Growth or Profitability First? The Case of Small and Medium-Sized Enterprises in Canada

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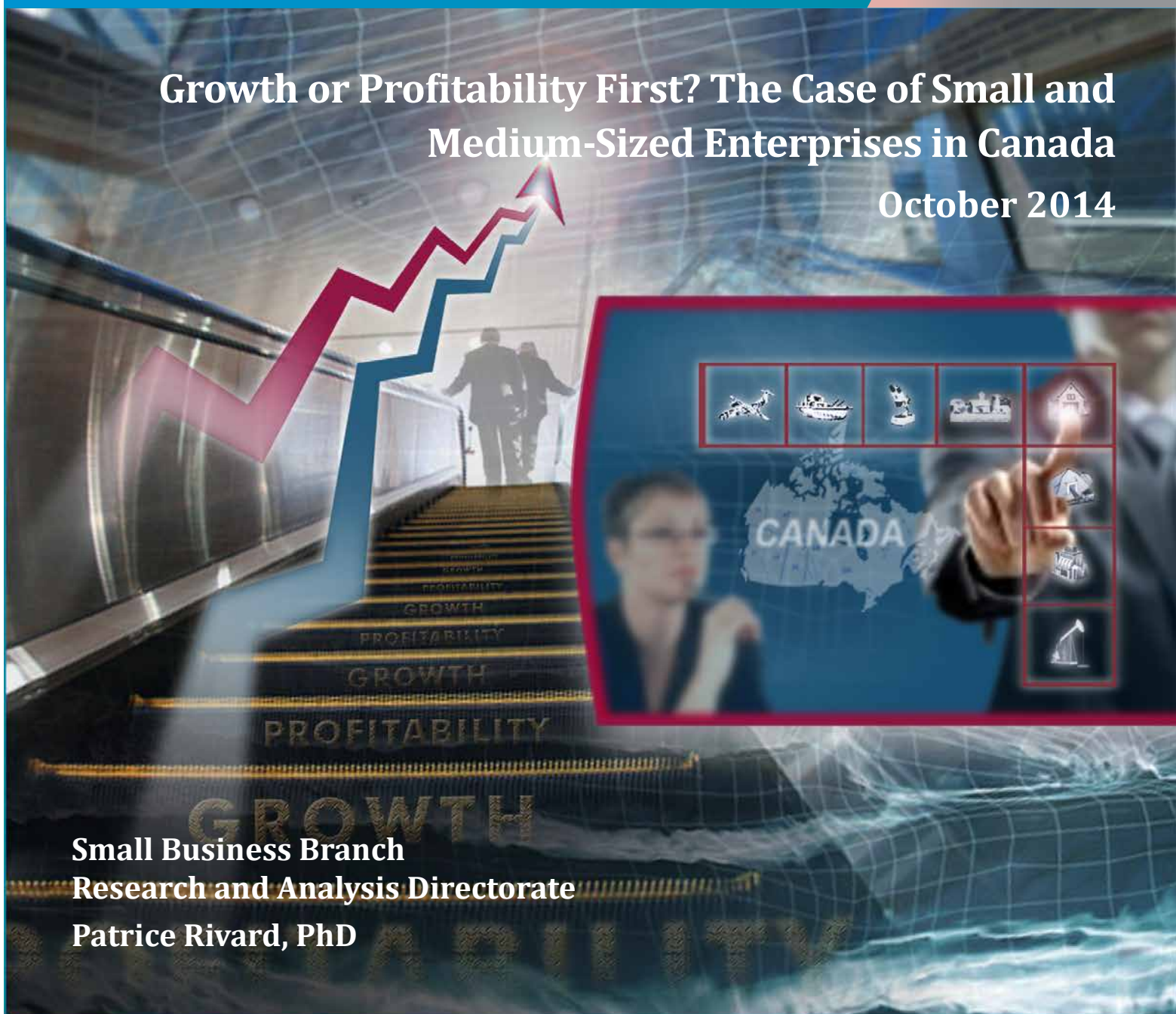
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Abstract

Based on a sample of small and medium-sized enterprises in Canada, we examine the relationship between a firm's growth and profitability for the period from 2006 to 2011. Using a dynamic probit model with random effects, we show that a firm with a high level of profitability and a low level of growth has a greater chance of subsequently achieving high growth and high profitability than a firm with a high level of growth and a low level of profitability. In addition, this study shows that human capital is a determining factor as it plays a positive role in a firm achieving superior performance in both growth and profitability. A firm's debt is also a significant factor that can slow progress. Finally, the results of model estimations show that a firm's age has no effect on the evolution of its situation in terms of growth and profitability.

1.   Introduction

Growth is a topic that is increasingly the focus of government concern. However, the prerequisites for sustainable growth are still poorly understood, and particularly the relationship between growth and profitability. Governments often concentrate on financing or barriers to entry, but there is recognition that a firm's growth strategies are just as important. Given the conclusions of our research, creating the conditions for profitability appears essential to sustainable growth.

According to the empirical findings of Coad (2007), there is little research on the relationship between growth and profitability. This relationship is rather complex and researchers disagree on its nature. In fact, certain studies show that the two are unrelated, while others show a negative or positive relationship.[1] For example, Penrose (2009) suggests that the relationship between growth and profitability may be negative. This assertion refers to the fact that a growing firm may reach a point where it becomes ineffective, subjected to ever higher administrative costs that eat away profits.

More recently, Davidsson et al. (2009) studied the nature of the relationship between growth and profitability by establishing how firms fit into categories based on these two variables and by examining the transition of firms from one category to another over time. This method, called *transitional analysis*, shed new light on the subject. The authors established that highly profitable firms with low growth are most likely to achieve both high growth and high profitability, the category of the most successful firms. In addition, these firms are also less likely to become less profitable and to see their growth decline, the category of the least successful firms. Brännback et al. (2009), building on the work of Davidsson et al. (2009), arrived essentially at the same results. They concluded, in particular, that prior growth is a poor parameter for determining a firm's future performance. The results and conclusions of Davidsson et al. (2009) are also supported by the work of Jang (2011). The work of Davidsson et al. (2009) is essentially limited to a descriptive study of a firm's transition every year, and their analysis does not explicitly identify other potential causes with a significant influence on a firm's situation.[2]

The general purpose of this study, therefore, is to improve our empirical understanding of the applicable transitions in existing relationships between growth and profitability for small and medium-sized enterprises (SMEs) in Canada. To do so, we propose a twofold process.

1)   We use the transitional analysis methodology of Davidsson et al. (2009) to compare our respective data banks.

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1. See Table 10 in Appendix A, which provides an overview of research on the relationship between growth and profitability.

2. The authors declare that they only conducted additional analyses using a multiple logistic model and conclude that the model's (unspecified) control variables are not significant. However, their conclusions are supported by their model.

2) We take the analysis further by using a dynamic probit model with random effects. In this econometric model, the independent and control variables are integrated and allow us to determine their influence on a firm's probability of being in one category or another.

Use of the latter model also allows for calculating a firm's probability of being in the most successful or least successful category based on its previous situation. This is an interesting aspect that is not addressed in the work of Davidsson et al. (2009).

We begin this study by defining the terms *growth* and *profitability*. We then present the measures that are commonly used to determine growth and profitability and that serve as indicators of the relationship between these two variables. Next, we describe the data underlying this work, as well as the methodology we use, which is based on that of Davidsson et al. (2009). We explain the primary results and their consequences for Canadian SMEs. Finally, we conclude this work with a discussion on future research that might be undertaken in the area of growth and profitability.

2.   Definitions and Measures

In the classic work by Penrose (2009), *The Theory of the Growth of the Firm*, two meanings are generally attributed to the term *growth*.[3] On the one hand, growth is an increase in quantity, which can be applied, for example, in reference to growth in sales or exports. On the other hand, a second connotation refers to an increase in size or in quality and is seen as the result of a development process similar to a biological process, where a series of internal changes leads to an increase in the size and to a change in the characteristics of the growing object. For our own work, we consider the first definition of growth. The term *profitability* relates to a firm's ability to generate profits.

Growth of a business can be measured in various ways. Three measures are commonly used: *total sales*, *number of employees* and *total assets*. Studies on growth use one or another of these measures. These may be correlated, but are conceptually different. That is why it is sometimes difficult to compare them and to determine which is the most appropriate. However, Weinzimmer et al. (1998) present alternatives for measuring growth, as well as a few suggestions to help researchers choose the most suitable measure based on the data used. In their view, sales growth is an appropriate measure in many situations.[4]

A number of indicators can also be used to measure profitability. The profit margin ratio or the return on capital ratio (Lafrance, 2012) is generally used for this purpose. The first corresponds to the ratio between

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3. See also Davidsson et al. (2007).

