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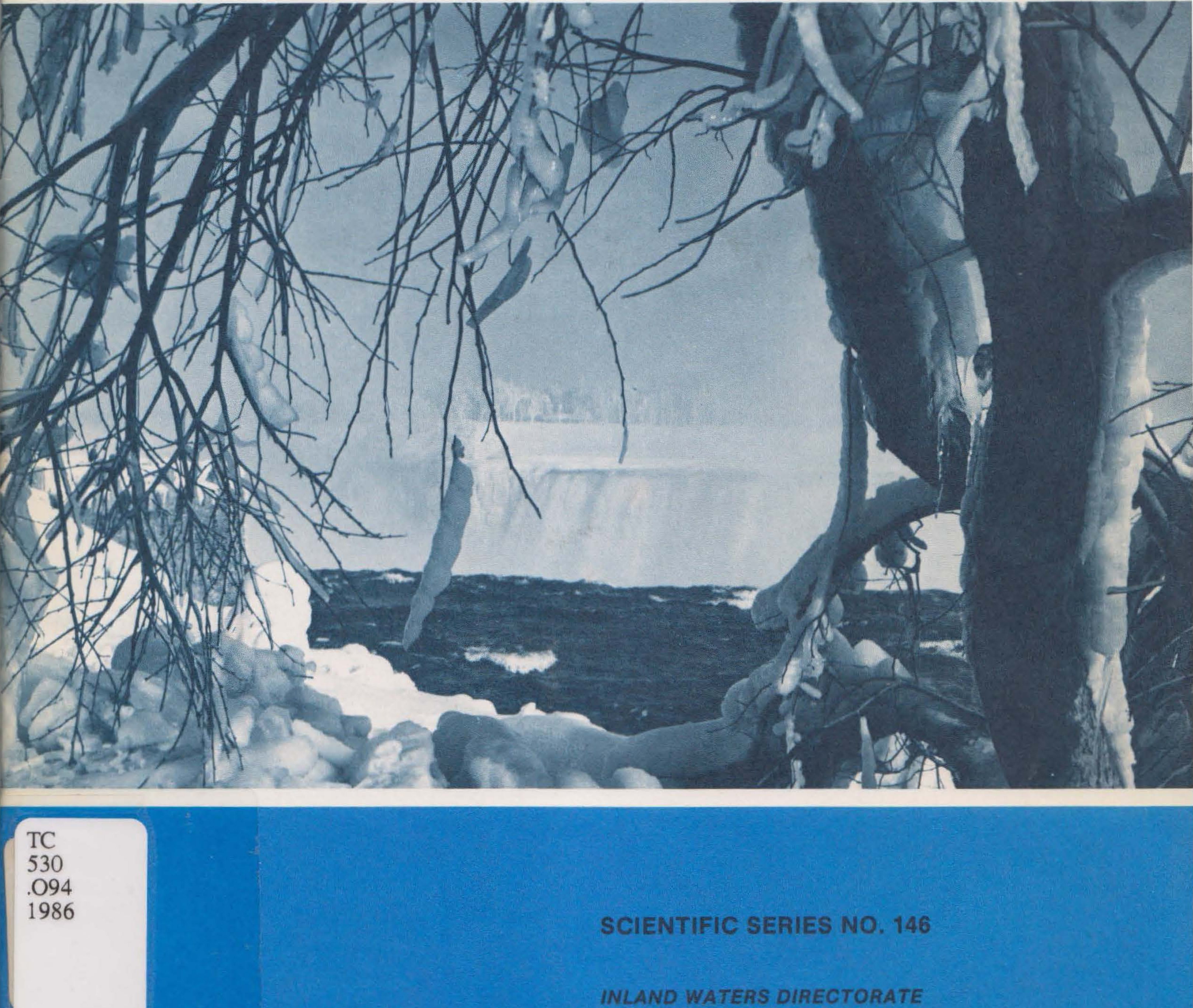
Pierre Ouellette, Nassir El-Jabi and Jean Rousselle



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INLAND WATERS DIRECTORATE
QUEBEC REGION
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SAINTE-FOY, QUÉBEC, 1986

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The Analysis of Flood Damage Time Series

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Abstract

Effective flood plain management requires estimation of the costs and benefits of all contemplated projects. In this study the focus is on estimating the benefits of such schemes.

Starting from the similarity between flood flow and flood damage time series, the authors take a probabilistic approach to flood damage estimation. They first develop a hydroeconomic model to assess flood-related damages and then derive a damage distribution function by applying the theory of extreme values and the sum of the random number of random variables to the estimated damage series. It is assumed that the values of extreme damages are independent and identically distributed over the time interval (0,t] of one year and one season simultaneously. The distribution function can then be used to estimate the benefits of flood plain management projects.

The Richelieu River basin has been used for a numerical application because of its combined rural and urban characteristics and the fairly large amount of information acquired in previous studies.

Résumé

L'aménagement des plaines inondables nécessite une estimation des coûts et des bénéfices reliés aux diverses interventions dont on envisage l'implantation. Dans cette étude, on s'intéresse plus particulièrement à l'estimation des bénéfices.

Cette étude partant de la similitude existant entre la série temporelle des débits de crues et la série temporelle des dommages d'inondation, a pour but d'appliquer l'approche probabiliste aux dommages d'inondation. À cette fin, on développera, dans un premier temps, un modèle hydro-économique permettant d'estimer les dommages d'inondation associés à une crue. Ensuite, on dérivera une fonction de répartition des dommages en appliquant la théorie des valeurs extrêmes et la somme du nombre aléatoire de variables aléatoires à la série de dommages d'inondation précédemment estimée. On considérera les valeurs des dommages maximaux indépendantes et identiquement distribuées dans l'intervalle de temps (0,t] d'un an et d'une saison simultanément. De cette fonction, on peut extraire une estimation des bénéfices des projets.

Pour l'application numérique, on a retenu le bassin versant de la rivière Richelieu étant donné ses caractéristiques urbaines et rurales, et pour le grand nombre d'informations disponibles.

List of Symbols

$U_i(x_{i1} \ldots x_{im})$	Satisfaction function for individual i
x_{ij}	Quantity of good j consumed by i, j=1...m
P_j	Price of good j
R_i	Revenue of individual i
W	Social utility function
$\Psi(x_2 \ldots x_m)$	Production function
θ	Factor representing the impact of the government's budgetary constraint on the choice of projects
B	Benefits of the project
C	Costs of the project
d	Variable representing physical and nonphysical damage
K	Variable representing all of the physical capital affected by the flood
KT	Variable representing the aggregate physical capital in the area
Y	Vector describing the hydrologic characteristics of a flood
α, γ	Coefficients of the global damage function of the Gompertz curve
z	Depth of submersion
Q_B	Base flow
Q_ν	Peak flow of rank ν in time interval (0,t] such that $Q_\nu > Q_b$

List of Symbols (Cont.)

