In Situ Determination of the Thermal Diffusivity of Sea Ice

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1983

Canadian Technical Report of Hydrography and Ocean Sciences No. 27

Fisheries and Oceans Pêches et Océans

Canada

Canadian Technical Report of Fisheries and Aquatic Sciences

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Cat. No. Fs 97-18/27 ISSN 0711-6764

Correct citation for this publication:

Melling, H. 1983. In Situ Determination of the Thermal Diffusivity of Sea Ice.
Can. Tech. Rep. Hydrogr. Ocean Sci. No. 27:24p.

CONTENTS

ABSTRACT

Melling, H. 1983. In Situ Determination of the Thermal Diffusivity of Sea Ice. Can. Tech. Rep. Hydrogr. Ocean Sci. 27:24p.

A general method is described which utilizes measurements of temperature, T, and salinity, S, within undisturbed sea ice to deduce its thermal diffusivity, K(T,S). In applying the method, the observations are analysed within the framework of the thermal diffusion equation so as to generate an overdetermined system of linear equations in the unknown thermal parameters, which may be solved by least-squares techniques. Observational limitations, and statistical constraints on sampling and reliability are discussed. Computations utilizing data from first-year sea ice 1.8 m in thickness are presented and compared with existing experimental data and theory.

key words: sea ice, diffusivity, temperature.

RESUMÉ

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Le présent rapport décrit une méthode générale permettant d'utiliser la température T et la salinité S à l'intérieur de la glace de mer non perturbée pour en déduire sa diffusivité thermique K(T,S). Pour appliquer cette méthode, il faut analyser les observations à l'aide de l'équation de diffusion thermique afin de générer un système surdéterminé d'équations linéaires dans les paramètres thermiques inconnus, équations pouvant être résolues par les techniques des moindres carrés. On élabore les restrictions qui proviennent de l'observation et les contraintes statistiques sur l'échantillonnage et la fiabilité. Des calculs réalisés à partir des données recueillies sur la glace de mer de moins d'un an et de 1.8 m d'épaisseur sont présentés et comparés aux données et à la théorie expérimentales existantes.

Mots-clés: glace de mer, diffusivité, température.

ACKNOWLEDGEMENTS

I thank E. Lyn Lewis for making available his 1966 observations. Computer work by Stephen Smith and manuscript preparation by Pat Perras and Lorena Quay and drafting by Dennis Richards are greatly appreciated.

1. INTRODUCTION

Sea ice is a material composed of pure ice, brine, solid salts and air. The proportions of these components depend upon conditions during growth of the ice, on its age and on its temperature (Assur, 1958; Nakawo and Sinha, 1981). The arrangement of the components is both non-homogeneous and anisotropic. In first-year ice, the uppermost 10-50 cm consists of small randomly oriented ice crystals and brine pockets and has relatively high bulk salinity (10-20). The region below, the columnar zone, consists of long vertically oriented ice crystals separated by layers of brine pockets which merge to form channels near the ice-water interface. Bulk salinity generally increases with depth in this region from a minimum (2-5) just below the uppermost layer. The lowest 10-15 cm is the interfacial layer where vertical growing plates of ice are separated by layers of seawater melt. The horizontal scale of organization in sea ice is large (0.3 - 1.0 m) relative to ice samples customarily studied (Lake and Lewis, 1970). More detailed discussion and explanation of sea-ice structure and composition may be found in Weeks and Ackley (1983).

The thermodynamic behaviour of a solid material is determined by its thermal diffusivity and the diffusion equation. Thermal diffusivity K, is the ratio of thermal conductivity, k, to the product of density, ρ , and specific heat, C. However, because sea ice is not a pure solid and because its individual components differ in their values of k, ρ and C, representative bulk values of these parameters must be determined if the overall thermodynamic behaviour of sea ice is to be modelled without excessive complexity. Bulk density is simply a volume-weighted average of the component densities. The bulk specific heat, in the absence of phase change, is a mass-weighted average of the component specific heats. However, in the case of sea ice the specific heat is a more complex quantity, since cooling results in a progressive freezing of water from the brine inclusions (with a consequent release of heat of fusion), in the concentration of brine (with a release of heat of dilution), in addition to a cooling of the constituents (with release of sensible heat). Bulk thermal conductivity depends on the geometrical arrangement of the constituents as well as

on their conductivities. A suitable conductive model for sea ice has vertically-oriented conductors made of bubbly ice connected in parallel with conductors of brine where the cross-sectional areas of the conductors are proportional to the volume ratios of the constituents. Calculations of the bulk thermodynamic properties of sea ice from a knowledge of the constituents are reported by Anderson (1958), Schwerdtfeger (1963) and Ono (1968).

Experimental determinations of the bulk thermodynamic properties of sea ice fall into 2 categories: laboratory studies in which samples of sea ice are removed from an ice sheet, or manufactured for analysis; and in situ studies which rely on ice-sheet temperature measurements to deduce the thermal diffusivity directly. Studies by Malmgren (1927), Nazincev (1959) and Dixit and Pounder (1975) fall into the first category, while those of Schwerdtfeger (1964, 1966), Weller (1968) and Ono (1965, 1966, 1968) fall into the second.

The laboratory environment facilitates accurate measurement of the dependence of parameters on ice temperature and bulk salinity. However, application of laboratory results to natural ice sheets is not straightforward since temperature and salinity profiles are in general not known. Moreover, because of the large horizontal scale of structural organization in sea ice, and because of the drainage of brine from ice samples withdrawn for analysis, representative values of bulk salinity and density are difficult to determine. In addition, convective transfer of heat occurs within the brine channels of natural sea ice (Lake and Lewis, 1970; Niedrauer and Martin, 1979), thereby increasing the effective thermal conductivity above laboratory derived molecular values. In situ techniques measure the effective values of thermal parameters without requiring adjustment for convective effects and with little more effort than that involved in measuring the temperature and salinity profiles required for utilization of laboratory data.

