Low flow characterization in New Brunswick using the Deficit Below Threshold approach

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by

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

Savoie, N., N. El-Jabi, F. Ashkar, and D. Caissie. 2004. Low flow characterization in New Brunswick using the Deficit Below Threshold approach. Can. Tech. Rep. Fish. Aquat. Sci. 2545: 48p.

Low flows play an important role in water management as well as for aquatic ecosystems. Although low flow remains an important component of the natural flow regime, it is often responsible for limiting aquatic habitat. Annual low flow analysis, a common method of low flow frequency analysis, considers only the most severe low flow event within a given time period. This method does not consider secondary low flow events within a year/season, therefore it is not suitable when characteristics of low flow such as duration, volume and intensity are important. In order to quantify low flows in terms of intensity, duration and volume deficit, the Deficit Below Threshold approach (i.e. DBT) is becoming increasingly important in both water and aquatic resources management, because this approach better quantifies low flow events. This approach considers all low flow below a certain threshold and considers many low flow events during a specific year/season.

Predictions of low flow events and frequencies are made using hydrologic data. To characterize low flow in terms of duration, volume and intensity, the Deficit Below Threshold (DBT) method was applied. 31 hydrometric stations in New Brunswick were selected for low flow analysis. Low flow event duration, volume and intensity data was fitted to generalized Pareto, exponential and Weibull distributions. The duration, volume and intensity were then calculated for recurrence intervals of 2, 10, 20 and 50 years using the best fitted distribution. In general, for the univariate analysis, the best fitted distribution was the Weibull distribution. The data was also fitted to a bivariate model following the Weibull distribution where the intensity was conditioned by durations of 7 and 14 days. Data for different recurrence intervals were also generated for the bivariate analysis. A regional analysis was then performed using the univariate and bivariate results. There are significant relationships between volume and drainage area as well as between low flow and drainage area. However, there does not seem to be a significant relationship between duration and drainage area.

RÉSUMÉ

Savoie, N., N. El-Jabi, F. Ashkar and D. Caissie. 2004. Low flow characterization in New Brunswick using the Deficit Below Threshold approach. Can. Tech. Rep. Fish. Aquat. Sci. 2545: 48p.

Les débits faibles jouent un rôle important au niveau de la gestion des ressources hydriques et des écosystèmes aquatiques. Les débits faibles sont non-seulement identifiés comme facteur limitant de l'habitat du poisson, mais, ceux-ci demeurent un élément important au niveau du régime fluvial des cours d'eau. La caractérisation de l'intensité, du volume et de la durée des débits d'étiage devient de plus en plus importante en aménagement des ressources hydrauliques et aquatiques. La méthode la plus utilisée pour l'analyse des débits d'étiages considère seulement l'événement à faible débit le plus sévère à l'intérieur d'une période donnée. Alors, cette méthode élimine des événements secondaires et n'est pas appropriée lorsque les caractéristiques telles que la durée, le volume et l'intensité sont importantes. Pour caractériser les étiages en volume, durée et intensité, l'analyse des séries de dépassement incomplète peut être appliquée. Cette approche considère tous les débits en dessous d'un niveau de référence et considère plusieurs événements pendant une période donnée.

Les données hydrologiques sont l'information de base utilisée par les hydrologistes pour prédire les événements de faible débit et leur fréquence. Pour caractériser le volume, la durée et l'intensité des débits d'étiages, une analyse de séries de dépassement incomplète a été appliquée. 31 stations hydrométriques ont été sélectionnées pour l'analyse des débits d'étiage. Les données de durée, volume et intensité ont été ajustées aux distributions suivantes: généralisée de Pareto, exponentielle et Weibull. Par la suite, les durées, volumes et intensités pour des périodes de récurrence de 2, 10, 20 et 50 ans ont été calculés en utilisant la distribution qui s'ajustait le mieux aux données. En général, pour l'analyse univariée, la distribution qui s'ajustait le mieux aux données était la distribution de Weibull. Un modèle bivarié suivant la distribution de Weibull a également été utilisé pour l'ajustement des données d'intensité conditionnées par des durées de 7 et 14 jours. Des données pour plusieurs périodes de récurrence ont également été calculées pour l'analyse bivariée. Une analyse régionale a été effectuée avec les résultats des analyses univariée et bivariée. Il y a une relation entre le volume et l'aire du bassin versant ainsi qu'entre le débit et l'aire du bassin versant. Cependant, il ne semble pas avoir de relation entre la durée et l'aire du bassin versant.

1.0 INTRODUCTION

The conflict between the ever-increasing demand for water withdrawal from rivers and water availability during drought and low flow periods is a recurring problem in water resources management. Low flow events are also known to limit available fish habitat and in some severe conditions can also prevent the connectivity between habitats (Lake 2003). In the 1990s, many rivers in eastern Canada experienced low flow conditions coupled with record high water temperatures (Caissie 1999; Caissie 2000). Also, during recent years of drought conditions, an increase in water withdrawal demand (offstream use), especially for irrigation, has been observed in New Brunswick. Increased water demand and low flow occurrence are factors that need to be considered when establishing instream flow requirements or minimum flows to protect aquatic habitat (Caissie and El-Jabi 1995).

Hydrologic data constitutes the basis of information used by hydrologists to make predictions of low flow events and frequencies. The probabilistic approach is useful in the analysis of such events due to the random nature of the low flows and the flexibility of this approach in characterizing low flow events. A low flow frequency analysis can be carried out following one of two methods. The first method, which has been widely applied in low flow studies, consists of analyzing the annual extreme low flow events (i.e. the annual minimums). When applied to river flows, this method considers only the most severe low flow event within a given time period, often chosen annually or by season. Therefore, this method eliminates secondary low flow events within a year/season and it is not suitable when characteristics of low flow such as duration, volume and intensity are important. To remedy this situation and to better characterize low flow in terms of duration, volume and intensity, a second approach can be applied. This approach considers all low flow below a certain threshold, also called a partial duration series analysis or Deficit Below Threshold (DBT) method. The DBT approach has not only the advantage of better characterizing low flow events (e.g. duration, volume, etc.), it can also consider many low flow events during a specific year/season.

Quantifying low flows in terms of intensity, duration and volume deficit (i.e. DBT) is becoming increasingly important in both water and aquatic resources management. In fact, not only intensity of low flow has been shown to be important to aquatic habitats, the duration of low flow events has been observed to be equally important (e.g. stress index to fish).

The objective of this research was to evaluate and report low flow characteristics in New Brunswick (NB) using hydrometric data and the partial duration analysis or DBT method.

2.0 DATA AND METHODOLOGY USED

2.1 Station Data

Hydrometric stations in New Brunswick were selected for the low flow analysis using Environment Canada's hydrometric station database (HYDAT) based on years of record and data quality. The following criteria were used: (i) natural flow at the gauging station, (ii) a time series of at least 20 years, and (iii) station in operation in the year 2000. The 31 selected stations represent all areas of the province with the exception of north central New Brunswick (Figure 1). These stations have an average drainage area of 1576 ± 526 km^2 (mean $\pm$ 1 SE). Table 1 provides a summary of the selected stations, station identification, latitude, longitude and the years used in the analysis. Daily river flows for each station were used for the analysis.

2.2 DBT Model

Two methods can be used in the analysis of hydrologic data for frequency analysis (Ashkar et al. 1998). The first and most widely applied method consists of analyzing the annual extreme low flow events or the annual minimums. The use of this method, which only considers the most severe low flow event within a given time period, results in a loss of valuable information by eliminating the secondary low flow events within a time period. Also, this method is not suitable to consider characteristics of low flows such as low flow duration, and volume and intensity. The second method, Deficit Below Threshold (DBT) or partial duration series analysis, is the stochastic analysis of low flow series below a certain threshold, Q_R. This method better characterizes low flow in terms of duration, volume and magnitude and can consider many low flow events during a specific time period. The DBT approach was used in the present study.

In this study, the threshold value or the river-flow reference value (Q_R) was chosen as the median monthly flow (Q_{50}) for August (i.e. 50th percentile of August daily river flows classified in descending order). This threshold, also referred to as the Aquatic Base Flow, has been used as a minimum flow in instream flow evaluation. The threshold value for each station was determined using Atlantic Canada Flow Analysis Software Version 1.0, ACFA 1.0 (Université de Moncton, 1994). All low flow events below this threshold were identified for the period of analysis using in-house software. This software identifies the low flow events below the threshold value with the following characteristics: year, season, month, start (days), peak (days), end (days), peak exceedance (m^3/s), real volume (m^3/s*days), triangular volume (m^3/s*days), rise duration (days), fall duration (days).

For the low flow analysis, events were characterized in terms of duration (T), volume (D) and magnitude or intensity (I). Figure 2 shows the following characteristics of a low flow event ν: the reference value Q_R (m^3/s), the duration T_ν (days) i.e. number of consecutive days for which the flow is below the reference value, the volume D_ν (m^3/s*days) i.e. cumulative deficit of streamflow for the duration, the intensity I_ν (m^3/s) i.e. the maximum flow deficit, the time of the beginning of the event $\tau_b(\nu)$ and the time of the end of the event $\tau_e(\nu)$. It should also be noted that the maximum observed volume deficit is represented by max D_{rec}.

Some of the low flow events may be close to each other and therefore mutually dependent. To avoid the dependency between events from a practical point of view, three simplifying assumptions were made in the present study based on Zelenhasic and Salvai (1987):

(i) very minor low-flow events with volumes D_i (i=1,2…) satisfying the following inequality $D_i < 0.005$ max D_{rec} (i=1,2…) were neglected because these events are insignificant compared to severe low-flow events with respect to volume;

(ii) for the remaining events, it was possible that the time period between two events $\Delta T_{\nu,\nu+1}$ is relatively short. In the case where $\Delta T_{\nu,\nu+1} \leq 6$ days, the events E_ν and $E_{\nu+1}$ can be assumed mutually dependent and the volume and duration become the following

$$D_\nu' = D_\nu + D_{\nu+1}, \text{ and} \tag{1}$$

$$T_\nu' = T_\nu + \Delta T_{\nu,\nu+1} + T_{\nu+1}\,; \tag{2}$$

(iii) it was possible a low-flow event begins in one year and ends in the following year. The time of occurrence of the event is then calculated as $\tau = ½(\tau_b + \tau_e)$ and the event is placed in the year that τ belongs to.

2.3 Univariate Analysis

The total number of low-flow events k per time interval [0,t], and the largest volume deficit, sup D_ν, the largest duration, sup T_ν, and the largest intensity, sup I_ν, during this time interval [0,t] are also important parameters. In this study, the time interval [0,t] was taken to be a one-year period. Zelenhasic and Salvai (1987) derived the distribution function of the number of low-flow events during the interval [0,t] using the results obtained by Todorovic and Zelenhasic (1970). For instance, if $\eta(t)$ is the number of low flow events in the interval [0,t] and for a fixed t, $\eta(t)$ is a non-increasing function of Q_R. The E^t_ν event is defined as:

$$E^t_\nu = \{\eta(t) = \nu\} \tag{3}$$

i.e., E^t_ν is the event that ν low-flow events occur in [0,t] and ν is a particular numerical value for $\eta(t)$. If $\Lambda(t)$ is the expected value of $\eta(t)$, then:

$$\Lambda(t) = \sum_{\nu=1}^{\infty} \nu P\left(E^t_\nu\right) \tag{4}$$

In most cases, $\Lambda(t)$ is a non-linear function of time because of the seasonal variation in streamflow or low flow events. Todorovic and Zelenhasic (1970) obtained the following solution for the distribution of the number of low-flow events in the interval [0,t]:

$$P\left(E^t_k\right) = [\Lambda(t)]^k \exp[-\Lambda(t)] \;/\; k! \tag{5}$$

under the assumption that low-flow events arrive according to a time dependent Poisson process.

