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Abstract

This paper assesses the expectations theory for the longer end of the term structure of Canadian interest rates using three empirical approaches that have received attention in the literature: (i) cointegration tests of the long-run unbiasedness hypothesis; (ii) simulations of a theoretical long-term yield that is consistent with the expectations hypothesis, and (iii) *ex post* tests of the rational expectations hypothesis. The empirical results in this paper show that the expectations theory has considerable economic and statistical content for explaining movements in Canadian long-term yields.

The cointegration results from a vector error-correction model find a long-run relationship between short- and long-term interest rates; the term spread is an unbiased predictor of changes in short-term rates over the long run. The multi-period forecast of changes in future short-term rates from a Campbell–Shiller vector autoregression model can account for most of the variance of long-term yields; the actual long-term yield moves almost one for one with its theoretical counterpart under the expectations hypothesis. The tests of the rational expectations hypothesis on bond yields from 1 to 5 years' maturity find that the term structure beyond 2 years resembles a rational forecast of the weighted average of changes in future short rates.

JEL classification: E43
Bank of Canada classification: Interest rates

Résumé

L'auteur de l'étude cherche à établir si la théorie des attentes se vérifie dans le cas des taux à moyen et long terme canadiens. Il a recours pour cela à trois méthodes empiriques utilisées dans la littérature : i) la réalisation de tests de cointégration afin de vérifier l'absence de biais en longue période; ii) la simulation d'un rendement à long terme théorique, fondé sur la théorie des attentes; iii) l'exécution de tests *ex post* portant sur l'hypothèse de rationalité des attentes. Selon les résultats empiriques obtenus, la théorie des attentes s'appuie sur des fondements économiques et statistiques très solides pour expliquer les mouvements des rendements à long terme canadiens.

Les tests de cointégration menés à l'aide d'un modèle vectoriel à correction d'erreurs font ressortir l'existence d'une relation de longue durée entre les taux d'intérêt à court terme et ceux à long terme. L'écart entre les taux courts et longs est une prévision non biaisée des variations des taux courts en longue période. Les prévisions multipériodes des variations des taux courts tirées d'un modèle VAR du type Campbell-Shiller parviennent à rendre compte de la majeure partie de la variance des rendements à long terme; le rendement effectivement observé à long terme varie à peu près dans les mêmes proportions que le rendement fondé sur la théorie des attentes. D'après les tests visant à valider l'hypothèse de rationalité des attentes dans les cas des rendements obligataires de un à cinq ans, la portion de la structure des taux correspondant aux échéances supérieures à deux ans peut être assimilée à une prévision rationnelle de la moyenne pondérée des variations futures des taux à court terme.

JEL : E43
Classification de la banque : Taux d'intérêt

Introduction

The most popular theory to explain long-term yields is the expectations theory of the term structure of interest rates. The expectations theory states that long-term yields are equal to current and expected future short-term rates plus a term premium. However, there is considerable anecdotal evidence that long-term yields are "excessively" volatile, in the sense that they vary more than is warranted by the theory. The conventional wisdom, based mainly on the U.S. experience, is that the expectations model can easily be rejected on statistical grounds.[1] Nevertheless, some recent research for other countries, including Canada, suggests that the expectations hypothesis may be adequate for explaining the term structure of interest rates, even though it might be rejected statistically for some specifications of the hypothesis.[2]

This paper assesses the expectations theory for Canada using three empirical approaches that have received attention in the literature: (i) cointegration tests of the long-run unbiasedness hypothesis, (ii) simulations of a theoretical long-term yield that is consistent with the expectations hypothesis, and (iii) ex post tests of the rational expectations hypothesis. The empirical analysis assesses the robustness of the information content in longer term yields about future short-term interest rates, and the stability of the expectations relationship both along the term structure and over different periods. The term structure relationship is also used to decompose recent movements in the long-term yield into expected changes in short-term interest rates and a time-varying term premium.

It is important to understand the strengths and weaknesses of the expectations hypothesis, because the term structure is used to extract information about financial market expectations of the future path of interest rates and inflation. Understanding the link between longer-term yields and financial market expectations about the path of short-term rates is also important for anticipating the response of long-term yields to monetary policy changes and for understanding the interest-rate channel of the monetary transmission mechanism.

Section 1 briefly outlines a general specification of the expectations theory of the term structure. In section 2, the results for a vector error-correction model (VECM) find a long-run cointegration relationship; short- and long-term rates share a common stochastic trend and the term spread is an unbiased predictor of changes in short-term rates over the long run. The relationship is found to be stronger during the fixed exchange-rate period, when short rates are generally more stable.

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1. See for example the survey of the term structure literature by Shiller (1990).
2. Hardouvelis (1994) undertakes a relatively broad empirical investigation of the expectations hypothesis for the G7 countries.

Section 3 uses the Campbell–Shiller vector autoregression (VAR) model to simulate the expected changes in short-term interest rates and to construct a theoretical long-term yield. The multi-period forecast of future short-term rates can account for most of the variance in long-term yields; the actual long-term yield moves almost one for one with its theoretical counterpart under the expectations hypothesis.

Section 4 tests the rational expectations hypothesis of the term structure for bond yields from 1- to 5-year maturities. The results show that the term structure beyond 2 years resembles a rational forecast of the weighted average of changes in future short-term rates. The relatively large bias in the forecast of future short rates from the shorter term structure reflects the poorer predictability of the short-term rate over these horizons; this is also consistent with the presence of a time-varying term premium.

The final section briefly summarizes the results from all three empirical approaches and suggests some avenues for future research. The appendix outlines the Johansen–Juselius estimation methodology.

1. The expectations theory of the term structure

One specification of the expectations theory of the term structure states that the long-term yield is equal to a weighted average of current and expected future short-term rates plus a term premium. A general form of this specification may be written as:

$$R_t^n \equiv \sum_{i=0}^{n-1} w_i E_t r_{t+i} + E_t \theta_t, \tag{1}$$

where R_t^n is the yield on a bond with a maturity of n periods, rt is the interest rate on a 1-period debt instrument, E_t is the expectation operator conditional on information available at time t, and w_i is a geometric declining weight that sums to 1.[3] In the modern version of the expectations hypothesis, the term premium on an n-period bond is time invariant, $E_t\theta_t = \theta_n$. In the pure version, the term premium is 0, $E_t\theta_t = 0$.

3. The declining weights $w_i \equiv g^i(1-g)/(1-g^n)$ are needed because long-term coupon bonds have a duration that is shorter than their maturity. The duration of a bond is often approximated in the literature with $g = (1+\bar{R}^l)^{-1}$, where $\bar{R}$ is the mean level of the long-term yield over the sample.

After subtracting the short-term rate from both sides of the term structure equation and rearranging, the term spread can be related to expected changes in the short-term rate:

$$\sum_{i=1}^{n-1} \frac{n-i}{n} \Delta r_{t+i} = R_t^n - r_t + \theta_t + \gamma_t, \tag{2}$$

where w_i is set equal to $1/n$ for simplicity and γ_t is a random forecast error in the term structure relationship.

The term spread can also be used to express the market's forecast of a 1-period change in the long-term yield by subtracting equation (1) from the next period's long-term yield R_{t+1}^{n-1}

$$(n-1)(R_{t+1}^{n-1} - R_t^n) = R_t^n - r_t + E_t \theta_{1, t+1}, \tag{3}$$

where $w_i = 1/n$ and the left-hand-side is the 1-period change in the *n*-period yield.[4] If the expectations hypothesis holds and the 1-period term premium $\theta_{1, t+1}$ is constant, then changes in the long-term yield reflect changes in the term spread. Intuitively, equation (3) states that if the *n*-period yield is expected to rise next period, which will result in a capital loss, then the *n*-period bond has to have a higher current yield than the 1-period instrument in order to equate expected returns over the next period.