ξ_ν	Exceedance of rank ν in time interval (0,t]
z_ν	Submersion depth associated with ξ_ν
D_ν	Damage associated with flood ξ_ν
$\tau(\nu)$	Time of occurrence of damage of rank ν
t	Time
$\Lambda(t)$	Average number of damages in time interval (0,t]
β	Parameter of the exponential distribution function of the damage values
$\chi(t)$	Maximum damage in time interval (0,t]
H(x)	Distribution function for damage values
$F_t(x)$	Distribution function for maximum damage
MAD	Mean annual damages
MAUD	Mean annual unit damage
r	Actualization rate
$\rho(x)$	Probability of damage x
MUD	Mean unit damage
f(x)	Percentage of residences affected by an exceedance x

Introduction

As there is a similarity between flood flow and flood damage time series, in this study probability theory is applied to the estimation of flood damage. The methodology used is based on stochastic processes which permit the analysis of independent random variables that are identically distributed over a given time interval.

Application of this methodology requires a long-range flood damage series, which can be derived in several different ways, i.e., by post-flood survey, correlation or simulation. Although this study is primarily intended to apply the extreme value theory to flood damage, it also presents a procedure for estimating flood damages from hydrologic and economic variables (Chapter 3). The appeal of this approach stems from the total absence or else the inadequacy of damage series permitting the use of the extreme value theory.

The occupation of flood plains entails serious risks of flooding. It is argued, however, that the advantages--both tangible and intangible--of living near water outweigh the potential risks. Throughout history mankind has developed brilliant civilizations near the water. Historically, the proximity of water was a major benefit. Rivers and streams were the most efficient means of long-distance transportation; their proximity guaranteed a more reliable supply of food and drinking water, and flooding, its adverse effects notwithstanding, was often the means of fertilizing the soil. We need only think of the Nile and its natural activity, so crucial to the well-being and indeed the survival of Egypt. However, as society evolved, some of these benefits lost much of their significance or became negligible. For example, the development of road and rail networks has brought opportunities that water transport cannot always rival. Nor is the food supply function of water the vital factor it once was. Very few people now catch their own fish, as the development of commercial fishing and food distribution networks has made buying fish a more attractive option. As for fertilization, the silt washed up by flood waters has been replaced by commercial products, which, given the random occurrence of flooding, are a more reliable source of supply.

Given that the beneficial aspects of living near water have lost some or much of their impact (although waste disposal and a number of other activities remain important advantages) and flooding remains a problem, we would logically expect a gradual move away from flood plains and consequent eradication of the flood problem. In fact, this is not at all the case. On the contrary, we are seeing an increase in flood-caused damage, as demonstrated in the study by Perrier (1978) on the chronological evolution of annual flood damage in Quebec. Ironically, the present situation can be explained in that the advantages of living in a riverside community still outweigh the disadvantages stemming from floods. Note, however, that such advantages are no longer attributable solely to the proximity of water but are increasingly linked to urban settlement.

This is made clearer when we take a broad look at the urbanization process. It has been said that people settled near bodies of water, some of which were prone to flooding, to reap the advantages offered by the site. Thus, to increase the chances of survival, they instituted riverside communities which in time matured into towns and villages. This had a major impact because the growth of urban populations justified the development of services and infrastructures--the most striking examples being roads and aqueducts--which had once been economically infeasible. Towns thus became more attractive, not only because of the proximity of water but also because of the availability of services. This led to even greater urban population density. At the time of the industrial revolution, industries settled in towns as a matter of course to take advantage of established services, tap the large workforce potential and consumer markets and, naturally, exploit such water-related services as processing and energy supplies. This was a dynamic process, the industrial presence making towns even more attractive to outsiders, which in turn further increased urbanization, a decisive factor in a town's ability to attract more industry. From that point on, urbanization no longer depended entirely on the proximity of water but increasingly on the ability of a town to draw people interested in urban services, job opportunities, etc.

In sum, the fewer advantages of living near water, coupled with the continuing flood problem, do not deter people from settling in towns situated in flood plains, as the decline in the benefits derived from the proximity of water is more than offset by the increased advantages of urbanization.

Foreseeably, then, the flood problem will continue to exist, making it both interesting and useful to acquire as much knowledge as possible about flood damage. This is the contribution which this study intends to make.

Intervention Criterion

2.1 Cost-Benefit Analysis

To establish a criterion for flood plain intervention, we assume that the government, seeking to maximize the satisfaction of all members of society, will call upon its departments and agencies to set up programs designed to achieve this goal. This leads us to develop a theoretical framework inspired by the work of Eckstein (1958).

(a) An individual will allocate his income to the acquisition of various consumer goods to obtain maximum satisfaction. The utility function (U) of an individual (i) can be expressed as

$$U_i = U_i(x_{i1}, x_{i2} \ldots x_{im}) \tag{2.1}$$

Function $U_i(x_{i1} \ldots x_{im})$ is continuous and increasing; it is also strictly quasiconcave (Malinvaud, 1977). The level of satisfaction achieved will be a function of the quantities of goods (x_j, j=1...m) consumed by individual i, who will therefore seek to maximize U_i, subject to the constraint that the sum of his spending must not exceed his income (R_i). The optimum is determined by the following solution of the Lagrangian function (L_i):

$$L_i = U_i(x_{i1}, x_{i2} \ldots x_{im}) + \lambda_i(R_i - \sum_{j=1}^{m} P_j x_{ij}) \tag{2.2}$$

where λ_i = the Lagrangian multiplier for individual i;
x_{ij} = the good j consumed by individual i, j=1...m;
P_j = the price of good j (it is assumed from here on that the price system P_j is the same for all individuals);
R_i = the revenue of individual i;
$R_i - \sum_{j=1}^{m} P_j x_{ij}$ = the budgetary constraint on individual i.

Taking the partial derivatives relative to x_{ij} (j=1...m) and isolating λ_i, we obtain the following conditions of optimality:

$$\lambda_i = \frac{\partial U_i}{\partial x_{i1}} \cdot \frac{1}{P_1} = \ldots = \frac{\partial U_i}{\partial x_{im}} \cdot \frac{1}{P_m} \qquad (2.3)$$

This gives the usual conditions of optimality, i.e., an individual will apportion his revenue among various goods in such a way that the marginal utility weighted by the inverse of their price is the same for all of the goods.