The time of occurrence of the k^{th} event in a series of occurrences following a Poisson process obeys a gamma probability law (Parzen, 1966). According to Todorovic and Zelenhasic (1970), the distribution function of the time of occurrence of the k^{th} low-flow event may be written as:

$$F_k(t) = P\{\tau(k) \le t\} \tag{6}$$

where $\tau(k)$ is the time of occurrence of the k^{th} low-flow event measured relative to an arbitrary origin. From equations (3) and (6) we have:

$$F_k(t) = \sum_{j=k}^{\infty} P\left(E^t_j\right) \tag{7}$$

which can also be written as

$$F_k(t) = 1 - \sum_{j=0}^{k-1} P\left(E^t_j\right) \tag{8}$$

Denote $f_k(t)$ as the density function corresponding to $F_k(t)$. Considering (5) and after differentiation with respect to t, Todorovic and Yevjevich (1969) obtained:

$$f_k(t) = \frac{\lambda(t)}{\Gamma(k)} \left\{\int_0^t \lambda(s)\; ds\right\}^{k-1} \exp\left\{-\int_0^t \lambda(s)\; ds\right\},\ t \ge 0 \tag{9}$$

where $\lambda(t)$ is the mean number of low-flow events in the time interval [0,t]. This function $\lambda(t)$ is a deterministic periodic function of time with a one year period. From (4), it follows that:

$$\Lambda(s) = \int_0^t \lambda(s)\; ds \tag{10}$$

The largest deficit volume, sup D_ν, the largest deficit duration, sup T_ν, and the largest deficit intensity, sup I_ν, in a time interval [0,t] (e.g. annually), are also variables of interest. Denote by $\chi(t)$ the largest value of the deficit variable of interest (volume, duration or intensity), and let X_ν denote this variable. $\chi(t)$ is defined as follows:

$$\chi(t) = \sup\{X_\nu\},\ \tau(\nu)\leq t \tag{11}$$

The corresponding distribution function is:

$$F_t(x) = P\{\chi(t) \leq x\},\ t > 0,\ x \geq 0 \tag{12}$$

Todorovic and Zelenhasic (1970) have given the expression for the function $F_t(x)$ as the mathematical expectation of the following conditional probability:

$$P\{\sup X_\nu \leq x/\eta(t);\ \tau(\nu) \leq t\} \tag{13}$$

which results in

$$F_t(x) = P\left(E_0^t\right) + \sum_{k=1}^{\infty} P\left[\bigcap_{\nu=1}^{k} \left(X_\nu \leq x\right) \cap E_k^t\right] \tag{14}$$

This distribution function represents the probability that all deficits X_ν in an interval of time [0,t] will be less than or equal to x. If x=0, we have:

$$F_t(0) = P(E_0^t) \tag{15}$$

which can be defined as the probability that there will be no deficits in the time interval [0,t]. For the particular case where the deficit volumes, durations or intensities X_ν are independent, identically distributed random variables and where the vectors $\{X_1, X_2, \ldots, X_k\}$ and $\{\tau(k), \tau(k+1)\}$ are mutually independent, equation (14) becomes (Zelenhasic and Salvai, 1987):

$$F_t(x) = P\left(E_0^t\right) + \sum_{k=1}^{\infty} \left\{ \left[H(x)\right]^k P\left(E_k^t\right)\right\} \tag{16}$$

where H(x) is the distribution function of the deficit variable of interest (volume, duration or intensity) in the interval [0,t].

Once the low flow events were identified, the volume, duration and intensity data were fitted by a probability distribution function. Three different distribution functions were investigated: the exponential distribution, the generalized Pareto distribution and the Weibull distribution. The exponential and generalized Pareto distributions were chosen because they have been widely used in the study of extreme hydrological phenomena such as floods (Todorovic, 1978; Cunnane, 1979; North, 1980; Ashkar and Rousselle, 1981; Davidson and Smith, 1990; Madsen et al., 1997). The Weibull distribution was chosen because it has been widely applied for studying minimal extremes or low flows.

For the exponential distribution, the cumulative distribution function (cdf) of the variable X representing the volume, duration or intensity is given by:

$$H(x) = 1 - \exp\left(\frac{x}{\alpha}\right) \tag{17}$$

where α is a scale parameter. For the largest duration, volume or intensity in a time interval [0,t], which we shall denote by X_G, the distribution function is given by:

$$F(x_G) = \exp\left[-\Lambda(t)\exp\left(-\frac{x_G}{\alpha}\right)\right] \tag{18}$$

where $\Lambda(t)$ is the average number of low-flow events in the interval [0,t] .

For the generalized Pareto distribution, the cdf of X is given by:

$$H(x) = 1 - \left(1 - k\frac{x}{\alpha}\right)^{\frac{1}{k}}, \ k \neq 0 \tag{19}$$

where $\alpha > 0$ is a scale parameter and k is a shape parameter. The maximum volume, duration or intensity $\chi(t)$ in a time interval [0,t] in this case follows a generalized extreme value distribution, which has the following distribution function (Rosbjerg et al., 1992):

$$F(x_G) = \exp\left[-\Lambda(t)\left(1 - \frac{kx_G}{\alpha}\right)^{\frac{1}{k}}\right] \tag{20}$$

For hydrologic applications, the values of the shape parameter k in equations (19) and (20) range between –0.5 and 0.5 (Hosking and Wallis, 1987).

With the method of moments, the parameters α and k of cdf (19) are estimated by relating them to the mean $\overline{x}$ and the variance s^2 of X. These relations are given by:

$$\overline{x} = \frac{\alpha}{(1+k)} \tag{21}$$

$$s^2 = \frac{\alpha^2}{(1+k)^2(1+2k)} \tag{22}$$

So, the parameters are estimated by:

$$\hat{\alpha} = \frac{1}{2}\overline{x}\left(\frac{\overline{x}^2}{s^2} + 1\right) \tag{23}$$

$$\hat{k} = \frac{1}{2}\left(\frac{\overline{x}^2}{s^2} - 1\right) \tag{24}$$

For the Weibull distribution, the cdf of X is given by:

$$H(x) = 1 - \exp\left[-\left(\frac{x-b}{m-b}\right)^n\right] \tag{25}$$

For the largest volume, duration or intensity in the interval [0,t], the distribution function is as follows:

$$F(x_G) = \exp\left\{-\Lambda(t)\exp\left[-\left(\frac{x-b}{m-b}\right)^n\right]\right\} \tag{26}$$

By the method of moments, the parameter estimators for m and b are given by:

$$m = \bar{x} + sA(n) \tag{27}$$

$$b = m - sB(n) \tag{28}$$

The values for A(n), B(n) and can be found in Table 2 from El-Jabi and Rousselle (1990) after the coefficient of skewness is calculated as follows:

$$C_s = \frac{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^3\right]}{\left[\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2\right]^{3/2}} \tag{29}$$

2.4 Bivariate Analysis

When the joint occurrence of two or more variables is involved, the frequency analysis of these variables should be based on their joint probability distribution (bivariate or multivariate). In the present study two general methods for developing bivariate distributions were considered. They will be referred to as methods A and B. Ashkar et al. (1998) have used these two methods.

a) Method A

Referring to the work of Cohen (1984) and the Finch-Groblicki method (1984), Singh and Singh (1991) developed a joint probability density function (pdf) for two variables (X_1, X_2) to study rainfall. This function is as follows:

$$f(x_1, x_2)= f_1(x_1)f_2(x_2)\{1+c\rho\{F_1(x_1), F_2(x_2)\}] \tag{30}$$

where $F_1(x_1)$ and $F_2(x_2)$ are the marginal cdf's of X_1 and X_2, and $f_1(x_1)$ and $f_2(x_2)$ are the marginal densities. We can define $\rho(u, v;\ u = F_1(x_1),\ v = F_2(x_2))$ to be a function on the unit square $\{(u,v):0 \le u,v \le 1\}$ that is "0-marginal":

$$\int_0^1 \rho\ (u,v)\ dv = 0\ ;\ \int_0^1 \rho\ (u,v)\ du = 0 \tag{31}$$

where $\rho(u,v) \ge -1$. The constant c in (30) is chosen so that $f(x_1, x_2)$ is positive. In the present study, $f_1(x_1)$ and $f_2(x_2)$ are densities of positive random variables representing the intensities and durations of low flow events.

The joint density function $f(x_1, x_2)$ can be given for different forms of r(u,v), which is a function of two variables u and v defined on the unit square and normalized to one (Singh and Singh, 1991):

$$\int_0^1\int_0^1 r\ (u,v)\ du\ dv = 1 \tag{32}$$

The function $\rho\{u, v;\ u =F_1(x_1),\ v = F_2\ (x_2)\}$ is obtained as follows:

$$r_1\ (u) = \int_0^1 r\ (u,v)\ dv\ ;\ r_2\ (v) = \int_0^1 r(u,v)\ du \tag{33}$$

and

$$\rho\ (u,v) = r(u,v) - r_1(u) - r_2(v) + 1 \tag{34}$$

The function r(u,v) can take many forms, but one that seems sufficiently flexible is (Ashkar and El-Jabi, 2002):

$$r\ (u,v) = (n+1)\ (m+1)\ u^n\ \ v^m \tag{35}$$

which gives

$$\rho\ \ (u,v) = \left[(n+1)\ \ u^n\ -1\right]\ \ \left[(m+1)\ \ v^m\ -1\right] \tag{36}$$

Combining equations (30) and (36) gives the following joint pdf of (X_1, X_2):

$$f(x_1, x_2) = f(x_1) f_2(x_2) \left\{1 + c\left[(n+1)u^n\ -1\right]\left[(m+1)v^m\ -1\right]\right\} \tag{37}$$

which corresponds to the following cdf

$$F(x_1, x_2) = F_1(x_1) F_2(x_2) + c\left[(F_1(x_1))^{n+1} - F_1(x_1)\right]\left[(F_2(x_2))^{m+1} - F_2(x_2)\right] \tag{38}$$

The conditional pdf of X_1 given $X_2 = x_2$ is given by:

$$f_1(x_1|x_2) = f_1(x_1)\ \left[1 + c\rho\ \ \{F_1(x_1)\ ,\ F_2(x_2)\}\right] \tag{39}$$

and the conditional cdf is

$$F_1(x_1|x_2) = F_1(x_1) + c\ \ \left[(m+1)\ (F_2(x_2))\ ^m - 1\right]\ \ \left[(F_1(x_1))\ ^{n+1} F_1(x_1)\right] \tag{40}$$

The constant c is chosen so that $f(x_1, x_2)$ is positive; a condition that is equivalent to:

$$\frac{-1}{\max(1, mn)}\ \le c \le\ \frac{1}{\max(m,n)} \tag{41}$$

As mentioned above, three different distribution functions were used as marginals for the random variable X (volume, duration or intensity). These are the exponential distribution, the generalized Pareto distribution and the Weibull distribution.

When the marginal distribution of X is exponential, we have:

$$f(x) = \frac{1}{\alpha} \exp\ \left(-\frac{x}{\alpha}\right) \tag{42}$$

and

$$F(x) = 1 - \exp\ \left(-\frac{x}{\alpha}\right) \tag{43}$$

In the case of the generalized Pareto distribution, the pdf of X is easily obtained from the pdf of a standard exponential random variable, Y, using the following transformation (Hosking and Wallis, 1987):

$$X = \alpha\left[1 - \exp\left(-\frac{Y}{\alpha}\right)\right] \tag{44}$$

which gives the following pdf and cdf for X

$$f(x) = \frac{1}{\alpha}\left(1 - k\,\frac{x}{\alpha}\right)^{\left(\frac{1}{k}\right)-1};\ k\neq 0 \tag{45}$$

$$f(x) = \frac{1}{\alpha}\exp\left(-\frac{x}{\alpha}\right);\ k=0 \tag{46}$$

and

$$F(x) = 1 - \left(1 - k\,\frac{x}{\alpha}\right)^{\frac{1}{k}};\ k\neq 0 \tag{47}$$

$$F(x) = 1 - \exp\left(-\frac{x}{\alpha}\right);\ k=0 \tag{48}$$

(Note that when k=0, X becomes an exponential random variable).

For the Weibull distribution, the pdf and cdf are respectively given by (Ashkar and Bayentin, 2001):

$$f(x) = \alpha\exp\left[-(\alpha x)^s\right](\alpha x)^{s-1} \tag{49}$$

$$F(x) = 1 - \exp\left[-(\alpha x)^s\right] \tag{50}$$

where α is a scale parameter and s is a shape parameter.

