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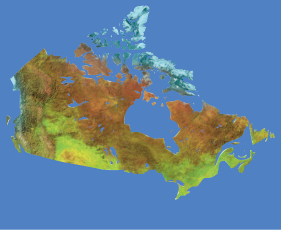

Review of formation resistivity factor equations related to new pore-structure concepts

T.J. Katsube

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Critical review

G. Oldenborger

Author

T.J. Katsube
(J.Katsube@nrcan.gc.ca)
Geological Survey of Canada
601 Booth Street
Ottawa, Ontario
K1A 0E8

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Abstract: Formation resistivity factor (F) is the ratio of a rock's electrical resistivity (ρ_r) to its saturating fluid resistivity (ρ_f): $F = \rho_r/\rho_f$. Since ρ_r represents electric current flow, F adds information to the rock's porosity (ϕ) and fluid-flow properties, and is used for hydrocarbon seal characterization. This study reviews the F-equation evolution related to pore-structure concept developments since the 1940s, and concludes the recent general format of $F = b_f\tau^2/\phi$ ($b_f = 1.5$, τ is connecting pore tortuosity) is adequate, except that the current b_f values need reconsideration in certain cases. For example, $b_f = 1.5$ is adequate if ϕ represents the total connecting-pore porosity (ϕ_C) for certain sheet-like pores, but if ϕ represents effective porosity (ϕ_E), as most former cases, or if it represents ϕ_C of circular and/or tubular pores, the b_f value must be changed. This paper provides some new b_f values and methods to determine them.

Résumé : Le facteur de résistivité de formation (F) est le rapport entre la résistivité électrique de la roche (ρ_r) et la résistivité des fluides qui la saturent (ρ_f) : $F = \rho_r/\rho_f$. Étant donné que ρ_r représente la circulation du courant électrique, F apporte de l'information supplémentaire sur la porosité de la roche (ϕ) et sur les propriétés d'écoulement des fluides, permettant ainsi de caractériser l'étanchéité aux hydrocarbures. La présente étude examine l'évolution de l'équation de F en rapport avec l'évolution des concepts sur la structure des pores depuis les années 1940, et conclut que le format général récent de l'équation $F = b_f\tau^2/\phi$ (où $b_f = 1{,}5$ et τ est la tortuosité des pores connectés) est approprié, bien que les valeurs actuelles de b_f doivent être réexaminées dans certains cas. Par exemple, $b_f = 1{,}5$ convient si ϕ représente la porosité totale des pores connectés (ϕ_C) pour certains pores en feuillets, mais si ϕ représente la porosité efficace (ϕ_E), comme dans la plupart des cas précédents, ou s'il représente ϕ_C de pores circulaires ou tubulaires, la valeur de b_f doit être modifiée. Le présent article fournit de nouvelles valeurs pour b_f et offre un aperçu des méthodes servant à les déterminer.

INTRODUCTION

The formation resistivity factor (F) is the ratio of the bulk electrical resistivity of a rock (ρ_r) to the electrical resistivity of the fluid (ρ_f) that saturates that rock (Archie, 1942; Sheriff, 1974):

$$F = \rho_r/\rho_f \tag{1}$$

The pore-surface conductivity effects (Patnode and Wyllie, 1950) are considered to be excluded from ρ_r. If a cube consisting of water that has an electrical resistivity of ρ_f, then the electrical resistivity of that cube (ρ_r) would equal ρ_f, implying that F = 1.0; however, in these studies it is considered that the cube is filled with rock-forming material. Assuming that there are no interconnected layers of electrically conductive minerals in that cube of rock, the electrical current can flow only through the fluid in its interconnected pores. This implies that F is related to the porosity (ϕ) of that rock. For this reason, F is considered to represent an important characteristic of fluid-flow pathways, and is often used to add information related to fluid migration and hydrocarbon seal characteristics of rocks (e.g. Sheriff, 1974; Jain and deFigueirdo, 1982; Walsh and Brace, 1984; Bowers and Katsube, 2002; Katsube et al., 2006).

The formula first used to express the relationship between F and ϕ was Archie's Law (e.g. Archie, 1942; Wyllie and Gregory, 1953; Keller and Frischknecht, 1966; Gomez-Rivero, 1976; Keller, 1982; Jain and deFigueirdo, 1982):

$$F = a\phi^{-m} \tag{2}$$

where a and m are coefficients. Rocks have often been characterized by these two coefficients, based on F versus ϕ measurements (e.g. Keller and Frischknecht, 1966). For example, Archie's Law (Archie, 1942) assumes that a = 1 and m = 2, the Humble formula (Winsaur et al., 1952; Sheriff, 1974) assumes that a = 0.65 and m = 2.15. Wyllie and Gregory (1953) and Gomez-Rivero (1976) assumed that m = 1.8log(a) to about 1.29log(a) for sand and m = 2.03log(a) to about 0.9log(a) for carbonate rocks.

