

**Working Paper 2000-7/Document de travail 2000-7**

**Non-Parametric and Neural Network Models  
of Inflation Changes**

by

**Greg Tkacz**

**Bank of Canada**



**Banque du Canada**

ISSN 1192-5434  
ISBN 0-662-28938-2

Printed in Canada on recycled paper

**Bank of Canada Working Paper 2000-7**

**April 2000**

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**Greg Tkacz\***

Department of Monetary and Financial Analysis  
Bank of Canada  
Ottawa, Canada K1A 0G9

Tel: (613) 782-8591 Fax: (613) 782-7508  
[gtkacz@bank-banque-canada.ca](mailto:gtkacz@bank-banque-canada.ca)

The views expressed in this paper are those of the author.  
No responsibility for them should be attributed to the Bank of Canada.



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## Acknowledgements

Financial assistance from the McGill Institute for the Study of Canada and the *Fonds pour la Formation de chercheurs et l'aide à la recherche* (Québec) is gratefully acknowledged. I thank John W. Galbraith, Victoria Zinde-Walsh, Agathe Côté, Kevin Clinton, Ben Fung, Mingwei Yuan, and seminar participants at the Bank of Canada, McGill University, and the meetings of the Canadian Economics Association for several useful comments.

## Abstract

Previous studies have shown that interest rate yield spreads contain useful information about future changes in inflation. However, such studies have for the most part focused on linear models, ignoring potential non-linearities between interest rates and inflation. Using two different non-linear models, we find that the relationship between interest rate yield spreads and inflation changes for policy-relevant horizons in the United States is most pronounced at negative long-short yield spreads, and almost non-existent at positive values of the spread. These findings are consistent with studies noting asymmetric effects of monetary policy on the real economy.

*JEL classification codes: C51, E31*

*Bank classifications: Economic models; Inflation and prices*

## Résumé

Les travaux antérieurs ont montré que les écarts de taux de rendement fournissent une information utile sur l'évolution future de l'inflation. Ces travaux étaient toutefois fondés pour la plupart sur des modèles linéaires et faisaient fi de la non-linéarité possible de la relation entre les taux d'intérêt et l'inflation. Dans cette nouvelle étude, l'auteur a recours à deux modèles non linéaires différents. Il constate que la relation entre les écarts de taux de rendement et les variations de l'inflation dans le cas des horizons de prévision pertinents pour la conduite de la politique monétaire aux États-Unis est au plus fort lorsque les écarts entre taux longs et taux courts sont négatifs et qu'elle est presque nulle lorsqu'ils sont positifs. Ces résultats sont conformes à ceux obtenus par d'autres auteurs qui ont souligné les effets asymétriques de la politique monétaire sur l'économie réelle.

*Classifications JEL : C51, E31*

*Classifications de la Banque : Inflation et prix; Modèles économiques*

## 1. Introduction

In the conduct of monetary policy, it is crucial for policy-makers to be forward-looking given the lags with which policy actions ultimately affect the target variables. In virtually all developed countries, such policy actions are implemented through the control by the central monetary authority of a short-term—usually overnight—interest rate, while the primary (or lone) target variable is the rate of inflation. Consistent with the interest rate channel of monetary policy, changes in the overnight rate lead to changes of varying magnitudes to interest rates along the maturity spectrum, with the longest rates being the least responsive to policy actions. The levels of such long-term rates are most heavily influenced by factors such as the supply and demand of funds, inflation expectations, and country-specific default risk premia.

In response to movements in interest rates, economic agents alter their purchasing and investment decisions. This affects the level of real economic activity after a suitable lag, which is usually of the order of 12 months. Finally, through a short-run Phillips curve, the aggregate price level adjusts in response to any excessive or deficient demand. The time lapse between a policy action and the resulting maximal impact on the target variable via the interest rate channel is therefore approximately 18 to 24 months. This version of the interest rate channel is roughly consistent with Blinder's (1998) discussion of the transmission mechanism.

Given the lags with which policy actions impact the target variable, central bankers require reliable models of inflation that can guide their policy-making. Because of potential initial measurement errors and the usual one- to two-quarter publication delay of macroeconomic data, policy-makers often supplement forecasts from formal macroeconomic models with information that can be extracted from simple indicator models constructed around monetary and financial variables. Such reduced-form models are useful for obtaining initial inflation forecasts and can be used for comparisons with the formal forecasts.

In recent years, several authors have found empirically that interest rate yield spreads contain useful information about the future directions of inflation. In particular, Mishkin (1990b) and Frankel and Lown (1994) find for the United States that the information content is greatest at the middle and long ends of the yield curve. Mishkin (1990a) also remarks that yield differentials on securities of 1-year or less contain virtually no information about inflation changes. Similar information-content patterns are found for several other countries, e.g., Jorion and Mishkin (1991) and Mishkin (1991) for selected



OECD countries, Robertson (1992) for the United Kingdom, Lowe (1992) for Australia and Day and Lange (1997) for Canada.

All the above studies concentrate on linear models, with the result that the impact on inflation of incremental changes in the yield spread is similar regardless of the level of the yield spread. The assumption of linearity, however, may not be justified if it is believed that the effects of monetary policy on the economy are asymmetric. Asymmetries are most likely to arise if the economy is less responsive to a positive policy stimulus than to a negative stimulus. As Friedman (1968) notes, when interest rates rise, the costs of financing a purchase or investment project also rise, which may induce agents to delay their purchases of large items or shelve to indefinitely an investment project. Conversely, when interest rates fall, there is no immediate incentive to either increase consumption or invest in a new project. If the real side of the economy responds in an asymmetric fashion to policy actions, then through the Phillips curve we may expect inflation to respond asymmetrically as well. Cover (1992), Morgan (1993), and Rhee and Rich (1995), for example, all find evidence in favour of asymmetric effects of monetary policy on real output, which justifies our line of research. Apart from Tkacz (1999a), who examines whether the relationship between inflation changes and yield spreads can be improved using a two-regime threshold model, few studies have pursued the issue of non-linearities between these specific variables. The present study extends Tkacz (1999a) by examining more general non-linear models in order to obtain more accurate estimates of the underlying non-linearities. We consider two types of non-linear models in order to verify the robustness of our results.

