

# Two-Echelon Supply Chain Management – Task Authorization #1:

## Final Report

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## Abstract

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Adaptive and dispersed military and humanitarian missions are complex operations that require significant resources, including personnel, equipment, and medical and food supplies. For these operations to be effective, it is important to ensure that resources are provided and supported in a cost effective and timely fashion. This report describes the design and development of models and algorithms to improve distribution and inventory management of resources for such military and humanitarian missions. The scope addressed is an extended two-echelon tactical Supply Chain Network (SCN). Challenges addressed and novelties of the models include: (a) integration of inventory management and distribution management, (b) simultaneously addressing several realistic features of supply chain management within one problem, (c) having an extended network that includes the tactical SCN and a portion of the operational SCN, with different scales for transportation times and (d) the intrinsic complexities of multi-echelon, multi-period discrete optimization problems. Both deterministic and stochastic problems are addressed. The work done is an advancement on providing tools for military or humanitarian logistics leaders to optimise supply chain management and use of resources and hence reduce time and costs in supplying forward operating bases with supplies such as food, water, medical provisions and equipment.

## Résumé

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Les missions militaires et humanitaires adaptatives et dispersées sont des opérations complexes qui nécessitent d'importantes ressources, y compris le personnel, l'équipement et les fournitures médicales et l'approvisionnement alimentaire. Pour que ces opérations soient efficace, il est important de s'assurer que les ressources sont fournis et pris en charge de manière efficace et rentable. Ce rapport décrit la conception et le développement de modèles et d'algorithmes pour améliorer la distribution et la gestion des stocks pour les missions militaires et humanitaires. Le champ d'application visé est le réseau d'approvisionnement (SCN) tactique à deux échelons. Les défis abordés et les nouveautés des modèles incluent: (a) l'intégration de la gestion des stocks et de gestion de la distribution, (b) examiner simultanément plusieurs caractéristiques réalistes de la gestion du réseau d'approvisionnement au sein d'un seul problème, (c) ayant un réseau étendu qui inclut le SCN tactique et une partie du SCN opérationnel, avec différentes échelles de temps de transport et (d) les complexités intrinsèques des problèmes d'optimisation discrète aux échelons et périodes multiples. Les problèmes à la fois déterministes et stochastiques sont abordés. Le travail accompli est un avancement des outils pour les meneurs de la logistique militaire ou humanitaire afin d'optimiser la gestion du SCN et l'utilisation des ressources et réduire le temps et les coûts dans l'approvisionnement de bases d'opérations avancées avec des fournitures telles que la nourriture, l'eau, des fournitures médicales et du matériel.

## Executive Summary

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### Introduction

Adaptive and dispersed military and humanitarian missions are complex operations that require significant resources, including personnel, equipment, and medical and food supplies. For these operations to be effective, it is important to ensure that resources are provided and supported in a cost effective and timely fashion. This project is focused on developing models and algorithms to improve distribution and inventory management of resources for such military and humanitarian missions. The goal is to support decision makers and analysts of the Canadian Operational and Tactical Support Commands. The scope is an extended two-echelon tactical Supply Chain Network (SCN), i.e., one that includes the initial leg between Canada and the place of operation (which could be in another part of Canada or elsewhere in the world). Challenges addressed and novelties of the models include: (a) integration of inventory management and distribution management, (b) simultaneously addressing several realistic features of supply chain management within one problem, (c) having an extended network that includes the tactical SCN and a portion of the operational SCN, with different scales for transportation times and (d) the intrinsic complexities of multi-echelon, multi-period discrete optimization problems.

The project was undertaken by a team consisting of Project Engineer (Dr. Minic), Project Manager (Dr. Dutkiewicz), and software analysts and developers (Dr. Ghazel and Mr. Thomson) at MDA, two academic experts from the University of Montreal (Prof. Gendreau and Prof. Potvin), and a Subject Matter Expert in the field of humanitarian and military logistics (Mr. Conrad). Combining a wide breadth of expertise in industry, academia, and the military provided the project with a strong and broad set of skills and experience, enabling novel work to be done, new models to be designed, and solutions approaches to be implemented and tested using data based on realistic scenarios.

### Results

The project was able to successfully perform problem definition and characterization, design a new mathematical programming formulation (model) of the problem, propose algorithms for solving the deterministic extended two-echelon supply chain management problem, and do preliminary work on devising algorithms for solving the stochastic version of the problem. One solution approach (algorithm) was implemented using a combination of in-house developed source code and the Gurobi general purpose Mixed-Integer Programming (MIP) solver software. The work for the deterministic problem necessarily had to be done first, since this provides the foundation from which more complicated problems, such as stochastic version of the problem, can be developed. Most of this study concentrated on the deterministic case, but sufficient initial work on the stochastic case was undertaken to show the potential for developing it further. The results presented in the report are thus mainly for the deterministic scenarios.

In addition, the project team designed and implemented a software tool that is a discrete event simulator for a supply chain network. This can be used to test the solutions of the SCM problem and to plot inventory changes throughout the time horizon.

### **Deterministic Problem Results:**

The designed mathematical programming model and the implemented software deals with inventory levels, routes for the transportation assets, transportation costs, inventory costs, met and unmet demand at the nodes for different scenarios. Input parameters that can be set include: number of time periods, duration of the time period, number of satellite nodes, number of demand nodes for each satellite, number of people at the demand node, number of products, transportation asset types for each echelon (e.g. cargo plane and ship at the far supply node, variety trucks at the close supply nodes, and the satellite nodes), transportation asset capacities, number of each transportation type at each node, capacities of warehouses, and initial inventory levels for each product at each node (warehouse). The output of the implemented software states for each time period the routes for the transportation assets, the amount of each product that travels on each transportation asset, the levels of inventories, and unmet demand, if any.

The types of products considered include food, water, petroleum and oil, as well as equipment, medical supplies and other military / relief operations consumable materiel. Our model addresses the compatibility between a product and a vehicle type (e.g. making sure that petroleum travels on a crude oil truck, and that products that need refrigerator trucks are properly transported), the compatibility between the product and the inventory type (e.g. fridge, oil storage etc.), and the compatibility between two products in one transportation asset type (e.g. whether two products can travel together on a plane).

The implemented solution was first run with a set of simple cases to verify that the code operates as expected, that it is able to converge to an optimal solution for this level of problem, to provide insights into the problem complexity, and to get an idea of the times needed for solving larger problems. Results showed that the implemented solution is indeed working and suitable for application to more complicated scenarios.

A number of more complex scenarios, simulating disaster relief scenarios, were then run. The disaster relief problem instances were randomly generated based on the earthquake data provided by the project SME, John Conrad. During the computational study, the software was limited to running for a maximum of 5 minutes, to make it feasible to do a large number of test runs. However, even with this limitation, although optimal solutions were not found, the optimality gaps between the best MIP solutions found and the best lower bounds found were all low (less than 3%) indicating that these solutions are reasonable. The software also allows the user to observe and track inventory levels at nodes, distances traveled, and to see the effects on costs.

### **Stochastic Problem Results:**

The project team was able to implement the explicit form of the stochastic model, and succeeded in solving the problem with the solution approach generated for solving deterministic model. The stochastic problem was run for 100 minutes and generated a feasible solution with an optimality gap of 0.06%. Due to project time and budget constraints, it was not possible to do further testing

with the stochastic model. Thus, although a considerable amount of progress was made on this, further testing and potentially debugging are still needed for the stochastic problem solver.

## Significance

This study undertook novel work in the domain of Supply Chain Management, to create new models and algorithms for adaptive and dispersed military and humanitarian missions. The models and algorithms developed allow optimisation of distribution and inventory management of resources for these missions. The work done is an advancement on providing tools for military or humanitarian logistics leaders to optimise use of resources and hence reduce time and costs in supplying forward operating bases with supplies such as food, water, medical provisions and equipment.

The novelties of the proposed models include:

- Integration of inventory management into a multi-echelon vehicle routing problem (VRP).
- Multiple realistic problem characteristics – multiple products, heterogeneous fleet, multiple periods, compatibility constraints – that have been considered separately in various other multi-echelon VRP or SCM models, but all of these sources of complexity have not previously been considered simultaneously in the same problem.
- Stochastic demand combined with all the characteristics and features listed in the previous bullet.
- Extension of the two-echelon network with an additional level – made of a unidirectional link between a far supply node and a close supply node – that must be handled in a particular way, when compared to the rest of the network, due to the different travel time scales.
- Consideration of heterogeneous fleets at each level: multi-echelon routing problems often involve different types of vehicles, but the fleets considered are usually homogeneous by level.

## Future Plans

This study focussed on foundational analysis and development of tools needed for extended two-echelon supply chain management. Now that the initial work has been accomplished, the models and tools exist to do much more interesting work, such as: modeling of risks and safety issues; exploring effects such as demand level fluctuations; incorporating uncertainty in travel times; including order priorities; implementing advanced solution approaches to solving the stochastic problem; refining the modeling of stochastic demand and considering dependencies. The tools developed in this study will expedite further work in this domain.

S. Mitrovic Minic, M. Gendreau, J.-Y. Potvin, and D. Thomson (2013) “To-Echelon Supply Chain Management – Task Authorization #1: Final Report”, DRDC Valcartier CR 2013-??



## Sommaire

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### Présentation

Les missions militaires et humanitaires adaptatives et dispersées sont des opérations complexes qui nécessitent d'importantes ressources, y compris le personnel, l'équipement et les fournitures médicales et l'approvisionnement alimentaire. Pour que ces opérations soient efficace, il est important de s'assurer que les ressources sont fournis et pris en charge de manière efficace et rentable. Ce projet est axé sur le développement de modèles et d'algorithmes pour améliorer la distribution et la gestion des stocks et des ressources pour les missions militaires et humanitaires. L'objectif est d'aider les décideurs et les analystes des commandes canadiennes de soutien opérationnel et tactique. Le champ d'application visé est le réseau d'approvisionnement (SCN) tactique à deux échelons - celui qui comprend le trajet initial entre le Canada et le lieu de l'opération (qui pourrait être dans une autre partie du Canada ou d'ailleurs dans le monde). Les défis abordés et les nouveautés des modèles incluent: (a) l'intégration de la gestion des stocks et de gestion de la distribution, (b) examiner simultanément plusieurs caractéristiques réalistes de la gestion du réseau d'approvisionnement au sein d'un seul problème, (c ) ayant un réseau étendu qui inclut le SCN tactique et une partie du SCN opérationnel, avec différentes échelles de temps de transport et (d ) les complexités intrinsèques des problèmes d'optimisation discrète aux échelons et périodes multiples .

Le projet a été entrepris par une équipe composée d'ingénieur de projet (Dr. Minic), gestionnaire de projet (Dr. Dutkiewicz), et les analystes et développeurs de logiciels (Dr. Ghazel et M. Thomson) chez MDA, deux experts universitaires de l'Université de Montréal (Les professeurs Gendreau et Potvin), et un expert en la matière dans le domaine de la logistique humanitaire et militaire (M. Conrad). La combinaison d'une vaste gamme d'expertise provenant l'industrie et des milieux universitaires et militaires a apporté les compétences nécessaires au projet, permettant la réalisation de nouveaux travaux, de nouveaux modèles, et de solutions mettant en œuvre des approches testées à l'aide données basées sur des scénarios réalistes.

