Modelling the Rise of Hydrothermal Plumes

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ABSTRACT

Middleton, J.H. and R.E. Thomson. 1986. Modelling the rise of hydrothermal plumes. Can. Tech. Rep. Hydrogr. Ocean Sci. No. 69: 18 p.

This report presents models for the rise of turbulent hydrothermal plumes in neutral or stably stratified oceanic environments. Different models are used for the cases of zero ambient crossflow and significant ambient crossflow. Specific examples are presented to provide insight into the way in which different source characteristics affect the resulting plume rise.

Key words: plumes, hydrothermal plumes, plume model

RÉSUMÉ

Le présent rapport présente des modèles pour la montée de panaches hydrothermaux turbulents dans des milieux océaniques neutres ou ayant une stratification stable. On a utilisé différents modèles dans les cas d'écoulement transversal ambiant nul et important. On donne des exemples précis pour permettre de comprendre la facon dont les différentes caractéristiques des sources influent sur la montée du panache qui en résulte.

Mots-clés: panaches, panaches hydrothermaux, modèles de panache.

1. INTRODUCTION

Hydrothermal vents are regional sources of superheated recirculated crustal water located within the ridge complexes of seafloor spreading centres. The buoyant plumes emanating from such vents result in distinct temperature and chemical anomalies that are detectable for tens of kilometres from the source (Crane *et al*., 1985). Although the physical, biological and chemical processes that determine the off-axis dispersion and composition of the plume are not well understood, hydrothermal emissions are considered important in the redistribution of trace metals and in their eventual concentration in sediments away from the vent field. Hydrothermal plumes may also have a sufficient dynamical effect on the regional oceanography to drive a mean abyssal circulation (Reid, 1982; Stommel, 1982) and may be important in the recolonization of benthic animals living near the vents.

The Endeavour segment of the Juan de Fuca Ridge is an area of particularly active research by Canadian and U.S. scientists. Multidisciplinary investigations of the biological, chemical, geological and physical oceanographic aspects of the region have been under way for approximately three years and various plans exist for continued work within the ridge domain. As yet, however, there is no comprehensive understanding of the mechanisms controlling the formation, dispersion and chemical "aging" of hydrothermal plumes in this area. The absence of information on the spatial and temporal variability of the oceanic circulation in the general vicinity of the ridge is also a major factor contributing to this lack of understanding.

The purpose of this report is to identify and outline those plume theories which appear to be most appropriate for modelling the rise of the plume from the vent source into the ocean interior, and to identify the important parameters which require measurement in order to apply the models. Salient features to be considered are:

i The plumes are emitted from vent fields having a radius of the order of tens of metres or more;

ii The surrounding ocean is stably stratified;

iii Semidiurnal tidal currents of up to 0.2 ms^{-1} provide a crossflow of varying strength and direction.

Based on the above features we will require plume theories capable of predicting the plume rise from sources of finite size in neutral or stably stratified environments which may or may not have a significant crossflow.

Morton, Taylor and Turner (1956) investigated the rise of turbulent buoyant plumes from point sources into an otherwise motionless but stably stratified environment. For constant Brunt-Väisälä frequency, N, the equations of motion may be reduced to a set of three coupled linear differential equations representing conservation of mass, momentum and buoyancy. Morton (1959) and Morton and Middleton (1973) extended this work to study the rise of plumes from real (finite size) sources of mass flux, momentum flux and buoyancy flux. Middleton (1979) has determined times of rise for these plumes. Such models may be used to predict vertical velocity, buoyancy and plume radius as a function of height for plumes issuing from real sources into a stably stratified environment with no crossflow.

In a crossflow of velocity, U, plumes are accelerated to crossflow velocity in the immediate neighbourhood of the source, resulting in a 'bent-over' plume (Priestley, 1956). Equations representing conservation of mass, momentum and buoyancy per unit length of plume were found for bent-over plumes from point sources by Slawson and Csanady (1967, 1971). More recently, Middleton (1985) has examined the rise of plumes from real (finite size) sources into both neutral and stably-stratified crossflows. Middleton's (1985) extension to real sources of Slawson and Csanady's (1971) theory is analagous to Morton and Middleton's (1973) extension to real sources of the theory of Morton, Taylor and Turner (1956).

The purpose of this report is to outline models pertinent to the 'first-stage' rise of turbulent forced plumes in a stably stratified or neutral crossflow, where buoyancy generated turbulence dominates the plume mixing. Near the maximum height of rise in a stable environment, the level of buoyancy generated turbulence within the plume will drop until it becomes comparable to that of the ambient turbulence. At this stage the ambient turbulence begins to control the plume dilution and dispersion, and the problem becomes that of diffusion of a passive scalar.

