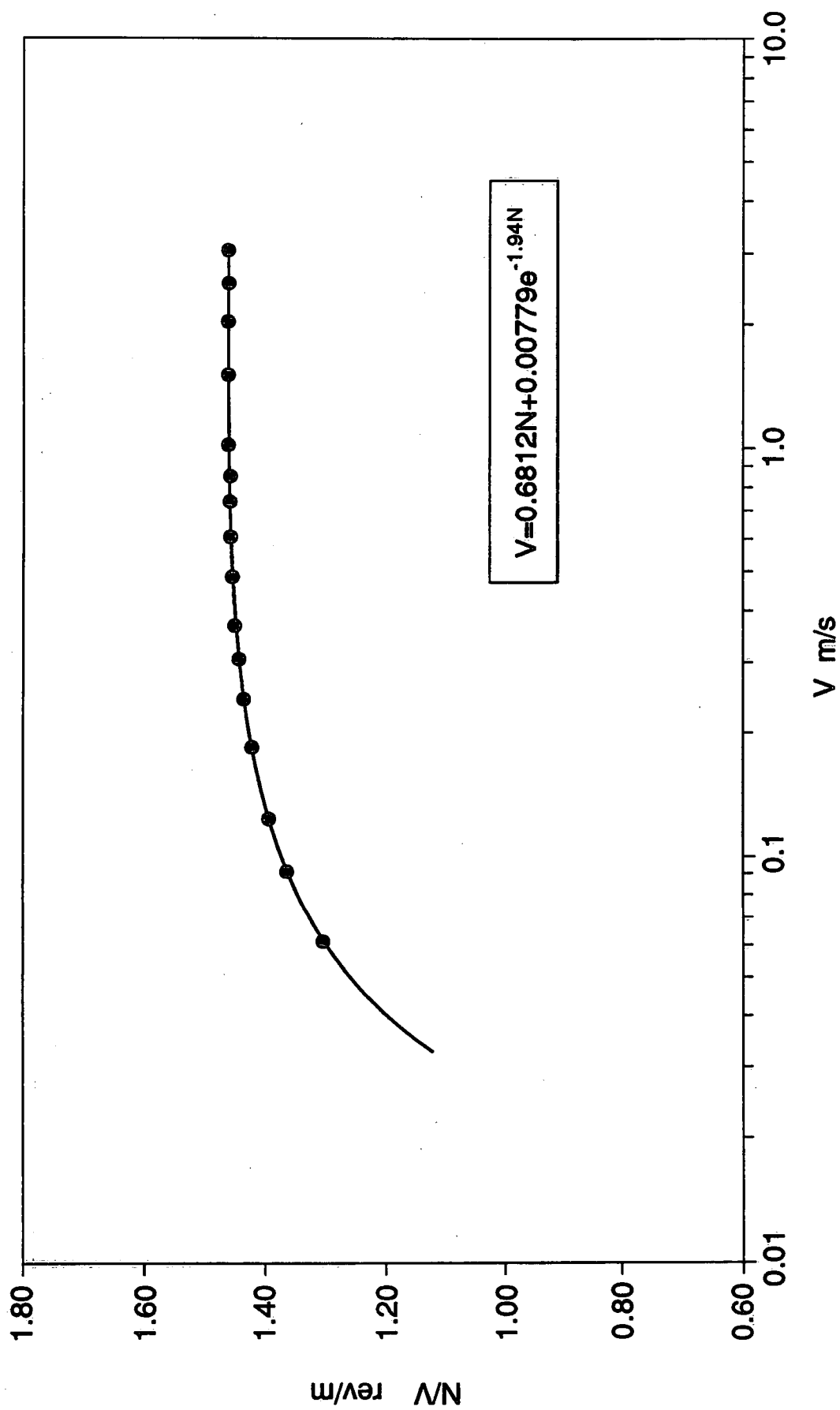
FIGURE 10 TYPICAL CALIBRATION FOR METER No. 6-449