### Résultats

Le projet a permis de réaliser avec succès la définition du problème et de caractérisation, la conception d'une nouvelle formulation de programmation mathématique (modèle) du problème, proposer des algorithmes pour résoudre le déterminisme du problème de gestion de la chaîne d'approvisionnement à deux échelons, et faire un travail préliminaire sur des algorithmes pour résoudre la version stochastique du problème. Une approche (algorithme) a été implémenté en utilisant une combinaison de code source développé à l'interne et le logiciel Gurobi de programmation Mixed-Integer (MIP). Le travail pour le problème déterministe devait nécessairement être fait en premier, puisque cela fournit la base à partir de laquelle les problèmes les plus complexes, telles que la version stochastique du problème, peuvent être développées. La plupart de cette étude s'est concentrée sur le cas déterministe, mais un travail initial suffisant sur le cas stochastique a été entrepris pour en démontrer sont potentiel, qui pourra être développé

d'avantage dans des travaux futures. Les résultats présentés dans le rapport sont donc principalement pour les scénarios déterministes.

En outre, l'équipe du projet a conçu et mis en œuvre un outil logiciel qui est un simulateur à événements discrets pour un réseau d'approvisionnement. Ceci peut être utilisé pour tester les solutions du problème SMC et de tracer les variations de stocks tout au long de l'horizon temporel.

### **Résultats de problèmes déterministes**

Le modèle de programmation mathématique conçu et les logiciels développés traitent des niveaux de stocks, des itinéraires pour les actifs de transport, les coûts de transport, les coûts d'inventaire, se sont réunis et la demande non satisfaite au niveau des nœuds pour différents scénarios. Les paramètres d'entrée qui peuvent être définies comprennent: le nombre de périodes de temps, la durée de la période de temps, le nombre de nœuds satellites, le nombre de nœuds de demande pour chaque satellite, le nombre de personnes au niveau du nœud de la demande, le nombre de produits, les types d'actifs de transport pour chaque échelon (par exemple avion cargo et navire au nœud d'alimentation lointain, les camions de variété près des nœuds d'approvisionnement et satellites), les capacités de transport d'actifs, le nombre de chaque type de transport à chaque nœud, la capacité des entrepôts, et les niveaux de stocks initiaux pour chaque produit à chaque nœud (entrepôt). La sortie des états logiciels mis en œuvre pour chaque période de temps, les itinéraires pour les actifs de transport, la quantité de chaque produit qui se déplace sur chacun des actifs de transport, les niveaux des stocks et de la demande non satisfaite, le cas échéant.

Les types de produits considérés comprennent la nourriture, l'eau, le pétrole et l'huile, ainsi que du matériel, des fournitures médicales et d'autres matériel consommable pour les opérations militaires et de secours. Notre modèle porte sur la compatibilité entre un produit et un type de véhicule (par exemple en s'assurant que le pétrole se déplace sur un camion de pétrole brut, et que les produits qui ont besoin camions frigorifiques sont correctement acheminés), la compatibilité entre le produit et le type d'inventaire (par exemple réfrigérateur, stockage de l'huile, etc), et la compatibilité entre les deux produits en un seul type d'élément de transport (par exemple, si les deux produits peuvent se déplacer ensemble sur un avion).

La solution mise en place a été exécuté avec un ensemble de cas simples pour vérifier que le code fonctionne comme prévu, qu'il est capable de converger vers une solution optimale pour ce niveau de problème, pour donner un aperçu de la complexité du problème, et pour avoir une idée du temps nécessaire pour résoudre des problèmes plus importants. Les résultats ont montré que la solution mise en place fonctionne et est en effet approprié pour une application à des scénarios plus complexes.

Un certain nombre de scénarios plus complexes, simulant un scénario de secours aux sinistrés, ont ensuite été exécuté. Les problèmes de secours aux sinistrés ont été générés de façon aléatoire sur la base des données sismiques fournies par notre expert en la matière, John Conrad. Au cours de l'étude de calcul, le logiciel a été limité à courir pour un maximum de 5 minutes, pour qu'il soit possible de faire un grand nombre d'essais. Cependant, même avec cette restriction,

bien que des solutions optimales sont introuvables, les écarts optimaux entre les meilleures solutions de MIP trouvés et les meilleures bornes inférieures trouvés étaient tous faibles (moins de 3%) indique que ces solutions sont raisonnables. Le logiciel permet également à l'utilisateur d'observer et de suivre les niveaux de stocks au niveau des nœuds, les distances parcourues, et de voir les effets sur les coûts.

### **Résultats de problèmes stochastique**

L'équipe du projet a réussi à mettre en œuvre la forme explicite du modèle stochastique, et a réussi à résoudre le problème avec l'approche de la solution générée pour résoudre le modèle déterministe. Le problème stochastique a été exécuté pendant 100 minutes et a généré une solution réalisable avec un écart d'optimalité de 0,06%. En raison de la durée du projet et les contraintes budgétaires, il n'était pas possible de faire d'autres essais avec le modèle stochastique. Ainsi, même si un progrès considérable a été accomplis sur ce point, des essais supplémentaires et potentiellement du débogage sont encore nécessaires pour la résolution des problèmes stochastiques.

## **Importance**

Cette étude a entrepris de nouveaux travaux dans le domaine de la gestion du réseau d'approvisionnement, afin de créer de nouveaux modèles et algorithmes adaptatifs et dispersés pour les missions militaires et humanitaires. Les modèles et algorithmes développés permettent d'optimiser la distribution et la gestion de l'inventaire des ressources pour ces missions. Le travail accompli fourni des outils pour les meneurs de la logistique militaire ou humanitaire afin d'optimiser l'utilisation des ressources et de réduire les délais et donc les coûts en fournissant les bases d'opérations avancées avec des fournitures telles que la nourriture, l'eau, des fournitures médicales et du matériel.

Les nouveautés des modèles proposés sont les suivantes:

- Intégration de la gestion des stocks dans un problème de routage de véhicule à multi-échelon (VRP).
- Plusieurs caractéristiques de problèmes réalistes - plusieurs produits, la flotte hétérogène, de plusieurs périodes, les contraintes de compatibilité - qui ont été considérées séparément dans divers autres modèles SCM VRP à multi-échelon ou, mais toutes ces sources de complexité n'ont pas encore été examinées simultanément dans le même problème.
- Demande stochastique combiné avec toutes les caractéristiques et fonctions énumérées au point précédent.
- Extension du réseau à deux échelons avec un niveau supplémentaire - en une liaison unidirectionnelle entre un nœud d'alimentation lointain et des nœuds d'alimentation plus près - qui doit être manipulé d'une manière particulière, par rapport au reste du réseau, en raison des différentes échelles de temps de voyage.

- Examen des flottes hétérogènes à chaque niveau: VRP à multi-échelon impliquent souvent différents types de véhicules, mais les flottes sont généralement considérées comme homogènes par niveau.

## Plans pour l'avenir

Cette étude a porté sur l'analyse et le développement des outils nécessaires à la gestion de la chaîne d'approvisionnement prolongée à deux échelons . Maintenant que le travail initial a été accompli, les modèles et les outils existent pour faire un travail beaucoup plus intéressant, tels que: la modélisation des risques et les questions de sécurité, les effets explorer tels que les fluctuations du niveau de la demande, intégrant l'incertitude dans les temps de déplacement, y compris la priorité des commandes; la mise en œuvre de solution avancée pour résoudre le problème stochastique; affiner la modélisation de la demande stochastique et des dépendances correspondantes. Les outils développés dans cette étude permettront d'accélérer les travaux dans ce domaine.

S. Mitrovic Minic, M. Gendreau, J.-Y. Potvin, and D. Thomson (2013) "To-Echelon Supply Chain Management – Task Authorization #1: Final Report", DRDC Valcartier CR 2013-??

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## ACRONYMS AND ABBREVIATIONS

AHSVS	Armoured Heavy Support Vehicle System
ALNS	Adaptive Large Neighborhood Search
API	Application Programming Interface
APOD	A Point of Disembarkation
CANOSCOM	Canadian Operational Support Command
CFJOSG	Canadian Forces Joint Operational Support Group
CPLEX	A commercial MIP solver developed by IBM ILOG
CPU	Central Processing Unit
DES	Discrete Event Simulation
DND	Department of National Defence
DRDC	Defence Research and Development Canada
EVPI	Expected Value of the Perfect Information
FOB	Forward Operating Base
G&T	General and Technical
GUROBI	An MIP solver
HLVW	Heavy Logistic Vehicle Wheeled
IMP	Individual Meal Pack
JSON	Javascript Serialized Object Notation
JVM	Java Virtual Machine
LAV	Light Armoured Vehicle
MDA	MDA Systems Ltd.
MIP	Mixed-Integer Programming
NF	Network Flow
NSE	National Support Element
PHA	progressive hedging algorithm
PLS	Palletized Loading System
POL	Petroleum, Oil, and Lubricants
SCM	Supply Chain Management
SCN	Supply Chain Network
SME	Subject Matter Expert
SVG	Scalable Vector Graphics
UAV	Unmanned Aerial Vehicle

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UI	User interface
VRP	Vehicle Routing Problem
VSS	Value of the Stochastic Solution
YAML	YAML Ain't Markup Language

# 1 Introduction

This document represents the Final Report of the Task Authorization #1 for project “In-theatre 2-echelon supply chain management – preliminary models and algorithms”. The work has been executed by MDA Systems Ltd. (MDA) under the Defence Research and Development Canada (DRDC) contract W7701-125337 “In-Theatre Sense and Respond Logistics for Adaptive Dispersed operations – adaptive planning models and algorithms” [A-1], Task Authorization #1 W7701-4500989924 “In theatre 2 echelon supply chain management – preliminary models and algorithms” A-2].

The focus of the project for this Task Authorization is on preliminary models and algorithms for decision support of the transportation and inventory management activities related to the tactical and possibly operational logistics. The aim is to support decision makers and analysts of the Canadian Operational and Tactical Support Commands.

The objective is to research new problems, concepts, models, and algorithms addressing supply chain management issues such as stochastic demands, and chaotic or hostile environments. This document reports on the research study on the preliminary models and the solution approaches for the transportation and inventory management problems in the two-echelon supply chain management.

The layout of this report is:

- Section 1 introduces the 2-echelon supply chain management and defines our problem and its characteristics
- Section 2 provides the background and the motivation for this project
- Section 4 provides literature review
- Section 5 introduces our model for the deterministic version of the problem
- Section 6 introduces one possible model of the stochastic version of the problem
- Section 7 describes implemented solution approaches
- Section 8 gives initial outlines of few other potential solution approaches
- Section 9 describes the software package developed
- Section 10 presents the computational study and the experimental results
- Section 11 concludes the document and provides directions for future work
- Appendix A describes the Military Scenario
- Appendix B presents the More on Military Transportation Assets
- Appendix C presents the Capacities and/or Stock Levels for in Theatre Modeling
- Appendix D presents the Current Distribution Policies

- Appendix E describes Data Sets
- Appendix F presents the SCM Solver: Master Input File
- Appendix G presents the SCM Solver: Output
- Appendix H lists the References

## 1.1 Applicable Documents

- |     |                                     |  |
|-----|-------------------------------------|--|
| A-1 | RV-24692 (1)                        | Contract for “In-Theatre Sense and Respond Logistics for Adaptive Dispersed operations – adaptive planning models and algorithms” September 27, 2012.                |
| A-2 | W7701-4500989924 TA1                | Task Authorization #1, “In theatre 2 echelon supply chain management – preliminary models and algorithms”, 28 November, 2012.  |
| A-3 | TA-1 W7701-125337_4500989924_Mod001 | Amendment #1 to Contract A-2 W7701-4500989924 TA1  |
| A-4 | 01-7368                             | MDA Proposal for “In-Theatre Sense and Respond Logistics for Adaptive Dispersed Operation— Adaptive Planning Models and Algorithms”, Nov 15, 2012 and June 11, 2013. |

## 1.2 Reference Documents

As mentioned above, References are listed in Appendix H.