The in situ techniques of Schwerdtfeger and of Weller derive thermal diffusivity from the amplitude and phase changes with depth of indenti-

fiable temperature fluctuations propagating into an ice sheet. The technique used by Ono deduces diffusivity from profile data at a depth where the vertical temperature gradient is zero. Neither technique is suitable for general application.

In this paper, a general technique is developed for the determination, in situ, of the thermal parameters of sea ice. The limitations on the technique imposed by sampling requirements are discussed. For illustrative purposes the method is applied to an existing sea ice data set and a major difficulty related to horizontal inhomogeneity in ice temperature is so identified.

2. MATHEMATICAL FORMULATION

The thermodynamical model of the ice sheet used in the determination of the thermal parameters is one-dimensional in z where the z-axis is directed upwards. Any temperature variations in x or y, due to changes in ice composition and thickness or in snow cover are ignored since the potential value of a simple model is to be explored.

Thermal parameters of the ice sheet (conductivity, k; density, ρ; specific heat, C; thermal diffusivity, $K = k/(\rho C)$) are assumed to be functions of temperature T and salinity S alone. There are variations in these parameters with density (equivalently, air content) but these are of lesser importance relative to those associated with the large temperature and salinity ranges typically found in sea ice (-40 to -1.5^{o}C; 0 to 15).

The effect of solar heating of the interior of the ice is not incorporated. In polar regions, insolation is small or absent throughout the winter and penetration of solar radiation through snow cover is small (Grenfell and Maykut, 1977).

The thermal diffusion equation describing temperature changes within a solid is:

$$\rho C \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z}\right), \qquad (1)$$

where t is time. Temperature is a function of both z and t. Therefore:

$$dT = \frac{\partial T}{\partial z} dz + \frac{\partial T}{\partial t} dt. \qquad (2)$$

On an isotherm, dT = 0, so that:

$$\frac{\partial T}{\partial t} = - \frac{\partial T}{\partial z} \frac{dH_\theta}{dt}, \qquad (3)$$

where $H_\theta(t)$ is the vertical position of the θ isotherm. The right-hand side of (1) can be expanded to:

$$\frac{\partial}{\partial z}\left(k\,\frac{\partial T}{\partial z}\right) = k\,\frac{\partial^2 T}{\partial z^2} + \frac{\partial T}{\partial z}\left(\frac{\partial T}{\partial z}\,\frac{\partial k}{\partial T} + \frac{\partial S}{\partial z}\,\frac{\partial k}{\partial S}\right). \tag{4}$$

Substituting (4) and (3) into (1) yields:

$$\frac{\partial^2 T}{\partial z^2}\left(\frac{k}{\rho C}\right) + \left(\frac{\partial T}{\partial z}\right)^2 \frac{\partial k/\partial T}{\rho C} + \frac{\partial T}{\partial z}\,\frac{\partial S}{\partial z}\,\frac{\partial k/\partial S}{\rho C} = -\frac{\partial T}{\partial z}\,\frac{\partial H_\theta}{\partial t}\,. \tag{5}$$

This linear equation on isotherm θ has unknown coefficients $k/(\rho C) = K$, $(\partial k/\partial T)/(\rho C)$ and $(\partial k/\partial S)/(\rho C)$. The other terms in (5) may be evaluated, if measurements of temperature and salinity at positions through the thickness of the ice and at several times are available. Because the values of the temperature terms will change in time in response to varying snow cover and atmospheric temperatures and the value of the salinity term will change due to brine drainage, substitution into (5) of data at each time when isotherm θ crosses isohaline σ produces a number of equations in the three unknown thermal parameters at (θ,σ). In general the system of equations obtained will be overdetermined and application of standard least-squares techniques will yield the optimum solution.

A graphical interpretation of (5) is possible if the equation is rearranged into the form:

$$Y_\theta = -\frac{\partial \ln k}{\partial T}\,X_\theta - \frac{\partial \ln k}{\partial S}\,Z_\theta - \frac{1}{K}\,, \text{ for } \frac{\partial T}{\partial z}\,,\ \frac{\partial H_\theta}{\partial t} \neq 0. \tag{6}$$

Here $(X_\theta, Y_\theta, Z_\theta)$ is observable where: $X_\theta = (\partial T/\partial z)/(\partial H_\theta/\partial t)$, $Z_\theta = (\partial S/\partial z)/(\partial H_\theta/\partial t)$, and $Y_\theta = (\partial^2 T/\partial z^2)/\{(\partial T/\partial z)(\partial H_\theta/\partial t)\}$ is related to the curvature of the temperature profile within the ice. A plot of $(X_\theta, Y_\theta, Z_\theta)$ from the various times when isotherm θ crosses isohaline σ will scatter about a plane with y-intercept $-1/K$ and slopes $-(\partial \ln k/\partial T)$ and $-(\partial \ln S/\partial T)$ in the YX and YZ planes, respectively.

3. SAMPLING LIMITATIONS

Three conditions must be satisfied by observations used to deduce the thermal parameters of sea ice in the manner described above. First, the sampling rate in time must be adequate to resolve the highest frequency in $\partial T/\partial t$ which has significant amplitude. Second, the spacing of temperature sensors must be adequate to resolve the vertical wavelength corresponding to fluctuation at this frequency. Third, the data used to compute the least-squares solution for thermal parameters on each isotherm must be separated sufficiently in time to be statistically independent. Sampling for salinity must be sufficiently rapid to resolve temporal changes associated with brine drainage, and sufficiently closely spaced in the vertical to resolve variations related to the rate that the ice was formed.