2. The VECM

2.1 The long-run unbiasedness hypothesis

The changes in future short-term rates on the left-hand side of equation (2) is an I(0) series, with the level of the short rate being non-stationary and integrated of order 1. The right-hand side is a linear combination of two I(1) variables, R_t and r_t, plus a term premium θ_t and forecast error γ_t.[5] If the expectations hypothesis holds, then the term premium and the forecast error are stationary. This implies that the term spread is stationary, and thus long- and short-term rates are cointegrated. The term spread is an unbiased predictor of changes in future short-term rates over the long run if there is a stable one-to-one relationship between short- and long-term interest rates. However, the

4. See Evans and Lewis (1994) and Campbell and Shiller (1991) for this specification.
5. However, in practice, it is unlikely that inflation or real interest rates would rise or fall without limit in the long run as the presence of a unit root would allow. The non-stationary result could be an artifact of the short estimation period and may reflect economic disequilibrium or the lack of a constant nominal anchor over the estimation period. The assumption of a random walk for interest rates may also not apply if a lower bound exists at low levels of interest rates. Figure 1 suggests that a lower bound was not a problem for the 42-year sample period.

existence of a cointegration relationship is a necessary condition for the expectations hypothesis to hold, though it is not an explicit test of the hypothesis itself—this is discussed below.

The research on the existence of a long-run equilibrium relationship between Canadian interest rates as implied by the expectations hypothesis can be traced to Boothe (1991). Using residual-based tests, he found that cointegration between short- and long-term interest rates is always rejected at the 5 per cent level for the 1972–89 period. Furthermore, the coefficient on the short-term rate was always significantly below the theoretical value of 1.0 that is required for the term spread to be an unbiased predictor of changes in short rates over the long run. We re-estimated Boothe's equation for a longer sample period, from 1956 to 1998, and obtained

$$R_t^n = 2.73 + 0.72 r_t,$$

where R_t^n is the 10-year-and-over government bond yield and r_t is the 90-day commercial paper rate. As in Boothe, the augmented Dickey–Fuller test statistic at –3.21 suggests that the null hypothesis of no cointegration cannot be rejected by the data at the 10 per cent level. The coefficient on the short rate is slightly larger than the 0.59 in Boothe's regression for the 1972–89 period, but still noticeably less than 1.

Hejazi and Parkinson (1997) obtained similar results using weekly data for the 1982–96 period and a fractional cointegration specification. Similarly, Tkacz (1997a) generally could not reject cointegration between short-term rates and medium- or long-term bond yields at the 10 per cent level for the 1972–96 period using the Johansen–Juselius estimation technique; he estimated long-term coefficients between 0.79 and 0.88. However, after adjusting short- and long-term interest rates for inflation, Côté and Fillion (1997), rejected cointegration for a similar period, but found that the restriction of 1 on the slope coefficient could not be rejected.

Gravelle (1997) estimated a VECM on the money-market term structure for Canada on daily data for the 1982–97 period. He found that 3-month spot and forward rates were cointegrated, but that the cointegration vector could not be restricted to equal the spread between the forward and spot rates. He attributed this to a time-varying term premium, which he found to be related to interest rate volatility.

In general, the previous research for Canada either finds weak evidence of a cointegration relationship between short- and longer term interest rates or rejects the long-run unbiasedness hypothesis. The focus has generally been on the post-1972 period, presumably because the exchange rate was allowed to float over this period. A longer sample period, however, may be needed to conduct cointegration tests of the expectations hypothesis for the longer end of the term structure. The purpose of the following section is to re-estimate the cointegration relationship for a longer sample period and to compare the relationship for the fixed and floating exchange-rate regimes. The estimations also test the conditions for the expectations hypothesis at the long end of the term structure for each sample period.

2.2 Data and estimation

The empirical work uses the Johansen–Juselius (JJ) estimation technique to test the conditions for the expectations hypothesis for the 1956–98 period. The JJ methodology is chosen over the Engle and Granger (1987) residual-based tests for cointegration because it captures all of the information available in each endogenous variable.[6] The JJ estimation technique is outlined briefly in the appendix.

The tests of the long-run cointegrating implications of the expectations hypothesis are based on three conditions, assuming that interest rates are I(1). First, the necessary but not sufficient condition is that the long- and short-term rates must be cointegrated so that the interest rates are driven by a common permanent or long-run component.[7] Second, for long-run unbiasedness, the sum of the cointegration coefficients should equal 0, so that the short and long rates cointegrate with a cointegrating vector [1,–1] after normalization. Third, the coefficient on the error-correction term in the dynamic equations for changes in short- and long-term rates should be positive and statistically significant. For short-run unbiasedness, the error-correction coefficient should equal –1 in the dynamic equation for the short-term rate.[8]

The 10-year-and-over Government of Canada bond yield is used as the long-term yield in the estimations. It is the simple average of all bond yields with a maturity of 10 years and over, and is available back to 1956. The 90-day commercial paper rate is used as the short-term rate.[9]

Figure 1 presents the short- and long-term rates and the term spread for the 1956–98 period. It suggests that these interest rates were non-stationary over the sample period.[10] Since the interest rates do not display deterministic trends, there is no need to have a drift term in the dynamic equations to capture changes in the stochastic process. As a result, the intercept enters as a constant in the cointegration vector, and can be interpreted as the mean of the process.

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6. Davidson and Mackinnon (1993) note a number of other reasons for using the JJ-cointegration tests over the residual-based tests.
7. In essence, this is a joint test of the null hypothesis that the term premium and forecast error are stationary and interest rates contain unit roots.
8. See Rossi (1996) for the restrictions implied by short- and long-run unbiasedness.
9. The commercial paper rate is believed to capture the behaviour of short-term rates better than the treasury bill rate during the 1970s when secondary reserve requirements created a captive market for treasury bills.
10. Augmented Dickey–Fuller *t*-tests for unit roots cannot reject non-stationarity at the 95 per cent level for the short- and long-term interest rates. Although a unit root cannot be rejected in the interest rates, it should be noted that unit root tests cannot distinguish between roots equal to one or very close to one in finite samples.

The lag selection process for the vector error-correction models was guided by sequential likelihood-ratio tests, as well as tests for normality, serial correlation, and heteroskedasticity.[11] Overall, the tests chose two lags, which indicates that the financial dynamics beyond two quarters are not particularly relevant for interest rates for Canada.[12] The two-quarter lag structure is also generally in line with that of previous research.[13]

2.3 Cointegration results

Panel A of Table 1 presents the two JJ tests for cointegration and the coefficients on the loading factors (α). The cointegrating vectors are presented according to the magnitude of the corresponding eigenvalues. Both the maximal eigenvalue statistic and trace test are significant at the 99 per cent confidence level for the 1956–98 period. This shows that the data can easily reject the null hypothesis of no cointegration of short- and long-term rates. The cointegration results are stronger than that of previous research, owing to the longer sample period. The results for the two subperiods indicate that the relationship was slightly stronger in the years when the exchange rate was fixed, prior to 1972.

The cointegration results imply that there is a common stochastic trend driving both interest rates. Since, according to the Fisher hypothesis, nominal interest rates have a one-to-one relationship with expected inflation in the long run, inflation could be the common trend driving short- and long-term rates. Permanent innovations in the world real interest rate could also drive the level of interest rates for an open economy in the long run.

