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Critical Initiation of Oblique Detonation Waves by Finite Wedges

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Contract Number: 11418NG
Contract Scientific Authority: Robert Stowe 418-844-4000 x 4318

Defence R&D Canada – Valcartier

Contract Report
DRDC Valcartier CR 2011-277
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Abstract

In a shock-induced combustion ramjet, combustion relies on a standing oblique detonation wave at the entrance of the combustor. Assuming that the fuel is uniformly mixed with the air stream prior to reach this location, it is essential to predict the conditions (type of fuel, mixture equivalence ratio, flow velocity and deflection angle) to initiate such a detonation wave for various flight regimes.

In this investigation, hypersonic reactive flows over finite wedges are simulated with a method of characteristics program. The chemistry is modeled with a single-step Arrhenius equation. The independent parameters are the freestream velocity, the wedge height and the wedge angle. At the critical conditions to initiate an oblique detonation wave, the critical wedge drag is calculated. In an attempt to develop a universal prediction tool without having to solve the flowfield, the Lee-Vasiljev model is considered. This model equates the work done by the wedge (the wedge drag) to the critical energy to directly initiate a cylindrical detonation wave. According to this model, the energy transfered by the wedge to the mixture is the only parameter that determines whether detonation initiation occurs or fails.

The simulation results show three regimes (subcritical, critical and supercritical) that are typical to direct initiation of a detonation. The critical drag is obtained for a series of test cases. When the wedge angle is fixed and the wedge height is varied to find the critical drag, this parameter depends on the value of the wedge angle. However, the Lee-Vasiljev model predicts a constant critical drag for any values of wedge angle. Alternatively, the decay of the oblique shock wave for all critical cases (i.e. for all values of wedge angle) is very similar. This suggests the use of a critical decay rate model for the prediction tool.

Résumé

Pour un superstatoréacteur, la combustion est accomplie par une onde de détonation oblique localisée à l'entrée de la chambre de combustion. En supposant que le carburant est uniformément mélangé avec l'écoulement d'air en amont de la chambre de combustion, il est essentiel de prédire les conditions (type de carburant, le rapport d'équivalence, la vitesse d'écoulement et l'angle de déflection) pour initier une telle onde de détonation pour différents régimes de vol.

Dans cette étude, des écoulements hypersoniques réactifs autour de dièdres de longueur finie sont simulés avec la méthode des caractéristiques. La cinétique chimique est simulée avec une équation simple de type Arrhénius. Les paramètres indépendents sont la vitesse de l'écoulement libre, la hauteur du dièdre et l'angle du dièdre. Aux conditions critiques pour initier une onde de détonation oblique, la traînée critique du dièdre est calculée. Afin de développer un outil de prédiction universel sans avoir à simuler l'écoulement, le modèle Lee-Vasiljev est considéré. Ce modèle relie le travail fait par le dièdre (la traînée du dièdre) et l'énergie critique pour initier directement une onde de détonation cylindrique. Selon ce modèle, l'énergie transférée par le dièdre au mélange est le seul paramètre qui détermine si l'initiation d'une détonation a lieu ou non.

Les résultats démontrent trois différents régimes (subcritique, critique et supercritique) typiques de l'initiation directe d'une détonation. La traînée critique est obtenue pour différents cas. Lorsque l'angle du dièdre est tenu constant et que la hauteur du dièdre est variée pour trouver la traînée critique, ce paramètre dépend de la valeur de l'angle du dièdre. Cependant, le modèle Lee-Vasiljev prédit une traînée critique constante pour toute valeur de l'angle du dièdre. Alternativement, l'atténuation de l'onde de shock oblique pour tous les cas critiques (c'est-à-dire pour toute valeur de l'angle du dièdre) est très similaire. Ceci suggère l'utilisation d'un modèle de taux d'atténuation comme outil de prédiction.

Executive summary

Critical Initiation of Oblique Detonation Waves by Finite Wedges

J. Verreault; DRDC Valcartier CR 2011-277; Defence R&D Canada – Valcartier; September 2011.

In this report, critical initiation of oblique detonation waves from finite wedges is investigated. An incoming hypersonic reactive flow is considered. Two regimes of freestream velocity are investigated: a freestream Mach number of 13.7 and 22.8. The flowfields are solved with the method of characteristics and the chemistry is modeled with a single-step Arrhenius equation with $\gamma = 1.2$, $Q = 50$ and $E_a = 20$ as parameters. In order to identify the critical flow conditions for detonation initiation, the wedge height, the wedge angle and the freestream velocity are independently varied. For all cases, the drag of the wedge is calculated and used as the primary parameter.

In order to use a simple model to predict the critical conditions for detonation initiation for any mixture and any projectile shape and size, the Lee-Vasiljev model is considered. Furthermore, since this model has shown good agreement with a number of experiments using blunt projectiles, it appears to be a good candidate for a universal model. The Lee-Vasiljev model assumes that the hypersonic equivalence principle applies and that detonation initiation occurs according to the blast initiation model. These two assumptions are verified in the present analysis. According to the Lee-Vasiljev model, the only parameter that determines whether detonation initiation occurs or fails is the energy deposited by the source, or simply the drag of the wedge. In this sense, the details of the projectile (its geometry and size) do not contribute to the detonation initiation criterion. The main objective of this analysis is thus to verify this hypothesis by varying the wedge height, the wedge angle and the freestream velocity.

For the baseline case where the freestream Mach number is $M = 13.7$ and the wedge angle is $\phi = 30^\circ$, the critical drag to initiate an oblique detonation wave is sought by varying the wedge height h: $D^* = 190$. For the cases where $h = 4$ and the freestream velocity is varied, the critical drag is similar: $D^* = 180$. However, for the cases where the wedge angle is either $\phi = 20^\circ$ or $\phi = 45^\circ$, the critical drag takes a different value: D^*=169 for $\phi = 20^\circ$ and $D^* = 280$ for $\phi = 45^\circ$. To ensure that the hypersonic equivalence principle applies for the simulated flowfields, a freestream Mach number of $M = 22.8$ is considered with wedge angles of $\phi = 15^\circ, 20^\circ, 25^\circ, 30^\circ$ and 45°. The critical drag for detonation initiation is

calculated for each wedge angle: $D^* = 180, 220, 280, 330$ and 440, respectively. Therefore, the critical drag alone seems insufficient to provide a criterion for detonation initiation, at least when the chemistry is modeled with a single-step Arrhenius equation. By tracing the evolution of the oblique shock angle for all critical cases (i.e. for a given freestream Mach number and any wedge angle), it is observed that the decay of the oblique shock from the tip of the wedge to the initiation location is very similar for all the cases. This suggests that a critical decay rate model might be a better prediction tool to find critical conditions for detonation initiation.

Sommaire

Critical Initiation of Oblique Detonation Waves by Finite Wedges

J. Verreault ; DRDC Valcartier CR 2011-277 ; R & D pour la défense Canada – Valcartier ; septembre 2011.

Dans ce rapport, l'initiation critique d'ondes de détonation obliques par des dièdres de longueur finie est étudiée. Un écoulement libre hypersonique réactif est considéré. Deux régimes de vitesse d'écoulement sont étudiés : un nombre de Mach de 13.7 et 22.8. L'écoulement est simulé avec la méthode des caractéristiques et la cinétique chimique est modélisée avec une équation simple de type Arrhénius avec $\gamma = 1.2$, $Q = 50$ et $E_a = 20$ comme paramètres. Afin d'identifier les conditions d'écoulement critiques pour l'initiation d'une détonation, la hauteur du dièdre, l'angle du dièdre et la vitesse de l'écoulement libre sont variés indépendemment. Pour tous les cas, la traînée du dièdre est calculée et utilisée comme paramètre principal.

Afin d'utiliser un modèle simple pour prédire les conditions critiques pour l'initiation d'une détonation pour tout mélange et pour toute forme et dimension de projectile, le modèle Lee-Vasiljev est considéré. De plus, puisque ce modèle a été validé avec quelques expériences utilisant des projectiles émoussés, il semble être un bon candidat pour un modèle universel. Le modèle Lee-Vasiljev suppose que le principe d'équivalence hypersonique s'applique et que l'initiation de la détonation se produit selon le modèle d'initiation par choc. Ces deux hypothèses sont valides pour les simulations présentées dans cette étude. Selon le modèle Lee-Vasiljev, le seul paramètre qui détermine si l'initiation d'une détonation se produit ou non est l'énergie déposée par la source, ou simplement la traînée du dièdre. Par conséquent, les détails du projectile (sa géométrie et dimension) ne contribue point au critère d'initiation d'une détonation. L'objectif principal de cette étude est de vérifier cette hypothèse en variant la hauteur du dièdre, l'angle du dièdre et la vitesse de l'écoulement libre.

Pour les cas où l'écoulement libre est à un nombre de Mach de $M = 13.7$ et que l'angle du dièdre est de $\phi = 30^\circ$, la traînée critique pour initier une onde de détonation oblique est obtenue en variant la hauteur du dièdre h : $D^* = 190$. Pour les cas où $h = 4$ et que la vitesse d'écoulement libre est variée, la traînée critique est similaire : $D^* = 180$. Cependent, pour les cas où l'angle du dièdre est soit $\phi = 20^\circ$ ou $\phi = 45^\circ$, la traînée critique prend une valeur différente : $D^* = 169$ pour $\phi = 20^\circ$ et $D^* = 280$ pour $\phi = 45^\circ$. Pour s'assurer que le principe d'équivalence hypersonique est valide pour les écoulements simulés, un nombre de Mach de $M = 22.8$ est considéré avec des angles de dièdre de $\phi = 15^\circ, 20^\circ, 25^\circ, 30^\circ$ et 45°. La traînée critique pour l'initiation de détonation est calculée pour chaque angle de dièdre : $D^* = 180, 220, 280, 330$ et 440, respectivement. Par conséquent, la traînée critique seule semble insuffisant pour prédire l'initiation de détonation, du moins lorsque la cinétique chimique est modélisée par une équation simple de type Arrhénius. En traçant l'évolution de l'angle de l'onde de choc oblique pour tous les cas critiques (c'est-à-dire pour une vitesse d'écoulement donnée et pour tout angle de dièdre), il est observé que l'atténuation de l'onde de choc oblique entre le nez du dièdre et la position d'initiation de détonation est très similaire pour tous les cas. Ceci suggère qu'un modèle basé sur le taux d'atténuation de l'onde de choc oblique serait plus approprié afin de prédire les conditions critique pour l'initiation de détonation.