The temperature of the lower surface of sea ice is maintained very close to the freezing temperature of salt water. Thus the major fluctuations in temperature within the ice originate in the atmosphere and propagate downwards. The ice acts as a low-pass filter on the temperature fluctuations reaching any level $z = -D$ ($z = 0$ at the upper surface). If the ice sheet is approximated as a semi-infinite solid of constant diffusivity, the transfer function of the filter is:

$$H_D(\omega) = \exp\left\{ -(1 + j) \frac{D}{\sqrt{2K}} \sqrt{\omega} \right\}, \tag{7}$$

where $\omega = 2\pi f$ is angular frequency. The vertical wavelength, λ, of a fluctuation of frequency, ω, is computed as the value of D at which the shift in phase by $H_D(\omega)$ reaches 2π:

$$\lambda = 2\pi \sqrt{2K}/\sqrt{\omega} \tag{8}$$

Determination of the interval separating statistically independent values of $\partial T/\partial t$ requires knowledge of the spectrum of $\partial T/\partial t$ over a range of

periods from perhaps 1 hour to 30 days. This information is not available. It may be surmised that the spectrum of T over this range at z = 0 is "red", so that of $\partial T/\partial t$ (spectrum of T multiplied by ω^2) is approximately "white". Thus the shape of the spectrum of $\partial T/\partial t$ at z = -D will resemble $H_D^*(\omega)H_D(\omega)$. The corresponding auto-correlation function of $\partial T/\partial t$ is then:

$$B_D(\tau) = \int_{-\infty}^{\infty} H_D^*(\omega)H_D(\omega)e^{i\omega\tau}\, d\omega, \tag{9}$$

and the decorrelation time (interval between independent values), which may be defined as the lag at which the parabolic approximation to $B_D(\tau)$ for small τ is zero, is $\tau_D = \sqrt{2}/\omega_D$. $\omega_D = \sqrt{30}\ K/D^2$ is the root-mean-square width of $H_D^*(\omega)H_D(\omega)$, so that

$$\tau_D = \frac{D^2}{\sqrt{15}\ K} \tag{10}$$

Values of the decorrelation time at various depths are listed in Table 1 for ice of constant diffusivity ($K = 10 \times 10^{-7} m^2 s^{-1}$).

Sampling of temperature must occur at intervals shorter than τ_D since the values used to evaluate temporal derivatives must be correlated. Since a cubic fit to data requires 4 points, a sample interval equal to $\tau_D/3$ is appropriate. Values are shown for various depths in Table 1 with corresponding values of sensor spacing.

The decorrelation time is very long (~1-2 weeks) near the base of sea ice of thickness typically found in late winter (1-2m). As a consequence, the acquisition of an appreciable number of statistically independent data at these levels requires a very long observational period. Even at a depth of 0.5 m, only 10 independent values can be obtained per week. In natural sea ice the temperature of the ice/water interface is about -2°C and the

Table 1: Estimated Decorrelation Times, and Suitable Sample Intervals and Sensor Spacings for Temperature Measurements in Sea ice ($K = 10 \times 10^{-7}\ m^2 s^{-1}$).

Depth (m)	Decorrelation Time (hours)	Sample Interval (hours)	Sensor Spacing (m)
0.05	0.18	0.06	0.05
0.10	0.72	0.24	0.10
0.20	2.9	0.96	0.21
0.30	6.5	2.2	0.31
0.50	18	6.0	0.52
0.75	40	13	0.78
1.00	72	24	1.0
1.25	110	37	1.3
1.50	160	54	1.6
2.00	290	96	2.1

thermal diffusivity decreases by a factor of 10 or more towards that level. Thus decorrelation times are generally even greater than those computed above using a constant value of K.

The time and space intervals listed in Table 1 are only a rough guide. Since the estimation of vertical derivatives requires data from several levels, then the sampling intervals (spatial and temporal) at the level of interest must be the smallest of those appropriate at all levels used in the computation. Moreover, near the ice-water interface, derivatives must be computed by extrapolation downward from sensor levels within the ice, and a smaller spacing than indicated in Table 1 is actually required. Higher than minimum sampling rates will permit some reduction in error variance through data smoothing.

Estimates of the required accuracy and precision of temperature measurement can be made on the basis of Table 1 and data to be discussed in the next section. Table 2 shows the ranges in value of first derivatives observed near the upper and lower surfaces of a sea-ice sheet. Suitable values of Δt and Δz, the sample interval and the sensor spacing from Table 1, are also listed. For an accuracy in $\partial T/\partial t$ and $\partial T/\partial z$ equal to 1% of the range in value, the accuracy, δT, required for temperature is listed in the lower part of Table 2. Both Δt and Δz must also be accurate to 1%. Such an accuracy in time measurement is not problematic but the necessary accuracies of 1 mm in position and 0.001^{o}C in temperature pose experimental challenges.

The physics which determine gradients and variations with time of salinity in sea ice are not well understood, and sampling requirements are thus difficult to specify. Recent observations by Nakawo and Sinha (1982) do provide some general guidance, however. Rapid (over 1-10 days) desalinization by brine drainage occurs near freeze/thaw levels, whereas at levels with lower temperature, salinity changes are very slow. Thus near freeze/thaw levels spatial and temporal resolution should probably be high (perhaps $\sim$5 cm and $\sim$1 day), whereas at interior levels resolution may be reduced (to perhaps $\sim$20 cm and $\sim$10 days). Because the destructive

Table 2: Observed Ranges of Temperature Derivatives in Sea Ice (Lewis, 1967) and the Measurement Accuracy Required for 1% Accuracy in their Estimation.

