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Abstract

The observed predictability of excess returns in equity and foreign exchange markets has largely been attributed to the presence of time-varying risk premiums in these markets. For example, excess equity returns were found to be explained by various financial and economic variables. Similarly, in the foreign exchange market, the forward rate was found *not* to be an unbiased predictor of the future spot rate, and excess foreign exchange returns were shown to be partially explained by other variables of the foreign exchange market, notably the forward premium. However, notwithstanding the extensive empirical evidence on the above, theoretical models of international asset pricing have not been entirely successful in producing equilibrium conditions that replicate the actual behaviour of the different asset moments in empirical tests for reasonable parameter values. In fact, these models had limited success despite either rich preference structures or general driving processes for the exogenous environment of the model.

In this paper, we evaluate excess asset returns in equity and foreign exchange markets by combining generalized preferences to a heteroscedastic driving process in the same model. We do so by extending the international asset-pricing model of Bekaert, Hodrick, and Marshall (1997) in which the authors adopt disappointment-aversion-type preferences and a homoscedastic exogenous environment. We show that our very general framework, with plausible parameter values, is fairly successful in generating predictability and moment levels of excess returns that are consistent with the sample data.

JEL classification: E44, F31, G12, G15
Bank classification: Exchange rates; Financial markets; Market structure and pricing

Résumé

On attribue généralement la prévisibilité observée des excédents de rendement sur les marchés des actions et des changes à la présence de primes de risque variables sur ces marchés. Ainsi il s’est avéré possible d’expliquer les excédents de rendement sur les portefeuilles d’actions au moyen de diverses variables financières et économiques. De même, sur le marché des changes, le taux à terme *ne s’est pas* révélé un indicateur non biaisé du taux au comptant futur, car des études ont montré que d’autres variables, notamment la prime de risque à terme, peuvent rendre compte en partie de la présence d’excédents de rendement sur ce marché. En dépit des nombreuses études empiriques étayant ces résultats, les modèles théoriques d’évaluation des actifs internationaux ne parviennent pas à recréer entièrement les conditions d’équilibre permettant de reproduire le comportement effectif des différents moments des actifs dans des tests empiriques, et ce, pour des valeurs plausibles des paramètres. En fait, le succès de ces modèles est mitigé que les préférences soient de type « généralisées » ou qu'un processus d'impulsion assez général y soit défini pour les variables exogènes.

Afin d'évaluer les excédents de rendement observés sur les marchés des actions et des changes, les auteurs de l'étude combinent une fonction d'utilité généralisée à un processus d'impulsion héréroscédastique au sein d'un modèle élargi qui s'inspire du modèle d'évaluation des actifs internationaux de Bekaert, Hodrick et Marshall (1997); ces derniers postulaient plutôt des préférences de type « aversion pour la déception » et un processus homoscédastique pour l'environnement exogène du modèle. Le cadre très général retenu par les auteurs réussit assez bien, pour des valeurs plausibles des paramètres, à prévoir et à générer des excédents de rendement dont les moments sont conformes à ceux calculés à partir de l'échantillon étudié.

Classification JEL : *E44, F31, G12, G15*
Classification de la Banque : Marchés financiers; Structure de marché et fixation des prix; Taux de change

1. Introduction

Much of the research on financial markets in the last decade was prompted by the realization that first and second moments of various financial asset returns appear predictable. To explain the above, attention was duly focused on assumptions concerning information processing, the formation of expectations and different risk considerations. The emerging consensus maintained that, if expectations are rational, the observed predictability above could largely be attributed to the presence of time-varying risk premiums. That is, agents demand a higher return for incurring additional risk in these markets.

In the case of equity markets, for example, it was documented that a proportion of excess equity returns (that is, equity returns minus the yield on a risk-free rate) could be explained by certain financial variables (such as dividend yields, earnings to price ratios, default and term spreads) as well as by economic variables (for example, inflation and interest rates, changes in industrial production and in fiscal deficits). Similarly, in the foreign exchange market, tests of the forward-market efficiency hypothesis revealed that the forward rate is not an unbiased predictor of the future spot rate. That is, hypothesis testing of the regression coefficients of the exchange rate change on the forward premium yielded a non-zero intercept and a significantly negative slope. Consequently, it was shown that excess foreign exchange returns (defined as the future spot rate minus the forward rate) could partially be explained by other variables of the foreign exchange market, notably the forward premium.

While this observed predictability of excess returns seems to be robust to the frequency of data used, to the time spans adopted, as well as to the equity and foreign exchange markets considered, theoretical models of international asset pricing have not yet been successful in producing equilibrium conditions that replicate the actual behaviour of the various asset moments in empirical tests.[1] In fact, these models have fared poorly despite either rich preference structures or general driving processes for the exogenous environment of the model.

In this paper, we evaluate excess asset returns in equity and foreign exchange markets by combining generalized preferences to a heteroscedastic driving process. We do so by extending the international asset-pricing model of Bekaert, Hodrick, and Marshall (1997) who assume disappointment-aversion-type preferences and a homoscedastic exogenous environment. We show that our very general framework is quite successful in generating predictability and moment levels of excess returns that are consistent with the sample data. This outcome concurs with that of

1. See, for instance, Canova and Marrinan (1995) and Bekaert, Hodrick, and Marshall (1997).

Bonomo and Garcia (1994) who were able to closely replicate the level of moments of variables in the context of a closed economy, using a framework similar to ours.

2. Theoretical risk premium models

In the case of equity markets, it was shown by Mehra and Prescott (1985) that, for reasonable configurations of preference and endowment processes, an exchange-economy equilibrium model could not reproduce the secular difference between the average return on stocks and that on Treasury bills (this is the so-called equity premium puzzle).

Indeed, typical asset-pricing Euler equations imply that the conditional expectation of discounted asset returns equals a constant. In this context, the key to reproducing the required behaviour of assets is specifying a suitable form for the stochastic discount factor (hereafter SDF), that is, the intertemporal marginal rate of substitution. In particular, it was suggested that a successful model must produce a sufficiently variable SDF to be able to reproduce the level and dynamics of second moments of asset returns. Since the type of preferences and endowment processes adopted in the model both influence the form and variability of this discount rate, these features have been the focus of recent research in asset pricing.

Among those who focused on specifying preferences better are Epstein and Zin (1989, 1991) and Weil (1989) who introduced the notion of non-expected utility functions. With this type of preference, the hypothesis of independence of marginal utility of consumption across different states is loosened so that marginal utility of consumption in a good state is allowed to be influenced by the consumption level in a bad state. In addition, and contrary to the isoelastic preferences case, these new generalized preferences also permit disentanglement of the distinct notions of risk aversion and intertemporal substitution. This implies that a risk-averse consumer will not necessarily see consumption in different periods as highly complementary goods. The advantage of both of these properties of non-expected utility preferences is that they allow greater flexibility in the intertemporal marginal substitution function, thereby inducing more variation in it. Indeed, when applied to U.S. equity data, these studies found that their results provided some improvement on the expected utility framework outcomes. However, these gains were still not sufficient to solve the equity premium puzzle.

Other researchers focused on the time dimension of preferences instead, that is, on the time non-separability of preferences. Studies by Constantinides (1990), Ferson and Constantinides (1991), and Campbell and Cochrane (1999) used the idea of habit formation to address this issue. In this case, marginal utility of current consumption increases until consumption attains the level the individual is accustomed to consuming. In addition, when current consumption drops relative to the habitual level, risk aversion increases, inducing the agent to increase saving in the face of a possible further decline in consumption. Thus, with this set-up, risk aversion is time varying. In addition, similar to the non-state-separable utility case, these preferences also relax the link between intertemporal substitution and risk aversion. Yet, despite the improved dynamics of returns over the isoelastic preferences case, habit formation could still not match the high levels of equity risk premiums unless very high risk aversion was assumed.

The second strand of research chose to concentrate on the time-series specification of the model endowment process to try to match observed moments of returns. For instance, in the case of equity markets, Bonomo and Garcia (1994a) and Cecchetti, Lam, and Mark (1993) both proposed a bivariate Markov-switching process for consumption and dividend growths series to describe the endowment component in an expected utility framework. The first authors showed that, on the one hand, loosening the usual assumption that dividends equal consumption and, on the other hand, using a three-state bivariate Markov-switching model where the means and variances changed with the state yielded good results on real returns in such models. The second authors used a two-state homoscedastic specification with isoelastic preferences, with and without leverage effects. However, for both studies, the adopted endowment process could, once again, not fully explain the facts related to excess returns in equity markets.

Enriching either the preference structure or the driving process of the basic model therefore improved results somewhat for the case of equity returns. However, Bonomo and Garcia (1994) showed that, when both features of the model were addressed simultaneously, it was possible to substantially improve on the single-featured model results, thus better reproducing magnitudes of U.S. equity and risk-free rate moments in the data. In the Bonomo and Garcia (1994) model, agents were assumed to have a particular type of non-expected utility function resulting from disappointment-aversion-type preferences. As for the endowments, a bivariate consumption and dividend growth process was assumed and made to follow a three-state Markov regime-switching dynamics with state-dependent means and variances. The authors stressed the fact that disappointment-aversion preferences, coupled with a joint random walk endowment process, could produce only half the magnitude of the average U.S. equity risk premium estimated using the more general model. Similarly, isoelastic preferences combined with a bivariate three-state Markov-switching model yielded an even lower value for the average equity risk premium and a very high value for the mean of the risk-free rate.

Turning to the international returns case now, an added complication arises in that moments of excess returns on the foreign exchange market should also be explained. Indeed, so far, basic international models of consumption asset pricing have not fared so well in trying to reproduce the dynamics of international returns data. These are mostly cash-in-advance type models where money is introduced in CCAPM models through a cash-in-advance constraint.[2] The general conclusion arising from these models is that, while inflation risk does affect the stochastic properties of asset returns, with isoelastic preferences, the resulting variability in the stochastic discount rate is too small to match asset returns dynamics.

As in the single-country models, a few studies tried to modify the basic expected utility structure in order to have more complex driving processes. For instance, Canova and Marrinan (1993, 1995) used heteroscedastic endowment processes in the representative agent cash-in-advance model. They found that, while the resulting excess asset returns were variable, heteroscedastic, and serially correlated, the magnitudes of these second moments were still considerably smaller than those observed in actual data.

An attempt was also made at integrating generalized preferences in a two-country monetary model. The study by Bekaert, Hodrick, and Marshall (1997) uses non-expected utility preferences in a two-country model to explain excess returns in the international market. More specifically, the authors use disappointment aversion in order to characterize agents' preferences. With this set-up, agents are more sensitive to a disappointing state of the endowment than to an elating one so that a fairly small amount of uncertainty in the agents' exogenous environment can lead to potentially large fluctuations in the stochastic discount rate. While preferences are different than in a typical consumption CAPM model, the authors maintain the assumption of a homoscedastic driving process in their economy. Finally, in this model, the incentive to hold money comes from the assumption that, while real consumption is costly, these transaction costs are alleviated by holding real money balances.