In recognition of specific application to hydrothermal plumes in the ocean, account is taken of contributions to plume buoyancy from both temperature and salinity anomalies at the source, and contributions to the overall environmental stability from ambient vertical temperature and salinity gradients.

This report is organised as follows. The next section deals with plumes rising from real sources into a stably stratified, but otherwise still, environment. Section 3 is concerned with plume rise from real sources into a stably stratified crossflow, while in Section 4 we discuss examples of these models.

2. PLUME RISE IN A MOTIONLESS ENVIRONMENT

Similarity solutions for turbulent plume rise in both neutral and stably stratified, but otherwise still environments, were derived by Morton, Taylor and Turner (1956). They assumed that profiles of mean vertical velocity and buoyancy are similar at all plume cross-sections, that the rate of entrainment of fluid at any height is proportional to a characteristic vertical velocity at that height, and that local variations in density throughout the incompressible fluid are small compared to some reference density.

We designate z as the vertical axis of the axisymmetric plume, b the plume radius, w the mean vertical velocity and ρ the plume density (Figure 1). If $\rho_e(z)$ is the environmental density, ρ_o the reference density and the entrainment constant, then equations representing conservation of mass (or volume) and momentum can be written as

$$\frac{d}{dz}(b^2 w) = 2\alpha b w \quad , \tag{2.1}$$

$$\frac{d}{dz}(b^2 w^2) = b^2 g\left(\frac{\rho_e - \rho}{\rho_o}\right) . \tag{2.2}$$

For oceanographic applications the buoyancy is in general a function of at least two variables, temperature T and salinity S, such that

$$\left(\frac{\rho_e - \rho}{\rho_o}\right) \simeq \beta(T - T_e) + \varepsilon(S_e - S) \tag{2.3}$$

Here β is the coefficient of thermal expansion and ε the coefficient of haline contraction obtainable, for example, from Gill (1982, Appendix 3), and T_e and S_e represent environmental background values of temperature and salinity, respectively.

Equations representing conservation of buoyancy for each component are

$$\frac{d}{dz}[b^2 w g \beta (T - T_e)] = - b^2 w g \beta \frac{dT_e}{dz} \quad , \tag{2.4}$$

$$\frac{d}{dz}[b^2 w g \varepsilon (S_e - S)] = b^2 w g \varepsilon \frac{dS_e}{dz} \quad . \tag{2.5}$$

Here we have chosen the quantities of temperature excess $T-T_e$ and salinity deficit S_e-S in such a way that positive values of each contribute to positive (upward) buoyancy.

Some simplification of (2.1) to (2.5) is preferable to facilitate solution. Bulk dimensional quantities of volume flux, momentum flux and buoyancy flux may be defined by

$$V = b^2 w,\quad M = b^2 w^2 ,\quad F^T = b^2 wg\beta(T-T_e) ,\quad F^S = b^2 wg\varepsilon(S_e-S) . \tag{2.6}$$

Writing

$$N_T^{\ 2} = g\beta\frac{dT_e}{dz} \qquad\qquad N_S^{\ 2} = -g\varepsilon\frac{dS_e}{dz} \tag{2.7}$$

the conservation equations then become

$$\frac{dM}{dz} = 2\alpha V \qquad\qquad \frac{dV}{dz} = \frac{(F^T + F^S)}{2V^3}$$

$$\frac{dF^T}{dz} = -\ N_T^{\ 2}M \qquad\qquad \frac{dF^S}{dz} = -\ N_S^{\ 2}M \tag{2.8}$$

Morton et al. (1956) solved these equations numerically for the case $F^S = 0$, $N_S = 0$ from a source at $z = 0$ categorized by $F_o^{\ T} \neq 0$, $V_o = 0$ and $M_o = 0$. Morton (1959) and Morton and Middleton (1973) showed how these equations could be integrated to find plume rise as a function of finite source fluxes V_o, M_o and F_o and cast the problem in dimensionless form.

Integration of (2.8) may be achieved using a Runge-Kutta technique (for example, IMSL routine DVERK) from the source at $z = 0$ where $V = V_o$, $M = M_o$. $F^T = F_o^{\ T}$ and $F^S = F_o^{\ S}$ to the level of maximum rise where $M = 0$. At each level required, plume values of radius b, vertical velocity w, temperature excess $T-T_e$ and salinity deficit S_e-S may be found from the bulk quantities using (2.6). Although it is commonly assumed that the stratification parameters $N_S^{\ 2}$ and $N_T^{\ 2}$ are constant, any functional form of $N_T^{\ 2}(z)$ and $N_S^{\ 2}(z)$ may be used when integrating (2.8). For deep ocean applications, constant values do, however, provide satisfactory approximations. An appropriate value for the entrainment constant for plume rise into a motionless environment is $\alpha = 0.1$ (Morton and Middleton, 1973).