4. An heuristic argument would be to say that sales growth often precedes other indicators: an increase in sales frequently requires more assets and more employees. More recently, Shepherd and Wiklund (2009) delved deeply into the relationships between the various measures cited. In particular, they show empirically cases where the measures are equivalent. It is also a good reference for researchers who wish to use an appropriate measure for growth in a specific context.

profits and total operating revenues (gross sales or gross revenues), whereas the second is calculated as being the profits on total capital or total assets. In this case, we refer to *return on assets* or *return on investment*.[5] For the purposes of this study, we have chosen the profit margin ratio.

3.   Data and Methodology

In this section, we present relevant information on the data used in this study as well as on the methodology.

3.1   Data

The data used for this work are sourced from Statistics Canada's 2007 *Survey on Financing of Small and Medium Enterprises*.[6] The initial sample examined consists of 15,808 firms. In the present study, SMEs are defined as having from 1 to 499 employees.[7] Moreover, financial information on participating SMEs, provided by the Canada Revenue Agency (CRA), was matched with the Statistics Canada data for every year from 2002 to 2011.

This information has the advantage of being highly reliable and accurate given its official nature. As such, we created a longitudinal data set (panel data) based on data from Statistics Canada's survey and from the CRA. In addition, the sample is *balanced*, that is, all of the data for each firm are known for every variable and for each year. When this is not the case, the sample is said to be *unbalanced*.[8]

To optimize the number of firms in our sample, we limited our study to the years 2006 to 2011 as certain financial information was missing for several firms between 2002 and 2005. The results of this study, therefore, must be interpreted based on this sample. Finally, we processed the data to eliminate extreme values as well as observations where total sales, total assets or the number of employees were nil.

3.2   Methodology

This study involves two steps.

1) The first step consists of classifying the SMEs into five categories based on characteristics related to growth and profitability. Then, a study on the SMEs' transition over the years will be conducted to determine the proportion of firms changing from one category to another.

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5. Note that Schmalensee (1989) (Table 1, p. 340) uses 12 different indicators for profitability. Profits can also be calculated before or after tax in all cases. According to Hall and Weiss (1967), it is better to calculate profits after tax as taxes vary widely across industry sectors. The same argument can be made when considering Canadian provinces and territories individually as each has its own taxation system.

6. Statistics Canada chose 35,055 SMEs from the Business Register. Of these, 18,532 were contacted and 15,808 agreed to fill out the questionnaire.

7. In Statistics Canada's survey, SMEs are defined as businesses with fewer than 500 full-time employees and gross revenue of less than $50 million.

8. As the results of unbalanced samples are similar to those of balanced samples, they are not presented in this study.

2) For the second step, we use the unordered and ordered dynamic probit models with random effects for panel data to estimate a firm's probability of being in a category based on certain control variables. We compare the various results in this case and determine whether giving an order to the various potential situations for the firms every year has a notable effect on a firm's probability of being in one category or another.

3.2.1 Classification

As the general purpose of this study is to shed light on the relationship between growth and profitability for small and medium-sized enterprises in Canada, we first present the various measures of growth and profitability used in our work.

For the purposes of this study, three growth indicators are considered: total sales, number of employees and total assets. We use these measures to test whether or not similar results are obtained. If C, one of these three measures, is considered, growth is determined by the following equation:

$$\frac{C_t - C_{t-1}}{C_{t-1}} \times 100$$

As we must calculate relative growth rates, the first year cannot be considered in the analysis. As we are using only observations from 2006 to 2011, however, we can use 2006 to calculate a firm's rate of growth.

To measure profitability, we use the return on assets of Davidsson et al. (2009), which is defined as follows:

$$\frac{\text{Net income after tax}}{\text{Total assets}}$$

Using the definitions of growth and profitability, SMEs can be broken down into five categories:

1) *Mediocre*: low profitability and low growth (below the median for both variables and in the lowest quartile for at least one of the two);

2) *Average*: average performance (in the second or third quartile for profitability and growth);

3) *Growth*: low profitability and high growth (below the median for profitability and above for growth, but without qualifying for the *Average* category);

4) *Profit*: high profitability and low growth (above the median for profitability and below for growth, but without qualifying for the *Average* category); and

5) *Star*: high profitability and high growth (above the median for both variables and in the highest quartile for at least one of the two).

Table 1 shows this classification in detail, where (a, b) represents the quartile for profitability (a) and growth (b).

Table 1: Classification of SMEs based on growth and profitability

		Quartile for Growth			
		1	2	3	4
Quartile for profitability	1	(1, 1) *Mediocre*	(1, 2) *Mediocre*	(1, 3) *Growth*	(1, 4) *Growth*
	2	(2, 1) *Mediocre*	(2, 2) *Average*	(2, 3) *Average*	(2, 4) *Growth*
	3	(3, 1) *Profit*	(3, 2) *Average*	(3, 3) *Average*	(3, 4) *Star*
	4	(4, 1) *Profit*	(4, 2) *Profit*	(4, 3) *Star*	(4, 4) *Star*

The specific objectives of this study are to determine the category in which a Canadian SME must be at time $t-1$ to be in the *Star* category on the one hand and the *Mediocre* category on the other hand at time t. The *Star* category represents the most successful firms in terms of profitability and growth, whereas the *Mediocre* category represents the least successful firms. It is clear that our attention must focus on these two categories of firms. Based on the results of Davidsson et al. (2009), we also assert the two following hypotheses:

H_1: Firms with high profitability and low growth (those in the *Profit* category) at time $t-1$ are more likely to achieve high growth and high profitability (i.e., to be part of the *Star* category) at time t than firms with high growth and low profitability (those in the *Growth* category).

H_2: Firms with high growth and low profitability (those in the *Growth* category) at time $t-1$ are more likely to experience low growth and low profitability (i.e., to be part of the *Mediocre* category) at time t than firms with high profitability and low growth (those in the *Profit* category).

3.2.2 Transition matrices and Markov chains

The first method we employ to verify the validity of our two hypotheses (H_1 and H_2) is to consider the situation of the businesses every year and to track their evolution using the methodology of Davidsson et al. (2009). As mentioned earlier, SMEs were classified for the years 2006 to 2011 inclusively. As a result, we know whether each firm changed categories from year to year. This is what we call the *transition matrix*. We calculate the proportion of firms that change situations for every possible transition combination and every year from 2006 to 2011. In addition, we present the firms' transitions by aggregating the data.

Our first analysis of the behaviour of Canadian SMEs is very similar to the study of variables following a *discrete time stochastic process*. For every year examined, a firm's situation may be considered a variate, the value of which may have a finite number of possibilities corresponding to the five categories defined earlier. In addition, to analyze a firm's potential transitions over time, we find ourselves in the general context of *Markov chain theory*, more specifically, that of the order of one process.

Thus, the stochastic process related to a firm's situation over the years forms an order-one *Markov chain* if a firm's probability of being in a particular category depends only on the category to which it belonged over the previous period. This is a reasonable hypothesis as at time $t-1$ the category to which the firm belongs is determined by its growth and profitability, which may have an effect on the firm's situation at time t.

After calculating the proportion of firms in each category for transitions in the aggregate manner, we statistically test the difference between category proportions by using standard tests to verify the validity of hypotheses H_1 and H_2.[9]

3.2.3 Ordered and unordered dynamic probit models with random effects for panel data

The models we consider in this study are the *ordered dynamic probit model with random effects* and the *unordered dynamic probit model with random effects*. We refer the reader to Appendix B for the details of this model as well as our hypotheses. To conduct this study, we also based ourselves largely on the work of Contoyannis et al. (2004a) in the health field. We used a similar model, but adapted it to the context of Canadian SME performance defined on the classification method of Davidsson et al. (2009). The estimated models are based on the following equation:

$$s_{it}^* = \beta x_{it} + \gamma S_{it-1} + c_i + \varepsilon_{it}$$

where $i = 1,..., n$ and $T = 1,..., T_i$; x_{it} represents the independent variables and does not contain a constant term; S_{it-1} constitutes a set of dichotomous variables indicating that the firm belongs to a category at time $t-1$; and c_i is the firm's unobserved specific individual heterogeneity, which does not vary over time. Variable s_{it}^* is a latent variable of the firm's possible category and s_{it} is the observed variable. For the ordered model, we establish the order of the categories as follows:

<p align="center">*Mediocre ≺ Average ≺ Growth ≺ Profit ≺ Star*</p>

where ≺ denotes the direction of the order relation: if $a ≺ b,$ then a is considered a situation inferior to b. The order of these situations can be justified by the results of Davidsson et al. (2009) and the manner in which each situation is defined. Thus, dependent variable s_{it} takes the value of 0, 1, 2, 3 or 4 depending on whether the firm belongs to the *Mediocre, Average, Growth, Profit* or *Star* category respectively.[10]

For the unordered model, dependent variable s_{it} will be equal to 1 if the firm belongs to the *Star* category, 0 in all other cases, and s_{it} will be equal to 1 if the firm belongs to the *Mediocre* category, 0 in all other cases. As the hypothesis of an ordered model suggests a rigid structure that may not be representative of the data, this justifies use of the unordered model.