This paper is structured in the following manner. In the next section, we discuss the basic linear model developed by Mishkin (1990a) that is derived from the Fisher equation. In Section 3, we estimate non-parametric and neural network models and plot the fitted curves. Inference to statistically determine whether the non-linear models are preferred by the data is conducted in Section 4, while Section 5 concludes and suggests avenues for future research.

## **2. Theory**

The basic model developed in Mishkin (1990a) that links interest rate yield spreads to inflation changes is derived from the Fisher equation, which simply states that any  $m$ -period nominal interest rate can be specified as the sum of the expected  $m$ -period real interest rate and the expected  $m$ -period inflation rate. Given two Fisher equations, one

each for  $m$ - and  $n$ -period nominal interest rates where  $m > n$ , the final form of Mishkin's model is

$$p_t^m - p_t^n = a_{m,n} + b_{m,n}(R_t^m - R_t^n) + h_t^{m,n}, \quad (1)$$

where  $p_t^i = \log(P_{t+i}/P_t) \times 1200/i$  for  $i=m$  or  $n$ , and where  $m$  and  $n$  can take the values 3 (three months), 6 (six months), 12 (one year), 36 (three years), 60 (five years), or 120 (ten years).  $R_t^m$  and  $R_t^n$  are the  $m$ - and  $n$ -period interest rates on government securities. Since  $m > n$ , the variable  $(R_t^m - R_t^n)$  represents the yield spread between a longer-term and shorter-term government security. Positive values therefore represent upward-sloping yield curves, and negative values correspond to negative-sloping yield curves.

As previously mentioned, monetary policy is conducted through the control of short-term interest rates, and the effects of policy actions on interest rates diminish as we move along the maturity spectrum. Long-term interest rates largely reflect market beliefs of future inflation, and the monetary authority can affect such rates only as far as it can affect inflation expectations.<sup>1</sup> As such, the term  $(R_t^m - R_t^n)$  can act as a sensible proxy for the stance of monetary policy if  $m$  is large and  $n$  is small, since only one of the two rates,  $R_t^n$ , will move immediately in response to a policy action. For example, for  $m=120$  and  $n=3$ , a positive stimulus on behalf of the monetary authority will cause the short-term rate to fall immediately. The long-term rate, on the other hand, will remain relatively unchanged in the short-run, with the result that the yield curve will steepen and  $(R_t^{120} - R_t^3)$  will increase. A negative stimulus would have the opposite effect, with the yield curve inverting if the increase in the short-term rate is pronounced enough to cause  $R_t^3$  to exceed  $R_t^{120}$ . Interpretations of this sort on the potential economic meaning of the yield spread have been forwarded by, for example, Laurent (1988), Bernanke and Blinder (1992), and Rudebusch (1995).

There is no consensus as to what horizon constitutes the dividing line that distinguishes long-term from short-term interest rates, which is required if we wish to attach policy interpretations to some of our yield spreads. Recognizing that 3- and 6-month rates almost certainly fall within the short-term and that 5- and 10-year rates can almost surely be considered long-term rates, for convenience, and based on some shared

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<sup>1</sup> See Mehra (1996) for some empirical evidence that supports this claim.

time series properties that are presented in Tkacz (1999b), we assume that the 1-year rate is a short-term rate and that the 3-year rate is a long-term rate. As such, any yield spread for which  $m, n$  is the difference between a long and short rate (i.e.,  $m=36, 60, 120$  and  $n=3, 6, 12$ ) will be considered a policy-relevant horizon, that is, a horizon where the monetary authority has some influence over the yield spread ( $R_t^m - R_t^n$ ).

In cases where we consider the difference between either two short rates (both  $m$  and  $n$  are small) or two long rates (both  $m$  and  $n$  are large), we cannot attach any useful policy interpretation to the yield spread. When a policy action is initiated, the short rates move in very similar magnitudes, implying that their spreads change little in response to a policy action and therefore cannot be considered a relevant policy variable. In the case where both  $m$  and  $n$  are large, a policy action is unlikely to be felt immediately in either rate. Therefore such a horizon is also of little use as a measure of the stance of policy. Nevertheless, we document the results for such horizons since they can be useful as distant early warnings of future inflation.

Of course, regardless of the values of  $m$  and  $n$ , it is always possible that movements in yield spreads may be prompted by non-policy factors. Because of such instances, it is difficult to attribute all movements in the yield spreads to policy actions. However, it is well-known that short-term rates are more volatile than long-term rates, and as such, are responsible for most of the movements in the yield spread. This is one reason why Bernanke and Blinder (1992) find that either changes in short-term rates or a long-short yield spread can act as good measures of policy. In spite of the ongoing debate on the interpretation of the yield spread, it is still useful to pursue this line of research and to document the possible non-linearities between this variable and inflation changes if only to improve our understanding of the underlying relationship.

### **3. Estimation**

#### **3.1 Data and linear models**

The data for this study are obtained from the Federal Reserve Bank of St. Louis FRED database, with samples consisting of monthly observations beginning as far back as January 1953 and extending through October 1996.<sup>2</sup> Interest rates consist of 3 and 6-month Treasury bills, as well as 1-, 3-, 5-, and 10-year government bond rates. The

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<sup>2</sup> Six-month T-bills were first issued in 1958; therefore any model for which  $m$  or  $n$  equals 6 is constrained to begin in 1958.

dependent variable consists of changes in CPI inflation. All the yield spreads and inflation change series are either I(0) or long-run stationary, with the time series properties reported in Tkacz (1999b).