Some straightforward results are obtainable for plumes emanating from point sources where $b_o \to 0$, $M_o \to 0$. For 'top-hat' profiles (used for the calculations in this section), where the vertical velocity and buoyancy are assumed to be uniform over the plume cross-section, the plume equations may be written in dimensionless form (c.f. Morton et al., 1956, equation 11) with the vertical axis scaled as

$$z = 2^{-5/8}\ \alpha^{-\frac{1}{2}}\ F_o^{\ \frac{1}{4}}\ N^{-3/8}\ z_1 \tag{2.9}$$

where z_1 takes a value of $z_1 = 2.1$ at the level of zero buoyancy and $z_1 = 2.8$ at the level of zero momentum. Using $\alpha = 0.1$ the predicted dimensional heights are

$$z = 5.7\ F_o^{1/4}\ N^{-3/8} \qquad \text{(zero momentum)} \tag{2.10}$$

$$z = 4.3\ F_o^{1/4}\ N^{-3/8} \qquad \text{(zero buoyancy)} \tag{2.11}$$

These compare favourably with measurements of smoke rise in the atmosphere by Briggs (1969) where the observed height of rise is

$$z = 5.0\ F_o^{1/4}\ N^{-3/8} \tag{2.12}$$

The observed height of rise is therefore approximately mid-way between the predicted heights of zero momentum and zero buoyancy. This is probably due to the increasing importance of the ambient turbulence to plume dilution toward the maximum height of rise, and because of the fact that the plume might be expected to fall on account of its negative buoyancy at the maximum height of rise, while continuing to entrain. Middleton (1979) shows times of rise for such plumes to be

$$t \simeq \pi\ N^{-1} \tag{2.13}$$

Approximations (2.9)-(2.13) allow ready calculation for plumes from point sources of buoyancy, and may also be used to check the numerically integrated solutions to (2.8).

3. PLUME RISE IN A UNIFORM CROSSFLOW

Similarity solution may be derived for turbulent plume rise in a stratified crossflow for the case where the crossflow is sufficiently strong that the plume is bent-over in the immediate vicinity of the source (Slawson and Csanady, 1971; Middleton, 1985). As for the case of vertical plume rise, we assume that profiles of buoyancy and velocity are similar at all plume cross-sections, that the rate of entrainment of fluid into the plume at any height is proportional to the mean vertical velocity at that height, and that local variations in plume density are small compared with the reference density.

We consider a nearly horizontal plume with uniform cross-section having mean radius b, mean vertical velocity w and mean density ρ in a stratified crossflow of velocity U. Defining the origin at the plume source (as in Figure 2) with the horizontal x axis in the direction of crossflow gives the following kinematic relationships:

$$\frac{dz}{dx} = \frac{w}{U} \quad , \quad w = \frac{dz}{dt} \quad , \quad x = Ut \tag{3.1}$$

where z(x) is the plume trajectory.

Equations representing conservation of volume and momentum per unit length following the bent-over plume are:

$$w \frac{d}{dz}(b^2) = 2\alpha bw \quad , \tag{3.2}$$

$$w \frac{d}{dz}(b^2 w) = b^2 g\left(\frac{\rho_e - \rho}{\rho_o}\right) \tag{3.3}$$

where, as in §2, the buoyancy has contributions from both the temperature excess $(T-T_e)$ and the salinity deficit (S_e-S) with $(\rho_e-\rho)/\rho_o$ determined via (2.3).

Equations representing conservation of buoyancy for each component are:

$$\frac{d}{dz}[b^2 g\beta(T-T_e)] = - b^2 g\beta\frac{dT_e}{dz} \tag{3.4}$$

$$\frac{d}{dz}[b^2 g\varepsilon(S_e-S)] = b^2 g\varepsilon\frac{dS_e}{dz} \tag{3.5}$$

Solutions of (3.2) gives

$$b = b_o + \alpha z \tag{3.6}$$

where b_o is the plume radius at $z = 0$, the origin of the bent-over plume. To facilitate solution of (3.3), (3.4) and (3.5), we define dimensional bulk quantities of momentum flux and buoyancy flux per unit length of plume by

$$m = b^2 w \quad , \quad f^T = b^2 g\beta(T-T_e) \quad , \quad f^S = b^2 g\varepsilon(S_e-S) \tag{3.7}$$