The *t*-tests for the null hypothesis that the α-coefficients on the loadings are 0 are rejected at the 1 per cent level for the long-rate equation and at the 5 per cent level for the short-rate equation for the 1956–98 period. The significant *t*-statistics imply the rejection of weak exogeneity of both the short- and long-term rates, and that single-equation cointegration techniques of estimating the expectations relationship may be invalid. It also suggests that, in general, both short- and long-term rates adjust in order to correct a long-run disequilibrium between the two rates. The size of the adjustment coefficients in both equations suggest that both rates adjust by about the same amount to a shock to their common stochastic trend.

However, the statistical significance of the coefficients on the cointegration vector in the two dynamic equations differs for the two subperiods. The coefficient in the equation for the short rate is

11. Inspection of the residuals indicated the problem of non-normality was due in part to three outliers in the data for the long-term yield - 1980(Q2), 1981(Q4), and 1982(Q4). Consequently, dummy variables were added to the short-run dynamics of the long yield for these dates.
12. Sequential likelihood-ratio tests against 4 lags could not reject the null hypothesis of 2 lags at the 5 per cent significance level.
13. Campbell and Shiller (1987, 1991) choose 4- and 6-month lags, respectively; Engsted and Tanggaard (1994) find a 6-month lag, and Engsted (1995) and Hardouvelis (1994) choose 4-quarter lag structures.

positive and significant only in the pre-1972 period and the coefficient in the equation for the long yield is significant (and negative) only in the post-1972 period. The positive sign in the equation for the short rate is consistent with the expectations hypothesis, which implies that the spread between long and short rates should be able to predict changes in short-term rates. The high level of significance in the pre-1972 period is also consistent with the view that the short-term rate is more predictable during periods of fixed exchange rates, because it would follow the foreign interest rate one for one and the differential would reflect changes in the risk premium.[14] The relatively small size of the error-correction coefficients indicates that the term spread is a biased predictor of changes in short-term interest rates during any particular short run.

The negative response of the long rate to non-stationary movements in the short rate runs counter to the expectations hypothesis, and is more consistent with a causal interpretation of the error-correction term. A causal reaction would be expected if the short-term rate is exogenously determined. Engsted and Tanggaard (1994), for example, found that medium- and long-term rates adjusted to correct their disequilibrium with short rates when the Fed targeted the short rate during the period up to October 1979. In a small, open economy like Canada, the reaction of the long-term yield to changes in the short rate may reflect the transmission of changes in world interest rates.[15] The negative coefficient on the error-correction term could also arise because of the Fisher effect and monetary policy credibility, so that a credible rise in short rates would reduce inflation expectations and lead to a fall in the long-term yield.

Panel B of Table 1 presents the coefficients for the restricted and unrestricted cointegrating vectors, along with the *p*-value for the Chi-squared distribution for the likelihood-ratio test of the parameter restrictions. The cointegrating relationship is shown in vector format and is normalized on the long-term interest rate. The zero-sum restriction on the cointegration coefficients (1, –1) cannot be rejected at the 5 per cent level for all periods. This means that there is a one-to-one relationship between short- and long-term interest rates; the term spread is an unbiased predictor of changes in short-term interest rates over the long run. The constant term in the restricted model suggests that the term premium is about 75 basis points in long-run equilibrium. Recent empirical work by Fung et al. (1999) for Canada and by Campbell and Shiller (1996) for the United States found risk premiums of a similar size.

Panel C presents some diagnostic tests on the residuals of the VECM. The null hypothesis of no serial correlation is generally rejected for all periods. However, the null of normality of the residuals is rejected by the data, so the cointegration results should be treated with some caution.

14. This is consistent with Gerlach and Smets (1997b), who find that it is more difficult to reject the expectations hypothesis in countries that have conducted monetary policy using fixed exchange rates.
15. In fact, the null hypothesis of no cointegration between the Canadian short-term rate and the U.S. long-term bond yield can be rejected at the 5 per cent significance level.

Table 1: Johansen–Juselius results

A: Vector error-correction model					
Cointegration tests				**Loading factors (α)**	
Period	**H_0:**	**λ-max**	**Trace**	**Δ *rl* (*t*-statistic)**	**Δ *rs* (*t*-statistic)**
1956–98	r = 0 $r \leq 1$	29.9* 1.9	31.8* 1.9	–0.09 (3.31)	0.11 (2.17)
1956–71	r = 0 $r \leq 1$	24.1* 1.8	27.9* 1.8	–0.03 (0.91)	0.20 (3.27)
1972–98	r = 0 $r \leq 1$	18.4** 1.6	19.9** 1.6	–0.11 (3.01)	0.09 (1.29)

* Indicates that the null of *r* cointegrating vectors can be rejected at the 1 per cent level, ** at the 5 per cent level.

B: Cointegration vector							
Unrestricted vector (β coefficients)				**Restricted vectors (β coefficients)**			
Period	***rl***	***c***	***rs***	***rl***	***c***	***rs***	***p*-value**
1956–98	1	–1.33	–0.92	1	–0.74	–1	0.36
1956–71	1	–0.29	–1.13	1	–0.90	–1	0.51
1972–98	1	–1.90	–0.86	1	–0.67	–1	0.41

C: Diagnostic tests (probability values)				
Period	**L–B (N/4)**	**LM (1)**	**LM(4)**	**Normality**
1956–98	0.26	0.02	0.24	0.00
1956–71	0.63	0.59	0.94	0.00
1972–98	0.69	0.04	0.33	0.01

The Ljung–Box test is based on the estimated auto- and cross-correlations of the first N/4 lags (42), where N is the number of observations; the LM-type tests of Breusch–Godfrey are for first- and fourth-order autocorrelation; the Shenton–Bowman is a test for normality.

Following the methodology of Johansen and Juselius, the hypothesis of structural stability of the cointegrating parameters is tested using a recursive regression technique. The procedure statistically tests whether the parameters estimated from a sequentially updated subsample beginning in 1967 equal those from the full sample of 1956 to 1998. For each subsample, the test statistic is distributed as χ^2 with n–1 degrees of freedom, where n is the number of interest rates in the cointegration vector.

Figure 2 plots these χ^2 test statistics for the restricted VECM. The numbers are normalized so that 1.0 represents the 5 per cent critical level. The BETA_Z plots the χ^2 test statistic when all the parameters are estimated recursively. The BETA_R plots the statistic when all the short-run parameters are fixed and the long-run parameters are estimated recursively. The plots indicate that there was not a statistically significant difference between the subsample cointegrating vectors and the full sample vector. This suggests that the parameters for the long-run conditions for the expectations hypothesis were relatively stable over the 42-year period, which includes both fixed and flexible exchange-rate regimes.

2.4 Limitations of cointegration

The cointegration results for short- and long-term interest rates are consistent with the long-run implications of the expectations hypothesis, and suggest that the term spread is an unbiased predictor of changes in short rates over the long run. However, the cointegration methodology is a rather weak test of the expectations hypothesis, because it can only establish that short- and long-term interest rates share a common stochastic trend, which could be consistent with other theories of the term structure of interest rates. Also, the long-run equilibrium relationship may not necessarily result from market behaviour based on rational expectations.

In addition, although the residuals of the cointegration vector are stationary, there are periods of relatively large and persistent short-run deviations from the long-run relationship between short- and long-term rates, especially in the mid-1970s and the early 1980s.[16] These deviations indicate that the unbiased expectations hypothesis does not necessarily hold in any particular short run. Furthermore, the cointegration interpretation does not allow for a determination of whether the deviations are due to changing expectations about future interest rates or to changes in the term premium. For this reason, we now turn to the economic content of the term-structure relationship and to more explicit tests of the expectations hypothesis.

16. The residual series of the cointegration vector is quite similar to the fluctuations in the term spread in Figure 1 because of the restriction that the sum of the cointegration coefficients should equal 0.