Using this data set, we obtain estimates for  $\alpha_{m,n}$  and  $\beta_{m,n}$  from (1) that are very similar to the estimates in Mishkin (1990a, 1990b). In particular, for policy-relevant horizons (high  $m$  and low  $n$ , Figures 1 to 3), the slope is positive and statistically significant and usually lies between 1.0 and 1.5. This implies that a 100-basis-point widening of the spread between long and short interest rates results in an acceleration of price changes between periods  $m$  and  $n$  of between 1.0 and 1.5 per cent. Such changes could be initiated, for example, through monetary easing with the Federal Reserve lowering the short-term rate by 100 basis points. For the non-policy horizons where both  $m$  and  $n$  are low (Figure 4), the slopes are statistically insignificant, as was found by Mishkin (1990a). For the non-policy horizons with  $m$  and  $n$  large (Figure 5), we find that the slope is actually negative at the longest horizons, consistent with previous studies.<sup>3</sup>

### 3.2 *Non-parametric models*

Apart from specifying the variables that enter the relationship, the non-parametric models we consider impose no functional form on the data. The models are specified as

$$p_t^m - p_t^n = f(R_t^m - R_t^n) + \eta_t \quad (2)$$

where  $f(\cdot)$  is some unspecified function and  $\eta_t$  is a random disturbance. There are numerous methods to estimate (2). We choose to use the popular kernel method, using a Gaussian kernel. Each fitted value is found from

$$\hat{m}_h(R_t^m - R_t^n) = \frac{\sum_{i=1}^T K_h((R_t^m - R_t^n) - (R_{ii}^m - R_{ii}^n)) \times (p_{ii}^m - p_{ii}^n)}{\sum_{i=1}^T K_h((R_t^m - R_t^n) - (R_{ii}^m - R_{ii}^n))} \quad (3)$$

where the function  $K_h$  is the kernel with window width  $h$  (e.g., see Härdle [1990]). Naturally, the goodness-of-fit of the regression curve depends to a crucial degree on  $h$ ,

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<sup>3</sup> See Tkacz (1999a) for complete linear regression results.

with lower values of  $h$  fitting the data more closely. In our work, we set  $h=0.30$  for all the cases, which allows for easier comparison across models. We also select  $h$  by minimizing a cross-validation function, but only plot the curves for which  $h=0.30$ . Higher values for  $h$  produce smoother curves, whereas lower values will produce more jagged curves.

The estimated non-parametric regression curves are also shown in Figures 1 to 5. For the policy horizons (Figures 1 to 3), we find a consistent pattern in the curves, namely a steep portion for negative values of the yield spread and a flat portion for positive values of the spread. This can be interpreted as indicating that the relationship between yield spreads and inflation changes is far greater when the yield curve inverts, and that there is a weaker link between these variables for upward-sloping yield curves. If the yield curve inversions can be attributed to a tightening of monetary policy, then such observed non-linearities can be said to arise from monetary policy's asymmetric effect on inflation changes.

We note that, for the three short-term non-policy cases ( $m=6, n=3$ ;  $m=12, n=3$ ; and  $m=12, n=6$  in Figure 4), the curves are very flat, implying that inflation changes do not depend greatly on interest rate spreads at these horizons. This agrees with our linear models, namely that the yield spread is insignificant at explaining inflation changes at the short end of the yield curve. For policy purposes, the Fed should therefore not expect to extract noticeable information about future inflation using information contained solely within short-term interest rates.

However, since Fed actions are believed to take at least 12 months to impact the economy, the lack of any relationship at the shorter end of the term structure may simply be due to the ineffectiveness of Fed monetary actions in the short run. Thus, this is entirely consistent with the view that only interest rate spreads between longer and short rates represent good indicators of monetary policy.

Finally, it is worth noting the curves depicted for the cases  $m=60, n=36$ ;  $m=120, n=36$ ; and  $m=120, n=60$  (Figure 5). Here we find the curves to be either flat or sloping downwards. As we stated, the Fed has influence over these rates only so far as it can affect inflation expectations. The results show that when, say, the spread between the 5- and 10-year rates widen, inflation changes between these two horizons actually falls. This corroborates the negative slopes observed in the linear models.

### 3.3 Neural network models

With recent advances in computing technology, computationally intensive methods such as neural networks have begun to gain prominence as a method to capture non-linearities in complex data sets. It allows one to model relationships between one or several input (independent) variables and one or several output (dependent) variables.

The relationship between inputs and outputs, however, need not be direct. In the relationship between interest rates and inflation, for example, one can argue that there are most likely several intermediate variables in the transmission from interest rates to inflation changes. Interest rate changes can first affect durable consumption and investment, output, the output gap, and ultimately inflation. In neural network models, these intermediate stages can be captured by a “black box” in which weights between input variables and (unknown or unobserved) intermediate variables are computed, as are weights between the intermediate variables and the output variable. The unknown intermediate variables are commonly referred to as hidden units.

The number of intermediate variables and, indeed, the number of stages or layers, are not necessarily known. Therefore some experimentation is required on the part of the modeller in order to capture the salient features of the data adequately. The neural network model that we estimate has the form

$$(\rho_t^m - \rho_t^n) = \sum_{k=1}^K a_k g(b_{k1}(R_t^m - R_t^n)) \quad (4)$$

where  $g(u) = 1/(1 + e^{-u})$  is a logistic function that serves as a smooth squashing function (analogous to a smooth, as opposed to discrete, threshold model that allows for regime changes),  $b_{k1}$  is the connection strength (weight) between the yield spread and the hidden unit  $k$ ,  $K$  is the total number of hidden units ( $= 2$  in our work), and  $a_k$  is the weight of unit  $k$  on the output,  $(\rho_t^m - \rho_t^n)$ .

To estimate the parameters  $b_{k1}$  and  $a_k$ , one minimizes the sum of squared deviations between the output and the network:

$$SSD = \sum_{t=1}^T \left[ (\rho_t^m - \rho_t^n) - \sum_{k=1}^K a_k g(b_{k1}(R_t^m - R_t^n)) \right]^2 \quad (5)$$

Equation (5) is estimated using back propagation, which updates the parameter values until we achieve the pre-specified convergence level. In our applications, we specify a mean squared error that we wish to achieve. Through our experimentation, we found that convergence after some tens of thousands of epochs yielded sensible curves that balanced our desire to capture the underlying non-linearities in the data without overfitting, and therefore chose the convergence levels with this objective in mind.

The neural network curves for the policy horizons are also plotted in Figures 1 to 3. Two features are immediately apparent for all nine of these curves. First, they all closely resemble the non-parametric curves (although the neural network curves seem less influenced by the end-points, where data is scarcer). This is not entirely surprising since neural networks fall within a broader class of non-parametric models themselves. Second, they all have similar shapes, quite steep for values of the spread between -1.5 and 0.0, and flat for positive values of the spread. Again, if we assume the Fed can influence these spreads through its actions, then when the spread is positive and the Fed tightens (causing the long-short yield spread to fall), the effect on inflation changes is minimal. However, should it tighten when the spread is already negative (usually associated with a policy regime that is already considered tight), then the effect on inflation changes will be pronounced as we move down the steep portion of the fitted curve. Such information can be vital to policy-makers who view the yield curve as a good indicator of future inflation, as these findings show that most of the positive correlation between the yield spread and inflation changes are due to a strong relationship that prevails in a relatively narrow range of the yield spread.