Walsh and Brace (1984) related F to ϕ and the fluid- or electrical current-flow pathway characteristics by:

$$F = \tau^2/\phi \tag{3a}$$

where τ is the tortuosity of the fluid or electrical current-flow pathway, which is defined by the actual length (ℓ_C) of the flow pathway over the length (ℓ) or dimension of the cube of the rock (Fig. 1):

$$\tau = \ell_C/\ell \tag{3b}$$

A pore-structure concept for rocks (Fig. 2) was presented (Katsube and Collett, 1973) in the 1970s, which consisted of total connecting porosity (ϕ_C) for connecting pores that contribute mainly to fluid migration and of storage porosity (ϕ_S) for storage pores that contribute mainly to fluid storage, where their sum is effective porosity (ϕ_E):

$$\phi_E = \phi_S + \phi_C \tag{4}$$

Later, an equation based on this new storage-connecting pore concept was presented (Katsube and Agterberg, 1989), which included F:

$$\phi_E = \phi_S + b\tau^2/F \tag{5}$$

where b is a coefficient equal to 1.5 for an isotropic rock (Katsube et al., 1991) with sheet-like pores. In that publication ϕ_S was presented as pocket porosity (ϕ_P) which

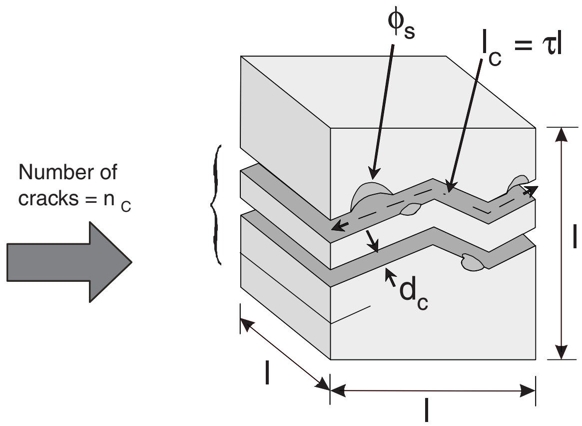


Figure 1. A single direction set of tortuous sheet-like pores (*modified from* Katsube and Kamineni, 1983). The d_C is the pore width or pore size, ℓ represents the dimension of the cube of rock, $\ell_C = \tau\ell$ is the actual length of the tortuous pore, where τ is the tortuosity. The n_C is the sheet-like pore density, implying the number of sheet-like pores per length ℓ, and ϕ_S represents a storage pore. The n_C is represented by n in the text of this paper. The arrow shows the electrical current flow direction.

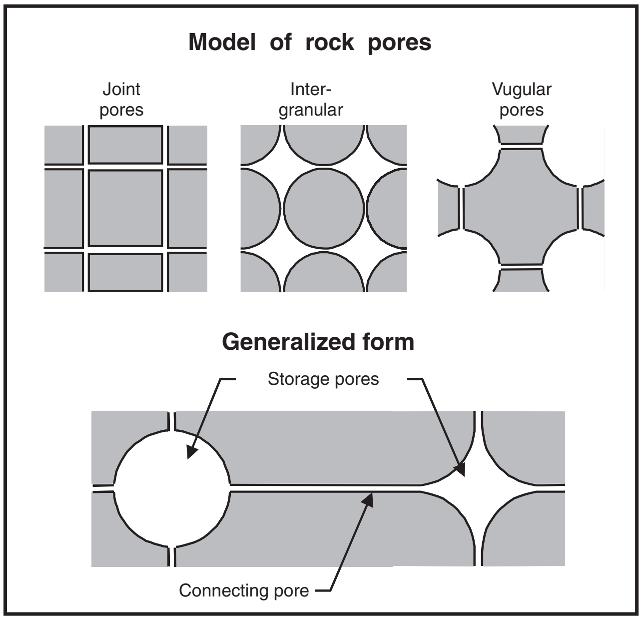


Figure 2. Storage and connecting pore model for shale or any other type of rock (Katsube and Collett, 1973; Katsube and Williamson, 1994, 1998) with interconnected pore systems.

by definition is identical to ϕ_S. A comparison between equations (4) and (5) indicate that essentially, the second term in equation (5) is equal to ϕ_C.

Later, a formation resistivity factor equation was presented as follows (Bowers and Katsube, 2002):

$$F = b_1\tau^2/\phi_C \tag{6}$$

in which ϕ, frequently used in previous studies (e.g. equations (2) and (3)), was replaced by ϕ_C. The b_1 is a coefficient equal to 1.5 for sheet-like pores, similar to that of b in equation (5). It was also indicated (Bowers and Katsube, 2002) that b_1 could take larger or smaller values depending on the three-dimensional sheet-like pore distribution, but no specifics of these conditions were discussed.