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1 Introduction

For the proper operation of the shock-induced combustion ramjet (shcramjet), or oblique detonation wave engine (ODWE), it is critical to predict the necessary conditions to initiate an oblique detonation wave (ODW) from a wedge or a cone. An experimental investigation was conducted at McGill University [1] to identify these conditions using hypersonic projectiles traveling into a quiescent reactive mixture. From these experiments, photographs of the flowfield around the projectiles were obtained. In order to understand the underlying physics and to predict the required conditions to initiate an ODW for any projectile velocity/size/shape and mixture composition/pressure, a universal model is needed. A model developed independently by Lee [2] and Vasiljev [3] can potentially be used for this purpose. In fact, this model has been validated in a number of studies using blunt projectiles [4, 5]. For hypersonic propulsion, the use of a wedge or a cone to initiate an ODW is preferable to reduce drag. The Lee-Vasiljev model was validated with the experimental results obtained at McGill University [1]. However, a validation over a wide range of wedge or cone angle, projectile velocity and projectile size is needed.

In this study, critical initiation of ODW from finite wedges is investigated theoretically using a Method of Characteristics (MoC) program. A simplified single-step Arrhenius equation is used to model the chemistry. The purpose of this study is to identify the critical conditions to initiate an ODW for a wide range of wedge angle, projectile velocity and size. It is also in the scope of this work to compare the Lee-Vasiljev model with the theoretical results and to verify the domain of applicability of this model.

In Section 2, background concepts are provided to highlight the key ideas of the Lee-Vasiljev model. The MoC program is described in Section 3 and the results are presented in Section 4. Conclusions of this analysis are given in Section 5.

2 Background Concepts

2.1 Energy of Initiation

Blast initiation of a detonation (or direct initiation of a detonation) refers to the generation of a strong blast wave from a source of energy that is capable of triggering chemical reactions in the wake through adiabatic compression, with the reaction front coupling with the blast to form a detonation wave in the farfield. The source of energy can be released from a point, a line or a plane to generate a spherical, cylindrical or planar blast wave, respectively.

In the theory of blast initiation of a detonation, it is assumed that the source generates a strong blast of sufficient duration. The only role of the source is to trigger the blast and shall not influence its propagation or decay. Therefore, the ideal case corresponds to an infinitely small source that deposits the complete amount of energy in an infinitely small duration. Then, the only parameter that determines a successful initiation or failure to initiate a detonation in the farfield is the magnitude of the energy released by the source. At the critical energy release, the radius at which the detonation is initiated is on the order of the critical explosion length given by:

$$R_o^* = \left(\frac{E_s^*}{p_o} \right)^{1/j} \tag{1}$$

where E_s is the energy of the source, p_o is the initial pressure of the gas and $j = 0, 1, 2$ for the rectangular, cylindrical and spherical geometries respectively. Different theoretical models were developed to predict the critical energy for detonation initiation. The critical curvature model by He and Clavin [6] treated the problem as quasi-steady. In this model, the curvature of the blast wave was responsible for quenching the chemical reactions in the case of failure to initiate a detonation. Eckett et al. [7] showed that the blast initiation problem cannot be assumed to be quasi-steady, since the magnitude of the unsteady terms were significantly larger than that of the curvature term. In their model, the critical decay rate of the blast determined the minimum amount of energy that initiated a detonation in the farfield.

Experimentally, blast initiation can be realized by using very small and powerful initiators. Benedick et al. [8], for example, measured the critical mass of explosives to directly initiate spherical detonations in unconfined hydrocarbon-air mixtures.

In order to directly initiate a cylindrical detonation, a strong cylindrical blast needs to be generated from a line source of energy. In this case, the energy is expressed per unit length. The phenomenon of detonation initiation by hypersonic projectiles can be related to the concept of direct initiation of a cylindrical detonation, assuming that the passage of the projectile plays the role of the line source of energy that generates a cylindrical blast. In other words, by using the blast wave analogy (or the hypersonic equivalence principle), the shock generated by the projectile can be viewed as a cylindrical shock in a plane perpendicular to the direction of propagation. The blast wave analogy is mathematically based on the fact that the governing equations for an unsteady two-dimensional flow are identical to that of a steady three-dimensional flow with the hypersonic small-disturbances assumption. For the hypersonic equivalence principle to be valid, the following requirements apply:

$$\left.\begin{array}{r}\tau << 1\\ M >> 1\end{array}\right\} \text{with } M\tau \sim 1 \text{ or } >> 1$$

where τ is the maximum slope of the body and M is the flow Mach number. Thus, in order to use the hypersonic equivalence principle, the maximum deflection angle must be very small and the flow Mach number very large. Figure 1 presents a schematic of the hypersonic equivalence principle and its application to the initiation of a cylindrical detonation.

To summarize the necessary conditions to use the blast wave analogy for direct initiation of a cylindrical detonation by hypersonic projectiles, the two requirements are as follows:

- The maximum deflection angle is very small and the flow Mach number is very large.
- The initiation of a detonation occurs according to the blast initiation model. In other words, the size of the source must be very small and the energy deposition from the source must be instantaneous (or very rapid). For the case of detonation initiation from a hypersonic projectile, the rate of energy deposition is determined by its velocity and shape.

To fully satisfy both requirements can be contradictory, and this is illustrated in Figure 2. In this figure, extreme cone half angles are considered (10° and 90°). For both cases, the evolution of the blast wave is shown in the perpendicular plane. One can observe that for the 90° cone half-angle, the expansion of the blast at the early stage is much faster than for the 10° cone half-angle. The initial strength of the blast is thus larger for the 90° case, which fulfills the requirement to apply the blast initiation model (instantaneous energy release). However, in this case, the hypersonic equivalence principle is invalid near the projectile since the deflection angle is very large (90°). On the other hand, this principle is valid for the 10° cone half-angle. However, since the initial strength of the blast is relatively low, the rate of energy release by the source is also low and the blast initiation model becomes less valid. The difficulty to fully satisfy both requirements motivates the investigation of ODW initiation from finite wedges and to explore the domain of applicability of the Lee-Vasiljev model.

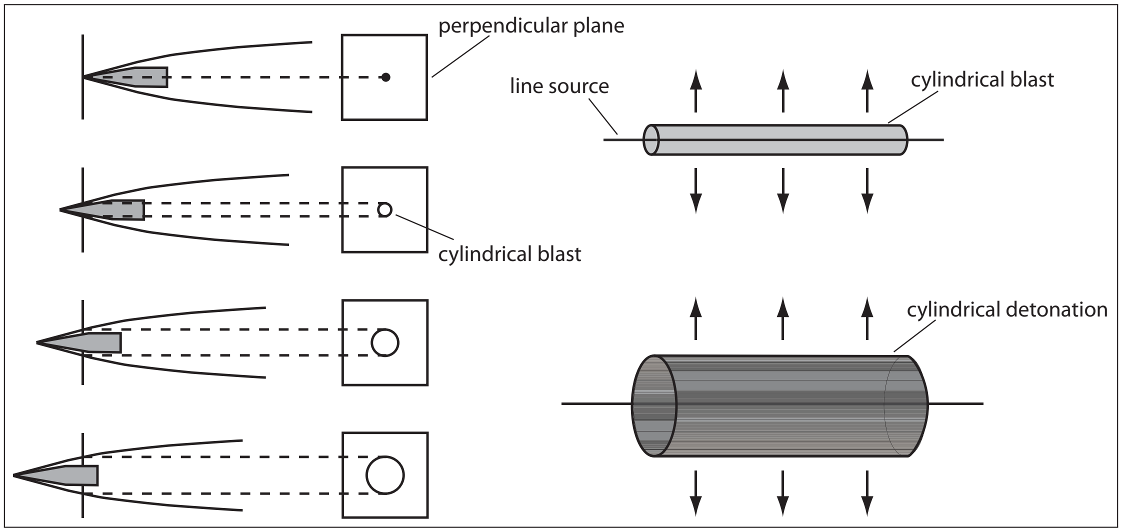


Figure 1: *Schematic of the hypersonic equivalence principle applied to the initiation of a cylindrical detonation*

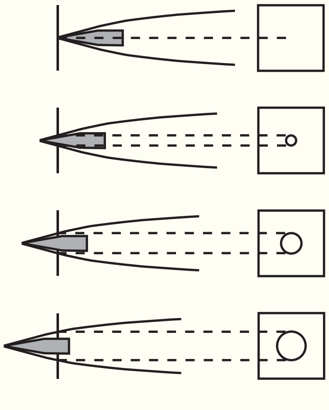
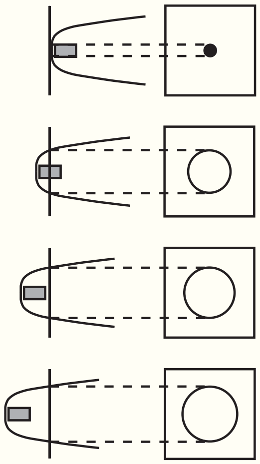
projectile cone angle	10°	90°
Schematic of the blast propagation in a perpendicular plane		
blast initiation of a detonation	slow rate of energy release	instantaneous energy release
hypersonic equivalence principle	valid	invalid near the projectile

Figure 2: *Illustration of the requirements of the Lee-Vasiljev model for extreme cone half-angles (10° and 90°)*