(b) By intervening in a given region, the project authorities supply the population with goods and/or services (Δx_{ij}). This variation in consumption will affect the level of satisfaction, which varies as follows:

$$\Delta U_i = \sum_{j=1}^{m} \frac{\partial U_i}{\partial x_{ij}} \Delta x_{ij} \qquad (2.4)$$

Integrating Equation 2.3 in Equation 2.4 yields

$$\Delta U_i = \lambda_i \sum_{j=1}^{m} P_j \; \Delta x_{ij} \qquad (2.5)$$

(c) With respect to the population affected by a given project, the variation in satisfaction ΔW will be defined as

$$\Delta W = \sum_{i=1}^{n} \Delta U_i \quad \text{(there are n individuals involved)} \qquad (2.6)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m} \lambda_i P_i \; \Delta x_{ij} \qquad (2.7)$$

In a first approximation, we assumed that the λ_i's are constant and the same for all individuals. Equation 2.7 becomes

$$\Delta W = \lambda \sum_{i=1}^{n} \sum_{j=1}^{m} P_j \Delta x_{ij} \tag{2.8}$$

Owing to its properties of growth and quasiconcavity, the consumer satisfaction function is unique to within a monotonic transformation (Malinvaud, 1977). We can therefore write

$$\Delta W = \sum_{i=1}^{n} \sum_{j=1}^{m} P_j \Delta x_{ij} \tag{2.9}$$

$$= \sum_{j=1}^{m} P_j \sum_{i=1}^{n} \Delta x_{ij} \tag{2.10}$$

Supposing $\Delta x_j = \sum_{i=1}^{n} \Delta x_{ij}$, one obtains

$$\Delta W = \sum_{j=1}^{m} P_j \Delta x_j \tag{2.11}$$

The variation in satisfaction of the population thus equals the sum of variations in real income.

(d) Let us now consider one possible application of this model to flood plain management. The project authorities will seek to maximize ΔW by converting production factors $(x_2 \ldots x_m)$ into consumer goods x_1. The production function will be represented by

$$x_1 = \Psi(x_2 \ldots x_m) \tag{2.12}$$

where x_1 is the aggregate of goods produced and $\Psi(x_2 \ldots x_m)$ is the production function.

The project authorities, while considering the production function, must also adhere to their budgetary constraint (D) and thus make certain that the costs (C) of their activities do not exceed D. The Lagrangian equation (L_2) is solved as follows:

$$L_2 = \sum_{j=1}^{m} P_j \ \Delta x_j - \mu(x_1 - \Psi(x_2 \ldots x_m)) - \theta(C-D) \qquad (2.13)$$

$$L_2 = P_1 \ \Delta x_1 - \sum_{t=2}^{m} P_t \ \Delta x_t - \mu(x_1 - \Psi(x_2 \ldots x_m)) - \theta(C-D) \qquad (2.14)$$

Since Δx_1 represents the variation in consumer goods, $P_1 \Delta x_1$ is equal to the real benefit (B) of the project. Since Δx_t represents the production factors used, $\sum_{t=2}^{m} P_t \Delta x_t$ represents the real costs (C) of the project. Equation 2.14 becomes

$$L_2 = B - C - \mu(x_1 - \Psi(x_2 \ldots x_m)) - \theta(C-D) \qquad (2.15)$$

At optimum, we have

$$\frac{\partial B}{\partial x_1} - \mu = 0 \qquad (2.16)$$

$$\frac{\partial C}{\partial x_t} (1+\theta) - \mu \frac{\partial \Psi}{\partial x_t} = 0 \quad \text{for any } t \qquad (2.17)$$

Isolating μ from (2.16) and (2.17) gives

$$(1+\theta) \ \frac{\partial C}{\partial x_t} - \frac{\partial B}{\partial x_1} \frac{\partial \Psi}{\partial x_t} = 0 \qquad (2.18)$$

$$\frac{\frac{\partial B}{\partial x_1} \frac{\partial \Psi}{\partial x_t}}{\frac{\partial C}{\partial x_t}} = (1+\theta) \quad \text{for any } t \qquad (2.19)$$

In a simplified form,

$$\frac{\partial B/\partial x_t}{\partial C/\partial x_t} = 1 + \theta \quad \text{for any } t \tag{2.20}$$

Equation 2.20 forms the intervention criterion for flood plain management authorities. The term $(\partial B/\partial x_t)$ represents the variation in consumer benefits (and satisfaction) generated by a project yielding x_1 produced by means of x_t. In our case, x_1 may represent an increase in flood protection; $(\partial C/\partial x_t)$ is the marginal cost of the project; θ is a factor that takes into account the scarcity of funds available to project authorities. Where only one project is contemplated, the criterion determines the optimum scope of the intervention, i.e., the marginal benefit should exceed the marginal cost by a factor $(1+\theta)$. When a number of independent projects are being considered, the individual marginal projects should be selected such that the benefits of the project exceed the cost of the project by a factor $(1+\theta)$. If the projects are incompatible or the benefits and costs of one project are affected by the implementation of another project, some other criterion will have to be used, in which case authorities will opt for a combination of compatible projects that maximizes the economic impact while adhering to the government's budgetary constraint. Further information on the evaluation of public projects can be found in the work of Lévy-Lambert and Dupuis (1973).

2.2 Remarks on the Intervention Criterion

We have just established a flood plain intervention criterion based on a relatively elementary theoretical framework. This criterion is dependent upon (a) the budgetary constraint, which determines the parameter θ, (b) the project production function, which determines costs, and (c) the function for the satisfaction of the individuals affected by the project, which determines benefits. Note that although this criterion is conceptually clear, its use poses a number of problems, particularly with respect to flood plain management projects. While budgetary constraint is fairly easy to obtain and the costs of the different interventions such as dikes and reservoirs can be calculated with satisfactory accuracy, we must contend with an extreme lack of accuracy when it comes to the evaluation of project benefits. Indeed, project evaluation is complicated by a lack of information on quantification of the benefits of flood plain intervention. Research efforts should therefore focus on maximizing information about these benefits. We attempt here to provide some initial solutions in this regard.

Theoretical Considerations

The purpose of this Chapter is to present the theory of extreme values which we will use to analyse the flood damage series. In view of the virtual nonexistence of flood damage series, we will also present a methodology for estimating flood-related damages by means of hydrologic and economic parameters.

3.1 Hydroeconomic Model

The damages associated with any given flood comprise two types of variables: natural random variables (flood flow, depth of submersion, etc.) and nonnatural determinist or random variables (economic development of the study area). The damage function can generally be written as follows (El-Jabi *et al*., 1982b):

$$d = f(K,Y) \tag{3.1}$$

where d = a variable describing various possible types of physical and nonphysical damage;

K = a variable describing all of the physical capital such as residences, commercial and industrial buildings, and stocks, and associated activities such as production flow and domestic services;

Y = a vector of the elements that describe the flood characteristics such as water depth and velocity and duration of submersion.