Some of the discussions on fluid-flow paths in a previous publication (Walsh and Brace, 1984) implied that the values of the coefficients equivalent to b or b_1 could vary depending on the shape of the pore cross-section (sheet-like or circular and/or tubular); however, little specific information related to those values or the methods to determine them were discussed. For example, there was no information presented that would imply that the coefficients b or b_1 would be a value other than equal to 1.0 in equation (3a).

The pore shapes considered, to date, for the F-equations have mainly been for sheet-like pores, and for their distribution in one or two directions. The purpose of this study is to review the mathematical equations for F related to the sheet-like pores, develop new F-related equations for the circular and/or tubular pores, and determine how the three-dimension distribution of these two pore types might affect the values of the coefficient b_1.

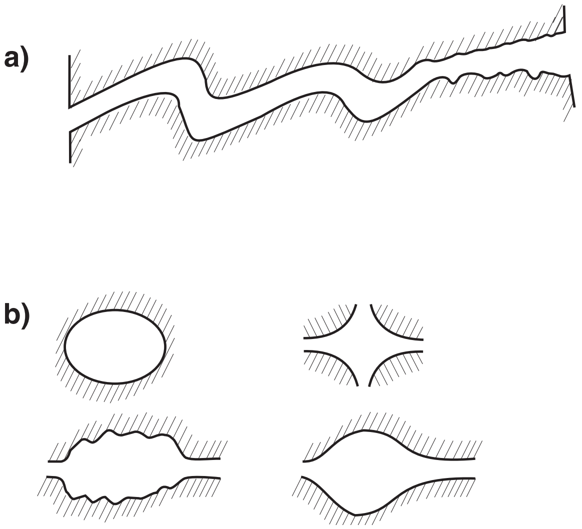


Figure 3. The various cross-sections of connecting pores **a)** represented by a tortuous sheet-like pore, and **b)** represented by various shapes of tubular pores.

BASIC PORE-STRUCTURE MODELS

The rock texture consists of mineral grains of various shapes and sizes and its pore structure is extremely complex. The most important factors of the pore structure are how much space there is between these grains and what are their shapes. That is because the space between these grains take the shapes either to mainly transport fluids forming connecting pores, or mainly to store the fluids forming storage pores. The transportation of fluids and electrical current is controlled mainly by the connecting pores. The storage pore shapes can be characterized by vugular or intergranular, as shown in Figure 2. There are two major types of connecting pores: sheet-like and circular and/or tubular pores. Many more types of connecting pores can be considered, but these two are most representative for describing the extreme differences between their types. For example, the cross-section of connecting pores can have many shapes, as shown in Figure 3. A representative cross-section of a sheet-like pore is shown in Figure 3a. Cross-sections of circular and/or tubular pores can take many shapes as shown in Figure 3b; however, in this study the author considers all these shapes, such as multiple circles or elliptic pores to be represented by circular and/or tubular pores in order to simplify the outstanding differences they have on the formation resistivity factor (F) equations. This implies that only two types of connecting pore cross-sections are considered in this study, sheet-like (Fig. 3a, 4a) and circular and/or tubular (Fig. 3b, 4b) pores.

The total connecting porosity (ϕ_C) value of a rock includes connecting pores in all three directions (Fig. 4); however, when considering fluid or electrical current flow through the rock, only pores in two directions are considered for sheet-like pores (Fig. 4a), and only in one direction for circular and/or tubular pores (Fig. 4b). The electrical-current flow in a rock is controlled mainly by connecting pores. Connecting pores imply pores that interconnect all pores in a rock, except isolated pores; however, some of these connecting- or storage-pore systems can be dead-ended. The connecting pores that contribute to electrical current- or fluid flow through the rock have to be interconnected from one end of the rock to the other, and not include dead-ended pores. These are distinguished as end-to-end connecting pores. The actual porosity of the end-to-end connecting pores should be smaller than the value of ϕ_C, since it does not include the porosity of the dead-ended pores. The storage-connecting pore system and its porosity descriptions are shown in Figure 5 for a rock section that includes vugular storage and isolated pores. The end-to-end connecting porosity of the pore system in Figure 5 is represented by ϕ_{CF}. The ϕ_C and storage porosity (ϕ_S) are determined by mercury injection and extrusion porosimetry (Wardlaw and Taylor, 1976; Bowers and Katsube, 2002). The total volume of mercury intruded by forcing it into a rock sample by pressure is used to determine its effective porosity (ϕ_E), and the volume of mercury trapped in the sample after releasing the pressure is used to determine ϕ_S. The ϕ_C is determined by the amount of mercury expelled from the sample after releasing the pressure, which is equivalent to taking the difference between ϕ_E and ϕ_S. Therefore, ϕ_C represents all interconnected porosity such as ϕ_{CF} and blind connecting-pore porosity (ϕ_{CB}) in Figure 5; however, since techniques to determine ϕ_{CB} do not currently exist, for convenience ϕ_C can be considered to equal ϕ_{CF}.