2.2 Derivation of the Lee-Vasiljev Model

The blast wave model applied to hypersonic projectiles was developed independently by Vasiljev [3] and Lee [2]. This theory states that the energy deposited by the projectile must be at least equal to the critical energy to directly initiate a detonation ($E_{proj} \geq E_c$). In a gas, one can assume that the hypersonic projectile produces a cylindrical blast wave propagating outwards from the flight axis. In a combustible mixture, this blast wave must be of sufficient strength to trigger a cylindrical detonation wave. Lee used the following equation to relate the velocity of the blast wave to its radius:

$$U_s^2 = \left(\frac{E_0}{2\pi I \rho_\infty} \frac{1}{r_s^2} \right) \tag{2}$$

where U_s is the blast wave velocity, E_0 is the blast energy, I is an integral that takes a value of 0.626 for $\gamma = 1.4$, ρ_∞ is the initial density and r_s is the blast radius. Lee stipulated that in order to initiate a cylindrical detonation, the blast radius at which it has decayed to the CJ velocity of the mixture must be at least some critical radius of the form $\kappa\lambda$, where κ is a constant and λ is the characteristic detonation cell size of the mixture. Lee used $\kappa = 3.2$ in his theory. Equation 2 can thus be expressed as:

$$E^* = 10\gamma p_0 M_{CJ}^2 \lambda^2 \tag{3}$$

where the relation $M_{CJ} = U_{CJ}\sqrt{\rho_0/(\gamma p_0)}$ was used. This minimum energy can be equated to the energy per unit length deposited by the projectile, which is simply its drag:

$$E_{proj} = D = \frac{1}{2}\rho U_{proj}^2 A C_D = \frac{\pi d_{proj}^2}{8} \gamma p M_{proj}^2 C_D \tag{4}$$

where d_{proj} is the projectile diameter. Equations 3 and 4 can be combined to give:

$$\frac{d_{proj}}{\lambda} = \left(\frac{80}{\pi C_D} \right)^{1/2} \left(\frac{M_{CJ}}{M_{proj}} \right) \tag{5}$$

This equation shows that for a given projectile size (d_{proj}) and a given mixture (λ and M_{CJ}), the critical projectile velocity to initiate a detonation can be predicted (and vice-versa). Equation 3 is intended to be used for real mixtures, as opposed to simplified chemistry models. Nevertheless, the derivation of the Lee-Vasiljev model is provided here for completeness. In the following analysis, this model is considered on a qualitative basis. This implies that it will be verified whether a critical projectile drag can predict critical detonation initiation for any projectile size and shape.

3 Description of the Method of Characteristics Program

The derivation of the procedure used in the MoC program closely follows the method outlined in Zucrow and Hoffman [9]. The MoC is applied to steady, two-dimensional rotational isentropic flow, which implies that the entropy is constant along any streamline, but can vary from one streamline to another. The chemistry is included in the system and via a single irreversible reaction of Arrhenius form. The governing equations are given by:

$$\frac{\partial}{\partial x}\begin{bmatrix} \rho u \\ \rho u^2+p \\ \rho uv \\ (\rho e+p)\,u \\ \rho Zu \end{bmatrix} + \frac{\partial}{\partial y}\begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2+p \\ (\rho e+p)\,v \\ \rho Zv \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \sigma \end{bmatrix} \tag{6}$$

where u and v are the $x-$ and $y-$velocity components, respectively, p is the pressure, ρ is the density, e is the specific energy and Z is the reaction progress variable from the reactant ($Z = 1$) to the product ($Z = 0$). The source function for the reaction progress variable is defined as:

$$\sigma = -\rho k Z \exp\left(-E_a \rho / p\right) \tag{7}$$

The equation of state can be expressed as:

$$p = (\gamma - 1)\,\rho\left[e - \frac{u^2+v^2}{2} + ZQ\right] \tag{8}$$

where γ is the ratio of specific heats. The chemistry parameters are the heat release Q, the activation energy E_a and the pre-exponential factor k, which is a spatial scaling factor. The set of partial differential equations described by Eqs. 6 are manipulated with the equation of state (Eq. 8) to obtain their characteristic form. The compatibility equations (or total differential equations) to be solved along the streamlines are:

$$dp + \rho U dU = 0 \tag{9}$$

$$dp - a^2 d\rho = \frac{\psi}{u} dx \tag{10}$$

$$\rho u dZ = \sigma dx \tag{11}$$

where a is the speed of sound, U is the magnitude of the velocity and $\psi = -Q(\gamma - 1)\sigma$. The compatibility equation to solve along the right-running characteristic C^- and the left-running characteristic C^+ (or equivalently along the Mach lines) is:

$$\frac{\sqrt{M^2-1}}{\rho U^2} dp_{\pm} \pm d\theta_{\pm} + \left[\frac{\mu \sin\theta}{yM} - \frac{\psi}{\rho U^2 a}\right] \frac{dx_{\pm}}{\cos(\theta \pm \alpha)} = 0 \tag{12}$$

In this equation, the ± sign is positive along a C^+ and negative along a C^-. M represents the Mach number, θ the flow angle with respect to the x axis, μ the problem symmetry ($\mu = 0, 1$ for the rectangular and axisymmetric configuration, respectively) and $\alpha = \sin^{-1}(1/M)$ the Mach angle. Note that the velocity components u and v from the set of equations 6 were transformed into the velocity magnitude U and the flow angle θ. In addition to the total differential equations to be solved for along the streamlines and the C^+ and C^- characteristics, their trajectory needs to be solved to construct the characteristic network. The slope of the streamlines is:

$$\Lambda = \frac{u}{v} = \tan\theta \tag{13}$$

and the slope of the characteristics is:

$$\Lambda = \tan(\theta \pm \alpha) \tag{14}$$

As explained by Zucrow and Hoffman, there are different methods to build the network in the domain of interest. A common method is to solve the flow conditions at the intersection of the C^- and C^+ characteristics. The streamline characteristic is extended backward

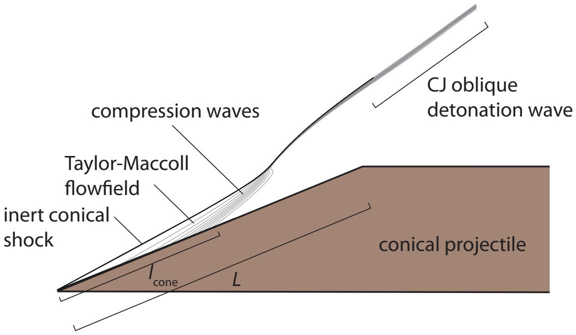


Figure 3: *Schematic of an ODW initiation from a conical projectile*

from the solution point and the flow conditions are determined by interpolation between previously calculated characteristics. However, when the chemistry is included in the system, Eq. 11 can be very stiff and even a small error in the interpolation of the species concentration can lead to a significant deviation of the solution from the true one. To avoid such problems, the flow conditions are solved at the intersection of the C^+ characteristics with the streamlines and the C^- characteristics are extended backward. The interpolation is thus made on the other flow conditions. The total differential equations are integrated using a modified Euler predictor-corrector method.

Figure 3 presents a schematic of an ODW initiation from a conical projectile. An inert shock wave is attached to the cone tip. Downstream of the shock, a fluid particle is compressed from the shock to the cone surface (described by a Taylor-Maccoll flowfield). The autoignition location of the mixture along the cone surface occurs a certain distance downstream of the cone tip, which determines the induction length l_{cone}. The reaction front may eventually couple with the shock to initiate an ODW which will be at a greater angle than the inert shock. The chemical length scale behind the oblique shock can vary by orders of magnitude due to the coupling between the reaction front and the oblique shock. There is therefore a need to increase the number of characteristics (i.e. the spatial resolution) in the regions of short induction distance. The MoC program is capable of automatically doubling the number of characteristics in a recursive method based on a specified number of points per half-reaction length. The user can set this criterion along any streamline behind the oblique shock and at the tip of the wedge along its surface. The refinement method also doubles the number of characteristics anywhere in the flowfield if the distance between two of them exceeds a value specified by the user. For the simulations presented in this study, the specified value is $10\eta_{tip}$, where η_{tip} is the distance between two characteristics at the tip of the wedge (where the simulation is initialized). This distance is defined as $\eta_{tip} = \Delta_{hrl}/n$, where Δ_{hrl} is the half-reaction length of a CJ detonation and n is the number of characteristics per half-reaction length. The parameter n varies from one simulation to another and was used to evaluate the sensitivity of the characteristic density on the results. A typical simulation size varies between 1 and 5 millions calculation points.

The results from the MoC program were validated against other sources. For a well-established overdriven ODW, the detonation angle given by the MoC program agreed very well with the detonation angle calculated with the conservation laws. Furthermore, the formation of an ODW was validated against Computational Fluid Dynamics (CFD) by comparing the shock trajectory.

The problem setup is illustrated in Fig. 4. The parameters U_o, ϕ and h are the freestream velocity, wedge angle and wedge height, respectively. The flow conditions are normalized with the freestream pressure $\tilde{p}_o$ and density $\tilde{\rho}_o$. The tilde sign (˜) refers to a dimensional quantity. The non-dimensional variables are defined as:

$$p = \frac{\tilde{p}}{\tilde{p}_o}, \qquad \rho = \frac{\tilde{\rho}}{\tilde{\rho}_o}, \qquad U = \tilde{U}\sqrt{\frac{\tilde{\rho}_o}{\gamma \tilde{p}_o}} = \frac{\tilde{U}}{\tilde{a}_o}, \qquad a = \sqrt{\frac{\gamma p}{\rho}},$$

$$u = U\cos\theta, \qquad v = U\sin\theta, \qquad M = \frac{U}{a}$$

The freestream conditions are set as:

$$p_o = 1, \quad \rho_o = 1, \quad u_o = 13.7, \quad v_o = 0, \quad Z_o = 1 \tag{15}$$

The freestream velocity u_o corresponds to the freestream Mach number. The choice of the chemistry parameters is based on the study of Watt and Sharpe [10]:

$$\gamma = 1.2, \qquad Q = 50, \qquad E_a = 20$$

These parameters correspond to a stable behavior for a planar 1D unsteady detonation. The pre-exponential factor k (in Eq. 7) is adjusted such that the half-reaction length of a planar ZND detonation (the distance between the shock front and the location where $Z = 0.5$) is unity.