Given the characteristics of flood damages, the damage function is a monotonic, continuous, non-decreasing function that takes the shape of an S-curve (Dantzig, 1956) (Fig. 3.1). The Gompertz curve reflects these characteristics and was thus the one used. If, in addition, we take only the depth of submersion as the hydrologic variable of damage, the damage function is expressed as

$$\frac{d}{K} = \frac{1}{e^{e^{-\alpha}} - 1} \{e^{[1-e^{-\gamma z}]e^{-\alpha}} - 1\} \quad (3.2)$$

where z = the depth of submersion;

α and γ = the parameters to be estimated.

In Equation 3.2, the units of the damage variable are regarded as unitary for the various economic sectors ($/unit). Simply knowing the depth of submersion of any given economic unit is therefore sufficient to calculate the damage that it incurs.

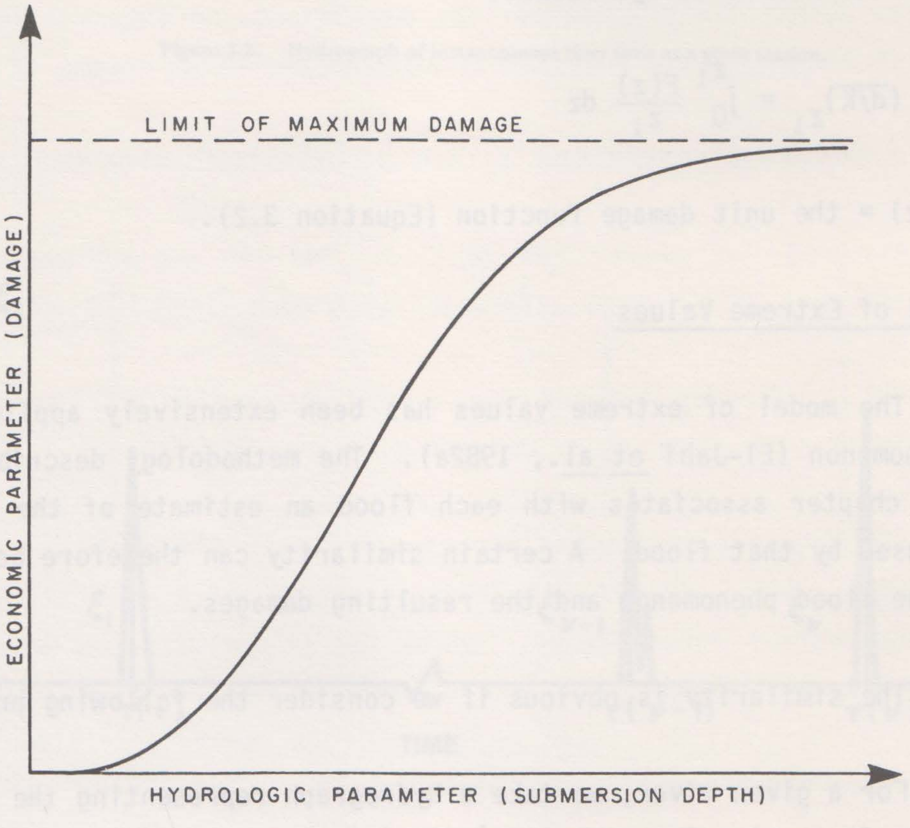

Figure 3.1. Damage function.

Note that Equation 3.2 considers a constant depth of submersion for the entire stock of physical capital. It is highly unlikely, however, that all of the economic units are situated on the same level and thereby subjected to the same submersion depth. The enormous task of calculating the depth of submersion for each individual unit is alleviated by determining the mean unit damage corresponding to a particular depth. This mean unit damage corresponds to the damage that would occur if the economic units were uniformly distributed within a sector lying between the base level (z = 0) and the elevation considered ($z = z_1$). This hypothesis is valid for certain economic units (residences) but does not apply in every case. Industrial sectors, for instance, must be dealt with on a per unit basis. The mean unit damage becomes

$$(\overline{d/K})_{z_1} = \int_0^{z_1} \frac{F(z)}{z_1}\, dz \qquad (3.3)$$

where F(z) = the unit damage function (Equation 3.2).

3.2 Model of Extreme Values

The model of extreme values has been extensively applied to the flood phenomenon (El-Jabi et al., 1982a). The methodology described in the preceding chapter associates with each flood an estimate of the mean unit damage caused by that flood. A certain similarity can therefore be expected between the flood phenomenon and the resulting damages.

The similarity is obvious if we consider the following process:

- For a given river, we take a hydrograph representing the instantaneous flows for a time interval (0,t] (Fig. 3.2).

- Given the time discontinuity of the flood phenomenon, the hydrograph can be derived by applying the model below (Fig. 3.3).

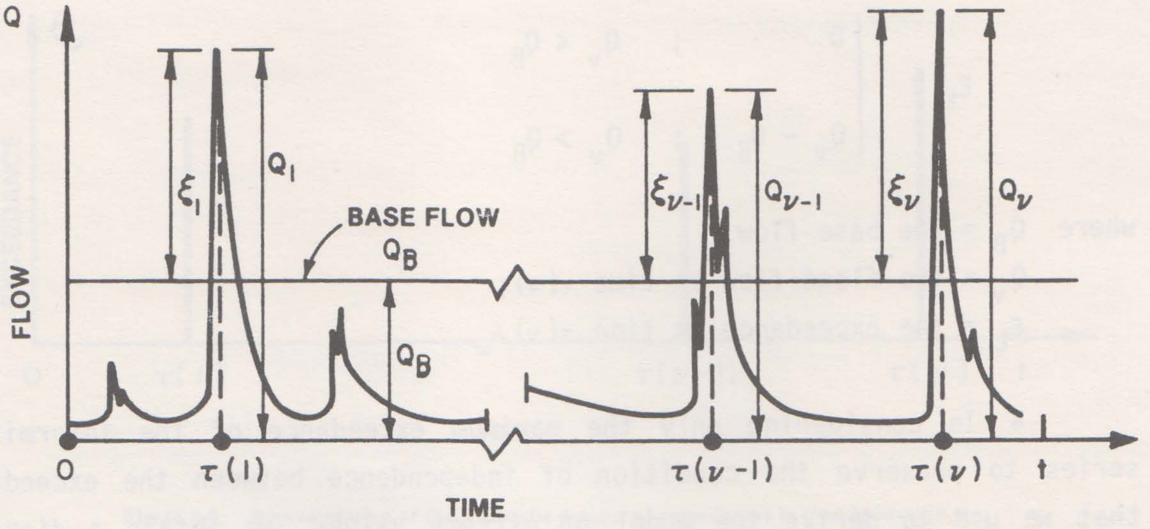

Figure 3.2. Hydrograph of instantaneous river flow at a given station.