The parameters α and s can be estimated using the method of maximum likelihood (ML). Specifically, the ML estimator, $\tilde{s}_{ML}$, of s, can be obtained numerically by the following equation (Ashkar and Bayentin, 2001):

$$\tilde{s}_{ML} = \left[\left(\sum_{i=1}^{n} x_i^{\tilde{s}_{ML}}\ln(x_i)\right)\left(\sum_{i=1}^{n} x_i^{\tilde{s}_{ML}}\right)^{-1} - \frac{1}{n}\sum_{i=1}^{n}\ln(x_i)\right]^{-1} \tag{51}$$

after which, the ML estimator of α can be calculated explicitly from the equation

$$\tilde{\alpha}_{ML} = \left[\frac{n}{\sum_{i=1}^{n} x_i^{\tilde{s}_{ML}}} \right] \tag{52}$$

b) Method B

A bivariate distribution with exponential marginals can easily be obtained from a bivariate distribution with gamma-distributed marginals, by equating the shape parameters of the gamma marginals to 1. A practical bivariate density with gamma marginals has been presented by Nagao and Kadoya (1971), from which a bivariate density with exponential marginals takes the following form:

$$f(x_1, x_2) = \frac{\lambda_1 \lambda_2}{(1-\rho)} \exp\left\{ -\frac{\lambda_1 x_1}{(1-\rho)} - \frac{\lambda_2 x_2}{(1-\rho)} \right\} I_0 \left(2 \frac{\sqrt{\rho}}{1-\rho} \sqrt{x_1 x_2 \lambda_1 \lambda_2} \right) \tag{53}$$

where X_1 and X_2 are random variables with exponential marginals of parameter λ_1 and λ_2 respectively, and I_0 is the modified Bessel function with argument 0, which can be expressed as:

$$I_0 \left(2 \frac{\sqrt{\rho}}{1-\rho} \sqrt{\lambda_1 \lambda_2 x_1 x_2} \right) = \sum_{j=0}^{\infty} \left[\frac{\rho \lambda_1 \lambda_2 x_1 x_2}{(1-\rho)^2} \right]^j \frac{1}{(j!)^2} \tag{54}$$

In this equation, ρ measures the correlation between the random variables X_1 and X_2. The equation representing the density of the variable X_1 conditioned by $X_2 = x_2$ is:

$$f(x_1 \mid x_2) = \frac{\lambda_1}{(1-\rho)} \exp\left\{ -\frac{\lambda_1 x_1}{(1-\rho)} - \frac{\rho \lambda_2 x_2}{(1-\rho)} \right\} I_0 \left(\frac{2\sqrt{\rho}}{1-\rho} \sqrt{\lambda_1 \lambda_2 x_1 x_2} \right) \tag{55}$$

When the variables X_1 and X_2 are standardized, i.e. divided by their means, the following conditional cdf is obtained:

$$F(\xi \mid \eta) = \int_0^{\xi} f(\xi \mid \eta)\, d\xi = \frac{1}{1-\rho} \exp\left(\frac{\rho \eta}{1-\rho} \right) \int_0^{\xi} \exp\left(-\frac{\xi}{1-\rho} \right) I_0 \left(\frac{2\sqrt{\rho}}{1-\rho} \sqrt{\xi \eta} \right) d\xi \tag{56}$$

where ξ and η correspond to the standardized values of X_1 and X_2, respectively. The integral in (56) can be calculated by a numerical approach, such as by the trapezoidal method (Ashkar et al., 1998).

Using the method of moments, the parameters in (55) are estimated as follows (Nagao and Kadoya, 1971):

$$\hat{\lambda}_1 = \frac{1}{\bar{x}_1} \; ; \; \hat{\lambda}_2 = \frac{1}{\bar{x}_2} \tag{57}$$

$$\hat{\rho} = \frac{\overline{x_1 x_2}}{\bar{x}_1 \bar{x}_2} - 1 \tag{58}$$

To obtain a bivariate distribution with generalized Pareto marginals for (X_1, X_2), the following procedure can be applied.

Consider a pair of random variables (Y_1, Y_2) with the joint pdf (Ashkar and Bayentin, 2001):

$$g(y_1, y_2) = \frac{1}{1-\rho} \exp\left(\frac{y_1 + y_2}{1-\rho}\right) \; I_0\left(\frac{2\sqrt{\rho y_1 y_2}}{1-\rho}\right) \tag{59}$$

i.e., Y_1 and Y_2 are random variables with standard exponential marginals. Note that (59) can be simply obtained from (53) by letting λ_1 = λ_2 = 1. Now by considering the following variable transformations:

$$X_1 = \frac{\alpha_1\left[1 - \exp(-k_1 Y_1)\right]}{k_1} = u_1(Y_1) \tag{60}$$

$$X_2 = \frac{\alpha_2\left[1 - \exp(-k_2 Y_2)\right]}{k_2} = u_2(Y_2) \tag{61}$$

whose inverses are given by:

$$Y_1 = -\frac{1}{k_1}\ln\left(1 - \frac{k_1 x_1}{\alpha_1}\right) = w_1(X_1) \tag{62}$$

$$Y_2 = -\frac{1}{k_2}\ln\left(1 - \frac{k_2 x_2}{\alpha_2}\right) = w_2(X_2) \tag{63}$$

the joint pdf of X_1 and X_2 is obtained as:

$$f(x_1, x_2) = g\left[w_1(x_1), w_2(x_2)\right] | J | \tag{64}$$

where $| J |$, the Jacobian of the transformation, is given by

$$| J | = \left|w_1'(x_1) \bullet w_2'(x_2)\right|; \; w_1'(x_1) = \alpha_1 - k_1 x_1 \text{ and } w_2'(x_2) = \alpha_2 - k_2 x_2 \tag{65}$$

The conditional pdf's and cdf's are developed as follows:

$$f(x_1 \mid x_2) = \frac{f(x_1, x_2)}{f_2(x_2)} \tag{66}$$

where $f_2(x_2) = \frac{1}{\alpha_2}\left(1 - \frac{k_2 x_2}{\alpha_2}\right)^{\frac{1}{k_2} - 1}$, and

$$F_1(x_1 \mid x_2) = \int_{-\infty}^{x_1} f_1(x_1 \mid x_2) \; dx_1 \tag{67}$$

which is equivalent to

$$F(x_1|x_2) = \int_{-\infty}^{y_1} g_1\ (y_1|y_2)\ \ dy_1 = \int_{-\infty}^{w_1(x_1)} g_1\ [y_1|y_2 = w_2(x_2)]\ \ dy_1 \tag{68}$$

where

$$g_1(y_1|y_2) = \frac{g(y_1, y_2)}{g_2(y_2)} = e^{y_2} g(y_1, y_2) \tag{69}$$

Finally, the joint cdf is given by:

$$F(x_1, x_2) = \int_{-\infty}^{x_1}\int_{-\infty}^{x_1} f\ (x_1, x_2)\ dx_1\ dx_2 \tag{70}$$

which is equivalent to

$$F(x_1, x_2) = \int_{-\infty}^{w_2(x_2)}\ \int_{-\infty}^{w_1(x_1)} g\ [u_1\ (y_1), u_2\ (y_2)]\ dy_1\ dy_2 \tag{71}$$

To obtain a bivariate distribution with Weibull marginals for (X_1, X_2), the following procedure can be applied.

A pair of random variables (Y_1, Y_2) is considered with the joint pdf (59), and the following variable transformations:

$$X_1 = \frac{(Y_1)^{\frac{1}{s_1}}}{\alpha_1} = u_1(Y_1) \tag{72}$$

$$X_2 = \frac{(Y_2)^{\frac{1}{s_2}}}{\alpha_2} = u_2(Y_2) \tag{73}$$

Solving for the Y_i's gives:

$$Y_1 = (\alpha_1 X_1)^{s_1} = w_1(X_1) \tag{74}$$

$$Y_2 = (\alpha_2 X_2)^{s_2} = w_2(X_2) \tag{75}$$

The joint and conditional distributions can then be developed as was done for the generalized Pareto distribution.

3.0 RESULTS AND DISCUSSION

3.1 DBT Model

Low flow events were identified for the 31 selected stations following the DBT method (as described in the Data and Methodology used section). The years of data used for each station is indicated in Table 1. Using ACFA, the threshold value was identified as the median monthly flow for August (50^{th} percentile of August daily river flows classified in descending order) also referred to as the Aquatic Base Flow. The low flow events were then identified using EXCRUE (D. Caissie, unpublished software). The simplifying assumptions to avoid the dependency between events described in the Data and Methodology used section were then applied. The number of low flow events below the threshold value that were identified for each station ranged from 42 to 178. Table 3 gives a summary of the threshold values (Q_R) and the number of low flow events identified for each station.

3.2 Univariate Analysis

Once the low flow events were identified in terms of volume, duration and intensity, the univariate analysis was carried out. For each station, the average number of events per year (in time period [0,t]) was identified. The theoretical and observed distributions of the low flow events were then compared. The theoretical distribution for the number of low-flow events in the interval [0,t] follows equation (5). A chi-squared test was then used to determine the difference between the theoretical distribution and the observed distribution. Station 01BU002 (Petitcodiac River) will be used as an example throughout this section to show the procedure / calculations applied to all stations. The average number of events per year for this station was 2.26 and the distribution results are presented in Figure 3. At the 5% level, no significant difference was observed between the theoretical and observed or calculated distributions. A significant difference between the theoretical and observed distributions was found for 6 of the 31 stations (01AD002, 01AD003, 01BC001, 01BE001, 01BJ007, 01BL003).

The volume, duration and intensity as well as the maximum volume, maximum duration and maximum intensity were modeled with the exponential, generalized Pareto and Weibull distribution. For the observed distribution, the data was divided into 12 to 16 classes. To evaluate the goodness of fit, Kolmogorov-Smirnov tests accompanied by graphical representations were used. For station 01BU002, the best fit was from the generalized Pareto for the volume and duration and from the Weibull distribution for the intensity. Figures 4, 5 and 6 show the graphical representation for the volume, duration and intensity for station 01BU002.

Table 4 gives a summary of the best fitted distributions H(x) for the volume, duration and intensity for the 31 stations. For volume, no distribution fitted the data for station 01BU003. For duration, no distribution fitted the data for station 01AD002. For intensity, no distribution fitted the data for stations 01AJ003 and 01BO001. For these stations, according to the Kolmogorov-Smirnov test, the maximum difference between the theoretical and observed distributions was larger than the critical difference. Therefore, the data were not distributed following the exponential, generalized Pareto or Weibull functions. It is possible that the data would fit other distributions.

The annual maximum volume, maximum duration and maximum intensity were modeled as well. For station 01BU002, the best-fitted distribution for the maximum volume, maximum duration and maximum intensity was the Weibull distribution. Figures 7, 8 and 9 show the graphical representation of the best-fitted theoretical and observed distributions for station 01BU002.

Table 5 gives a summary of the best fitted distributions F(x) for the largest volume, largest duration and largest intensity on an annual basis. For volume, no fit was found for one station (i.e., 01AQ001). For intensity, no fit was found for stations 01AD003, 01AQ001 and 01BV006. For these stations, according to the Kolmogorov-Smirnov test, the maximum difference between the theoretical and observed distributions was larger than the critical difference. Therefore, the data were not distributed following the exponential, generalized Pareto or Weibull functions. It is possible that the data would fit other distributions.

The volume, duration and intensity for recurrence intervals of T years were calculated using the following equation:

$$T = \frac{1}{1 - F(x)} \tag{76}$$

The distribution function F(x) to calculate low flow characteristics used was based on the best fitted distribution (exponential, generalized Pareto or Weibull), as presented in Table 5. As such, the Weibull distribution seemed to provide an overall best fit according to Kolmogorov-Smirnov tests and graphical representations. Table 6, 7 and 8 respectively give the volume, duration and low flow calculated for recurrence intervals of 2, 10, 20 and 50 years. It should be noted that data in Table 8 include the reference value Q_R (m^3/s) as well as the low flow (m^3/s), which was calculated from the intensity (m^3/s) value for each station.

3.3 Bivariate Analysis

The bivariate analysis was used to describe the distribution for variable X_1 conditioned by a variable X_2. In this study, intensity conditioned by 7-day and 14-day durations were calculated using a bivariate distribution with Weibull marginals. Two methods can be used for the bivariate analysis. When Method A was applied, results were not good because of the high correlation between the variables as was observed by Ashkar et al. (1998). However, it is possible that other forms of r(u,v) than the one presented here would be adequate. In this study, estimates of the correlation ρ, varied between 0.62 and 0.87. For station 01BJ007, the relationship between intensity and duration is shown in Figure 10. The estimated correlation in this case is $\sqrt{R^2}$ = 0.87.