The authors concluded that, while first-order aversion substantially increases the variance of risk premiums, once again, this increase is insufficient to match the excess return predictability in the data. They therefore suggested that excess return predictability of international financial assets cannot be explained only by modifying the preference assumptions of the model and that learning, peso problems, and Markov-switching models might be more suitable frameworks for examining the problem at hand.

Based on the good results of the Bonomo and Garcia (1994) model for the closed economy case, and following the suggestion of Bekaert, Hodrick, and Marshall (1997) (hereafter BHM), in this paper we evaluate international asset returns by adopting the basic framework of BHM but

2. Examples are Lucas (1982), Svensson (1985), Labadie (1989), and Giovannini and Labadie (1991).

modifying the endowment process to allow for multivariate regime switching. Our very general framework thus allows us to examine empirically the relative roles of the preference and endowment parameters, as well as their various combinations, for the simultaneous reproduction of the behaviour of equity and foreign exchange excess returns.

We find that our model presents substantial improvements over the BHM study. In particular, we are able to generate fairly closely the predictability of all excess returns considered. Furthermore, we are also able to reproduce quite well the mean and standard deviations of some of the financial series considered in this study. Nevertheless, we are somewhat less successful in matching moments of certain variables in the foreign exchange market. On the basis of our experiments, and similar to BHM, we therefore suggest that some form of market segmentation should be imposed in the model. Indeed, the extensive pricing-to-market (PTM) literature suggests that imperfect competition among firms does segment markets by country, thereby increasing exchange rate volatility relative to a situation where the law of one price holds.

3. The structure of preferences

A representative agent is endowed with disappointment-aversion preferences and maximizes intertemporal utility over home and foreign goods. This generalized preference structure was axiomatized by Gul (1991) to be the most restrictive formulation that is consistent with Allais' paradox and that also includes the expected utility theory as a special case. Therefore, the corresponding utility function is not the usual expected utility function, but rather the recursive functional form of Epstein and Zin (1989), which was developed to accommodate such generalized preferences in an infinite horizon setting.

As explained above, the reason for adopting these preferences is as follows. An unattractive feature of the Von Neumann-Morgenstern expected utility preferences is that relative risk aversion and intertemporal substitutability are intertwined; that is, the coefficient of relative risk aversion is the inverse of the elasticity of intertemporal substitution. This arises from the assumption of time and state separability that expected utility theory imposes. Thus, the respective roles of risk attitudes and intertemporal substitution cannot be fully exploited in consumption-savings and portfolio choice equations in the typical consumption asset-pricing model. It has been suggested that this inflexibility of the preference structure has been a possible cause for the failure of such models. On the other hand, since generalized preferences permit these two distinct aspects of preferences to be disentangled, it is a desirable feature to have in an intertemporal general-equilibrium model.

Secondly, disappointment-aversion preferences display first-order risk aversion. This means that, even when faced with a lottery close to perfect certainty, agents are substantially risk averse. In contrast, individuals having expected utility preferences (which exhibit second-order risk aversion instead) behave as if they are risk neutral in the face of the same lottery. With the latter specification, the equilibrium conditional variance of next period's consumption is small.

From these explanations we can see the importance of having first-order risk aversion for the purposes of obtaining higher variability in the stochastic discount rate. This, in turn, implies larger movements in expected asset returns.

Recursive Utility

The Epstein and Zin recursive utility is a CES function of the form

$$\Omega(c_0, \mu[\kappa(m)]) = \left\{c_0^{\rho} + \beta(\mu[\kappa(m)])^{\rho}\right\}^{1/\rho} \tag{1}$$

where c_0 is consumption in period zero, $\kappa(\ )$ is the probability measure for future utility, m is an element of the admissible lottery space, and μ is a certainty equivalent measure for random future utility (i.e., it is a weighted mean). The form of μ is decided by the type of preferences assumed. In our case, the certainty equivalent function for disappointment-aversion preferences is implicitly defined as

$$\frac{\mu(P)^{\alpha}}{\alpha} \equiv \frac{1}{KK}\left(\int_{(-\infty,\, \mu(P))} \frac{z^{\alpha}}{\alpha} dP(z) + A \int_{(\mu(P),\, \infty)} \frac{z^{\alpha}}{\alpha} dP(z)\right) \tag{2}$$

where $KK = A \cdot prob(z > \mu) + prob(z \le \mu)$, $A \le 1, \alpha < 1$. When A is less than one, the elation region is downweighted relative to the disappointment region; that is, outcomes that are below the certainty equivalent are more heavily weighted than those that are above $\mu(P)$. Notice that for $A = 1$ and $\alpha = \rho$, the above equation becomes the usual expected utility certainty equivalent function definition. Thus, using this framework, one can explicitly test the expected utility model conclusions to its generalized counterpart. We also point out that, when $A = 1$ and $\alpha \ne \rho$, we obtain Kreps-Porteus preferences that are of the non-expected utility type and were used by Weil (1989).

4. The two-country model

Our starting point is the economic framework described in Bekaert, Hodrick, and Marshall (1997) (hereafter BHM), which we detail in this section. As mentioned above, the main difference between our study and theirs is that our endowment processes and solution methods for the model Euler equations differ. These modifications will be discussed in this and the next section.

Let C_t^x denote the representative agent's consumption of the good produced in country x at time t and C_t^y his consumption of the good produced in country y at the same time period. We suppose that consuming involves real transaction costs. These are given by ψ_t^x for good x and ψ_t^y for good y. However, agents can reduce these costs by holding real money balances. Let M_{t+1}^x and M_{t+1}^y be the amounts of currencies of countries x and y respectively acquired by the agent at time t and held until time $t+1$. Defining P_t^x as the price of good x and P_t^y as that of good y, the transaction cost equations, evaluated in units of x and y respectively, are defined as

$$\psi_t^x \equiv \gamma\left(C_t^x\right)^{\nu}\left(\frac{M_{t+1}^x}{P_t^x}\right)^{1-\nu} \tag{3}$$

and

$$\psi_t^y \equiv \zeta\left(C_t^y\right)^{\xi}\left(\frac{M_{t+1}^y}{P_t^y}\right)^{1-\xi} \tag{4}$$

where $\nu > 1, \gamma > 0, \xi > 1, \zeta > 0$.

In addition to the currencies, agents can also hold n capital assets. We designate $z_{i,\,t+1}$ as the real value of the agent's investment in asset i, which pays off $R_{i,\,t+1}$ at time $t+1$. Finally, we denote W_t as the agent's wealth at the beginning of period t and J_t as the information available to the agent in the same time period. With this structure and with the retained preferences from equations (1) and (2), the maximum value function $V(W_t, J_t)$ can be written as:

$$V(W_t, J_t) = \underset{C_t^x, C_t^y, M_{t+1}^x, M_{t+1}^y, \{z_{i,\,t+1}\}}{MAX} \left\{\left(\left[C_t^x\right]^{\delta}\left[C_t^y\right]^{1-\delta}\right)^{\rho} + \beta(\mu[P(V(W_{t+1}, J_{t+1}))|J_t])^{\rho}\right\}^{\frac{1}{\rho}} \tag{5}$$

where $0 < \delta < 1, \rho < 1$. This maximization is subject to the individual's budget constraint given below.

Letting S_t denote the exchange rate, the budget constraint (evaluated in units of good x) is given by:

$$C_t^x + \psi_t^x + \frac{S_t P_t^y}{P_t^x}\left(C_t^y + \psi_t^y\right) + \sum_{i=1}^{n} z_{i,t+1} + \frac{M_{t+1}^x + S_t M_{t+1}^y}{P_t^x} \le W_t \tag{6}$$

with the wealth being defined as:

$$W_t = \sum_{i=1}^{N} R_{i,t} z_{i,t} + \frac{M_t^x + S_t M_t^y}{P_t^x}. \tag{7}$$

Solving the agent's problem—that is, maximizing the value function subject to the budget constraint, the wealth and transaction cost equations, as well as the market clearing conditions and purchasing power parity—we obtain the following set of Euler equations:[3]

$$E_t\{I_A(Z_{t+1})[Z_{t+1}^{\alpha} - 1]\} = 0 \tag{8}$$

$$E_t\left\{I_A(Z_{t+1})\left[Z_{t+1}^{\alpha}\frac{R_{i,t+1}}{R_{t+1}}\right]\right\} = E_t\{I_A(Z_{t+1})\} \tag{9}$$
$$\forall i = x, y, 1, \ldots, N$$

with

$$Z_{t+1} \equiv \left[\beta\left(\frac{C_{t+1}^x}{C_t^x}\right)^{\rho\delta - 1}\left(\frac{C_{t+1}^y}{C_t^y}\right)^{\rho(1-\delta)}\left(\frac{1 + \psi_{1t}^x}{1 + \psi_{1t+1}^x}\right)R_{t+1}\right]^{\frac{1}{\rho}}. \tag{10}$$

Here ψ_{it}^x is the derivative of the function ψ_t^x with respect to its ith argument, R_{t+1} is the real return on the market portfolio, and $I_A(Z)$ is an indicator function and defined as

$$I_A(Z) = \begin{Bmatrix} A & if & Z \ge 1 \\ 1 & if & Z < 1 \end{Bmatrix}. \tag{11}$$

Finally, the exchange rate is obtained from the first-order conditions and is given by

$$S_t = \frac{P_t^x}{P_t^y}\left(\frac{1 + \psi_{1t}^x}{1 + \psi_{1t}^y}\right)\left(\frac{C_t^x}{C_t^y}\right)\left(\frac{1 - \delta}{\delta}\right). \tag{12}$$

3. For the derivations of these equations, see BHM.

Equation (8) is the Euler equation for the market portfolio. Substituting the expression for Z_{t+1} and introducing the $I_B(Z)$ indicator function notation as

$$I_B(Z) = \begin{Bmatrix} 0 & if & Z < 1 \\ Z & if & Z \geq 1 \end{Bmatrix} , \tag{13}$$

the market portfolio equation becomes

$$\begin{gathered} E_t\left\{\left[\beta^{\frac{\alpha}{\rho}}\left(\frac{C^x_{t+1}}{C^x_t}\right)^{(\rho\delta-1)\frac{\alpha}{\rho}}\left(\frac{C^y_{t+1}}{C^y_t}\right)^{(1-\delta)\frac{\alpha}{\rho}}\left(\frac{1+\psi^x_{1t}}{1+\psi^x_{1t+1}}\right)^{\frac{\alpha}{\rho}}\right]\left(R^{\frac{\alpha}{\rho}}_{t+1}\right)-1\right\}+ \\ (A-1)E_t\left\{I_B\left(\left[\beta^{\frac{\alpha}{\rho}}\left(\frac{C^x_{t+1}}{C^x_t}\right)^{(\rho\delta-1)\frac{\alpha}{\rho}}\left(\frac{C^y_{t+1}}{C^y_t}\right)^{(1-\delta)\frac{\alpha}{\rho}}\left(\frac{1+\psi^x_{1t}}{1+\psi^x_{1t+1}}\right)^{\frac{\alpha}{\rho}}\right]\left(R^{\frac{\alpha}{\rho}}_{t+1}\right)-1\right)\right\} = 0 \end{gathered} \tag{14}$$