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9. See Appendix D.
10. The value assigned to the categories is arbitrary, but must respect the set order.

We also assume that the unobserved individual heterogeneous effects[11] are such that

$$c_i = c_0 + \alpha_1 \boldsymbol{S}_{i0} + \alpha_2 \bar{\boldsymbol{x}}_i + u_i \qquad (1)$$

where $\bar{\boldsymbol{x}}_i$ is the average of the variables by firm based on time and with the same hypotheses as for the theoretical model. Note that $\boldsymbol{S}_{i0}$ represents all the dichotomous variables for the firm's initial situation.

Earlier, we assumed that a firm's situation over time would follow a particular stochastic process defined as being a Markov chain. In this case, that means that a firm's probability of reaching a situation at time t depends only on its situation at time $t - 1$. Davidsson et al. (2009) obtained their results in a context similar to that of Markov chain theory as the authors analyzed the firm's transition over the years and calculated the proportion of firms whose situation changed. The model we use presents many advantages. First, it is possible to measure the impact of a firm's position in a category at time $t - 1$ on the probability of being in a category at time t. This will be given by the estimation of coefficients $\boldsymbol{S}_{it-1}$. This is the dynamic aspect of the model represented here. Next, we can also analyze the effect of independent and control variables on the probability that the firm will be in a particular situation. This is given by estimating the coefficients of $\boldsymbol{x}_{it}$. Finally, applying the results obtained with this model, we calculate the *average partial effects*.[12] Using these, we can, among other uses, quantify the effect on a firm's probability of being in a category when its previous situation corresponds to any of the five defined categories following the method of Davidsson et al. (2009). The various aspects arising from this study's model represent the significant contributions of this work as they allow us to examine in greater depth the performance of the SMEs and the link between a firm's growth and profitability.

3.2.4   Model variables

We now present the variables that are part of the models used in this study. The choice of these variables is based on the work of researchers who analyzed the determinants of growth with a clear influence on the firms' performance and, in particular, on their situation from year to year. Table 11, in Appendix D, provides a summary of this work and defines the variables that were incorporated into our study's models based on the availability of data in our sample.

- Dichotomous variables for provinces or regions: Quebec, Ontario, British Columbia, Atlantic (Nova Scotia, Newfoundland and Labrador, Prince Edward Island, New Brunswick), Prairies (Manitoba, Alberta, Saskatchewan), Territories (Yukon, Northwest Territories and Nunavut);

- Dichotomous variables for industry sectors:[13] agriculture; mining; construction; manufacturing; wholesale trade; retail trade; transportation and warehousing; information and cultural industries;

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11. See Appendix B for more information.

12. See Appendix B, section B.1.2.

13. According to the North American Industry Classification System (NAICS), 2007.

real estate and rental and leasing; professional, scientific and technical services; administrative services; health care and social assistance; arts, entertainment and recreation; accommodation and food services; other services;

- Dichotomous variables for the years considered: 2006 to 2011;

- Characteristics of firm:
  - Age of firm (Age)[14]
  - Number of employees (Emp)[15]
  - External financing (Debt):[16]

$$\frac{\text{Total liabilities}}{\text{Total assets}}$$

  - Human capital (Hum Cap):[17] to estimate human capital, we determine the ratio between the annual wages paid to employees by the business and the average annual wages paid to employees,[18] calculated by industry sector;

- Dichotomous variable for each category of firms at time $t - 1$;

- Dichotomous variable for each category of firms at time t_0, that is, 2006;

- Average observations from 2006 to 2011 for the variables number of employees (where applicable), age of firm, debt and human capital. These variables are used in equation (1) (and in equation (4) in Appendix B).

Total sales, assets and liabilities are expressed in millions of Canadian dollars. Profit is expressed in tens of thousands of Canadian dollars. Also, all amounts were adjusted based on 2006 prices using the consumer price index.[19]

Tables 2, 3 and 4 provide information on the sample used in this study when the firms' total sales are used as a measure of growth.[20]

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14. Firm age is estimated using the date at which the firm first appears in the Business Register.

15. This is the firm's average number of employees as reported to the Canada Revenue Agency. This variable is excluded from certain regressions, where the number of employees is used as a measure of growth.

16. To define certain financial variables, we consulted Statistics Canada's *Financial Performance Indicators for Canadian Business* (1995).

17. We estimated human capital in the same manner as Lopez-Garcia and Puente (2012).

18. Firms report their employees' annual wages to the Canada Revenue Agency.

19. Source: Statistics Canada, CANSIM, Table 326-0021.

20. Refer to Appendix E for other measures considered (total number of employees and total assets).

Table 2 provides information on certain variables. We note that for firms in the sample, on average, liabilities represent three quarters of assets. Table 2 also shows that the firms' average age is about 25 years and that the average number of employees is just over 30.

Table 2: Average of selected variables for models

Variable	Average
Debt	0.73 (0.76)
Hum Cap	1.00 (1.77)
Age	25.00 (16.60)
Emp	33.05 (55.34)
TN*	20,920

Standard deviation in parentheses.
*Number of observations x number of years.

Table 3 breaks down the firms by province or region. It shows that Ontario and Quebec account for almost half of all firms in Canada, that is, 27 percent for Ontario and 22 percent for Quebec, whereas the three territories together have the fewest SMEs in Canada.

Table 3: Distribution of firms by province or region

Province/region	Percentage
Ontario	27.56
Quebec	22.80
Prairies	19.93
British Columbia	12.40
Atlantic	13.86
Territories	3.44
NT*	20,920

*Number of observations x number of years.

Finally, Table 4 breaks down firms in the sample by industry sector. It shows that the greatest proportion of firms is found in three sectors: professional, scientific and technical services; manufacturing; and retail trade. The professional, scientific and technical services sector accounts for 17.3 percent of all firms, followed by the manufacturing sector (15.5 percent of all firms) and the retail trade sector (12.8 percent of all firms).

Table 4: Distribution of firms by industry sector

Industry sector	Percentage
Professional, scientific and technical services	17.30
Manufacturing	15.54
Retail trade	12.79
Construction	9.99
Accommodation and food services	9.75
Mining	8.13
Wholesale trade	7.36
Transportation and warehousing	4.45
Agriculture	3.61
Administrative services	3.08
Other services	2.84
Information and cultural industries	1.74
Health care and social assistance	1.58
Arts, entertainment and recreation	0.96
Real estate and rental and leasing	0.88
TN*	20,920

*Number of observations x number of years.

4.   Results

This section present the results. As three measures are used for growth, and to avoid repetition, this section provides only results for which the measure is the total number of sales. Results for other measures are presented in Appendix E.

4.1   Transition matrices of firms from 2006 to 2011

This subsection presents the transition matrix observed for aggregated data from 2006 to 2011 (see Table 5). Firm position at time $t - 1$ is found in the columns, while firm position at time t is found in the rows. The transition matrices for each year have been omitted as the results bear close resemblance to those of the aggregated data. We note that the proportion of firms in the *Profit* category at time $t - 1$ and in the *Star* category at time t is much higher than that of firms in the *Growth* category at time $t - 1$ and in the *Star* category at time t (nearly double). However, the proportion of firms in the *Profit* category at time $t - 1$ and in the *Mediocre* category at time t is much lower than that of firms in the *Growth* category at time $t - 1$ and in the *Mediocre* category at time t (two times smaller). These findings are also valid for every transition year considered (see Appendix E). Furthermore, we note that, in general, firms tend to remain in the same category from year to year.