In Figure 4, neural network curves are fitted for short-run non-policy horizons. They coincide with the other models, and we find the neural net curves to be very flat, indicating that there does not appear to be any discernible relationship between yield spreads and inflation changes at these horizons. By contrast, in the long-run non-policy horizons (Figure 5), we find some relationships in the data. Confirming what we had found earlier using non-parametric models, the relationship between yield spreads at the long-end of the yield curve and inflation changes appears to be highly non-linear. For  $m=120$  and  $n=36$  we find the fitted curve to be bell-shaped, indicating that some quadratic function might be appropriate in modeling the relationship.

#### 4. Inference

Although the fitted non-parametric and neural network curves appear to be non-linear for several horizons, it is useful at this point to determine whether the non-linear specifications

are statistically preferred by the data. To this end, we conduct two different model specification tests; both stipulate the linear model (1) under the null, and either a non-parametric or neural network model under the alternative.

Wooldridge (1992) shows that a Davidson and MacKinnon (1981) J-type test used for testing non-nested hypotheses is asymptotically valid when used to test the null of a linear model versus a non-parametric alternative. With the null hypothesis given by (1), the alternative is given by (2). To implement the test we need to run the following artificial regression:

$$(p_t^m - p_t^n) - (\hat{p}_t^m - \hat{p}_t^n) = a [\hat{m}_h(p_t^m - p_t^n) - (\hat{p}_t^m - \hat{p}_t^n)] + u_t \quad (6)$$

where  $(\hat{p}_t^m - \hat{p}_t^n)$  are the fitted values from (1), and  $\hat{m}_h$  denotes the fitted values of the non-parametric regression for bandwidth  $h$ . The test is simply a t-test on the parameter  $a$ ; if it is significant, then we reject the linear null in favour of the non-parametric alternative.

In Table 1, we present the results of the Wooldridge test for both an optimal bandwidth chosen through cross-validation,  $h=h^*$ , and for a fixed bandwidth  $h=0.30$ , to verify whether the results are robust to the selected bandwidth. The bandwidth of 0.30 is a rough average of the computed optimal bandwidths. For the policy horizons, we reject the linearity null for all cases when  $h=0.30$ , but are unable to do so at the 5 per cent level for policy horizons where  $h=h^*$  and  $m=120$ . For the short non-policy horizons (rows 10 through 12), we are unable to reject the null regardless of the selected bandwidth. Finally, for the long non-policy horizons (rows 13 through 15), both tests favour linearity for  $(m=60, n=36)$ , the non-parametric alternative for  $(m=120, n=36)$ , but are divided for the case of  $(m=120, n=60)$ . Based on this exercise, we can conclude that there is substantial evidence that non-parametric models are of use in modelling the relationship between inflation changes and yield spread for all but the shortest horizons.

White (1989) proposes a test that can be used as a model diagnostic with a neural network model under the alternative hypothesis. The regression used in the test is

$$p_t^m - p_t^n = a_{m,n} + b_{m,n}(R_t^m - R_t^n) + \sum_{j=1}^q d_j y (g'_j(R_t^m - R_t^n)_j) + h_t^{m,n} \quad (7)$$



where  $y(g'_j(R_t^m - R_t^n)) = [1 + \exp(-g'_j(R_t^m - R_t^n))]^{-1}$  is the logistic function as in (4). The test involves testing the significance of the parameters on the non-linear terms,  $\delta_j$ , and an LM statistic is used for this purpose, which follows a  $\chi^2(q)$  distribution.

The results are also presented in Table 1. Since the test statistic is based on random draws from a uniform distribution, we perform it three times in order to verify the robustness of our results. However, we report only the first test since the test statistics do not vary much. For the policy horizons (rows 1 through 9), we find that the linearity null is rejected for all but one case. More specifically, we notice that for policy horizons with  $m=120$ , the p-values tend to rise noticeably, the same phenomenon witnessed for the Wooldridge test with  $h=h^*$ . For the non-policy horizons, the same pattern emerges as with the previous tests, with the exception of the case ( $m=12, n=6$ ) where we detect evidence in favour of the non-linear alternative.

