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An Up-to-Date and Improved BVAR Model of the Canadian Economy

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CONTENTS

ABSTRACT

In this paper, we estimate a fully optimized BVAR model of the Canadian economy for the period 1971-87. The model is well-adapted to the features of a small open economy. We show how it can be used as an input in the monetary policy process either as a forecasting instrument or an analytical tool. In general, forecast results over the 1988-92 period compare well with those of univariate autoregressive models. The results from the variance decomposition exercise show a rather weak influence of monetary aggregates on macroeconomic variables, at least in a short-run context. However, foreign variables, particularly commodity prices, play an important role.

RÉSUMÉ

Dans cette étude, nous présentons l’estimation, pour la période 1971-1987, d’un modèle BVAR de l’économie canadienne particulièrement bien adapté aux caractéristiques d’économie ouverte du pays. La méthode utilisée permet d’optimiser globalement l’estimation de l’ensemble des hyperparamètres du modèle. Nous montrons comment le modèle peut être utilisé comme outil d’analyse ou de prévision dans le domaine de la politique monétaire. Les résultats des exercices de prévision réalisés à l’aide du modèle se comparent avantageusement à ceux qui sont obtenus pour la même période avec des modèles autorégressifs univariés (AR). Par ailleurs, les exercices de décomposition de variances indiquent que les agrégats monétaires ont, du moins à court terme, une influence plutôt faible sur les grandes variables macroéconomiques, mais que les variables étrangères, en particulier les prix des denrées de base, jouent un rôle important.

1 INTRODUCTION

The conduct of monetary policy is plagued by the well-known problem of long and variable lags between changes in the policy instruments and the effects of those changes on the ultimate goals. Policy makers have thus had to rely on monitoring strategies in order to adjust their policy as promptly as possible to unforeseen events. In this context, the monetary policy process has been presented as a sequence that proceeds from policy instruments (that is, very short-term interest rates, settlement balances or the monetary base) through intermediate economic variables (such as interest rates, financial aggregates, the exchange rate), be they indicators or proximate and intermediate targets, and then finally on to the ultimate goals of the policy (that is, economic growth, price stability). The experience with monetary targeting has been relatively disappointing in most countries where it has been tried. Be it because of financial innovations brought about by technological advances or because of the Lucas critique, central banks have had to adjust their targets frequently to shifts in economic agents' behaviour or, as in the case of Canada, abandon a policy that could not serve its purpose anymore.

Most central banks thus deserted strict monetary targeting in the 1980s and have proceeded to monitoring of the information on a large set of economic indicators. The exercise has to be based on reliable forecasting models that embody in some way the transmission mechanism of monetary policy, since the authorities have to be able to clearly identify and react promptly to the forecasted shifts in the policy indicators and thus try to prevent cumulative one-way errors.

After six years of targeting M1 growth, the Bank of Canada abandoned this procedure in 1982. In 1988, it began to focus on price stability as its principal ultimate objective. Then, in February 1991, the Bank, jointly with the Minister of Finance, announced a specific path for reducing inflation over the next few years. It is quite clear that the Bank is also monitoring the movements of a large set of economic variables in its effort to reach price stability (see, for instance, the Bank of Canada *Annual Report* for 1991, 31).

Policy makers at the Bank thus use a mixture of many possible methods for the conduct of their policy: medium and short-term forecasting exercises based on Bank models (until recently RDXF and now QPM), which use the principal economic variables related to the monetary policy process; monitoring of relevant economic series available at a higher frequency to readjust the forecasts made; use of recent atheoretical models that take into account the possible information content of financial aggregates (see Muller 1992) and, finally, judgmental adjustments to all these forecasts.

In this context, Racette and Raynauld (1992) (hereafter referred to as RR) recently proposed bivariate vector autoregressive (BVAR) modelling as an alternative forecasting and analytical instrument for the purpose at hand. Besides being very flexible forecasting tools, BVAR models allow a richness of dynamic interrelations between the variables that is missing in most other models. They also postpone imposing exclusion restrictions until after the dynamic interrelations have been taken into account. Moreover, they have been used for policy analysis purposes, either through the imposition of some identifying restrictions for the decomposition of the variances of the variables or through conditional forecasting exercises (Litterman 1986).

RR’s BVAR modelling exercise was in many ways a pioneering work in the complex context of an open economy. In this sense, the exercise had to be conducted as an experiment, since they were tackling new problems with this relatively recent instrument: a large sets of variables (as compared to the U.S. models that had been estimated); the co-existence of two separate sets of variables, some strictly exogenous to the others; the cointegration of some of the variables. Yet, the model showed excellent forecasting performance both in and out of the sample. Most of the uncovered dynamic interrelations were reasonable and led to interesting interpretations of the context of Canadian monetary policy over the last few years.

However, as with any such experimentation, the exercise was not totally satisfactory or comprehensive. On the one hand, RR had to deal with difficult methodological problems: for example, various hyperparameter estimation criteria were available without any comprehensive experiments to provide guidance for choice. On the other hand, given the tight schedule for the experiment, they could not solve all the puzzles that were left in the end with the results. In this paper, we pursue the experiment undertaken in RR and offer a much more detailed and thorough estimation strategy. We also solve the main loose ends that were left in the previous paper.

2 THE CONTEXT

Table 1 (p. 30) lists the variables that were chosen for this new round of BVAR modelling and Figure 1 (pp. 40-41) presents charts for each of the variables. One must note here that it was decided at the outset that the exercise was to be carried out with quarterly series so as to conform with the medium-term context in which policy is conducted.[1] Since the size of the available sample (over which consistent data can be collected for all variables and for which there is no critical regime break), is not large, given the number of lags required to allow for a certain richness of the dynamic interrelations between the variables, we chose a highly selective set of variables to represent the Canadian monetary policy context. The model includes variables that represent instruments, targets, indicators and ultimate goals of the policy and its international environment.

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[1] Racette and Raynauld (1991) estimate a model of a different nature trying to capture, in the very short run (weekly context), the dynamics between the specific actions of the Bank of Canada and their effects on indicators of monetary policy.

On the Canadian side of the model, we include the two components of M2 (M1 and M2-M1), since they may play some role in the monetary policy process, given the results of the studies at the Bank on their information content.[2] A short-term interest rate (TXCAN) is also critical, since apart from being very close to the true instruments of monetary policy (settlement balances and overnight rates), it is part of the monetary conditions that the Bank closely monitors through its policy. The exchange rate (TXCHA) is the other component of monetary conditions and it has always played a crucial role in the Canadian economy as well as in the monetary policy process.[3] Finally, aggregate output (GDP) and prices (GDPD) are at the same time potential ultimate goals and the components of nominal spending, which is part of the transmission mechanism and a likely intermediate target.

As for "international" variables, we kept the same ones as RR: aggregate U.S. output (GNP); prices (GNPD); and a short-term interest rate (TXUS). Given the crucial role played by real supply shocks within our sample, we also included the international price of oil (RPOIL) and the price of other basic commodities (RPCOM).[4]

Since RR (1992) describe in detail the movements of all the above variables over most of the current sample period, we will make only a few comments at this stage on the additional observations. The sample used for estimation still runs from

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[2] See Caramazza and Slawner (1991). We have dropped the monetary base from the set of variables since RR's (1992) results confirmed that it played a trivial role.

[3] The Bank monitors an index of monetary conditions constructed from a three-month interest rate and the G-10 *effective* exchange rate. However, the bilateral Canadian-U.S. dollar exchange rate was chosen for this study, since our "international" variables are mostly U.S. variables.

[4] RR highlight the distinction between auction variables (TXCAN, TXUS, RPOIL, RPCOM and TXCHA), which are characterized by rather choppy movements, and the other variables (M1, M2-M1, GDP, GDPD, GNP and GNPD), which are more trend-like series. The specific variables used to measure the two commodity prices are not the same as those used in RR's study.

1971 to the end of 1987 for possible comparison purposes, the rest of the sample (1988:1 to 1992:1) being reserved for forecasting exercises. The latter years include, of course, the long and taxing recession, which started somewhere in the middle of 1990 both in Canada and the United States, and events such as the Gulf War crisis at the beginning of 1991 and the turning point after the long appreciation of the Canadian dollar in November 1991.

3 THE METHODOLOGY

RR describe BVAR modelling in its general context and present this methodology comprehensively. In this section, we review the key features of the BVAR methodology, borrowing extensively from RR's presentation, and in the last section, we introduce the improvements in the current research.

3.1 Structural and vector autoregressive (VAR) models

As a starting point, we assume that the structure under investigation can be described by the following equations:

(1) $$D_{11}(L)\,X_{1t} + D_{12}(L)\,X_{2t} = D_1 + e_{1t}$$

$$D_{21}(L)\,X_{1t} + D_{22}(L)\,X_{2t} = D_2 + e_{2t}$$

where X_{1t} and X_{2t} are k1 and k2 (k1+k2 = k) vectors (as we will see later, X_{1t} will correspond to Canadian variables, while X_{2t} will represent world or U.S. ones);

e_{1t} and e_{2t} are error terms, with $Ee_{it}e_{jt}' = \Sigma_{ij}$;

$D_{ij}(L) = D_{ij,0} - D_{ij,1}L - D_{ij,2}L^2 - \ldots - D_{ij,m}L^m$ is a set of matrix polynomials of order m.

Note that in this specification, contemporaneous relationships within and between X_{1t} and X_{2t} can be captured by the $D_{ij,0}$ matrices. As usual, the e_{1t} are part of the structure and correspond to economically meaningful concepts. D_1 and D_2 are vectors of constants. In a more general form, we can write:

(2) $$D(L)\,X_t = D + e_t \qquad (t = 1, ..., T)$$

where $D(L) = D_0 - D_1L - ... - D_mL^m$ and $Ee_te_t' = \Sigma$. Structural models such as equation (2) are estimated under various maintained assumptions such as exogeneity of a block of variables (for example, X_{2t}) or exclusion restrictions implied by theory.

It is now well accepted that the basic structural phenomena can be recovered through the estimation of a vector autoregressive model (VAR), where each variable of the system is linked to its own past values as well as the lagged values of all the other variables in the system. More formally, VAR models correspond to the reduced form of equation (2):

(3) $$H(L)\,X_t = H + u_t \qquad (t = 1, ..., T)$$

where $H(L) = D_0^{-1}D(L)$, $H = D_0^{-1}D$ and $u_t = D_0^{-1}e_t$; u_t is constructed as a linear combination of structural errors e_t and cannot be interpreted from an economic viewpoint. However, with minimal identifying assumptions, estimates of e_t can be obtained from u_t. As usual, u_t is centred on zero with a variance-covariance matrix given by $\Omega = D_0^{-1}\Sigma D_0^{-1}{}'$. We can write a representative equation using the first variable of the system as an example, as:

(4) $$\begin{aligned} x_{1t} = {} & h_{11,1}\,x_{1t-1} + ... + h_{11,m}\,x_{1t-m} + h_{12,1}\,x_{2t-1} + ... + h_{12,m}\,x_{2t-m} \\ & + ... + h_{1k,1}\,x_{kt-1} + ... + h_{1k,m}\,x_{kt-m} + h_{10} + u_{1t} \end{aligned}$$

where h_{10} is the usual constant.

In this formulation, VAR models impose few constraints and require the estimation of a large number of parameters, thus rapidly exhausting the available degrees of freedom. For models as large as the one considered in this paper, some restrictions are required, for example, to exclude potentially important variables or restrict the number of lags. As well, large unrestricted regressions often suffer from "over-parameterization" and lead to poor forecasts: they tend to pick up sample-specific and temporary relationships which are a source of poor performance outside the sample period.