Equations representing conservation of momentum (3.3) and buoyancy (3.4) and (3.5) then become

$$\frac{dm}{dz} = \frac{b^2}{m}\,(f^T + f^S) \tag{3.8}$$

$$\frac{df^T}{dz} = - b^2 N_T^{\,2} \quad , \quad \frac{df^S}{dz} = - b^2 N_S^{\,2} \tag{3.9}$$

where N_T^2 and N_S^2 are defined as in (2.7)

Solution of (3.6)-(3.9) may be accomplished given initial conditions $m = m_o$, $f^T = f_o^T$, $f^S = f_o^S$ at the origin $z = 0$ of the bent-over plume. To facilitate solution it is useful to relate plume quantities from the real source to those at the origin of the bent-over plume (Figure 2). Without attempting to model the complex dynamics associated with the transition from vertical to almost horizontal flow, we argue that this transition should be rapid (and the following relations valid) in cases where the free stream velocity exceeds that of the vertical velocity at the real source. Denoting the buoyancy and momentum fluxes per unit time from the real source as in (2.6) of the previous section by F_o^T, F_o^S and M_o, respectively, and denoting the buoyancy and momentum fluxes per unit length at the origin of the bent-over plume by f_o^T f_o^S and m_o, respectively, then following Middleton (1985)

$$f_o^T = U^{-1} F_o^T \quad , \quad f_o^S = U^{-1} F_o^S \quad , \quad M_o = U^{-1} M_o \tag{3.10}$$

While (3.8) and (3.9) may be integrated directly using (3.6) to find the variation of the bulk quantities (3.7) with height, the time dependence remains unknown and a better method is to formulate the problem using time as the independent variable. Using the kinematic relationships (3.1), the governing equations become

$$\frac{db^3}{dt} = 3\alpha m \tag{3.11}$$

$$\frac{dm}{dt} = f^T + f^S \tag{3.12}$$

$$\frac{df^T}{dt} = -N_T^2 m \quad , \tag{3.13}$$

Analytical solutions may be found for these for either unstratified or uniformly stratified environments.

(a) Unstratified Environment

In this case $N_T^2 = N_S^2 = 0$ and solutions to (3.11)-(3.13) are,

$$f^T = f_o^T = U^{-1}F_o^T \quad , \quad f^S = f_o^S = U^{-1}F_o^S \quad \text{(constants)} \tag{3.14}$$

$$m = U^{-1}[M_o + (F_o^T + F_o^S)t] \tag{3.15}$$

$$b^3 = b_o^3 + 3\alpha U^{-1}[M_o t + \tfrac{1}{2}(F_o^T + F_o^S)t^2] \tag{3.16}$$

$$z = \alpha^{-1}(b-b_o) \quad , \quad x = Ut \tag{3.17}$$

The buoyancy fluxes f^T and f^S remain constant and the momentum flux increases linearly with time. The radius b and height z increase monotonically with time and distance downstream. To apply the solutions, the environmental crossflow U and the source fluxes F_o^T, F_o^S and M_o must be determined. Solution for m, b, z and x as a function of time then proceeds using (3.15) to (3.17). A suitable choice for the entrainment constant is $\alpha = 0.33$ (Middleton, 1985).

(b) Stratified Environment

Where the environmental stability parameters N_T^2 and N_S^2 are constant, analytical solutions for (3.11)-(3.13) may be found. Writing

$$f = f^T + f^S \quad , \quad F_o = F_o^T + F_o^S \quad , \quad N^2 = N_T^2 + N_S^2 \tag{3.18}$$

the solutions are

$$m = U^{-1}(M_o^2 + F_o^2 N^{-2})^{\frac{1}{2}} \sin(Nt + \delta) \tag{3.19}$$

$$f = U^{-1}(F_o^{\ 2} + M_o^{\ 2}N^2)^{\frac{1}{2}} \cos(Nt + \delta) \tag{3.20}$$

$$f^T = U^{-1}F_o^{\ T} + U^{-1}N_T^{\ 2}N^{-2}\{(F_o^{\ 2} + M_o^{\ 2}N^2)^{\frac{1}{2}}\cos(Nt + \delta) - F_o\} \tag{3.21}$$

$$f^S = U^{-1}F_o^{\ S} + U^{-1}N_S^{\ 2}N^{-2}\{(F_o^{\ 2} + M_o^{\ 2}N^2)^{\frac{1}{2}}\cos(Nt + \delta) - F_o\} \tag{3.22}$$

$$b^3 = b_o^{\ 3} + 3\alpha(UN)^{-1}(M_o^{\ 2} + F_o^{\ 2}N^{-2})^{\frac{1}{2}}\{\cos\delta - \cos(Nt + \delta)\} \tag{3.23}$$