For Method B, the first step was to estimate the parameters (α_1, s_1, α_2 and s_2) using ML. The data was then transformed from a Weibull distribution to a standard exponential distribution via equations (74) and (75), by replacing (α_1, s_1, α_2 and s_2) by their ML estimates. The transformed data was then used in the analysis from this point. The data was divided into three classes using equal probabilities to guarantee a large number of observations within each class to construct the empirical cdf's (Ashkar et al., 1998). The classes were divided such that:

Class 1: $0 \le F_2(x_2) \le 1/3$
Class 2: $1/3 \le F_2(x_2) \le 2/3$
Class 3: $2/3 \le F_2(x_2) \le 1$

where $F_2(x_2)$ is the marginal cdf of the random variable X_2. In the space of the variable X_2, these three classes correspond to classes of the form $a_i < X_2 < b_i$, where a_i and b_i are the bounds of class i. The mean of each class is used as the value of x_2 to be placed in the cdf's $F(x_1 \mid x_2)$. The mean for each class is as follows:

Class 1: -ln(5/6)
Class 2: -ln(3/6)
Class 3: -ln(1/6)

To estimate ρ, a 3x3 contingency table was constructed from the observed couples (x_1, x_2) ("observed contingency table") and compared to a series of 3x3 contingency tables calculated from the hypothesized model [i.e., based on Equations (71) and (59)], for different correlation coefficients (ρ) ("theoretical contingency tables"). The chosen ρ value was the one that minimized the chi-squared distance between the cell counts from the observed and the theoretical contingency tables (Ashkar and El-Jabi, 2002).

The empirical cdf's conditioned by duration corresponding to the different classes were calculated using a plotting position formula:

$$p(q) = \frac{q - 0.35}{N} \tag{77}$$

where q is the rank for values of the class (arranged in ascending order), N is the sample size, and p(q) is the empirical probability.

The intensity conditioned by durations of 7 and 14 days were calculated for recurrence periods of 2, 10, 20 and 50 years. For these calculations, data were chosen so that the median was respectively 7 and 14 days. For some stations, it was impossible to calculate the intensity conditioned by a 7-day duration because of insufficient data. Kolmogorov-Smirnov tests accompanied by graphical representations were used to determine the goodness of fit. Figures 11 and 12 show the plotted theoretical and empirical distribution for intensity conditioned by 7-day and 14-day durations for station 01AJ003. The results for each station are presented as low flow (m^3/s) calculated from intensity values conditioned by 7- and 14-day durations (Tables 9 and 10).

3.4 Regionalization

In New Brunswick, low flow characteristics are not available for all drainage basins due to the absence of gauging stations or to the poor quality of collected streamflow data. Therefore, regional relationships can be developed for drainage basins within homogeneous low flow zones having similar physiographic and climatic characteristics (Environment Canada and New Brunswick Department of the Environment, 1990). Many physiographic and climatic characteristics such as the area of lakes and swamps, the average water content of snow cover, the basin perimeter, the drainage area, the latitude, the longitude, the mean annual precipitation, and the mean annual runoff. However, these characteristics are not readily available or easily calculated at drainage basins of interest in New Brunswick. In a recent low flow frequency study using the annual minimum series approach and the same stations as the present study, drainage area and mean annual precipitation were used for the regionalization of low flow characteristics. Results showed that the inclusion of precipitation in the regionalization models only slightly improves the coefficients of determination (Hébert et al., 2003). Hence, in this study, only drainage area was used in the regionalization.

After the univariate and bivariate analyses were complete, the regionalization of low flow characteristics was carried out. The intensity data were transformed into low flow data (i.e. intensity subtracted from reference value) before the regionalization was undertaken. For the univariate results, the values for volume, duration and low flow were related to the drainage area. Stations 01AK006 and 01AL004 were not used in this analysis because their drainage areas were small (<100km^2). Two types of linear regressions were done. The first was performed with the untransformed data (Y= mDA + b), whereas the second was done using a logarithmic transformation (logY= βlogDA + α). Y corresponds to volume, duration or intensity and DA corresponds to drainage area. Figures 13, 14, 15, 16, 17 and 18 show the regressions for volume, duration and intensity. The estimates of m, b, β and α for the regressions as well as the R^2 and the p value are given in Tables 11 and 12.

From Figures 13 to 18 and Tables 11 and 12, it was observed that the regression analysis gave slightly better results without the logarithmic transformation for volume and low flow. The R^2 values were slightly higher for untransformed data (normal scale rather than logarithmic). However, for volume, when the data is not in a logarithmic scale, the regression equations will give negative values for smaller basins (<500km^2). For this reason, it may be better to use the regression with logarithmic scale. For duration, there does not seem to be a significant relationship between duration and drainage area, i.e. size of basin. The p values show that the slope (m, in the case of untransformed data, and β, in the case of logarithmically transformed data) is not significantly different than 0. Although the low flow duration data did not show a level of association with basin size, duration data clearly showed that low flows of higher return events were of longer durations. For instance, the 2-year average low flow represented approximately 50-60 days in duration, while the 50-year average low flow was in the range of 160-170 days (Figure 15). More data for basins with drainage areas ranging from 4000 to 15000km^2 would be necessary to truly determine whether or not a relationship existed between duration and drainage area.

For the bivariate analysis, the intensity conditioned by duration data was transformed into low flow conditioned by duration data and then related to the drainage area. The regressions were developed in the same manner as for the univariate analysis. Figures 19, 20, 21 and 22 show the regressions for the low flow conditioned by 7- and 14-day durations. The estimates of m, b, β and α for the regressions as well as the R^2 and the p value are given in Tables 13 and 14.

The regression results for the bivariate analysis are also better without using the logarithmic scale R^2 criterion . However, the R^2 values are lower (62-74%) for the 7-day durations than the 14-day durations (83-94%). The 14-day duration probably gives better results than the 7-day duration because there is more volume, duration and intensity data points for the 14-day regression.

A test similar to that presented in Caissie et al. (2002) was performed to determine if there was a significant difference between the recurrence intervals for the regionalization results presented in each graph. In some cases, there was no significant difference between recurrence intervals. However, the regression lines were still presented for illustration purposes.

It should also be pointed out that the choice of the reference value Q_R and the simplifying assumptions may influence the results of the analysis. If these were modified, the results may differ. This would be interesting to study but may be difficult to generalize for a large number of stations as was studied here.

4.0 CONCLUSION

A low flow analysis was performed using the Deficit Below Threshold (DBT) model, which considers all flows below a reference value Q_R. This method better describes low flow events in terms of volume, duration and intensity, than a classic approach for low flow analysis. The DBT approach was applied on 31 hydrometric stations in New Brunswick. The reference value in this study was chosen as the median monthly flow for August.

Once the low flow events were identified, univariate and bivariate analyses were carried out. For the univariate analysis, data were fitted to the exponential, generalized Pareto and Weibull distributions. The volume, duration and intensity were then calculated for recurrence intervals of 2, 10, 20 and 50 years using the best-fitted distribution. In general, the best fitted distribution according to the Kolmogorov-Smirnov test and visual graphical evaluations was the Weibull distribution.

For the bivariate analysis, the intensity conditioned by durations of 7 and 14 days was modeled using the Nagao and Kadoya model with the Weibull distribution. The intensity conditioned by duration was calculated for recurrence intervals of 2, 10, 20 and 50 years as was done for the univariate analysis.

A regionalization was thereafter performed for the province of New Brunswick. For the univariate analysis, the regression for low flow and volume with basin drainage area gave good results especially with untransformed data. However, for volume, it may be necessary to use the regression with the logarithmic scale to avoid negative values for smaller basins. For duration, a very weak relationship was observed between duration and drainage area, which suggests that the duration is somewhat independent of basin size. However, a clear relationship was nevertheless observed between duration and the severity of low flow events. For the bivariate analysis, the regression for low flow conditioned by duration and drainage area gave better results for 14-day durations than the 7-day durations probably because of the lack of volume, duration and intensity data for the 7-day durations.

The Deficit Below Threshold Approach can provide a better analysis for engineering design projects because it characterizes low flows in terms of volume, duration and intensity which provides more information than typical low flow analyses. This method can also provide a better description of low flows for fish habitat studies and assessments.

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Table 1. Selected hydrometric stations.

River	ID	Latitude, Longitude	Drainage Area (km^2)	Analysis Period
1. Saint John R. at Fort Kent	01AD002	47 15 N, 68 35 W	14700	1927-1999
2. St. Francis R. at outlet of Glacier Lake	01AD003	47 12 N, 68 57 W	1350	1952-1999
3. Fish R. near Fort Kent	01AE001	47 14 N, 68 35 W	2260	1981-1999
4. Grande R. at Violette Bridge	01AF007	47 15 N, 67 55 W	339	1977-1999
5. Meduxnekeag R. near Belleville	01AJ003	46 13 N, 67 44 W	1210	1968-1999
6. Big Presque Isle Stream at Tracey Mills	01AJ004	46 26 N, 67 45 W	484	1968-1999
7. Becaguimec Stream at Coldstream	01AJ010	46 20 N, 67 28 W	350	1974-1999
8. Shogomoc Stream near TCH	01AK001	45 57 N, 67 19 W	234	1919-1940, 1944-1999
9. Middle Branch Nashwaaksis Stream near Sandwith's Farm	01AK006	46 05 N, 66 44 W	5.7	1967-1999
10. Nackawic Stream	01AK007	46 03 N, 67 14 W	240	1968-1999
11. Nashwaak R. at Durham Bridge	01AL002	46 08 N, 66 37 W	1450	1962-1999
12. Narrows Mountain Bk. near Narrows Mountain	01AL004	46 17 N, 67 01 W	3.89	1972-1999
13. North Branch Oromocto R. at Tracy	01AM001	45 40 N, 66 41 W	557	1963-1999
14. Salmon R. at Castaway	01AN002	46 17 N, 65 43 W	1050	1974-1999
15. Canaan R. at East Canaan	01AP002	46 04 N, 65 22 W	668	1926-1999, 1963-1999
16. Kennebecasis R. at Apohaqui	01AP004	45 42 N, 65 36 W	1100	1962-1999
17. Lepreau R. at Lepreau	01AQ001	45 10 N, 66 28 W	239	1919-1999
18. Restigouche R. below Kedgwick R.	01BC001	47 40 N, 67 29 W	3160	1963-1999
19. Upsalquitch R. at Upsalquitch	01BE001	47 50 N, 66 53 W	2270	1919-1932, 1944-1999
20. Jacquet R. near Durham Centre	01BJ003	47 54 N, 66 02 W	510	1965-1999
21. Restigouche R., Rafting Ground Bk.	01BJ007	47 54 N, 66 57 W	7740	1969-1999
22. Middle R. near Bathurst	01BJ010	47 37 N, 65 43 W	217	1982-1999
23. R. Caraquet at Burnsville	01BL002	47 42 N, 65 09 W	173	1970-1999
24. Big Tracadie R. at Murchy Bridge Crossing	01BL003	47 26 N, 65 06 W	383	1971-1999
25. SW Miramichi R. at Blackville	01BO001	46 44 N, 65 50 W	5050	1919-1932, 1962-1999
26. Little SW Miramichi R. at Lyttleton	01BP001	46 56 N, 65 54 W	1340	1952-1999
27. NW Miramichi R. at Trout Bk.	01BQ001	47 06 N, 65 50 W	948	1962-1999
28. Coal Branch R. at Beersville	01BS001	46 27 N, 65 04 W	166	1965-1999
29. Petitcodiac R. near Petitcodiac	01BU002	45 57 N, 65 10 W	391	1962-1999
30. Turtle Creek at Turtle Creek	01BU003	45 57 N, 64 52 W	129	1963-1999
31. Point Wolfe R. at Fundy National Park	01BV006	45 34 N, 65 01 W	130	1965-1999

Table 2. Functions to calculate the type III Weibull distribution. (El-Jabi and Rousselle, 1990).

C_s	1/n	A(n)	B(n)
-1.000	0.02	0.446	40.005
-0.971	0.03	0.444	26.987
-0.917	0.04	0.442	20.481
-0.867	0.05	0.439	16.576
-0.638	0.10	0.425	8.737
-0.254	0.20	0.389	4.755
0.069	0.30	0.346	3.370
0.359	0.40	0.297	2.634
0.631	0.50	0.246	2.159
0.896	0.60	0.193	1.815
1.160	0.70	0.142	1.549
1.430	0.80	0.092	1.334
1.708	0.90	0.044	1.154
2.000	1.00	0.000	1.000
2.309	1.10	-0.040	0.867
2.640	1.20	-0.077	0.752
2.996	1.30	-0.109	0.652
3.382	1.40	-0.136	0.563
3.802	1.50	-0.160	0.486
4.262	1.60	-0.180	0.418
4.767	1.70	-0.196	0.359
5.323	1.80	-0.208	0.308
5.938	1.90	-0.217	0.263
6.619	2.00	-0.224	0.224
7.374	2.10	-0.227	0.190
8.214	2.20	-0.229	0.161

Table 3. Threshold values and number of low flow events for hydrometric stations.