In principle, the one-direction connecting-pore porosity and connecting-pore tortuosity (τ) values can vary for the three different directions. That is because the pore structure or connecting pore distribution could be anisotropic; however, in this study they are considered identical, or isotropic, in order to simplify the mathematical model of the rock under consideration. Rock texture varies considerably, depending on the rock type, such as unconsolidated sedimentary rocks, compacted sedimentary rocks, cemented sedimentary rocks, igneous rocks, volcanic rocks, and altered rocks of all these types. The considerable variation in pore shapes and distributions are a result of these rock types and their varied history. Pore shapes of the storage pores also could vary, but intergranular and vugular pores (Fig. 2) are considered the most representative extremes in this study.

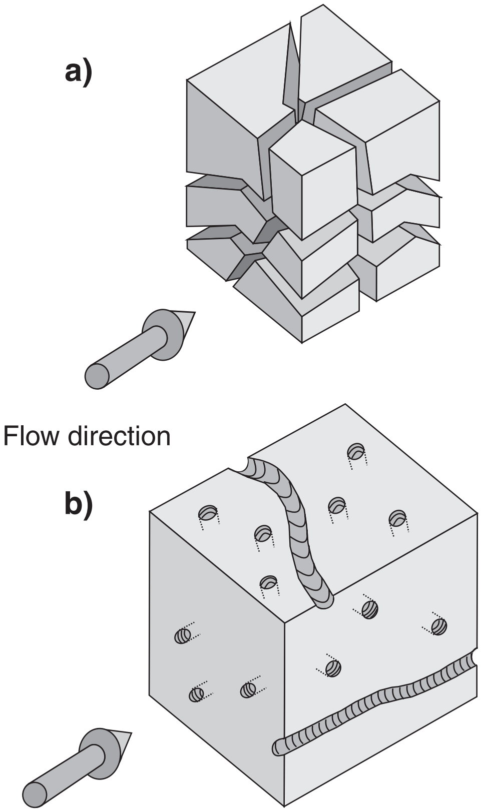


Figure 4. Three-dimension distribution of connecting pores for **a)** sheet-like connecting pores and for **b)** circular and/or tubular connecting pores. The arrow shows the flow direction of the fluid or electrical current considered in this study. The single direction flow in Figure 4a involves the horizontal set of sheet-like connecting pores and one of the vertical set of sheet-like connecting pores in the direction of the arrow, but not the vertical set of sheet-like connecting pores normal to the flow direction of the arrow.

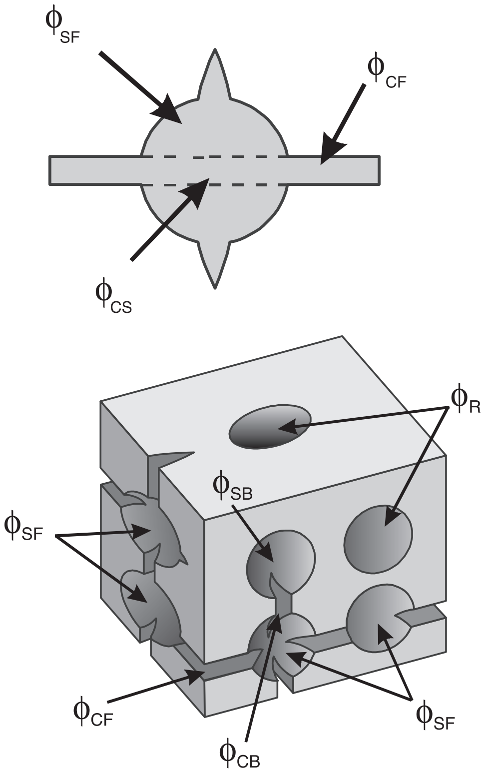
DEVELOPMENT OF FORMATION RESISTIVITY FACTOR EQUATIONS

Connecting-pore systems that contribute to the electrical-current flow

The connecting pores contribute to the flow of electrical currents through a rock, as previously indicated. The total connecting porosity (ϕ_C) values represent the porosity of the

Storage-flow pore relationship

$\phi_T = \phi_E + \phi_R$: total porosity
$\phi_E = \phi_S + \phi_C$: effective porosity
$\phi_S = \phi_{SF} + \phi_B$: storage porosity
$\phi_B = \phi_{SB} + \phi_{CB}$: blind porosity
$\phi_C = \phi_{CF} + \phi_{CS}$: connecting porosity