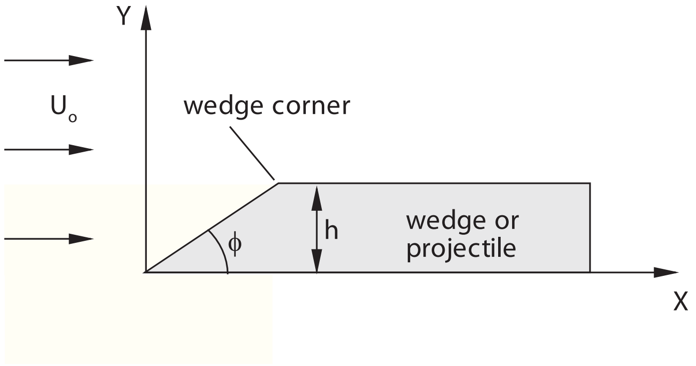


Figure 4: Problem schematic for the MoC simulations with finite wedges

4 Simulation Results

Simulations of reacting flows over finite wedges provide a means to determine critical conditions for oblique detonation initiation. The expansion waves emanating from the corner of the wedge constitute a quenching mechanism that competes with the chemical exothermic reactions. The expansion (or rarefaction) waves tend to weaken the shock to a Mach wave in the farfield and the chemical energy release tends to steepen the oblique shock to a CJ ODW angle. In a subcritical case (where the expansion waves dominate the chemical energy release), the chemical reactions are turned off and the oblique shock decays to a Mach wave in the farfield. In a supercritical case (where the chemical energy release dominates the expansion waves), an overdriven ODW generated by the wedge weakens to a CJ ODW in the farfield. In this case, the expansion waves are unable to quench the chemical reactions behind the shock. A more interesting and complex case occurs when both terms are equally important; this is a critical case. Therefore, the goals of the next MoC simulations are to determine the critical conditions for detonation initiation from finite wedges and to identify the governing mechanism that controls the onset of a detonation by varying the freestream velocity, wedge height and wedge angle.

Since the chemistry is modeled with a single Arrhenius equation, in principle it is not possible to determine a well-defined critical source of energy for detonation initiation. If one considers a domain that is sufficiently large, the reaction is always brought to completion. Nevertheless, as the magnitude of the energy deposited decreases, there is a range where the distance between the source and the initiation location increases dramatically. Therefore, instead of identifying a definite critical energy value, the qualitative trend is investigated by varying the amount of energy deposited.

According to the blast model (or the Lee-Vasiljev model), the amount of energy deposition is the only parameter that controls the onset of a detonation. This means that keeping the energy deposited constant, changing the details of the source (the wedge height or the wedge angle, for example) do not have any influence on the detonation initiation criteria. As mentioned in Section 2.1, the size of the source (the wedge height) needs to be very small (generally much smaller than the critical explosion length $R_o^* = (E_s^*/p_o)^{1/j}$). In this work, the planar geometry is used ($j = 1$), the freestream pressure is $p_o = 1$, and hence $R_o^* = E_s^*$. Whether this assumption is valid in this analysis will be verified.

For each case investigated, the energy per unit area provided by the wedge is calculated. Since the wedge is infinitely wide (perpendicular to the $X-Y$ plane), the energy per unit area, or drag, is given by:

$$D = \frac{1}{2}\rho_o U_o^2 h C_D$$

The drag coefficient for a wedge in a nonreacting supersonic flow is expressed as [11]:

$$C_D = \frac{4}{\gamma+1}\left(\sin^2\beta - \frac{1}{M_o^2}\right)$$

where β is the inert shock angle.

Figure 5 presents the flowfield for a subcritical case (where the expansion waves quench reaction and cause failure to initiate an ODW). For this case, the parameters are $u_o = 13.7$, $\phi = 30^\circ$ and $h = 3.5$. The wedge is shown at the bottom left of the graph at $X < 110$ and $Y < 3.5$. The contours of the reaction progress variable are shown with the reactant in white and the product in black. Decoupling between the reaction front and the shock front is clearly shown. The calculation domain is bounded by the shock front and the last C^+ characteristic. For all simulations, the domain of interest is limited to $X \leq 1000$. A supercritical case is illustrated in Fig. 6 where the parameters are $u_o = 13.7$, $\phi = 30^\circ$ and $h = 5$. In this case, coupling between the reaction front and the oblique shock (hence the onset of an ODW) occurs at approximately $X = 150$. A self-supported ODW extends in the farfield. A critical case is obtained for $u_o = 13.7$, $\phi = 30^\circ$ and $h = 3.8$ and is presented in Fig. 7. The reaction front decouples from the oblique shock and reaccelerates at approximately $X = 350$ to trigger the initiation of an ODW far from the wedge.

The evolution of the wave angle for the three cases is shown in Fig. 8. The corner of the wedge is located at approximately $X = 8$. For $0 < X < 8$, the wave angle increases due to