3.2 BVAR models

To overcome problems associated with VAR models, Litterman (1979), Doan, Litterman and Sims (1984), and Sims (1989) suggested the incorporation, in a Bayesian fashion, of relevant *a priori* information in the estimation. *A posteriori* estimates obtained after combining priors and sample information are the cornerstone of Bayesian VAR models.

Here, we use the Kalman filter framework proposed in Doan, Litterman and Sims (1984), and Sims (1989). For presentation purposes, however, we follow Harvey (1981) to simplify and unify the notation. All the coefficients of a given VAR equation are stacked in the vector α_t with individual elements $h_{j,\ell}$ (ℓ=1, ..., m, j=1, ..., k) and h_0 (the constant). In fact, with the representative equation (4) in mind, the first m coefficients correspond to all the lags of the first variable, and then subsequent m coefficients match all the lagged values of the second, etc. Since α is indexed by t, this formulation implies that the VAR coefficients $h_{j,\ell}$ are now time-varying (for ease of exposition, the individual coefficients will not be indexed by t). A Kalman filter setup is defined by a pair of measurement and transition equations corresponding to each equation of the VAR model:

(5) $$y_t = Z_t\alpha_t + \xi_t$$

$$\alpha_t = \alpha_{t-1} + n_t \qquad (t = 1, ..., T)$$

where y_t is the dependent variable and Z_t is a vector of all the right-hand variables (the lagged values of all the variables of the system plus the constant term). For example, for the first VAR equation (4), $y_t = x_{1t}$ and $Z_t = [x_{1t-1}, ..., x_{1,t-m}, ..., x_{kt-1}, ..., x_{kt-m}, 1]$. By providing a convenient set of recursive predictions and updating equations (see Harvey, 1981), the Kalman filter framework fits most forecasting tasks nicely. However, to start the Kalman filter iterations and obtain the minimum mean square linear estimator (MMSLE) predictions of y_t given information available at t-1, we have to specify the initial α_0 (more precisely, its mean a_0 and its associated variance-covariance matrix P_0), as well as Q, the variance-covariance matrix of n_t. For simplicity, it is assumed that the scalar σ^2, the variance of ξ_t, is a known fixed quantity (to be replaced in practice by the estimated variance of a fixed-coefficients autoregressive model for variable y). The vector a_0 and the matrices P_0 and Q will be referred to loosely as the initial *a priori* information.

Following Litterman, these priors are not based on economic theory but rest on some empirical regularities. For example, according to Litterman (1979) and Nelson and Plosser (1982), most macroeconomic variables can be approximated by the simple discrete random-walk model with drift. For a representative equation i of system (3), this takes the form

(6) $$X_t = h_0 + X_{t-1} + u_t$$

More specifically, the prior mean a_0 for the ith equation will be centred on the elements $h_{j,\ell}$ (ℓ=1, ..., m; j = 1, ..., k) equal to 1 for j = i and ℓ=1, and 0 otherwise.[5]

At first glance, this information appears to be rather restrictive and may not be appropriate to all the series investigated. The degree of constraint will depend on the dispersion associated with each coefficient. Ideally, this *a priori* uncertainty should reflect some basic requirements. In addition, the procedure chosen should be practical, since it would be impossible to determine (separately) the values of the coefficients. To meet these requirements, Sims (1989) suggests the following general formulation for SQRT(P_0):

7) $$\text{SQRT}\{\ \text{VAR}\,[h(j,\ell)]\ \} = \text{TIGHT}\cdot\text{OTHER}\cdot D(\ell)\cdot(\hat{\sigma}_i/\hat{\sigma}_j)$$

where

TIGHT	is the overall tightness parameter. It is the standard deviation of the coefficient of variable i in equation i lagged once, for example, X_{t-1} in equation (6).
OTHER	allows the imposition of a tighter standard error on the jth variable in equation i; it operates only when i ≠ j.
D(ℓ)	is a distributed-lag function that tightens the standard errors as the lag length increases. We use the harmonic specification $g(\ell) = \ell^{-\text{DECAY}}$ where DECAY is a constant. Note that D(1) = 1.
($\hat{\sigma}_i$/ $\hat{\sigma}_j$)	adjusts the *a priori* information to the relative scale of the variables where $\hat{\sigma}_i$ is the estimated standard error of a univariate AR model for variable i and $\hat{\sigma}_j$ is the same statistic computed for variable j.

The standard error associated with the constant term is given by

(8) $$\text{SQRT}\{\text{VAR}[h_0]\} = \text{TIGHT}\cdot\text{CONREL}\cdot\hat{\sigma}_i$$

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[5] In practice, h_0 is set to zero, but since its initial prior is kept loose, we will eventually get the form described by equation (4).

and is thus proportional to the standard error of the error term of the left-hand-side variable i.

Variables determined in auction markets (the exchange rate, commodity prices, etc.) must be given special treatment. For example, instead of using a discrete random-walk prior, Sims (1989) proposed a discrete specification resulting from time-averaging a random-walk process operating on a shorter interval. Indeed, if the true random-walk process is defined over a daily (or weekly) interval, but the estimations are carried out on quarterly averages, we will get, approximately, the coefficients given by the autoregressive (AR) representation (in levels) of an ARIMA(0,1,1) model with a moving-average coefficient equal to 0.2679. For the m=4 case, the prior mean given by equation (5) is thus replaced by

$$(9) \qquad X_t = h_0 + 1.2679\, X_{t-1} - 0.3397\, X_{t-2} + 0.0910\, X_{t-3} - 0.0244\, X_{t-4} + u_t$$

In fact, Sims (1989) uses this form of prior for all the variables in his BVAR system and reports interesting improvements in forecast performances; we will follow the same strategy. In addition, the auction nature of these series implies a behaviour that is often independent of observed past information. Consequently, it is preferable to introduce additional tightening around the initial prior mean. Specifically, if we have such a left-hand variable, we will set the prior as

$$(10) \qquad \mathrm{WIENER}^2 \cdot P_0$$

and force the associated prior mean of the constant to be close to zero (by multiplying its prior variance by WIENER) to confine any possible drift. We thus have a stringent random-walk prior for these variables.

3.3 New features of the BVAR methodology

Time-varying prior

In this research, the variance-covariance matrix of the disturbance n_t of the transition equation is set to

$$Q = TVAR^2 \cdot I \qquad (11)$$

where I is the identity matrix. Q controls the speed at which the coefficients are allowed to change over time. Equation (11) is in sharp contrast to Sims's (1989) and RR's formulation, where $Q = TVAR^2 \cdot P_0$: this latter specification assumes that the speed at which a coefficient is changing is proportional to its initial *a priori* variance. Practical experimentation has shown that this intuitively very plausible formulation was the source of difficult problems in the estimation stage.[6] The proposed formulation (11) assumes a constant speed of change for all coefficients and leads to a much more tractable estimation process.

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[6] If the $Q = TVAR^2 \cdot P_0$ setup is used, the likelihood function could always be increased by reducing the value of TIGHT (recall that the TIGHT hyperparameter controls the scale of the P_0 matrix). This leads to systems with almost no cross-effects between variables, since the random-walk prior was in fact the posterior model obtained. Intuitively, we achieved a small Q best (according to all estimation criteria used) by reducing TIGHT instead of lowering TVAR: in fact, TVAR was always increasing in our search exercises. The new $Q = TVAR^2 \cdot I$ setup avoids completely this corner solution and leads to much more reasonable results. In RR, this problem was not detected, since we restricted our search exercise to a narrow hyperparameter space, building on previous results reported in the literature.

Cointegration prior

The BVAR specification adopted so far does not address the important issue of cointegration as raised by Engle and Granger (1987) and illustrated by the behaviour of Canadian and U.S. interest rates. For example, in our BVAR specification, Canadian and U.S. interest rates are each associated with a random-walk prior:

(12.1) $$TXCAN_t = TXCAN_{t-1} + u_{TXCANt}$$

(12.2) $$TXUS_t = TXUS_{t-1} + u_{TXUSt}$$

Following a shock, lasting divergences could be observed - a characteristic at odds with Canadian-U.S. historical evidence. The rather harmonious monetary policies followed in the two countries have led to a form of parity of nominal interest rates, at least in the long term. However, even if the BVAR methodology allows other variables to influence Canadian interest rates, the impact is usually much too tenuous to produce a long-run convergence. To put all the variables on an equal footing, we have modified the usual random-walk BVAR setup by using a prior mean of zero for all the coefficients of the Canadian interest rate equation. However, we have given some extra room to the $TXCAN_{t-1}$ and $TXUS_{t-1}$ coefficients by multiplying the prior variances applied in all other equations (the P_0 on the right-hand side of equation 13) by the same hyperparameter COINT (to be estimated), giving the specific P_0 for these variables:

(13) $$P_0 = COINT \cdot INDEX \cdot P_0$$

where INDEX is a vector of dummy variables equal to one to designate the appropriate variables. Equation (13) puts all the Canadian and U.S. lagged coefficients on an equal footing and will lead to estimates that will more faithfully respect the long-run features discussed above: no divergent paths will be observed. More precisely, before applying (13), we have turned off the effect of OTHER on all the U.S. interest rate coefficients. Compared to the cointegration setup used in RR, the one advocated in this paper is much more versatile and can be applied quite easily to more than one pair of long-run relationships.

3.4 Estimation

Sims estimates the unknown hyperparameters by maximizing the likelihood function but argues that "maximum likelihood is justifiable only as an approximation to a Bayesian procedure or as a device for summarizing a likelihood function" (Sims 1989, 13). More precisely, the log-likelihood (excluding constants) of the representative equation can be written as

$$\text{(14)} \qquad \text{Log } L(y) = -\frac{1}{2}\sum_{t=1}^{T} \log f_t - \frac{1}{2}\sum_{t=1}^{T} \frac{v_t^2}{f_t}$$

where $v_t = y_t - y_{t/t-1}$ and f_t is the variance of v_t (see Harvey 1981, 14). In fact, Sims maximizes the sum of the k individual likelihood functions ignoring, for computational reasons, the covariance between the error terms of the different equations. It is interesting to note that the last term of equation (14) consists of the sum of one-step-ahead forecast errors squared-weighted by their variance. Large errors will not penalize the likelihood as long as they are associated with large variances.

As an alternative estimation procedure, Doan, Litterman and Sims (1984) seek hyperparameters that maximize the negative of the log-determinant of the matrix of the cross-product of the measurement error vector V_t:

$$\text{(15)} \qquad S = -\, 1\text{n} \left| \sum_{t=1}^{T} V_t V_t^{\prime} \right|$$

where $V_t = [v_{1t}\ v_{2t}\ \ldots\ v_{kt}]'$ with the v_{it}'s defined above. As indicated by Harvey

(1989, 129), S is a good approximation of the likelihood function of the system in the steady state. RR used the square-root of the trace of S, which corresponds to the sum of the root-mean-squared errors. In this paper, we use the multivariate criteria (15) which, we believe, is more in the spirit of a Kalman filter *cum* VAR estimation. Furthermore, our rather extensive experience with this criteria shows that it is rather well-behaved and not prone to problems related to the inclusion of off-diagonal covariance terms, as was feared before.

3.5 Other considerations

The matrix P_0 specified so far is diagonal, as we have not introduced any covariance terms. This independence assumption implies a much higher degree of uncertainty for the forecasts of the left-hand variable than our random-walk prior mean usually delivers in practice. Sims (1989), much concerned by the potential difficulties raised by the diagonal assumption, introduced an intuitive and simple solution based on a *dummy* observation (see Sims 1989, 8, for a complete discussion of this feature). Our setup does not need such adjustment, since the chosen estimation criteria do not react to any change of the hyperparameter governing the inclusion of this dummy observation.

A final word on institutional considerations: in this paper we make the assumption that U.S. and world variables are exogenous with respect to Canada: accordingly, we set the hyperparameter OTHER to zero when Canadian variables were part of a U.S. or world equation.