$$V = 0.6812N + 0.00779e^{-1.94N}$$

V m/s

N/V rev/m
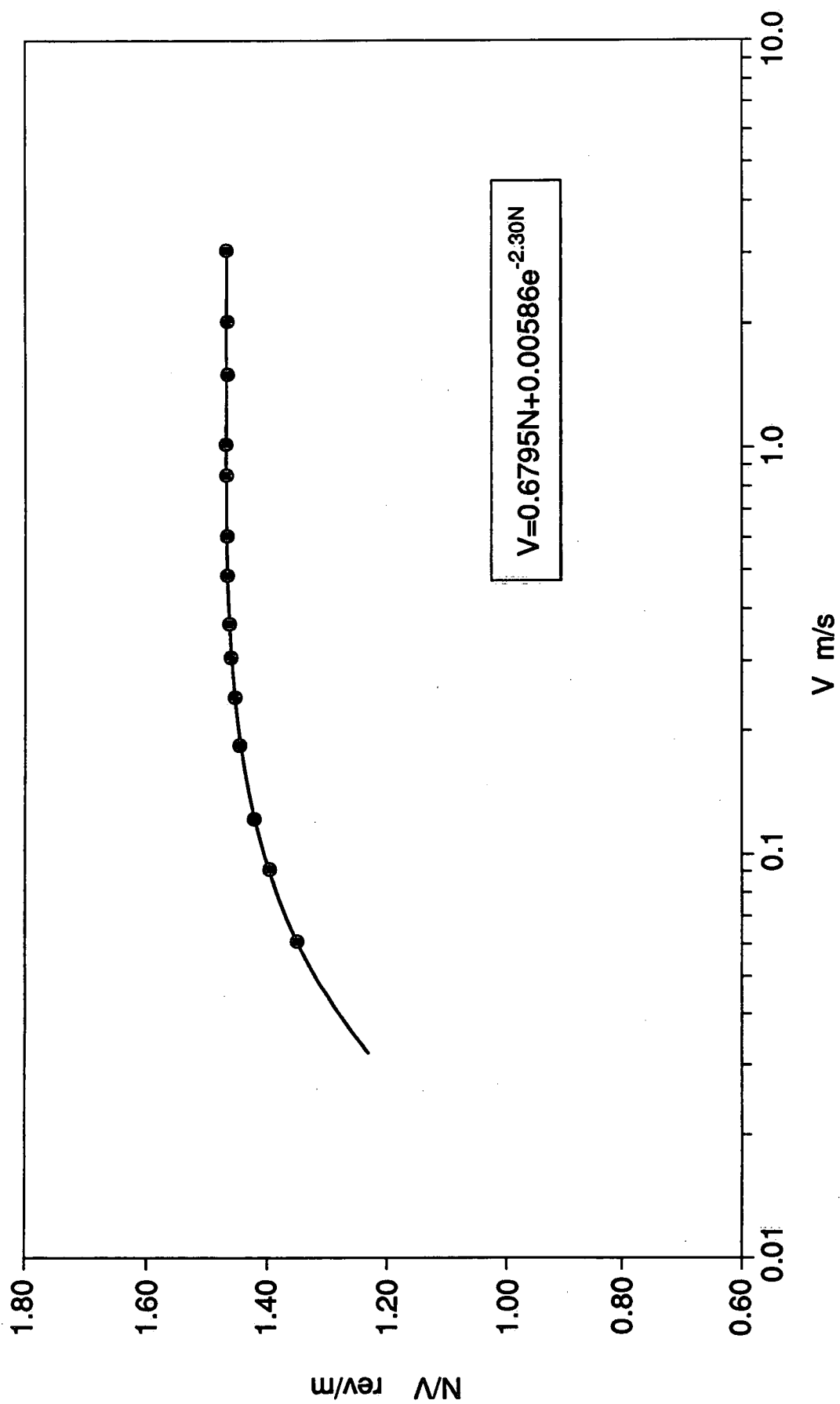
FIGURE 11 TYPICAL CALIBRATION FOR METER No. 6-322