River	ID	Threshold Value (Q_R) (m^3/s)	Number of events
1. Saint John R. at Fort Kent	01AD002	92.94	178
2. St. Francis R. at outlet of Glacier Lake	01AD003	7.733	110
3. Fish R. near Fort Kent	01AE001	13.24	42
4. Grande R. at Violette Bridge	01AF007	2.153	90
5. Meduxnekeag R. near Belleville	01AJ003	4.191	82
6. Big Presque Isle Stream at Tracey Mills	01AJ004	2.167	89
7. Becaguimec Stream at Coldstream	01AJ010	1.440	108
8. Shogomoc Stream near TCH	01AK001	0.588	115
9. Middle Branch Nashwaaksis Stream near Sandwith's Farm	01AK006	0.011	80
10. Nackawic Stream	01AK007	0.344	83
11. Nashwaak R. at Durham Bridge	01AL002	8.581	100
12. Narrows Mountain Bk. near Narrows Mountain	01AL004	0.018	73
13. North Branch Oromocto R. at Tracy	01AM001	1.312	55
14. Salmon R. at Castaway	01AN002	4.210	67
15. Canaan R. at East Canaan	01AP002	1.098	100
16. Kennebecasis R. at Apohaqui	01AP004	4.720	80
17. Lepreau R. at Lepreau	01AQ001	1.183	156
18. Restigouche R. below Kedgwick R.	01BC001	25.01	95
19. Upsalquitch R. at Upsalquitch	01BE001	13.88	162
20. Jacquet R. near Durham Centre	01BJ003	2.483	86
21. Restigouche R., Rafting Ground Bk.	01BJ007	61.66	78
22. Middle R. near Bathurst	01BJ010	0.849	59
23. R. Caraquet at Burnsville	01BL002	1.439	82
24. Big Tracadie R. at Murchy Bridge Crossing	01BL003	3.041	64
25. SW Miramichi R. at Blackville	01BO001	38.45	173
26. Little SW Miramichi R. at Lyttleton	01BP001	11.95	162
27. NW Miramichi R. at Trout Bk.	01BQ001	6.396	128
28. Coal Branch R. at Beersville	01BS001	0.472	76
29. Petitcodiac R. near Petitcodiac	01BU002	0.899	86
30. Turtle Creek at Turtle Creek	01BU003	0.516	64
31. Point Wolfe R. at Fundy National Park	01BV006	1.169	103

Table 4. Best fitted distribution for volume, duration and intensity.

	Pareto	Exponential	Weibull	none
Volume	25	1	4	1
Duration	21	4	5	1
Intensity	0	0	29	2

Table 5. Best fitted distribution for the largest volume, largest duration and largest intensity in a one year period.

	Pareto	Exponential	Weibull	none
Largest volume	4	0	26	1
Largest duration	2	1	28	0
Largest intensity	0	0	28	3

Table 6. Volume (m^3/s*days) of low flow events at recurrence periods of 2, 10, 20 and 50 years.

ID	Recurrence Period (yrs.)			
	2	10	20	50
01AD002	2222	7510	10428	15234
01AD003	146	498	650	857
01AE001	186	527	672	872
01AF007	102	214	259	318
01AJ003	55.4	191	253	340
01AJ004	27.0	86.1	119	173
01AJ010	17.2	43.3	56.0	75.5
01AK001	6.0	21.1	26.8	34.2
01AK007	8.6	20.4	25.2	31.5
01AL002	112	280	342	419
01AM001	20.2	72.1	88.9	109
01AN002	58.2	167	208	261
01AP002	11.6	44.1	57.4	75.1
01AP004	37.0	133	184	269
01BC001	625	1724	2165	2747
01BE001	327	1040	1349	1770
01BJ003	19.8	57.9	72.1	90.4
01BJ007	1734	4290	5234	6437
01BJ010	22.9	57.0	70.2	87.4
01BL002	28.6	84.2	108	139
01BL003	53.8	160	203	260
01BO001	587	1389	1691	2078
01BP001	238	594	735	919
01BQ001	119	309	386	488
01BS001	4.1	17.1	24.0	34.5
01BU002	9.7	34.4	44.7	58.5
01BU003	4.1	13.7	16.9	21.0
01BV006	20.4	55.5	69.1	86.8

Table 7. Duration (days) of low flow events at recurrence periods of 2, 10, 20 and 50 years.

ID	Recurrence Period (yrs.)			
	2	10	20	50
01AD002	60	136	166	203
01AD003	57	142	176	220
01AE001	53	93	105	119
01AF007	83	152	178	211
01AJ003	37	88	107	131
01AJ004	42	99	120	147
01AJ010	35	67	79	94
01AK001	28	88	107	130
01AK007	41	83	99	120
01AL002	44	90	105	123
01AM001	32	87	103	122
01AN002	40	93	112	135
01AP002	28	81	101	126
01AP004	35	90	109	133
01AQ001	30	79	95	115
01BC001	68	150	181	219
01BE001	73	177	215	265
01BJ003	65	137	162	193
01BJ007	75	144	166	193
01BJ010	67	135	159	189
01BL002	65	149	181	221
01BL003	64	133	156	184
01BO001	52	102	120	141
01BP001	59	124	147	177
01BQ001	56	114	135	161
01BS001	33	90	113	143
01BU002	32	87	108	134
01BU003	35	94	114	138
01BV006	41	93	111	134

Table 8. Low flow (m^3/s) at recurrence periods of 2, 10, 20 and 50 years.

ID	Ref.Value Q_R (m^3/s)	Recurrence Period (yrs.)			
		2	10	20	50
01AD002	92.9	34.8	18.2	13.9	9.23
01AE001	13.2	6.71	3.73	2.93	2.03
01AF007	2.15	0.17	0.00	0.00	0.00
01AJ003	4.19	1.50	0.43	0.15	0.00
01AJ004	2.17	0.93	0.30	0.12	0.00
01AJ010	1.44	0.52	0.22	0.13	0.03
01AK001	0.59	0.31	0.10	0.04	0.00
01AK007	0.34	0.06	0.00	0.00	0.00
01AL002	8.58	4.32	2.51	2.01	1.44
01AL004	0.02	0.01	0.00	0.00	0.00
01AM001	1.31	0.48	0.01	0.00	0.00
01AN002	4.21	2.15	1.18	0.92	0.61
01AP002	1.10	0.48	0.18	0.10	0.02
01AP004	4.72	2.82	1.65	1.32	0.93
01BC001	25.0	10.8	5.61	4.19	2.58
01BE001	13.9	6.33	3.55	2.84	2.05
01BJ003	2.48	1.75	1.37	1.26	1.14
01BJ007	61.7	26.7	14.5	11.2	7.56
01BJ010	0.85	0.32	0.12	0.06	0.00
01BL002	1.44	0.74	0.47	0.39	0.31
01BL003	3.04	1.64	1.05	0.90	0.72
01BO001	38.5	18.9	9.15	6.30	3.02
01BP001	12.0	5.50	3.13	2.45	1.68
01BQ001	6.40	3.00	1.74	1.38	0.98
01BS001	0.47	0.22	0.11	0.09	0.05
01BU002	0.90	0.41	0.21	0.15	0.09
01BU003	0.52	0.33	0.25	0.23	0.20

Table 9. Low flow (m^3/s) conditioned by a 7-day duration for recurrence intervals of 2, 10, 20 and 50 years.

Station	Ref. Value Q_R (m^3/s)	Recurrence Period (yrs)			
		2	10	20	50
01AF007	2.15	1.27	0.94	0.86	0.76
01AJ003	4.19	2.76	2.00	1.79	1.55
01AJ010	1.44	1.00	0.79	0.73	0.67
01AK001	0.59	0.43	0.35	0.33	0.30
01AK007	0.34	0.20	0.13	0.11	0.09
01AL002	8.58	6.48	5.26	4.91	4.52
01AN002	4.21	3.19	2.64	2.48	2.30
01AP002	1.10	0.75	0.57	0.52	0.46
01AP004	4.72	3.70	3.12	2.95	2.77
01AQ001	1.18	0.83	0.65	0.61	0.55
01BL002	1.44	1.11	0.94	0.89	0.84
01BP001	12.0	9.10	7.52	7.12	6.37
01BS001	0.47	0.32	0.24	0.22	0.20
01BU002	0.90	0.61	0.48	0.45	0.41
01BV006	1.17	0.83	0.67	0.62	0.57

Table 10. Low flow (m^3/s) conditioned by a 14-day duration of for recurrence intervals of 2, 10, 20 and 50 years.

Station	Ref. Value Q_R (m^3/s)	Recurrence Period (yrs)			
		2	10	20	50
01AD002	92.9	53.1	38.3	34.3	30.1
01AD003	7.73	5.76	4.72	4.43	4.12
01AE001	13.2	9.76	7.43	7.09	6.46
01AF007	2.15	1.09	0.55	0.44	0.30
01AJ003	4.19	2.43	1.67	1.46	1.23
01AJ004	2.17	1.42	1.03	0.92	0.81
01AJ010	1.44	0.86	0.65	0.59	0.53
01AK001	0.59	0.38	0.30	0.28	0.26
01AK007	0.34	0.17	0.10	0.08	0.05
01AL002	8.58	6.00	4.75	4.41	4.03
01AM001	1.31	0.68	0.48	0.43	0.37
01AN002	4.21	2.91	2.35	2.20	2.02
01AP002	1.10	0.66	0.48	0.43	0.37
01AP004	4.72	3.41	2.82	2.66	2.47
01AQ001	1.18	0.71	0.54	0.49	0.43
01BC001	25.0	17.2	14.4	13.6	12.8
01BE001	13.9	9.66	7.60	7.04	6.42
01BJ003	2.48	1.69	1.37	1.29	1.20
01BJ007	61.7	43.3	36.2	34.3	32.2
01BJ010	0.85	0.53	0.36	0.31	0.26
01BL002	1.44	1.05	0.88	0.83	0.78
01BL003	3.04	2.30	1.94	1.84	1.73
01BO001	38.5	28.1	22.7	21.2	19.4
01BP001	11.9	8.42	6.86	6.42	5.96
01BQ001	6.40	4.52	3.58	3.32	3.03
01BS001	0.47	0.29	0.22	0.20	0.17
01BU002	0.90	0.54	0.42	0.39	0.35
01BU003	0.52	0.38	0.32	0.31	0.29
01BV006	1.17	0.72	0.56	0.51	0.46

Table 11. Linear regression analysis of volume, duration and low flow at recurrence intervals of 2, 10, 20, and 50 years and basin drainage area upstream of the gauging station (Y= b + mDA).

Period	b	m	R^2	p
		Volume (m^3/s*days)		
2 years	-45.17	0.166	0.943	<0.001
10 years	-192.3	0.515	0.970	<0.001
20 years	-292.3	0.694	0.968	<0.001
50 years	-473.7	0.978	0.957	<0.001
		Duration (days)		
2 years	46.32	0.0018	0.109	0.080
10 years	105.1	0.0030	0.100	0.095
20 years	126.0	0.0036	0.097	0.100
50 years	152.3	0.0042	0.089	0.115
		Low flow (m^3/s)		
2 years	0.192	0.0027	0.945	<0.001
10 years	0.126	0.0014	0.941	<0.001
20 years	0.098	0.0011	0.939	<0.001
50 years	0.072	0.0007	0.929	<0.001

Table 12. Linear regression analysis of volume, duration and low flow (logarithmic scale) at recurrence intervals of 2, 10, 20, and 50 years and basin drainage area (logarithmic scale) upstream of the gauging station (log Y= α+ β logDA).

Period	α	β	R^2	p
		Volume (m^3/s*days)		
2 years	-1.865	1.255	0.818	<0.001
10 years	-1.348	1.239	0.853	<0.001
20 years	-1.249	1.242	0.858	<0.001
50 years	-1.151	1.246	0.862	<0.001
		Duration (days)		
2 years	1.357	0.110	0.168	0.027
10 years	1.823	0.072	0.123	0.063
20 years	1.914	0.068	0.110	0.079
50 years	2.008	0.063	0.093	0.107
		Low flow (m^3/s)		
2 years	-3.407	1.238	0.843	<0.001
10 years	-3.566	1.191	0.680	<0.001
20 years	-3.610	1.174	0.729	<0.001
50 years	-3.397	1.070	0.618	<0.001

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Table 13. Linear regression analysis of low flow conditioned by 7- and 14-day durations at recurrence intervals of 2, 10, 20, and 50 years and basin drainage area upstream of the gauging station (Y= b + mDA).