4 ESTIMATION AND FORECASTING RESULTS

4.1 Hyperparameters

Let us first summarize our BVAR apparatus. The objective of the BVAR model is to account for the movements of six Canadian variables (M1, M2-M1, TXCAN, TXCHA, GDPD and GDP), taking into account the exogenous international environment captured by three U.S. variables (GNP, GNPD and TXUS) and two real prices (RPOIL and RPCOM). The prior means are based on a discrete time-averaged random walk, except for Canadian interest rates, where the prior mean vector is equal to zero. The Canadian interest rate variable was treated in a special way, since it is, by assumption, cointegrated (COINT) with U.S. interest rates. Since the exchange rate, the real price of oil and the prices of commodities are considered to be speculative variables, their *a priori* variance is multiplied by the WIENER parameter and their constant pushed to zero to limit any drift. All coefficients are assumed to evolve over time at a speed controlled by TVAR. With TIGHT, OTHER, DECAY, CONREL, TVAR, WIENER and COINT, we have seven hyperparameters to estimate. Since we have 11 explanatory variables with four lags for each variable, a constant, and a postal strike dummy, each equation has a total of 46 right-hand variables (Z_t). Actual estimation was for the period 1970:01 to 1987:04; we reserved the 1988:01-1992:01 period for an *ex ante* forecasting exercise. All computations were carried out with RATS 3.0.

To find the hyperparameter values that maximize S, we proceeded by grid-search evaluations, a method especially well-suited to the problem at hand. The set of admissible values for the coefficients to be estimated is fairly well defined, and we get, as a by-product, a picture of the likelihood function at the maximum. The estimation results are given in Table 2 (p. 31). It is difficult to proceed to a meaningful comparison, since Sims (1989) and RR did not use the same criteria. However, our results suggest a relatively loose system for the Canadian and U.S.

variables (especially if we consider the value of TIGHT = 0.42) but a relatively tight one for auction variables. The hyperparameter estimates for DECAY (1.42) and OTHER (0.24) are broadly in line with published work in this area. With an estimate of TVAR = 0.4E - 04, we detected little time variation (of course, the sample period is relatively short) and policy evaluation exercises can be safely conducted. Finally, as suspected, the COINT (1.77) coefficient reflects the importance of both Canadian and U.S. lagged interest rates in the Canadian interest rate equation.

Figure 2 (p. 42) illustrates the contours of the likelihood function for various combinations of the hyperparameters. They convey the clear impression that we have found a well-defined local maximum that, as far as we can tell, is also a global one.

4.2 Reduced-form impulse responses

Since the estimation results obtained with the model at this stage can only be considered as correlations, we will only make some brief comments on the general patterns of dynamic interactions that emerge from the reduced-form impulse response coefficients given by

$$X_t = u_t + \psi_1 u_{t-1} + \psi_2 u_{t-2} + \ldots \tag{16}$$

where the ψ_ℓ matrices are calculated recursively from the H_ℓ ones. The matrix of coefficients $\psi_{ij,\ell}$ gives the impact after ℓ periods of a one-standard-error shock of variable j on variable i. Even if the causal pattern is not strictly interpretable, these coefficients will be an important ingredient in the policy simulation exercises. The results are illustrated in Figures 3 to 5 (pp. 43-45) and show the dynamic interactions between variables over a 32-quarter period. For instance, the graph on the second line, first column of Figure 3 shows $\psi_{21,\ell}$ and illustrates the impact of a M1 shock on M2-M1. As indicated previously, being reduced-form errors, the u’s cannot lead to any economic interpretations. For example, an M1 shock cannot be associated with a single source; it could be the result of demand, supply or multiplier shocks.

An examination of the general patterns of the graphs shown in Figures 3 to 5 reveals certain interesting features:

- The only truly persistent Canadian shock is the shock to exchange rate, a feature that is not really surprising, since the shock has been treated as an auction variable. The exchange rate is almost totally isolated, showing no clearly visible reactions to any of the other variables, be they domestic or foreign.
- Shocks to M1 and GDP show a certain degree of persistence, but after an initial outburst, they weaken rapidly.
- The shock to Canadian interest rates is particularly short-lived.
- Interrelations between Canadian variables are weak, the GDP appearing as the only variable to be systematically affected by all the others.
- There is much more action in Figure 4, which shows the effects of foreign shocks on the Canadian economy. In particular, the two last shocks (to RPOIL and especially to RPCOM), which are singularly persistent, produce very clearly visible effects on Canadian macro-variables.

4.3 Within-sample forecasting results

We conducted a within-sample forecasting exercise with the model for the 1980-87 period.[7] Table 3 (p. 32) presents the forecasting performance over the above period

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[7] The forecasting period was chosen at first to allow for comparison with RR's results, but we became aware at a later stage through an examination of the forecasts with ARs that the measures of many of the variables (supplied by the Bank) had changed so much that the comparison was not warranted any more.

of the BVAR model in terms of Theil (one-period horizon) and root mean square error (1-, 4- and 8-period horizons) statistics. For comparison purposes, Table 4 (p. 33) presents the same statistics for an AR(4) model. To the 11 variables of the model, we added the forecasts for the Canadian (GDPN) and U.S. (GNPN) nominal expenditures constructed by combining real income and the implicit price deflator for each country.

In general, by taking into account dynamic interrelations between the variables, the BVAR model forecasts much better than the AR one. Within the few exceptions (GDPD, 1-quarter horizon; TXUS, 1-year horizon; and, at the 8-quarter horizon, GNPN and TXUS), the only cases where the differences are clearly in favour of the AR model are those for the U.S. interest rate. For the GDPD at the 1-quarter horizon, the difference is only minor and the forecasting performance of both the BVAR and AR models are excellent.

Cast on an annual basis, the root mean square error (RMSE) statistics generally improve as the horizon lengthens, which is reassuring given the horizon adopted for the policy process. The only exceptions to this pattern are the U.S. GNP deflator, which has very low RMSEs for all horizons, and M2-M1, whose RMSEs show a small increase at the 8-quarter horizon.

The BVAR model seems to have an excellent forecasting ability for some variables closely related to the monetary policy process. Hence, the goal variables (GDPD and GDP) are easily forecasted at all horizons and we get good results predicting some possible intermediate targets (M2-M1 and GDPN). It, however, seems that M1 moves around so much that it is difficult to trust its forecasts in a monitoring process.

5 ANALYSIS OF THE DYNAMIC INTERACTIONS

5.1 Recovering structural shocks

Except for a few well-defined cases, the u_t shocks underneath the ψ_ℓ impulse response matrices (see equation 16) do not yield any economic interpretations. The motivation is well-known and can be found in any standard econometric textbook. In most cases, these shocks are linear combinations of unobservable structural shocks. Under certain conditions, Sims (1986) and Bernanke (1986) have shown that it is possible to recover the e's by exploiting the relationship

$$D_0 u_t = e_t \tag{17}$$

implicit in the BVAR reduced-form specification. The general problem lies in estimating the $[(k \cdot k) - k]$ coefficients of the D_0 matrix and the k variances of Σ (the structural shocks are assumed to be orthogonal) from the $[k \cdot (k+1)/2]$ estimates of Ω, the variance-covariance matrix of the BVAR residuals. As usual, we have to make sure that the number of parameters estimated is less than or equal to the number of elements in matrix Ω. We have modified a RATS procedure written by Doan (1990), where the estimates are chosen to ensure that $D_0^{-1} \Sigma D_0^{-1\prime}$ corresponds as closely as possible to Ω. Simple examples of this type of estimation can be found in Sims (1986), Bernanke (1986), Blanchard (1989) and Johnson and Schembri (1989).

5.2 Identification of the structural model

The BVAR residuals were used to calculate the (11x11) reduced-form variance-covariance matrix Ω (see Table 5, p. 34). From these correlations and with the help of some identifying restrictions, we must estimate a set of equations expressing the contemporaneous interactions between the innovations, that is, D_0. The estimation method is non-linear, and we must ensure that the estimation converges, that the overidentifying conditions are satisfied, and finally, that the values of the parameters are “reasonable.” The structural model that appears in Table 6 (p. 35) must thus be examined with all these aspects of the methodology in mind.

We now examine each of the estimated equations that come out of the above identification process. The innovations in the two components of M2 faithfully behave like money demand equations. In fact, the estimates are very close to those of a regular money demand equation and indicate very high elasticities in such a short-run context. The innovations to both M1 and M2-M1 are related positively to real income and prices. The estimate of the price coefficient in the M1 equation indicates homogeneity. The income elasticity in this same equation is also very high. As expected, the M1 shock is related negatively to the short-run interest rate. The real income and price coefficients in the M2-M1 equation are lower than those of the M1 equation. The positive coefficient on the interest rate can easily be explained by the omission of an own rate of interest.

As in RR, the interest rate equation behaves like a monetary policy reaction function. Hence the rate of interest is pushed up in reaction to unanticipated depreciations, increases in the price level or in real income. As expected, an innovation to U.S. interest rates leads to an increase in Canadian rates. Moreover, the coefficient estimates of the exchange rate and the U.S. rates are almost identical, indicating a symmetrical reaction of the Canadian rate to the two innovations.

The exchange rate equation is difficult to interpret as a demand or supply relationship as such, but the signs of the estimates are generally easily interpretable. An unexpected increase in the Canadian price level leads to a depreciation. The exchange rate reacts to the same extent (but of course in opposite directions) to an increase in Canadian and U.S. interest rates. An unexpected increase in the price of the two composite commodities leads to an appreciation of the Canadian dollar, in line with the fact that Canada is a producer of those commodities. All these reactions are comforting. However, we could not get rid of a stubborn perverse positive (but fortunately not significant) reaction of the price of foreign exchange to a U.S. price innovation. This is one of the very few troublesome stylized facts that are left in this new BVAR estimation.

We will interpret shocks to the Canadian price level as domestic supply innovations, since they are positively related to real income. In this equation, the coefficient on the exchange rate indicates that an unexpected depreciation provokes a fall in the *domestic* price level, possibly revealing a negative real income effect coming from a sluggish import reaction.

Finally, the equation concerning innovations to real income can be associated with a demand relationship, with a negative relation to the price level capturing a likely real wealth effect. We also get a positive reaction to U.S. real income and a negative relation to interest rates. Since Canada is a net commodity supplier, the positive impact of the two commodity prices probably arises from some real income effects.[8]

The identification of foreign shocks is of less concern to us than that of their Canadian counterparts, and we will not spend too much time commenting on the very compact equations that we got. Innovations to the U.S. real income and to the two commodity prices are taken as exogenous to the system. The U.S. price level is positively related to the U.S. income variable and to the two real commodity price shocks, possibly revealing a supply relationship.

Finally, the innovation to the U.S. interest rate reacts positively to a shock to U.S. real income, which we will interpret as a monetary policy reaction function.

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[8] We tried our best to incorporate a monetary variable in this equation in order to get a real balance effect, but the model consistently rejected this effect.

5.3 Impulse responses to structural shocks

Many of the general comments that we made on the reduced-form results are still true when we examine the impulse responses to structural shocks in Figures 6 to 8 (pp. 46-48). Hence, apart from the shock to the exchange rate, most other Canadian disturbances show little persistence, although the shock to real income does show up more strongly than in the reduced-form model. The shock to the interest rate is still relatively short-lived. Figure 7 shows that most of the action comes from foreign variables and, particularly, the two commodity prices and TXUS. On the Canadian side, the shock to real GDP seems to be the most influential, while that to the deflator has meagre effects on other variables.

As the following paragraphs suggest, identification of structural shocks reveals interactions between variables that do not appear in the reduced-form model.