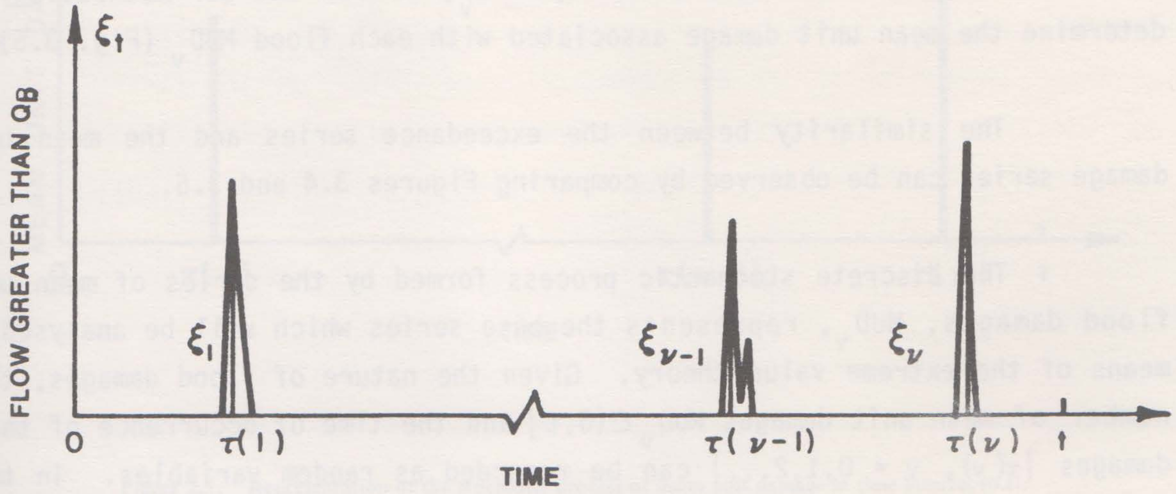

Figure 3.3. Flood flow hydrograph.

$$\xi_t = \begin{cases} 0 & ; \quad Q_\nu \leqslant Q_B \\ Q_\nu - Q_B & ; \quad Q_\nu > Q_B \end{cases} \tag{3.4}$$

where Q_B = the base flow;

Q_ν = the flood flow at time $\tau(\nu)$;

ξ_t = the exceedance at time $\tau(\nu)$.

- In considering only the maximum exceedance of the intermittent series to preserve the condition of independence between the exceedances that we use to derive the model of extreme values, we obtain a discrete, non-negative stochastic process for the exceedances in interval $(0,t]$. The ξ_ν series is the one often analysed by the extreme value theory (Fig. 3.4).

- A depth of submersion z_ν can be associated with each exceedance ξ_ν.
- Knowing the submersion depth z_ν, we can use our methodology to determine the mean unit damage associated with each flood MUD_ν (Fig. 3.5).

The similarity between the exceedance series and the mean unit damage series can be observed by comparing Figures 3.4 and 3.5.

The discrete stochastic process formed by the series of mean unit flood damages, MUD_ν, represents the base series which will be analysed by means of the extreme value theory. Given the nature of flood damages, the number of mean unit damages $MUD_\nu \in (0,t]$ and the time of occurrence of those damages $\{\tau(\nu),\ \nu = 0,1,2...\}$ can be regarded as random variables. In this study, their values are also regarded as random.

It has been demonstrated (Todorovic, 1970) that the maximum exceedances follow a stochastic process wherein the number and values of the exceedances are combined in a model in which they are independent and identically distributed over a time interval (T_{k-1},T_k), giving the distribution function of the maximum exceedance $\chi(t)$:

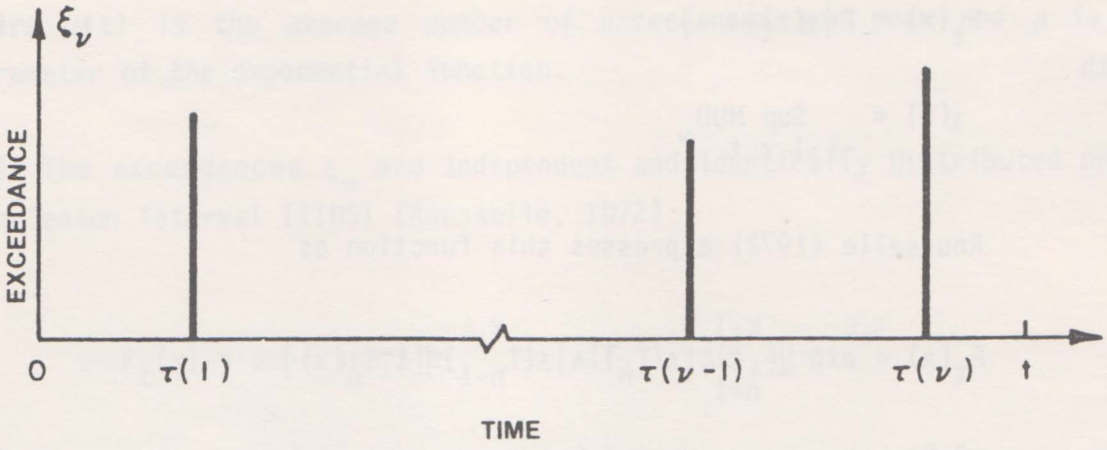

Figure 3.4. Representation of the stochastic process of exceedances in time interval (0,t].

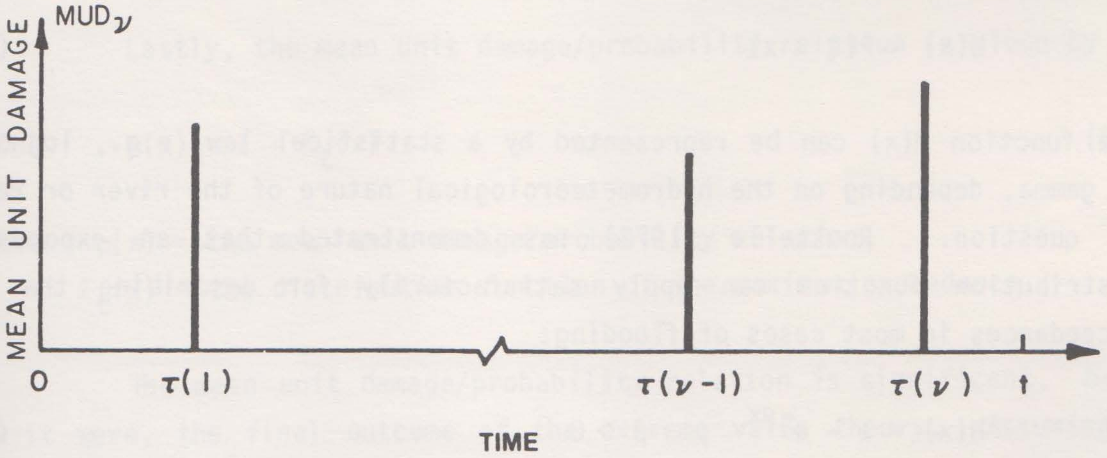

Figure 3.5. Representation of the stochastic process of mean unit damage in time interval (0,t].