Period	b	m	R^2	p
>Low Flow (m^3/s) conditioned by 7- day duration				
2 years	-0.584	0.0046	0.735	<0.001
10 years	-0.498	0.0037	0.718	<0.001
20 years	-0.474	0.0035	0.709	<0.001
50 years	-0.446	0.0032	0.700	<0.001
Low Flow (m^3/s) conditioned by 14- day duration				
2 years	0.138	0.0042	0.939	<0.001
10 years	0.299	0.0032	0.898	<0.001
20 years	0.351	0.0029	0.880	<0.001
50 years	0.384	0.0026	0.858	<0.001

Table 14. Linear regression analysis of low flow conditioned by 7- and 14-day durations (logarithmic scale) at recurrence intervals of 2, 10, 20, and 50 years and basin drainage area (logarithmic scale) upstream of the gauging station (log Y= α+ β logDA).

Period	α	β	R^2	p
Low Flow (m^3/s) conditioned by 7- day duration				
2 years	-2.714	1.062	0.673	<0.001
10 years	-2.861	1.078	0.645	<0.001
20 years	-2.913	1.084	0.633	<0.001
50 years	-2.981	1.093	0.616	0.001
Low Flow (m^3/s) conditioned by 14- day duration				
2 years	-3.013	1.174	0.879	<0.001
10 years	-3.190	1.194	0.858	<0.001
20 years	-3.256	1.204	0.847	<0.001
50 years	-3.351	1.219	0.830	<0.001

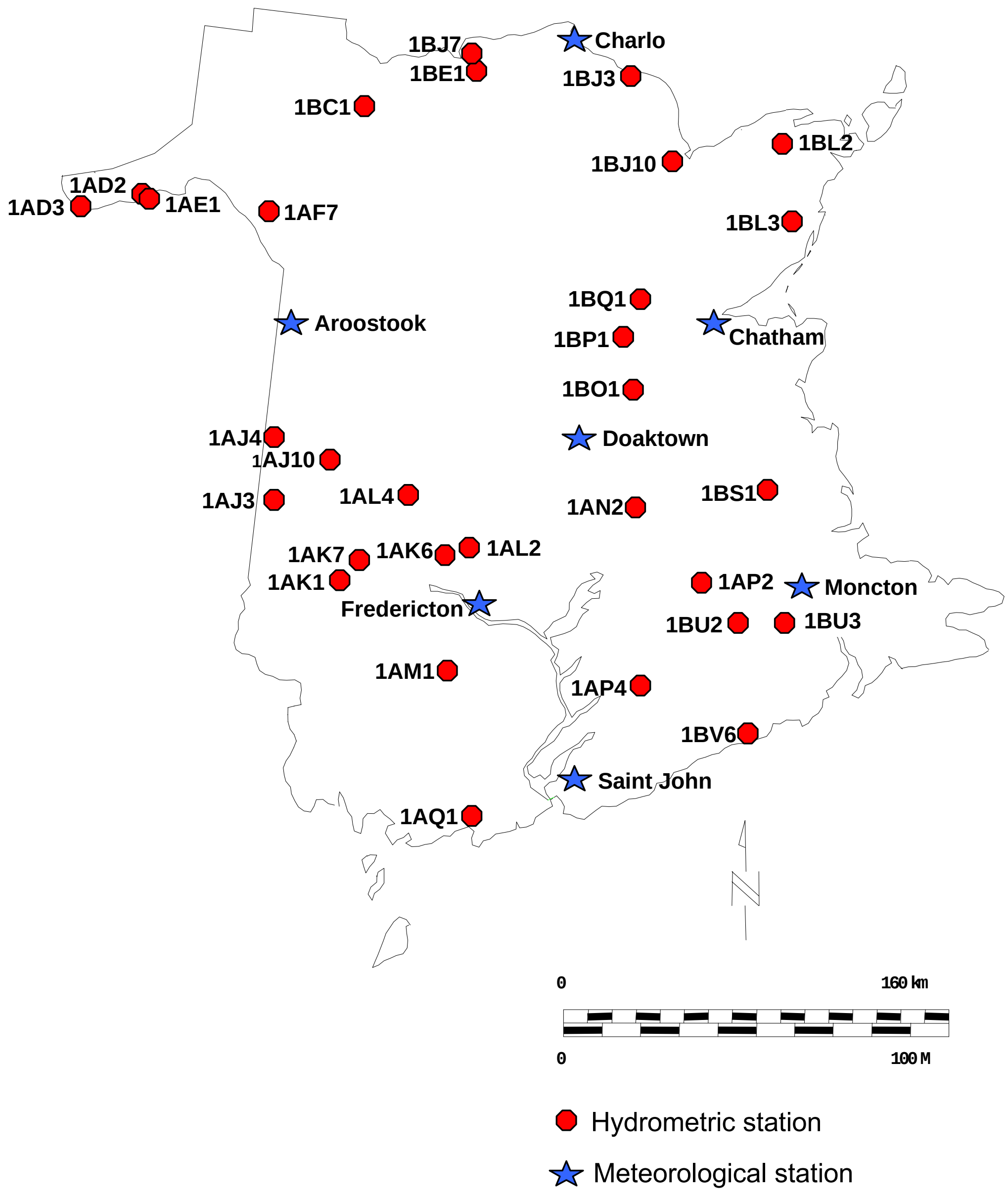

Figure 1. Location of selected hydrometric stations in New Brunswick.

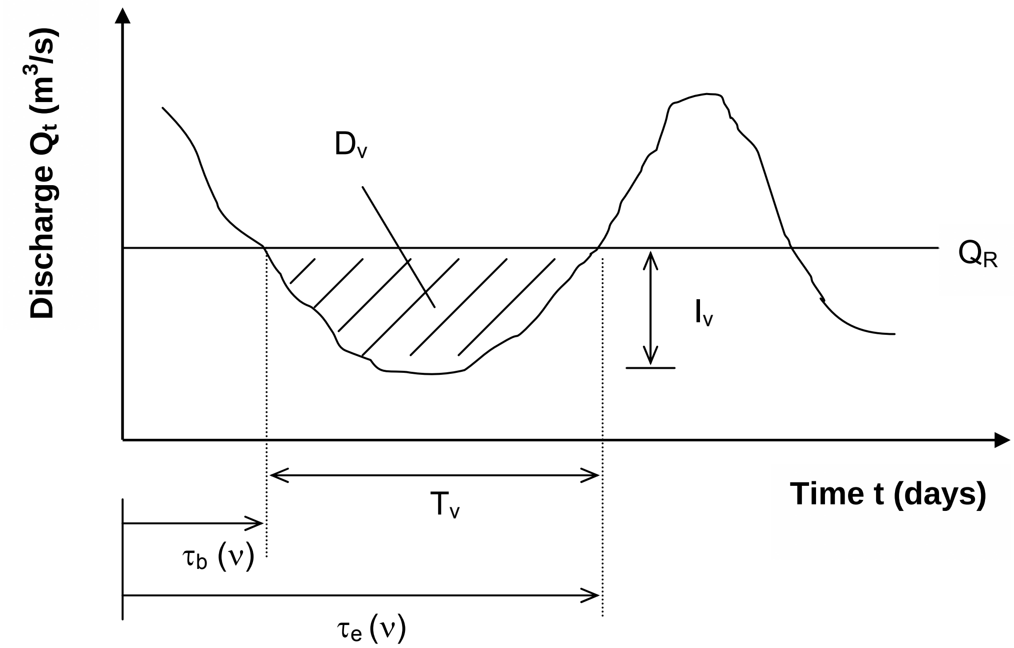

Figure 2. Hydrograph representing low flow event characteristics.

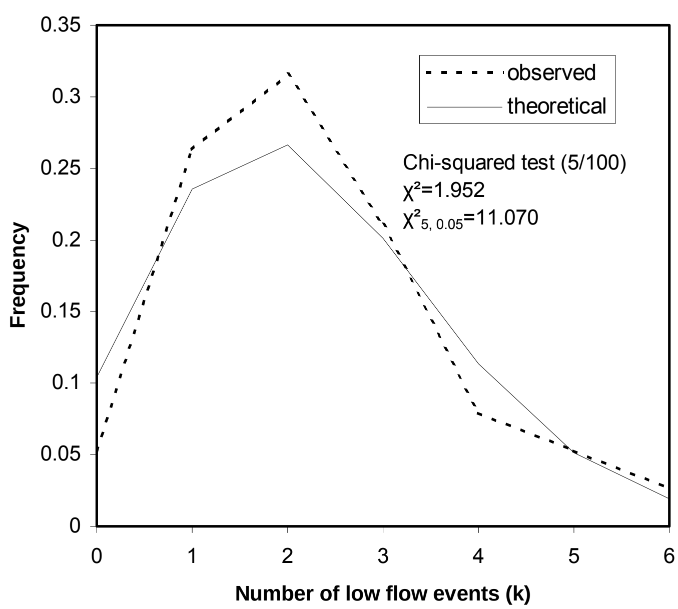

Figure 3. Theoretical and observed distribution of low flow events per year for station 01BU002.

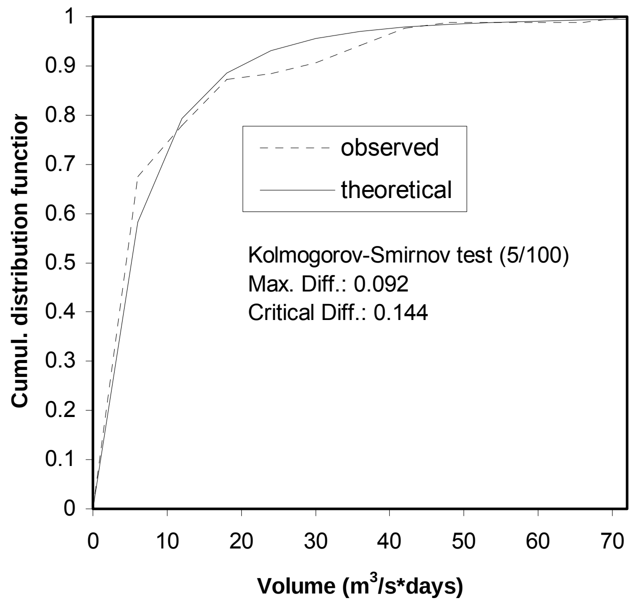

Figure 4. Theoretical (generalized Pareto) and observed volume distribution for station 01BU002.

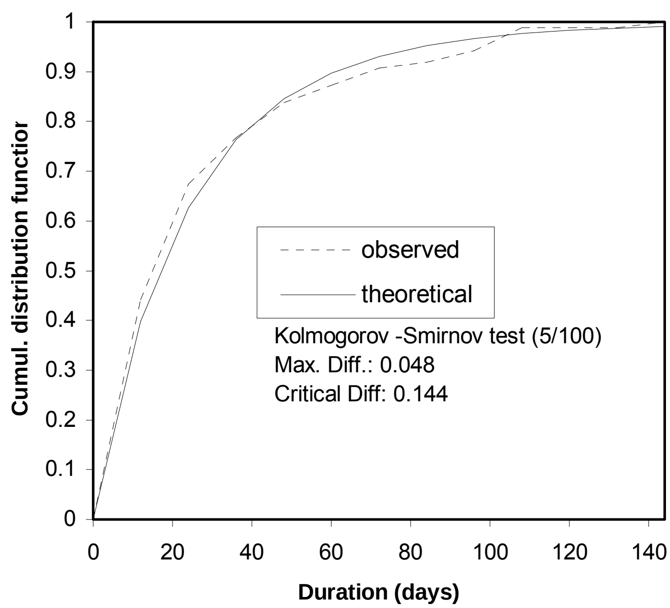

Figure 5. Theoretical (generalized Pareto) and observed duration distribution for station 01BU002.