ϕ_{SF} : storage-flow porosity
ϕ_{SB} : storage-blind porosity
ϕ_{CF} : connecting-flow porosity
ϕ_{CS} : connecting-storage porosity
ϕ_{CB} : connecting-blind porosity
ϕ_R : isolated porosity

Figure 5. Complete storage-connecting pore system (Bowers and Katsube, 2002) and porosity descriptions for a rock section that includes vugular storage and isolated pores, as explained in the diagram.

sheet-like and circular and/or tubular connecting pores in all three directions of the rock (Fig. 4), but not all of these connecting-pore sets contribute to the flow of the electrical current. If considering three sets of sheet-like connecting-pore systems in this model, one horizontal and two vertical sets as shown in Figure 4a, then the horizontal set and one of these vertical sheet-like pore sets will contribute to the one-direction flow of the electrical current. The vertical set that is normal to the electrical current-flow direction will not contribute to the flow, as can be implied from Figure 4a. For the circular and/or tubular connecting pores, there are two horizontal sets and one vertical set of pores (Fig. 4b), but only the one set that is in the same direction of the electrical current-flow will contribute to that flow.

For sheet-like connecting pores, the tortuosity characteristics for both the electrical-current flow direction and for the direction normal to that flow need to be known. That is for both the horizontal and vertical sheet-like pores; however, in the development process of the formation factor (F) equations, single-direction tortuosity pores (e.g. equation (1)) will also be included. That is to help understand the relationship to previous F-equations (e.g. Katsube and Kamineni, 1983; Walsh and Brace, 1984). For the circular and/or tubular pores, only the single-direction tortuosity requires to be considered.

Resistivity equations for sheet-like connecting pores

Since the connecting pores control the flow of electrical current through the rock, their length in the direction of the flow, the number of pores, and size of their cross-section area determine the resistivity values. Since the actual length of the sheet-like pores in the horizontal flow direction of the rock cube in Figure 1 is $\tau\ell$, where τ and ℓ are the tortuosity and dimension of that rock cube, and since the total area of the cross-sections of the pores is $nd_S\ell$ where n and d_S are the number of parallel sheet-like pores per length ℓ and the size of the pores, the resistivity in the horizontal flow direction (ρ_{S1}) is

$$\rho_{S1} = \{\tau\ell/(nd_S\ell)\}\rho_f = \tau\rho_f/(nd_S) \qquad (7)$$

If these horizontal sheet-like connecting pores are tortuous in two directions, which implies in the flow direction and in the direction normal to the flow direction shown by the arrow in Figure 4a, then the total cross-section area of n sheet-like connecting pores in that horizontal direction will be $n\tau d_S\ell$. The resulting resistivity (ρ_{S2}) will be

$$\rho_{S2} = \{\tau\ell/(n\tau d_S\ell)\}\rho_f = \rho_f/(nd_S) \qquad (8)$$

If the electrical current flows through the horizontal sheet-like pores as well as one set of the vertical sheet-like pores, shown in Figure 4a, which have similar pore characteristics as the horizontal two-direction tortuous pores, then the resistivity (ρ_{S3}) of the combination of the two sets of sheet-like connecting pores will be

$$\rho_{S3} = \rho_{S2}/2 = \{\tau\ell/(n\tau d_S\ell)/2\}\rho_f = \rho_f/(2nd_S) \qquad (9)$$

The value of ρ_{S3} is half that of ρ_{S2}, because in this case, it is considered that there are two sets of similar sheet-like pores with resistivities of ρ_{S2} in a parallel relationship (Bowers and Katsube, 2002). The vertical set of sheet-like pores normal to the direction to the flow direction (Fig. 4a) is not considered to have any effect on the resistivity.

Resistivity equations for circular and/or tubular connecting pores

The actual length of the circular and/or tubular pores in the horizontal flow direction (direction of the arrow of the rock cube in Fig. 4b) is $\tau\ell$. Since the cross-section area of a circular and/or tubular connecting pore is $(d_T/2)^2\pi$, where d_T is the diameter of the pore, the total cross-section area of n circular and/or tubular connecting pores is $n(d_T/2)^2\pi$. Therefore, the resistivity (ρ_{T1}) of the circular and/or tubular connecting pores in the flow direction is

$$\rho_{T1} = \ell\tau\rho_f/\{n(d_T/2)^2\pi\} = 4\ell\tau\rho_f/(nd_T^2\pi) \qquad (10)$$

Connecting-porosity equations for sheet-like and circular and/or tubular connecting pores