Table 5: Transition matrix for firms, aggregated data from 2006 to 2011 (percentage)

		Position at time $t-1$				
		Mediocre	Average	Growth	Profit	Star
Position at time t	Mediocre	33.65	19.26	**30.34**	**16.42**	15.60
	Average	22.15	45.24	23.16	20.82	20.18
	Growth	23.32	10.16	25.10	5.28	5.17
	Profit	5.58	8.54	6.03	26.97	23.97
	Star	15.29	16.80	**15.37**	**30.50**	35.08

Table 6 presents the results (as a percentage) of the tests of hypotheses H_1 and H_2 for each transition year and for the aggregated data from 2006 to 2011.

Table 6: Hypothesis testing (percentage)

Final situation	Star			Mediocre		
Initial situation	*Growth*	H_1	*Profit*	*Growth*	H_2	*Profit*
2006–2007	15.26	***	26.55	30.51	***	15.00
2007–2008	14.80	***	27.77	28.23	***	16.36
2008–2009	17.85	***	31.28	29.64	***	18.90
2009–2010	14.07	***	36.17	33.02	***	14.20
2010–2011	14.73	***	31.39	30.55	***	17.34
2006–2011	15.37	***	30.50	30.34	***	16.42

***p<0.001.

In every case, we find that hypotheses H_1 and H_2 are true for each transition year and for the aggregated data. In short, a greater proportion of firms initially in a *Profit* situation reaches the highest success category, *Star*, than firms initially in a *Growth* situation. The proportion of firms initially in a *Growth* situation that end up in the *Mediocre* category, the category of least success, is greater than the proportion of firms initially in a *Profit* situation.

4.2 Estimation of models

Table 7 presents the results of estimations based on the ordered and unordered dynamic probit models with random effects.

Certain control variables, such as dichotomous variables for years and for industry sectors, have been omitted. In addition, reference categories for the corresponding dichotomous variables are Ontario for the provinces or regions, firms in the *Growth* category for the firm's situation at time $t-1$ and the manufacturing sector for the industry sector variable. In the ordered model, the approximated threshold parameters[21] are called Threshold1, Threshold2, Threshold3 and Threshold4.

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21. See Appendix B.2.

Table 7: Results of estimations based on the ordered and unordered dynamic probit models with random effects

	Ordered model	Unordered model	
	RE (1)	RE-*Star* (2)	RE-*Mediocre* (3)
Mediocre$_{t-1}$	0.0863***	-0.0131	0.122**
	(3.03)	(-0.32)	(-3.25)
Profit$_{t-1}$	0.299***	0.288***	0.272***
	(8.84)	(6.43)	(-6.13)
Average$_{t-1}$	0.0728**	-0.00620	-0.193***
	(2.62)	(-0.15)	(-5.28)
Star$_{t-1}$	0.291***	0.208***	-0.308***
	(9.20)	(4.55)	(-7.78)
Debt	-0.202***	-0.334***	0.174***
	(-8.73)	(-8.23)	(6.20)
Emp	0.00286***	0.00416***	-0.00430***
	(3.31)	(3.36)	(-3.40)
Age	0.00609	0.00183	-0.0254
	(0.26)	(0.06)	(-0.78)
Hum Cap	0.121***	0.140***	-0.184***
	(4.83)	(3.79)	(-4.87)
Prairies	0.0853**	0.0994**	-0.00469
	(2.78)	(2.60)	(-0.12)
Quebec	0.0563*	0.0816*	-0.0625
	(2.01)	(2.32)	(-1.72)
Threshold1	-0.688***		
	(-12.51)		
Threshold2	0.166**		
	(3.04)		
Threshold3	0.562***		
	(10.27)		
Threshold4	1.024***		
	(18.60)		
Log likelihood	-31,707.211	-10,329.857	-10,614.673
TN†	20,920	20,920	20,920

Statistic *t* in parentheses.
*p<0.05, **p<0.01, ***p<0.001.
(1) Dynamic probit model with random effects (RE). (2) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Star* and 0 otherwise. (3) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Mediocre* and 0 otherwise.
†Number of observations x number of years.

First we note that a firm in the *Profit* category at time $t-1$ is more likely to achieve the *Star* category at time t than a firm in the *Growth* category for the ordered model (1). As we imposed an order of potential situations for firms, it was to be expected that the estimated coefficients for situations at time $t-1$ would follow a gradient of values, that is, they would be negative for *Mediocre* and *Average* situations and positive for *Profit* and *Star* situations, all considered with respect to the *Growth* situation. The estimations obtained did not do so, except for the *Profit* and *Star* situations. In fact, a firm in the *Mediocre* category at time $t-1$ has a better chance, all other things being equal, of achieving the *Star* category at time t than a firm in

the *Growth* category. The same rule applies to firms in the *Average* category at time $t − 1$. As such, this situation is not an absolute indicator of future performance.

Moreover, as the estimated coefficient of *Profit*$_{t−1}$ is positive and the context is an ordered model, we can conclude that a firm in this category is less likely to end up in the *Mediocre* category than a firm in the *Growth* category at time $t − 1$. Thus, for these models, hypotheses **H**$_1$ and **H**$_2$ are verified for the Canadian firms in our sample.

Table 8 presents the average partial effects for the ordered model, which indicate the effect on the probability of achieving the *Star* and *Mediocre* categories based on the firm's category at time $t − 1$. If we consider model (1a), we find that if a firm is in the *Profit* category at time $t − 1$, its probability of being in the *Star* category at time t is about 8 percentage points higher than if it is in the *Growth* category at time $t − 1$. Thus, the *Profit* category is among those that foster the most chances for a firm to subsequently achieve greater success. In addition, a firm in the *Profit* category at time $t − 1$, is 7 percentage points less likely to be in the *Mediocre* category, according to model (1b).

Table 8: Average partial effects on the probability of achieving the *Star* and *Mediocre* categories for the ordered dynamic probit model with random effects

	Ordered model	
	RE (1a)	RE (1b)
	Star	*Mediocre*
Mediocre$_{t-1}$	0.0218	-0.0220
	(0.00489)	(0.00494)
Profit$_{t-1}$	0.0803	-0.07143
	(0.0145)	(0.0155)
Average$_{t-1}$	0.0183	-0.0187
	(0.00409)	(0.00414)
Star$_{t-1}$	0.0770	-0.0712
	(0.0135)	(0.0141)
TN*	20,920	20,920

Standard deviation in parentheses.
(1) Dynamic probit model with random effects (RE).
*Number of observations x number of years.

In terms of the unordered model, that is, models (2) and (3), hypotheses **H**$_1$ and **H**$_2$ are also verified. For model (2), firms in the *Profit* category at time $t − 1$ are more likely to achieve the subsequent *Star* category than if they are in the *Growth* category. Model (3) reveals that a firm in the *Profit* category at time $t − 1$ is less likely to end up in the *Mediocre* category at time t than a firm in the *Growth* category. Table 9 indicates that for model (1), a firm in the *Profit* category at time $t − 1$ is about 8 percentage points more likely to be in the *Star* category at time t than a firm in the *Growth* category. On the other hand, model (2) shows that being in the *Profit* category at time $t − 1$, makes a firm 7 percentage points less likely to be in the *Mediocre* category at time t.

Table 9: Average partial effects on the probability of reaching the *Star* category and of being in the *Mediocre* category for the unordered dynamic probit model with random effects

	Unordered model	
	RE-*Star* (1)	RE-*Mediocre* (2)
Mediocre$_{t-1}$	-0.00340	-0.0313
	(0.000830)	(0.00686)
Profit$_{t-1}$	0.0804	-0.0669
	(0.0163)	(0.0144)
Average$_{t-1}$	-0.00161	-0.0495
	(0.000394)	(0.0105)
Star$_{t-1}$	0.0565	-0.0765
	(0.0119)	(0.0156)
TN*	20,920	20,920

Standard deviation in parentheses.
(1) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Star* and 0 otherwise. (2) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Mediocre* and 0 otherwise.
*Number of observations x number of years.

In short, the ordered and unordered models give the same results for the effect of the *Profit* and *Growth* situations at time $t-1$ on the probability of achieving the highest success category (*Star*) or being in the least successful category (*Mediocre*).

4.3   Other results

External financing or debt

Another important result concerns the variable for firms' external financing or debt, expressed as the ratio of total liabilities to total assets. In all models, this variable is significant and the estimated coefficient is negative. Therefore, we can conclude that excessive debt may impede achievement of the *Star* category and favours the probability of being in the *Mediocre* category. In terms of the number of employees, Table 7 reveals that this variable is significant and favours a firm's probability of being in the *Star* category. Hence, the size of a business appears to have a substantial effect on achieving success.