Domestic shocks

The shocks to M1 and M2-M1 have been identified as money demand disturbances, which lead in the case of M1 to the expected increase in the short-run rate of interest, but not in the case of M2-M1. In both cases, there is a positive correlation between the two shocks, which indicates some complementarity between the two components of M2. At first glance, it seems difficult to interpret the increase in both prices and output that results from these supposed money demand shocks. Yet these effects are consistent with indicator models' stylized facts. They may catch some influences that are absent from the short list of variables of the model. For instance, fiscal or money multiplier influences could find their way through these disturbances (see Sims 1986).[9]

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[9] It has been suggested that non-linear effects of interest rates on the demand for M1 could also be responsible for this result.

The monetary policy shock (to the interest rate) leads to the reactions a central banker would expect: following a short-lived increase in the interest rate, the exchange rate appreciates persistently and the *levels* of prices and especially of output fall. There is a drop in M1 (which subsides somewhat after a short while). M2-M1 increases at first, probably because economic agents leave M1 for components yielding somewhat higher interest returns) and then decreases as a result of the movements of the other variables in the chain of reactions.

The very persistent shock to the exchange rate has relatively small but easily interpretable effects on the other variables. The depreciation induces an immediate but short-lived policy reaction with an increase in the interest rate. The latter seems to push M1 down for a little while. After a short delay, prices and output increase, leading to increases in the two components of M2.

The price shock, which was identified as a supply shock, has the weakest influences on all the other variables of the system. It leads, as expected, to a depreciation, to a faint and short increase in the interest rate (policy reaction?) rapidly reversed and to a fall in income that also rapidly turns down, possibly as a result of the depreciation and the fall in the interest rate. The monetary aggregates are only very weakly affected by this shock.

The shock to real output, identified as a demand shock, is also generally well behaved. It induces an increase in prices and leads to a higher interest rate, a possible policy reaction once more. It seems that the demand for the two components of M2 increases. The exchange rate immediately and persistently appreciates, in line with the relatively steep increase in the interest rate (see the reaction of the exchange rate to a shock to this variable). This could reflect a tightening of monetary conditions and/or an increase in the excess demand for savings.

Overall, the system of reactions to the different shocks is easily interpretable in terms of standard theoretical reasoning. We seem to find relatively well-behaved reactions to supply, demand and policy shocks. The two central columns show the effects of unexpected changes in monetary conditions and leave no doubt as to the fact that a tightening of these conditions will have the desired restrictive (though weak) effects on output prices and the money stock.

Foreign shocks

Foreign shocks also generally lead to interesting interpretative stories (Figures 7 and 8, pp. 47, 48). A shock to U.S. real income that induces an increase in prices and the interest rate at home is transmitted to Canada in the same form. GDP, prices and the interest rate all rise and the demand for money increases. We can even see a certain hesitation at first in the increase in M1 as a result of the conflicting influences of the rising interest rate, prices and output.

The structural origin of the shock that gives rise to an increase in the U.S. price level is not particularly clear, since it has negligible effects on any other variable in the United States. However, it seems to be detrimental to Canada, where it gives rise to inflation and leads at first to a fall in income and a depreciation of the dollar. Canadian interest rates rise, again possibly as a policy reaction to the depreciation or to inflation, which leads to a small fall in M1. Eventually, the two components of M2 increase as the interest rate falls back, prices go up and real output turns up.

The sharp increase in the U.S. interest rate produces the expected immediate fall in real output in the U.S. This effect is surprisingly strong and long-lasting. Prices in the U.S. increase at first, an unexpected movement, but eventually fall. Movements in real output and prices in Canada parallel those of their U.S. counterparts. The dollar depreciates despite the strong reaction of the Canadian interest rate. The two components of M2 eventually fall with a small increase at first for M2-M1, possibly because of the shifts to these components with the initial rise in interest rates.

In reaction to an oil price shock, U.S real output weakly decreases, while prices increase strongly and persistently. U.S. interest rates rise. The Canadian dollar appreciates (after all, Canada is a net producer) but the supply shock and the slowdown in the United States eventually induce a fall in real income in Canada after a short-lived increase. Prices increase (but surprisingly less than in the United States). Interest rates increase defensively in reaction to the U.S. price and interest rate increases. M1 falls deeply, responding to the increase in the interest rate and fall in real income. The movement of M2-M1 is almost imperceptible.

Finally a real (non-oil) commodity price shock has strong effects on all the variables of the system, possibly because it gathers together a group of supply influences not taken into account by our short list of exogenous variables. In the United States, inflation rises in an increasing fashion, while real income is depressed and the interest rate increases. The Canadian dollar again appreciates persistently and our prices, income and interest rates follow the movements of their U.S. counterparts. Real income in Canada does not fall as much as in the United States, since Canada is a producer of those commodities. As for monetary aggregates, it seems that the increase in prices dominates the other effects (falling output and increasing interest rates), since the two aggregates rise persistently. In fact, the real price of non-oil commodities is the dominant influence on the two components of M2, a difficult result to explain with standard theory.

5.4 Variance decomposition

Tables 7 and 8 (pp. 36 and 37) present the variance decomposition for the Canadian variables respectively for the 8- and 32-quarter horizons. Briefly, this decomposition provides a kind of ranking of the relative importance of the forecast error of variable j on the overall forecast error of variable i under study (see Sims 1980, and Doan and Litterman 1984, for detailed explanations of the variance decomposition technique). Given the estimated BVAR coefficients, the estimated contemporaneous effects of the foreign variables, the structural-disturbance coefficients, we have derived the $G_{ij,\ell}$ coefficients that give the impact after ℓ periods of a one-standard-error structural shock of variable j on variable i. The weight of variable j in the decomposition of variable i after s periods is given by

$$\text{(18)} \qquad \text{Weight}_{ij}(S) = \frac{\sum_{\ell=1}^{S} G_{ij,\ell}}{\sum_{j=1}^{k} \sum_{\ell=1}^{S} G_{ij,\ell}}$$

Our comments will use the 8-quarter decomposition results to sort out the main influences on each of the Canadian variables on a relatively short-term horizon and we will refer to the 32-quarter results for a longer-run perspective.

- In the first eight quarters, the explained variance of M1 shocks is dominated by its own past disturbances (41 per cent) and by shocks to the U.S. interest rate (31 per cent). Smaller contributions come from Canadian real output (7.6 per cent), M2-M1 (6.2 per cent) and TXCAN (4 per cent). These influences seem to amount to a demand for money, with the U.S. interest rate playing a major role through the estimated co-integration. At the longer horizon, the results are even more difficult to interpret, with the two real commodity prices taking most of the importance (RPOIL, 23.4 per cent; RPCOM, 43.1 per cent) and the U.S. interest rate keeping a substantial contribution (11 per cent).

- At a short horizon, the main contributors to the explained variance of M2-M1 are U.S. real income (34.5 per cent), its own past disturbances (20.7 per cent), Canadian GDP and, again, RPCOM (17.1 per cent). This last variable accounts for almost all of the explained variance of M2-M1 at the 32-quarter horizon, with the U.S. GNP coming second (13.1 per cent) and leaving little for other influences. The results confirm the very modest influence of interest rates on this large portion of M2 and give credence to the view that M2 would be difficult to control in the short-run in Canada.

- As for the explained variance of the Canadian interest rate, the U.S. interest rate accounts for 50 per cent at the short and 35 per cent at the long horizon. Other main contributors are the two commodity prices and the U.S. and Canadian real output. These results indicate some interest rate parity effects with some elements of a possible reaction function.

- The contributors to the variance of the exchange rate warrant little comment, since this is evidently a random-walk variable with most of the variance self-explained at the short and long horizons. For the rest, the two interest rates and commodity prices explain the remaining action, which seems quite natural.

- The explained variance of the GDP deflator is led mainly by RPCOM both in the short (43 per cent) and long (87 per cent) run. This gives a major role to real supply shocks during our sample period and leaves little room for monetary explanations of inflation, with the two components of M2 amounting to a meagre portion of the contribution.

- For real GDP, once its own past disturbances have been taken into account, most of the remaining variance is explained by U.S. GNP (31 per cent) and interest rates (26 per cent) at the short horizon. At the longer horizon, these variables are again among the main contributors (respectively 12 per cent and 26 per cent), but so also are shocks to the price of oil. These results give an important role to real shocks, the U.S. business cycle and the Federal Reserve’s policy in the determination of Canadian business cycles. It is noteworthy that M1 is, for the long horizon, the main Canadian contributor.

- The variance decomposition of Canadian nominal GDP shows a mixture of the two latter series of effects. Again, the U.S. GNP and interest rates are important, with RPCOM taking a major role at the longer horizon.

- In general, the variance decomposition exercise shows very little influence of monetary variables in our model. Most of the contributors to the variance of Canadian variables are foreign variables, particularly U.S. GNP and U.S. interest rates, and at the longer horizon, the two commodity prices. In this respect, RPCOM plays an important role for many of the variables, a result that comes out in many other studies (Hamilton 1988; Sims 1992; Amano and van Norden 1993).

5.5 Ex ante forecasting exercise (1988:1-92:1)

Overall, as indicated by Tables 9 and 10 (pp. 38, 39), the BVAR *ex ante* forecasts compare well with those of the AR(4) models. More specifically, considering only Canadian variables, the BVAR performs best for GDP, GDPN and M2-M1. The two methods give similar results for the exchange rate, while the AR(4) is more precise than the BVAR for M1 and the Canadian interest rate. Further exercises could test whether improvements could be achieved in forecasting performance by exploiting information available at the time of the forecast.

6 CONCLUSION

As a conclusion, let us highlight the main contributions of this new round of BVAR estimation:

- At the methodological level, we now have a well-tested and fully optimized BVAR apparatus adapted to important features of a small open economy (for example, cointegration).
- The methodological improvements show up in the well-behaved empirical results in forecasts, easy estimation of structural equations, and simulated structural impulse responses.
- After further fine-tuning and adjustments to the list of variables, the model could effectively be used as an input in the monetary policy process either as a forecasting instrument or an analytical tool. In this last respect, the plausible results that we get for the two components of Canadian monetary conditions (the short-term interest rate and the exchange rate) could serve as a basis for conditional forecasting experiments.

TABLE 1: THE VARIABLES

Canadian variables

M1 = narrow monetary aggregate (B1627)

M2-M1 = deposits in M2 excluding demand deposits = M2-M1 (B1630-B1627)

TXCAN = short-term nominal interest rate - 90-day commercial paper rate (B14017)

TXCHA = Canadian dollar/U.S. dollar exchange rate, average noon spot rate (B3400)

GDPD = implicit price index of real gross domestic product, 1981=100 (D20337)

GDP = real gross domestic product (D20031)

GDPN = nominal GDP (D20000)

World variables

GNP = real U.S. gross national product (1987)

GNPD = U.S. GNP deflator

GNPN = U.S. nominal GNP

TXUS = short-term U.S. 90-day commercial paper rate (B54412)

RPOIL = index of the real price of oil in U.S. dollars (series supplied by the Bank of Canada)

RPCOM = international index of real commodities prices in U.S. dollars, excluding energy (series supplied by the Bank of Canada)

Note: For M1, M2, GDP, GDPD, GNP, and GNPD, we used the seasonally adjusted measures. For comparison purposes and to conform with the monetary policy process, we used seasonally adjusted quarterly time series.