Similarly, the Euler equation for any asset i return is given by

$$\begin{gathered} E_t\left\{\left[\beta^{\frac{\alpha}{\rho}}\left(\frac{C^x_{t+1}}{C^x_t}\right)^{(\rho\delta-1)\frac{\alpha}{\rho}}\left(\frac{C^y_{t+1}}{C^y_t}\right)^{(1-\delta)\frac{\alpha}{\rho}}\left(\frac{1+\psi^x_{1t}}{1+\psi^x_{1t+1}}\right)^{\frac{\alpha}{\rho}}\right]\left(R^{\frac{\alpha}{\rho}-1}_{t+1}\right)R_{i,t+1}\right\}+ \\ (A-1)E_t\left\{I_B\left(\left[\beta^{\frac{\alpha}{\rho}}\left(\frac{C^x_{t+1}}{C^x_t}\right)^{(\rho\delta-1)\frac{\alpha}{\rho}}\left(\frac{C^y_{t+1}}{C^y_t}\right)^{(1-\delta)\frac{\alpha}{\rho}}\left(\frac{1+\psi^x_{1t}}{1+\psi^x_{1t+1}}\right)^{\frac{\alpha}{\rho}}\right]\left(R^{\frac{\alpha}{\rho}-1}_{t+1}\right)R_{i,t+1}-1\right)\right\} \\ = 1 \end{gathered} , \tag{15}$$

As for the returns to holding currencies, since money provides transaction services in the period it is acquired but is held until the next period, some loss to purchasing power occurs because of accrued inflation. Thus, the real return to holding the currencies of countries x and y are

$$R_{x,t+1} \equiv \left(\frac{P^x_t}{P^x_{t+1}}\right)\left(\frac{1}{1+\psi^x_{2t}}\right), \qquad R_{y,t+1} \equiv \left(\frac{S_{t+1}P^x_t}{S_t P^x_{t+1}}\right)\left(\frac{1}{1+\psi^y_{2t}}\right). \tag{16}$$

Finally, the continuously compounded nominal interest rates in this two-country economy are functions of the marginal cost functions and are given by

$$i_t^x = \ln\left(\frac{1}{1+\psi_{2t}^x}\right), \qquad i_t^y = \ln\left(\frac{1}{1+\psi_{2t}^y}\right). \tag{17}$$

The Endowment Process

In this section we explain the exogenous environment of the economy. Our purpose is to see whether explicitly allowing for changing means and variances in the endowment process will help improve the model fit.

For this first experiment, we use the simplest possible extension. We will assume that (1) each country's real dividend equals a proportion of its consumption (see Section 5), and (2) that there is no correlation between the endowment processes of the two countries.

Thus, for each country, we assume the existence of a bivariate exogenous endowment growth process g^l, $l = x, y$, with means and variances that change depending on the value taken by the state variable s_{t+1}. This state variable can take one of two values (1 or 2) and follows a Markov process so that information available at time t encompasses all previous periods' information. The stationary transition probability matrix for the probability of passing from one state to the next for country l is given by Φ^l with elements $\left[P_{ij}^l\right] = \left[P^l(s_{t+1} = j | s_t = i)\right]$ for $i, j = 1, 2$.

The bivariate process for each country is composed of the growth rates of the logarithms of real consumption and money supply. Thus, for $s_{t+1} = j$ and for country l, $l = x, y$, we have that

$$\ln\left(\frac{C_{t+1}^l}{C_t^l}\right) = \mu_j^{C,l} + \omega_j^{C,l}\varepsilon_{t+1}^{C,l} \tag{18}$$

and

$$\ln\left(\frac{M_{t+1}^l}{M_t^l}\right) = \mu_j^{M,l} + \omega_j^{M,l}\varepsilon_{t+1}^{M,l} \tag{19}$$

The residuals in the above equations, given by vector ε_{t+1}^l, are assumed to have a joint normal distribution with mean zero and correlation matrix ρ^l so that the process g^l is also normally distributed with mean μ_j^l and variance-covariance matrix Ω_j^l with non-zero off-diagonal terms.

Thus we have that

$$\begin{bmatrix} \varepsilon^{C,l}_{t+1} \\ \varepsilon^{M,l}_{t+1} \end{bmatrix} \sim N(0, \rho^l) \qquad with \qquad \rho^l = \begin{bmatrix} 1 & \rho^{CM,l} \\ \rho^{CM,l} & 1 \end{bmatrix} \tag{20}$$

so that $g_{t+1} \sim N\left(\mu^l_j, \Omega^l_j\right)$ with

$$g^l_{t+1} = \begin{bmatrix} \ln\left(\dfrac{C^l_{t+1}}{C^l_t}\right) \\ \ln\left(\dfrac{M^l_{t+1}}{M^l_t}\right) \end{bmatrix} \qquad and \qquad \Omega^l_j = \begin{bmatrix} \left(\omega^{C,l}_j\right)^2 & \omega^{CM,l}_j \\ \omega^{CM,l}_j & \left(\omega^{M,l}_j\right)^2 \end{bmatrix}. \tag{21}$$

5. Data and solution method

The purpose of this paper is to attempt to explain excess returns in the Japanese and U.S. equity markets, as well as the return on the US$/Yen exchange rate. We therefore require data on stock returns, spot and forward exchange rates, interest rates, as well as money supply and real consumption for both countries. In order to make our results comparable to those of BHM, we try to obtain data similar to theirs and make use of some of their calibrated parameters.

We obtain monthly equity returns data for the United States and Japan from Morgan Stanley Country Indexes and convert these to quarterly returns. Our monthly interest rate data for the United States and Japan are LIBOR 90-day Eurodollar and Euroyen rates respectively and are obtained from the International Financial Statistics (IFS) data base. From the same source, we also obtain end-of-period spot and 90-day forward dollar/yen exchange rates.

With regards to the exogenous environment, we obtain quarterly consumption and money supply data from the OECD and divide by population to obtain per capita data. Our quarterly U.S. consumption is defined as real U.S. consumption of non-durables and services. However, for lack of comparable data, we define Japanese consumption as the sum of real total government and private consumption. Naturally, this series is bound to be smoother than one describing only non-durables and services data for Japan. However, as pointed out by BHM, consumption data is, at any rate, only an approximation to endowment because it excludes locally produced goods that get exported to the rest of the world and includes imported goods. Finally, we use U.S. and Japanese quarterly deseasonalised nominal M1 data to characterize money supplies.

Although our exogenous process parameters are estimated, a few other parameters in the model are calibrated. These are the parameters related to preferences and the transaction cost technology parameters. With regards to taste parameters, we experiment with a range of possible values as given by the intervals $A \in \{1.0, 0.75, 0.5, 0.25\}$, $\alpha \in \{-1, -2\}$, $\delta = \{0.5, 0.8\}$, and $\rho \in \{-1, -3, -6, -9\}$. Finally, similar to BHM, we fix the subjective quarterly discount rate β to $(0.96)^{0.25}$.

To provide values for our transaction cost function parameters, we borrow the calibrated values for these from BHM. The authors calculated them by applying linear regression on the model's implications for money demand that were derived from equation (16). Thus, for the United States, we set $\gamma = 0.001$ and $\nu = 4.35$, while, for Japan, $\zeta = 0.017$ and $\xi = 2.01$.

Next, we describe a brief overview of our solution method for which the intuition is the following. The model implies expressions for the first and second moments of asset returns. These can be written as functions of a few state-dependent endogenous variables. Given particular preference and estimated endowment process parameter values, we numerically solve for the values of these endogenous variables in the different states. Using these in conjunction with the laws of motion specified in the endowment process, we can then simulate different financial returns series. From here, we obtain first and second moments that can subsequently be compared to those obtained from actual data for the same series.

In somewhat more detail, at first we estimate two different versions of two-state bivariate Markov-switching processes for both United States and Japan using maximum likelihood. The estimation period is 1975q1 to 1995q4. Model A is a two-mean, single-variance model while Model B also allows the variances to change with the state. At this stage, we do not try to describe the particular specification in this class of models that best fits the data. Rather, we are trying simply to assess whether introducing the simplest form of heteroscedasticity helps to improve the model fit. Tables 1 and 2 summarize the values of these estimates.

Table 1: Bivariate two-state Markov estimation results for the United States

	Model A Two-mean, one-variance		Model B Two-mean, two-variance	
	State 1	State 2	State 1	State 2
$\mu_j^{C, US}$	0.5784 (9.80)	0.2560 (3.94)	0.5872 (10.96)	0.2532 (3.05)
$\mu_j^{M, US}$	0.8403 (6.86)	2.1759 (6.13)	0.8607 (5.65)	2.1016 (11.46)
$\omega_j^{C, US}$	0.3808 (15.31)		0.3380 (9.28)	0.4802 (9.38)
$\omega_j^{M, US}$	0.7823 (21.90)		0.9139 (8.60)	1.0895 (9.79)
p_{11}^{US}	0.838 (24.55)		0.877 (15.2)	
p_{22}^{US}	0.880 (13.04)		0.906 (17.7)	
ρ^{US}	0.779 (17.26)		0.715 (11.6)	
U.S. llf	-11.5		2.4	

From these tables, we can see that the means of consumption and money supply growths are quite different in the two states.[4] Thus, average U.S. consumption growth in state 1 is almost twice its value in state 2 whereas the reverse is true in Japan. Average money growth, however, seems to exhibit a similar pattern in both countries.

Turning now to the value of the maximized log likelihood function for these models, it is clear that Model B, with the state-dependent variances, is the preferred model for both countries. More specifically, state 2 is the more volatile regime with Japan exhibiting a much greater difference in variances across the two states than the United States. Overall, then, state 1 is characterized by low means and low variances, while state 2 is the opposite.

4. Although one cannot apply the usual *t*-test in these types of models, the large values of these statistics for most of the estimated parameters indicate that they may well be significant.