## 5. Conclusion

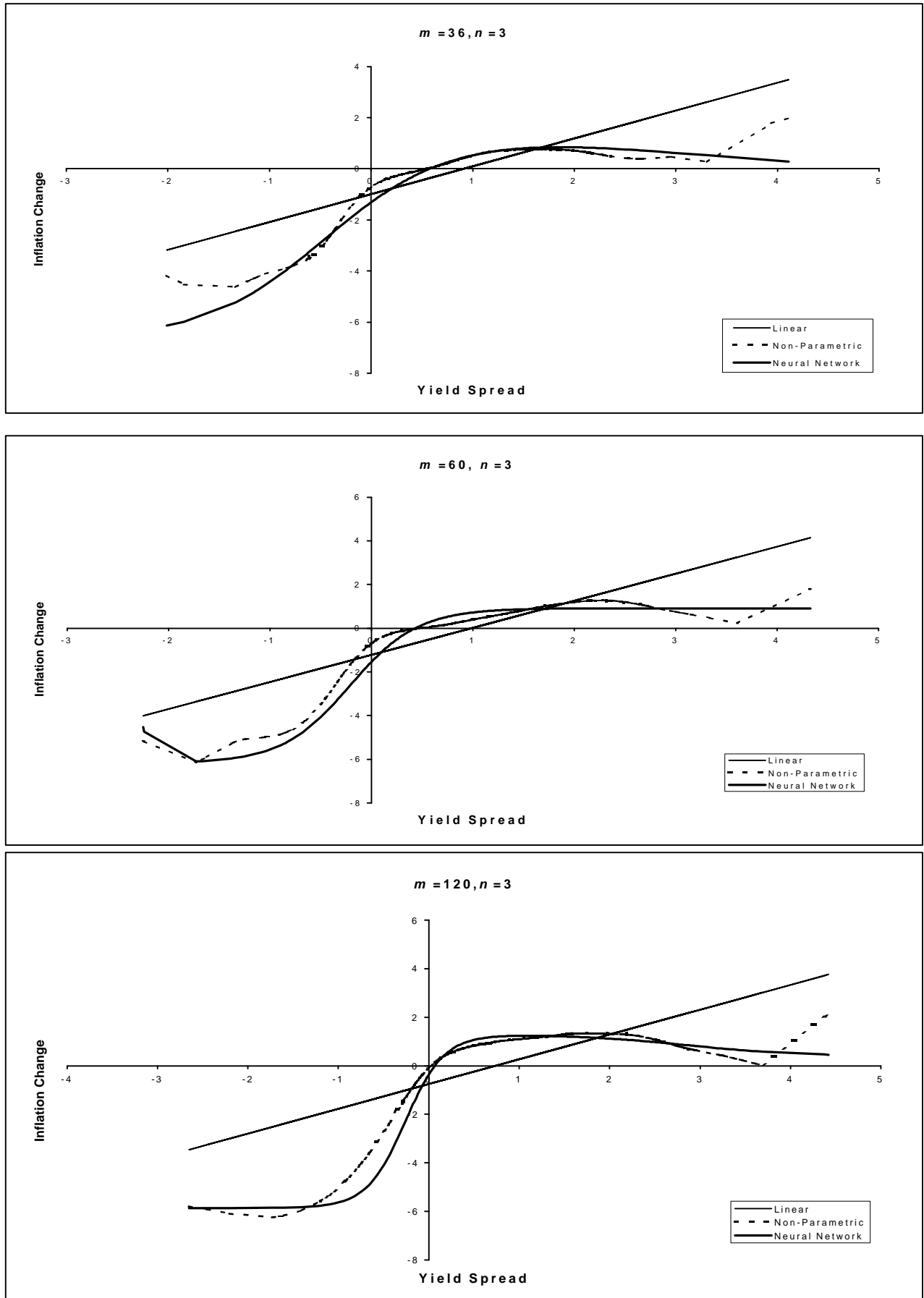
The objective of this paper is to determine whether the relationship between interest rate yield spreads and changes in the inflation rate can be better captured using non-linear models. Our results suggest that, for medium and long horizons, the non-linear models prevail, indicating that negative yield spreads have a marginally higher impact on inflation than positive yield spreads. From a policy-maker's perspective, this implies that when policy is already tight, a further tightening would result in a marginally greater reduction in inflation. A similar tightening would have a lower impact, if any, on inflation were it implemented during an expansionary policy regime. At the shorter end of the yield curve, however, where policy has little or no impact on yield spreads, there does not appear to be any relationship, linear or otherwise, between yield spreads and inflation changes.

If inflation is affected by monetary policy through a Phillips-curve-type relationship, then our results would appear to be consistent with the claim that the effects of monetary policy on the inflation rate are non-linear. Furthermore, the form of the non-linearities observed here are consistent with the direction of asymmetry between monetary policy actions and output growth uncovered by Morgan (1993) and others, namely that tight policy is more potent than expansionary policy. As the empirical evidence in favour of non-linearities accumulates, policy-makers should be aware of the potentially asymmetric effects of their policy actions.

**Table 1: Non-linearity tests**

$m,n$ (months)	Non-parametric			Neural Network
	Estimated Bandwidth ( $h^*$ )	Wooldridge t-stat ( $h=h^*$ )	Wooldridge t-stat ( $h=0.30$ )	Lee, White & Granger F(2, $T-4$ )
36,3	0.13	-4.008 (0.000)	6.225 (0.000)	7.547 (0.000)
60,3	0.46	-4.049 (0.000)	7.241 (0.000)	14.36 (0.000)
120,3	0.72	-1.456 (0.146)	10.31 (0.000)	1.535 (0.217)
36,6	0.16	-2.555 (0.011)	6.851 (0.000)	7.115 (0.001)
60,6	0.21	-2.848 (0.005)	8.124 (0.000)	19.85 (0.000)
120,6	0.12	-1.953 (0.052)	10.66 (0.000)	3.248 (0.040)
36,12	0.02	-3.559 (0.000)	4.554 (0.000)	15.48 (0.000)
60,12	0.24	-4.147 (0.000)	6.003 (0.000)	15.70 (0.000)
120,12	0.40	-0.748 (0.455)	9.352 (0.000)	4.112 (0.017)
6,3	0.51	-0.167 (0.868)	-0.403 (0.687)	0.071 (0.931)
12,3	3.01	0.029 (0.977)	0.979 (0.328)	2.536 (0.080)
12,6	0.28	-0.116 (0.907)	0.339 (0.735)	5.641 (0.004)
60,36	0.22	-1.670 (0.096)	-0.114 (0.910)	1.750 (0.175)
120,36	0.28	3.759 (0.000)	3.546 (0.000)	7.127 (0.001)
120,60	0.04	7.067 (0.000)	1.254 (0.210)	13.73 (0.000)

The null hypothesis for each test is linearity, with p-values in parentheses. We present two different sets of Wooldridge (1992) non-parametric tests: The first is for a bandwidth  $h^*$ , which was chosen through cross-validation, and the second for a constant bandwidth  $h=0.30$ .