## 2 Background

This project deals with the supply chain management arising in military and humanitarian operations. The main focus is on the combined inventory management and distribution (transportation) management problem. The problem is assumed to be centralized. The demand is assumed to be stochastic.

The introduction, motivation, and main definitions relevant to this project may be found in [MC\_2011] where both tactical and operational logistics for DND operations are discussed. This section contains some excerpts from this document.

### 2.1 Supply Chain Network

The DND Supply Chain Network (SCN) consists of the:

- Operational SCN – the portion of the SCN from the locations in Canada to a Point of Disembarkation (the APOD), and
- Tactical SCN – the portion of the SCN from the APOD to the forward operating DND units.

The Operational SCN was managed by the Canadian Forces Joint Operational Support Group (CFJOSG) since October 2012, earlier known as the Canadian Operational Support Command (CANOSCOM). The Tactical SCN is managed by the National Support Element (NSE) in conjunction with the Canadian Task Force Headquarters G4 (Logistics) Staff..

This project considers the tactical SCN extended by including the main supply node in Canada (Figure 3-1). We refer to such an SCN as the extended two-echelon SCN. The motivation comes from: (a) the differing origin location of the supplies (Section 2.2.3), (b) the complexity of the problem, (c) the differences between the Supply Chain Management (SCM) issues in the operational and tactical SCNs, and (d) the challenges arising from the division of the SCN into operational and tactical portions.

As such the solution approaches would address the following two situations: when supplies are originating in Canada or far from the disaster area, and when supplies are originating at an APOD.

For the CF operations, the path of supplies through the SCN is determined by their location of origin. Thus, the situation is as follows:

- The tactical SCN is sufficient to consider for the locally purchased supplies:
  - Food (80% local, 20% Canada)
  - Water (90% local)
  - Petroleum, Oil, and Lubricants (POLs) (85-90% local)

- The entire SCN has to be considered for the supplies coming from Canada or from a far-away supply location:
  - Medical supplies (100% Canada)
  - Equipment (100% Canada)
  - Spare parts (100% Canada)
  - Other military operations consumable materiel (100% Canada)

Locally purchased supplies sometimes come through the APOD, and sometimes are delivered directly to Forward Operating Bases (FOBs). For example, around 90% of the locally purchased water is bottled water, and it is delivered to the APOD first before it is transported to FOBs by the NSE vehicles.

The percentages are based on the experiences of the team SME, John Conrad.

In disaster relief operations, the situation is similar when dealing with international relief operations for disasters abroad, or when dealing with potential mega-disasters within Canada. For example, in the case of a mega-earthquake that may happen in Western Canada, all of Canada may be involved in the relief operation, and the origins of some supplies may be far away. Thus, the portions of the SCN that need to be considered are as follows:

- The tactical SCN is sufficient to consider for the locally purchased supplies:
  - Food (80% local or from close-by locations)
  - Water (90% local or from close-by locations)
- The entire SCN has to be considered for the supplies coming from Canada-at-large or from a far-away supply location:
  - Medical supplies (100% from further locations in Canada)
  - Equipment (100% from further locations in Canada)

## 2.2 Gaps and Requirements

Proposed enhancements to the Canadian tactical in-theatre supply chain management include [MC\_2011]:

1. Merging Operational and Tactical supply chains.
2. New technologies should be considered for implementation:
  - a) Faster collection of requests (via radio),
  - b) Better forecasting (including consideration of future mission plans),
  - c) Improved inventory management (incorporating automated systems),
  - d) Transportation (improved planning, routing and scheduling).
3. Transportation services for materiel and personnel have to be considered together.

From these proposed enhancements, this project will address points 2c and 2d – inventory management and transportation strategies. Also, the project will consider the issues (point 1) relevant to merging of the tactical and operational DND Supply Chain Networks.

### **2.2.1 Relevant Tactical Logistics IT Gaps**

From the gaps identified in Section 6 of [MC\_2011] report, this project will address gaps from Operations Management (G4 group) shown in Table 1:

- G4.1 Distribution management gaps:
  - G4.1.1 Distribution planning, including entire supply chain
  - G4.1.2 Computer support for routing and scheduling in order to fulfill transportation requests or to resolve trade-offs between time, costs, and risks; heterogeneous fleet, heterogeneous loads; pre-configured loads
  - G4.1.3 Real-time distribution planning
  - G4.1.4 Dynamic routing with re-routing capabilities in order to react to unexpected demands
  - G4.1.5 Replenishment cycle changes
- G4.2 Inventory management gaps:
  - G4.2.5 Re-supply, i.e., replenishment

**Table 1 Tactical Logistics IT Gaps to be considered in this project [MC\_2011, Section 6.3]**

Gap category	Gap	Description	Research in progress	Implementation in progress	Fielding in progress	Currently operational	Achieved	Priority/ importance	Potential technology for enhancement
G4. Operations Mgmt.	G4.1	Distribution management	✓	✗	✗	✗	●	■ (2/4)	Optimization tools
	G4.2	Inventory management	✓	✗	✗	✗	●	■ (2/4)	Optimization tools

### 2.2.2 Relevant Requirements

The general requirements are taken from [MC\_2011, Section 7.2]. “DA” is the term used within this document for the proposed future Decision Support system for Operational and Tactical Logistics.

These general requirements are listed here as a source of information that could be used for solving inventory management and distribution management problems. The specific requirements or portions of the requirements that are to be addressed in the project are underlined.

Requirement	Description	Gaps
<b>Distribution management</b>		
DA-REQ-600	The DA should have tools/algorithms that provide <u>lists of transportation requests</u> based on forecasts, demand, and min/max stock levels at each node of the distribution network. Real-time/ dynamic aspects of the problem have to be supported. Changes to replenishment cycle have to be supported.	G4.1.1, G4.1.3, G4.1.5
DA-REQ-603	The DA should have tools/ <u>algorithms for dynamic routing and scheduling</u> of the heterogeneous fleet of vehicles to satisfy requests for transportation of people and materiel. Optimization and multi-criteria decision making should be included. The tools should provide multiple solutions/ alternatives among which a decision maker will choose. Re-routing, transshipment, pre-configured loads should be supported.	G4.1.2, G4.1.3, G4.1.4
<b>Inventory management</b>		
New	The DA should have <u>inventory management tools for finding and managing the stock levels</u> at all warehouse locations.	G4.2.4



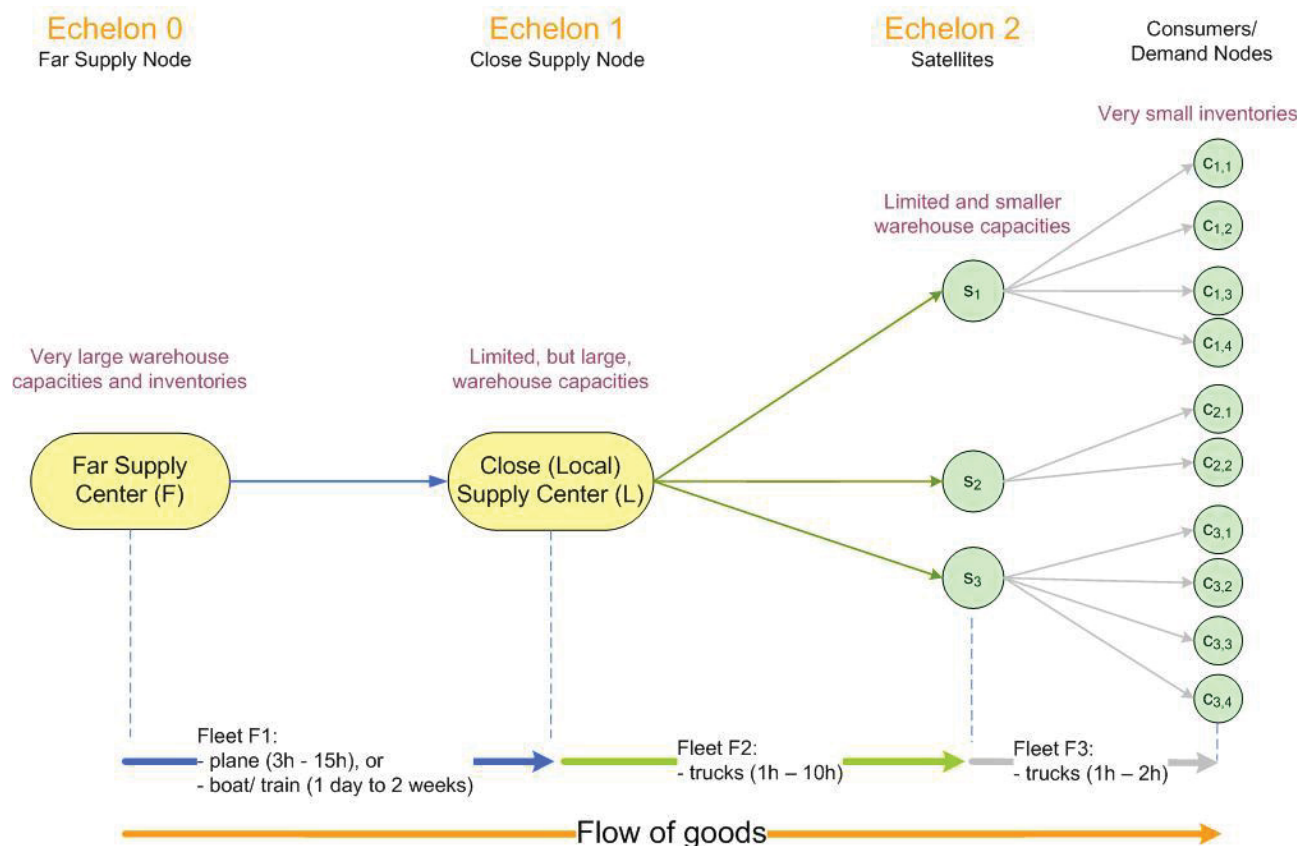
## 3 Two-Echelon Supply Chain Management

### 3.1 Supply Chain Network

#### 3.1.1 The Extended Two-Echelon SCN

The Extended Two-Echelon SCN, as shown in 1, is to be considered for:

- Military operations: for the supplies coming from Canada
- Disaster relief operations:
  - Disaster relief abroad
  - Domestic mega-disaster: for the supplies coming from far-away locations in Canada.