TABLE 2: HYPERPARAMETER ESTIMATES: 1971:1-1987:4

TIGHT	0.42
OTHER	0.24
DECAY	1.42
CONREL	135.0
TVAR	0.0004
WIENER	0.001
COINT	1.77

TABLE 3: FORECASTING PERFORMANCE OF THE BVAR MODEL: 1980-87

	Horizon (quarters)					
	1			4	8	
	RMSE	RMSEA	THEIL	RMSE	RMSE	RMSEA
Canada						
M1	1.89	7.78	0.78	5.56	8.89	4.35
M2-M1	0.92	3.73	0.32	3.43	8.45	4.14
TXCAN	1.62	6.64	0.94	3.83	5.25	2.60
TXCHA	1.43	5.84	0.99	3.75	5.75	2.83
GDPD	0.50	2.02	0.31	1.55	2.87	1.42
GDP	0.90	3.65	0.68	2.96	4.83	2.39
GDPN	0.78	3.16	0.33	3.27	5.12	2.53
U.S.						
GNP	0.96	3.90	0.80	2.42	3.63	1.80
GNPD	0.32	1.29	0.22	1.38	3.81	1.89
GNPN	1.12	4.56	0.52	3.43	6.10	3.00
TXUS	2.10	8.67	1.07	4.02	5.95	2.93
World						
RPOIL	10.82	50.82	0.93	28.38	33.48	15.53
RPCOM	3.88	16.45	0.96	9.79	16.12	7.76

Note: RMSE is the usual root-mean-square error for the indicated forecasting horizon (1, 4 or 8 quarters); RMSEA is the same statistic expressed on an annual basis (compounded rate); THEIL refers to the Theil statistics.

TABLE 4: FORECASTING PERFORMANCE OF AR MODELS 1980-87

	Horizon (quarters)					
	1			4	8	
	RMSE	RMSEA	THEIL	RMSE	RMSE	RMSEA
Canada						
M1	2.34	9.69	0.96	5.92	7.39	3.63
M2-M1	1.00	4.06	0.34	4.67	12.27	5.96
TXCAN	1.86	7.65	1.08	4.39	6.20	3.05
TXCHA	1.50	6.14	1.03	3.99	6.33	3.12
GDPD	0.49	1.97	0.30	1.94	4.81	2.38
GDP	1.11	4.51	0.87	3.73	5.96	2.94
GDPN	1.05	4.27	0.44	3.89	5.79	2.85
U.S.						
GNP	1.07	4.35	0.89	3.19	5.54	2.73
GNPD	0.36	1.45	0.26	1.74	4.71	2.33
GNPN	1.20	4.89	0.55	3.77	5.94	2.93
TXUS	2.20	9.10	1.12	3.66	4.12	2.04
World						
RPOIL	11.39	53.95	0.98	29.45	40.45	18.51
RPCOM	4.59	19.66	1.14	14.16	29.83	13.94

Note: RMSE is the usual root-mean-square error for the indicated forecasting horizon (1, 4 or 8 quarters); RMSEA is the same statistic expressed on an annual basis (compounded rate); THEIL refers to the Theil statistics.

TABLE 5: CORRELATION MATRIX OF THE ESTIMATED BVAR RESIDUALS

	M1	M2-M1	TXCAN	TXCHA	GDPD	GDP	GNP	GNPD	TXUS	RPOIL	RPCOM
M1	1.0	-0.15	-0.26	-0.05	0.09	0.05	-0.03	0.09	0.14	0.14	0.14
M2-M1		1.0	0.43	0.19	0.10	0.17	0.03	0.23	0.30	0.21	0.20
TXCAN			1.0	0.18	0.10	0.11	0.33	0.01	0.65	-0.07	0.12
TXCHA				1.0	-0.22	0.02	0.11	0.16	0.22	-0.21	-0.16
GDPD					1.0	-0.32	-0.01	-0.06	-0.09	0.14	0.16
GDP						1.0	0.24	-0.04	0.16	0.05	0.07
GNP							1.0	0.09	0.26	-0.13	0.13
GNPD								1.0	0.03	0.06	0.04
TXUS									1.00	-0.08	0.28
RPOIL										1.0	-0.02
RPCOM											1.0

TABLE 6: CONTEMPORANEOUS STRUCTURAL MODEL

$$\text{M1} = -\underset{(.16)}{0.35}\ \text{TXCAN} + \underset{(.35)}{0.90}\ \text{GDPD} + \underset{(.18)}{0.63}\ \text{GDP} + e$$

$$\text{M2-M1} = \underset{(.09)}{0.34}\ \text{TXCAN} + \underset{(.19)}{0.48}\ \text{GDPD} + \underset{(.09)}{0.43}\ \text{GDP} + e_2$$

$$\text{TXCAN} = \underset{(.20)}{0.37}\ \text{TXCHA} + \underset{(.27)}{0.61}\ \text{GDPD} + \underset{(.14)}{0.57}\ \text{GDP} + \underset{(.09)}{0.39}\ \text{TXUS} + e_3$$

$$\text{TXCHA} = -\underset{(.30)}{0.42}\ \text{TXCAN} + \underset{(.62)}{0.37}\ \text{GDPD} + \underset{(.40)}{0.62}\ \text{GNPD} + \underset{(.18)}{0.45}\ \text{TXUS} - \underset{(.01)}{0.03}\ \text{RPOIL} - \underset{(.05)}{0.11}\ \text{RPCOM} + e_4$$

$$\text{GDPD} = -\underset{(.13)}{0.16}\ \text{TXCHA} + \underset{(.11)}{0.21}\ \text{GDP} + e_5$$

$$\text{GDP} = -\underset{(.17)}{0.27}\ \text{TXCAN} - \underset{(.42)}{0.87}\ \text{GDPD} + \underset{(.16)}{0.79}\ \text{GNP} + \underset{(.01)}{0.02}\ \text{RPOIL} + \underset{(.04)}{.02}\ \text{RPCOM} + e_6$$

$$\text{GNP} = e_7$$

$$\text{GNPD} = +\underset{(0.03)}{0.08}\ \text{GNP} + \underset{(.004)}{0.003}\ \text{RPOIL} + \underset{(0.01)}{0.001}\ \text{RPCOM} + e_8$$

$$\text{TXUS} = \underset{(0.05)}{0.19}\ \text{GNP} + e_9$$

$$\text{RPOIL} = e_{10}$$

$$\text{RPCOM} = e_{11}$$

TABLE 7: VARIANCE DECOMPOSITION (PER CENT) - 8-PERIOD HORIZON

	VARIABLES						
SHOCKS	**M1**	**M2-M1**	**TXCAN**	**TXCHA**	**GDPD**	**GDP**	**NGDP**
Canada							
M1	40.8	0.6	1.7	0.0	2.1	4.0	5.1
M2-M1	6.2	20.7	0.7	0.0	2.0	0.3	1.2
TXCAN	4.1	1.0	14.6	7.3	0.1	5.6	3.7
TXCHA	0.7	0.2	2.4	67.5	1.6	0.1	0.4
GDPD	0.2	0.9	1.7	0.8	20.6	4.6	1.2
GDP	7.6	19.7	6.6	1.4	15.2	25.9	32.8
U.S.							
GNP	0.8	34.5	9.7	0.0	8.3	31.3	32.1
GNPD	0.2	3.4	1.3	2.5	4.7	1.3	0.1
TXUS	30.8	1.6	49.5	9.3	0.6	25.7	13.3
World							
RPOIL	2.9	0.2	8.4	4.1	1.4	0.9	0.9
RPCOM	5.9	17.1	3.2	6.9	43.1	0.2	9.2

Note: Each column corresponds to a variable i; each line gives the relative importance of variable j in the explained variance of variable i. Columns do not add exactly to 100 because of rounding.

TABLE 8: VARIANCE DECOMPOSITION (PER CENT) - 32-PERIOD HORIZON

	VARIABLES						
SHOCKS	**M1**	**M2-M1**	**TXCAN**	**TXCHA**	**GDPD**	**GDP**	**NGDP**
Canada							
M1	10.1	0.6	1.3	0.1	0.6	3.9	3.1
M2-M1	2.7	2.6	0.5	0.0	0.4	1.6	1.5
TXCAN	1.5	0.5	9.9	7.3	0.2	2.9	1.6
TXCHA	1.4	0.8	2.7	67.7	0.4	2.8	2.0
GDPD	0.2	0.2	1.2	0.8	1.0	1.5	0.4
GDP	3.7	2.9	4.5	1.4	1.4	10.3	7.9
U.S.							
GNP	2.0	13.1	7.5	0.0	5.0	12.2	13.2
GNPD	1.0	4.4	0.9	2.5	3.1	0.9	2.2
TXUS	10.9	1.8	34.6	9.3	0.4	26.5	10.2
World							
RPOIL	23.4	0.3	23.9	3.9	0.6	33.8	7.2
RPCOM	43.1	72.9	12.9	6.9	86.8	3.5	50.5

Note: Each column corresponds to a variable i; each line gives the relative importance of variable j in the explained variance of variable i. Columns do not add exactly to 100 because of rounding.

TABLE 9: FORECASTING PERFORMANCE OF THE BVAR MODEL: 1988:1-1992:1

	Horizon (quarters)					
	1			4	8	
	RMSE	**RMSEA**	**THEIL**	**RMSE**	**RMSE**	**RMSEA**
Canada						
M1	1.56	6.39	0.94	5.03	8.65	4.24
M2-M1	0.59	2.38	0.21	1.87	2.60	1.29
TXCAN	0.88	3.57	1.05	2.42	3.43	1.70
TXCHA	1.55	6.35	0.94	2.72	4.08	2.02
GDPD	0.34	1.37	0.37	1.36	4.11	2.03
GDP	0.48	1.93	0.81	1.78	3.59	1.78
GDPN	0.58	2.34	0.43	2.51	7.51	3.69
U.S.						
GNP	0.48	1.93	0.81	1.70	2.30	1.14
GNPD	0.22	0.88	0.22	0.73	1.60	0.80
GNPN	0.51	2.06	0.36	1.60	2.30	1.14
TXUS	0.76	3.07	1.12	2.33	3.89	1.93
World						
RPOIL	17.61	91.33	1.03	25.73	33.84	15.69
RPCOM	5.14	22.20	0.99	10.35	17.27	8.29

Note: RMSE is the usual root-mean-square error for the indicated forecasting horizon (1, 4 or 8 quarters); RMSEA is the same statistic expressed on an annual basis (compounded rate); THEIL refers to the Theil statistics.

TABLE 10: FORECASTING PERFORMANCE OF THE AR(4) MODEL: 1988:1 - 1992:1

	Horizon (quarters)					
	1			4	8	
	RMSE	RMSEA	THEIL	RMSE	RMSE	RMSEA
Canada						
M1	1.53	6.26	0.92	3.77	4.41	2.18
M2-M1	0.67	2.71	0.24	3.74	8.30	4.07
TXCAN	0.70	2.83	0.83	2.38	2.89	1.43
TXCHA	1.47	6.01	0.89	2.62	4.06	2.01
GDPD	0.37	1.49	0.41	1.20	1.71	0.85
GDP	0.73	2.95	0.90	2.34	4.53	2.24
GDPN	0.73	2.95	0.53	2.66	5.90	2.91
U.S.						
GNP	0.43	1.73	0.72	1.55	2.72	1.35
GNPD	0.23	0.92	0.23	0.73	1.02	0.51
GNPN	0.52	2.10	0.37	1.72	2.84	1.41
TXUS	0.73	2.95	1.07	1.90	2.52	1.25
World						
RPOIL	18.78	99.06	1.10	23.50	26.22	12.35
RPCOM	4.88	21.00	0.94	11.25	19.56	9.34

Note: RMSE is the usual root-mean-square error for the indicated forecasting horizon (1, 4 or 8 quarters); RMSEA is the same statistic expressed on an annual basis (compounded rate); THEIL refers to the Theil statistics.