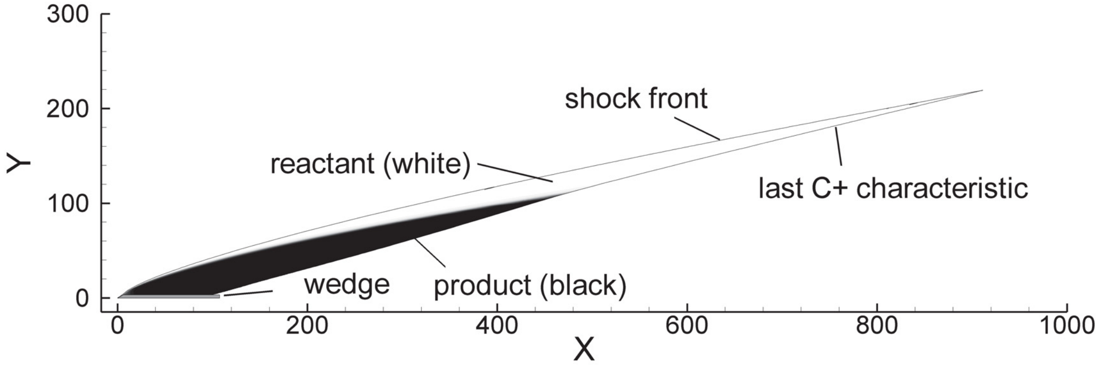


Figure 5: *Contours of the reaction progress variable for a subcritical case*

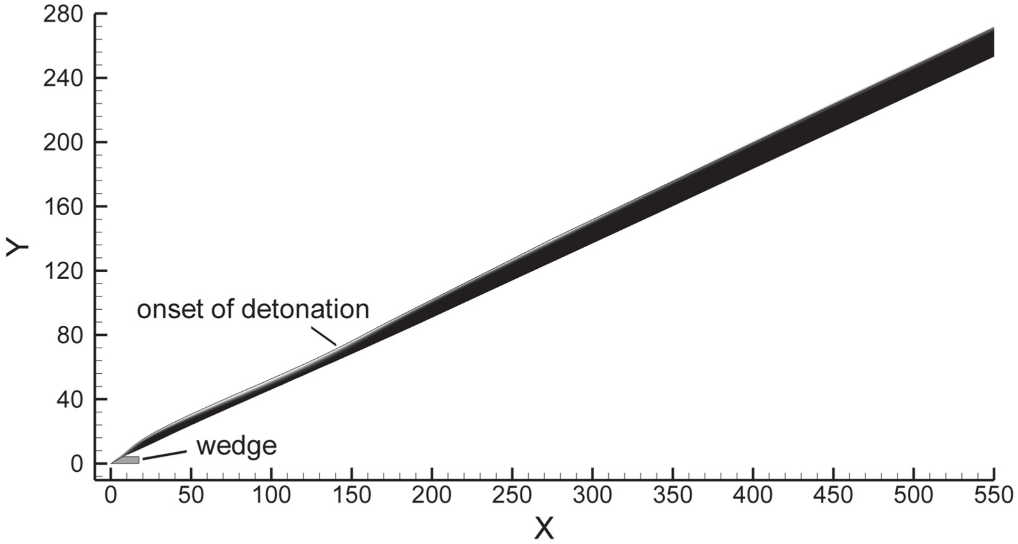


Figure 6: *Contours of the reaction progress variable for a supercritical case*

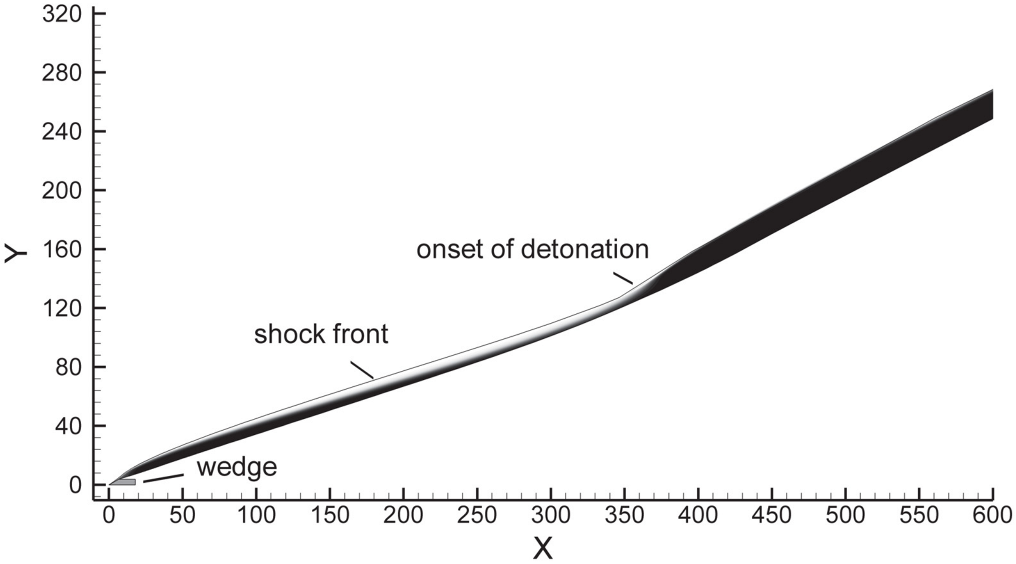


Figure 7: *Contours of the reaction progress variable for a critical case*

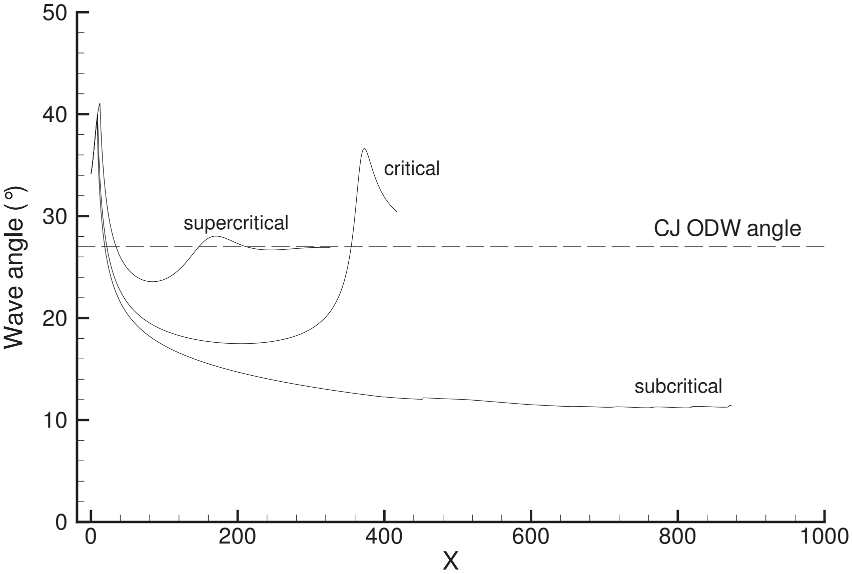


Figure 8: *Evolution of the wave angle for subcritical, critical and supercritical cases*

the formation of an overdriven ODW in front of the wedge. The expansion waves interact with the ODW for $X > 8$. In the subcritical case, the shock angle monotonically decays. In the supercritical case, the wave angle decays to about 3° lower than the CJ ODW angle and increases back to this value. In the critical case, the shock decays to an angle 9° lower than the CJ value and sharply increases at the initiation location. These three cases are qualitatively similar to the three regimes observed in blast initiation of a detonation.

4.1 Effect of the Wedge Height

The cases considered to study the effect of the wedge height on the detonation initiation phenomenon are listed in Table 1. For each height, the corresponding drag is provided. Also, the initial number of characteristics per half-reaction length (hrl) is varied for all cases between $n = 5$ and 10 pts/hrl to evaluate the sensitivity of this parameter on the results.

Table 1: *Cases to study the effect of the wedge height*

ϕ	u_o	h	D	Outcome
30°	13.7	3.5	185	no detonation
		3.7	195	detonation
		3.8	200	
		3.9	206	
		4.0	211	
		5.0	264	
		6.0	316	
		8.0	422	

Figure 9 displays the evolution of the wave angle for the considered cases. The labels on the graph refer to the drag values. Only the simulations with a resolution of $n = 10$ pts/hrl are shown. As the wedge drag decreases, onset of detonation occurs farther away from the source. In the present simulations, the initiation distance δ refers to the distance between the source and the initiation location, defined as the location where the wave angle crosses the CJ ODW angle in the reacceleration process (or downstream of the quasi-steady state region). For example, for the case $D = 200$, the initiation location is $X = 350$, which corresponds to $Y = 131$ (see Fig. 7). Hence, the initiation distance for this case is $\delta = 131$. Note that the initiation location could also be defined where the ODW angle decays to the ODW CJ angle downstream of the overshoot. However, in most cases, the wave angle asymptotes to the CJ value and it becomes difficult to determine a well-defined initiation distance. Figure 10 presents the initiation distance for all cases. For large values of drag (at

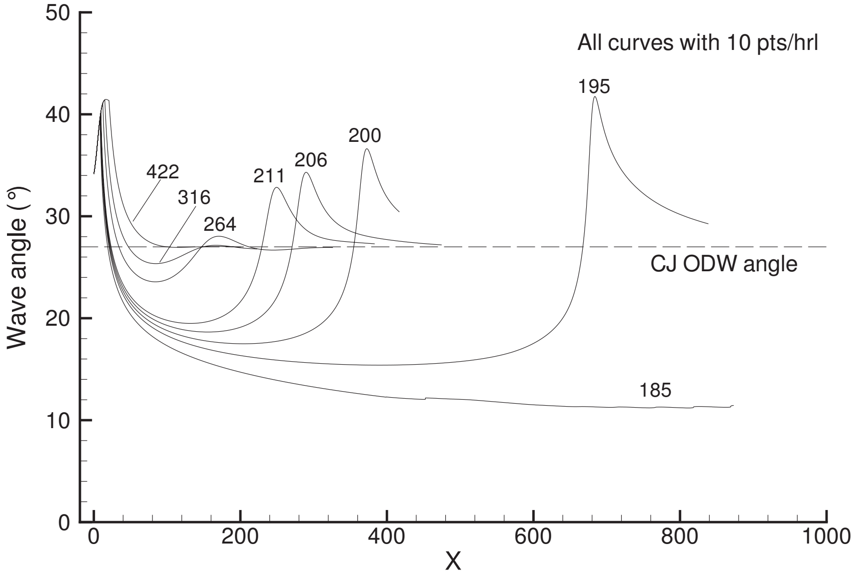


Figure 9: *Evolution of the wave angle when varying the wedge height (the labels refer to the wedge drag)*

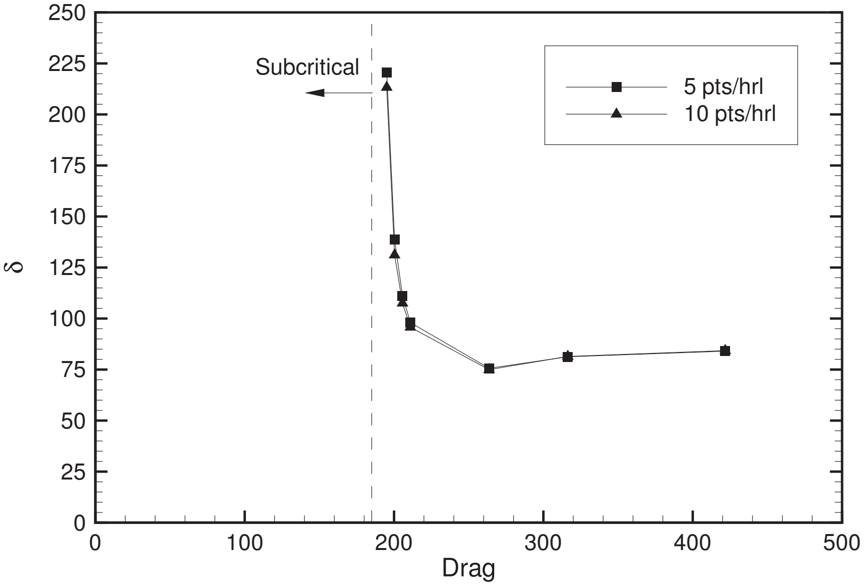


Figure 10: *Initiation distance as a function of the wedge drag when varying the wedge height*

supercritical conditions), the results for $n = 5$ and 10 pts/hrl are identical and the initiation distance is between 70 and 80 (about 14 times greater than the wedge height). As the critical conditions are approached, the initiation distance increases to about 220 (60 times greater than the wedge height) for $D = 195$ and $n = 10$ pts/hrl. Also, the resulting flowfield is slightly more influenced by the initial number of characteristics at critical conditions. The critical drag is in the range $185 < D^* < 195$. The critical explosion length is therefore $R_o^* = D \approx 190$ which is about 50 times larger than the wedge height. The assumptions of the Lee-Vasiljev model are thus valid. Furthermore, at critical conditions, $\delta \approx R_o^*$, which was also observed in experimental studies on direct initiation of cylindrical detonations [12].

4.2 Effect of the Freestream Velocity

In order to study the effect of the freestream velocity on the initiation of a detonation, this parameter is varied from 13.5 to 22.0 and the cases are listed in Table 2. The evolution of the wave angle for all the cases with $n = 10$ pts/hrl are displayed in Fig. 11. Due to the varied freestream velocity, the CJ ODW angle also varies for each case and is shown by horizontal dashed lines. The curve with $D = 172$ is a subcritical case (the corresponding CJ ODW angle for this case is 30.3°). Initiation of a detonation is obtained for $D \geq 184$. The initiation distance for all cases is presented in Fig. 12. The results follow closely the trend observed in Fig. 10 and the critical drag is in the range $172 < D^* < 184$. This result agrees very well with the critical drag when the wedge height is varied.