	DEPTH WITHIN ICE	
	0.1m	1.5m
Observed Range $\partial T/\partial z$ [deg/m]	10-20	15-16
Observed Range $\partial T/\partial t$ [deg/s]	$\left.\begin{matrix}-30\\+30\end{matrix}\right\} \times 10^{-6}$	$\left.\begin{matrix}1\\2\end{matrix}\right\} \times 10^{-6}$
Sensor Spacing Δz [m]	0.1	0.1
Sample Interval Δt [s]	10^3	2×10^5
Temperature Accuracy δT [^oC]	.007 (for $\partial T/\partial z$) .0004 (for $\partial T/\partial t$)	.001 (for $\partial T/\partial z$) .001 (for $\partial T/\partial t$)
Position Accuracy δz [m]	.001	.001
Time Accuracy δt [s]	10	2×10^3

nature of salinity determination for sea ice prohibits observation of temperature and salinity at the same points, precision of salinity measurement exceeding 0.5 is unnecessary since the horizontal inhomogeneity in brine distribution for practically sized ice samples is of this order.

4. OBSERVATIONAL DATA

Observations suitable for application of the procedure discussed above do not appear to exist. However, a study by Lewis (1967) comes close to satisfying the considerations of the preceding section with regard to temperature. In Lewis' experiment thermistor chains were frozen into sea ice approximately 1.8 m thick in Cambridge Bay in late winter. The vertical separation of thermistors was 0.15 m, which is adequate for estimation of $\partial T/\partial z$ at depths below 0.3 m. The accuracy of emplacement was ± 0.5 mm and meets the constraints of Table 2. Temperatures were sampled every hour, an interval adequate again below 0.3 m. However, accuracy of temperature measurement was only $0.01^{\circ}C$, a value that falls far short of the $0.001^{\circ}C$ requirement in Table 2. A salinity profile was determined by Lewis (1967) at 15 cm resolution, but temporal changes in salinity were not followed. For this reason salinity dependence is ignored in this illustrative calculation, so that (6) is simplified to a linear dependence of Y_θ on X_θ, (Lewis observed salinity to decrease to a minimum of 0.8 at 40 cm depth from values of 1.6 and 5.0 in the uppermost and lowest 15 cm layers, respectively).

The observations were prepared for analysis in the following manner. Time sequences at each level were manually despiked and passed through a low-pass filter which limited the bandwidth to a value of 0.15 cph commensurate with the vertical thermistor spacing. Selected isotherms were located within the ice sheet at each observation time by interpolating between measurement levels using cubic Lagrange Polynomials. Values of the first-order quantities $\partial T/\partial z$ and $(\partial^2 T/\partial z^2)/(\partial T/\partial z)$ were calculated at the depths of the isotherms using the same polynomials. Isotherm depths were subsampled at intervals of time appropriate to each isotherm (Table 1) and values of $\partial H_\theta/\partial t$ computed using a quadratic Lagrange Polynomial. At times when $\partial H_\theta/\partial t$ fell within the range of experimental noise, data were not used because of their detrimental effect on the least-squares solution of (5). Zero values of $\partial T/\partial z$ which cause similar problems were not encountered in these observations.

Each set of derivatives at a particular time on an isotherm formed an independent two-parameter version of (5). The systems of independent equations for each isotherm were passed to a standard subroutine for solution. Data from two thermistor chains, separated by 20 m, were so treated.

Isotherms within the ice sheet at chain #30 are depicted in Figure 1. The attenuation of fluctuations with depth, the preferential attenuation of high-frequency fluctuations and the growing phase lag with depth are evident.

Figure 2 presents a selection of scatter plots of the variables in (6) for various isotherms. A linear arrangement of the data indicates that the mathematical formulation described in Section 2 is appropriate to the physical situation. The intercept of such a line is $-1/K$, and the slope $-\partial(\ln k)/\partial T$. The scatter plots in Figure 2 appear linear for higher temperature isotherms near the ice-water interface, but bilinear for lower temperature isotherms near the ice-snow interface.

The bilinear behaviour is startling. Simple arguments based on (6) indicate that it must result from systematic underestimates of $\partial^2 T/\partial z^2$ for some portion of the record. An argument is presented in the Appendix suggesting that uneven and variable snow cover is the cause. Lewis (1967) records the presence of an uneven snow cover of up to 50 cm thickness at the experiment site. Because snow has such low thermal conductivity, a snowdrift 20 cm high and 2 m across will generate near the ice-snow interface horizontal first and second derivatives of temperature comparable to those in the vertical. Thus the one-dimensional formulation in Section 2 may not be appropriate for parameter estimation in many observational situations. In the scatter plots which show bilinear behaviour, data define one line during the first portion of the record, and then switch to the other line for the remaining portion. It is hypothesized that the switch occurs during a significant rearrangement of the snow cover, presumably by strong winds. Deep in the ice where the horizontal gradients are much reduced by horizontal heat conduction at higher levels, the effect of