Table 2: Bivariate two-state Markov estimation results for Japan

	Model A Two-mean, one-variance		Model B Two-mean, two-variance	
	State 1	State 2	State 1	State 2
$\mu_j^{C,\,JAP}$	0.4713 (4.79)	0.9956 (8.26)	0.3021 (3.24)	0.8635 (7.24)
$\mu_j^{M,\,JAP}$	0.6040 (5.41)	2.6514 (22.66)	0.7864 (6.31)	1.7237 (7.90)
$\omega_j^{C,\,JAP}$	0.6547 (12.23)		0.3809 (5.52)	0.6413 (9.36)
$\omega_j^{M,\,JAP}$	0.7268 (24.34)		0.5333 (5.24)	1.5186 (9.89)
p_{11}^{JAP}	0.662 (7.37)		0.916 (19.4)	
p_{22}^{JAP}	0.768 (12.07)		0.733 (9.09)	
ρ^{JAP}	-0.0585 (0.03)		0.052 (0.44)	
JAP llf	-111.8		-56.378	

These tables also report the values of the correlation coefficients between the growth rates of consumption and money supply for the United States and Japan. In general, the dynamics are quite different for each country. Thus, in the case of the United States, the growth rate in consumption and money exhibit a high degree of correlation over time. This is not the case for Japan where it would seem that the different variables evolve almost independently of each other.

It seems, then, that there are two quite distinct regimes with different means and variances for both countries. However, as mentioned above, it should be kept in mind that we have imposed a two-state model here and that it would have been more rigorous to have carried out various specification tests in order to choose the appropriate number of states.

Substituting these estimated parameter values for their expressions in the equilibrium Euler equations, and selecting values for the preference parameters from the specified grids above, we solve for the values of the state-dependent endogeneous variables. These are solved using numerical techniques. All asset returns can then be expressed as functions of these state-dependent endogenous variables.[5] Thus, the market portfolio real return is given by

$$R_{t+1} = \left(\frac{\lambda(S_{t+1})+1}{\lambda(S_t)}\right)\left(\frac{1+\psi_1^x(S_{t+1})}{1+\psi_1^x(S_t)}\right)\exp\left(\mu^{C,x}(S_{t+1})+\left(\omega^{C,x}(S_{t+1})\right)\varepsilon_{t+1}^{C,x}\right) \quad (22)$$

where λ_t is ratio of aggregate invested wealth at time t to real consumption inclusive of transaction costs in the same period, ψ_{1t}^x is the time t derivative of the cost function of country x with respect to its first argument, $\mu_{t+1}^{C,x}$ is the mean of consumption in country x at time $t+1$, and $\omega_{t+1}^{C,x}$ is its standard deviation for the same time period.

As for real equity returns for the two countries, remember that we have assumed that each country's real dividend is a proportion of its consumption. For our empirical analysis, we obtained an estimate of this proportion by regressing each country's real dividend series on consumption over our usual data span. For the United States, this estimate is 0.2 while it is 0.03 for Japan. Consequently, real equity returns for country x (the United States) and y (Japan) are respectively given by

$$R_{t+1}^x = \left(\frac{\varphi^x(S_{t+1})+0.2}{\varphi^x(S_t)}\right)\left(\frac{1+\psi_1^x(S_{t+1})}{1+\psi_1^x(S_t)}\right)\exp\left(\mu^{C,x}(S_{t+1})+\left(\omega^{C,x}(S_{t+1})\right)\varepsilon_{t+1}^{C,x}\right) \quad (23)$$

and

$$R^y{}_{t+1} = \left(\frac{\varphi^y(S_{t+1})+0.03}{\varphi^y(S_t)}\right)\left(\frac{1+\psi_1^x(S_{t+1})}{1+\psi_1^x(S_t)}\right)\exp\left(\mu^{C,x}(S_{t+1})+\left(\omega^{C,x}(S_{t+1})\right)\varepsilon_{t+1}^{C,x}\right) \quad (24)$$

where φ_t^x and φ_t^y are the respective ratios of country equity prices to their consumptions. Finally, the nominal risk-free rates, simply compounded, can be written as

$$i_t^x = \left(\frac{1}{1+\psi_2^x(S_t)}\right), \quad \text{and} \quad i_t^y = \left(\frac{1}{1+\psi_2^y(S_t)}\right) \quad (25)$$

5. See the appendix in this text, and that of BHM, for the details of the calculations.

while the forward premium can be obtained from the covered interest rate parity condition. That is,

$$\frac{F_t}{S_t} = \frac{1 + i_t^x}{1 + i_t^y}. \tag{26}$$

Accordingly, given the laws of motion specified in the endowment process, we can simulate these various returns and calculate moments.

6. Stylized facts

Let us briefly summarize some relevant stylized facts that will help us assess the model performance.

The substantial literature that exists in the field of asset pricing, and, more generally, of international asset pricing, has noted that excess returns are predictable in almost all types of assets and for many different countries. Moreover, when continuously compounded excess equity returns or excess forward returns are regressed on a forward premium of that country, the slope coefficient is normally negative.[6] These facts are not consistent with the joint hypotheses of market efficiency, rational expectations, and risk neutrality. However, if agents are assumed to be rational but not risk neutral, they will demand a premium for incurring additional risk in these markets. In this case, predictable excess returns can be interpreted as being equal to the risk premium in that market.

Define F_t as the dollar/yen 90-day forward rate and S_t as the dollar/yen spot exchange rate. Then, excess forward returns are defined as $(S_{t+1} - F_t)/S_t$. Similarly, excess equity returns for the United States and Japan are given respectively by $(R_{t+1}^{US} - i^{us})$ and $(R_{t+1}^{JAP} - i^{JAP})$. Finally, we define market excess returns, $(R_{t+1}^{W} - i_t^{US})$, according to the approximate relation given by $R_{t+1}^{W} - i_t^{US} \approx \frac{1}{2}\left[\left(R_{t+1}^{US} - i^{us}\right) + \{(R_{t+1}^{JAP} - i^{JAP}) + (S_{t+1} - F_t)/S_t\}\right]$, which would hold exactly if returns were continuously compounded over the considered period.[7]

6. Although other financial variables were also found to predict excess returns, the forward premium is frequently used as a regressor because it has good predictive power for many different asset markets.
7. See Bekaert, Hodrick, and Marshall (1997) for the log-form definition.

Table 3: Regression results with actual data

Dependent Variable	Constant c	Slope Coeff. b	R^2
$(S_{t+1} - F_t)/S_t$	14.14	-4.15	0.244
$R^w_{t+1} - i^{us}$	15.64	-2.89	0.105
$R^{US}_{t+1} - i^{us}$	14.75	-2.93	0.139
$R^{JAP}_{t+1} - i^{JAP}$	2.35	1.30	0.017

Table 3 reports the results of the regressions of excess returns on a constant and the forward premium, which is defined as $fp_t = (F_t - S_t)/S_t$. As expected, most of the slope coefficients are significantly negative and R-squares are relatively high. The only exception is the regression with Japanese data, which yields a positive sign for the slope for the sample period considered.[8] However, our regression for this country seems to be in concordance with most existing empirical works that suggest that the R^2 is lower for Japan than for other country excess returns. In the above regressions, the fitted values can be interpreted as the risk premiums associated with these markets.

In Table 4, we also report some descriptive statistics on the sample data. From here, we can see that the means of the various equity excess returns are around 6 per cent while those of the foreign exchange market are approximately 1 per cent. Similarly, we note that standard deviations of all of these returns are relatively high while the fitted values exhibit half (in the case of the foreign exchange returns) to one-eighth of the total variation in the data. Finally, the growth rate of the exchange rate yields a mean of 4 per cent and a standard error of 26 per cent.

8. This slope is, however, significantly negative in the sample considered by BHM (1976q1 to 1990q4).

Table 4: Descriptive statistics on sample data

Series	mean	std. error
$(S_{t+1} - F_t)/S_t$	1.24	27.93
fitted $(S_{t+1} - F_t)/S_t$	1.24	13.8
$R^{w}_{t+1} - i^{us}$	6.64	29.61
fitted $R^{w}_{t+1} - i^{us}$	6.64	9.61
$R^{US}_{t+1} - i^{us}$	5.63	26.11
fitted $R^{US}_{t+1} - i^{us}$	5.63	9.74
$R^{JAP}_{t+1} - i^{JAP}$	6.4	32.82
fitted $R^{JAP}_{t+1} - i^{JAP}$	6.4	4.32
$(S_{t+1} - S_t)/S_t$	4.35	26.44

All figures are annualized quarterly returns in percentage. Fitted values are from regressions of excess returns on the forward premium.

7. Model results

The purpose of our model is to explain the behaviour of excess returns in equity and foreign exchange markets. This means that not only are we attempting to explain the different levels of the variables of interest, but we are also concerned with the dynamic interactions between these. Thus, we are attempting to go a step further than typical studies in international finance, which are predominantly concerned with explaining the predictability of excess returns.

In order to get an overall sense of the strengths and weaknesses of our model and not to get entangled in details, we focus on two general measures. First, we examine the extent to which our generated forward premium can explain the dynamic facts of various generated excess returns with respect to predictability. This criterion should mostly capture the dynamic dimension of our model.

Second, we compare moments of returns obtained from the model to those documented in Table 4 above. In this case, we are concerned mainly with matching levels.

7.1 Implications of the model for excess return predictability

In order to assess the goodness of fit of our model through time, we examine the extent to which the model can predict excess returns. These results also permit us to gauge our model performance relative to others. This will be discussed in detail in Section 7.3.

For a given combination of preference parameters and for each of the returns of interest, we generate 3,000 series of 63 observations[9] with our model. For each replication, we regress the generated excess returns on the generated forward premium. We then collect various statistics, notably, the constant, the slope, and the R-squared value from these regressions, and calculate their distributions. We report the median values of these distributions for particular parameter combinations in Tables 5 to 8 below and compare these to the corresponding statistics tabulated in Table 3. In addition to the above, and to give a better idea of the model outcomes, we also report the p-value of sample data statistics in the model-generated distributions of these respective statistics.

Although we examine a wide variety of parameter combinations, in the tables below we report only a selected few. These include the case of the expected utility preferences (that is, $A = 1, \alpha = \rho$), and the two forms of generalized preferences: Kreps-Porteus preferences ($A = 1, \alpha \neq \rho$) and disappointment preferences ($A < 1, \alpha \neq \rho$). The latter are presented under the two scenarios for the evolution of the bivariate endowment process; the two-mean single-variance case (referred to as Model A), and the two-mean, two-variance situation (referred to as Model B). As for the remaining parameter combinations, we noticed, for example, that changing the value of α from -1 to -2 does not matter too much for the results. Similarly, varying the choice of the weighting parameter A is not extremely decisive for the outcomes. Having said this, we now turn to the tables.[10]

Looking ahead, results are quite encouraging on the whole. Thus, for all excess returns regressions and for most of the parameter combinations reported, actual regression constants and slopes fall inside the respective model-generated distributions. In addition, the model-generated medians often have the correct signs. Finally, R-squared values obtained from regressions on these generated series are also acceptable compared to their values calculated from regressions on actual data. We also find that, as we argued before, the two-variance specification yields substantially

9. The number of observations corresponds to that in our actual sample data.
10. Results for the non-reported parameter combinations are available upon request.

better results than the homoscedastic case, as do generalized preferences over the expected utility case.