$$z = \alpha^{-1}(b - b_o) \quad , \quad x = Ut \tag{3.24}$$

where
$$\tan\delta = NM_oF_o^{\ -1} \tag{3.25}$$

The general features of the solutions as outlined by Middleton (1985) are:

i. the plume radius increases linearly with height;

ii. the momentum first increases as a result of the generation of momentum by buoyancy, then decreases becoming zero at the maximum height of rise;

iii. the buoyancy flux reduces with height, eventually becoming negative, with the upward momentum then being responsible for continued rise from the level of zero buoyancy to the level of zero momentum;

iv. the volume flux is proportional to b^2 and increases monotonically with time from its source value.

Solution is achieved using time t as the independent variable; the method is as follows. Values of b_o, M_o, F_o^T and F_o^S at the real source are specified, along with the crossflow velocity U and the stability parameters N_T^2 and N_S^2. The source flux F_o, the overall stability N^2 and the phase δ are found from (3.18) and (3.25). Momentum and buoyancy fluxes per unit length m and f are found from (3.19) and (3.20) while the contributions to the buoyancy flux from temperature and salinity are found from (3.21) and (3.22). The plume radius, height and downstream distance, x, are found from (3.24) and (3.25) while the temperature excess and salinity deficit are recovered from f^T and f^S through (3.10).

Relatively simple solutions for heights of rise are obtainable for plumes from point sources where $b_o \to 0$ and $M_o \to 0$. In this case $\delta = 0$ and the height of rise (3.24) becomes approximately

$$z = \left(\frac{6\ F_o}{\alpha^2 UN^2}\right)^{1/3} \ . \tag{3.26}$$

Observations by Slawson and Csanady (1967, 1971) in the atmosphere indicate that $\alpha \simeq 0.33$.

Inspection of (3.22) shows that times of rise from a point source to the maximum height of rise are

$$t \simeq \pi N^{-1} \tag{3.27}$$

The approximation (3.26) may be used for plumes issuing from sources which are small, and may also be used as a check calculation for the full solutions (3.18)-(3.25).

4. APPLICATION TO PLUMES FROM HYDROTHERMAL VENTS

In this section we consider plume rise in conditions typical of the Juan de Fuca ridge region. The source of the plumes is around 2000 m depth where the environmental temperature gradient is approximately $dT_e/dz \approx 10^{-3}\ C°m^{-1}$, $dS_e/dz \approx 0$ (Crane et al., 1983, Figure 3). From Gill (1982, Table A.3.1) the coefficient of thermal contraction is $\beta \approx 1.25 \times 10^{-4}(°C)^{-1}$ and $\varepsilon = 7.5 \times 10^{-4}$ ppt^{-1} (ppt ≡ parts per thousand), so that $N^2 \approx 1.23 \times 10^{-6}\ s^{-2}$ and the time of rise of plumes

$$t \approx \pi N^{-1} \approx 1 \text{ hour}$$

Crossflow velocities are typically $0.2\ ms^{-1}$ so that the horizontal distance traversed by the plume while rising is

$$x \approx 0.7 \text{ km}$$

To examine the effect that variation of the source parameters has on the plume rise, we have chosen eight examples, each having the same buoyancy flux. The plumes differ as a result of differing radii, vertical velocities, temperature excesses and salinity deficits. Examples 1 to 4 have no crossflow while examples 5 to 8 have a crossflow of $0.2\ ms^{-1}$. Examples 1 to 3 have source dimensions that increase with increasing example number so that Example 1 has effectively a point source. Example 2 has the same source size and overall buoyancy flux as Example 1, but has contributions from the salinity deficit S_e-S as well as the temperature excess T-T_e. Examples 5-8 use the same source parameters as Examples 1-4 but the plumes rise in a crossflow.

For the case of no crossflow, the maximum height of rise (2.10) for plumes from a point source is z = 372 m. Using the full equations, integrated solutions for examples 1 to 4 are shown in Figure 3 where plume radius is plotted as a function of height. Examples 1 and 2 are essentially for point sources and the calculated height of rise (365 m) is almost that of the point source. As the initial plume radius becomes larger in examples 3 and 4 the height of rise is reduced substantially below that for a point source.

With a crossflow of $U = 0.2\ ms^{-1}$, the maximum height of rise (3.26) is z = 176 m for plumes from an idealised point source. Using the full equations, integration for examples 5 to 8 provides the plume trajectories shown in Figure 4. For examples 5 and 6 where the plume rise is essentially from a point source the maximum height of rise (171 m) is only slightly below that for the idealised point source. In general for larger plume source radii the trajectories are flatter and the maximum height of rise is lower.