FIGURE 1: VARIABLES OF THE SYSTEM

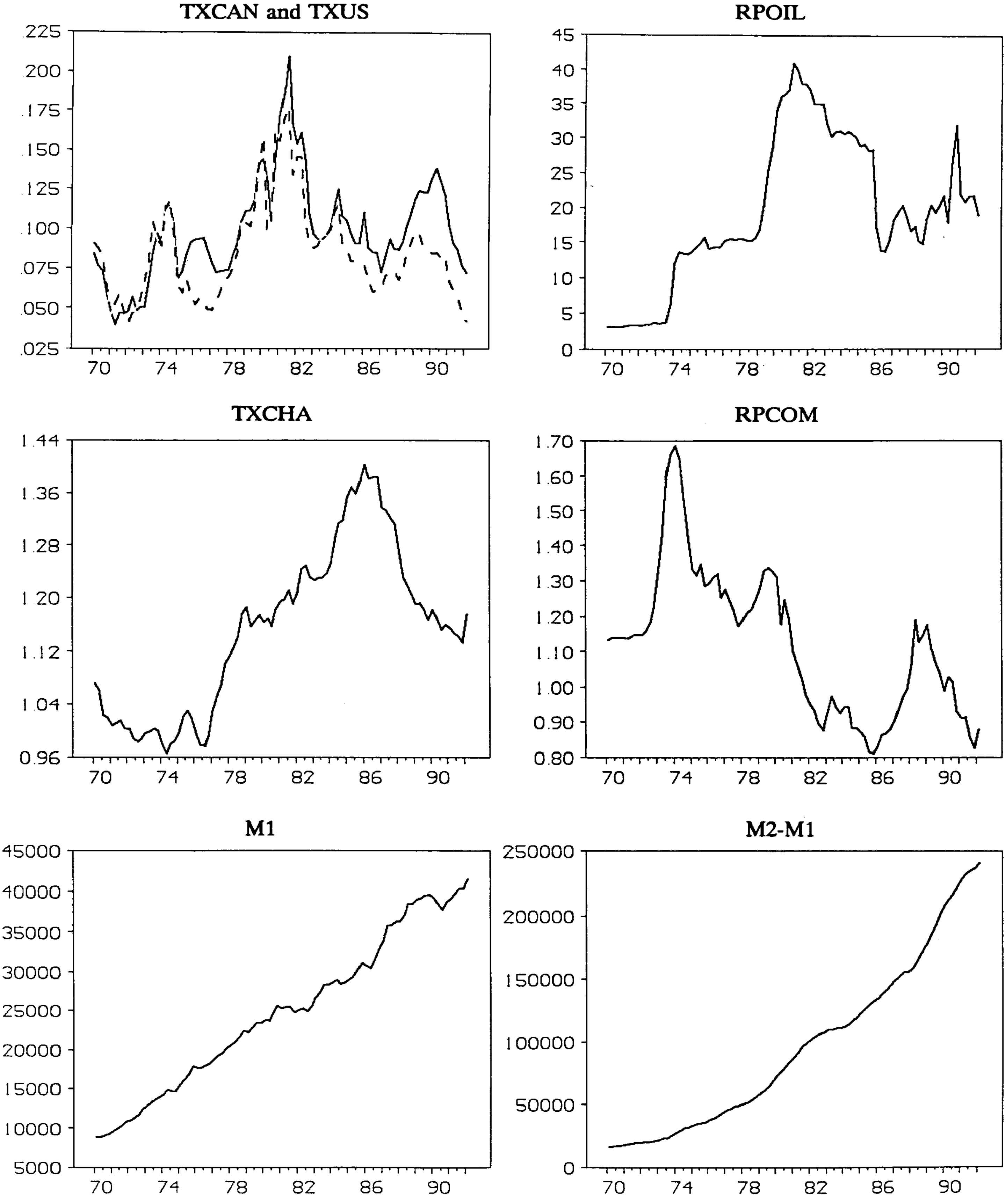


FIGURE 1 ***(CONTINUED)***

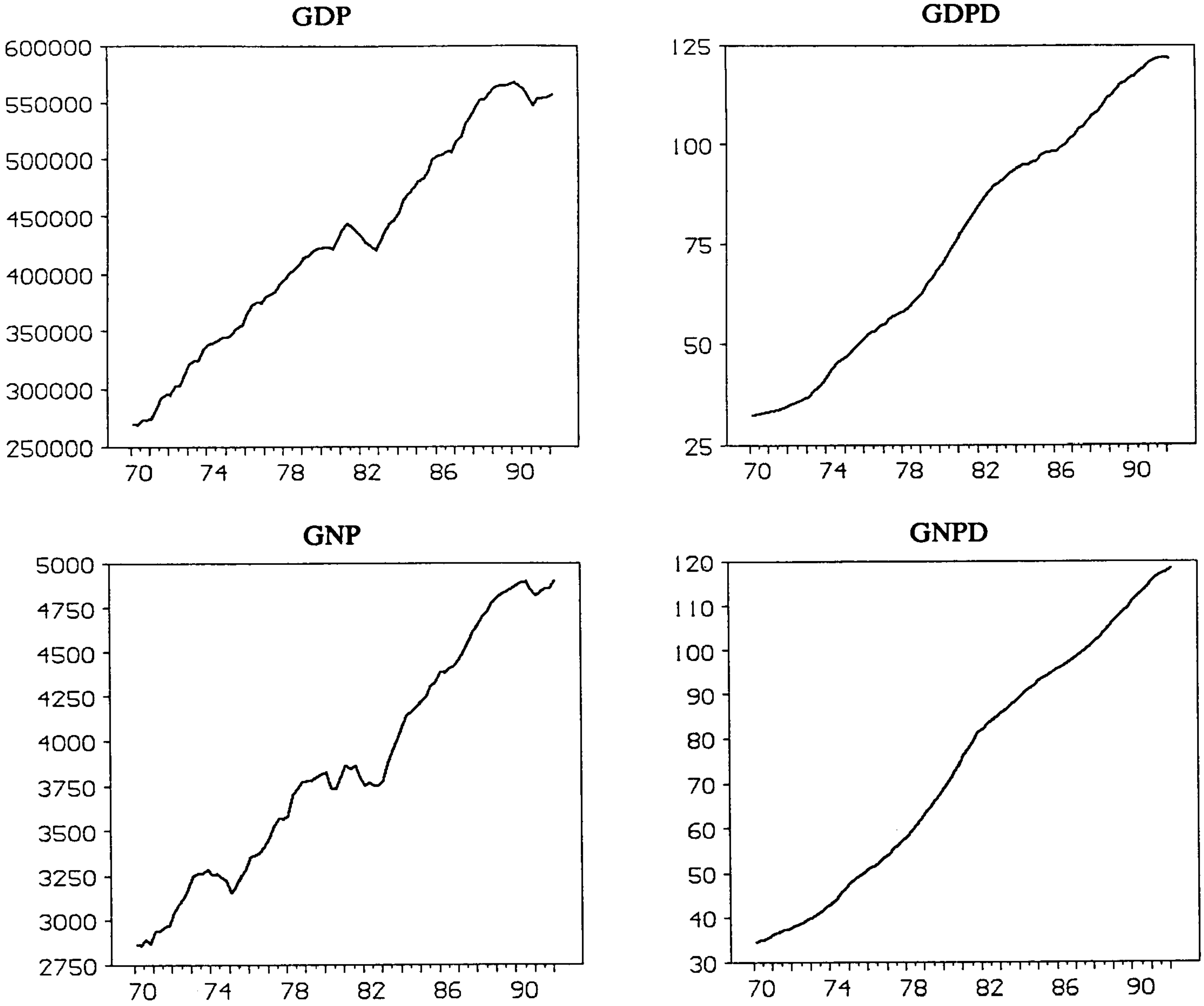
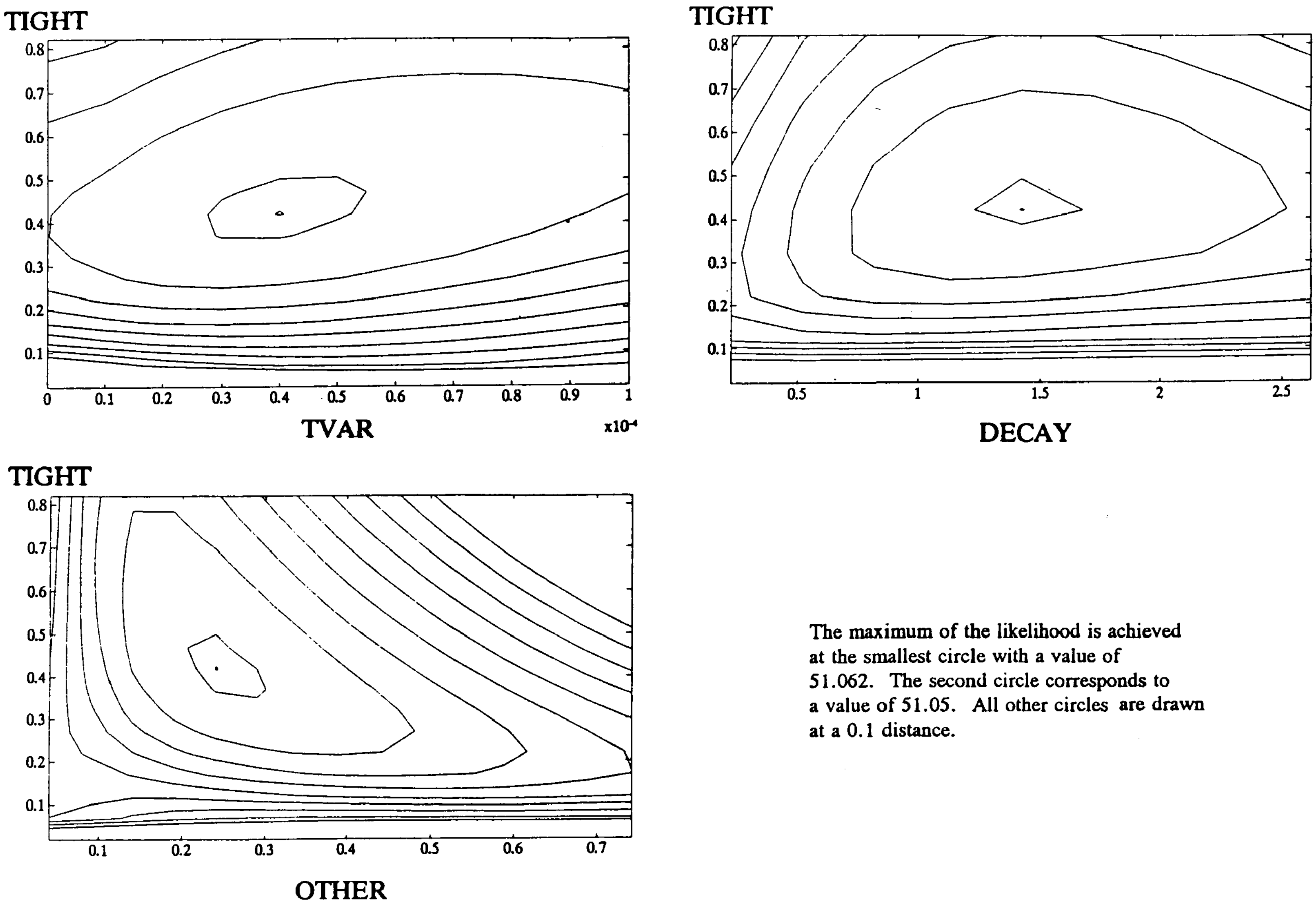


FIGURE 2: LIKELIHOOD FUNCTION CONTOURS



The maximum of the likelihood is achieved at the smallest circle with a value of 51.062. The second circle corresponds to a value of 51.05. All other circles are drawn at a 0.1 distance.