Table 2: *Cases to study the effect of the freestream velocity*

ϕ	h	u_o	D	Outcome
30°	4	12.3	172	no detonation
		12.8	184	detonation
		13.2	197	
		13.7	211	
		14.6	239	
		16.4	302	
		20.1	449	

4.3 Large Wedge Angle

The next simulations are carried out with a wedge angle of 45° and various wedge heights. The cases considered to study the effect of a larger wedge angle are listed in Table 3. The evolution of the wave angle for all cases with $n = 10$ pts/hrl are presented in Fig. 13 and

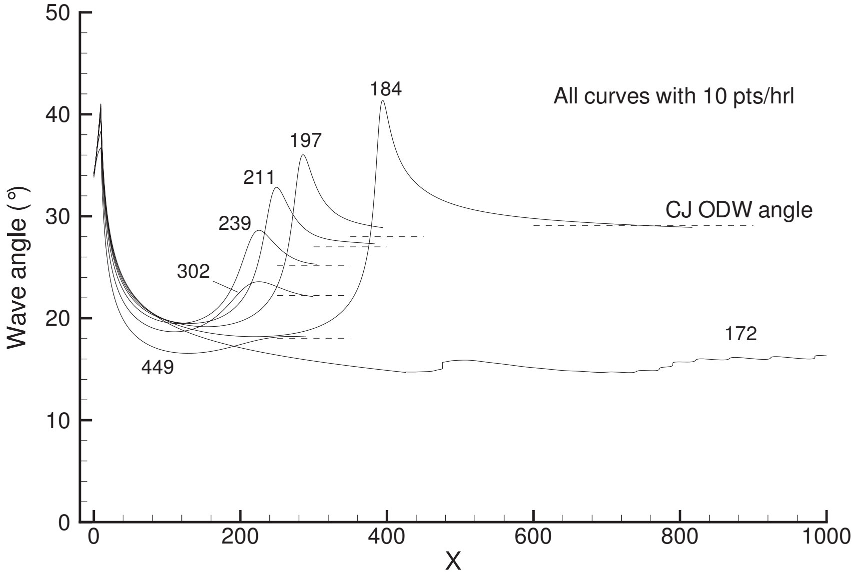


Figure 11: *Evolution of the wave angle when varying the freestream velocity (the labels refer to the wedge drag)*

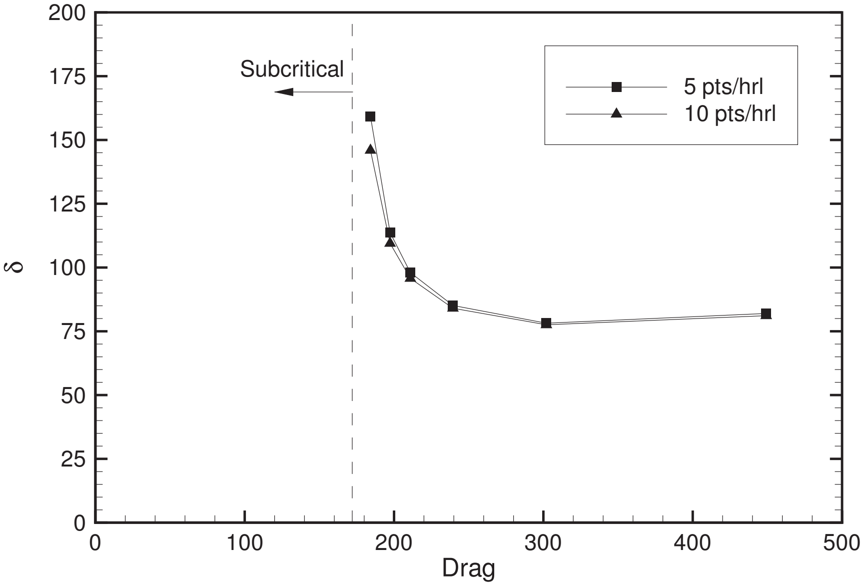


Figure 12: *Initiation distance as a function of the wedge drag when varying the freestream velocity*

the corresponding initiation distance in Fig. 14. The same trend as for the previous results can be observed here: the initiation distance becomes much larger than the wedge height near the critical conditions and the critical drag is in the range $270 < D^* < 280$.

Table 3: *Cases for a wedge angle of 45°*

ϕ	u_o	h	D	Outcome
45°	13.7	2.55	270	no detonation
		2.65	280	detonation
		2.7	285	
		2.8	296	
		2.9	307	
		3.0	317	
		3.5	370	
		4.0	423	

4.4 Small Wedge Angle

Considering a wedge angle of 20°, the cases investigated are listed in Table 4. The evolution of the wave angle is displayed in Fig. 15. For these cases, the so-called quasi-steady state region is no longer characterized by a quasi-constant wave angle, but instead exhibits oscillations followed by either a globally decreasing behavior for the subcritical case or a sudden increase at the initiation location for critical and supercritical cases. The initiation distance for all cases is illustrated in Fig. 16. As opposed to the previous re-

Table 4: *Cases for a wedge angle of 20°*

ϕ	u_o	h	D	Outcome
20°	13.7	6.4	160	no detonation
		6.75	169	detonation
		6.8	170	
		6.9	173	
		7.0	175	
		7.5	188	
		8.0	201	
		9.0	226	
		10.0	251	
		12.0	301	

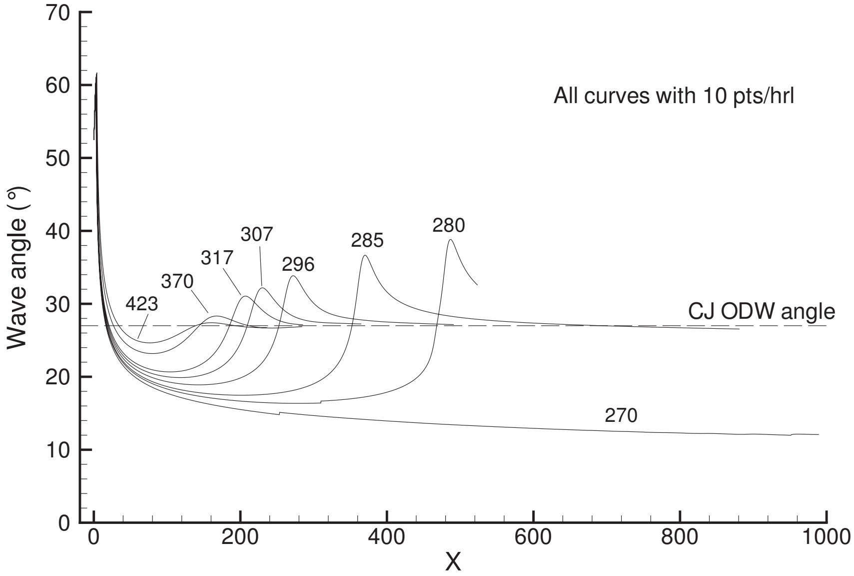


Figure 13: *Evolution of the wave angle for a 45° wedge angle (the labels refer to the wedge drag)*

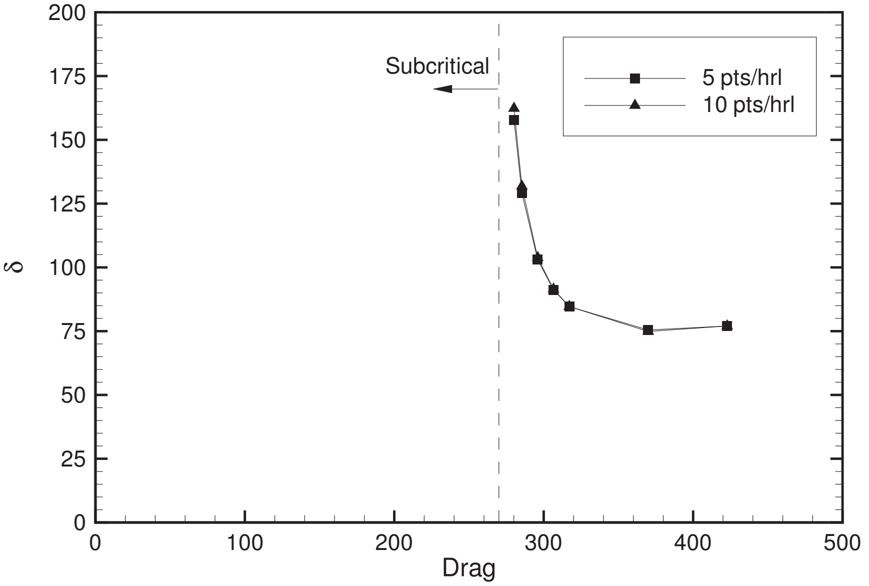


Figure 14: *Initiation distance as a function of the wedge drag for a 45° wedge angle*

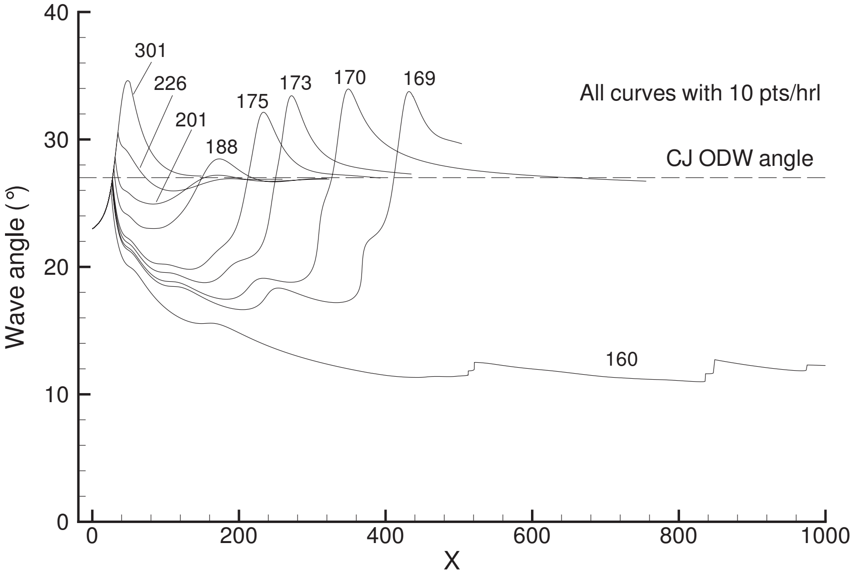


Figure 15: *Evolution of the wave angle for a 20° wedge angle (the labels refer to the wedge drag)*

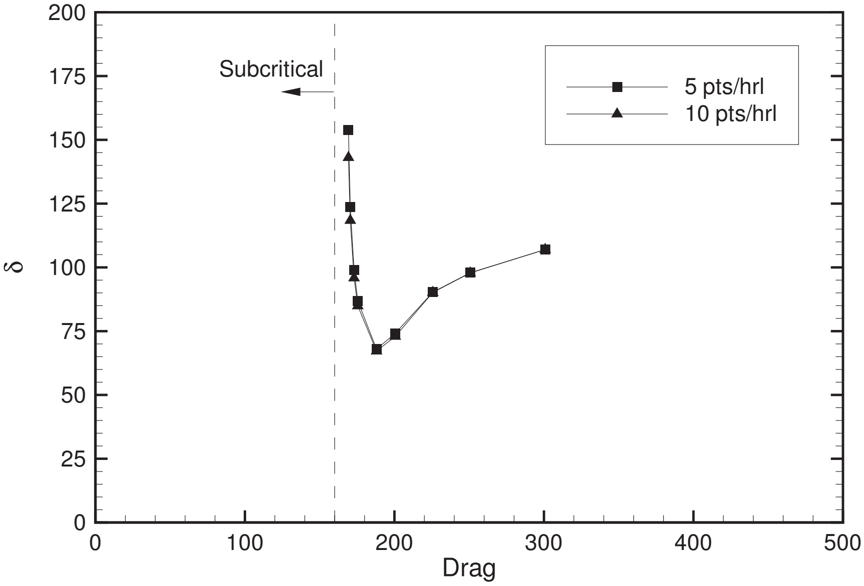


Figure 16: *Initiation distance as a function of the wedge drag for a 20° wedge angle*

sults, the initiation distance does not converge at supercritical conditions. As the drag increases for $D > 188$, the initiation distance also increases. The critical drag is in the range $160 < D^* < 169$.