$V = 0.6795N + 0.00586e^{-2.30N}$

V=0.6794N+0.01014e$^{-1.28N}$

N/V rev/m

V m/s

0.01   0.1   1.0   10.0

0.60   0.80   1.00   1.20   1.40   1.60   1.80
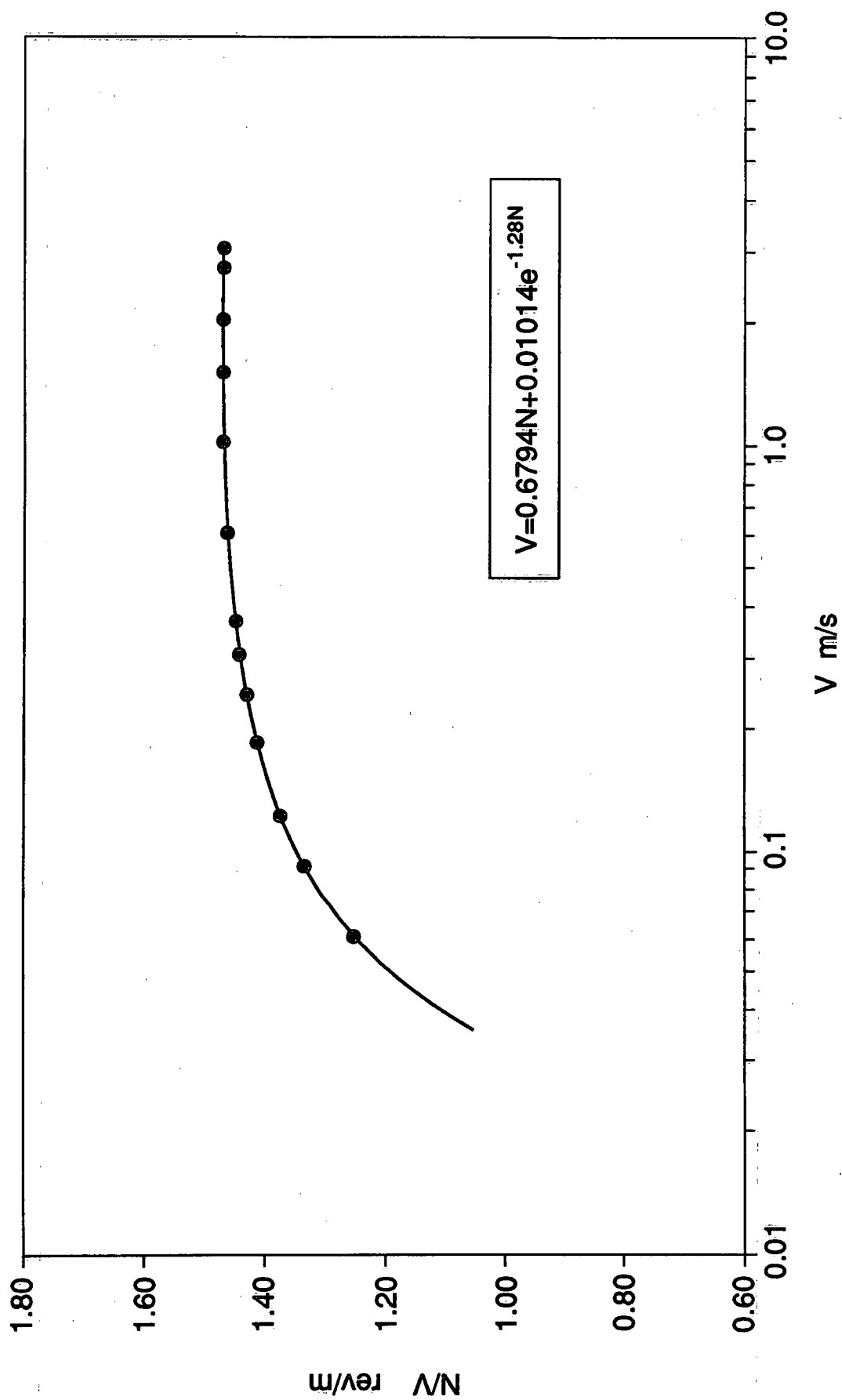
FIGURE 12 TYPICAL CALIBRATION FOR METER No. 6-273
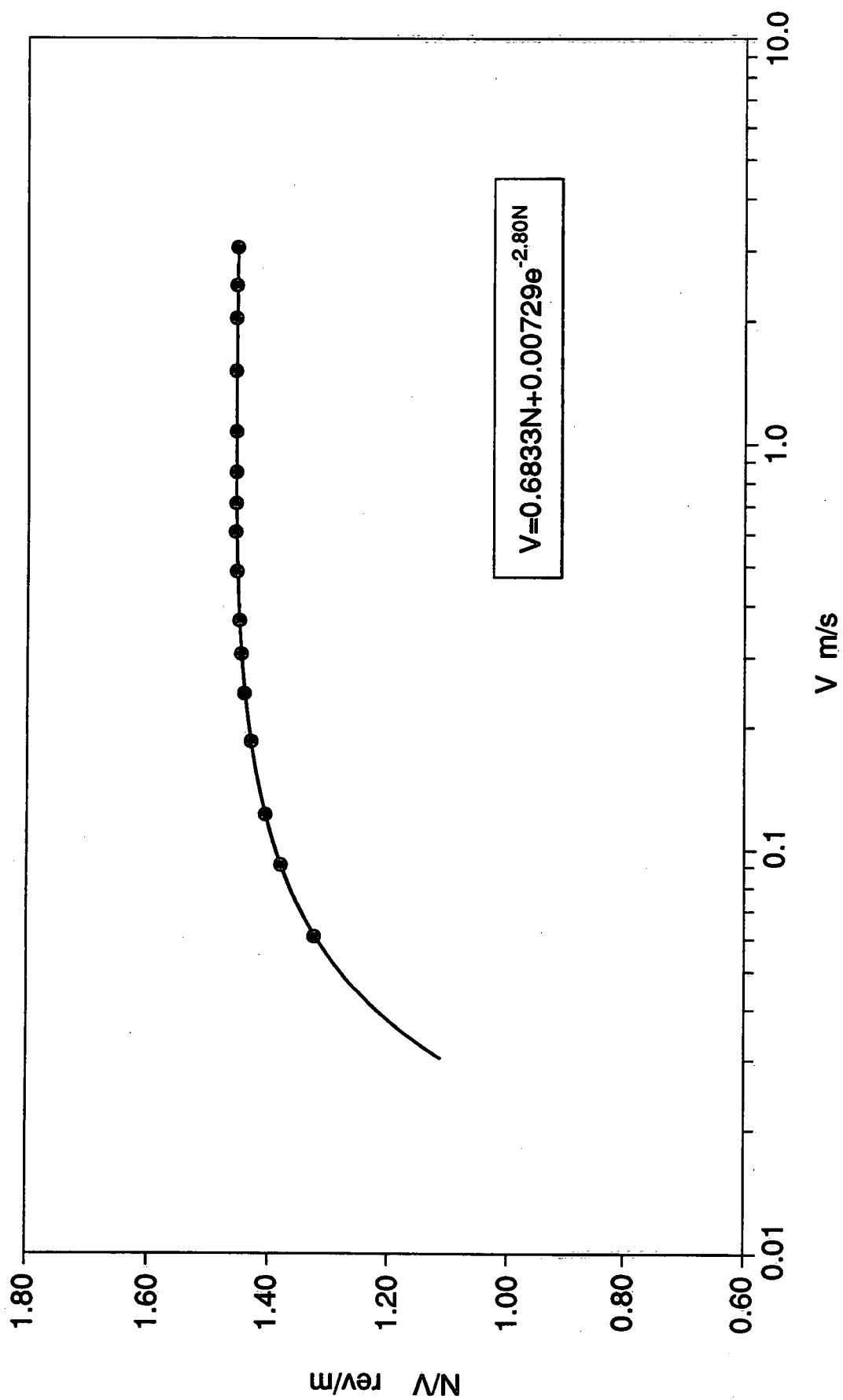
FIGURE 13   TYPICAL CALIBRATION FOR METER No. 6-487

The figure shows a calibration plot with N/V rev/m on the vertical axis (ranging from 0.60 to 1.80) and V m/s on the horizontal axis (logarithmic scale from 0.01 to 10.0). The fitted equation shown on the plot is:

$$V = 0.6833N + 0.007729e^{-2.80N}$$