**Figure 3-1 The extended two-echelon Supply Chain Network. The arrows are showing from where the supplies/goods are coming from.**

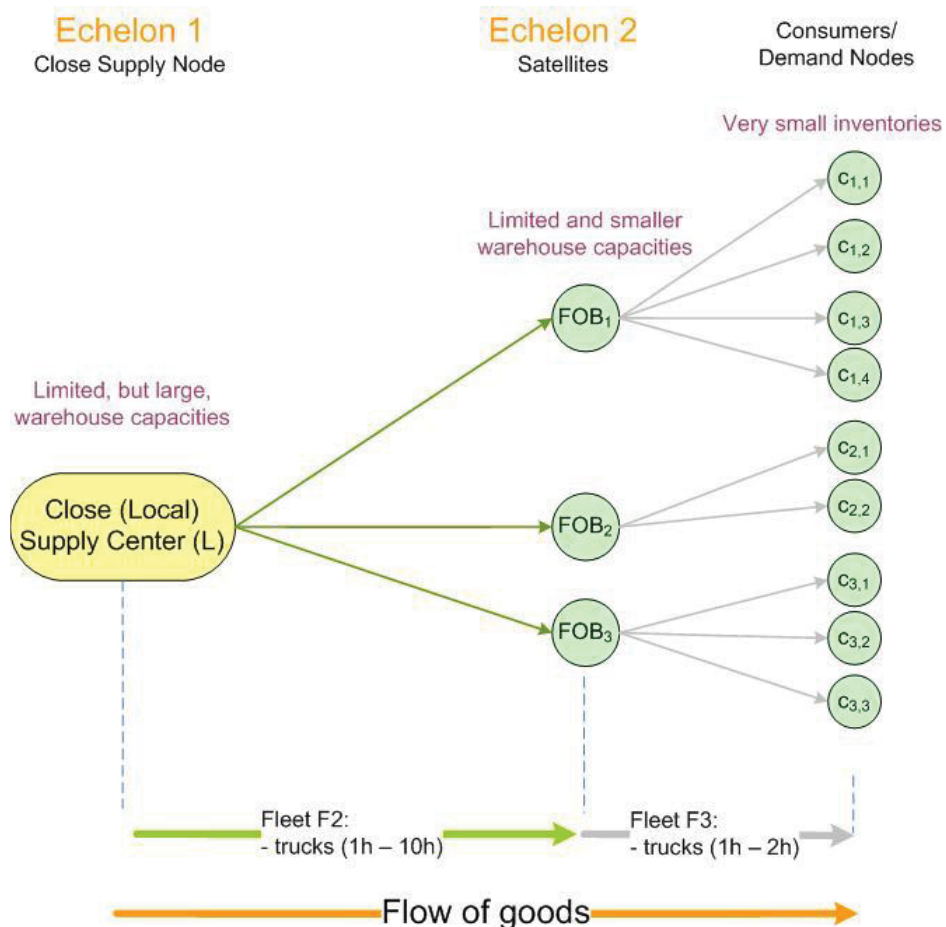
Going from left to right, we have the following nodes at each level of the Extended Two-Echelon SCN: a far supply node, a close supply node (APOD), satellite nodes (FOBs) and customer nodes. The most right echelon is labeled Echelon 0, the middle echelon is Echelon 1, and the right most echelon is Echelon 2. The terms “consumers” and “customers” are used interchangeably throughout the document to represent the nodes in the right-most level of the supply chain network.

### 3.1.2 The Two-Echelon SCN

The two-echelon SCN, shown in Figure 3-2, would be sufficient to consider when dealing with:

- Military operations: for the supplies purchased locally (water, food, and POLs)
- Relief operations for domestic disasters that can be dealt with within one province

It is assumed that all the supplies purchased locally are delivered to APOD and then transported to FOBs by the NSE vehicles.



**Figure 3-2 The two-echelon SCN – the tactical SCN. The arrows are showing from where the supplies/goods are coming from.**

## 3.2 Problem Definition and Characterization

The problem is integrated inventory management and distribution management arising in the supply management for humanitarian and military operations.

The characteristics of our supply chain management problem that are novel compared to the state-of-the-art literature include:

- Inventories at each node of the SCN
- Transportation times in the first level of the SCN are measured in weeks, while the travel times in the last level of the SCN are measured in hours or minutes, resulting in the situation that this portion of the network must be handled in a particular way, when compared to the rest of the network, due to this different time scale.
- Integration of inventory management in a multi-echelon vehicle routing problem
- Multiple realistic problem characteristics – multiple products, heterogeneous fleet, multiple periods, compatibility constraints – that have been considered separately in various multi-echelon VRP or SCM models, but never all of these sources of complexity simultaneously in the same problem.
- Stochastic demand with combination with all characteristics and features listed above.
- Heterogeneous fleets of vehicles at each echelon.

We provide the mathematical programming formulation of the merged inventory management and distribution problems.

The problems we tackled may arise in both disaster relief and military operations. We developed disaster relief problem instances (Section F) based on data provided by the team SME. For the reference, Table 3-1 compares logistics scenarios and its logistics problem characteristics.

**Table 3-1 Logistics Scenario Examples and Their Similarities and Differences**

<b>Logistics Scenarios Examples</b>	<b>Tight constraints</b>	<b>Objectives</b>	<b>Vehicle Fleet</b>	<b>Network</b>	<b>Stochastic</b>	<b>Fluctuating Demand</b>
<b>Commercial/ Retail</b>	Vehicle fleet Warehouse capacities	Min cost	Fixed size Homogenous	Stable and predictable travel times	Demand Travel times	Yes, but not very high variability
<b>Mega-Earthquake</b>	Supplies (medical, water, food) Transportation network Warehouse capacities; gradually decreasing from left to right in the SCN	Min risk Min losses of supplies	Not tight constraint Heterogeneous	Variable travel times Risks Not entire network available	Demand Travel times Lost supplies Risks	High variability
<b>Man-made disaster: virus spread/ disease outbreak</b>	Supplies (medical) Warehouse capacities	Min risk Min losses of supplies	Not tight constraint Heterogeneous	Variable travel times Risks	Demand Travel times Lost supplies Risks	High variability
<b>Peace-keeping</b>	Supplies (POLs, equipment, medical) Transportation network Warehouse capacities	Min risk Min losses of supplies	Not tight constraint Heterogeneous	Variable travel times Risks Not entire network available	Demand Travel times Lost supplies Risks	High variability

The demand for certain supplies can fluctuate greatly, and the demands are rarely known very far in advance.

When you face a complex problem, it is quite natural to address first a simplified variant and move on from there. The same approach is taken here: the deterministic variant is considered first, before addressing the full complexity of the problem with stochastic demand. Thus, in the deterministic model we assume that demands are known in advance, and in the stochastic model we assume that we know possible scenario realizations of the uncertain demand.

Other problem characteristics:

- Demand for food, water, and POLs is much more predictable than demand for medical supplies and certain military consumables.
- Some supplies require specialized transportation assets.
- It seems that the transportation activities in the tactical SCN are very rarely characterized by unavailability of transportation assets: the number of trucks available is usually larger than the number of trucks needed for each particular day of the week. The reasons are the high probability of the truck breakdowns in a hostile, uncertain environments and the high need for the backup trucks that have to be available on short notice. A similar situation might be expected in the disaster relief operations. For example, in natural disasters, the transportation assets would be available after a very short time period and the main shortages would be in the medications supplies.

The following sections provide elements and identify those that potentially have a stochastic nature.

### **3.2.1 Problem elements and characteristics**

The problem characteristics include:

- Different modes of transportation
- Vehicle fleet
  - Separate vehicle fleets between any two echelons
  - Heterogeneous vehicle fleets
  - Depots
  - Vehicle capacities
  - Re-fueling issues
  - Compatibility between vehicles and supply types
  - Compatibility between supplies travelling on the same vehicle

- Routes
  - Each route starting at the far supply node (F) visits the close supply node (L) and comes back to the far supply node
  - Each route starting at the close supply node (L) visits one or more satellite nodes (FOBs) and comes back to the close supply node (L), as illustrated in 1. Each satellite node (FOB) can be visited at most once in each route.
  - Each route starting at a satellite (FOB) visits one or more demand nodes (c) and comes back to the satellite (as illustrated in 1). Each demand node can be visited at most once in each route. Each demand node is assigned to one satellite. Each satellite nodes can serve only the demand nodes assigned to it.
- Demand
  - Demands for each time period: deterministic and stochastic (see further details in 5.1). Each demand is associated with the time period and it can be assumed that the supply has to arrive and be in the inventory at the demand node at the end of the previous time period so that it can be consumed and it can satisfy the demand. There is no a time window explicitly associated with the demand.
  - Different types of supplies, some requiring specialized transportation assets
  - Unexpected sudden changes in demand (surge)
- Inventories/ Stores
  - Inventories (stores) at each location
  - Different types of inventories (stores) at one location
  - Different cost of inventories at each echelon or at each location
- Travel times
  - Deterministic
- Service times and Loading/ unloading times
  - Service times may vary at each node
  - Unloading/ loading proportional to the quantities transported
- Transportation networks: each edge has
  - Distance
  - Cost (associated with mileage, time, fuel, journey, load, and potentially safety)

### 3.2.2 Stochastic problem characteristics

Problem characteristics that potentially could be stochastic:

- Demand – being more stochastic going from left to right in the SCN

### 3.2.3 Metrics/ Measures of Performance

We use the most commonly used metrics including:

- Inventory costs
- Transportation costs
- Service level, *i.e.*, unmet demand

## 3.3 Assumptions

This section lists the assumptions on which we will rely when solving the selected supply chain management problem:

1. Distribution Network Structure is given: the number and location of the SCN nodes.
2. Distribution strategy is centralized.
3. The following logistics activities will not be considered:
  - Recovery of damaged vehicles
  - Urgent direct flight from APOD beyond FOBs
  - Hazardous material transport
  - Vehicle unavailability due to repair
  - Repair duration
  - Fleet size decrease due to assignment to other activities
4. Since the following will not be modeled explicitly, perturbation in demand could be later used to model:
  - Materiel provided to other nations by DND – increase in demand
  - Materiel provided by other nations to DND – decrease in demand
5. Other problems that will not be considered in this project:
  - Bin packing for filling up the vehicles/ aircrafts, *i.e.*, planning of the loading of packages into trucks/ aircrafts.
  - Scheduling and routing of protection support vehicles and staff.
  - Possible multi-role of an Unmanned Aerial Vehicle (UAV) to ensure transportation and some form of protection (due to reconnaissance capability)
  - Scheduling of regular maintenance of vehicles so that there are very few periods when more than one vehicle is unavailable due to regular maintenance.
6. Cash-Flow is assumed to be resolved.

7. The problem of arranging and re-arranging the containers in the receiving yard at APOD will not be considered.

### 3.4 Multi-Objective Optimization

We address the multi-objectives using a weighted sum in order to deal with the problem complexity and the stochastic aspects first. Since we are considering only transportation and inventory costs, using the weighted sum is a commonly used approach. The terms of our objective function are similar in nature.

“A multi-objective problem is often solved by combining its multiple objectives into one single-objective scalar function. This approach is in general known as the *weighted-sum* or *scalarization* method.” [Caramia and Dell’Olmo, 2008].

More details about the weights and their relations is provided in Section 5.2.



## **4 Literature Review**

### **4.1 Humanitarian Logistics**

This section provides a review of papers found that deal with humanitarian logistics problems.

The papers have been sorted into two groups. The first group of papers does not provide rigorous models and sometimes does not provide explicit statements of the optimization problems, but deals with analysis of the issues and different policy aspects involved with humanitarian logistics. These policy papers could be used when generating the problem instances because they often provide statistical data relevant to operational problems in the humanitarian logistics.

[Kovacs and Spens, 2007], [Martinez et al, 2011], [VanWassenhove et al, 2012] discuss humanitarian logistic at a very high level and provide global views and analysis of the issues and policies without providing a model or a solution approach. [Martinez et al, 2011] provides data on humanitarian vehicle fleet, routing, maintenance, etc. The data might be used for generating random problem instances.

The second group of papers provides rigorous modeling potentially with mathematical programming formulations of the problem.