**Figure 1: Fitted curves, policy horizons (short rate = 3 months)**

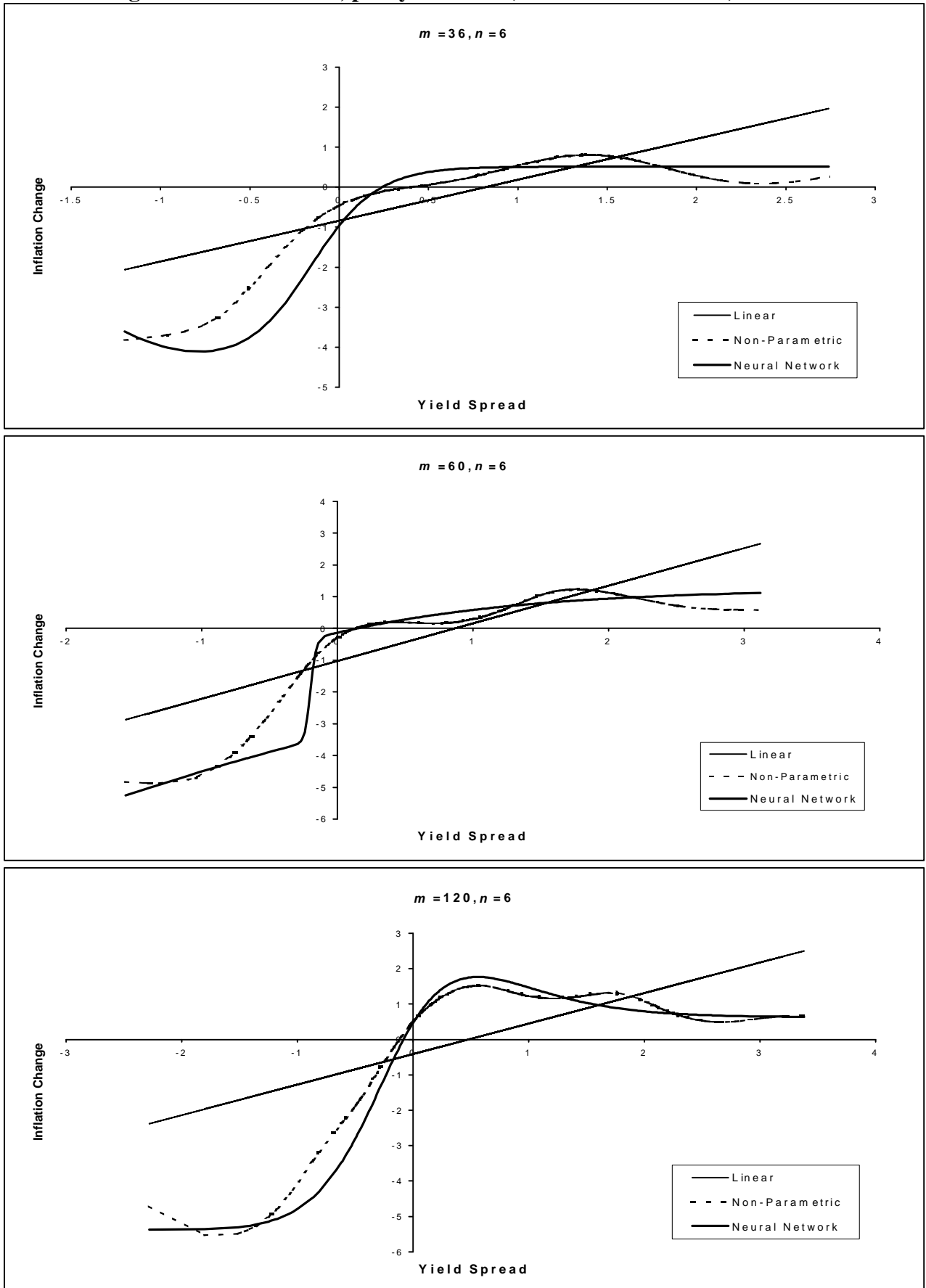
**Figure 2: Fitted curves, policy horizons (short rate = 6 months)**

Figure 3: Fitted curves, policy horizons (short rate = 12 months)

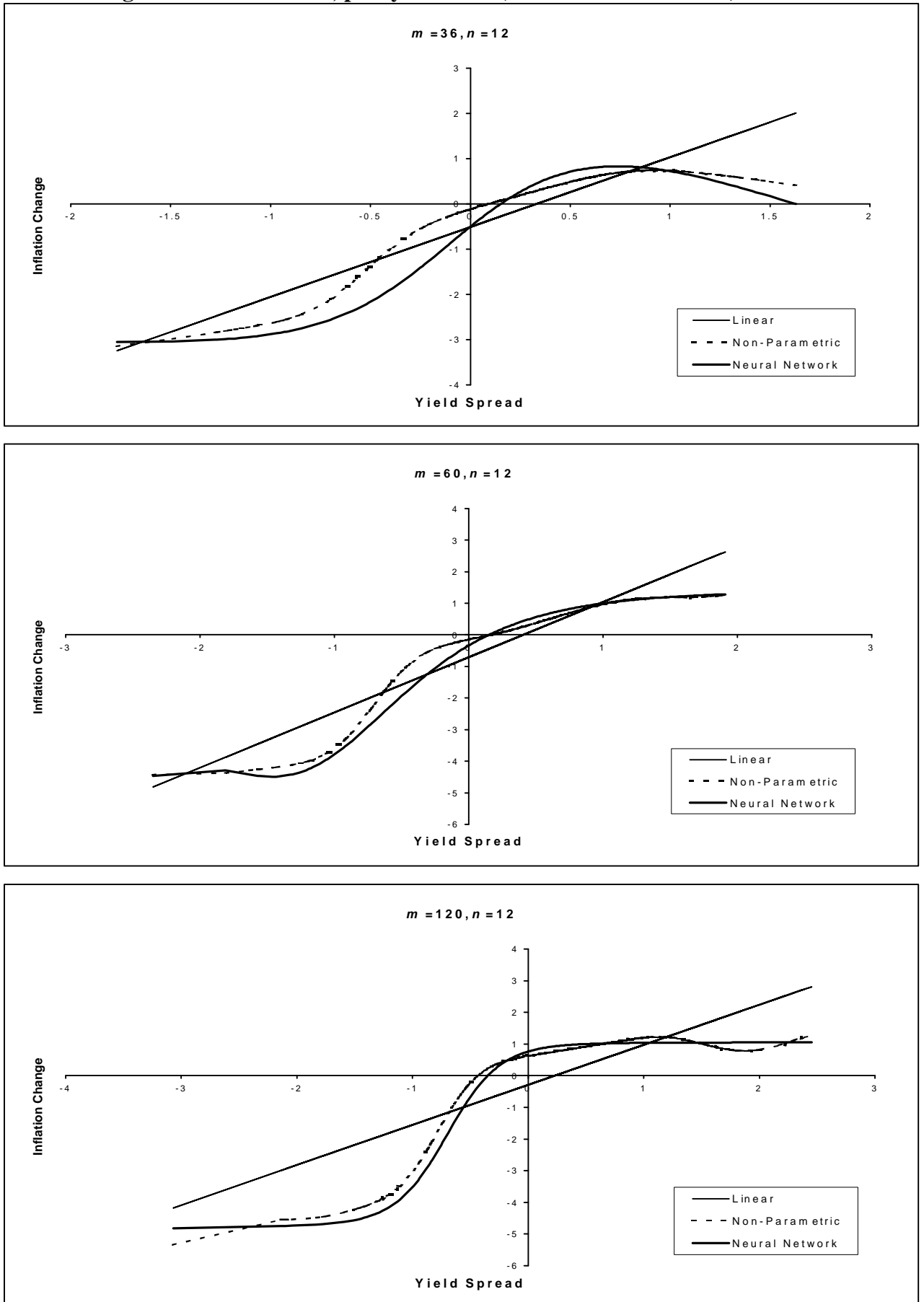


Figure 4: Fitted curves, non-policy horizons (short-end of yield curve)