The connecting porosity of a single connecting pore is its length multiplied by its cross-section area divided by the volume of the rock cube, which is ℓ^3. Therefore, the one-flow-direction connecting porosity (ϕ_{CS1}) of a rock with sheet-like single-direction tortuous pores (Fig. 1) for equation (7) and the connecting porosity (ϕ_{CS2}) of a rock with sheet-like two-direction tortuous pores (part of Fig. 4a) for equation (8) are

$$\phi_{CS1} = (\ell\tau)(nd_S\ell)/\ell^3 = nd_S\tau/\ell \qquad (11)$$

and

$$\phi_{CS2} = (\ell\tau)(nd_S\tau\ell)/\ell^3 = nd_S\tau^2/\ell \qquad (12)$$

The connecting porosity (ϕ_{CS3}) for the combination of sheet-like two-direction tortuous pores of a horizontal set and a vertical set (Fig. 4a), related to equation (9), is

$$\phi_{CS3} = 2(\ell\tau)(nd_S\tau\ell)/\ell^3 = 2nd_S\tau^2/\ell \qquad (13)$$

The connecting porosity ϕ_{CS3} is two times ϕ_{CS2}, because the characteristics of both horizontal and vertical pore sets are considered to be identical and in a parallel relationship (Bowers and Katsube, 2002). The connecting porosity (ϕ_{CT1}) of the circular and/or tubular pores in one flow direction (Fig. 4b) for equation (10) is

$$\phi_{CT1} = (\ell\tau)\{n(d_T/2)^2\pi\}/\ell^3 = nd_T^2\pi\tau/(4\ell^2) \qquad (14)$$

F-equations for sheet-like and circular and/or tubular connecting pores

Based on equation (1), the formation resistivity factor (F) is simply the resistivities in equations (7) to (10) divided by ρ_f. Therefore, if the F for the sheet-like single-direction tortuous pores, sheet-like two-direction tortuous pores, combination of sheet-like two-direction tortuous pores for horizontal and vertical pore sets, and circular and/or tubular pores in one flow direction are F_{S1}, F_{S2}, F_{S3}, and F_{T1}, then

$$F_{S1} = \rho_{S1}/\rho_f = \tau/(nd_S) \tag{15a}$$

$$F_{S2} = \rho_{S2}/\rho_f = 1/(nd_S) \tag{16a}$$

$$F_{S3} = \rho_{S3}/\rho_f = 1/(2nd_S) \tag{17a}$$

and

$$F_{T1} = \rho_{T1}/\rho_f = 4\ell\tau/(nd_T{}^2\pi) \tag{18a}$$

If the porosities in equations (11) to (14) are considered in equations (15a) to (18a), then one derives

$$F_{S1} = \tau/(nd_S) = \tau^2/(\phi_{CS1}\ell) \tag{15b}$$

$$F_{S2} = 1/(nd_S) = \tau^2/(\phi_{CS2}\ell) \tag{16b}$$

$$F_{S3} = 1/(2nd_S) = \tau^2/(\phi_{CS3}\ell) \tag{17b}$$

$$F_{T1} = 4\ell\tau/(nd_T{}^2\pi) = \tau^2/\phi_{CT1}\ell \tag{18b}$$

Consideration of the total connecting porosity (ϕ_C)

The number of pore sets, n, for all three directions in a rock are considered to be equal in this study, although that is not necessarily the case as shown in Figure 4a. As previously indicated (*see* 'Basic pore-structure models' section), the total connecting porosity (ϕ_C) represents the connecting porosity of all three directions of a rock. Therefore its relationship to the connecting porosities in equations (13) and (14) above are as follows:

$$\phi_C = 1.5\phi_{CS3} \tag{19a}$$

or

$$\phi_C = 3\phi_{CT1} \tag{19b}$$

Therefore from equations (17b), (18b), (19a), and (19b) one derives

$$F_{S3} = 1/(2nd_S) = \tau^2/(\phi_{CS3}\ell) = 1.5\tau^2/(\phi_C\ \ell) \tag{17c}$$

and

$$F_{T1} = 4\ell\tau/(nd_T{}^2\pi) = \tau^2/\phi_{CT1}\ell = 3\tau^2/(\phi_C\ell) \tag{18c}$$

The characteristics of the sheet-like pores considered in equations (11) and (15b) are slightly different from those in equations (13) and (17b), because those in the last two equations are tortuous in two directions, whereas those in equations (11) and (15b) are tortuous in only one direction. In addition, the sheet-like pores considered in equations (12) and (16b) are slightly different from those in equations (13) and (17b), because those in the last two equations represent the pore-structure characteristics for two directions (horizontal and vertical sheet-like pores) in the rock, whereas the first two represent the characteristics for only one direction in the rock. Therefore, using ϕ_C related to these single-set sheet-like pores could be erroneous under the current definition of ϕ_C. In previous studies, such as when Figure 1 (Katsube and Kamineni, 1983) or equation (3a) (Walsh and Brace, 1984) were used for the one-direction flow of fluids or electrical currents in sheet-like pores, the connecting-porosity concept for ϕ_{CS1} and ϕ_{CS2} could have been considered equal to the concept of ϕ_C; however, now with sheet-like and circular and/or tubular pores, and with sheet-like pores with two-direction tortuosities and a combination of horizontal and vertical sheet-like pores considered, the basic connecting-porosity concept has evolved into a more complex one, with their relationships now represented by equations (19a) and (19b).