The MoC results presented above show that when the wedge height or the freestream velocity is varied for $\phi = 30°$, the critical drag to initiate an ODW in the farfield is around $D^* \approx 185$. However, when the wedge angle is varied, the critical drag takes a different value ($D^* = 169$ for $\phi = 20°$ and $D^* = 280$ for $\phi = 45°$). These different trends are shown collectively in Fig. 17, in which the initiation distance is displayed as a function of the wedge drag for the three wedge angles. According to the Lee-Vasiljev model, the details of the projectile (hence the wedge angle) do not influence the critical drag for detonation initiation, assuming that the hypersonic equivalence principle is valid and the blast initiation model applies. Therefore, it is possible that the dependence of the critical drag on the wedge angle is due to the violation of the hypersonic equivalence principle assumption, especially for a wedge angle of 45°. To verify this hypothesis, simulations are conducted at a much higher Mach number ($M = 22.8$) and for a wider range of wedge angle (from 15° to 45°). Figure 18 presents the initiation distance as a function of the wedge drag for different values of wedge angles and for a Mach number of 22.8. It can be observed that even for a very large Mach number (22.8) and low wedge angles (15° to 25°), the critical drag for detonation initiation is influenced by the wedge angle. Figure 19 presents a visual representation of the shock trajectory for a Mach number of 22.8 and wedge angles of 15°

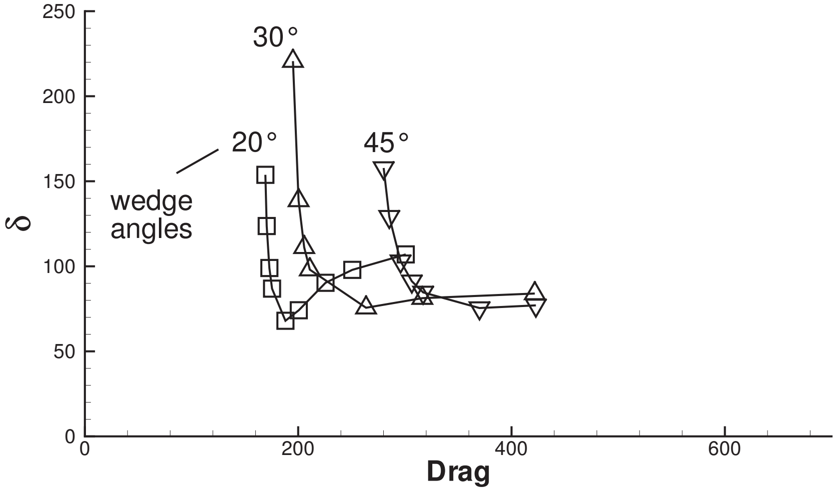


Figure 17: *Initiation distance as a function of the wedge drag for a Mach number of 13.7 and for various wedge angles*

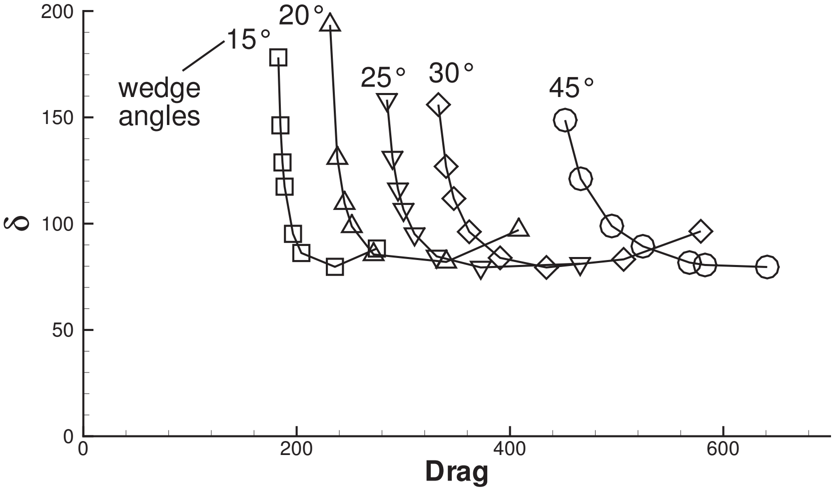


Figure 18: *Initiation distance as a function of the wedge drag for a Mach number of 22.8 and for different wedge angles*

and 45°. For these cases, the onset of detonation occurs far from the source, which is typical to the blast initiation model. Therefore, although the assumptions of the Lee-Vasiljev model are verified, the critical drag for detonation initiation depends on the wedge angle.

Eckett et al. [7] used a decay rate model to determine the critical conditions for direct initiation of a spherical detonation. Therefore, criticality for direct initiation of a detonation can be determined according to a critical decay rate. With an ideal source of energy (a source infinitely small that releases the energy instantaneously), the decay rate of the blast depends only on the amount of energy deposited by the source. Figure 20 shows the evolution of the wave angle for a Mach number of 13.7 and for three wedge angles at constant drag ($D = 211$). As expected from the results presented above, detonation initiation occurs for $\phi = 20^\circ$ and 30°, but fails to occur for $\phi = 45^\circ$. It can be observed that even though the drag is the same for the three curves, the decay of the oblique shocks differs. Figure 21 illustrate the evolution of the wave angle for the three wedge angles at their respective critical drag (for a Mach number of 13.7). The decay of the blast from the source to the quasi-steady region is very similar for the three curves. The same trend is observed for a Mach number of 22.8. Figure 22 shows the evolution of the wave angle for a Mach number of 22.8 for different wedge angles at their respective critical drag. The decay of the oblique shock is very similar for all the curves. Therefore, in accordance with the decay rate model, critical initiation of an ODW seems to be governed by a critical decay rate of the oblique shock wave.

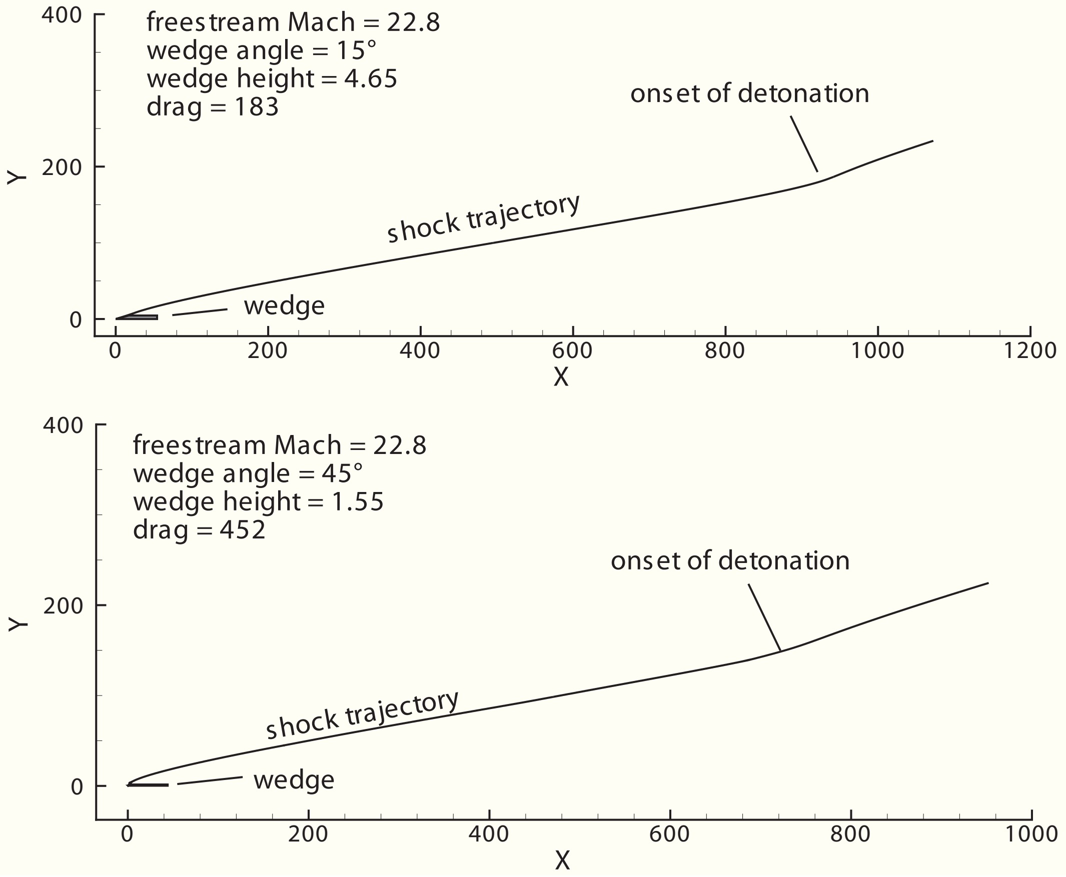


Figure 19: *Examples of shock trajectory for a Mach number of 22.7 and for a wedge angle of 15° and 45°*