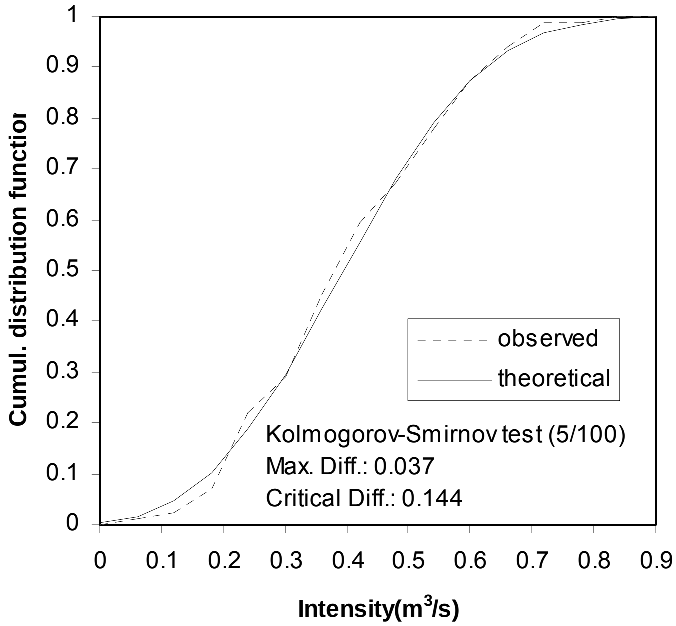

Figure 6. Theoretical (Weibull) and observed intensity distribution for station 01BU002.

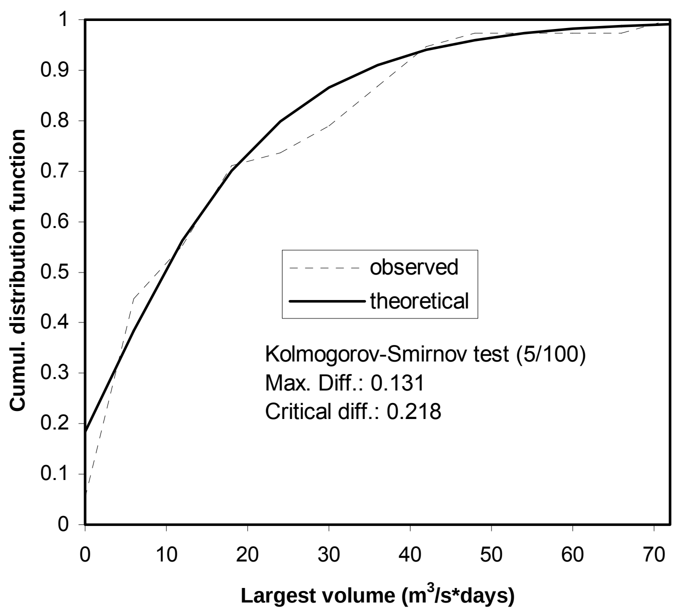

Figure 7. Theoretical (Weibull) and observed largest volume distribution for station 01BU002.

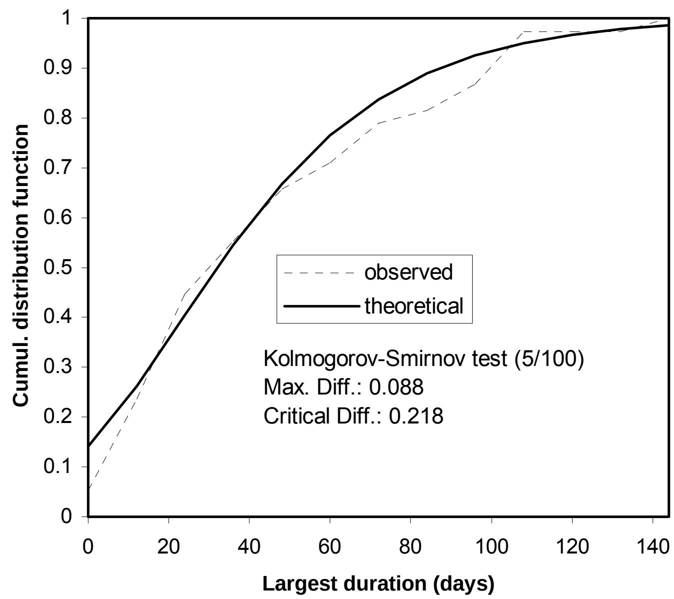

Figure 8. Theoretical (Weibull) and observed largest duration distribution for station 01BU002.

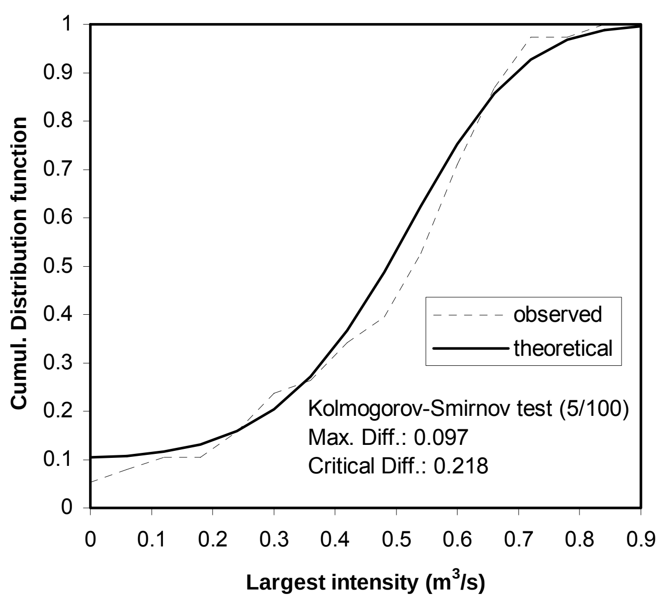

Figure 9. Theoretical (Weibull) and observed largest intensity distribution for station 01BU002.

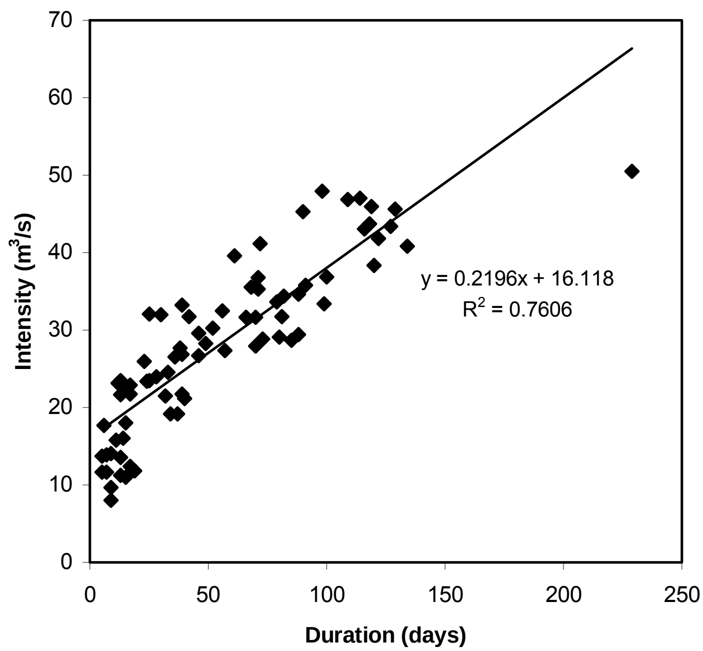

Figure 10. Intensity (m^3/s) vs. Duration (days) for station 01BJ007.

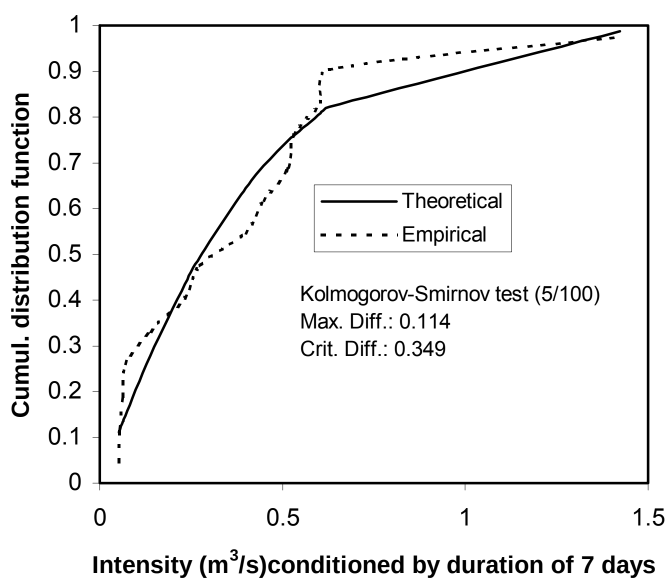

Figure 11. Theoretical (Weibull) and empirical distribution of the intensity (m^3/s) conditioned by a duration of 7 days for station 01BJ003.

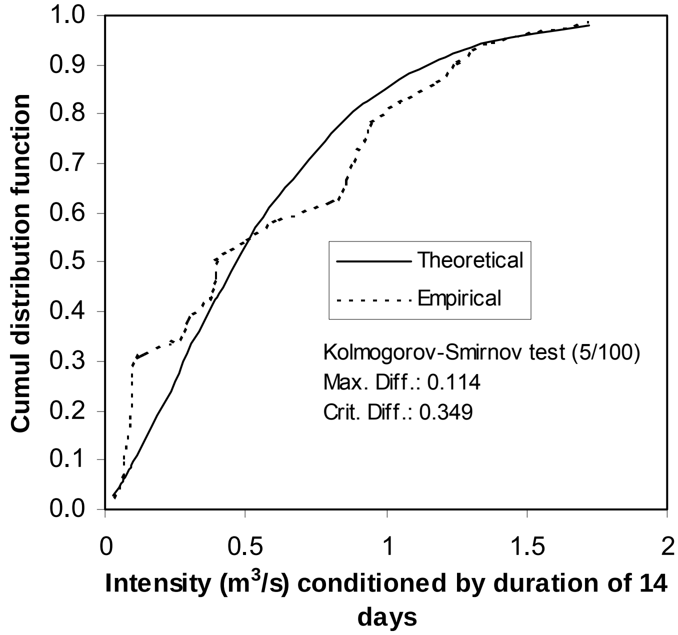

Figure 12. Theoretical (Weibull) and empirical distribution of the intensity (m^3/s) conditioned by a duration of 14 days for station 01BJ003.

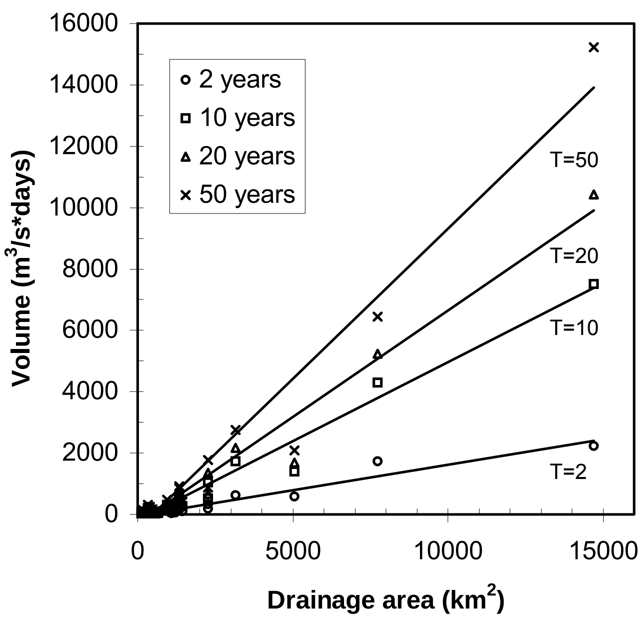

Figure 13. Regionalization of low flow volumes in N.B (28 stations).

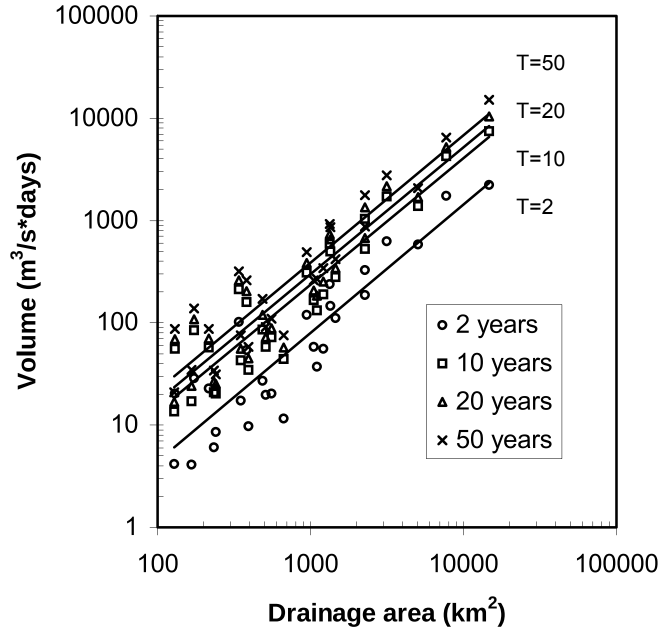

Figure 14. Regionalization of low flow volumes (logarithmic scale) in N.B (28 stations).