[Najafi et al, 2013] deals with a multi-objective robust optimization model for logistics planning in the earthquake response phase. The problem is multi-periods and multi-products. The SCN consists of the demand nodes, the supply nodes, and the nodes representing the emergency medical centers. The vehicle fleet is characterized by different modes of transport and different capacities with both weight and volume capacities. Demand has priorities. The transportation of people is also considered, and it does not allow mixing of commodities and people. The uncertainty is in the number of injured people, commodity demands, suppliers' capacities, and hospitals' capacities. The uncertain data is dealt with by an uncertain set defined by a nominal value and a permitted change. These sets can be unequal for different periods as they can follow from non-identical distributions.

[Doyen et al, 2012] present a two-echelon stochastic facility location model for humanitarian relief logistics and uses a two-stage stochastic programming model. Decisions are made for the location of the pre- and post-disaster rescue centers, the amount of relief items to be stocked at the pre-disaster rescue centers, and the amount of relief item flows at each echelon. The objective is to minimize the total cost of facility location, inventory holding, transportation and shortage. The deterministic equivalent of the model is a mixed-integer linear programming model which is solved by a heuristic method based on Lagrangean relaxation. For the experimental study, randomly generated test instances are used with up to 25 scenarios. The model is validated by

calculating the value of the stochastic solution and the expected value of perfect information.

[Nikbakhsh et al, 2011] discusses the humanitarian logistics planning in disaster relief operations. It provides the classification of disasters, occurrence statistics (1900-2005), distribution by type, and statistics of damages in China, India, Iran, and US. It also lists the world's five most important industrial accidents. The authors present the disaster management cycle, and discuss the differences and similarities between the humanitarian logistics and the commercial supply chain. For humanitarian logistics, the paper provides the humanitarian logistics supply chain structure and the list of goods.

Very simple separate Mixed-Integer Programming solutions (MIPs) are presented for: the facility location problem, the transportation and distribution problem (simple Vehicle Routing Problem (VRP)), and the inventory problem (deterministic multi-period model with multi-objective function). Also, an integrated location-routing model is presented. The paper also proposes performance measures for a humanitarian logistics system. Case-studies are also presented: analysis of weaknesses of response logistics in the last few disasters (Katrina, Asian Tsunami, earthquake in 2006).

## 4.2 Two-Echelon Vehicle Routing Problem

We searched for papers on two-echelon vehicle routing problems since they more often contain rigorous modeling, mathematical programming formulations, and solution approaches for the relevant optimization problems.

Again the papers were sorted into two groups. The first group of papers discusses the heuristics approaches without introducing the MIP formulation of the problems, while the second group of papers provides MIP formulations.

[Jacobsen and Madsen, 1980] is interesting from a historical perspective since it represents the first time a two-echelon VRP was addressed. The problem was:

- Location (two-echelon) vehicle routing problem.
- Distribution of newspapers via transfer points.
- Combined routing problem and location problem.
- Location of satellite nodes is to be determined.
- No split deliveries on the routes between the depot and satellites.

The solution approach presented is a simple heuristic for assignment of customers to satellites and route construction.

The characteristics of the introduced problem different to our problem:

- The combined routing problem and location problem.
- No inventory management.

- One product.
- No split deliveries on the routes between the depot and satellites.

[Crainic et al, 2008] describes the static and deterministic two-echelon vehicle routing problem with the following characteristics:

- Objective: Minimize the total traveling costs.
- SCN network: One depot, multiple satellites, multiple customers.
- Demand and warehouses: One product. No warehouses.
- Fleet: Different types of vehicles at each echelon.
- Routes: Split deliveries are allowed on the routes from the depot to the satellites, but not on the routes from satellites to customers.

The proposed solution approach includes:

- No mathematical programming model.
- Depot-to-satellite (first-level) and satellite-to-customer (second-level) deliveries are divided into two separate sub-problems.
- At the second-level, customers are clustered and assigned to satellites with heuristic rules. It leads to multiple independent single depot VRPs (one for each satellite) which are solved exactly with an ILOG dispatcher. Then, at the first-level, the obtained VRP is also solved with the ILOG dispatcher.
- Another variant is proposed by directly solving a multi-depot VRP at the second level.
- At the end, exchange heuristics are applied to improve the solution.

The characteristics of the introduced problem that are different to our problem:

- There is no inventory management.
- One product considered.

Subsequently, several papers address the same problem with improved heuristics approaches:

- [Crainic et al, 2008] This is an extended abstract that describes lower bounds for the first-level and second-level sub-problems (for example, by relaxing vehicle capacity). These bounds can be used to evaluate the performance of heuristic methods.
- [Crainic et al, 2010] Analysis of solutions produced by the clustering heuristic reported in Crainic et al. (2008) depending on the number of customers and satellites and their spatial distribution.
- [Crainic et al, 2011a] Multi-start local search heuristic. Starting with an assignment of customers to satellites, a local search heuristic is used to find better assignments. The local search is restarted through a perturbation of the best

assignment found so far. VRPs are (heuristically) solved with a branch-and-cut algorithm which is stopped after some given Central Processing Unit (CPU) time.

- [Crainic et al, 2012] Different assignments of customers to satellites are built with GRASP using a randomized variant of the clustering heuristic reported in [Crainic et al, 2008]. The obtained VRPs are solved with the hybrid meta-heuristic EVE-OPT, previously proposed by Perboli et al. (2008).
- [Hemmelmayr et al, 2012] proposed an Adaptive Large Neighborhood Search (ALNS), and introduced a new set of benchmark instances with up to 10 satellites and 200 customers.

[Jung and Mathur, 2007] introduced the problem where inventory and distribution decisions are considered jointly. The problem solution consists of the replenishment quantities, the associated replenishment (reorder) intervals for each retailer, and the delivery routes. The solution approach is a heuristic. The problem characteristics are:

- One warehouse and  $n$  retailers with constant demand
- Inventories at the warehouse and at retailers
- Objective: Minimize the long-run average inventory and the transportation costs

[Mancini, 2012] is a summary of Mancini's Ph.D. thesis on heuristic methods for solving the two-echelon VRP. Difference: No inventory management has been considered.

The following group of papers provide MIP formulations of their respective problems.

[Crainic et al, 2009] describes a two-echelon VRP in a city logistics context. They provide a mathematical model for a time-dependent variant with fleet synchronization at satellites and customer time windows. There is no algorithm proposed for solving the problem. Difference: no inventories, cross-docking.

[Gonzalez-Feliu et al, 2006] describes a flow-based mathematical programming model for the two-echelon capacitated vehicle routing problem. The authors also propose valid inequalities (cuts) that were used when the model is solved with an MIP solver (Xpress). A set of problem instances with up to 4 satellites and 50 customers was proposed. The medium-sized instances with 2 satellites and 21 customers are solved to optimality.

[Jepsen et al, 2013] propose a branch and cut algorithm for the symmetric capacitated two-echelon VRP. In their model the satellite nodes can be left unused. The objective is to minimize the traveling costs plus the handling costs at the satellites. The authors propose a new mathematical programming model for the two-echelon VRP, and another model for a relaxed version of the two-echelon VRP which breaks the symmetry of the first model and provides tighter lower bounds when linear relaxations are solved. The authors mention that the model in [Perboli et al, 2011] has a mistake and provide an example to support their claim. The branch-and-cut algorithm is applied on the relaxed

model. When an integer solution is found, it is checked for feasibility to the two-echelon VRP. If infeasible, a specialized branching scheme is applied. Linear relaxations are solved with CPLEX. The proposed exact algorithm can solve instances with up to 5 satellites and 50 customers. Difference: no inventories.

[Perboli and Tadei, 2010] and [Perboli et al, 2011] strengthen the MIP mathematical programming model proposed by [Gonzalez-Feliu et al, 2006] by adding new valid inequalities.

For [Gonzalez-Feliu et al, 2006], [Perboli and Tadei, 2010], [Perboli et al, 2011], and [Gonzalez-Feliu et al, 2006], the differences compared to our problem are:

- No inventory management
- No scheduling issues
- One product
- No warehouses (inventories) at satellites
- Homogeneous vehicles for each level

[Santos et al, 2013] considers the same two-echelon vehicle routing problem but addresses it with a branch-and-price approach.

### 4.3 Integrated SCM Problems

Most of the literature on the integrated problem in the supply chain management deals with network design with some limited routing considerations – the location-routing problems. Although some recent studies report on the integration of inventory management and routing.

[Mumtaz and Brah, 2010] studied the integrated location and inventory problem in a two-echelon SCN. The problem consists of finding the number and location of distribution centers (satellites) that will be used for serving a set of customers with a given (deterministic) demand for product made in one factory. Shortages are not allowed. The location of factory and customers is known. The authors proposed an MIP formulation of the problem and solved the problem using a heuristic. Vehicle routing is not explicitly considered. The objective is to minimize long run average total costs consisting of the location costs, transportation costs, and inventory holding costs. Differences from our problem: we do not have the network design and location problem. Also, they are assuming:

- Unlimited inventory capacities at the plant and distribution centers
- Unlimited vehicle capacities for the vehicles transporting goods between the plant and the distribution centers
- Homogenous fleet of vehicles

- One product
- No vehicle routing problem explored explicitly with all its issues

[Max Shen, 2007] is a survey paper on supply chain models dealing with the decisions on the inventory locations and the distribution and routing. There are mathematical programming formulations of different problems that are built progressively from easier to more complex.

[Anily and Federgruen 1993] dealt with the two-echelon distributions systems with vehicle routing costs and central inventories. The problem is static, deterministic, and characterized by one depot and multiple retailers with inventories at the depot and retailers. Difference: One product. There is no useful mathematical programming model. The proposed heuristic partitions the retailers into regions (super-retailers), reducing the problem to finding an inventory replenishment strategy for a classical one-depot – multiple retailers system. The retailers in the same region are visited by a single vehicle route.

[Jung, 2012] proposes an effective genetic algorithm for solving the integrated inventory and routing problem of supply chain composed of multi-warehouses and multi-retailers. The objective is to determine replenishment intervals for the retailers and warehouses as well as the vehicles routes so that the total cost of delivery and inventory cost is minimized. Evaluating the fitness of the objective function has a computational complexity close to linear.

[Dondo et al, 2011] extends previous work – [Dondo et al, 2009] dealing with management of distribution within a supply chain network – by including cross-docking. Thus, inventories and cross docking at intermediate facilities are both allowed. The problem is static and deterministic. This work is the most similar to ours. In some ways, it extends our problem by allowing direct shipping from suppliers to customers and by considering the allocation of vehicles to facilities. In other ways, it is more restrictive by considering a single time period. A mathematical model is provided and solved with an MIP solver (GUROBI). Tests on five instances derived from case studies with up to 3 satellites, 29 customers and 6 products are reported. Difference: direct shipping, no far supply node, no inventory at customer nodes, single time period.



## 5 Deterministic Model: Extended 2E-SCM

Clearly, addressing the full complexity of our problem is a challenging task. For example, the two-echelon vehicle routing problem is a much simpler problem but has only been recently addressed in the literature. The complexity of our problem comes from different dimensions such as demand for multiple products, the presence of a heterogeneous fleet of vehicles, inventories, and multiple periods. We also have an additional level made of a unidirectional link between a far supply node and a close supply node. This link must be handled in a particular way, when compared to the rest of the network, due to a different time scale. All these issues make for a novel problem that has not been previously considered in the literature.

For the sake of a comparison with recent studies, among the most comprehensive papers dealing with multi-period, multi-objective, and multi-commodity humanitarian logistics optimization we select [Najafi et al, 2013] which unfortunately does not include intermediate transportation nodes (called satellites in our problem) but does handle the inventory management problem explicitly.