Table 5: Selected results for U.S. excess equity returns; median of distribution & p-values

			Model A (Two-mean, one-variance)		Model B (Two-mean, two-variance)	
	Sample Regression Statistics		A=1	A=0.5	A=1	A=0.5
			EU	DA	EU	DA
$\alpha = -1$	14.75	c	5.7171	5.7568	9.013	9.598
$\rho = -1$			*(98.70)*	*(99.03)*	*(83.52)*	*(85.96)*
$\delta = 0.8$	-2.93	b	-0.3525	-0.2348	1.475	4.006
			(25.23)	*(24.23)*	*(32.84)*	*(25.96)*
	0.139	R-squared	0.0080	0.0080	0.0076	0.0088
			KP	DA	KP	DA
$\alpha = -1$	14.75	c	0.551	0.692	11.391	11.391
$\rho = -3$			*(97.20)*	*(96.57)*	*(65.76)*	*(65.68)*
$\delta = 0.8$	-2.93	b	-6.196	-6.245	-1.767	-1.781
			(82.07)	*(81.03)*	*(41.84)*	*(42.12)*
	0.139	R-squared	0.0054	0.0052	0.0095	0.0095
$\alpha = -1$	14.75	c	4.580	4.960	9.946	9.929
$\rho = -3$			*(93.93)*	*(94.03)*	*(77.32)*	*(79.16)*
$\delta = 0.5$	-2.93	b	-8.673	-7.626	-2.465	-2.194
			(59.47)	*(57.33)*	*(46.96)*	*(45.92)*
	0.139	R-squared	0.0083	0.0089	0.0089	0.0089

Model A is the two-mean, one-variance model while Model B is the two-mean, two-variance specification. The reported medians for c, b, and R-squared are the respective medians of the distributions of c, b, and R-squared obtained from the 3,000 regressions. The values in parentheses are the p-values of the actual data statistic in the relevant generated distribution. Where applicable, the notation “>”(“<”) indicates that the observed data sample statistic is larger (smaller) than the largest (smallest) model-generated statistic with that particular parameter combination. Preferences are defined as follows: EU is expected utility, KP is Kreps-Porteus, and DA is disappointment aversion.

In more detail, consider Table 5, which documents the regression outcomes for the U.S. excess equity returns. Compared with the single-variance case (all A models), we notice that the heteroscedastic endowment parameter combinations (all B models) help increase the values of both the constant and the slope to their observed levels (that is, to 14.75 and -2.93 respectively from Table 3). In addition, adopting generalized preferences further improves matters over the use of expected utility by yielding the correct sign for the slope as well. R-squared values also seem to improve, going from Model A to B, and from expected utility preferences to the generalized version. Finally, we note that, while there is some difference in results between Kreps-Porteus and disappointment-aversion preferences, these are not, in general, very substantial.

Table 6: Selected results for Japanese excess equity returns; median of distribution & p-values

			Model A (Two-mean, one-variance)		Model B (Two-mean, two-variance)	
	Sample Regression Statistics		A=1	A=0.5	A=1	A=0.5
			EU	DA	EU	DA
$\alpha = -1$	2.35	c	-33.411	101.41	9.725	11.237
$\rho = -1$			*(>)*	*(<)*	*(2.04)*	*(0.76)*
$\delta = 0.8$	1.30	b	-143.08	473.37	6.588	11.363
			(>)	*(<)*	*(29.72)*	*(16.96)*
	0.017	R-squared	0.8782	0.9127	0.0083	0.0173
			KP	DA	KP	DA
$\alpha = -1$	2.35	c	-8.311	-5.976	5.556	5.576
$\rho = -3$			*(98.30)*	*(94.73)*	*(25.60)*	*(27.00)*
$\delta = 0.8$	1.30	b	-25.072	-24.033	-21.797	-22.563
			(>)	*(>)*	*(99.68)*	*(99.68)*
	0.017	R-squared	0.0739	0.0667	0.113	0.117
$\alpha = -1$	2.35	c	8.479	10.031	6.242	6.928
$\rho = -3$			*(7.03)*	*(3.27)*	*(9.88)*	*(6.64)*
$\delta = 0.5$	1.30	b	13.430	18.304	-0.642	-0.739
			(15.67)	*(8.50)*	*(61.92)*	*(62.28)*
	0.017	R-squared	0.0182	0.0291	0.0054	0.0054

See notes in Table 5 for explanations of various symbols and notations.

Turning to the outcomes of the regressions on Japanese excess returns in Table 6, an examination of the p-values tells us, once again, that heteroscedastic endowments produce better results than their single-variance counterparts. Results also improve moving from expected utility to generalized preferences. Thus, for instance, the parameter combination in the second row comes closest to reproducing the observed value of 2.35 for the constant while the highest p-value for the slope is attained with the parameter set in row three. Indeed one can argue that the latter parameter combination seems to exhibit the best results overall, even if R-squared values are lower than with other cases. Interestingly, the Model A expected utility version yields particularly bad outcomes for the regression statistics (obviously, the obtained R-squared values cannot be relied on in this context). In addition, disappointment aversion does not improve matters at all. Finally, note that Kreps-Porteus preferences produce marginally better results than disappointment aversion in this Japanese excess returns case.

Table 7: Selected results for excess foreign exchange returns; median of distribution & P-values

			Model A (Two-mean, one-variance)		Model B (Two-mean, two-variance)	
	Sample Regression Statistics		A=1	A=0.5	A=1	A=0.5
			EU	DA	EU	DA
$\alpha = -1$	14.14	c	3.421	3.319	4.207	4.818
$\rho = -1$			*(99.47)*	*(99.73)*	*(95.48)*	*(96.84)*
$\delta = 0.8$	-4.15	b	1.274	1.288	0.239	2.907
			(0.80)	*(0.80)*	*(18.08)*	*(8.48)*
	0.244	R-squared	0.0152	0.0156	0.0079	0.0096
			KP	DA	KP	DA
$\alpha = -1$	14.14	c	-1.606	-1.216	2.254	2.247
$\rho = -3$			*(>)*	*(>)*	*(96.24)*	*(96.24)*
$\delta = 0.8$	-4.15	b	-3.195	-2.936	-1.993	-2.017
			(26.27)	*(23.83)*	*(11.80)*	*(12.56)*
	0.244	R-squared	0.0159	0.0137	0.0253	0.0256
$\alpha = -1$	14.14	c	1.902	2.671	2.848	3.014
$\rho = -3$			*(99.70)*	*(99.57)*	*(97.36)*	*(97.64)*
$\delta = 0.5$	-4.15	b	0.266	1.284	-2.749	-2.550
			(22.40)	*(14.00)*	*(32.64)*	*(30.48)*
	0.244	R-squared	0.0055	0.0070	0.0200	0.0186

See notes in Table 5 for explanations of symbols and notations.

Next, we examine the tabulated values for the foreign exchange excess returns, which are found in Table 7. The situation is similar to the previous two cases, with Model B specifications outperforming all Model A versions except for the case of Model A in row one, and with generalized preferences doing better than the expected utility case overall. In particular, disentagling α from ρ corrects the sign of the slope and helps increase R-squared values. However, for all reported cases, the value of the median of the model-generated constant is at least three times smaller than the observed constant in the regression that uses actual data.

Finally, with respect to the predictability of excess market returns, as evident from Table 8, the story is that, once again, the main improvement in results occurs when endowments are switched from a homoscedastic specification to a two-variance scenario. In addition, there is also the now-familiar correction in the sign of the slope when generalized preferences are adopted instead of the expected utility case.

Table 8: Selected results for excess world returns; median of distribution & P-values

			Model A (Two-mean, one-variance)		Model B (Two-mean, two-variance)	
	Sample regression statistics		A=1	A=0.5	A=1	A=0.5
			EU	DA	EU	DA
$\alpha = -1$	15.64	c	-11.846	55.816	11.799	12.990
$\rho = -1$			*(99.97)*	*(<)*	*(72.44)*	*(68.36)*
$\delta = 0.8$	-2.89	b	-71.213	237.80	4.611	9.876
			(>)	*(<)*	*(26.96)*	*(16.40)*
	0.105	R-squared	0.7646	0.9023	0.0076	0.0117
			KP	DA	KP	DA
$\alpha = -1$	15.64	c	-4.607	-3.291	10.126	10.167
$\rho = -3$			*(99.57)*	*(99.17)*	*(72.04)*	*(72.16)*
$\delta = 0.8$	-2.89	b	-17.358	-16.662	-12.615	-13.003
			(99.90)	*(99.83)*	*(88.80)*	*(89.24)*
	0.105	R-squared	0.0347	0.0316	0.0426	0.0440
$\alpha = -1$	15.64	c	7.528	8.753	10.088	10.423
$\rho = -3$			*(90.77)*	*(87.30)*	*(81.84)*	*(81.24)*
$\delta = 0.5$	-2.89	b	2.758	6.757	-2.677	-2.531
			(37.07)	*(29.53)*	*(48.84)*	*(48.20)*
	0.105	R-squared	0.0071	0.0087	0.0080	0.0080

See notes in Table 5 for explanations of symbols and notations.

Overall, then, results in this section have shown the complex interplay of preference and endowment process parameters. The most striking conclusions that have emerged from these tables is that it is necessary to impose heterocedastic endowment processes in conjunction with generalized preferences in order to be able to approximate fairly closely the observed excess returns regression statistics, and this with plausible parameter values.[11] This is the case for both the equity and the foreign exchange market. Indeed, it seems that the parameter combination in the last row,

11. In the literature, those models that had some success in reproducing the predictability of certain returns in the international context did so at the expense of calibrated values for the various risk parameters to levels that are regarded, in general, as being higher than plausible.

which features a relative risk-aversion coefficient equal to 2 (defined as $1-\alpha$) and less substitutability between present and future utility than that imposed by expected utility preferences,[12] is able to produce good results for the predictability of all the excess returns. Furthermore, we have found that there is no marked difference between adopting Kreps-Porteus or disappointment-aversion preferences. One gives slightly better results than the other, depending on the case examined.

So far, we put the emphasis on assessing the usefulness of our model for explaining excess returns and we have concluded that our model has good merit. But how does it perform with respect to matching the level of moments for different returns? This is the topic of the next section.