FIGURE 3: REDUCED FORM IMPULSE RESPONSES - CANADIAN SHOCKS ON CANADIAN VARIABLES

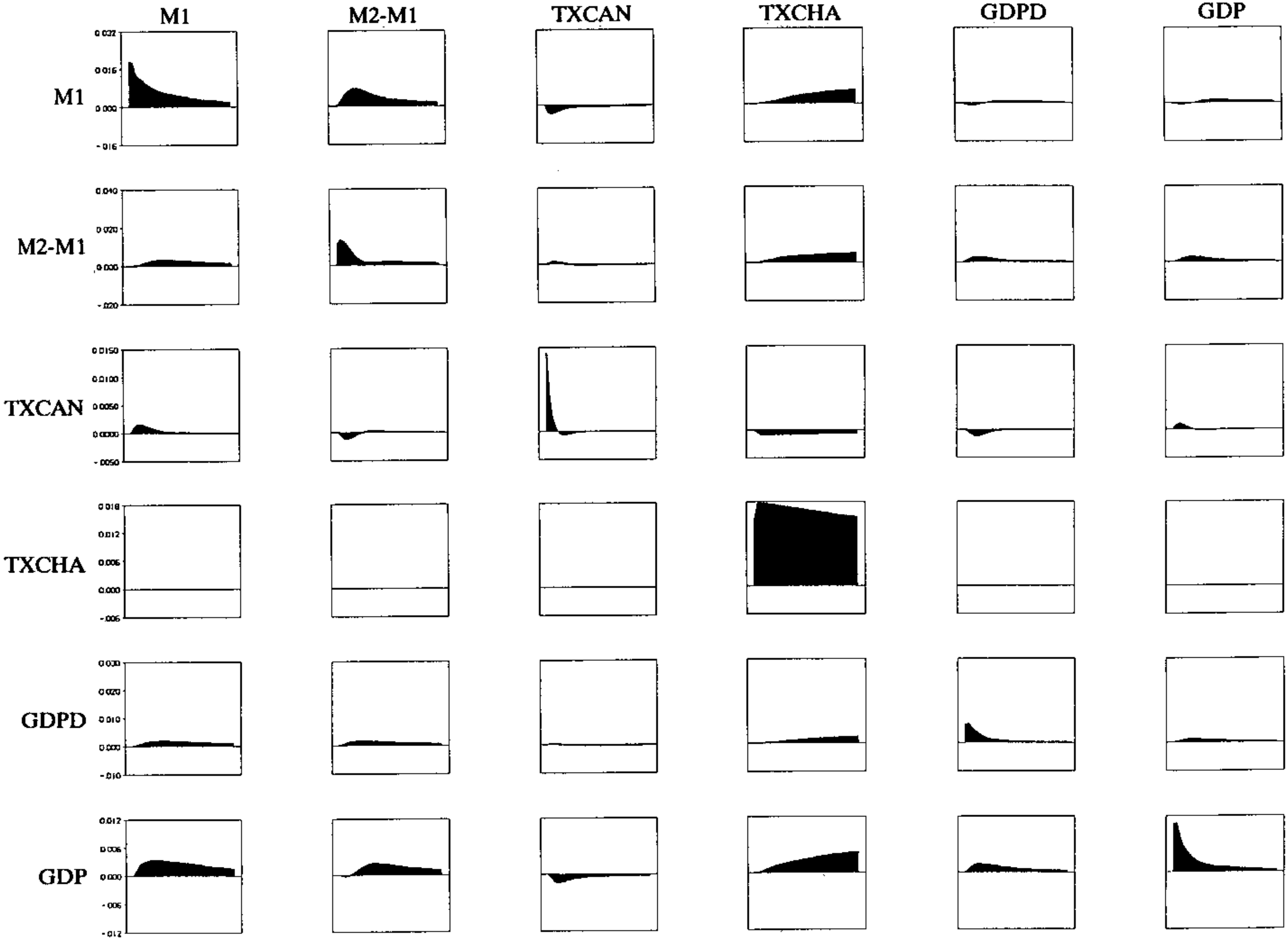


FIGURE 4: REDUCED FORM IMPULSE RESPONSES - FOREIGN SHOCKS ON CANADIAN VARIABLES

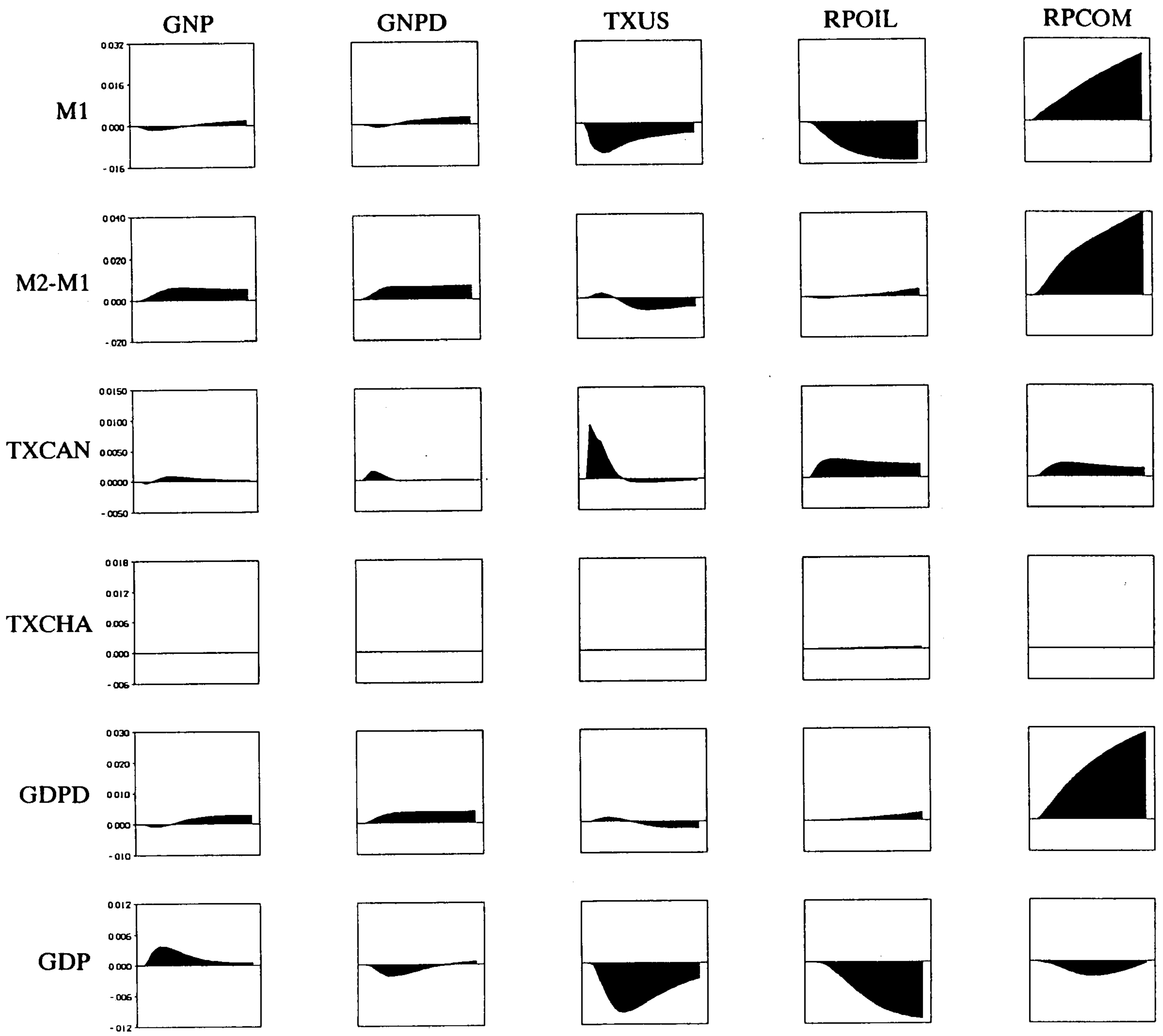


FIGURE 5: REDUCED FORM IMPULSE RESPONSES - FOREIGN SHOCKS ON FOREIGN VARIABLES

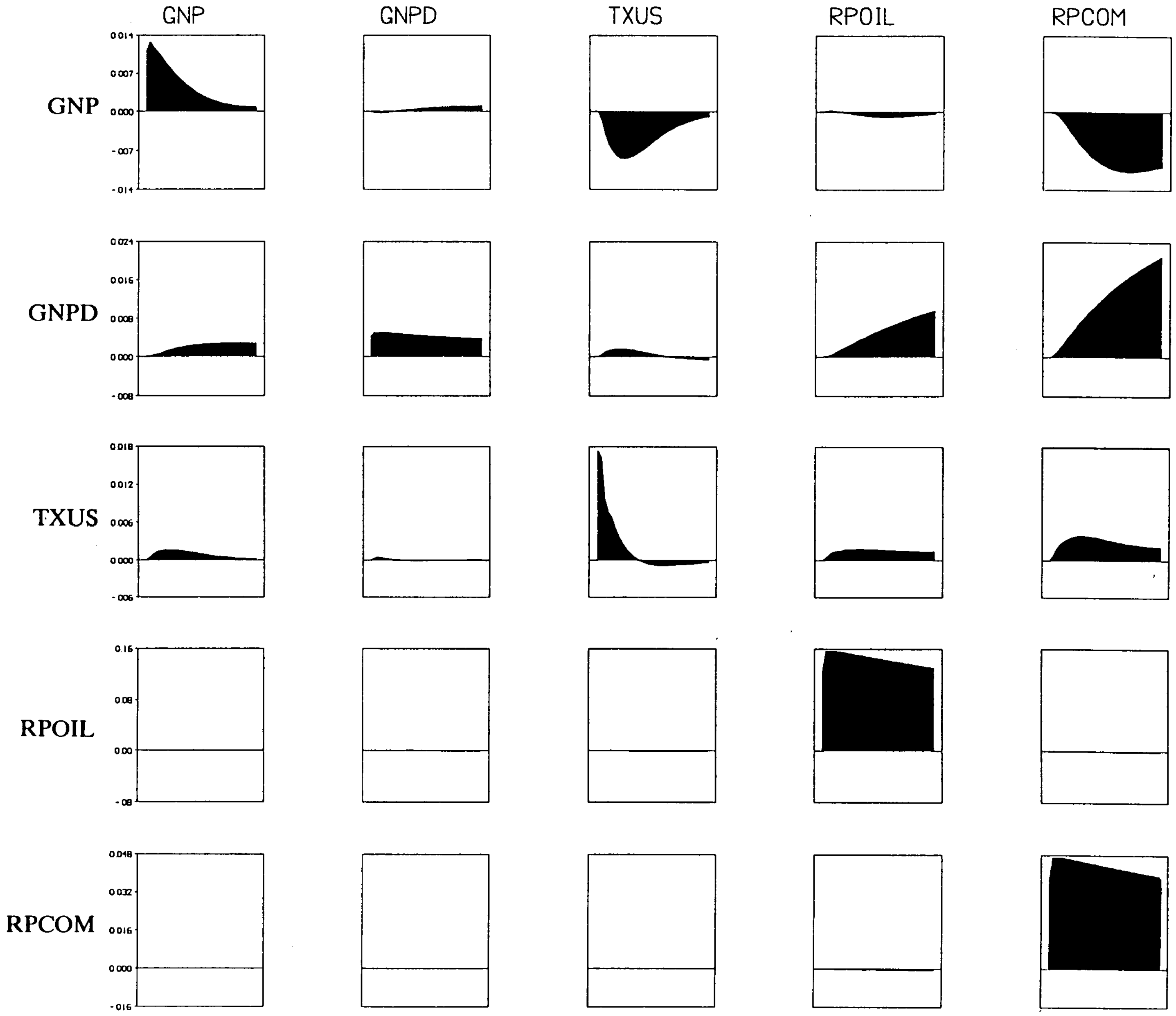