The main contribution and novelty anticipated in our project is the explicit integration of the transportation (distribution) management problem and the inventory management problem for the two-echelon or three-echelon supply chain networks.

In this section, we model the centralized, static, deterministic supply chain management problem within an extended two-echelon network. This network can be also called a simplified three-echelon SCN. The network design is known and we are considering the problem of inventory management and the problem of distribution management (vehicle routing). The distributed problem is not a topic of this Task Authorization. These distributed problems may arise when communication lines are broken between disconnected portions of the SCN, in which case, it could be assumed that each SCN subnetwork is using the same SCN management policies and strategies proposed in this study.

In the literature, the two-echelon supply chain network consists of three levels of nodes: supplier nodes, satellite nodes, and customer nodes. Normally, there is a vehicle fleet associated to the supplier nodes and a vehicle fleet associated to the satellite nodes. The corresponding depots are at the supplier nodes and at the satellite nodes. All the goods travel from the supplier nodes to the customer nodes, through the satellite nodes.

In order to model emergency management scenarios in the case of a domestic mega-disaster and in the case of international disaster relief operations, we are considering the extended two-echelon supply chain network that has one additional supply node far away from the disaster area and the customers [Berger et al, 2012]. Some or all of the goods originate at the far supply node and have to travel through the close supply node, further to the satellite nodes and the customer nodes. Although such a network has a three-echelon structure, we call it the extended two-echelon supply chain network since there are only two supply nodes connected by a single unidirectional link.

To summarize, the novelties of this research include:

- Integration of inventory management in a multi-echelon vehicle routing problem
- Multiple realistic problem characteristics – multiple products, heterogeneous fleet, multiple periods, compatibility constraints – that have been considered separately in various multi-echelon VRP or SCM models, but never all of these sources of complexity simultaneously in the same problem.
- Stochastic demand with combination with all characteristics and features listed in the previous bullet.
- Extension of the two-echelon network with an additional level – made of a unidirectional link between a far supply node and a close supply node – that must be handled in a particular way, when compared to the rest of the network, due to a different time scale.
- Multi-echelon routing problems often involve different types of vehicles, but fleets are homogeneous by level; our problem is different because it considers heterogeneous fleets at each level.

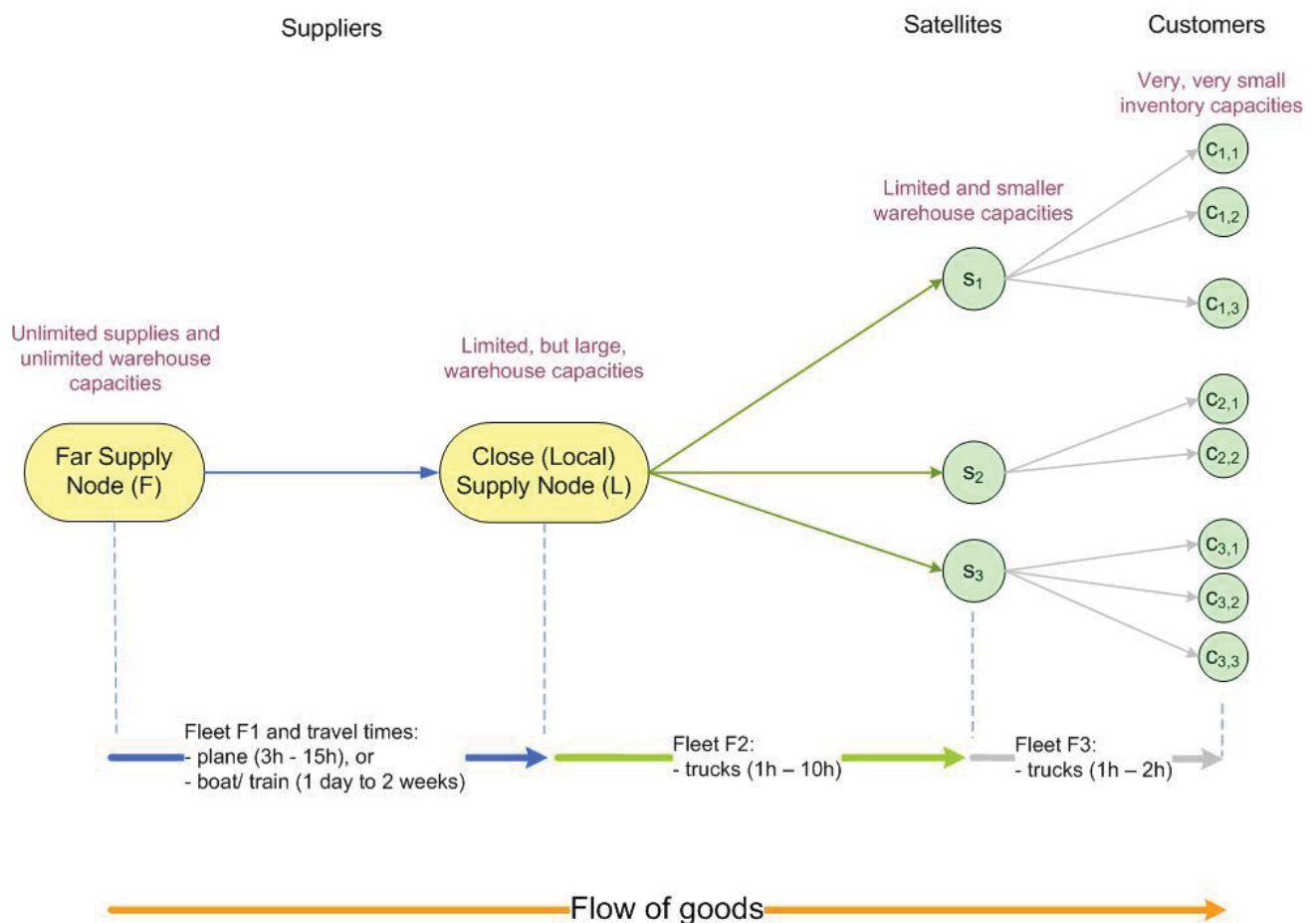
## 5.1 SCM in Disaster Relief Operations

The following are the characteristics of the deterministic problem considering the disaster relief scenario:

- Objective is the weighted sum of:
  - Transportation costs for Echelon 0 – from the far supply node (F) to the close supply node (L)
  - Transportation costs for Echelon 1 – from the close supply node (L) to the satellite nodes
  - Transportation costs for Echelon 2 - from the satellite nodes to the customer nodes
  - Inventory costs, and
  - Unmet demand penalties



- General
  - Centralized
  - The number of time periods and their duration
- The structure of the supply chain network is as shown in Figure 5-1:
  - One far supply node F
  - One close (local) supply node L
  - A number of satellite nodes,  $S = \{s_1, s_2, \dots, s_k\}$
  - A number of customer nodes with demand for each supply node  $i$ ,  $C_i = \{c_{i1}, c_{i2}, \dots, c_{im}\}$



**Figure 5-1 Extended Two-Echelon Supply Chain Network**

- Multi-products - there are 6 types of goods:
  - Petroleum, Oil, and Lubricants (POLs)
  - Water
  - Food

- Medication
- Medication requiring refrigerator
- Food requiring refrigerator
- Inventories: there are three types of warehouses/ inventories at each node of the SCN, and there is a maximum capacity for each warehouse type at each node:
  - POLs warehouse
  - Regular warehouse: Water, food, medications
  - Specialized warehouse: Refrigerator for special medications and food
- Fleet of vehicles:
  - Fleet F1 – for the transportation within the SCN first level – between F and L. The fleet depot is located at F. There are two modes of transportation, both with large capacities:
    - Planes: fast, expensive, not very many in the fleet
    - Ships or trains: slower and cheaper mode of transportation with relatively large fleet, with the capacities much larger than planes (10 times, 100 times, or even 1,000 times larger)
  - Fleet F2 – for the transportation within the SCN second level – between L and the satellite nodes in S. The fleet depot is located at L. There are three types of vehicles: POL trucks, regular trucks (for palletted goods), and specialized trucks (refrigerators).
  - Fleet F3 – for the transportation within the SCN third level – between the satellite nodes in S and the customer nodes in C. There is one depot at each satellite node  $s_i$ . There are also three types of vehicles: POL trucks, regular trucks (for palletted goods), and specialized trucks (refrigerators).
- Multi-period problem – the planning horizon is divided in several time periods that are used to synchronize the transportation between the SCN levels that have different travel times. Having decisions variables for each time period in an integrated model, assures that the decisions in one period take into account the decisions and expected outcomes from the previous period.
- Travel times:
  - Each routes between L and satellites  $s_i$  in S, and routes between satellites and the demand nodes can be completed within the same time period within which it starts.
  - There are speeds for each transportation asset, and the travel times along each link are calculated based on the distance of the link.
- Demand/ consumption:
  - There is a demand/ consumption for each product at each customer node in C for each period.

- (Optional) We can also consider that there is a demand for each product at the close supply node L and at each satellite node  $s_i$  in S.
- Supplies:
  - The supplies coming from the far supply node F.
  - We assume that the supplies and capacities of warehouses at the far supply node F are unlimited.
- Transportation routes:
  - First level routes – F to L – Fleet F1:
    - Direct routes between the far supply node F and the close supply node L.
  - Second level routes – L to S – Fleet F2:
    - A vehicle from fleet F2 starts its route at the close supply node and visits each satellite node at most once before returning to the close supply node (as illustrated in Figure 5-1).
    - Each route starts and finishes at the close supply node L and can visit several satellite locations.
    - Split deliveries to a satellite  $s_i$  are allowed, i.e., a portion of the demand at  $s_i$  can be filled with one truck (route) and the remaining portion can be filled with another truck (route).
  - Third level routes – S to C – fleet F3:
    - Each route starts and finishes at some satellite  $s_i$  and can visit several customer locations (as illustrated in Figure 5-1).
    - Each satellite has its subset of customers. No customer is served by two or more satellite locations.
    - Split deliveries are allowed due to demand for multiple products.
    - Multi-trip routes for one vehicle are allowed at this level.

The vehicles from fleet F1 can travel only between the far supply node F and the close supply node L. The vehicles from fleet F2 can visit only the close supply node L and the satellite nodes from S. The vehicles from fleet F3 can visit only the nodes in S and C. Each  $s_i$  has its own fleet F3<sub>i</sub> assigned to it.

Types of trucks:

- Special trucks for transport of POLs
- General trucks for transport of packaged water, food, and medications
- Fridge trucks for special medications and food

Fleets of vehicles:

- F1: very large capacities, two modes:
  - planes (fast and expensive)
  - ships/ trains (slow and cheap)
- F2: three types of trucks with medium capacities
- F3: three types of trucks with smaller capacities

For converting the disaster relief scenario to a military scenario, the vehicles with fridges can be considered as the specialized vehicles for the transportation of equipment and consumable military operation materiel.

## 5.2 Mathematical Programming Formulation

This section presents the deterministic model of the Extended Two-Echelon SCM (inventory and distribution problem). The problem is centralized.

The problem has five criteria, one of which is a penalty function. The criteria are:

- Transportation costs for Echelon 0 – from the far supply node (F) to the close supply node (L)
- Transportation costs for Echelon 1 – from the close supply node (L) to the satellite nodes
- Transportation costs for Echelon 2 – from the satellite nodes to the customer nodes
- Inventory costs, and
- Unmet demand penalties

The multiple criteria are dealt with by using an objective that is the weighted sum of the five listed criteria. The sum of the first four weights could be equal to 1, and the weight of the last criteria  $\omega_5$  is usually set to a large number to avoid an unmet demand. In the situations when an unmet demand is allowed and the decision maker is more concerned with minimizing the costs, this fifth weight  $\omega_5$  can be reduced. The right value for this weight can be evaluated after computational experiments and consultations with experts and decision makers. However, note that the high values for  $\omega_5$  are appropriate in order to address the “non-comparable” criteria contributions (*e.g.* travel cost vs. unmet demands proportion).