3. The Campbell–Shiller VAR model

3.1 Theoretical long-term yield

In a seminal paper, Campbell and Shiller (1987) use the cointegration property of short- and long-term interest rates to specify a VAR model that can simulate the expected future changes in short-term rates. The approach tests for the expectations hypothesis by generating the VAR forecasts of changes in future short-term rates, and then comparing the counterfactual long-term yield that is consistent with the expectations hypothesis with the behaviour of the actual long-term yield. The Campbell–Shiller methodology allows for multi-period forecasting of changes in short-term rates without having to both estimate regressions with overlapping errors and drop large portions of the estimation period in order to test for the *ex post* success of the term spread as a predictor of changes in future short-term rates.

Campbell and Shiller (1987, 1991) specify a bivariate VAR model for two stationary variables, the term spread, $S_t \equiv R_t - r_t$, and the change in the short interest rates, Δr_t:

$$\begin{bmatrix} \Delta r_t \\ S_t \end{bmatrix} = \begin{bmatrix} a(L) & b(L) \\ c(L) & d(L) \end{bmatrix} \begin{bmatrix} \Delta r_{t-1} \\ S_{t-1} \end{bmatrix} + \begin{bmatrix} \mu_{rt} \\ \mu_{st} \end{bmatrix}, \tag{4}$$

where $a(L),\ldots, d(L)$ are lagged polynomials. The model is estimated for demeaned values, which guarantees a non-varying component of the term premium. This constant component is accounted for by a non-zero difference of the unconditional means of the long- and short-term interest rates.

In the VAR model, changes in the short-term interest rate would only be dependent on past changes in the short rate if $b(L)$ were 0. On the other hand, if market participants have additional information beyond the history of past changes in the short rate (and therefore past S_t), then S_t will have incremental explanatory power. If agents do not have such information, then they form S_t as an exact linear function of current and lagged Δr_t.

The estimation methodology proceeds in three steps. First, a second-order VAR model is estimated for the change in the short-term rate, and the spread between long- and short-term interest rates is estimated as in equation (4).[17] Second, the VAR framework is used as a model for a multi-period forecast of changes in future short-term rates. Assuming a constant-term premium, the predicted changes in short-term rates, along with a set of declining geometric weights, are then used to compute a theoretical or counterfactual long-term yield or term spread that is consistent with the expectations hypothesis. Third, the theoretical yield or spread is compared with the historical

17. As with the VECM in Section 2, sequential likelihood-ratio tests against four lags could not reject the null hypothesis of two lags at the 5 per cent level.

behaviour of the actual series in order to assess how well the expectations hypothesis explains movements in long-term yields or the term spread over time.

Campbell and Shiller show that, although the restrictions implied by the expectations theory can easily be rejected on U.S. data, long- and short-term interest rates computed under the assumption of the expectations hypothesis evolve over time much as actual term spreads do.[18] Similarly, Hardouvelis (1994) and Gerlach (1996) found that interest rates evolve much as predicted by the expectations hypothesis for other countries. For Canada, they found that the variance of the theoretical term spread or long-term yield could account for about 90 per cent of the variance of the actual series since the mid-1950s, and that their correlation was virtually equal to 1. Sutton (1998) found that the variance accounted for a much smaller percentage for a more recent period, and that theoretical spreads tend to be positively correlated between countries.

In the following section, the Campbell–Shiller model is re-estimated for Canada to include the most recent developments in interest rates. The robustness of the predictive content of the long-term spread is assessed by including medium-term and U.S. long-term spreads against the short rate in the model in order to assess whether the long-term spread contains all of the relevant information of market participants. The robustness of the predictive content is also assessed across different periods and using rolling out-of-sample forecasts. Finally, the Campbell–Shiller model is used to decompose movements in the long-term yield over the 1990s.

3.2 Diagnostic statistics

The comparison of actual and theoretical long-term yields relies on two different volatility tests to assess the extent to which the expectations hypothesis can account for the movements in long yields. These tests are: the ratio of the standard deviations, σ_{R*} and σ_R; and the correlation between the yields, $\rho_{R*,\,R}$, over the sample period, where the star superscript indicates theoretical yield. In addition, Campbell and Shiller examined the coefficient γ of a regression of the theoretical yield on the actual yield, which is simply the multiplicative of the two volatility statistics, $\gamma \equiv \rho_{R*,\,R}(\sigma_{R*}/\sigma_R)$. If the behaviour of long-term yields is broadly defined by unbiased market expectations of changes in future short rates, then the standard deviations of the two yields is expected to be similar, the correlation between the two yields is expected to be close to 1, and γ is expected to be close to 1. Under the spread-overreaction hypothesis, the ratio of the standard deviations of the theoretical and actual term spreads, σ_{S*} and σ_S, is less than unity.

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18. The statistical rejection of the expectations hypothesis for the United States is often attributed to the change in monetary-policy operating procedures—see Hamilton (1988).

The theoretical long-term yield for a forecast horizon of 40 quarters[19] and the actual yield are presented in Figure 3. The two series moved closely together over the period from 1956 to 1998, including the first half of the 1980s, when interest rates were high and volatile. The mean discrepancy between the theoretical and actual yields at 59 basis points is close to the size of the estimated long-run equilibrium term premium of 75 basis in the previous section. However, the discrepancy between the two yields is at times as much as 200 basis points, which is consistent with the view that long-term yields contain a positive time-varying component of the term premium. The theoretical discrepancy also reflects the forecast errors of market participants.

The volatility statistics for the full sample period in Table 2 show that the ratio of the standard deviations of the theoretical and actual long-term yields is slightly above 1 and the correlation of the levels of the two yields is slightly below 1, yielding a slope coefficient of 1.05. Similarly, the $\bar{R}^2$ at 0.90 suggests that changes in the theoretical yield explain most of the movements in the actual yield.

The correlation between the theoretical and actual term *spreads* at 0.87 indicates that the two spreads also moved relatively closely together over the sample period. However, the ratio of the standard deviations of the two spreads is noticeably below 1, indicating some excessive volatility of the actual spread. This overreaction of the actual term spread reflects both a time-varying premium and systematic forecast errors.

The statistics in the second column for the pre-1972 period show, as in the previous section, that the expectations relationship between long- and short-term interest rates was slightly stronger during the fixed exchange-rate period; the slope coefficient is not statistically different from 1. The VAR system in the third column indicates that including the spread between the U.S. 10-year yield and the Canadian short-term rate in the estimations slightly reduces the standard deviation of the theoretical long-term yield, so that the slope coefficient can now easily be restricted to equal 1 for the full sample. This suggests that the U.S. long-term yield may contain some incremental information for explaining short-term deviations from the conditions for the expectations hypothesis that is not contained in the domestic long-term spread.[20]

Overall, the volatility statistics suggest that the expectations hypothesis can broadly account for movements in long-term Canadian yields. However, the statistics are informal and low-powered tests of the restrictions of the expectations hypothesis, because they only assess whether the currently observed long-term yield is equal to the future path of short-term interest rates as

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19. The results changed very little for horizons from 20 to 80 quarters because of the very small weights attached to changes in short-term rates beyond 5 years.
20. The diagnostic statistics for the VAR model that included medium-term spreads against the short rate were not noticeably different from those presented in column 1.

predicted by the VAR model. Section 4 tests more formally the restrictions of the rational expectations hypothesis.

3.3 The term structure in the 1990s

In this section, out-of-sample predictions for the past 10 years are used to decompose recent movements in the long-term yield into the expected path of changes in future short-term rates and the term premium. The decomposition will give some indication of how well the expectations hypothesis can account for the period of high and volatile interest rates in the late 1980s and the early 1990s.