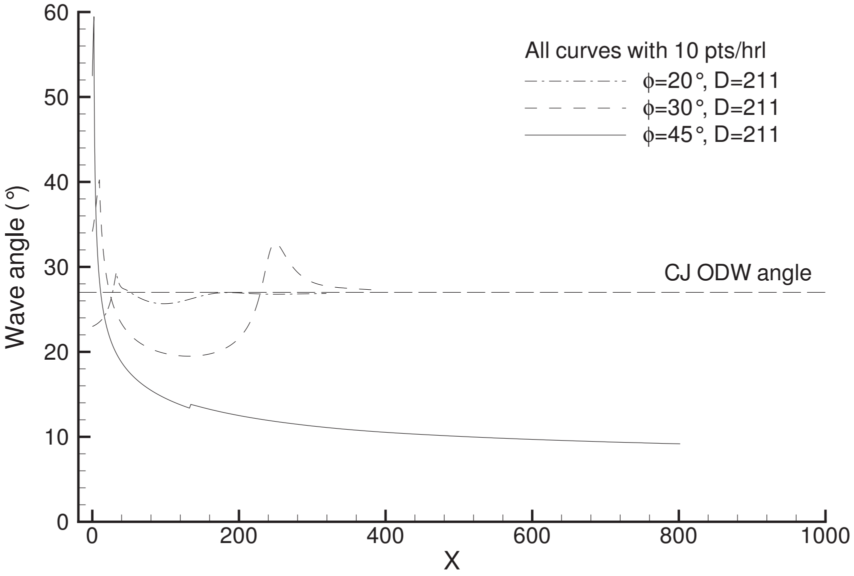


Figure 20: *Evolution of the wave angle for different wedge angles at constant drag and for a Mach number of 13.7*

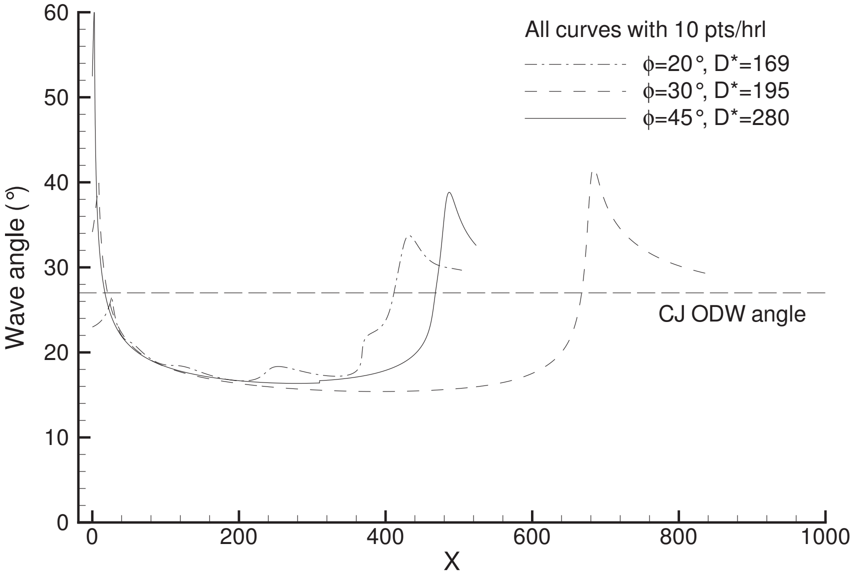


Figure 21: *Evolution of the wave angle for different wedge angles at their respective critical drag and for a Mach number of 13.7*

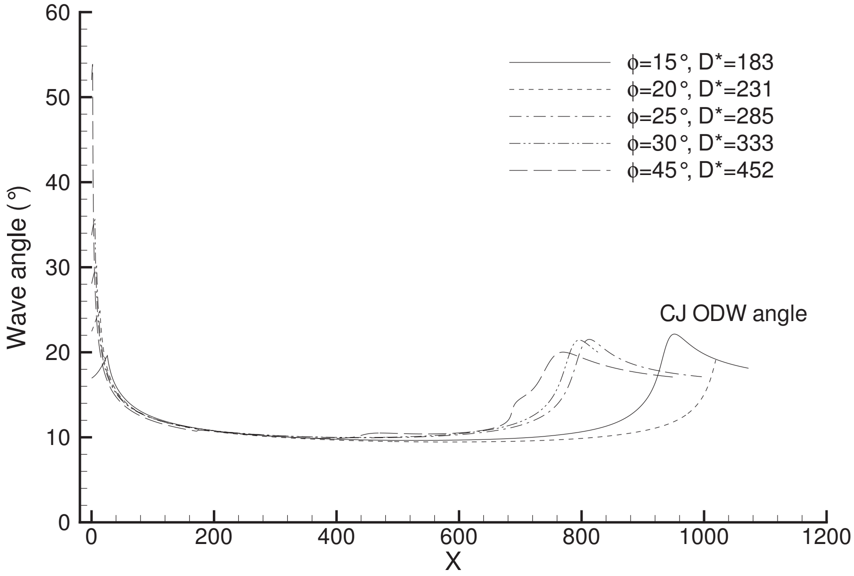


Figure 22: *Evolution of the wave angle for different wedge angles at their respective critical drag and for a Mach number of 22.8*

5 Conclusions

In this study, critical initiation of ODWs from finite wedges was presented. The three regimes observed from the simulated flowfields (subcritical, critical and supercritical) were qualitatively similar to those of direct initiation of spherical (or cylindrical) detonations. The Lee-Vasiljev model was considered to predict the critical conditions for detonation initiation. The simulation results showed that although the assumptions of the model are verified (the hypersonic equivalence principle was valid and blast initiation of a detonation applied), the critical drag varied with respect to the wedge angle, which was in disagreement with the Lee-Vasiljev model. However, the decay rate of the oblique shock was almost identical at the critical conditions for detonation initiation for the three wedge angles. This suggests the use of a critical decay rate model to predict the conditions for detonation initiation from hypersonic projectiles.

References

[1] Verreault, J. and Higgins, A. J. (2011), Initiation of detonation by conical projectiles, *Proceedings of the Combustion Institute*, 33(2), 2311–2318.

[2] Lee, J. H. S. (1997), Initiation of detonation by a hypervelocity projectile, *Progress in Astronautics and Aeronautics*, 173, 293–310.

[3] Vasiljev, A. A. (1994), Initiation of gaseous detonation by a high speed body, *Shock Waves*, 3(4), 321–326.

[4] Higgins, A. J. (1996), Investigation of detonation initiation by supersonic blunt bodies, Ph.D. thesis, University of Washington.

[5] Kasahara, J., Arai, T., Chiba, S., Takazawa, K., Tanahashi, Yu, and Matsuo, A. (2002), Criticality for stabilized oblique detonation waves around spherical bodies in acetylene/oxygen/krypton mixtures, *Proceedings of the Combustion Institute*, 29(2), 2817–2824.

[6] He, L. and Clavin, P. (1994), On the direct initiation of gaseous detonations by an energy source, *Journal of Fluid Mechanics*, 277, 227–248.

[7] Eckett, C. A., Quirk, J. J., and Shepherd, J. E. (2000), The role of unsteadiness in direct initiation of gaseous detonations, *Journal of Fluid Mechanics*, 421, 147–183.

[8] Benedick, W. B., Guirao, C. M., Knystautas, R., and Lee, J. H. S. (1986), Critical charge for the direct initiation of detonation in gaseous fuel-air mixtures, *Progress in Astronautics and Aeronautics*, 106, 181–202.

[9] Zucrow, M. J. and Hoffman, J. D. (1977), Gas Dynamics, Vol. 2: Multidimensional Flow, John Wiley & Sons Inc.

[10] Watt, S. D. and Sharpe, G. J. (2005), Linear and nonlinear dynamics of cylindrically and spherically expanding detonation waves, *Journal of Fluid Mechanics*, 522, 329–356.

[11] Jr., John D. Anderson (2006), Hypersonic and High-Temperature Gas Dynamics, 2nd ed, American Institute of Aeronautics and Astronautics Inc.

[12] Radulescu, M. I., Higgins, A. J., Murray, S. B., and Lee, J. H. S. (2003), An experimental investigation of the direct initiation of cylindrical detonations, *Journal of Fluid Mechanics*, 480, 1–24.

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In a shock-induced combustion ramjet, combustion relies on a standing oblique detonation wave at the entrance of the combustor. Assuming that the fuel is uniformly mixed with the air stream prior to reach this location, it is essential to predict the conditions (type of fuel, mixture equivalence ratio, flow velocity and deflection angle) to initiate such a detonation wave for various flight regimes.

In this investigation, hypersonic reactive flows over finite wedges are simulated with a method of characteristics program. The chemistry is modeled with a single-step Arrhenius equation. The independent parameters are the freestream velocity, the wedge height and the wedge angle. At the critical conditions to initiate an oblique detonation wave, the critical wedge drag is calculated. In an attempt to develop a universal prediction tool without having to solve the flowfield, the Lee-Vasiljev model is considered. This model equates the work done by the wedge (the wedge drag) to the critical energy to directly initiate a cylindrical detonation wave. According to this model, the energy transferred by the wedge to the mixture is the only parameter that determines whether detonation initiation occurs or fails.

The simulation results show three regimes (subcritical, critical and supercritical) that are typical to direct initiation of a detonation. The critical drag is obtained for a series of test cases. When the wedge angle is fixed and the wedge height is varied to find the critical drag, this parameter depends on the value of the wedge angle. However, the Lee-Vasiljev model predicts a constant critical drag for any values of wedge angle. Alternatively, the decay of the oblique shock wave for all critical cases (i.e. for all values of wedge angle) is very similar. This suggests the use of a critical decay rate model for the prediction tool.

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oblique detonation wave; method of characteristics; critical initiation

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