## Notation

$N$	set of nodes
$A$	set of arcs
$L$	close (local) supply node
$F$	far supply node
$S$	set of satellites
$S^+$	$S \cup \{L\}$
$C$	set of customers
$C_s$	set of customers served by satellite $s \in S$
$C_s^+$	$C_s \cup \{s\}$
$B$	set of bases ( $N \setminus C$ )
$T$	set of time periods (planning horizon)
$\eta^{\max}$	duration of a time period – this is also the maximum duration of all routes of one vehicle within a time period
$\bar{t}$	last period among the time periods in $T$
$P$	set of product types
$G$	set of store types
$G_j$	set of store types at node $j \in N$ . It is assumed that there is only one store per store type at each node in the distribution network
$R_g$	set of product types compatible with store type $g \in G$ .
$a_g^p$	=1 when $p \in R_g$ , 0 otherwise. This is the indicator parameter indicating the compatibility between the product type and the store type
$V$	set of vehicle types
$V_b$	set of vehicle types at base $b \in B$
$P_v$	set of product types compatible with vehicle type $v \in V$
$b_v^p$	=1 when $p \in P_v$ , 0 otherwise. This is the indicator parameter indicating the compatibility between the product type and the vehicle type
$n_{vb}$	number of vehicles of type $v \in V$ at base $b \in B$
$m_{vs}$	maximum number of routes of vehicle type $v \in V$ from satellite $s \in S$ during one time period
$d_{pc}^t$	demand of product type $p \in P$ of customer $c \in C$ in period $t \in T$
$\tau_{ijv}$	travel time on arc $(i, j) \in A$ for vehicle type $v \in V$ ; This is equal to the length of arc $(i, j)$ divided by the speed the vehicle $v$ can use on the arc $(i, j)$ : $\text{length}_{ij} / \text{speed}_{ijv}$
$c_{ijv}$	travel cost on arc $(i, j) \in A$ for vehicle type $v \in V$ ; This is equal to the length of arc $(i, j)$ multiplied by the fuel consumption per unit of length of vehicle type $v$ on the arc $(i, j)$ multiplied by the fuel cost per unit of fuel: $\text{length}_{ij} * \text{fuel\_consumption}_v * \text{fuel\_cost}$
$f_g^p$	inventory cost per unit of product type $p \in R_g$ per time period at store type $g \in G$ at customer node

$\tilde{f}_g^p$	inventory cost per unit of product type $p \in R_g$ per time period at store type $g \in G$ at satellite node
$\hat{f}_g^p$	inventory cost per unit of product type $p \in R_g$ per time period at store type $g \in G$ at the close supply node
$\rho_{vs}^p$	loading/unloading time of one unit of product type $p \in P$ at satellite $s \in S$ for vehicle type $v \in V$
$\delta_v$	number of time periods to deliver from F to L for vehicle type $v \in V$ . This is equal to the length of arc (F,L) divided by the speed of vehicle type $v$ along this arc: $\text{length}_{FL} / \text{speed}_{FLv}$ .
$\varphi_v$	delivery cost from F to L for vehicle type $v \in V$ . This is equal to the length of arc (F,L) multiplied by the fuel consumption per unit of length of vehicle type $v$ multiplied by the fuel cost per unit of fuel: $\text{length}_{FL} * \text{fuel\_consumption}_v * \text{fuel\_cost}$ . An additional fixed cost can be added, because the transportation assets used on this leg are major assets including planes, ships, and railcars.
$\varepsilon_v$	number of time periods of unavailability after delivery from F to L for vehicle type $v \in V$ . Equal to the time needed for the vehicle (transportation asset) to come back from the close supply node L to the far supply node F plus the maintenance time needed after each trip.
$\sigma_{iv}$	service time at node $i \in N$ for vehicle type $v \in V$ . This time is equal to the time needed for potential re-fueling, (potential) quick check-up, and administrative activities.
$\tau_v^{\max}$	maximum route duration for vehicle type $v \in V$ – this can be used for handling re-fueling issues
$u_v$	capacity of vehicle type $v \in V$
$\bar{u}_g$	capacity of store type $g \in G$
$\omega_l$	weights in the objective function, $l = 1, 2, 3, 4, 5$
$\gamma_v^{pp'}$	= 1 if product types $p \in P, p' \in P, p \neq p'$ , can be transported together (at the same time, on the same vehicle) by vehicle type $v$ ; 0 otherwise
$s(c)$	satellite for customer $c \in C$
$\alpha_{pc}$	penalty for not satisfying the demand for product $p$ at customer $c$ , <i>i.e.</i> , priority for satisfying the demand for product $p$ at customer $c$
$\mu$	small number $0 < \mu < 1$ to accommodate the loads $q$ between 0 and 1

### Decision variables

$x_{ijkvs}^{rt}$	= 1 if the $r^{\text{th}}$ route of the $k^{\text{th}}$ vehicle of type $v$ of satellite $s$ travels from $i$ to $j$ in time period $t$ for $i, j \in C_s^+, i \neq j, k = 1, \dots, n_{vs}, v \in V_s, s \in S, r = 1, \dots, m_{vs}, t \in T$ ; 0 otherwise.
$\tilde{x}_{ijkv}^t$	= 1 if the $k^{\text{th}}$ vehicle of type $v$ of close supply node L travels from $i$ to $j$ in time period $t$ for $i, j \in S^+, i \neq j, k = 1, \dots, n_{vL}, v \in V_L, t \in T$ ; 0 otherwise.
$\hat{x}_{kv}^t$	= 1 if the $k^{\text{th}}$ vehicle of type $v$ of remote supply node F is sent to close supply node L in time period $t, k = 1, \dots, n_{vF}, v \in V_F, t \in T$ ; 0 otherwise.

$y_{kvs}^{prt}$	= 1 if the $r^{\text{th}}$ route of the $k^{\text{th}}$ vehicle of type $v$ of satellite $s$ is used to transport product $p$ in time period $t$ for $k = 1, \dots, n_{vs}$ , $v \in V_s$ , $s \in S$ , $p \in P_v$ , $r = 1, \dots, m_{vs}$ , $t \in T$ ; 0 otherwise.
$\tilde{y}_{kv}^{pt}$	= 1 if the $k^{\text{th}}$ vehicle of type $v$ of close supply node $L$ is used to transport product type $p$ in time period $t$ for $k = 1, \dots, n_{vL}$ , $v \in V_L$ , $p \in P_v$ , $t \in T$ ; 0 otherwise.
$\hat{y}_{kv}^{pt}$	= 1 if the $k^{\text{th}}$ vehicle of type $v$ of remote supply node $F$ is sent to close supply node $L$ with product type $p$ in time period $t$ for $k = 1, \dots, n_{vF}$ , $v \in V_F$ , $p \in P_v$ , $t \in T$ ; 0 otherwise.
$q_{kvcs(c)}^{prt}$	quantity of product type $p$ delivered to customer $c$ on the $r^{\text{th}}$ route of the $k^{\text{th}}$ vehicle of type $v$ of satellite $s(c)$ in time period $t$ , for $k = 1, \dots, n_{vs(c)}$ , $v \in V_{s(c)}$ , $c \in C$ , $p \in P_v$ , $r = 1, \dots, m_{vs(c)}$ , $t \in T$ .
$\tilde{q}_{kvs}^{pt}$	quantity of product type $p$ delivered to satellite $s$ by the $k^{\text{th}}$ vehicle of type $v$ of close supply node $L$ in time period $t$ , for $k = 1, \dots, n_{vL}$ , $v \in V_L$ , $s \in S$ , $p \in P_v$ , $t \in T$ .
$\hat{q}_{kv}^{pt}$	quantity of product type $p$ sent to close supply node $L$ by the $k^{\text{th}}$ vehicle of type $v$ of remote supply node $F$ in time period $t$ , for $k = 1, \dots, n_{vF}$ , $v \in V_F$ , $p \in P_v$ , $t \in T$ .
$h_{gcs(c)}^{pt}$	inventory level of product type $p \in R_g$ in store type $g \in G_c$ at customer node $c$ (of the satellite node $s(c)$ ) at the beginning of period $t \in T$ . Note that $h_{gcs(c)}^{p0}$ should be fixed and equal to the initial inventory levels if those are known in advance.
$\tilde{h}_{gs}^{pt}$	inventory level of product type $p \in R_g$ in store type $g \in G_s$ at satellite node $s$ at the beginning of period $t \in T$ . Note that $h_{gs}^{p0}$ should be fixed and equal to the initial inventory levels.
$\hat{h}_g^{pt}$	inventory level of product type $p \in R_g$ in store type $g \in G_L$ at the close supply node $L$ at the beginning of period $t \in T$ . Note that $h_g^{p0}$ should be fixed and equal to the initial inventory levels.
$w_{kvcs(c)}^{rt}$	position of customer $c$ in the $r^{\text{th}}$ route of the $k^{\text{th}}$ vehicle of type $v$ of satellite $s(c)$ in time period $t$ for $k = 1, \dots, n_{vs(c)}$ , $v \in V_{s(c)}$ , $c \in C$ , $r = 1, \dots, m_{vs(c)}$ , $t \in T$ .
$\tilde{w}_{kvs}^t$	position of satellite $s$ in the route of the $k^{\text{th}}$ vehicle of type $v$ of close supply node $L$ in time period $t$ for $k = 1, \dots, n_{vL}$ , $v \in V_L$ , $s \in S$ , $t \in T$ .
$\underline{d}_{pc}^t$	amount of unsatisfied demand of product $p$ at customer $c$ in time period $t$ .