7.2 Implications of the model for moments of returns

A second way of evaluating our model performance is by comparing model-obtained moments of various returns to those obtained from the sample data. We are trying to see whether the parameter combination that was able to address the risk premium puzzle in the equity and foreign exchange markets can also yield the level of moments of the various financial variables equally satisfactorily.

Thus, for a particular parameter combination, and for each of the 3,000 generated series, we compute the mean and the standard error. We then calculate the "empirical" distribution of the means and that of the standard errors and check whether their medians are close to the statistics obtained from the sample data. In addition, to give a better sense of the model performance, once again we report the p-values of the actual sample data moments relative to the generated empirical distributions. If the model is a good representation of reality, the actual sample data mean and standard deviation should fall somewhere in these generated empirical distributions. Tables 9 and 10 below show the medians and p-values of the generated distributions.

Overall, results are quite good, although no one particular preference and endowment combination closely reproduces all of the means and the standard deviations observed in the sample data. Indeed, p-values under the Model B columns indicate that most sample means and standard deviations fall inside the respective generated distributions. Nevertheless, in a few cases, and especially with respect to the first moment, parameter combinations with the A specification yield better outcomes. Still, the combination that we designated as giving the best overall results for the dynamic dimension of excess returns (in the previous section) also performs fairly well in

12. The Epstein and Zin utility framework yields a slightly unconventional concept of intertemporal substitution in that it pertains to present and certainty equivalent of future utility as opposed to present and future consumption seen in standard functions.

reproducing the first and second moments of the series of interest. Having said this, let us now turn to Tables 9 and 10 and discuss the results in more detail.

Interestingly, the expected utility framework with a bivariate Markov heteroscedastic forcing process produces fairly good results for the means of excess returns. Notably, for Japan, the model-produced mean is 7.74 per cent per annum compared with 6.40 per cent in the sample data. In addition, for the United States, Table 9 reads a value of 8.97 per cent for mean excess returns, which is not too far from the 5.63 per cent obtained in the data (the rank of the latter in the distribution of sorted means is around the 12th percentile). This model also produces acceptable values for variables in the foreign exchange market. In particular, the mean growth rate of the exchange rate is at 3.86 per cent, which is extremely close to its value in the sample data of 4.35 per cent. The mean foreign exchange excess return, however, is more than three times higher than its value in the data. Not surprisingly, then, the median of the means of market excess returns is also higher (at 10.56 per cent) than in the data (with a value of 6.64 per cent).

These results are not much different from results in the third row for the generalized preferences case under the two-variance scenario. While the mean of Japanese excess equity returns is better with this parameter combination, that for the United States is higher than with the previous parameter combination, and the excess foreign exchange mean return is about the same as before.

Market excess returns have an even higher mean than under the expected utility case. Still, the latter produces a p-value of about 13, indicating that the sample statistic is still a distance from the tail of the generated distribution. Given that the mean growth rate produced with this parameter set is also not too different from its observed value, we could conclude that the preferred parameter and endowment combination in the regression experiments in Section 7.1 also yields fairly close values for the means of these returns. Surprisingly, the choice of the value of the aggregation parameter δ turns out to be important in producing this result, since outcomes in row two of the table are not as good.

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Table 9: Medians of distributed means of returns

			Model A (Two-mean, one-variance)		Model B (Two-mean, two-variance)	
		Sample Means	A=1	A=0.5	A=1	A=0.5
			EU	DA	EU	DA
$\alpha = -1$	xsFX	1.24	2.499 (3.90)	2.448 (4.73)	4.178 (0.36)	4.094 (0.44)
$\rho = -1$	xsRW	6.64	40.587 (<)	-106.81 (>)	10.556 (14.68)	10.792 (13.00)
$\delta = 0.8$	xsRUS	5.63	6.118 (37.73)	6.058 (39.07)	8.968 (12.08)	8.864 (12.32)
	xsRJAP	6.40	72.322 (<)	-222.486 (>)	7.741 (34.80)	8.401 (25.80)
	DELS	4.35	1.764 (96.90)	1.768 (96.87)	3.862 (61.00)	3.870 (60.92)
			KP	DA	KP	DA
$\alpha = -1$	xsFX	1.24	3.120 (0.07)	3.111 (0.07)	4.988 (0.04)	4.970 (0.04)
$\rho = -3$	xsRW	6.64	21.096 (<)	21.348 (<)	25.543 (<)	25.779 (<)
$\delta = 0.8$	xsRUS	5.63	10.204 (13.23)	10.221 (13.97)	14.240 (0.40)	14.236 (0.40)
	xsRJAP	6.40	28.685 (<)	29.184 (<)	31.595 (<)	32.056 (<)
	DELS	4.35	1.656 (97.70)	1.656 (97.70)	3.832 (61.52)	3.833 (61.56)
$\alpha = -1$	xsFX	1.24	2.004 (17.43)	1.990 (17.97)	4.293 (0.36)	4.253 (0.36)
$\rho = -3$	xsRW	6.64	6.990 (44.97)	7.015 (44.13)	11.521 (12.96)	11.667 (11.88)
$\delta = 0.5$	xsRUS	5.63	7.912 (13.23)	7.805 (13.97)	11.399 (3.80)	11.193 (3.84)
	xsRJAP	6.40	3.800 (89.13)	4.045 (86.90)	6.694 (46.12)	7.350 (40.52)
	DELS	4.35	1.647(97.63)	1.651 (97.63)	3.855 (61.12)	3.858 (61.00)

xsRW, xsRUS, xsRJAP denote excess returns in the Market, U.S. and Japanese equity markets while xsFX is the excess foreign exchange return. DELS is the growth rate of the exchange rate. Where applicable, the notation ">"("<") indicates that the observed data sample statistic is larger (smaller) than the largest (smallest) model-generated statistic with that particular parameter combination. Preferences are defined as follows: EU is expected utility, KP is Kreps-Porteus and DA is dissapointment aversion.

Comparing results across the two endowment specifications, from the third row of the table, it is clear that a homoscedastic forcing process in conjunction with generalized preferences will produce first moment values that are almost half their amounts under the heteroscedastic scenario, and, in most cases, closer to their observed values in actual sample data. For example, foreign exchange excess returns exhibit a mean of 2 per cent, the mean of U.S. excess equity returns is 7.9 per cent, while that for Japan is only 3.8 per cent. Even the mean growth rate of the exchange rate is halved in this case. These observations point to the fact that a bivariate endowment process with changing mean is sufficient to produce adequate first moments of excess returns in many cases.

Finally, we remark that, regardless of the endowment process, disappointment aversion decreases the mean of foreign exchange excess returns while, in general, increasing that of excess equity returns (although these changes are small in magnitude).

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Table 10: Medians of distributed standard deviations of returns

			Model A (Two-mean, one-variance)		Model B (Two-mean, two-variance)	
		Sample Std. err	A=1	A=0.5	A=1	A=0.5
			EU	DA	EU	DA
$\alpha = -1$	xsFX	27.93	5.340 (99.80)	5.350 (99.80)	8.632 (95.04)	8.659 (95.08)
$\rho = -1$	xsRW	29.61	30.413 (35.27)	96.353 (<)	21.713 (76.08)	21.817 (75.84)
$\delta = 0.8$	xsRUS	26.11	10.813 (98.83)	10.805 (98.83)	17.867 (79.04)	17.956 (78.96)
	xsRJAP	32.82	57.308 (<)	191.818 (<)	18.226 (90.48)	18.376 (98.40)
	DELS	26.44	5.387 (99.60)	5.396 (99.60)	8.625 (94.36)	8.652 (94.36)
			KP	DA	KP	DA
$\alpha = -1$	xsFX	27.93	5.054 (99.80)	5.054 (99.80)	8.612 (95.08)	8.613 (95.12)
$\rho = -3$	xsRW	29.61	18.896 (92.07)	19.066 (91.97)	35.453 (32.16)	35.601 (31.80)
$\delta = 0.8$	xsRUS	26.11	17.471 (86.57)	17.528 (86.40)	27.484 (46.12)	27.562 (45.76
	xsRJAP	32.82	18.486 (93.03)	18.669 (93.03)	38.587 (34.04)	38.840 (33.48)
	DELS	26.44	5.029 (99.70)	5.032 (99.70)	8.536 (94.28)	8.540 (94.28)
$\alpha = -1$	xsFX	27.93	5.026 (99.80)	5.038 (99.80)	8.689 (94.56)	8.694 (94.60)
$\rho = -3$	xsRW	29.61	13.312 (98.17)	13.260 (98.23)	26.095 (60.40)	25.853 (61.00)
$\delta = 0.5$	xsRUS	26.11	14.812 (90.87)	14.469 (92.37)	24.445 (54.92)	23.956 (56.88)
	xsRJAP	32.82	10.201 (99.63)	10.340 (99.67)	21.328 (81.56)	21.517 (81.56)
	DELS	26.44	5.029 (99.73)	5.037 (99.73)	8.646 (94.00)	8.656 (94.04)

xsRW, xsRUS, xsRJAP denote excess returns in the Market, U.S. and Japanese equity markets while xsFX is the excess foreign exchange return. DELS is the growth rate of the exchange rate. Where applicable, the notation “>”(“<”) indicates that the observed data sample statistic is larger (smaller) than the largest (smallest) model-generated statistic with that particular parameter combination. Preferences are defined as follows: EU is expected utility, KP is Kreps-Porteus and DA is dissapointment aversion.

We now analyze the results in Table 10, which report outcomes for the variability of our selected series. We argued, in the earlier sections of the paper, that the enrichment of both preference and endowment functions were motivated primarily by the desire to increase the variability of the endogenous stochastic discount rate of the model. An examination of the results in this table shows that this is indeed the case. On the one hand, moving from Model A to Model B almost doubles the standard errors of the different variables, thus yielding values that are closer to those observed in the actual sample data. That is, given generalized preferences, enriching the forcing process by rendering it heteroscedastic induces substantial fluctuations in the SDF, which, in turn, leads to important increases in the variances of all of the financial variables examined. On the other hand, comparing the first row to the remaining two, except for the case of foreign exchange variables, standard errors generally increase by an important quantity here as well. Also, as expected, decreasing the value of the preference parameter A from 1 to 0.5 will, in general, increase variances. Still, in terms of magnitude, the latter factor is the least important among the three aspects considered. The above facts are further confirmed when one also examines the p-values of the sample data statistics in their respective model-generated distributions.

As for the foreign exchange variables, the highest generated standard deviation for foreign exchange excess returns is of the order of 8.694, which is close to one-third of its value in the sample data. The situation is similar for the standard error of the growth rate of the exchange rate. Nevertheless, these results are highly encouraging given the unrealistic assumption of short-run purchasing power parity that is imposed in the model.