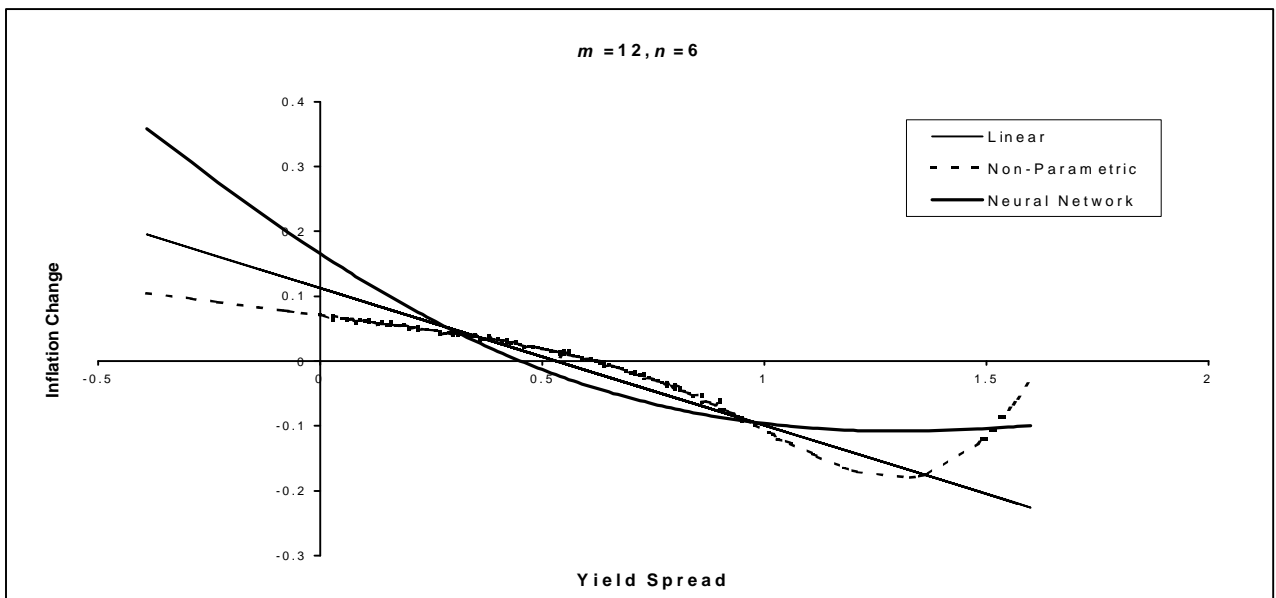
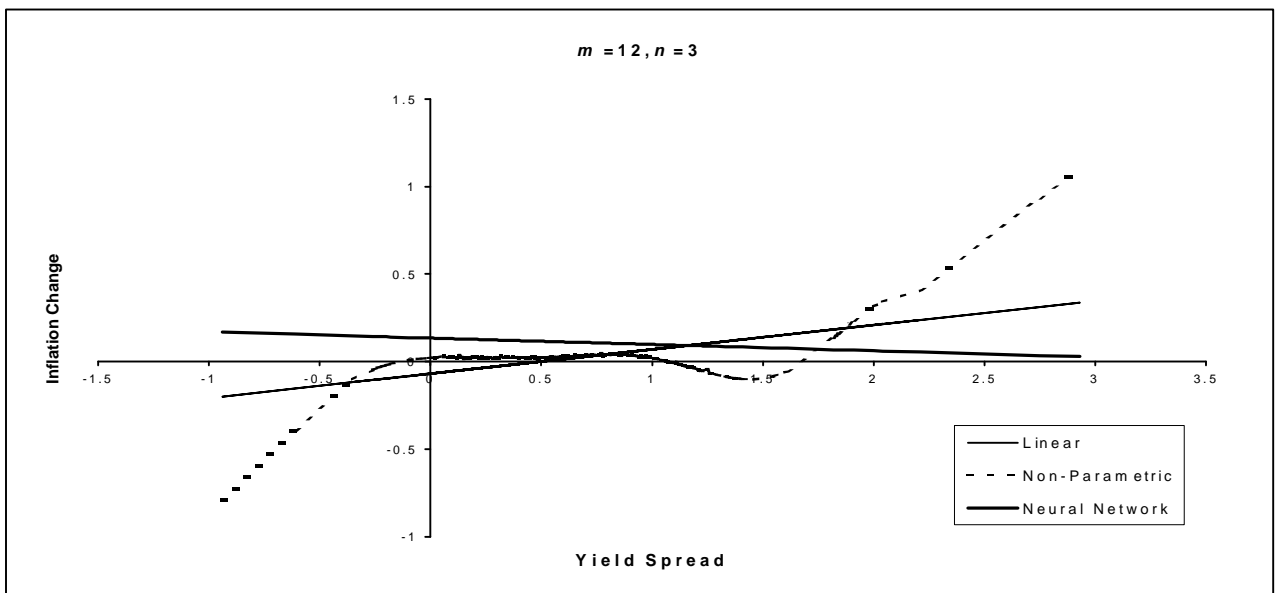
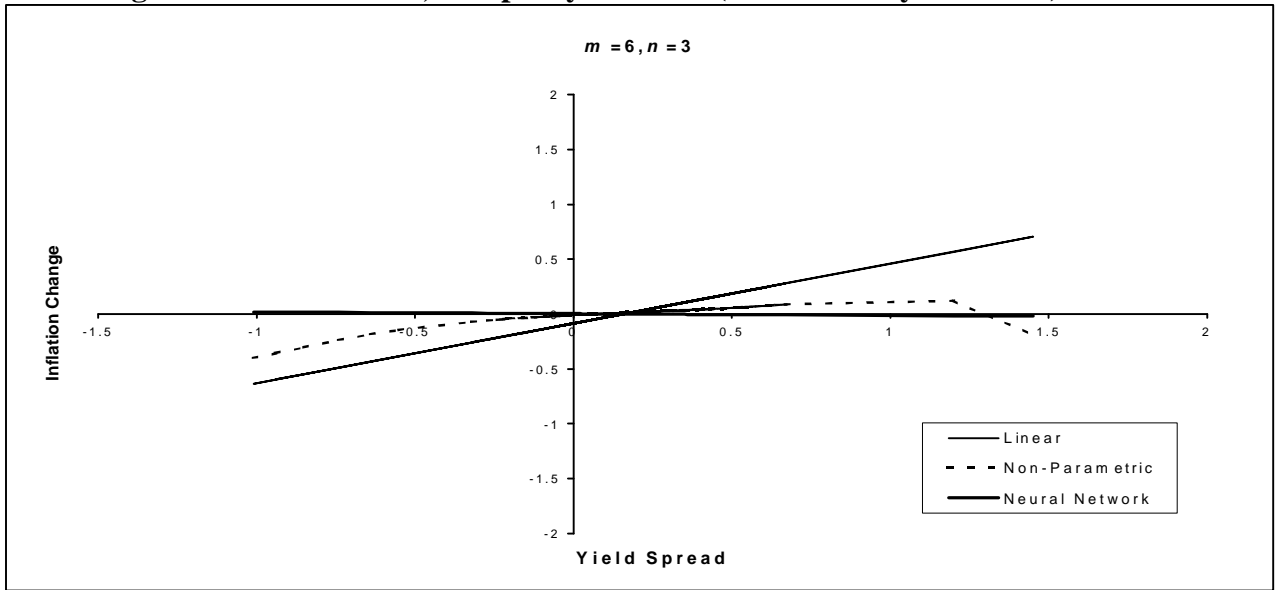
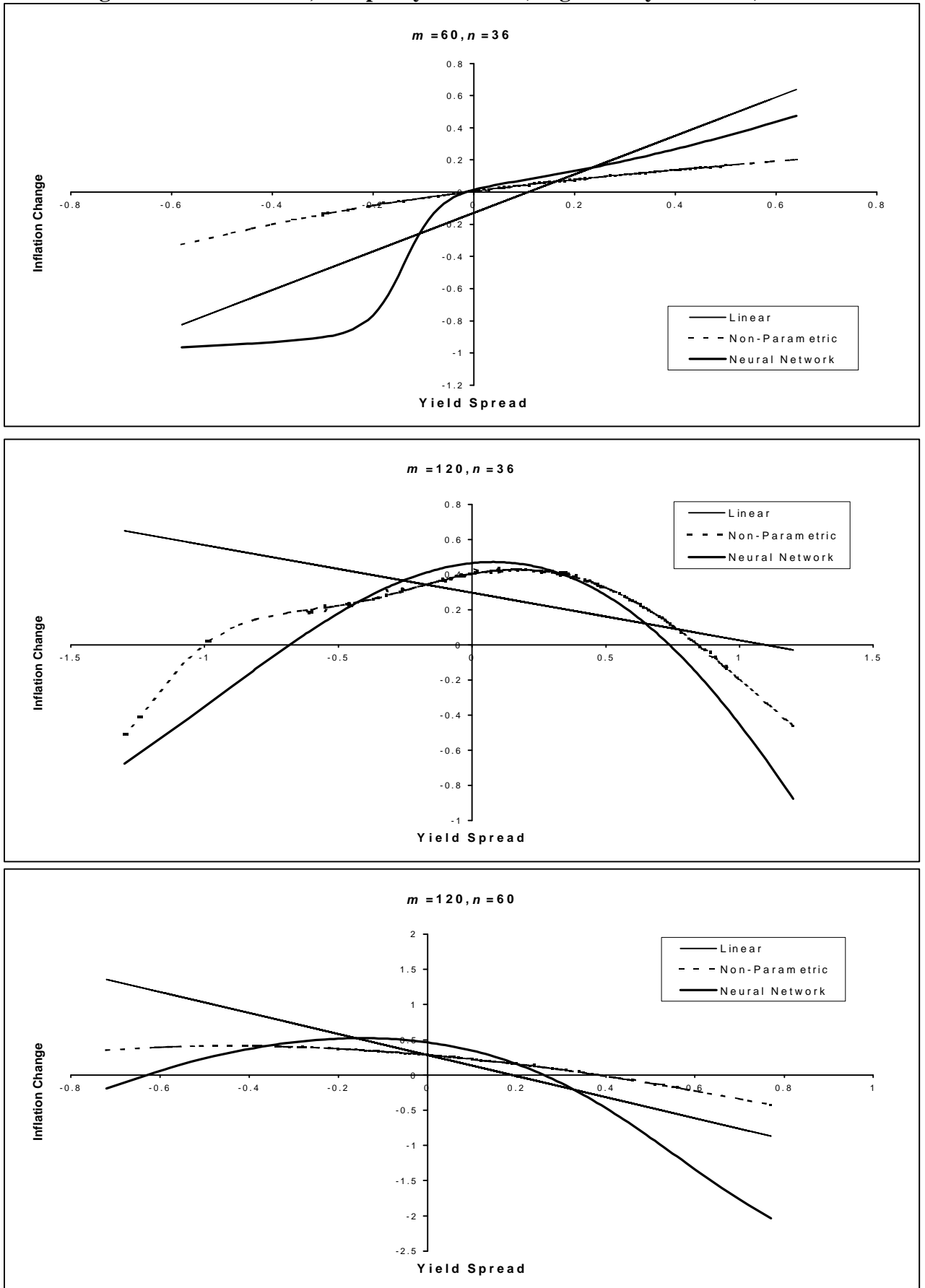


Figure 5: Fitted curves, non-policy horizons (long-end of yield curve)



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