A full description of the physics is not appropriate here and the reader is referred to the cited references for additional details, however, the main features of the solutions are:

i. the maximum height of plume rise increases with source buoyancy flux F_o and decreases with increased crossflow U and increased stratification N^2;

ii. for given values of F_o, U and N^2 the maximum height of rise decreases with increasing source radius b_o and increases with increasing source velocity w_o.

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TABLE 1

Environmental and source parameters for plume calculations

$$\frac{dT_e}{dz} = 10^{-3}(^{\circ}C)m^{-1} \quad , \quad \beta = 1.25 \times 10^{-4}(^{\circ}C)^{-1}$$

$$\frac{dS_e}{dz} = 0 \text{ ppt } m^{-1} \quad , \quad \varepsilon = 7.5 \times 10^{-4} \text{ (ppt)}^{-1}$$

$$N^2 = 1.23 \times 10^{-6} \; s^{-2}$$

$$F_o = 0.025 \; m^4 \; s^{-3}$$

Parameter	Example No.							
	1	2	3	4	5	6	7	8
Crossflow $U(ms^{-1})$	0	0	0	0	0	0	0	0
Source radius $b_o(m)$	2.0	2.0	10.0	30.0	2.0	2.0	10.0	30.0
Source velocity $w_o(ms^{-1})$	0.2	0.2	0.1	0.1	0.2	0.2	0.1	0.1
Source temperature excess $T-T_e$(°C)	25.0	31.0	2.0	0.222	25.0	31.0	2.0	0.222
Source salinity deficit S_e-S (ppt)	0	-1.0	0	0	0	-1.0	0	0

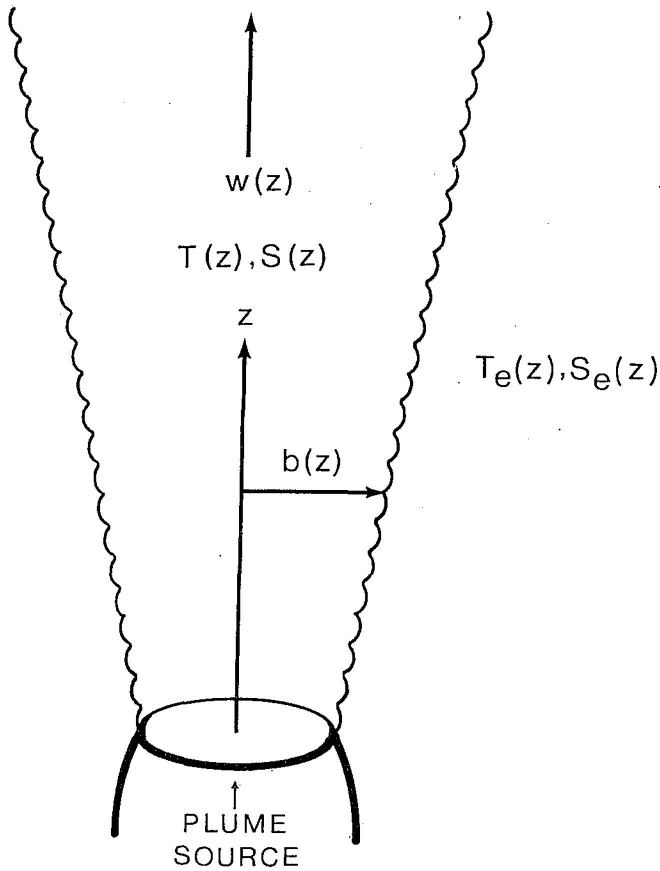

Figure 1. Schematic diagram of plume rise in a still ocean environment with environmental values of temperature $T_e(z)$ and salinity $S_e(z)$.