On the whole, then, results from this section confirm the expected and reveal that it takes the joint assumption of heteroscedastic endowments and generalized preferences to produce the most satisfactory outcomes, especially for the standard deviations of returns.

7.3 Model performance relative to BHM and discussion

With respect to excess returns predictability, our model performs much better than BHM. For all parameter combinations considered, and for regressions similar to ours, these authors obtain only negligible R-squared values. In addition, their model-generated slope coefficients are much smaller in magnitude than those obtained with their sample data. While they show that the variances of ex ante risk premiums do increase as first-order risk aversion increases (that is, as A decreases), given that the model-generated slopes are non-monotonic in A, this does not necessarily lead to increases in the slope magnitudes. On the basis of these results, these authors conclude that a risk-based explanation for the predictability of excess asset returns is not sufficient.

Based on our own results, we concur with BHM in saying that simply increasing the degree of aversion to small lotteries is insufficient to explain the predictability of excess returns. Indeed, this dimension of preferences seems to be the least important factor in enhancing the model performance. Intertemporal substitutability, overall attitudes towards risk, the way consumption goods are aggregated, and the amount of heteroscedasticity in the exogenous environment all contribute to the outcomes of the model. We were able to get these additional insights into the workings of the international asset-pricing model by following a different strategy than BHM. By adopting Markov-switching endowments in our model, integrating these in our Euler equations, and obtaining full analytical forms for these first-order conditions, we were able to shed some light on the respective roles of various parameters in the model. In particular, we show that it is possible to obtain satisfactory results by simultaneously adopting preferences that are of the generalized type and by assuming that the forcing process is heteroscedastic (hereafter, we will refer to this specification as our most general framework).

This is also true when it comes to matching moments, although homoscedastic endowments in conjunction with generalized preferences are sufficient to reproduce first moments quite closely. As expected, with our most general specification, standard deviations are substantially increased and attain values that are generally quite close to those obtained from the sample data. For example, we obtain a mean value of 25.85 for the standard deviation of market excess equity returns, which is quite close to the value of 29.61 observed in the sample data and approximately four times higher than the corresponding outcome in BHM. In addition, the latter obtain a value of 0.94 per cent for the first moment of these returns while our result is 11.67 per cent for this same statistic.

Nevertheless, one aspect of the model that could be improved is the foreign exchange market. Indeed, our most general parameter scenario produces a higher mean (4.29 compared with 1.24) and a smaller standard deviation (8.69 relative to 28) than in the sample data. In addition, because the standard deviation of the exchange rate growth rate also falls a bit short (8.6 compared with 26 in the data), we concur with BHM in saying that model assumptions need to be modified before better results can be obtained for these variables.

In both our views, a crucial assumption in the model is the adoption of purchasing power parity (PPP), which determines the level of the exchange rate in the model. Yet, numerous empirical studies have shown that this parity does not hold in the data for relatively short spans like ours. If PPP did not hold, agents in the different countries would evaluate their consumption bundles differently because of the varying relative prices of goods in their respective countries. This would result in a different value for the exchange rate, and therefore of excess foreign exchange returns. In turn, this has implications for the means of the growth rate of the exchange rate and for the foreign exchange excess returns.

As for the variabilities of these series, while these are indeed increased with the heteroscedasticity in the exogenous process, we feel that two explanations are possible for why we do not attain sample statistic values. First, even if the joint money and consumption growth process is better characterized by a two-state heteroscedastic series, it remains that all the shocks that are driving the returns do not necessarily originate from these two sources. Fiscal shocks may be just as important in shaping the behaviour of returns. For the moment, there is no fiscal side to our two-country economies. Yet, the study by Canova and Marrinan (1995) has shown that variation in the volatility of fiscal aggregates in their model contributes substantially to generating variability in their simulated returns series. Second, there is evidence that, on the one hand, the premium in the exchange market is quite distinct from that in equity markets, and on the other, that countries are segmented by price-discriminating monopolistic firms. Both of these lead to the conclusion that the various types of structural uncertainties in the model apply differently to these markets. For the moment, our model evaluates all returns in the same fashion, through a rich, time-varying, but identical, stochastic discount rate, and all risks are pooled equally between the two countries. Yet, this may not be an appropriate assumption as BHM also point out.

Finally, one caveat worth mentioning is that the time span considered for our stylized facts and for estimating the forcing parameters includes the transition period of 1978 to 1982. This is the period during which the growth of money supply was a policy instrument for most countries and was kept relatively smooth. On the other hand, interest rates were left to fluctuate to reflect nominal shocks in the economy. Since, in our theoretical model, all nominal innovations are generated in the money supply process and get transmitted into the economy via the quantity equation, our model perhaps encounters some difficulty in generating the amount of fluctuations in foreign exchange variables over this particular period.

8. Conclusion

In this paper, we use Markov-switching endowment processes in the two-country transaction cost model of Bekaert, Hodrick, and Marshall (1997) with disappointment-aversion-type preferences. We analytically integrate our assumed processes in the Euler equations of the model, solve these numerically for the various shadow prices and the endogeneous money velocities, and using these, simulate series of financial returns for various assets.

We show that many factors, interacting in a non-linear fashion, need to be united in order for an international asset-pricing model of this type to yield good results. By good results, we mean that the model can approximately reproduce, on the one hand, the observed predictability of excess returns using the forward rate, and, on the other, match the means and the standard deviations of various financial variables. We show that simply changing preferences from expected utility to a general recursive one is not enough. Nor is it sufficient to change only the endowment process. In fact, we show that it is necessary to adopt generalized preferences jointly with heteroscedastic endowments in order to generate the level of second moments of excess returns observed in the data, in addition to their predictability. Nevertheless, standard errors of variables in the foreign exchange market are not as close to the stylized facts as is the case with the other variables, and this is regardless of the combination of parameters retained.

With its present set of assumptions, our model results might well improve if we used better data for Japanese consumption, estimated our preference parameters, and integrated fiscal shocks to the model. Nevertheless, we feel that we would not be able to reconcile more closely outcomes for the foreign exchange market variables with their stylized facts unless the purchasing power parity assumption was abandoned or some form of agent heterogeneity was considered. These aspects are left open for future research.

Appendix

Derivation of relationships reported in Section 5

Euler equation for market portfolio

The real return to holding the market portfolio is given by

$$R_{t+1} = \frac{\hat{W}_{t+1} + \overline{C_{t+1}}}{\hat{W}_t} \tag{B.1}$$

where $\hat{W}_t$ is aggregate invested wealth and $\overline{C_{t+1}}$ is the real time t+1 payoff to this investment. Using Euler's theorem and defining $\lambda_{t+1} = \hat{W}_{t+1}/\overline{C_{t+1}}$, we have that

$$\frac{\overline{C_{t+1}}}{\overline{C_t}} = \left(\frac{C^x_{t+1}}{C^x_t}\right)\left(\frac{1+\psi^x_{1t+1}}{1+\psi^x_{1t}}\right) \tag{B.2}$$

so that the market return is written

$$R_{t+1} = \left(\frac{\lambda_{t+1}+1}{\lambda_t}\right)\cdot\left(\frac{C^x_{t+1}}{C^x_t}\right)\left(\frac{1+\psi^x_{1t+1}}{1+\psi^x_{1t}}\right). \tag{B.3}$$

Replacing terms for their expressions and knowing that, at equilibrium, all output must be consumed or spent as transaction costs, the market portfolio equation (14), in its simplified version, is written as

$$\begin{aligned} &E_t\left\{\beta^{\frac{\alpha}{\rho}}\left(\frac{C^x_{t+1}}{C^x_t}\right)^{\alpha\delta}\left(\frac{C^y_{t+1}}{C^y_t}\right)^{(1-\delta)\frac{\alpha}{\rho}}\left(\frac{\lambda_{t+1}+1}{\lambda_t}\right)^{\frac{\alpha}{\rho}} - 1\right\} + \\ &(A-1)E_t\left\{I_B\left(\beta^{\frac{\alpha}{\rho}}\left(\frac{C^x_{t+1}}{C^x_t}\right)^{\alpha\delta}\left(\frac{C^y_{t+1}}{C^y_t}\right)^{(1-\delta)\frac{\alpha}{\rho}}\left(\frac{\lambda_{t+1}+1}{\lambda_t}\right)^{\frac{\alpha}{\rho}} - 1\right)\right\} = 0 \end{aligned} \tag{B.4}$$

Now, re-writing the various variables in terms of state-dependent expressions, for $s_t = i$ and $s_{t+1} = j$, the above equation becomes

$$\beta^{\frac{\alpha}{\rho}} \cdot E_t\left\{\exp\left(a_j^x + a_j^y\right)\left(\frac{\lambda_j + 1}{\lambda_i}\right)^{\frac{\alpha}{\rho}} - 1\right\} + (A-1)E_t\left(I_B\left[\beta^{\frac{\alpha}{\rho}}\exp\left(b_j^x + b_j^y\right)\left(\frac{\lambda_j + 1}{\lambda_i}\right)^{\frac{\alpha}{\rho}} - 1\right]\right) = 0 \tag{B.5}$$

where the abbreviations stand for the following:

$$\begin{aligned} a_j^x &= \alpha\delta\mu_j^{C,x} + \frac{\alpha^2\delta^2\left[\omega_j^{C,x}\right]^2}{2} \\ a_j^y &= (1-\delta)\frac{\alpha}{\rho}\mu_j^{C,y} + \left(\frac{(1-\delta)^2\alpha^2}{2\rho^2}\right)\left[\omega_j^{C,y}\right]^2 . \\ b_j^x &= \alpha\delta\left(\mu_j^{C,x} + \omega_j^{C,x}\varepsilon_{t+1}^{C,x}\right) \\ b_j^y &= \frac{(1-\delta)\alpha}{\rho}\left(\mu_j^{C,y} + \omega_j^{C,y}\varepsilon_{t+1}^{C,y}\right) \end{aligned} \tag{B.6}$$

We now define z_j as

$$z_j = \alpha\delta\omega_j^{C,x}\varepsilon_{t+1}^{C,x} + \left(\frac{(1-\delta)\alpha}{\rho}\right)\omega_j^{C,y}\varepsilon_{t+1}^{C,y} \tag{B.7}$$

with marginal density $f(z_j)$ given by

$$f(z_j) = \frac{1}{\sqrt{2\pi}\cdot\sigma_j}\cdot\exp\left(\frac{-z_j^2}{2\sigma_j^2}\right) \tag{B.8}$$

and variance

$$\sigma_j^2 = \left(\alpha\delta\omega_j^{C,x}\right)^2 + \left(\left(\frac{(1-\delta)\alpha}{\rho}\right)\omega_j^{C,y}\right)^2 . \tag{B.9}$$