$$F_t(x) = P\{\chi(t) \leq x\} \tag{3.5}$$

with

$$\chi(t) = \sup_{\tau(\nu) \leq t} MUD_\nu$$

Rousselle (1972) expresses this function as

$$F_t(x) = \exp \{- \sum_{n=1}^{k-1} [\Lambda(T_n) - \Lambda(T_{n-1})][1-H_n(x)]$$

$$- [\Lambda(t) - \Lambda(T_{k-1})][1-H_k(x)]\} \tag{3.6}$$

for every k = 1,2... and $t \in (T_{k-1}, T_k)$ where $\Lambda(t)$ is the average number of exceedances $\xi \in (0,t]$ and H(x) is the distribution function of the value of the exceedances such that

$$H(x) = P(\xi \leq x) \tag{3.7}$$

The function H(x) can be represented by a statistical law, e.g., lognormal or gamma, depending on the hydrometeorological nature of the river or region in question. Rousselle (1972) has demonstrated that an exponential distribution function can apply satisfactorily for describing the flow exceedances in most cases of flooding:

$$H(x) = 1 - e^{-\beta x} \quad ; \quad \beta > 0 \tag{3.8}$$

where $\beta = \{E(\xi)\}^{-1}$

Therefore two cases are to be considered.

(1) The exceedances ξ_ν are Independent and Identically Distributed over a one-Year interval (IIDY) (Zelenhasic, 1970):

$$F_t(x) = \exp [-\Lambda(t)\, e^{-\beta x}] \tag{3.9}$$

where $\Lambda(t)$ is the average number of exceedances per year and β is the parameter of the exponential function.

(2) The exceedances ξ_ν are Independent and Identically Distributed over a one-Season interval (IIDS) (Rousselle, 1972):

$$F_t(x) = \exp\{-\Lambda(T_1)\, e^{-\beta_1 x} - [\Lambda(T_2) - \Lambda(T_1)]\, e^{-\beta_2 x}$$

$$- [\Lambda(T_3) - \Lambda(T_2)]\, e^{-\beta_3 x} - [\Lambda(t) - \Lambda(T_3)] e^{-\beta_4 x}\} \qquad (3.10)$$

where $\Lambda(T_1)$, $[\Lambda(T_2) - \Lambda(T_1)]$... represent the average number of exceedances per season and β_1, β_2... are the parameters of the exponential function.

Lastly, the mean unit damage/probability relation is given by

$$\rho(x) = 1 - F_t(x) \qquad (3.11)$$

where $\rho(x)$ = the mean unit damage/probability relation;
$F_t(x)$ = the distribution function of the maximum exceedance.

The mean unit damage/probability relation is significant, being, as it were, the final outcome of the extreme value theory. Assuming the independence of the number and value of mean unit flood damages, this model permits us to relate this variable to its probability of occurrence. This is particularly important because this relation can be used to derive the mean annual unit damage, a key variable in the profitability analysis of flood plain management projects. The mean annual unit damage (MAUD) is calculated by the following formula:

$$\text{MAUD} = \int_0^\infty \rho(x)\, dx \qquad (3.12)$$

Numerical Application

This Chapter presents a numerical illustration, beginning with the development of an intermittent series of mean unit flood damages in the Quebec town of Saint-Jean on the Richelieu River. Secondly, this series will be analysed by means of the extreme value theory. Lastly, we will comment on the utility of this approach. The application will be confined to the residential sector. Note, however, that a few slight modifications will render this methodology applicable for any other economic sector as well.

4.1 Estimation of the Mean Unit Flood Damage Series

Estimation of the mean unit flood damage series requires evaluation of parameters α and γ of Equation 3.2. These parameters are constant for all floods and are determined by the economic characteristics of the study area.

The data needed for this estimation were taken from the post-flood inquiry done in 1976 by the Centre de recherche en aménagement régional (CRAR, 1977) within the Richelieu River region. This inquiry permitted the estimation of the ratio d/K by giving the flood damage (d) relative to the flood of 1976 and to a hypothetical equivalent flood plus 0.305 m (1 ft) of additional submersion. Also, the K values for each damage unit could be estimated. Thus, the variables K, Y (= z, the submersion height in this case) and d are known. The estimation of the parameters α and γ from the Equation 3.2 is done by considering d/K as a dependent variable and z as an independent variable which corresponds to the least-squares method for nonlinear functions (Draper and Smith, 1966). Knowing these parameters (α = 0.953 and γ = 0.108), it is easy to find d/K for a certain submersion height.

The depth of submersion in the study area is then evaluated from the daily flow rates of the target river, supplied by Environment Canada (Table 4.1). The backwater curves are calculated by the HEC-2 program

Table 4.1 Peak Flows and Corresponding Exceedances of the Richelieu River: Hydrometric Gauging Station 02ØJ007 at Fryers Rapids, Flow = 25 000 cfs

Year.day*	Flow (cfs)	Exceedance (cfs)	Year.day*	Flow (cfs)	Exceedance (cfs)
1938.182	25 800	800	1955.199	40 700	15 700
1938.186	25 500	500	1956.224	32 500	7 500
1938.192	25 300	300	1958.211	39 400	14 400
1938.200	29 200	4 200	1959.211	34 200	9 200
1938.202	26 300	1 300	1960.213	36 800	11 800
1939.212	42 600	17 600	1961.216	25 900	900
1940.219	37 100	12 100	1961.218	25 400	400
1940.268	25 500	500	1961.228	26 800	1 800
1942.210	32 500	7 500	1962.214	30 100	5 100
1943.183	25 300	300	1963.212	37 600	12 600
1943.229	38 100	13 100	1968.186	29 900	4 900
1944.203	25 600	600	1969.192	25 700	700
1944.215	34 300	9 300	1969.212	39 200	14 200
1944.233	26 400	1 400	1970.213	39 100	14 100
1945.187	35 100	10 100	1971.224	40 300	15 300
1945.207	25 200	200	1972.223	42 300	17 300
1945.235	34 700	9 700	1972.255	26 100	1 100
1946.173	26 300	1 300	1972.258	25 600	600
1946.175	25 800	800	1973.188	36 600	11 600
1947.249	43 700	18 700	1973.240	30 100	5 100
1948.187	29 500	4 500	1973.276	25 800	800
1948.198	26 800	1 800	1974.081	26 900	1 900
1948.201	25 800	800	1974.105	25 500	500
1950.194	26 500	1 500	1974.189	26 900	1 900
1950.199	26 000	1 000	1974.227	35 600	10 600
1950.208	28 800	3 800	1975.215	27 000	2 000
1950.222	30 100	5 100	1976.188	41 900	16 900
1951.201	38 700	13 700	1977.185	35 600	10 600
1952.204	33 600	8 600	1977.223	25 800	800
1952.225	26 300	1 300	1978.213	37 400	12 400
1953.193	29 600	4 600	1979.188	35 000	10 000
1953.217	32 500	7 500	1981.151	27 900	2 900
1954.212	36 800	11 800	1981.196	26 900	1 900