Age

In the case at hand, a firm's age is not significant in explaining the transition over time. In the literature on the subject, empirical research has shown that the relationship between a firm's growth and its age is negative. This suggests that younger firms are more likely to record higher growth than older firms.[22] However, this does not appear to be the case for the sample of Canadian firms in this study. This may be due to sampling issues as the *Survey on Financing of Small and Medium Enterprises* is biased towards older firms as seen in Table 2.

22. See the work of Evans (1987), Coad et al. (2013), Lotti et al. (2009) and Nunes et al. (2013).

Human capital

This study's models highlight an important aspect of SMEs in relation to their employees and their human capital. As explained earlier, to estimate the latter we used the ratio of total wages paid to the average wages of firms in the same industry sector. While this is an approximation, highly educated and experienced workers generally tend to earn higher wages.[23] This can also be explained by the fact that the market attributes a higher productivity value to certain workers. These assumptions are consistent with the theory of human capital.

We find, in Table 7, that the independent variable related to human capital has a positive estimated coefficient. Thus, a firm with high human capital has a greater chance of achieving high growth and high profitability. This demonstrates, in particular, the link between human capital and a firm's performance.

Geography

The firms' geographic situation for certain provinces or regions also appears to have a non-negligible effect on their performance. Table 7 shows the estimated coefficients obtained in the models for two of the provinces whose coefficient was significant. Hence, we find that being based in Quebec or in the Prairies increases the probability that a firm will reach the *Star* category, for models (1) and (2), versus a firm based in Ontario, and diminishes the probability that a firm will be in the *Mediocre* category for model (1).

5.   Conclusions

The purpose of this study was to shed new light on the nature of the relationship between growth and profitability for Canadian SMEs. Like Davidsson et al. (2009), we found that a highly profitable firm has a greater chance of going on to reach the highest success category than a firm in a high-growth category. Perhaps the main contribution of this paper owes much to the use of a dynamic probit model with random effects, which allowed for a more in-depth analysis than that carried out by Davidsson et al. (2009).

This model enabled us to capture the effect of a firm's situation at a given time on the probability that it will be in a certain category at a subsequent point in time and to measure the effect of other independent variables on a firm's probability of being in a certain category. As such, we were able to show, for the sample in question, the following elements:

- Human capital is a positive and significant factor in firms reaching a high level of success, in terms of both growth and profitability. Conversely, human capital allows a firm to reduce its chances of being in the least successful category.

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23. See Weiss (1995).

- Debt is also a significant variable that can impede a firm's ability to perform well in terms of growth and profitability.

- Although numerous empirical studies have shown the considerable influence of a firm's age on its growth, this variable is not significant in the models we used.

- There appears to be a degree of difference among Canadian provinces or regions with respect to a firm's performance.

In terms of future research on the subject, a number of avenues could be explored. Our study considered the human capital of employees, but not the owners' characteristics. Indeed, several works[24] indicate that the characteristics of a firm's owner, notably his or her experience and level of education, can have an influence on a firm's growth. This research could be undertaken using the 2011 *Survey on Financing and Growth of Small and Medium Enterprises*, which contains information on owners' characteristics. A second subject could explore the relationship between a firm's performance and its exports of goods or services. This research could examine whether exports enable the firm to achieve a higher level of performance in terms of growth and profitability.

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24. See, for example, the work of Dobbs and Hamilton (2007), Hamilton and Lawrence (2001), Barkham (1994) and Kangasharju (2000).

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Appendices

A   Empirical Research on the Relationship between Growth and Profitability

Table 10: Empirical research on the relationship between growth and profitability

Reference	Measure of growth	Measure of profitability	Years	Sample size	Country	Sector	Growth–profitability relationship
Reid (1995)	Assets	N/A	1985–1988	73	Scotland	N/A	Negative
Glancey (1998)	Assets	Return on assets Assets to sales	1988–1990	38	Scotland	Manufacturing	None
Roper (1999)	Total sales	Return on assets Assets to sales	1993–1994	703	Ireland	Manufacturing	Low
Nakano and Kim (2011)	Assets	Return on investment	1987–2007	1,633	Japan	Manufacturing	Positive and negative
Markman and Gartner (2002)	Sales Employees	Profits	1992–1997 1993–1997 1994–1998	1,233	United States	All sectors	None
Cowling (2004)	Sales	Return on investment	1991–1993	256	United Kingdom	N/A	Positive
Coad (2007)	Sales Employees Value added	Gross operating surplus on value added	1996–2004	8,405	France	Manufacturing	Positive

B   Econometric Models

This appendix presents, in a general context, the econometric models used in this study.

B.1   Dynamic probit model for panel data

B.1.1   Theoretical elements of the model

One of the models we use in this project is based largely on the *dynamic probit model for panel data* (or longitudinal data). Details regarding this model can be found in the excellent work of Wooldridge (2010).

As the terminology indicates, the model combines three essential aspects. First, we will consider panel data. The data consist of individuals (i) that are observed over a period of time (T) . In this context, the notation y_{it} indicates that we observe individual[25] i at time t, for $i = 1,..., n$ and $t = 1,..., T$. [26] In general, n will be large and T relatively small. The term *dynamic* refers to the fact that we will use variables from the previous period (lagged variables) at time $t-1$. Finally, the term *probit* means that the model is probabilistic and that the error term follows a particular distribution, which is a normal distribution in the case at hand. The variable y_{it}^* is a *latent* variable. This is an unobserved variable for which an indicator, noted as y_{it}, is observed and linked to this variable in the manner explained below. Let us consider the following latent regression:

$$y_{it}^* = \boldsymbol{\beta}\boldsymbol{x}_{it} + \rho y_{it\text{-}1} + c_i + \varepsilon_{it} \qquad (2)$$

where $\boldsymbol{x}_{it}$ is a vector of dimension $1 \times K$ formed by independent variables, c_i represents the unobserved heterogeneous effects and ε_{it} is the error term, which follows a standardized normal distribution, noted as N(0,1). Given the relationship between c_i and $\boldsymbol{x}_{it}$, there are two types of model: the *random effects* model, if it is assumed that c_i and $\boldsymbol{x}_{it}$ are non-correlated, and the *fixed effects* model, if it is assumed that these terms are correlated. We will also hypothesize that ε_{it} is strictly exogenous, that is, $\boldsymbol{x}_{it}$ is non-correlated with ε_{is} for any time t and s. This hypothesis can be expressed as follows:

$$E(\varepsilon_{it}|\boldsymbol{x}_{i1},\boldsymbol{x}_{i2},..., \boldsymbol{x}_{iT}, c_i) = 0$$

The latent variable y_{it}^* and its indicator y_{it} are related as follows:

$$y_{it} = 1, \ \text{if } y_{it}^* > 0$$

$$y_{it} = 0, \ \text{if } y_{it}^* \leq 0$$

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25. The term *individual* is used in the broader sense of the term and includes, for example, firms.

26. Instead of T, we could consider T_i, which means the model is unbalanced. If the model is balanced, then $T_i = T$ for any i.

Considering the distribution of the error term, it follows that:

$$P(y_{it}^* > 0 | \boldsymbol{x}_{it}, y_{it-1}, c_i) = P(y_{it} = 1 | \boldsymbol{x}_{it}, y_{it-1}, c_i) = \Phi(\boldsymbol{\beta}\boldsymbol{x}_{it} + \rho y_{it-1} + c_i)$$

$$P(y_{it}^* \leq 0 | \boldsymbol{x}_{it}, y_{it-1}, c_i) = P(y_{it} = 0 | \boldsymbol{x}_{it}, y_{it-1}, c_i) = 1 - \Phi(\boldsymbol{\beta}\boldsymbol{x}_{it} + \rho y_{it-1} + c_i)$$

where Φ is the distribution function of the standardized normal distribution:

$$\Phi(\boldsymbol{x}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp\left(-\frac{1}{2}t^2\right) dt$$

Finally, it is also found that:

$$E\left[y_{it} | \boldsymbol{x}_{it}, y_{it-1}, c_i\right] = \Phi(\boldsymbol{\beta}\boldsymbol{x}_{it} + \rho y_{it-1} + c_i) \qquad (3)$$