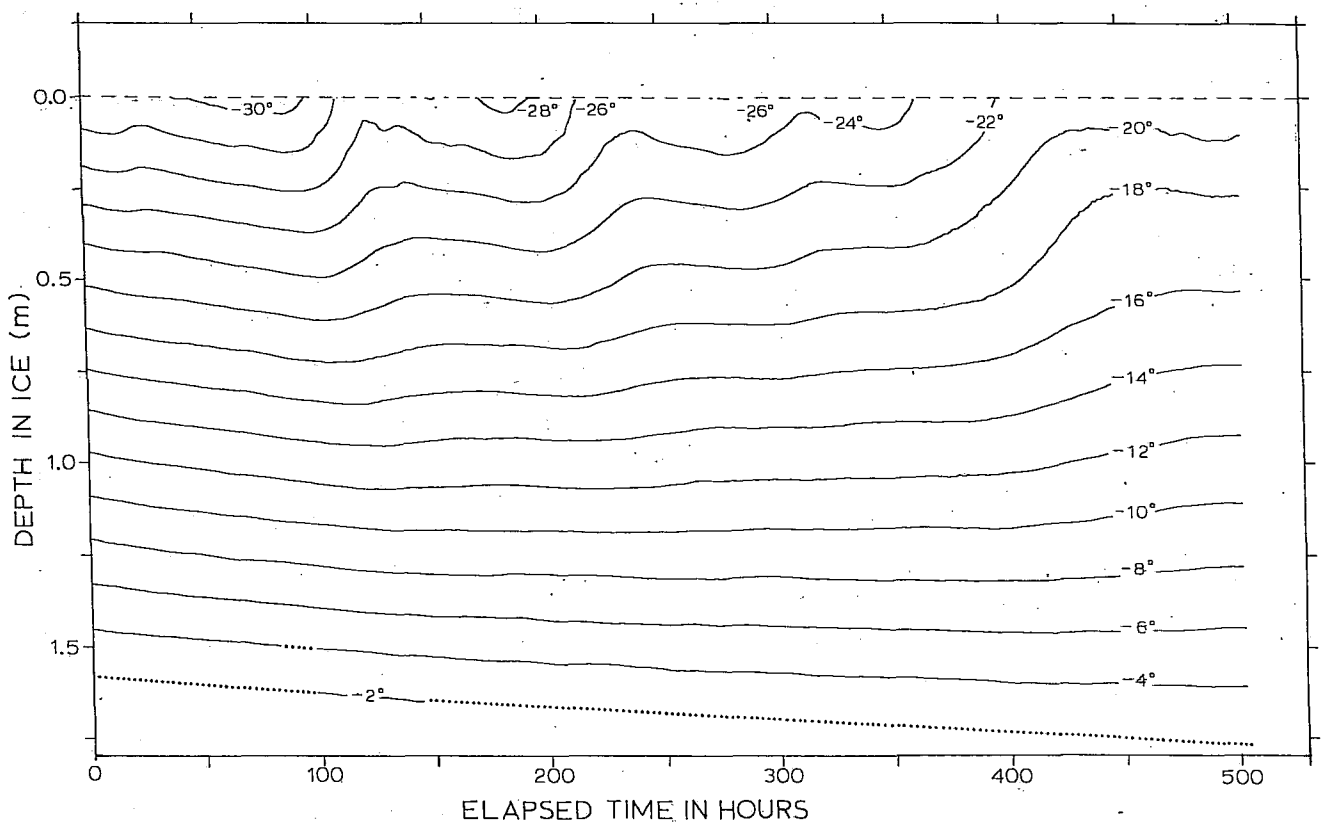

Figure 1. Depth of isotherms within the ice sheet versus time. Labels indicate degrees Celsius. The ice-water interface lies just beneath the extrapolated position of the -2°C isotherm (dotted line). These data were derived from Chain #30 (Lewis, 1967).