The diagnostic statistics on the rolling out-of-sample predictions of the theoretical long-term yield for the 1972–98 period are presented in the final column of Table 2. The statistics are quite similar to those for the in-sample estimation over the 1956–98 period, suggesting that the predictions from the model are reasonably robust.

Figure 4 plots the sum of the out-of-sample theoretical yield and an estimate of the constant portion of the term premium for the 1988–98 period, along with its discrepancy with the actual yield. The constant component of the term premium was approximated by the mean discrepancy between the actual and theoretical yields. Consequently, the actual long-term yield can be decomposed into the following three components:

$$R_t^n \equiv \hat{R}_t + \theta + \gamma_t, \tag{5}$$

where R_t^n is the actual long-term yield, $\hat{R}_t$ is the theoretical long-term yield that is equal to the weighted average of the (predicted) short-term yields, θ is the constant component of the term premium, and γ_t captures the time-varying portion of the term premium and systematic forecast errors. The size of γ_t is then equal to the discrepancy between the actual R_t^n and the adjusted theoretical yield, $\hat{R}_t + \theta$, which is theoretically white noise.

The adjusted theoretical yield in Figure 4 tracks the actual yield generally within one standard deviation (± 90 basis points) of the discrepancy between the two yields, including the decline in 1993 and relatively sharp increase over 1994.[21] The theoretical long-term yield was higher than the actual yield over the 1989–90 period and lower on average over the 1991–97 period. The negative discrepancy in 1989–90 shows that, based on historical relationships, the model was predicting larger increases in short-term rates than were realized. The overestimation mainly

21. The bands for the one standard deviation are not confidence bands, because they do not take into account parameter uncertainty.

reflects the increase in short-term rates of about 5 percentage points that began in early 1988.[22] In addition, the market likely attached some probability to a later shift to a regime of lower rates that is not captured in the theoretical yield.

The slightly higher actual long-term yield for the period from 1991 to mid-1997 mainly reflects a positive risk premium related to political uncertainty and the effects of Canada's fiscal position in the early 1990s. The spike in the theoretical yield in 1992(Q4) reflects an increase in short-term rates of about 1 1/4 percentage points that was related to the defeat of the Charlottetown Accord. The spike in 1995(Q1) reflects an increase in short rates of over 2 percentage points that was related to the Mexico crisis. Overall, the model appears to overpredict the theoretical long-term yield during periods of relatively large increases in short-term interest rates.

Since mid-1997, the discrepancy between actual and theoretical long-term yields has narrowed sharply, mainly reflecting the unwinding of the risk premium related to the improved political and fiscal situation in Canada, as well as lower inflation risk. The actual yield was slightly lower than the adjusted theoretical yield in 1998, which is consistent with the view that the relatively low level of long-term government yields was in part a result of some flight to quality over this period because of deteriorating economic conditions in some overseas countries.

4. The multi-period regression model

4.1 The rational expectations hypothesis

In this section, a much stronger test of the expectations hypothesis is undertaken by assuming that expectations are rational and by testing the *ex post* success of the term structure as a predictor of changes in future short-term interest rates over the relevant horizons. The expected value of the short-term rate for rationally formed expectations may be written as:

$$r_{t+i} = E_t[r_{t+i}] + \varepsilon_{t+i}, \tag{6}$$

where ε_{t+i} is a white noise process ($E_t[\varepsilon_{t+i}] = 0$). The assumption of rational expectations implies that forecast errors are orthogonal within the sample to all available information. The expectations hypothesis may then be expressed in terms of rational expectations by substituting for the expected value of the short-term rates $E_t[r_{t+i}]$ into equation (1) and rearranging

$$\sum_{i=0}^{n-1} w_i r_{t+i} - r_t = \alpha + \beta(R_t^n - r_t) + \mu_{t+n-1}, \tag{7}$$

22. A negative term premium could have existed over this period if market participants had had a preferred habitat to lend at longer horizons because of the high long-term yields, while borrowers preferred to finance investments at shorter horizons.

Table 2: Diagnostic statistics

System	***R–r***, Δr	***R–r***, Δr	***R–r***, ***R***^{us}***–r***, Δr	***R–r***, Δr
Period	**1956(Q4)–1998(Q4)**	**1956(Q4)–1971(Q4)**	**1956(Q4)–1998(Q4)**	**1972(Q1)–1998(Q4)#**
Theoretical long-term yield				
$\bar{R}^2$	0.90	0.85	0.88	0.90
γ **(*t*-statistic)**	1.05 (39.5)	0.96 (18.6)	1.03 (34.7)	1.15 (30.8)
$\gamma = 1$ **(*p*-value)**	1.00 0.04	1.00 0.47	1.00 0.32	1.00 0.00
σ_{R*}	3.05	1.20	3.01	2.76
σ_R	2.74	1.15	2.74	2.28
σ_{R*}/σ_R	1.11	1.04	1.10	1.21
$\rho_{R*, R}$	0.95	0.92	0.94	0.95
σ_{R-R*}	0.96	0.46	1.06	0.95
μ_{R-R*}	0.59	0.52	0.59	–0.15
Theoretical term spread				
σ_{S*}/σ_S	0.51	0.71	0.57	0.77
$\rho_{S*, S}$	0.87	0.91	0.77	0.87
$\bar{R}^2_{VAR, \Delta r}$	0.12	0.29	0.18	0.09
$\bar{R}^2_{VAR, S}$	0.76	0.83	0.77	0.75

R = actual long-term yield; *R** = theoretical long-term yield; *r* = short-term rate; R^{us} = long-term U.S. yield; *S* = term spread (*R*–*r*); *S** = theoretical term spread (*R**–*r*); # denotes out-of-sample forecasts; σ = standard deviation of the yields; ρ = correlation between the yields; γ = coefficient of a regression of the theoretical yield on the actual yield.

where $\alpha = -\theta_n$ and the error term μ_{t+n-1} is assumed to follow a moving average process of *n*–1 and has a mean of 0. The left-hand side of equation (7) is the rollover spread, which measures the *ex post* deviation of the current one-period yield from its weighted average level over the maturity of the bond. A test of $\beta = 1$ on the term spread is a joint test of the null hypothesis that expectations are rationally formed and that arbitrage between short- and long-term rates holds as assumed by the expectations hypothesis.

Although this multi-period regression approach is a stronger test of the expectations hypothesis, it has a drawback; a significant amount of the recent estimation period is lost at long forecast horizons. As a result, most previous research has focused mainly on very short-term maturities. For instance, Gerlach and Smets (1997a) used 1- to 12-month eurorates in 17 countries to show that, for many countries including Canada, the data cannot reject the hypothesis that the β coefficient is significantly different from 1. Similarly, Hejazi and Lai (1996) rejected the restriction for the structure of Canadian treasury bill yields. Fremont (1996) also found that the coefficients on most money market spreads against the overnight rate were significantly less than 1. Stréliski (1997), on the other hand, found that the restriction could not be rejected for most forward rate spreads, but that the relationship was unstable.

Hardouvelis (1994) used the multi-period regression approach to test the expectations hypothesis on the spread between the 10-year yield and the 90-day rate for the G-7 countries. He attempted to minimize the data loss for the long forecast horizon by regressing cumulative future returns on 3-month investments at various horizons on the long-term spread. For Canada, he found that the coefficient on the spread increased as the forecast horizon increased, reaching a value of close to 1 at 27 quarters.

4.2 Estimation results

In this section, the rational expectations hypothesis is tested using a recently constructed data set of hypothetical par-value yields for 1- to 5-year maturities by Day and Lange (1997).[23] Unlike Hardouvelis, these medium-term yields allow for estimations of precise forecast horizons beyond the money market term structure without losing a significant portion of the recent sample period. The expectations relationship is evaluated by testing the stability of the regression parameters and by relating the size of the β coefficients to the predictability of the short rate over the medium term.