Effect of storage and connecting pores

In an actual rock, the number of pores in each of the three directions and their characteristics are not necessarily identical; however, in this study, in order to simplify the mathematical model of the pore structure, it is assumed that they are identical in all three directions. Based on this assumption, as a conclusion from equations (17c) and (18c), the general form for the F-equation can be expressed by

$$F = b_f\tau^2/\phi_C \tag{20}$$

where b_f is a coefficient similar to b_1 in equation (6), although the value of $b_f = 1.5$ (Bowers and Katsube, 2002) in equation (6) is specifically for the combination of sheet-like connecting pores in horizontal and vertical directions, as expressed in equation (17b) and (17c) of this study. For other cases the value of b_f can be different. For example, this study shows that $b_f = 3.0$ for circular and/or tubular pores, as expressed in equation (19) and equation (18c). The value of ℓ is considered to equal 1.0 in equation (20).

A more generalized expression for equation (20) would be to use ϕ instead of ϕ_C, as in equations (2) and (3a), which is essentially the effective porosity (ϕ_E) which has been more widely used in the past, however, ϕ or ϕ_E is the sum of the storage porosity (ϕ_S) and ϕ_C, as expressed in equation (4). There is often a considerable difference between ϕ_C and ϕ_E as shown in some publications (e.g. Katsube, 2000; Bowers and Katsube, 2002; Katsube et al., 2005). These publications suggest that the value of ϕ_C can vary between 20% and 80% of ϕ_E for sedimentary rocks, implying that the value of b_f in equation (20) would have to be increased by a factor of 1.25 to 5.0 times, if ϕ or ϕ_E were to be used instead of ϕ_C for these rocks. In addition, in principle, the three direction components of ϕ_C and the τ values for the three directions in these rocks could vary considerably. That is a possibility if there is an anisotropic pore distribution in the three directions (e.g. Fig. 4a, b). In such cases these anisotropies could also significantly affect the value of b_f. In this study, however, they are considered identical, for convenience, in order to simplify the mathematical model of the rocks under consideration, as indicated above.

DISCUSSION

This study has reviewed past formation resistivity factor (F) equations, starting with Archie's Law (Archie, 1942) to recent equations, and concludes that the following general format for the F-equation presented in several publications (e.g. Walsh and Brace, 1984; Bowers and Katsube, 2002) is appropriate

$$F = b_f \tau^2 / \phi \qquad (21)$$

where b_f is a coefficient, τ is the flow path tortuosity and ϕ is the porosity. This study concludes that if ϕ represents the connecting porosity (ϕ_C) of a rock for all three directions, then $b_f = 1.5$ (equation (17c)) for the combination of vertical and horizontal flow paths consisting of sheet-like connecting pores, similar to b_1 in equation (6) of a previous study (Bowers and Katsube, 2002); however, if it represents the ϕ_C of the three-direction circular and/or tubular connecting pores, then $b_f = 3.0$ (equation (18c)). As a result, this study shows that some reconsiderations for the value of the coefficient equivalent to b_f in the previous F-equations is necessary, particularly, depending on the definition of ϕ. This study also shows that b_f takes considerably different values, if ϕ is equal to the effective porosity (ϕ_E) rather than ϕ_C. The ϕ_E is the sum of ϕ_C and the storage porosity (ϕ_S) (equation (4)).