* The days are based on the hydrologic year: October 1=1, September 30 = 365.

developed at the Hydrologic Engineering Center by the U.S. Army Corps of Engineers (1969). The depth of submersion for a given municipality is calculated by taking the average of the downstream and upstream levels for a particular flood and subtracting the average of the downstream and upstream levels for the base flow.

Knowing the submersion depth of a given flood enables the estimation of the associated unit damage. To take into account the dispersed layout of residences within the study area, however, the mean unit damage (MUD) must be determined. This analysis assumes that the units are uniformly spread over the flood area. Knowing parameters α and γ and the submersion depth for the floods having occurred during the period 1938-1981 makes it possible to generate the mean unit damage series. The integral of Equation 3.3 was calculated using the Continuous System Modelling Program (CSMP).

4.2 Application of Extreme Value Theory to the Mean Unit Flood Damage Series

The series of mean unit flood damages sustained by the town of Saint-Jean comprises 66 events (or damage incidents) over a 44-year period, an average of 1.5 events per year. The seasonal distribution of damages for the recording period is given in Table 4.2. As is shown, most of the damage occurred in the spring as a result of flooding brought on by snowmelt.

(a) Distribution Function of Mean Unit Damage Value

The distribution function of the mean unit damage value, H(x), given by Equation 3.8, is required to derive the damage distribution function. In the case of annual damage distribution, assuming an exponential distribution of the mean unit damage (MUD) value gives

$$H(x) = 1 - \exp\left[-21.671\ x\right] \qquad (4.1)$$

Table 4.2. Seasonal and Annual Variations of $\Lambda(t)$

Season	Period	Interval	$\Lambda(t)$
Autumn	Sept. 21 - Dec. 20	$(0,T_1]$	$\Lambda(T_1) = 0.023$
Winter	Dec. 21 - March 20	$(T_1,T_2]$	$\Lambda(T_2) - \Lambda(T_1) = 0.045$
Spring	March 21 - June 20	$(T_2,T_3]$	$\Lambda(T_3) - \Lambda(T_2) = 1.386$
Summer	June 21 - Sept. 20	$(T_3,T_4]$	$\Lambda(T_4) - \Lambda(T_3) = 0.045$
Year	Sept. 21 - Sept. 20	$(0,T_4]$	$\Lambda(t) = 1.500$

Table 4.3 Seasonal and Annual Variations of β

Season	Average of MUD	$\beta = \{E[MUD]\}^{-1}$
Autumn	0.0225	44.444
Winter	0.0150	66.667
Spring	0.0488	20.487
Summer	0.0075	133.333
Year	0.0461	21.671

The parameters of the exponential function for seasonal distribution of the mean unit damage are given in Table 4.3. Using the Kolmogorov-Smirnov test, adjustment of the observed values to the exponential function was accepted at a confidence level of 5%.

(b) Damage Distribution Function

Given the values of the parameters $\Lambda(t)$ (Table 4.2) and β (Table 4.3), the damage distribution function can be determined. For the town of Saint-Jean, an annual distribution of the mean unit damage (IIDY) gives

$$F_t(x) = \exp\{-1.5 \exp[-21.671\ x]\} \tag{4.2}$$

This function can be derived for different time periods by replacing $\Lambda(t)$ and β of the period considered in Equation 3.10. Adjustment of the MUD distribution to a double exponential function was verified at a confidence level of 1% using the Kolmogorov-Smirnov test.

(c) Mean Unit Damage/Probability Relation

The damage/probability relation is given by the equation $\rho(x) = 1-F_t(x)$. In the case of Saint-Jean, an annual distribution (IIDY) of the mean unit damage yields

$$\rho(x) = 1 - \exp\{-1.5 \exp[-21.671\ x]\} \tag{4.3}$$

and a seasonal distribution (IIDS) yields

$$\begin{aligned} \rho(x) = 1 - \exp\{&-0.023 \exp[-44.444\ x] \\ &-0.045 \exp[-66.667\ x] \\ &-1.386 \exp[-20.487\ x] \\ &-0.045 \exp[-133.33\ x]\} \end{aligned} \tag{4.4}$$

The results of Equations 4.3 and 4.4 are presented in Figure 4.1. The mean unit damage/probability analysis is the final step in the successful application of extreme value theory to Saint-Jean. This relation can be put to an important use, namely the estimation of the mean annual unit damage (MAUD).