As mentioned earlier, two types of model can be used depending on the hypotheses with respect to the correlation of independent variables and the unobserved heterogeneous effect. The interest in the random effects model resides essentially in the possibility of estimating the coefficients of variables that are set in time (e.g., gender, ethnicity, skill). This is not possible with fixed effects models. Thus, in this case, it is impossible to determine how this particular type of variable affects the dependent variable. Using a dynamic model may also pose a problem when estimating coefficients. Variable y_{it-1} is endogenous as it is correlated with the error term. This stems primarily from the fact that the "real" initial observation y_{i0} is not known as we begin to observe individuals from an arbitrary initial time. The prior information is unknown. This means that the initial observation is contained in the error term, hence the correlation with the lagged variable y_{it-1}. This is the *initial condition problem*. Wooldridge (2000, 2005) dealt with this problem in relation to dynamic non-linear random effect models. The solution consisted essentially of modelling the distribution of unobserved effects conditional to the initial values and to the exogenous independent variables. Based on the Wooldridge solution, we will therefore assume that:

$$c_i = c_0 + \alpha_1 y_{i0} + \alpha_2 \bar{\boldsymbol{x}}_i + u_i \qquad (4)$$

where $\bar{\boldsymbol{x}}_i$ is the average variables by individual at a given time, that is:

$$\bar{\boldsymbol{x}}_i = \frac{1}{T} \sum_{i=1}^{T} \boldsymbol{x}_{it}$$

It is assumed that the error term u_i is non-correlated with the variables and is distributed, conditional to $\boldsymbol{x}_{it}$, such that $N(0, \sigma_u^2)$. Note that the dichotomous (or binary) variables are excluded from the calculation of $\bar{\boldsymbol{x}}_i$ to avoid collinearity. Thus, equation (3) may be written:

$$E\left[y_{it} | \boldsymbol{x}_{it}, y_{it-1}, c_i\right] = \Phi(\boldsymbol{\beta}\boldsymbol{x}_{it} + \rho y_{it-1} + c_0 + \alpha_1 y_{i0} + \alpha_2 \bar{\boldsymbol{x}}_i + u_i)$$

and, in the form of latent regression:

$$y_{it}^* = \boldsymbol{\beta x}_{it} + \rho y_{it-1} + c_0 + \alpha_1 y_{i0} + \alpha_2 \bar{\boldsymbol{x}}_i + u_i + \varepsilon_{it}$$

The above solution entails a number of advantages. First, it can be applied easily by certain statistical software programs (e.g., Stata) to estimate the ordered dynamic probit model with random effects by the maximum likelihood method. This method can also be used to estimate the coefficients of variables that do not vary over time.

Note that this method has been used extensively in the literature, notably in the works of Contoyannis et al. (2004a, 2004b), Heiss (2011) and, more recently, Lopez-Garcia and Puente (2012).

B.1.2 Average partial effects

The interest in using the probit model resides in the fact that it is possible to quantify the potential effect of certain specific independent variables on the probability that the dependent variable will take on a certain value. The sign of the estimated coefficients of $\boldsymbol{\beta}$ will give the direction of the effect (positive or negative), but not the magnitude. That is why we will define the *average partial effects*, which allow us to obtain this information.

Generally, if we have the following model:

$$E(y_{it}|\boldsymbol{x}_{it}, c_i) = P(y_{it} = 1|\boldsymbol{x}_{it}, c_i) = \Phi(\boldsymbol{x}_{it} + c_i), \ t = 1,..., T$$

then, by simplifying the notation by dropping subscript i, the partial effect for a continuous variable x_{tj} is given by:

$$\frac{\partial P(y_t = 1|\boldsymbol{x}_t, c)}{\partial x_{tj}} = \beta_j \phi(\boldsymbol{x}_t + c)$$

where ϕ is the standardized normal distribution:

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \exp(-z^2/2)$$

For discrete variables, the partial effect is calculated based on

$$\Phi(\boldsymbol{x}_t^{(1)} + c) - \Phi(\boldsymbol{x}_t^{(0)} + c) \qquad (5)$$

where $\boldsymbol{x}_t^{(0)}$ and $\boldsymbol{x}_t^{(1)}$ are the respective values of the variable considered.[27]

The difficulty of calculating partial effects resides essentially in the fact that the heterogeneous effects, c, are not observed. A measure commonly used for the effect of independent variables consists of calculating

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27. We use the same notation as Papke and Wooldridge (2008).

the expectation on the partial effects based on the distribution of c. Thus, the *average partial effect*, noted as *APE*, evaluated in x_t is defined by:

$$APE(x_t) = E_c \left[\beta_j \phi(x_t + c) \right]$$

where the expectation is conditional to c. As a result, the average partial effect no longer depends on c. The average partial effect can be obtained for discrete variables by taking the average of the difference calculated in (5).

Similar to (4), we will assume that:

$$c_i = \Psi + \xi x_i + u_i$$

with u_i distributed based on N($0, \sigma_u^2$).

Wooldridge (2010) shows that the partial effects may be obtained by deriving, or by calculating, the difference for the following expression:

$$E_{\bar{x}_i} \left[\Phi(\Psi_\alpha + \beta_\alpha x_t + \xi_\alpha \bar{x}_i) \right] \qquad (6)$$

where subscript α indicates that the coefficients were divided by $\sqrt{1 + \sigma_u^2}$. The expression found in (6) can be estimated by:

$$\frac{1}{N} \sum_{i=1}^{N} \Phi(\Psi_\alpha + \beta_\alpha x_t + \xi_\alpha \bar{x}_i) \qquad (7)$$

Note that convergent estimators of the coefficients may be used directly in (7) to obtain convergent estimators of the average partial effects.

In short, a convergent estimator of the average partial effects is obtained by deriving, or by calculating, the difference for the following expression:

$$\frac{1}{N} \sum_{i=1}^{N} \Phi(\hat{\Psi}_\alpha + \hat{\beta}_\alpha x_t + \hat{\xi}_\alpha \bar{x}_i)$$

where the notation $\widehat{}$ means an estimation of the coefficient and subscript α means that the coefficients were divided by $\sqrt{1 + \hat{\sigma}_u^2}$.

In the context of the model specified in (2) and the hypothesis formulated on the unobserved heterogeneous effects in (4), a convergent estimator of the average partial effects is given by deriving, or by calculating, the difference:

$$\frac{1}{N} \sum_{i=1}^{N} \Phi(\hat{c}_{0\alpha} + \hat{\alpha}_{1\alpha} + \hat{\alpha}_{2\alpha} \bar{x}_i + \hat{\beta}_\alpha \bar{x}_{it} + \hat{\rho}_\alpha y_{it-1})$$

It is also possible to calculate the average partial effects for any time t and i. In this case, the difference must be derived or calculated:

$$\frac{1}{NT} \sum_{t=1}^{T} \sum_{i=1}^{N} \Phi(\hat{c}_{0\alpha} + \hat{a}_{1\alpha} + \hat{a}_{2\alpha}\bar{x}_i + \hat{\boldsymbol{\beta}}_\alpha x_{it} + \hat{\rho}_\alpha y_{it-1})$$

B.2   Ordered dynamic probit model for panel data

The theory we presented concerning the dynamic probit model for panel data can be generalized directly to an *ordered* model. This model will also be used in this study. As before, the latent variable is noted as y_{it}^* and the dummy variable as y_{it}. We assume that y_{it} takes its values in the set $\{0,1,..., J\}$, where J is a positive integer. The latent regression model is similar and is given by:

$$y_{it}^* = \boldsymbol{\beta} x_{it} + \rho y_{it-1} + c_i + \varepsilon_{it}$$

The same hypotheses as in the unordered case apply to this model as well. Let $\mu_1 < ... < \mu_J$ represent *threshold parameters* and let us define:

$$y_{it} = 0, \text{ if } y_{it}^* \leq \mu_1$$

$$y_{it} = 1, \text{ if } \mu_1 < y_{it}^* \leq \mu_2$$

$$\vdots$$

$$y_{it} = J, \text{ if } y_{it}^* > \mu_J$$

Thus, the value of y_{it} is determined based on the interval in which variable y_{it}^* is located. These intervals are given by the threshold parameters.