The estimations for the medium-term structure begin in 1967 and end in 1997 for the 1-year model, 1996 for the 2-year model, and so on (Table 3). The serial correlation due to overlapping

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23. The par-value yields were estimated on end-of-the month (last Wednesday) observations for all domestic-pay Government of Canada bonds going back to January 1967. The monthly observations are averaged to obtain a quarterly frequency.

observations and the problem of heteroskedasticity are corrected with the Newey and West (1987) adjustment methodology, which is now standard in the finance literature.

The regression estimates of equation (7) for the medium-term par-value yields are presented in the top panel of Table 3. The hypothesis that there is no predictive power in the term spread ($\beta=0$) is strongly rejected at the one per cent significance level for all forecast horizons.[24] The hypothesis that the expectations theory holds with $\beta=1$ can be rejected for the 1-year forecast horizon and, at the 10 per cent level, for the 2-year horizon, but it cannot be rejected for the 3- to 5-year horizons. The joint hypothesis that $\beta=1$ and $\alpha=0$ as predicted by the strong form of the expectations hypothesis is also not rejected for horizons beyond 2 years.

The bottom panel in Table 3 presents the estimation results for the spread between the 10+-year average yield and the short-term rate over various horizons as in Hardouvelis (1994). The long-term spread was found to be an optimal predictor of future short rates at the 7-year horizon.

The R^2 values from the multi-period regressions increase to a maximum of 0.43 at the 7-year horizon. The relatively low values of the R^2 indicate that most of the variation in the changes in short rates is unpredictable. However, the value of the β coefficients beyond 2 years are not significantly different than 1, indicating that the predictable part of the variation is efficiently predicted by the market for theses horizons. In short, the variation in the spread between medium- and short-term rates is an unbiased indicator of what the market expects to happen to future short rates, but not necessarily a precise predictor of future movements in short rates.

The Andrews supremum *F*-tests for structural change reject the null of parameter stability for all forecast horizons.[25] The *F*-statistics indicate parameter instability in January-February 1981.

4.3 The time-varying term premium

The possible causes of the rejection of the null hypothesis of a β-coefficient equal to 1 for the 1- and 2-year horizons can be seen with the aid of the theoretical formula for β in the forecast equation under the assumption of rational expectations.[26] For ease of exposition, a two-period case is considered so that the estimate of the β-coefficient converges to:

$$\hat{\beta} = \frac{\sigma^2(E_t \Delta r_{t+1}/2) + 2\rho\sigma(E_t \Delta r_{t+1}/2)\sigma(\theta_t)}{\sigma^2(E_t \Delta r_{t+1}/2) + 2\rho\sigma(E_t \Delta r_{t+1}/2)\sigma(\theta_t) + \sigma^2(\theta_t)}, \tag{8}$$

24. Although *t*-statistics are relatively large, they are based on Newey–West standard errors, which may be somewhat smaller than small sample errors.
25. The sup-*F* test by Andrews (1993) was programmed in RATS by Tkacz (1997b).
26. See Hardouvelis (1994), Gerlach and Smets (1997a, 1997b), among others, for the theoretical formula for the β-coefficient.

Table 3: Regression results

$$\sum_{i=0}^{n-1} w_i r_{t+i} - r_t = \alpha + \beta(R_t^n - r_t) + \mu_{t+n-1},$$

where $w_i \equiv g^i(1-g)/(1-g^n)$ and $g \equiv (1+\bar{R})^{-1}$.

R_t^n **= 1- to 5-year par-value yields: 1967(Q2) to 1998(Q4) minus *n* quarters**					
***n* quarters**	***n* = 4**	***n* = 8**	***n* = 12**	***n* = 16**	***n* = 20**
α **(*t*-statistic)**	–0.50 (3.56)	–0.55 (1.84)	–0.60 (1.45)	–0.63 (1.30)	–0.64 (1.20)
β **(*t*-statistic)**	0.47 (3.36)	0.69 (4.29)	0.84 (4.06)	0.94 (4.94)	1.05 (8.13)
H_o: β =1.0 (*p*-value)	1.0 (0.00)	1.0 (0.06)	1.0 (0.44)	1.0 (0.77)	1.0 (0.72)
H_o: β =1.0 and α =0 (*p*-value)	1.0 0.0 (0.00)	1.0 0.0 (0.02)	1.0 0.0 (0.26)	1.0 0.0 (0.42)	1.0 0.0 (0.44)
$\bar{R}^2$	0.12	0.18	0.25	0.31	0.37
R_t^n **= 10+-year average yield: 1956(Q2) to 1998(Q4) minus *n* quarters**					
***n* quarters**	*n* = 12	*n* = 20	*n* = 24	*n* = 28	*n* = 40
α **(*t*-statistic)**	–0.88 (2.57)	–0.86 (2.20)	–0.85 (2.08)	–0.80 (1.92)	–0.55 (1.37)
β **(*t*-statistic)**	0.60 (4.04)	0.83 (6.98)	0.91 (8.89)	1.00 (10.32)	0.95 (9.44)
H_o: β =1.0 (*p*-value)	1.0 (0.01)	1.0 (0.16)	1.0 (0.38)	1.0 (0.99)	1.0 (0.61)
$\bar{R}^2$	0.24	0.36	0.38	0.43	0.41

Coefficient *t*-statistics are based on Newey–West standard errors of the coefficients.
p-values for the restriction β =1 are distributed as $\chi^2(1)$.

where $E_t \Delta r_{t+1}$ is the expected change in the short-term rate, ρ is the correlation between the time-varying term premium (θ_t) and the expected change in the short rate, and σ is the standard deviation of θ_t.

The formula indicates that the size of the β coefficient under rational expectations depends on three terms: the variation of the expected changes in the short-term rate, the variation of the term premium $\sigma(\theta_t)$, and the correlation ρ between the term premium and expected changes in the short rate. The β coefficient is 0 if the short-term interest rate is not predictable ($E_t \Delta r_{t+1} = 0$) and is equal to 1 in the absence of a time-varying premium ($\sigma^2(\theta_t) = 0$). However, variations in the term premium $\sigma(\theta_t)$ will bias downwards the coefficient on the term spread; the size of the bias depends on the variance of the expected change in the future short rate. The bias also depends on ρ, the correlation between the term premium and expected changes in the short-term rate, so that β can be greater than 1 when ρ is sufficiently negative and the variation in expected changes in future short rates is low.[27]

In order to assess the extent to which the size of the β coefficients over the different forecast horizons can be attributed to the predictability of the short-term rate, the variances of the changes in the expected short rate were estimated for the various horizons. The expected changes in short rates were estimated using the Campbell–Shiller bivariate VAR framework outlined in Section 3.[28]

The variances of the (weighted) changes in the expected short rate are presented in Table 4. The variance of the expected changes in the short-term rate increase with the size of the β coefficient estimates, suggesting that the size of β varies directly with the predictability of the short rate. The ratio of the variance of the expected changes to the total changes, which is a measure of the degree of predictability of the short-term rate,[29] suggests that the rejection of the expectations hypothesis ($\beta = 1$) for the shorter maturity structure is due to the poorer predictability of the short rate over this horizon. The weaker correlation between the size of the β-coefficient and changes in the variance of expected changes in the short rate for these maturities is also consistent with the existence of a time-varying term premium.