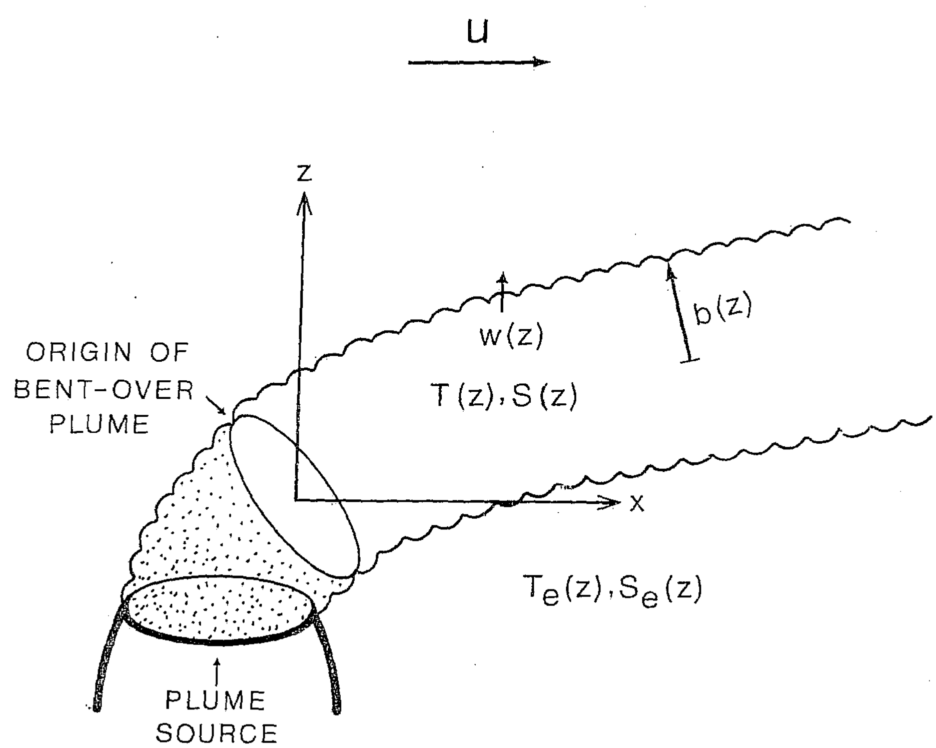

Figure 2. Schematic diagram of plume rise in an ocean crossflow of velocity U with environmental values of temperature $T_e(z)$ and salinity $S_e(z)$. The origin for the plume calculations is at z = 0. The stippled region between the real source and the origin is not explicitly modelled but the relation between source and origin values is given by (3.10).