Replacing these terms in equation (22) and after some algebraic manipulations, the market portfolio equation becomes

$$(\beta^{\alpha/\rho}) \cdot \sum_{j=0}^{K} p_{ij} \left(\exp\left(a_j^x + a_j^y\right)\left(\frac{\lambda_j + 1}{\lambda_i}\right)^{\frac{\alpha}{\rho}} \right) + (A-1) \cdot \sum_{j=0}^{K} p_{ij} \left\{ \kappa_j \int_{B(i,j)}^{\infty} [c_j \exp(z_j) - 1] f(z_j) dz_j \right\} = 0 \tag{B.10}$$

where the state dependent constant c_j is given by

$$c_j = \beta^{\frac{\alpha}{\rho}} \exp\left(\alpha\delta\mu_j^{C,x} + (1-\delta)\frac{\alpha}{\rho}\mu_j^{C,y}\right)\left(\frac{\lambda_j + 1}{\lambda_i}\right)^{\frac{\alpha}{\rho}} \tag{B.11}$$

and the term κ_j is defined as

$$\kappa_j = \left(\frac{(1-\delta)\alpha}{\rho}\omega_j^{C,y}\right), \tag{B.12}$$

and where $B(i, j)$ is the lower limit for z_j and is given by

$$B(i,j) = -\frac{\alpha}{\rho}\log(\beta) - \alpha\delta\mu_j^{C,x} - (1-\delta)\frac{\alpha}{\rho}\mu_j^{C,y} - \frac{\alpha}{\rho}\log\left(\frac{\lambda_j + 1}{\lambda_i}\right). \tag{B.13}$$

Euler equations for equity portfolios

Following the above steps and defining φ_{t+1}^{x} and φ_{t+1}^{y} as the respective ratios of country equity prices to their dividends, both evaluated in real terms, it can be shown that the country x equity portfolio can be written in terms of state-dependent variables as:

$$\begin{gathered}(\beta^{\alpha/\rho})\cdot\sum_{j=0}^{K}p_{ij}\left(\exp\left(a_j^x+a_j^y\right)\left(\frac{\lambda_{j+1}}{\lambda_i}\right)^{\frac{\alpha}{\rho}-1}\left(\frac{\varphi_{j+1}^x}{\varphi_i^x}\right)\right)\\ +\\ (A-1)\cdot\sum_{j=0}^{K}p_{ij}\left\{\kappa_j\int_{B(i,j)}^{\infty}\left[c_j^x\exp(z_j)-1\right]f(z_j)dz_j\right\}\\ =1\end{gathered} \tag{B.14}$$

where the state dependent constant c_j^x is given by

$$c_j^x = \beta^{\frac{\alpha}{\rho}}\exp\left(\alpha\delta\mu_j^{C,x}+(1-\delta)\frac{\alpha}{\rho}\mu_j^{C,y}\right)\left(\frac{\lambda_{j+1}}{\lambda_i}\right)^{\frac{\alpha}{\rho}-1}\left(\frac{\varphi_{j+1}^x}{\varphi_i^x}\right). \tag{B.15}$$

The country *y* equity portfolio equation, in turn, is given by

$$\begin{gathered}(\beta^{\alpha/\rho})\cdot\sum_{j=0}^{K}p_{ij}\left(\exp\left(a_j^x+a_j^y\right)\left(\frac{\lambda_{j+1}}{\lambda_i}\right)^{\frac{\alpha}{\rho}-1}\left(\frac{\varphi_{j+1}^y}{\varphi_i^y}\right)\right)\\ +\\ (A-1)\cdot\sum_{j=0}^{K}p_{ij}\left\{\kappa_j\int_{B(i,j)}^{\infty}\left[c_j^y\exp(z_j)-1\right]f(z_j)dz_j\right\}\\ =1\end{gathered} \tag{B.16}$$

where the state dependent constant c_j^y is given by

$$c_j^y = \beta^{\frac{\alpha}{\rho}}\exp\left(\alpha\delta\mu_j^{C,x}+(1-\delta)\frac{\alpha}{\rho}\mu_j^{C,y}\right)\left(\frac{\lambda_{j+1}}{\lambda_i}\right)^{\frac{\alpha}{\rho}-1}\left(\frac{\varphi_{j+1}^y}{\varphi_i^y}\right). \tag{B.17}$$

Euler equations for currency portfolios

The Euler equation for the return on holding the country x currency is defined next. Given the cost function in equation (3), its derivative with respect to the first and second arguments are $\psi_{1t}^{x} = \gamma\nu\left(v_t^x\right)^{\nu - 1}$ and $\psi_{2t}^{x} = \gamma(1-\nu)\left(v_t^x\right)^{\nu}$, where v_t^x is the money velocity of country x at time t. Using these definitions, define the term V_{ij}^{x} as

$$V_{ij}^{x} = \left(\frac{v_i^x}{v_j^x}\right)\left(\frac{1+\psi_{1i}^{x}}{1+\psi_{1j}^{x}}\right)\left(\frac{1}{1+\psi_{2i}^{x}}\right). \tag{B.18}$$

Given the definition of returns to currency in equation (16) and using the above equations, the country x currency holdings Euler equation is written

$$\begin{aligned}
&(\beta^{\alpha/\rho}) \cdot \sum_{j=0}^{K} p_{ij}\left(\exp\left(m_j^x + a_j^y\right)\left(\frac{\lambda_j + 1}{\lambda_i}\right)^{\frac{\alpha}{\rho} - 1}\left(V_{ij}^{x}\right)\right) \\
&+ \\
&(A-1) \cdot \sum_{j=0}^{K} p_{ij}\left\{h_j^x \int_{B(i,j)}^{\infty} c_j^{cx} \exp\left(z_j + \frac{\left[-2\alpha\delta\omega_j^{C,x}\omega_j^{M,x}\rho^{CM,x}\right]z_j}{\sigma_j^2}\right) f(z_j)dz_j\right\} \\
&- \\
&(A-1) \cdot \sum_{j=0}^{K} p_{ij}\left\{\kappa_j \int_{B(i,j)}^{\infty} f(z_j)dz_j\right\} \\
&= 1
\end{aligned} \tag{B.19}$$

where the abbreviated terms are

$$m_j^x = \exp\left(\alpha\delta\mu_j^{C,x} - \mu_j^{M,x} + \frac{\alpha^2\delta^2\left[\omega_j^{C,x}\right]^2 + \left[\omega_j^{M,x}\right]^2 - 2\alpha\delta\omega_j^{C,x}\omega_j^{M,x}\rho^{CM,x}}{2}\right), \tag{B.20}$$

$$h_j^x = \kappa_j \exp\left(\frac{\left(-\kappa_j\omega_j^{M,x}\rho^{CM,x}\right)^2}{\sigma_j^2}\right) \tag{B.21}$$

and where the state dependent variable c_j^{cx} is given by

$$c_j^{cx} = \beta^{\frac{\alpha}{\rho}}\left(\exp\left(\alpha\delta\mu_j^{C,x} + (1-\delta)\frac{\alpha}{\rho}\mu_j^{C,y} - \mu_j^{M,l} + \frac{\left(\omega_j^{M,x}\right)^2\left[1-(\rho^{CM,x})^2\right]}{2}\right)V_{ij}^x\left(\frac{\lambda_j+1}{\lambda_i}\right)^{\frac{\alpha}{\rho}-1}\right) \quad \text{(B.22)}$$

As for the country *y* currency portfolio Euler equation, given the cost function in equation (3), its derivative with respect to the first argument is

$$\psi_{1t}^y = \zeta\xi\left(v_t^y\right)^{\xi-1} \quad \text{(B.23)}$$

and its derivative with respect to the real money balances is

$$\psi_{2t}^y = \zeta(1-\xi)\left(v_t^y\right)^{\xi} \quad \text{(B.24)}$$

where v_t^y is the money velocity of country y at time t. Using these definitions, as before, define the term V_{ij}^y as

$$V_{ij}^y = \left(\frac{v_i^y}{v_j^y}\right)\left(\frac{1+\psi_{1i}^y}{1+\psi_{1j}^y}\right)\left(\frac{1}{1+\psi_{2i}^y}\right). \quad \text{(B.25)}$$

Given the definition of returns to the currency y in equation (15) and using the above expressions, the country *y* currency holdings Euler equation is written

$$(\beta^{\alpha/\rho}) \cdot \sum_{j=0}^{K} p_{ij}\left(\exp\left(m_j^y + a_j^x\right)\left(\frac{\lambda_j+1}{\lambda_i}\right)^{\frac{\alpha}{\rho}-1}\left(V_{ij}^y\right)\right)$$

$$+$$

$$(A-1) \cdot \sum_{j=0}^{K} p_{ij}\left\{h_j^y\int_{B(i,j)}^{\infty} c_j^{cy}\exp\left(z_j + \frac{\left[-2\alpha(1-\delta)\omega_j^{C,y}\omega_j^{M,y}\rho^{CM,y}\right]z_j}{\rho\sigma_j^2}\right)f(z_j)dz_j\right\} \quad \text{(B.26)}$$

$$-$$

$$(A-1) \cdot \sum_{j=0}^{K} p_{ij}\left\{\kappa_j\int_{B(i,j)}^{\infty} f(z_j)dz_j\right\}$$

$$= 1$$

where the abbreviated terms are

$$m_j^y = \exp\left((1-\delta)\frac{\alpha}{\rho}\mu_j^{C,y} - \mu_j^{M,y} + \frac{(1-\delta)^2\alpha^2\left[\omega_j^{C,y}\right]^2 + \left[\omega_j^{M,y}\right]^2 - 2\alpha(1-\delta)\omega_j^{C,y}\omega_j^{M,y}\rho^{CM,y}}{2\rho^2}\right) \quad \text{(B.27)}$$

and

$$h_j^y = \kappa_j \exp\left(\frac{\left(-2\alpha\delta\omega_j^{C,x}\omega_j^{M,y}\rho^{CM,y}\right)^2}{2\sigma_j^2}\right) \quad \text{(B.28)}$$

and where the state dependent variable c_j^{cy} is given by

$$c_j^{cy} = \beta^{\frac{\alpha}{\rho}} \exp\left(\alpha\delta\mu_j^{C,x} + (1-\delta)\frac{\alpha}{\rho}\mu_j^{C,y} - \mu_j^{M,y} + \frac{\left(\omega_j^{M,y}\right)^2\left[1-(\rho^{CM,y})^2\right]}{2}\right) \cdot V_{ij}^y \cdot \left(\frac{\lambda_j + 1}{\lambda_i}\right)^{\frac{\alpha}{\rho} - 1} . \quad \text{(B.29)}$$

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