The failure of the data to reject the expectations hypothesis beyond the 2-year forecast horizon is consistent with the finding by Fama and Bliss (1987) that, in the United States, spreads for maturities longer than 1 or 2 years have some predictive content for movements of future interest rates. The results for horizons beyond 2 years are also consistent with those in Day and Lange (1997), who found that the slope of the medium-term structure is an unbiased predictor of

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27. Jorion and Mishkin (1991) and Hardouvelis (1994) find that the correlation is negative for maturities beyond 1 year.
28. Mankiw and Miron (1986), Gerlach and Smets (1997a, 1997b) and others rely on a univariate forecasting equation comprising the current and lagged short- and longer-term interest rates as explanatory variables to estimate the expected short-term interest rates.
29. This follows Gerlach and Smets (1997a, 1997b).

Table 4: Predictability of the short-term rate

		Variance of			
		Future changes in short rates			Term spread
***n* quarters**	**β**	**Total**	**Expected***	**Ratio (%)**	
Medium-term (par value) spread, 1968(Q2) to 1998(Q4) minus *n* quarters					
4	0.50	1.42	0.15	10.6	0.81
8	0.69	3.26	0.48	14.7	1.27
12	0.84	4.46	0.80	17.9	1.54
16	1.06	5.27	1.01	19.2	1.85
20	1.05	5.72	1.00	17.5	1.96
Long-term (10+-year) spread, 1957(Q2) to 1998(Q4) minus *n* quarters					
28	1.00	4.98	0.75	15.1	2.15
40	0.95	4.12	0.71	17.2	1.90
* Estimated by a second-order Campbell–Shiller bivariate VAR model.					

future changes in inflation over these horizons for Canada. The rejection of the expectations hypothesis at the shorter horizons is consistent with the results on the money-market term structure by Fremont (1996) and Gravelle et al. (1998).[30]

Conclusion

The long-run equilibrium conditions for the expectations hypothesis could not be rejected in a VECM of short- and long-term interest rates at the 99 per cent confidence level for the period from 1956 to 1998. The one-to-one relationship between the short- and long-term rates indicates that the term spread is an unbiased predictor of changes in short-term rates over the long run. These results

30. The failure to reject the restrictions of the expectations hypothesis is only a necessary condition for the time-varying term premium to be invariant and the hypothesis to be true. The multi-period regressions would not reject the expectations theory if term premiums were time-varying but orthogonal to the term spreads.

are much stronger than those found in previous research, and reflect both the estimation of a VECM and a longer sample period. However, there are periods of relatively large and persistent deviations from the long-run relationship between short- and long-term rates, so the unbiased expectations hypothesis does not necessarily hold in any particular short run.

The theoretical long-term yield that was simulated from the Campbell–Shiller VAR model shows that movements in current and expected future short-term rates account for a large fraction of the variance of long-term yields. The long-term yield moves almost one for one with its theoretical counterpart under the expectations hypothesis. Movements in the U.S. long-term yield were found to have some incremental information about future short-term rates that is not contained in domestic long-term yields. The VAR model was also found to be useful for decomposing recent movements in the long-term yield into expected changes in future short-term rates and a time-varying term premium.

Similarly, the rational expectations hypothesis could not be rejected for the term structure beyond 2 years, with slope coefficients on the term spread not being significantly different from 1. The spread between the short rate and long end of the term structure was found to be an optimal predictor of future short rates at the 7-year horizon. The estimates of the expected paths of changes in the short rate suggest that the predictability of the short-term rate increases with the forecast horizon. Thus, the large *ex post* bias in the forecast of future short rates from the shorter term structure reflects the poorer predictability of the short-term rate over this horizon. This is also consistent with the presence of a time-varying term premium. The rational expectations relationship was found to be unstable across forecast horizons in early 1981, when interest rates were quite volatile.

Overall, the results from the three empirical approaches show that the expectations hypothesis has considerable economic and statistical content for explaining movements in long-term yields for Canada. The results also suggest that medium-term horizons are probably better than short-term horizons for extracting information about market expectations of the future path of interest rates and inflation.

However, the results also indicate that the unbiased expectations hypothesis does not hold in every period. Future research should focus on the sources of this shortcoming, such as time variations in the term premium and systematic forecast errors. Recent work by Lange (1999) using a regime-switching model suggests that short-run deviations from the expectations hypothesis may be due to the market's anticipation of a future shift in the interest rate process during periods of volatile short-term rates.

Figure 1

Short-term and long-term yields

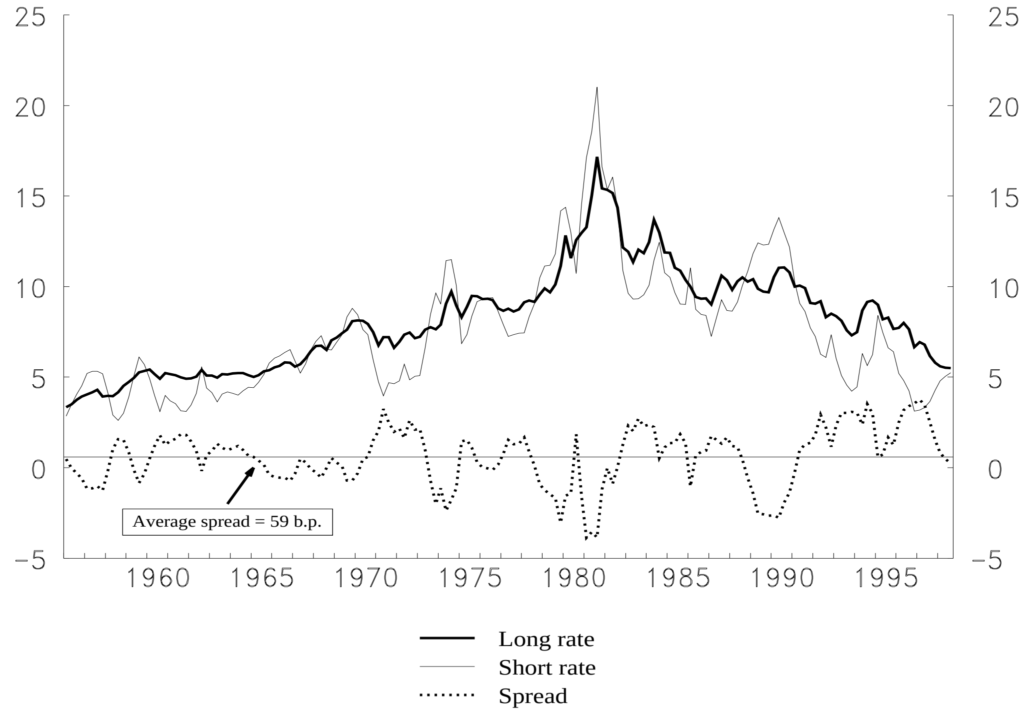
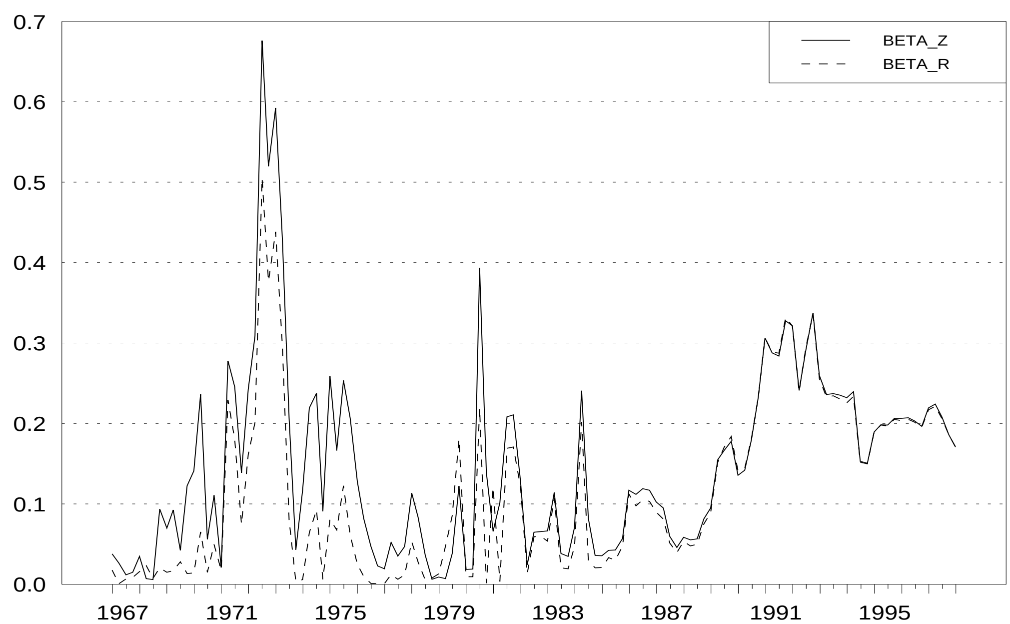