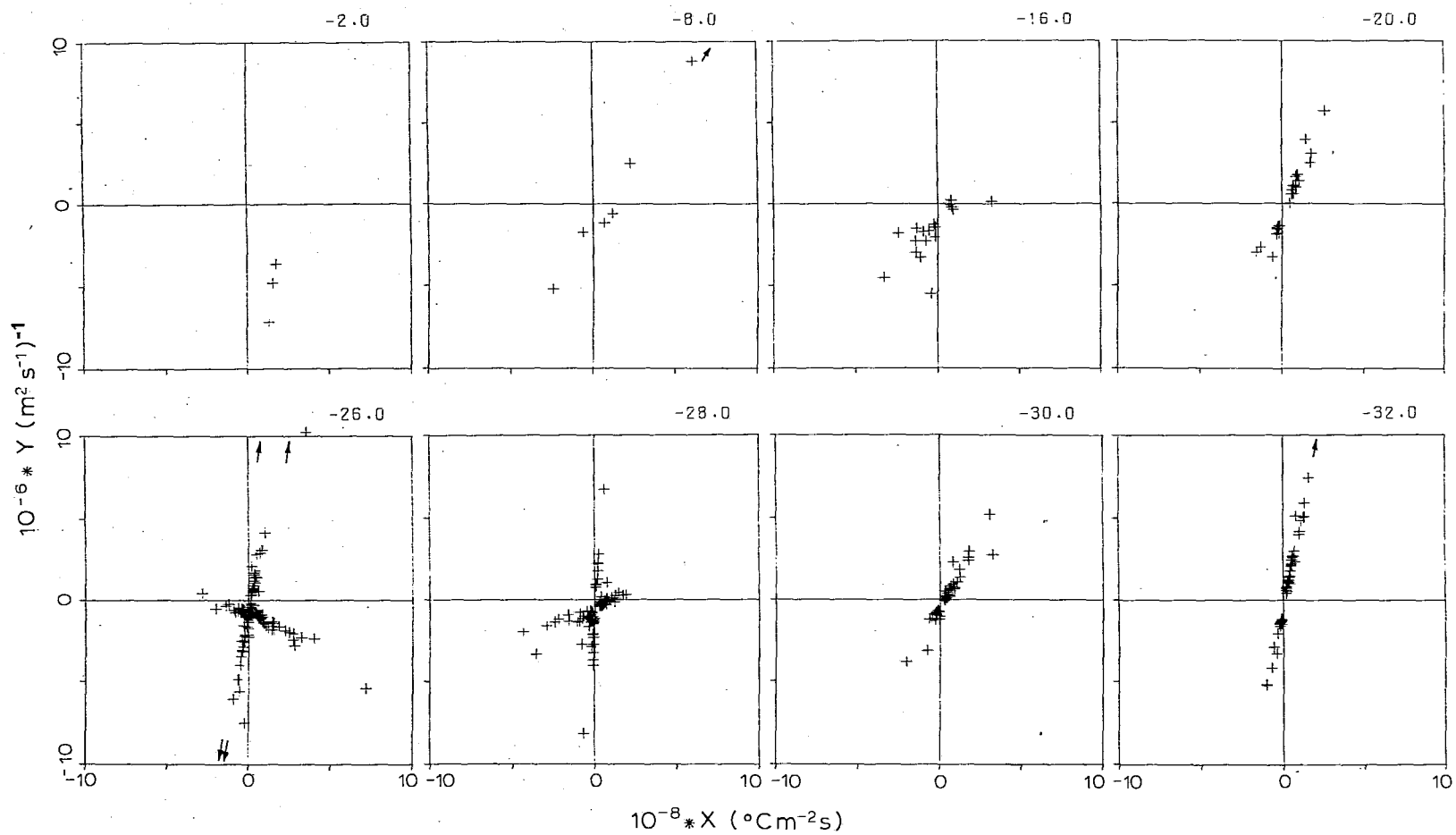

Figure 2. Scatter plots displaying observed relationships between temperature derivatives on various isotherms. The one-dimensional diffusion equation requires that data on these plots scatter about a straight line with vertical axis intercept equal to -1/K. The plot for -32°C is derived from Chain #10. The other plots are derived from Chain #30 (Lewis, 1967).

drifts is felt only weakly and the parameters estimated are more closely representative of the sea ice. The plot for -30°C (Figure 2) appears contrary to this argument, as it is closest to the ice-snow interface and yet shows only a single line. The explanation lies in the short sojourn of this isotherm in the ice (Figure 1), so that only a single snow drift configuration was encountered.

Figure 3 displays the derived values of thermal diffusivity versus isotherm temperature, and curves indicating Schwerdtfeger's (1963) values for two salinities in the range reported by Lewis (1967). In view of the large scatter for temperatures below about -20°C, attributable to the uncertain effects of uneven snow cover discussed, comment on the values in this range is superfluous. At higher temperatures where uneven snow cover is less influential, the agreement with Schwerdtfeger (1963) is encouraging. The scatter in plotted points in Figure 3 results from temperature measurement errors, from the neglect of salinity effects and from the small number of independent data obtained on isotherms at levels deep within the ice. Similar comments pertain to the values of $\partial \ln k/\partial T$ depicted in Figure 4.