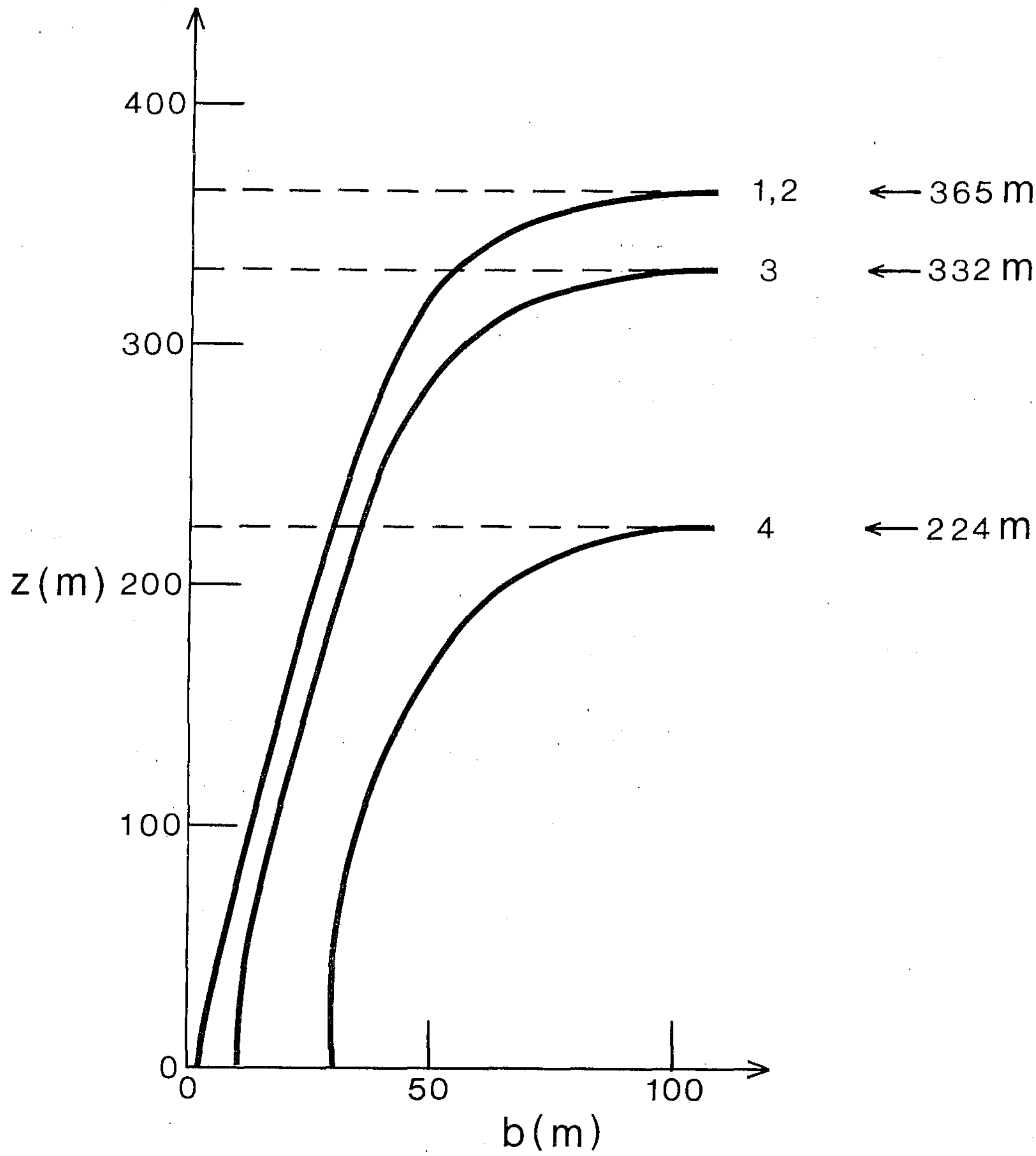

Figure 3. Plume profiles for examples 1-4 of Table 1. Each profile is identified by its example number and its maximum height of rise.

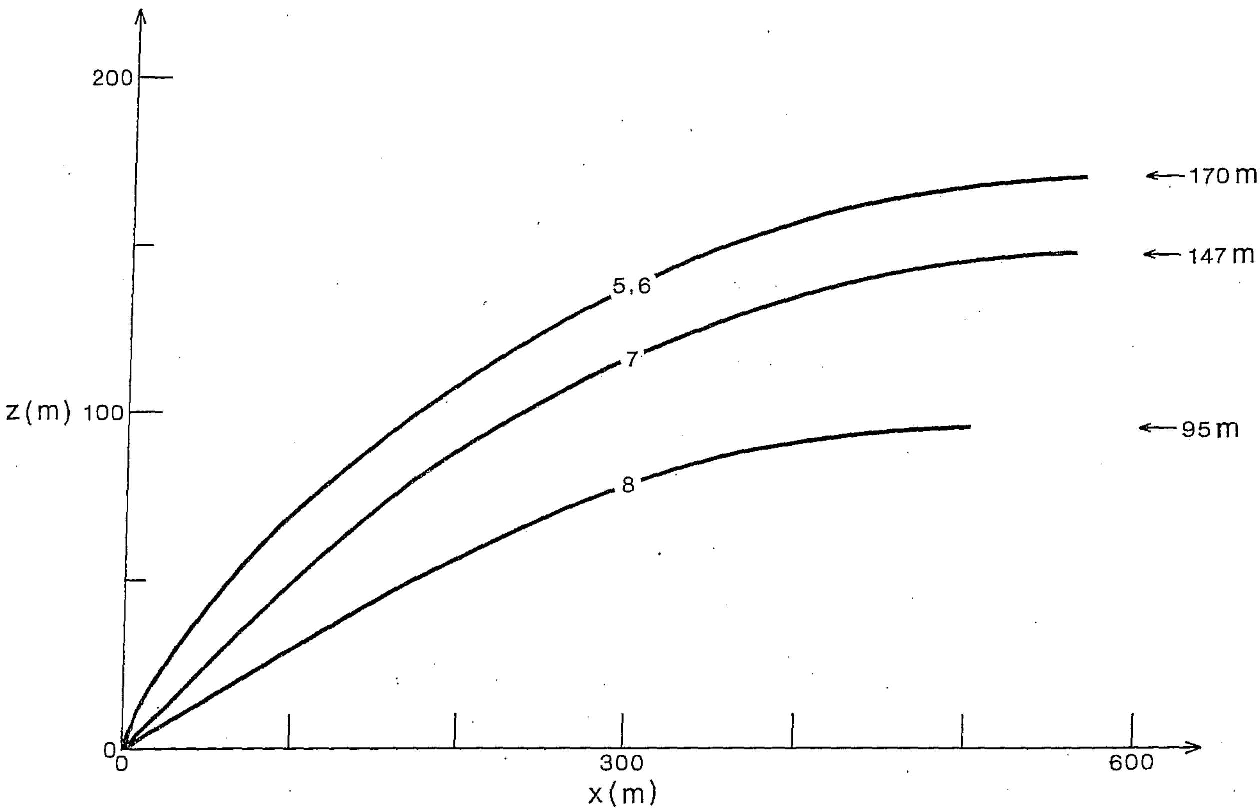

Figure 4. Plume trajectories for examples 5-8 of Table 1. Each trajectory is identified by its example number and its maximum height of rise.