Figure 2

Test of known beta eq. to beta(t)



1 is the 5% significance level

Figure 3

Actual and theoretical long-term yields
(In sample)

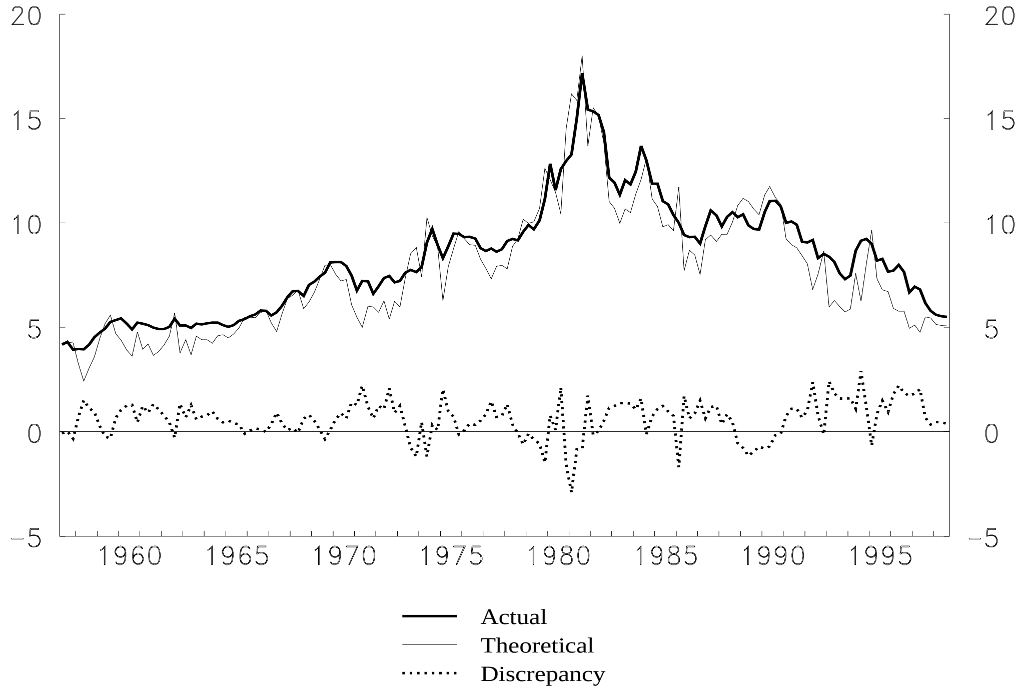
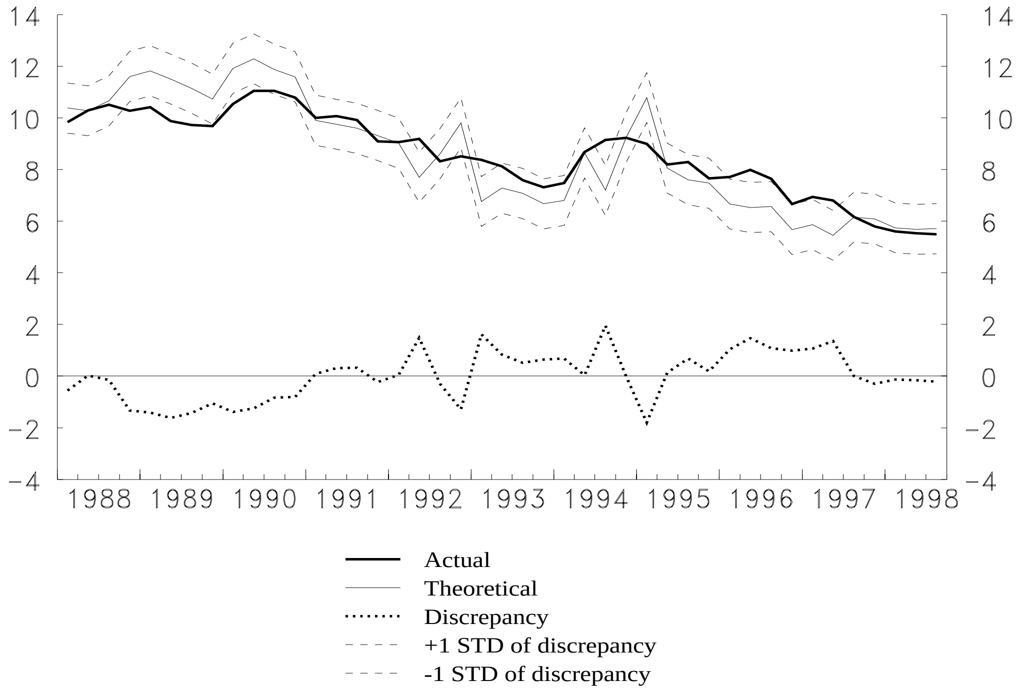


Figure 4

Actual and theoretical long-term yields
(Out of sample)



Appendix: Johansen–Juselius estimation methodology

The JJ maximum-likelihood technique is based on the following *p*-lag vector error-correction model

$$\Delta X_t = \Gamma_1 \Delta X_{t-1} + \ldots + \Gamma_{p-1} \Delta X_{t-p+1} + \Pi X_{t-1} + \mu + \varepsilon_t,$$

where X_t is a vector of n I(1) variables. The first p–1 elements on the right-hand side are I(0) and the next element is a linear combination of I(1) variables. The ε_t is assumed to be a vector white noise process.

The JJ technique decomposes the (*nxn*) matrix Π into an (*rxn*) matrix of *r* cointegrating vectors β' and an (*nxr*) matrix of *r* loading factors or speeds of adjustment coefficients α. Since the methodology is not able to exactly identify the matrices, it identifies a cointegrating space for which β is simply a set of basis vectors. If only one cointegrating vector is found, conclusions can be drawn about a unique long-run relationship between the variables. However, when more than one cointegration vector~~s~~ is present, any linear combination of the cointegration relations will preserve the stationarity property.

The parameters of the Γ_i matrices provide a measure of the short-run dynamics of the system. The μ is a *nx*1 vector of unrestricted constants. The constant can be restricted to be a common mean in the error-correction term, assuming no linear trend in the data.

The VECM is estimated under the unrestricted null hypothesis that there are up to *n* cointegrating relationships, which are equal to the number of endogenous variables in the system. The JJ procedure uses the n eigenvalues (factors) that solve the maximization problem to construct two versions of the likelihood-ratio statistic to test down the cointegrating rank of the long-run matrix Π. The maximal eigenvalue statistic compares the null hypothesis of H0(r) with an alternative hypothesis of H1(r+1), while the trace test compares the same null with the alternative of H1(n). Likelihood-ratio tests can also be used to perform hypothesis tests about the basis of the cointegrating space β and about the adjustment matrix α.

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