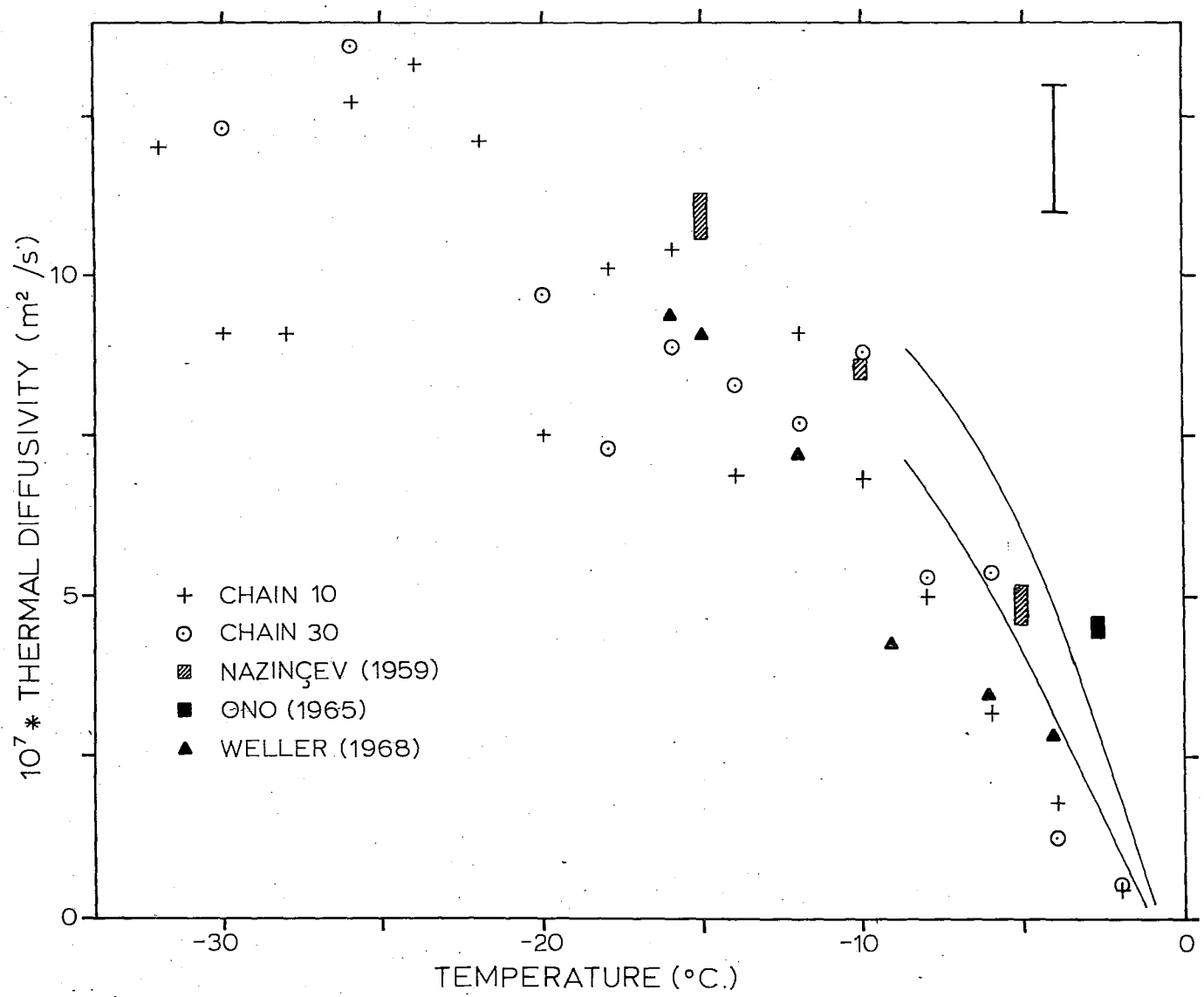

Figure 3. Plot of experimental values of thermal diffusivity against temperature. Curves are computed values from Schwerdtfeger (1963) for salinities of 2 (upper) and 4 (lower). Other comparison data are identified. The error bar indicates the accuracy estimate derived from the sum of squares. Observations from Lewis (1967).

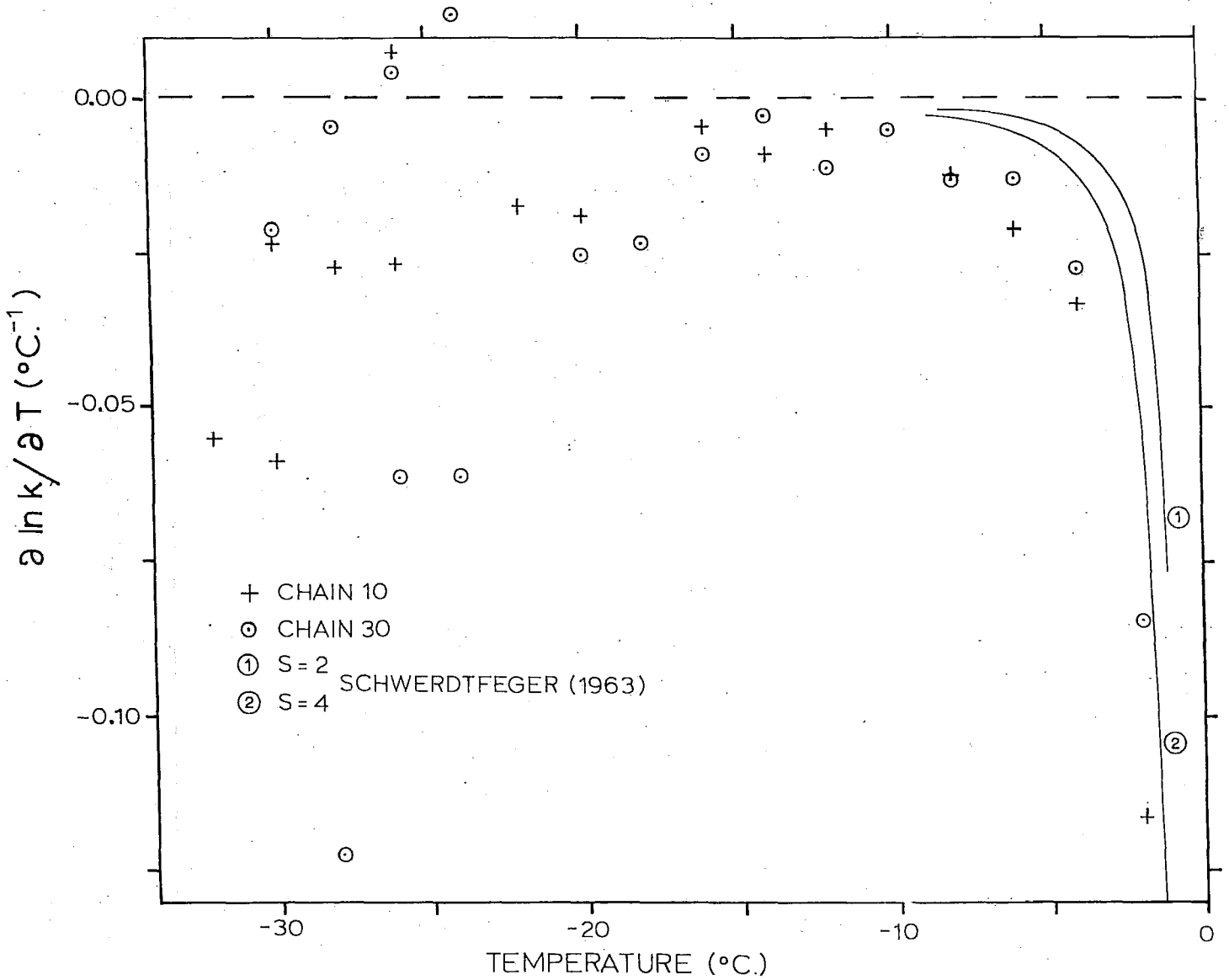

Figure 4. Plot of ∂lnk/∂T against temperature. Comparison curves are from Schwerdtfeger (1963). Observations from Lewis (1967).