Assuming that the error term is normally distributed, it follows that the probabilities that the dependent variable takes on either of the previous values, conditional to the independent variables, are given by:

$$P_{it0} = P(y_{it} = 0 | x_{it}, y_{it-1}, c_i) = \Phi(\mu_1 - \boldsymbol{\beta} x_{it} - \rho y_{it-1} - c_i) \qquad (8)$$

$$P_{it1} = P(y_{it} = 1 | x_{it}, y_{it-1}, c_i) = \Phi(\mu_2 - \boldsymbol{\beta} x_{it} - \rho y_{it-1} - c_i) - \Phi(\mu_1 - \boldsymbol{\beta} x_{it} - \rho y_{it-1} - c_i) \qquad (9)$$

$$\vdots$$

$$P_{itJ} = P(y_{it} = J | x_{it}, y_{it-1}, c_i) = \Phi(\mu_J - \boldsymbol{\beta} x_{it} - \rho y_{it-1} - c_i) \qquad (10)$$

Note, in this case, parameters μ_j are also to be estimated as for $\boldsymbol{\beta}$ and ρ. Again, this model may be estimated by the maximum likelihood method.[28]

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28. We used the reoprob.ado program, written by Guillaume R. Fréchette (*Stata Technical Bulletin*, Vol. 59, January 2001).

The hypotheses we formulated on the unordered model are transferable to the ordered model, particularly the hypothesis on the distribution of the unobserved heterogeneous effect of individuals (given by (4)). Generalization of the concepts presented in the previous section is almost direct. It is a matter of using the previous definitions, which are simply an extension of those of the unordered model. However, one exception concerns the significance of the estimated coefficients. For an ordered model, the sign of the coefficient indicates the effect on probability only for extreme cases. We can easily see by deriving (8) and (10) that a positive coefficient increases probability P_{itJ} and that a negative coefficient increases probability P_{it0}. For intermediate values, the sign of the coefficient does not generally indicate the effect on probability.[29] This can be observed by deriving expression (9).

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29. Refer to Wooldridge (2010) for more details.

C   Hypothesis Testing

Below we use $\hat{p}_1$ to signify the proportion of firms in the *Profit* situation at time $t-1$ and the *Star* situation at time t, and $\hat{p}_2$ to signify the proportion of firms in the *Growth* situation at time $t-1$ and the *Star* situation at time t. Our hypotheses are:

$$H_0 : \hat{p}_1 = \hat{p}_2$$

and

$$H_1 : \hat{p}_1 > \hat{p}_2$$

This corresponds to hypothesis $\boldsymbol{H}_1$. Let $\tilde{p}_1$ represent the proportion of firms in the *Profit* situation at time $t-1$ and the *Mediocre* situation at time t and $\tilde{p}_2$ represent the proportion of firms in the *Growth* situation at time $t-1$ and the *Mediocre* situation at time t. The hypotheses in this case are:

$$H_0 : \tilde{p}_1 = \tilde{p}_2$$

and

$$H_1 : \tilde{p}_1 < \tilde{p}_2$$

The latter are related to hypothesis $\boldsymbol{H}_2$.

Let p, s and z be defined, respectively, by:

$$p = \frac{p_1 \cdot n_1 + p_2 \cdot n_2}{n_1 + n_2}$$

$$s = \sqrt{p\,(1-p)\left(\frac{1}{n_1 + n_2}\right)}$$

$$z = \frac{p_1 - p_2}{s}$$

where p_1 corresponds to the estimated value of $\widehat{p}_1$ or $\tilde{p}_1$ and p_2 is the estimated value of $\widehat{p}_2$ or $\tilde{p}_2$. Since we have a one-tailed test, the statistic z_α can be found using a normal table with a significance level of $\alpha\%$, where $\alpha \in \{1, 5, 10\}$. If $z_\alpha < z$, this results in rejection of H_0 in favour of H_1 in the first case. If $z_\alpha > z$, H_0 is rejected in favour of H_1 in the second case.

D   Empirical Research on Determinants of Growth

Table 11: Empirical research on determinants of growth

Reference	Measure of growth	Years	Sample size	Country	Sector	Determinant of growth
Hart and Prais (1956)	Market value	1885–1896 1896–1907 1907–1924 1924–1939 1939–1950	Varies according to years considered	United Kingdom	Mining Manufacturing Distribution	Size
Simon and Bonini (1958)	Sales Assets Employees Value added Profits	1954–1955 1954–1956	500	United States	Manufacturing	Size
Hymer and Pashigian (1962)	Assets	1946–1955	1,000	United States	Manufacturing	Size
Singh and Whittington (1975)	Assets	1948–1960	2,000	United Kingdom	Manufacturing Construction Distribution Other services	Size
Evans (1987)	Employees	1976–1980	100	United States	Manufacturing	Size Age
Hall (1987)	Employees	1972–1979 1976–1983	1,349 1,098	United States	Manufacturing	Size
Heshmati (2001)	Employees Sales Assets	1993–1998	N/A	Sweden	N/A	Size Age External financing Human capital
Becchetti and Trovato (2002)	Employees	1989–1997	5,000+	Italy	Manufacturing	Size Age External financing
Lotti et al. (2009)	Employees	1987–1994	3,285	Italy	Radio Television Communications equipment	Size Age
Levratto et al. (2010)	Employees	1997–2007	12,811	France	Manufacturing	Age Size Human capital External financing
Nakano and Kim (2011)	Assets	1987–2007	1,633	Japan	Manufacturing	Size
Chandler (2012)	Wages Employees Revenues Profits	1996–2003	2,304	Canada	14 specific sectors	External financing Age Size
Lopez-Garcia and Puente (2012)	Employees	1996–2003	1,411	Spain	All sectors, except agriculture and finance	Human capital External financing Age
Coad et al. (2013)	Employees Sales	1998–2006	62,259	Spain	Manufacturing	Age
Daunfeldt and Elert (2013)	Employees Revenues	1998–2004	288,757	Sweden	All sectors	Size
Nunes et al. (2013)	Sales	1999–2006	495 and 1,350	Portugal	Agriculture, forestry and mining Construction Manufacturing Commerce Services Tourism	Age External financing

E   Results of Other Measures Used

This section provides the results for two other measures used in this study: total number of employees and total assets.

E.1   Total number of employees

Table 12: Average of selected variables for models

Variable	Average
Debt	0.73 (0.74)
Hum Cap	1.00 (1.78)
Age	25.12 (16.60)
Emp	32.16 (54.42)
TN*	22,800

Standard deviation in parentheses.
*Number of observations x number of years.

Table 13: Distribution of firms by province or region

Province/region	Percentage
Ontario	27.57
Quebec	22.85
Prairies	20.04
British Columbia	12.39
Atlantic	13.88
Territories	3.27
TN*	22,800

*Number of observations x number of years.

Table 14: Distribution of firms by industry sector

Industry sector	Percentage
Professional, scientific and technical services	16.82
Manufacturing	14.65
Retail trade	12.08
Construction	9.45
Accommodation and food services	9.28
Mining	7.59
Wholesale trade	7.00
Transportation and warehousing	4.10
Agriculture	7.46
Administrative services	3.11
Other services	2.74
Information and cultural industries	1.67
Health care and social assistance	1.56
Arts, entertainment and recreation	0.92
Real estate and rental and leasing	1.58
TN*	22,800

*Number of observations x number of years.

Table 15: Transition matrix for firms, aggregated data from 2006 to 2011 (percentage)

Position at time t		Position at time $t-1$				
		Mediocre	Average	Growth	Profit	Star
	Mediocre	34.45	16.09	31.11	11.30	11.46
	Average	22.23	45.87	20.41	30.74	19.42
	Growth	22.89	12.79	27.17	8.67	10.13
	Profit	10.31	12.38	9.89	29.57	24.86
	Star	10.11	12.87	11.41	29.72	34.13

Table 16: Hypothesis testing (percentage)

Final situation	Star			Mediocre		
Initial situation	Growth	H_1	Profit	Growth	H_2	Profit
2006–2007	11.01	***	25.52	29.80	***	12.11
2007–2008	12.21	***	28.38	28.19	***	10.20
2008–2009	10.23	***	30.63	33.02	***	11.39
2009–2010	10.96	***	30.89	33.56	***	12.25
2010–2011	12.64	***	33.29	31.11	***	10.62
2006–2011	11.41	***	29.72	31.11	***	11.30

***$p<0.001$.

Table 17: Results of estimations based on the ordered and unordered dynamic probit models with random effects, using the number of employees as a measure of growth

	Ordered model	Unordered model	
	RE (1)	RE-*Star* (2)	RE-*Mediocre* (3)
$Mediocre_{t-1}$	0.0667*	-0.0281	-0.0962**
	(2.53)	(-0.70)	(-2.71)
$Profit_{t-1}$	0.542***	0.572***	0.626***
	(18.96)	(14.57)	(-16.16)
$Average_{t-1}$	0.154***	0.0668	-0.398***
	(6.22)	(1.76)	(-12.19)
$Star_{t-1}$	0.489***	0.495***	-0.597***
	(16.64)	(11.45)	(-15.86)
Debt	-0.232***	-0.329***	0.196***
	(-9.93)	(-8.01)	(6.86)
Age	0.0135	0.0464	0.00597
	(0.60)	(1.42)	(-0.19)
Hum Cap	0.235***	0.276***	0.416***
	(12.43)	(9.82)	(-13.89)
Prairies	0.0458	0.0536	0.0463
	(1.66)	(1.48)	(1.32)
Quebec	0.0502*	0.0749*	-0.0610
	(1.97)	(2.25)	(-1.85)
Threshold1	-0.702***		
	(-14.40)		
Threshold2	0.165***		
	(3.40)		
Threshold3	0.637***		
	(13.11)		
Threshold4	1.241***		
	(25.30)		
Log likelihood	-34,614.743	-10,136.976	-10,675.734
TN[†]	22,800	22,800	22,800

Statistic *t* in parentheses.
*p<0.05, **p<0.01, ***p<0.001.
(1) Dynamic probit model with random effects (RE). (2) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Star* and 0 otherwise. (3) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Mediocre* and 0 otherwise.
[†]Number of observations x number of years.