FIGURE 6: STRUCTURAL FORM IMPULSE RESPONSES - CANADIAN SHOCKS ON CANADIAN VARIABLES

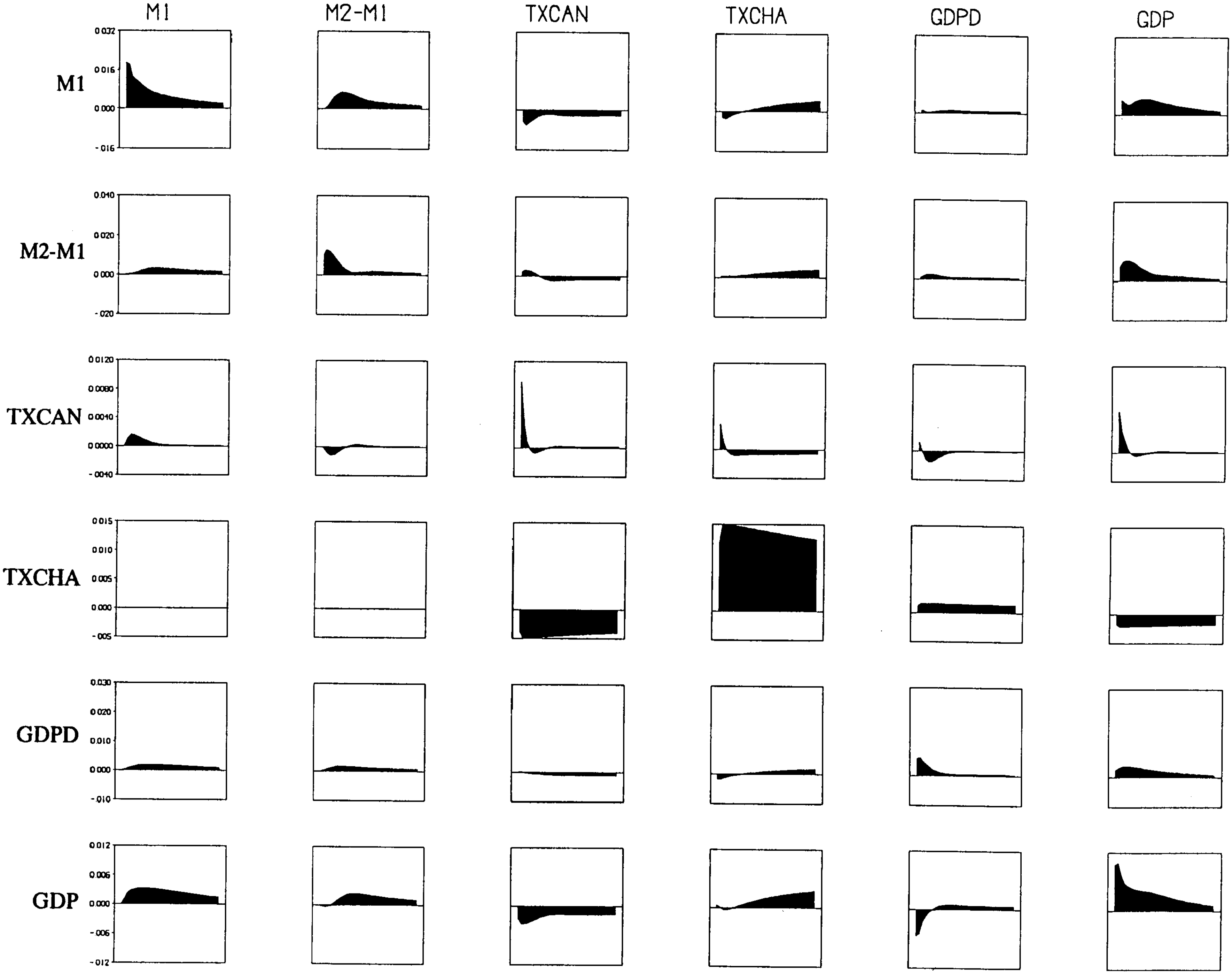


FIGURE 7: STRUCTURAL FORM IMPULSE RESPONSES - FOREIGN SHOCKS ON CANADIAN VARIABLES

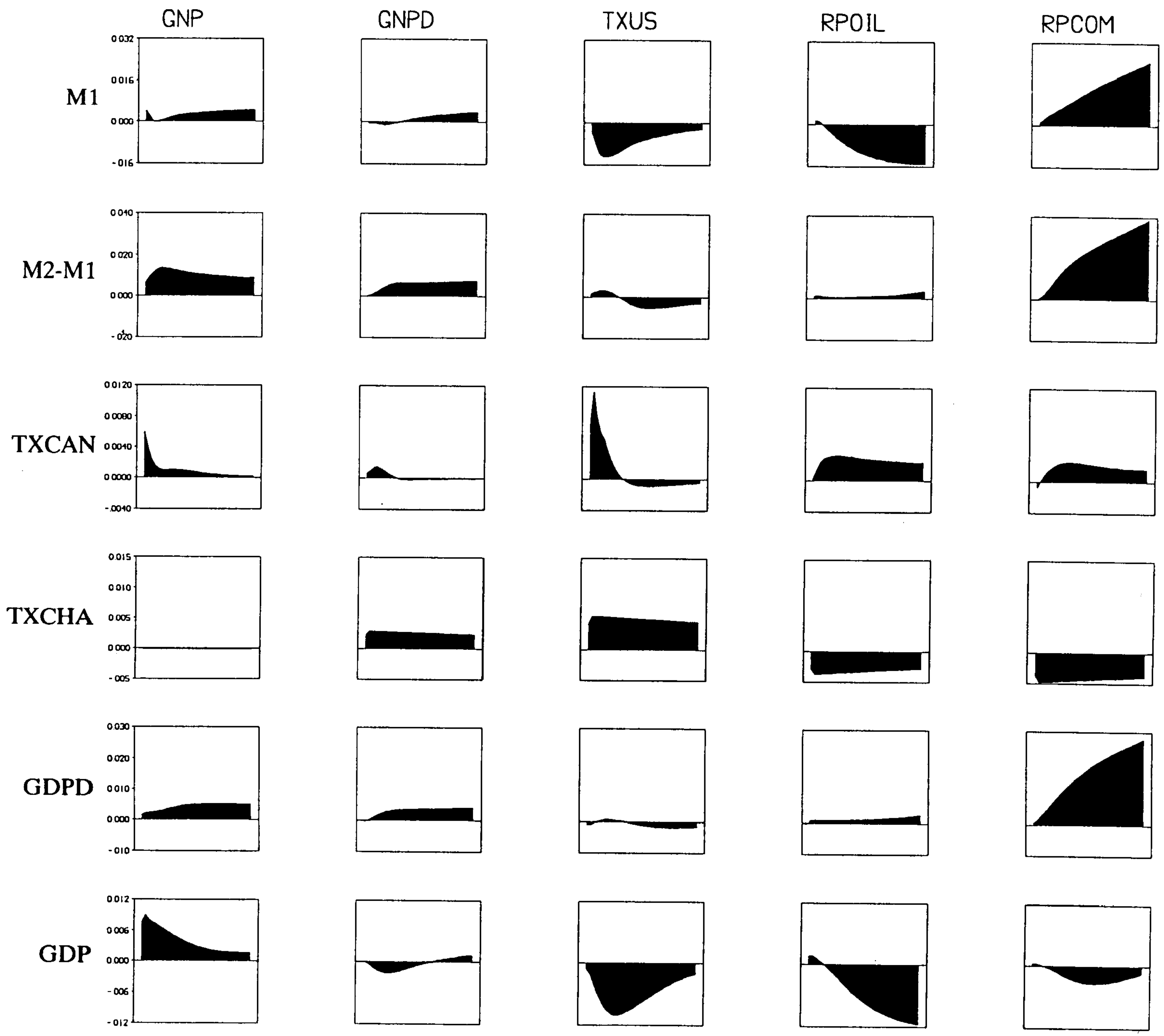
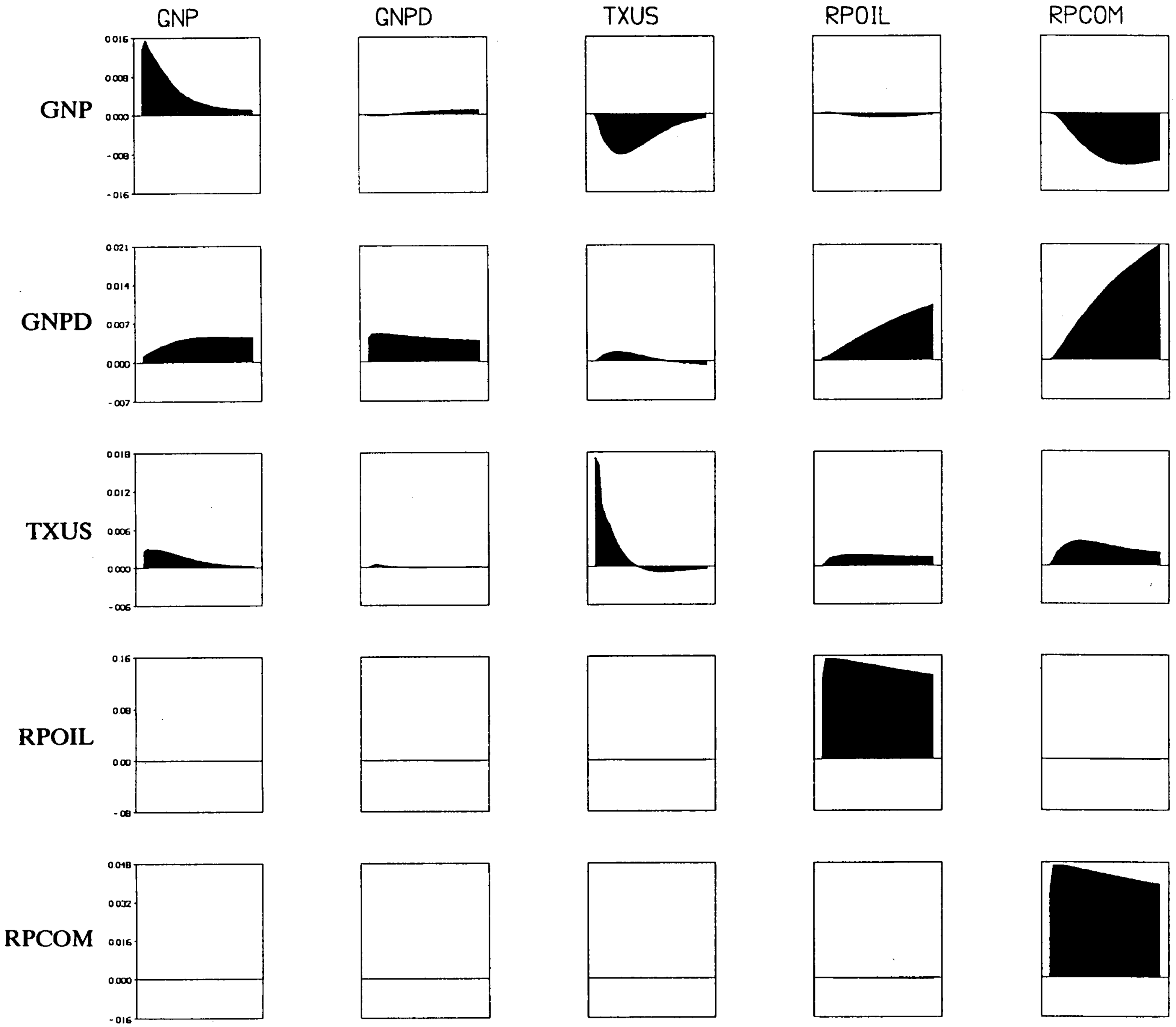


FIGURE 8: STRUCTURAL FORM IMPULSE RESPONSES - FOREIGN SHOCKS ON FOREIGN VARIABLES



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