Although ϕ_E is the sum of ϕ_C and ϕ_S (Katsube and Collett, 1973), only ϕ_C is the main source for electric-current transport through a rock; however, in the past, starting with the Archie's Law equation in 1942, often no distinction was made between these three porosities (ϕ_E, ϕ_S, ϕ_C) for the F-equations, and ϕ, which by definition is equivalent to ϕ_E, had represented the porosity of a rock (equations (2) and (3a)). The concept of the three porosities was first presented in the early 1970s (Katsube and Collett, 1973), but was not applied to the F-equations until the late 1980s (Katsube and Agterberg, 1989) where ϕ_C appeared in an equations related to F (equation (5) and (4)). This study does not recommend that ϕ_E be used instead of ϕ_C in the F-equations, but if it is used, then the values of b_f (or b_1 in equation (6)) must be increased by an order of 1.25 to 5.0 times the values presented above and in the previous presentations. This is because the ϕ_C data published to date are in the range of only 20–80% (e.g. Katsube, 2000; Bowers and Katsube, 2002; Katsube et al., 2005) of the value of ϕ_E. Another factor to be considered, is the connecting porosity of the end-to-end connecting pores that actually transports the electrical current through the rock. It could in fact be smaller than the connecting porosity represented by ϕ_C, as indicated before. That is because ϕ_C actually includes the porosity of the dead-ended connecting pores (Fig. 5), which do not contribute to fluid or electrical current transport across the rock. No reliable method to determine the porosity of the dead-end connecting pores is available at present; therefore, it is assumed that ϕ_C represents all of the connecting porosity that contributes to the transport of the electrical current.

In a rock formation, the connecting pores of both types, sheet-like and circular and/or tubular, are generally distributed in all three directions. In this study, it is considered that the fluids and electrical currents flow in only one direction, either along or across a rock formation. When considering these flows, two of the three-directional sets of sheet-like connecting pores will contribute to the flow (Bowers and Katsube, 2002), but only one set of the three-direction sets of circular and/or tubular connecting pores will contribute to that flow, as can be concluded from Figure 4. This is the main reason that the values of the coefficient b_f (in equation (21)) differ between the two types of connecting pores described above. In this study, the connecting pore characteristics of τ, n, d_S and ϕ_C, are considered to be similar in all three directions. Whereas that might be the case for the two horizontal directions of many sedimentary rock formations, that is unlikely to be the case for the horizontal versus vertical directions for most sedimentary rock formations, particularly for sedimentary rocks that have been compacted in the vertical direction. This implies that there is room to develop new F-equations that take into consideration these anisotropic effects.

The pore-surface electrical-resistivity or -conductivity effects are considered to be eliminated when determining the F values (Patnode and Wyllie, 1950), as previously indicated. The actual electrical resistivity or conductivity values of a rock reflect both the results of the electrical current flowing through the fluid in the connecting pores and through the adsorbed water layers on these pore surfaces; however, since the pore-surface layer thicknesses are only a few nanometres or a fraction of that (Katsube et al., 2000), they are usually considered insignificant compared to the larger pore sizes of the rock that contribute to fluid flow, unless the pore sizes are extremely small. Therefore, generally, the accuracy of the F measurements are considered to increase by eliminating the pore-surface electrical-resistivity or -conductivity effects, as indicated above (Patnode and Wyllie, 1950), although that is mainly when the purpose of the F determinations are for adding information to the fluid-flow characteristics. If there are cases where migration of certain chemicals in the rock are important, then information on the pore-surface characteristics would become necessary. That is because the pore-surface adsorbed-water layers can contribute to diffusion transport of certain chemicals.

CONCLUSIONS

- A review of the formation resistivity factor (F) equations, starting with Archie's Law (Archie, 1942) to recent ones, concludes that the general format of $F = b_f \tau^2/\phi$ (equation (21)) is adequate, where b_f is a coefficient, τ is the flow pore tortuosity, and ϕ is the porosity. The values used for the coefficient b_f, however, requires considerable reconsiderations.

- When ϕ represents the connecting porosity (ϕ_C) of sheet-like connecting pores, then the coefficient b_f can accepted to be 1.5 as previously published (Bowers and Katsube, 2002); however, this study shows that b_f must be 3.0 when the connecting pores are circular and/or tubular in shape, and in addition, if ϕ represents other porosities or if the connecting pore distributions are anisotropic, then b_f could take different values.
- Although ϕ or its equivalent which is effective porosity (ϕ_E) consists of ϕ_C which contributes to flow of electrical current and fluids, it also consists of storage porosity (ϕ_S) which has less of a contribution to electrical current and fluid flow. This study suggests that if ϕ_E is used instead of ϕ_C in the F-equations, which has often been the case in the past, the value of b_f must be increased by a factor of 1.25–5.0.
- All connecting pores in a rock, sheet-like or circular and/or tubular, are distributed in three directions. This implies that there can be considerable anisotropy in the pore structure and its distribution, depending on the history of the rock. In the past, however, little consideration has been given to the anisotropic affect of pore distributions on the F-equations. This implies that this is an area that will have to be given more consideration in the future.
- The electrical resistivity or conductivity effect of the pore-surface adsorbed-water layers have always been considered eliminated when determining the F values. If there are cases where the characteristics of the flow or migration of certain chemicals in the rock become important, however, then information on the pore-surface characteristics could become necessary. That is because the pore-surface adsorbed-water layers can contribute to diffusion transport of certain chemicals.

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