Table 18: Average partial effects on the probability of achieving the *Star* and *Mediocre* categories for the ordered dynamic probit model with random effects, using the number of employees as a measure of growth

	Ordered model	
	RE (1a)	RE (1b)
	Star	*Mediocre*
Mediocre$_{t-1}$	0.015	-0.016
	(0.00484)	(0.00482)
Profit$_{t-1}$	0.138	-0.114
	(0.0307)	(0.0322)
Average$_{t-1}$	0.0351	-0.0366
	(0.0114)	(0.0115)
Star$_{t-1}$	0.123	-0.105
	(0.0278)	(0.0289)
NT*	22,800	22,800

Standard deviation in parentheses.
(1) Dynamic probit model with random effects (RE).
*Number of observations x number of years.

Table 19: Average partial effects on the probability of reaching the *Star* category and of being in the *Mediocre* category for the unordered dynamic probit model with random effects, using the number of employees as a measure of growth

	Unordered model	
	RE-*Star* (1)	RE-*Mediocre* (2)
Mediocre$_{t-1}$	-0.00660	-0.0235
	(0.00217)	(0.00735)
Profit$_{t-1}$	0.155	-0.133
	(0.0340)	(0.0432)
Average$_{t-1}$	0.0160	-0.0933
	(0.00527)	(0.0307)
Star$_{t-1}$	0.131	-0.129
	(0.0300)	(0.0411)
NT*	22,800	22,800

Standard deviation in parentheses.
(1) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Star* and 0 otherwise. (2) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Mediocre* and 0 otherwise.
*Number of observations x number of years.

E.2   Total assets

Table 20: Average of selected variables for models

Variable	Average
Debt	0.72 (0.75)
Hum Cap	1.00 (1.78)
Age	25.21 (16.70)
Emp	32.23 (55.52)
TN*	22,695

Standard deviation in parentheses.
*Number of observations x number of years.

Table 21: Distribution of firms by province or region

Province/region	Percentage
Ontario	27.74
Quebec	22.74
Prairies	20.27
British Columbia	12.23
Atlantic	13.77
Territories	3.26
TN*	22,695

*Number of observations x number of years.

Table 22: Distribution of firms by industry sector

Industry sector	Percentage
Professional, scientific and technical services	17.01
Manufacturing	14.61
Retail trade	11.92
Construction	9.43
Accommodation and food services	9.28
Mining	7.78
Wholesale trade	7.01
Transportation and warehousing	4.12
Agriculture	7.42
Administrative services	3.11
Other services	2.67
Information and cultural industries	1.67
Health care and social assistance	1.52
Arts, entertainment and recreation	0.93
Real estate and rental and leasing	1.54
TN*	22,695

*Number of observations x number of years.

Table 23: Transition matrix for firms, aggregated data from 2006 to 2011 (percentage)

		Position at time $t-1$				
		Mediocre	Average	Growth	Profit	Star
Position at time t	Mediocre	36.79	17.78	33.95	12.94	12.85
	Average	24.23	48.37	26.85	17.93	21.94
	Growth	18.94	10.07	20.65	9.24	6.63
	Profit	7.37	9.01	8.27	22.55	20.57
	Star	12.66	14.77	10.27	37.34	38.01

Table 24: Hypothesis testing (percentage)

Final situation	Star			Mediocre		
Initial situation	Growth	H_1	Profit	Growth	H_2	Profit
2006–2007	10.96	***	35.42	32.23	***	12.33
2007–2008	10.54	***	36.01	36.74	***	11.62
2008–2009	10.17	***	40.30	34.14	***	14.80
2009–2010	8.93	***	38.64	33.33	***	12.52
2010–2011	10.53	***	36.27	32.98	***	13.38
2006–2011	10.27	***	37.34	33.95	***	12.94

***$p<0.001$.

Table 25: Results of estimations based on the ordered and unordered dynamic probit models with random effects, using total assets as a measure of growth

	Ordered model	Unordered model	
	RE (1)	RE-*Star* (2)	RE-*Mediocre* (3)
Mediocre$_{t-1}$	0.152***	0.125**	-0.170***
	(5.38)	(2.93)	(-4.59)
Profit$_{t-1}$	0.681***	0.782***	-0.612***
	(21.12)	(17.51)	(-14.00)
Average$_{t-1}$	0.231***	0.161***	-0.395***
	(8.67)	(3.89)	(-11.43)
Star$_{t-1}$	0.540***	0.560***	-0.566***
	(17.70)	(12.19)	(-14.60)
Debt	-0.311***	-0.515***	0.282***
	(-12.15)	(-11.48)	(9.00)
Emp	0.000904	0.00142	-0.00241
	(1.07)	(1.15)	(-1.95)
Age	-0.0327	-0.0173	0.0248
	(-1.44)	(-0.54)	(0.78)
Hum Cap	0.0906***	0.0629	-0.138***
	(3.80)	(1.78)	(-3.93)
Prairies	0.0466	0.0465	-0.00639
	(1.64)	(1.29)	(-0.17)
Quebec	0.000996	-0.0288	-0.0641
	(1.97)	(2.25)	(-1.85)
Threshold1	-0.578***		
	(-11.20)		
Threshold2	0.363***		
	(7.06)		
Threshold3	0.745***		
	(14.43)		
Threshold4	1.202***		
	(23.12)		
Log likelihood	-33,535.943	-10,661.585	-11,131.342
TN†	22,695	22,695	22,695

Statistic *t* in parentheses.
*p<0.05, **p<0.01, ***p<0.001.
(1) Dynamic probit model with random effects (RE). (2) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Star* and 0 otherwise. (3) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Mediocre* and 0 otherwise.
†Number of observations x number of years.

Table 26: Average partial effects on the probability of achieving the *Star* and *Mediocre* categories for the ordered dynamic probit model with random effects, using total assets as a measure of growth

	Ordered model	
	RE (1a)	RE (1b)
	Star	*Mediocre*
Mediocre$_{t-1}$	0.0375	-0.0376
	(0.0113)	(0.012)
Profit$_{t-1}$	0.193	-0.141
	(0.0357)	(0.0420)
Average$_{t-1}$	0.0569	-0.0571
	(0.0172)	(0.0181)
Star$_{t-1}$	0.145	-0.123
	(0.0289)	(0.0322)
TN*	22,695	22,695

Standard deviation in parentheses.
(1) Dynamic probit model with random effects (RE).
*Number of observations x number of years.

Table 27: Average partial effects on the probability of reaching the *Star* category and of being in the *Mediocre* category for the unordered dynamic probit model with random effects, using total assets as a measure of growth

	Unordered model	
	RE-*Star* (1)	RE-*Mediocre* (2)
Mediocre$_{t-1}$	0.03183	-0.0417
	(0.0101)	(0.0118)
Profit$_{t-1}$	0.231	-0.132
	(0.0427)	(0.0390)
Average$_{t-1}$	0.0407	-0.0960
	(0.0131)	(0.0275)
Star$_{t-1}$	0.155	-0.129
	(0.0326)	(0.0340)
TN*	22,695	22,695

Standard deviation in parentheses.
(1) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Star* and 0 otherwise. (2) Dynamic probit model with random effects and dependent variable = 1 if firm belongs to *Mediocre* and 0 otherwise.
*Number of observations x number of years.