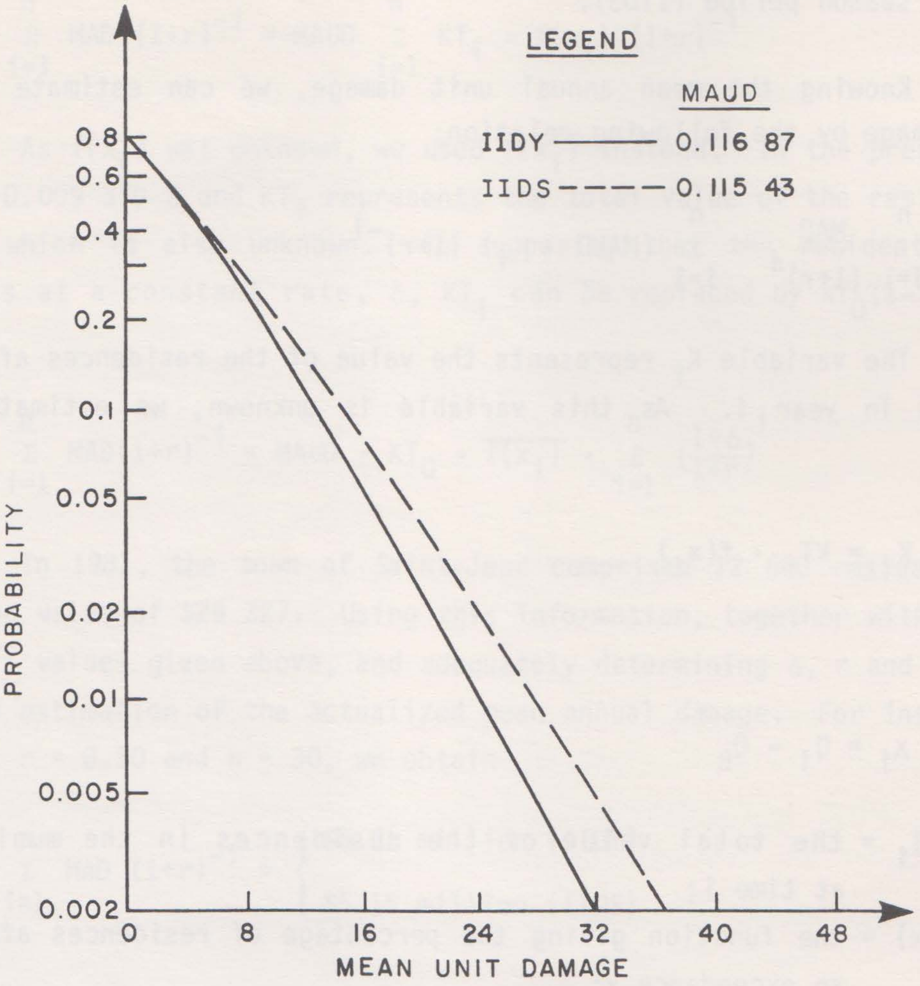

Figure 4.1. Mean unit damage/probability relation.

This particular estimate is made by calculating the area under the probability curve represented by Equation 3.11. For Saint-Jean, this gives the values below:

- A mean annual unit damage of .116 87 on the assumption that the floods are independent and identically distributed over a one-year period (IIDY);

- A mean annual unit damage of .115 43 on the assumption that the floods are independent and identically distributed over a one-season period (IIDS).

Knowing the mean annual unit damage, we can estimate the mean annual damage by the following relation:

$$\sum_{i=1}^{n} \frac{MAD}{(1+r)^{i}} = \sum_{i=1}^{n} (MAUD \cdot K_i)(1+r)^{-i} \qquad (4.5)$$

The variable K_i represents the value of the residences affected by the flood in year i. As this variable is unknown, we estimated it as follows:

$$K_i = KT_i \cdot f(x_i) \qquad (4.6)$$

with

$$x_i = Q_i - Q_B$$

where KT_i = the total value of the residences in the municipality at time i;

$f(x)$ = the function giving the percentage of residences affected by an exceedance x;

Q_i = the exceedance flow at time i;

Q_B = the base flow in the area.

Note that $f(x_i)$ is clearly a monotonic, non-decreasing function taking nonnegative values and tending asymptotically to the unit. Lacking any other preliminary information on the form of this relation, the form selected was

$$f(x_i) = \frac{x_i}{x_i+b} \quad (4.7)$$

The CRAR survey (1977) was used to estimate this equation. For Saint-Jean we found b = 656 400. Equation 4.5 becomes

$$\sum_{i=1}^{n} MAD\ (1+r)^{-i} = MAUD \sum_{i=1}^{n} KT_i \cdot f(x_i)\ (1+r)^{-i} \quad (4.8)$$

As $f(x_i)$ was unknown, we used $\overline{f(x_i)}$ instead. In the present case, $\overline{f(x_i)}$ = 0.009 370 8 and KT_i represents the total value of the residences at time i, which is also unknown. In supposing that the residential stock increases at a constant rate, δ, KT_i can be replaced by $KT_0(1-\delta)^i$, which gives

$$\sum_{i=1}^{n} MAD(1+r)^{-i} = MAUD \cdot KT_0 \cdot \overline{f(x_i)} \cdot \sum_{i=1}^{n} \left(\frac{1+\delta}{1+r}\right)^i \quad (4.9)$$

In 1981, the town of Saint-Jean comprised 12 680 residences with an average value of $29 327. Using this information, together with the MAUD and $\overline{f(x_i)}$ values given above, and adequately determining δ, r and n, we can derive an estimation of the actualized mean annual damage. For instance, if δ = 0.03, r = 0.10 and n = 30, we obtain

$$\sum_{i=1}^{n} MAD\ (1+r)^{-i} = \begin{cases} \$5.21 \text{ million (IIDY)} \\ \$5.15 \text{ million (IIDS)} \end{cases} \quad (4.10)$$

Conclusions

5.1 Methodology

This research effort purported to apply the theory of extreme values to mean unit flood damages to obtain an estimate of the mean annual damage, a key variable in the cost-benefit analysis of flood plain management projects. This theory should be applied to samples comprising numerous events. Since there exist virtually no such flood damage series, we resorted to a simulation method whereby an estimate of mean unit flood damage is associated with each flood. This approach has the advantage of taking into account the hydrologic characteristics of the floods and of the economic development of the study area.

Once the mean unit flood damage series has been calculated, we can apply the theory of extreme values, which exploits the characteristics of the flood damage phenomenon (random variables continued in time). Given these characteristics, stochastic processes are very useful and indeed necessary tools for the study of the phenomenon. The stochastic approach to estimating flood damage features a number of advantages:

(i) The possibility of choosing various distribution functions for the mean unit damage values;

(ii) The capacity to consider all damages having occurred in time interval (0,t], not just one single damage, as with other probabilistic methods; this advantage broadens our information base;

(iii) The flexibility in the choice of time interval (0,t], which may range from one day to one year; in this study we worked with an interval of one year and one season;

(iv) The capacity to combine the value and number of occurrences of flood damages for a representation of the maximum damage distribution.

5.2 Numerical Application

The conclusions derived from our numerical application pertain to two different areas: the method of estimating the damages associated with a given flood and application of the extreme value theory.

(i) Flood Damage Estimation Methodology

We have been successful in estimating flood damages by means of a relatively simple methodology that uses readily obtainable hydrologic and economic series as input variables. This approach, however, has one weakness; it does not take into account the time required to recapitalize an area that has been flooded. As a result, when the area suffers a second flood several days after the first, it is assumed that the local capital stock is the same for the second occurrence, and no consideration is given to post-flood capital depreciation or to the time required for capital revaluation. This oversight theoretically entails overassessment of the damages from the second flood. In the case of the Richelieu, however, this bias has only relative significance owing to the local flood profile (infrequent occurrence and broad hydrographs). Nevertheless, recapitalization time would be an important factor for areas subject to frequent major flooding.

(ii) Application of the Extreme Value Theory

In light of the objective of our research effort, which was to apply the theory of extreme values to the estimation of mean unit flood damages, our main conclusion is that the application was a definite success.

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