5. CONCLUSIONS

The computational approach discussed is workable and produces reasonable estimates of thermal parameters for sea ice down to $-20^{o}C$ in the test case chosen for simplified analysis. The values for K and $\partial(\ln k)/\partial T)$ obtained compare quite favourably with theory and other observational data, despite the poor (by present-day standards) absolute accuracy in temperature measurement. The statistical scatter with this data set is large ($\pm 20\%$), but some improvement may be anticipated with improvement in measurement accuracy.

Two severe observational constraints have been identified. First, the isotherm considered must not be so deep in the ice that the number of independent data acquired during the observational period, is insufficient for this analysis. Second, if the isotherm level is shallow the effect of uneven snow cover force a 3-dimensional treatment of the thermal diffusion problem. The first condition is surmountable (within seasonal limits) by extending the period of observation, while the second could only be avoided by tending the snow cover at the observational site, an unrealistic alternative. Thus a 3-dimensional observational array and analysis method are probably unavoidable.

The effect of salinity on thermal parameters was neglected in the test calculation. A comparison of the second and third terms of (5), using Schwerdtfeger (1963) demonstrates that in this case the influence of temperature in the equation is approximately four times that of salinity. Thus the neglect of salinity herein may be (weakly) justified. However such is generally not true. As mentioned above, the destructive nature of salinity analysis, the drainage of brine from ice samples withdrawn for analysis and the large scale of organization of brine distribution in sea ice all combine to make the precise measurement of the vertical and temporal variations of sea ice salinity a formidable experiment problem. It is worthwhile noting that both this problem, and the problem of uneven snow cover are not unique to the methods discussed, but are also encountered when values for thermal parameters appropriate for natural ice sheets must be

chosen from laboratory data expressed as functions of temperature and salinity.

Measurement constraints for temperature are also severe. The position of sensors frozen into the ice must be known to better than 1 mm, and temperatures must be accurate to 0.0005^{o}C. However, both these requirements are achievable with careful engineering and calibration.

The approach discussed permits evaluation of the thermal conductivity of sea ice only to within a multiplicative constant. A direct measurement of heat flux either within the ice sheet or in the atmospheric or oceanic constant-flux layers is thus necessary to compute this constant.

APPENDIX:

HORIZONTAL TEMPERATURE GRADIENTS IN SEA ICE ASSOCIATED WITH UNEVEN SNOW COVER

The analysis model discussed in the main body of the paper assumes that a one-dimensional treatment of thermal diffusion is appropriate for the analysis of observations in sea-ice temperature. However, since the thermal conductivity of dry wind-packed snow is only about 1/8 that of sea ice, uneven covers of thin snow can have drastic effects on ice temperatures. In the following simplistic calculation a snowdrift of depth, h, and width, 2L, is assumed to overlie the thermistor chain embedded in ice of thickness. H. Air and water temperatures are T_A and T_W respectively, while ice and snow conductivities are k_I and k_S. No horizontal heat transports are considered. The following discussion aims only to estimate the magnitude of thermal effects due to uneven snow cover. It does not strive to represent effects in a physically exact formulation.

The ice surface temperature beneath the drift, T_I, may be estimated by assuming vertical flux continuity,

$$k_s\,(T_A - T_I)/h = k_I(T_I - T_W)/H, \qquad \text{(A1)}$$

yielding,

$$T_I = \frac{T_A + (k_I/k_s)(h/H)T_W}{1 + (k_I/k_s)(h/H)} \qquad \text{(A2)}$$

The temperature of the nearby snowfree ice will be T_A. Thus the horizontal gradient of temperature at the ice surface is the order of $(T_I - T_A)/L$

Substituting from (A2) and setting $k_I/k_s \sim 8$ yields

$$\frac{\Delta T}{\Delta x} \sim \frac{8h/L}{1 + 8h/H}\;\frac{T_W - T_A}{H} \qquad \text{(A3)}$$

Thus, the horizontal gradient is proportional to the vertical gradient in the snow-free ice. The second derivative is the order of,

$$\frac{\Delta^2 T}{\Delta x^2} \sim \frac{16h/L^2}{1 + 8h/H} \; \frac{T_W - T_A}{H} \tag{A4}$$

again proportional to the vertical gradient. Inserting typical values: $h \sim 0.2$m, $H \sim 1.6$m, $L \sim 1$m, $T_W - T_A \sim 30^oC$, gives $\Delta T/\Delta x \sim 15^oC/m$ and $\Delta^2 T/\Delta x^2 \sim 30^oC/m^2$. Values of $\partial T/\partial z \sim 19^oC/m$ and $\partial^2 T/\partial z^2 \sim 50^oC/m^2$, as found in Lewis' (1967) observations, are of comparable magnitude. Values of the horizontal derivatives will decrease with increasing depth in the ice. It is apparent from these values that a one-dimensional treatment of heat diffusion in sea ice with uneven snow cover is not appropriate.

The lines with differing slopes delineated by data shown in Figure 2 can be rationalized if a snow drift centred over the thermistor chain is assumed (snow will tend to drift about any object projecting from the ice surface). Then $\Delta T/\Delta x \sim 0$ at the chain, while $\Delta^2 T/\Delta x^2 < 0$. Thus some considerable fraction of $\nabla^2 T$ will not be detected by the thermistor array and $\nabla^2 T$ will be underestimated relative to observed values of $\partial T/\partial t$. Since the degree of underestimate is approximately proportional to $\partial T/\partial z$ (and the abscissa in Figure 2), a straight line will be seen in the data, but its slope will change with surface snow conditions.

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