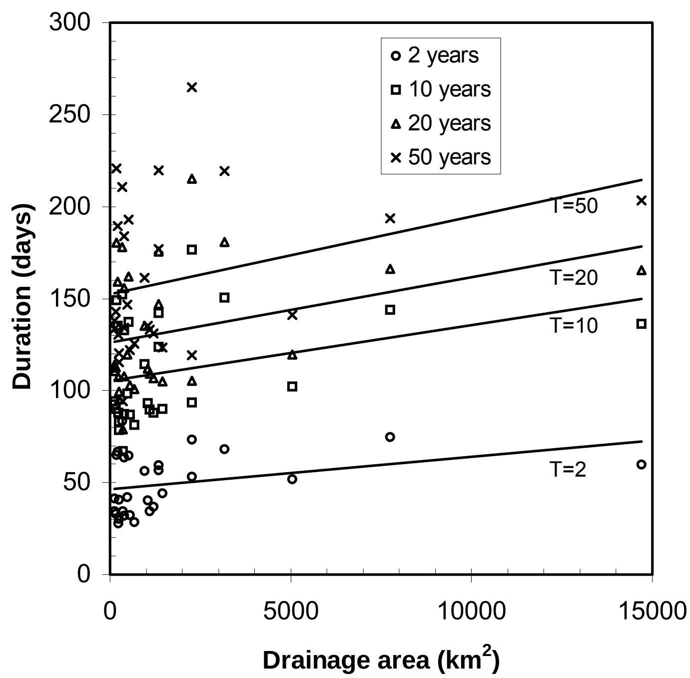

Figure 15. Regionalization of low flow durations in N.B (29 stations).

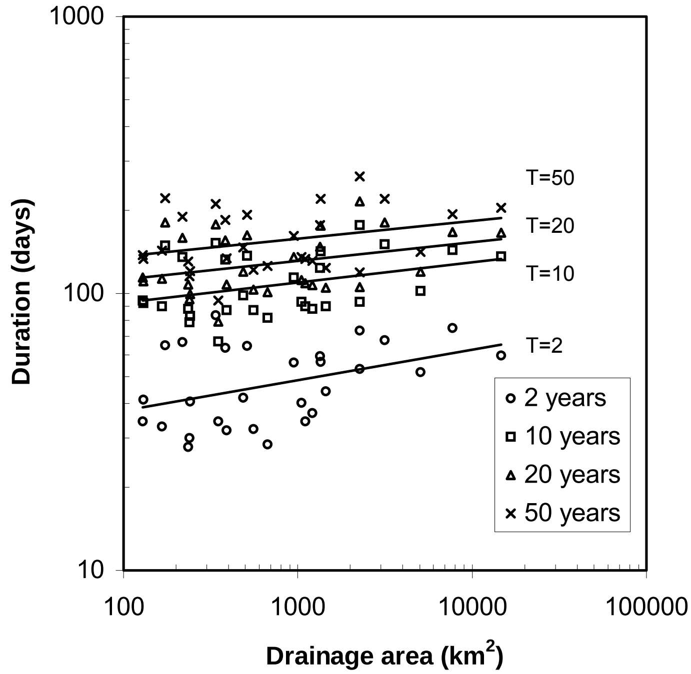

Figure 16. Regionalization of low flow durations (logarithmic scale) in N.B (29 stations).

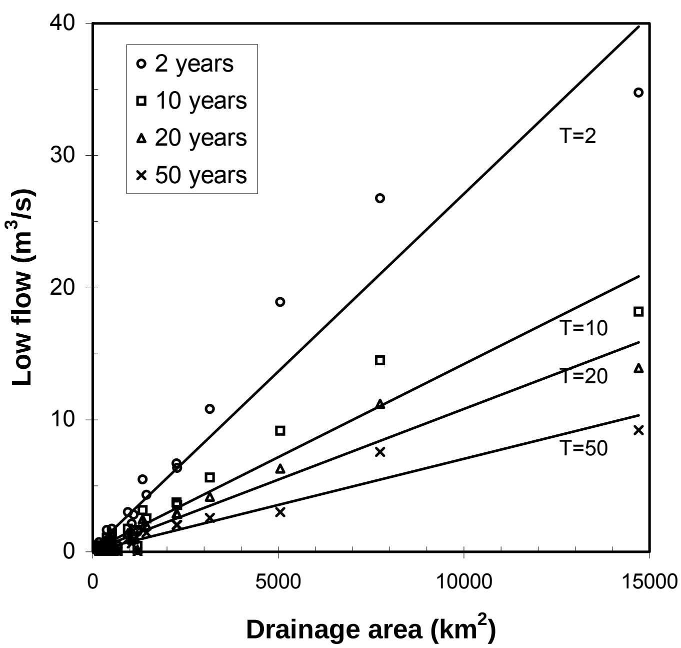

Figure 17. Regionalization of low flow in N.B (26 stations).

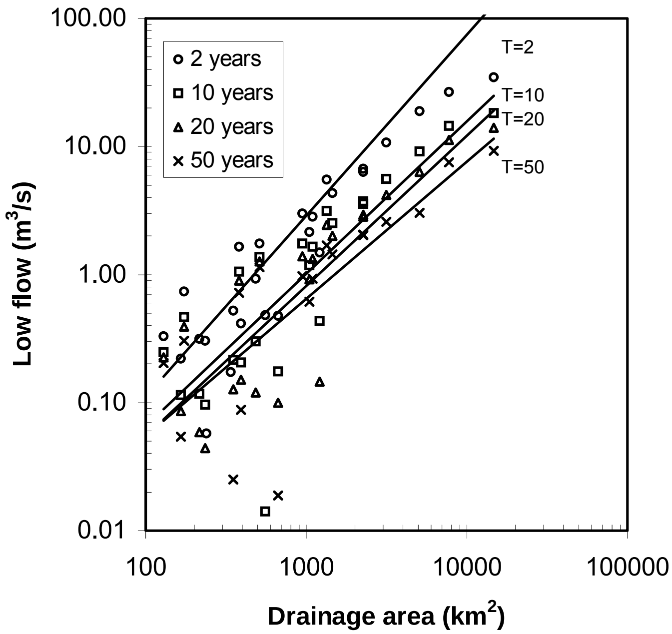

Figure 18. Regionalization of low flow (logarithmic scale) in N.B (26 stations).

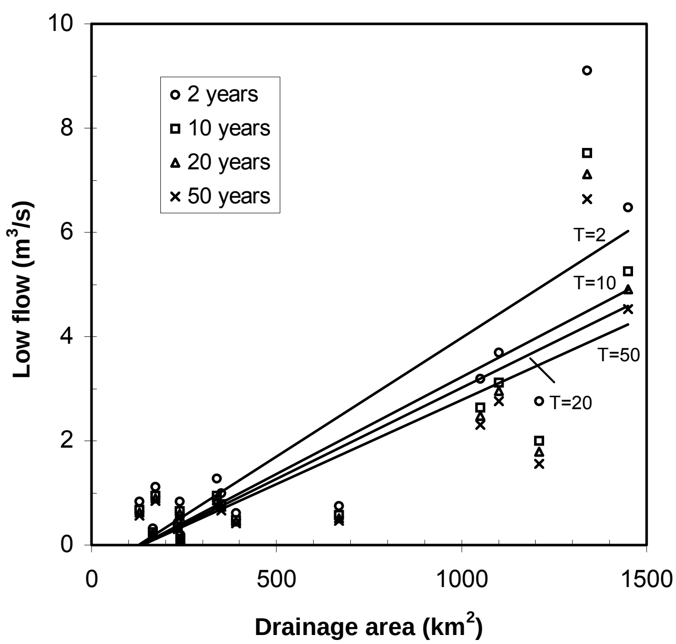

Figure 19. Regionalization of low flow conditioned by a 7-day duration in N.B (15 stations).

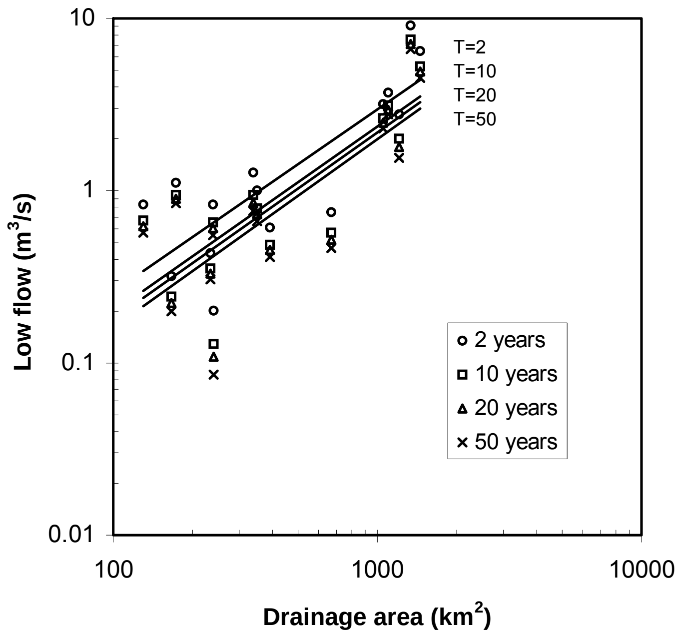

Figure 20. Regionalization of low flow conditioned by a 7-day duration (logarithmic scale) in N.B (15 stations).

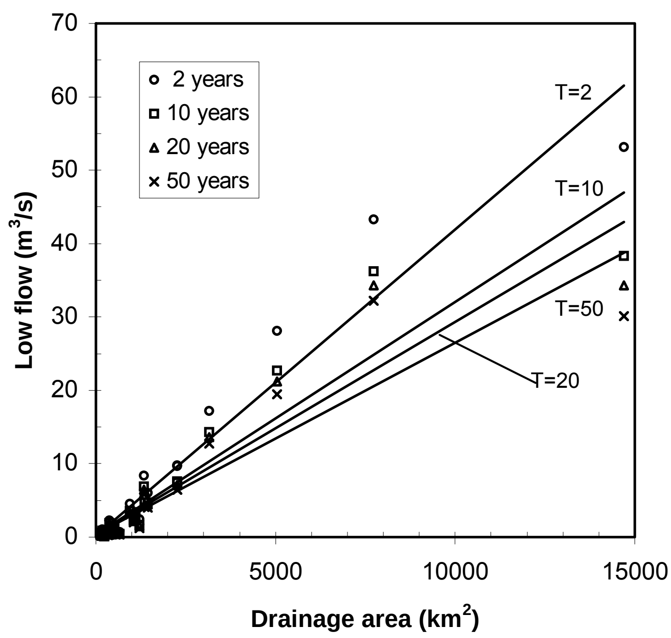

Figure 21. Regionalization of low flow conditioned by a 14-day duration in N.B (29 stations).

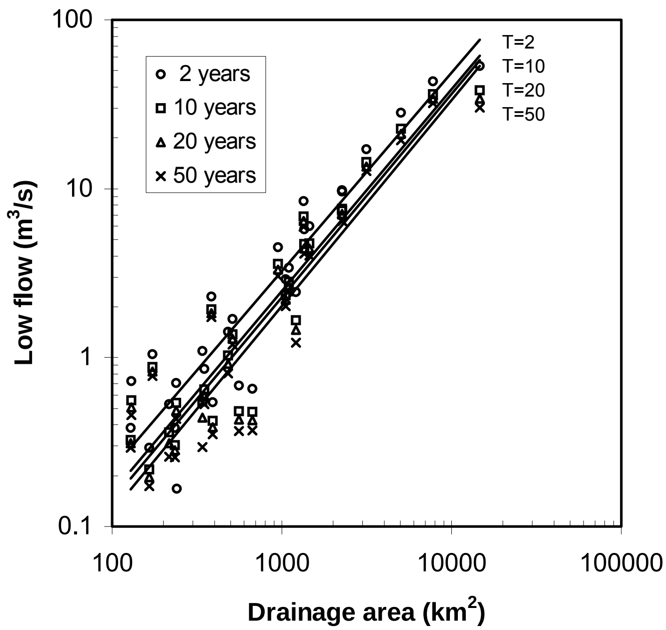

Figure 22. Regionalization of low flow conditioned by a 14-day duration (logarithmic scale) in N.B (29 stations).