### Objective

$$\begin{aligned}
\text{Min } \sum_{t \in T} \left( \omega_1 \sum_{s \in S} \sum_{v \in V_s} \sum_{k=1}^{n_{vs}} \sum_{r=1}^{m_{vs}} \sum_{i \in C_s^+} \sum_{\substack{j \in C_s^+ \\ i \neq j}} c_{ijv} x_{ijkvs}^{rt} + \omega_2 \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \sum_{s \in S^+} \sum_{\substack{s' \in S^+ \\ s \neq s'}} c_{ss'v} \tilde{x}_{ss'kv}^t \right. \\
+ \omega_3 \sum_{v \in V_F} \sum_{k=1}^{n_{vF}} \varphi_v \hat{x}_{kv}^t \\
+ \omega_4 \left( \sum_{s \in S} \sum_{c \in C_s} \sum_{g \in G_c} \sum_{p \in R_g} f_g^p h_{gcs(c)}^{pt} + \sum_{s \in S} \sum_{g \in G_s} \sum_{p \in R_g} \tilde{f}_g^p \tilde{h}_{gs}^{pt} \right. \\
\left. \left. + \sum_{g \in G_L} \sum_{p \in R_g} \hat{f}_g^p \hat{h}_g^{pt} \right) + \omega_5 \sum_{p \in P} \sum_{c \in C} \alpha_{pc} \underline{d}_{pc}^t \right)
\end{aligned}$$

### Constraints

Inventory balance at customer nodes

$$\sum_{g \in G_c} h_{gcs(c)}^{pt} + \sum_{v \in V_{s(c)}} \sum_{k=1}^{n_{vs(c)}} \sum_{r=1}^{m_{vs(c)}} q_{kvcs(c)}^{prt} - d_{pcs(c)}^t = \sum_{g \in G_c} h_{gcs(c)}^{p(t+1)} - \underline{d}_{pcs(c)}^t, \quad (1)$$

$$c \in C_s, s \in S, p \in P, t \in T \setminus \{\bar{t}\}.$$

Inventory balance at customer nodes for the last period  $\bar{t}$  in the planning horizon

$$\sum_{g \in G_c} h_{gcs(c)}^{p\bar{t}} + \sum_{v \in V_{s(c)}} \sum_{k=1}^{n_{vs(c)}} \sum_{r=1}^{m_{vs(c)}} q_{kvcs(c)}^{pr\bar{t}} - d_{pcs(c)}^{\bar{t}} \geq -\underline{d}_{pcs(c)}^{\bar{t}}, \quad c \in C_s, s \in S, p \in P. \quad (1a)$$

Inventory balance at satellites

$$\sum_{g \in G_s} \tilde{h}_{gs}^{pt} + \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \tilde{q}_{kvs}^{pt} - \sum_{c \in C_s} \sum_{v \in V_s} \sum_{k=1}^{n_{vs}} \sum_{r=1}^{m_{vs}} q_{kvcs(c)}^{prt} = \sum_{g \in G_s} \tilde{h}_{gs}^{p(t+1)}, \quad s \in S, p \in P, t \in T \setminus \{\bar{t}\}. \quad (2)$$

Inventory balance at satellites for the last period  $\bar{t}$  in the planning horizon

$$\sum_{g \in G_s} \tilde{h}_{gs}^{p\bar{t}} + \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \tilde{q}_{kvs}^{p\bar{t}} - \sum_{c \in C_s} \sum_{v \in V_s} \sum_{k=1}^{n_{vs}} \sum_{r=1}^{m_{vs}} q_{kvcs(c)}^{pr\bar{t}} \geq 0, \quad s \in S, p \in P. \quad (2a)$$



Inventory balance at close supply node L

Note that during the implementation in the second term when  $t - \delta_v < 0$ , assume  $\hat{q}_{kv}^{p(t-\delta_v)} = 0$

$$\sum_{g \in G_L} \hat{h}_g^{pt} + \sum_{v \in V_F} \sum_{k=1}^{n_{vF}} \hat{q}_{kv}^{p(t-\delta_v)} - \sum_{s \in S} \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \tilde{q}_{kvs}^{pt} = \sum_{g \in G_L} \hat{h}_g^{p(t+1)}, \quad p \in P, t \in T \setminus \{\bar{t}\}. \quad (3)$$

Inventory balance at close supply node L for the last period  $\bar{t}$  in the planning horizon

$$\sum_{g \in G_L} \hat{h}_g^{p\bar{t}} + \sum_{v \in V_F} \sum_{k=1}^{n_{vF}} \hat{q}_{kv}^{p(\bar{t}-\delta_v)} - \sum_{s \in S} \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \tilde{q}_{kvs}^{p\bar{t}} \geq 0, \quad p \in P. \quad (3a)$$

Demand constraints at customer nodes

$$\sum_{g \in G_c} h_{gcs(c)}^{pt} \geq d_{pcs(c)}^t - \underline{d}_{pcs(c)}^t, \quad c \in C_s, s \in S, p \in P, t \in T. \quad (4)$$

Demand constraints at satellites

$$\sum_{g \in G_s} \tilde{h}_{gs}^{pt} \geq \sum_{c \in C_s} \sum_{v \in V_s} \sum_{k=1}^{n_{vs}} \sum_{r=1}^{m_{vs}} q_{kvcs(c)}^{prt}, \quad s \in S, p \in P, t \in T. \quad (5)$$

Demand constraints at close supply node L

$$\sum_{g \in G_L} \hat{h}_g^{pt} \geq \sum_{s \in S} \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \tilde{q}_{kvs}^{pt}, \quad p \in P, t \in T. \quad (6)$$

Store capacity constraints at customer nodes

$$\sum_{p \in R_g} h_{gcs(c)}^{pt} \leq \bar{u}_g, \quad g \in G_c, c \in C_s, s \in S, t \in T. \quad (7)$$

Compatibility between store types and product types at customer nodes

$$h_{gcs(c)}^{pt} \leq a_g^p \bar{u}_g, \quad p \in P, g \in G_c, c \in C_s, s \in S, t \in T. \quad (7a)$$

Store capacity constraints at satellite nodes

$$\sum_{p \in R_g} \tilde{h}_{gs}^{pt} \leq \bar{u}_g, \quad g \in G_s, s \in S, t \in T. \quad (7')$$

Compatibility between store types and product types at satellite nodes

$$\tilde{h}_{gs}^{pt} \leq a_g^p \bar{u}_g, \quad p \in P, g \in G_s, s \in S, t \in T. \quad (7'a)$$

Store capacity constraints at the close supply node

$$\sum_{p \in R_g} \hat{h}_g^{pt} \leq \bar{u}_g, \quad g \in G_L, t \in T. \quad (7'')$$

Compatibility between store types and product types at the close supply node

$$\hat{h}_g^{pt} \leq a_g^p \bar{u}_g, \quad p \in P, g \in G_L, t \in T. \quad (7''a)$$

Product compatibility constraints aboard the vehicles (between satellites and customers)

$$y_{kvs}^{prt} + y_{kvs}^{p'rt} \leq 1 + \gamma_v^{pp'}, \quad (8)$$

$$k = 1, \dots, n_{vs}, v \in V_s, s \in S, p \in P_v, p' \in P_v, p \neq p', r = 1, \dots, m_{vs}, t \in T.$$

Product compatibility constraints aboard vehicles (between L and satellites)

$$\tilde{y}_{kv}^{pt} + \tilde{y}_{kv}^{p't} \leq 1 + \gamma_v^{pp'}, \quad k = 1, \dots, n_{vL}, v \in V_L, p \in P_v, p' \in P_v, p \neq p', t \in T. \quad (9)$$

Product compatibility constraints in vehicles (between F and L)

$$\hat{y}_{kv}^{pt} + \hat{y}_{kv}^{p't} \leq 1 + \gamma_v^{pp'}, \quad k = 1, \dots, n_{vF}, v \in V_F, p \in P_v, p' \in P_v, p \neq p', t \in T. \quad (10)$$

Vehicle capacity constraints (between satellites and customers)

$$\sum_{c \in C_s} q_{kvcs(c)}^{prt} \leq u_v y_{kvs}^{prt}, \quad k = 1, \dots, n_{vs}, v \in V_s, s \in S, p \in P, r = 1, \dots, m_{vs}, t \in T. \quad (11)$$

$$\sum_{p \in P} \sum_{c \in C_s} q_{kvcs(c)}^{prt} \leq u_v, \quad k = 1, \dots, n_{vs}, v \in V_s, s \in S, r = 1, \dots, m_{vs}, t \in T. \quad (12)$$

Other linking constraints between  $q$  and  $y$  variables ( $\mu$  is a small number,  $0 < \mu < 1$ )

$$\sum_{c \in C_s} q_{kvcs(c)}^{prt} + 1 - \mu \geq y_{kvs}^{prt}, \quad k = 1, \dots, n_{vs}, v \in V_s, s \in S, p \in P, r = 1, \dots, m_{vs}, t \in T. \quad (11a)$$

Compatibility between vehicle types and product types (between satellites and customers)

$$q_{kvcs(c)}^{prt} \leq b_v^p u_v, \quad k = 1, \dots, n_{vs}, v \in V_s, c \in C_s, s \in S, p \in P, r = 1, \dots, m_{vs}, t \in T. \quad (12a)$$

Vehicle capacity constraints (between L and satellites)

$$\sum_{s \in S} \tilde{q}_{kvs}^{pt} \leq u_v \tilde{y}_{kv}^{pt}, \quad k = 1, \dots, n_{vL}, v \in V_L, p \in P, t \in T. \quad (13)$$

$$\sum_{p \in P} \sum_{s \in S} \tilde{q}_{kvs}^{pt} \leq u_v, \quad k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (14)$$

Other linking constraints between  $q$  and  $y$  variables ( $\mu$  is a small number  $0 < \mu < 1$ )

$$\sum_{s \in S} \tilde{q}_{kvs}^{pt} + 1 - \mu \geq \tilde{y}_{kv}^{pt}, \quad k = 1, \dots, n_{vL}, v \in V_L, p \in P, t \in T. \quad (13a)$$

Compatibility between vehicle types and product types (between L and satellites)

$$\tilde{q}_{kvs}^{pt} \leq b_v^p u_v, \quad k = 1, \dots, n_{vL}, v \in V_L, s \in S, p \in P, t \in T. \quad (14a)$$

Vehicle capacity constraints (between F and L)

$$\hat{q}_{kv}^{pt} \leq u_v \hat{y}_{kv}^{pt}, \quad k = 1, \dots, n_{vF}, v \in V_F, p \in P, t \in T. \quad (15)$$

$$\sum_{p \in P} \hat{q}_{kv}^{pt} \leq u_v, \quad k = 1, \dots, n_{vF}, v \in V_F, t \in T. \quad (16)$$

Other linking constraints between  $q$  and  $y$  variables ( $\mu$  is a small number  $0 < \mu < 1$ )

$$\hat{q}_{kv}^{pt} + 1 - \mu \geq \hat{y}_{kv}^{pt}, \quad k = 1, \dots, n_{vF}, v \in V_F, p \in P, t \in T. \quad (15a)$$

Compatibility between vehicle types and product types (between F and L)

$$\hat{q}_{kv}^{pt} \leq b_v^p u_v, \quad k = 1, \dots, n_{vF}, v \in V_F, p \in P, t \in T. \quad (16a)$$

Maximum route length constraints – to handle re-fueling, if needed (between satellites and customers)

$$\sum_{i \in C_s^+} \sum_{\substack{j \in C_s^+ \\ i \neq j}} \tau_{ijv} x_{ijkvs}^{rt} \leq \tau_v^{max}, \quad (17)$$

$$k = 1, \dots, n_{vs}, v \in V_s, s \in S, r = 1, \dots, m_{vs}, t \in T.$$

The duration of all routes of one vehicle has to be smaller than the time period duration (for the vehicles traveling between satellites and customers)

$$\sum_{r=1}^{m_{vs}} \left( \sum_{i \in C_s^+} \sum_{\substack{j \in C_s \\ i \neq j}} (\tau_{ijv} + \sigma_{jv}) x_{ijkvs}^{rt} + \sum_{j \in C_s} \tau_{jsv} x_{jskvs}^{rt} + \sum_{c \in C_s} \sum_{p \in P_v} \rho_{vs}^p q_{kvcs(c)}^{prt} \right) \leq \eta^{max}, \quad (18)$$

$$k = 1, \dots, n_{vs}, v \in V_s, s \in S, t \in T.$$

Maximum route length constraints – to handle re-fueling, if needed (between L and satellites)

$$\sum_{i \in S^+} \sum_{\substack{j \in S^+ \\ i \neq j}} \tau_{ijv} \tilde{x}_{ijkv}^t \leq \tau_v^{max}, \quad k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (19)$$

The route of each vehicle has to finish within the time period (between L and customers)

$$\sum_{i \in S^+} \sum_{j \in S} (\tau_{ijv} + \sigma_{jv}) \tilde{x}_{ijkv}^t + \sum_{j \in S} \tau_{jLv} \tilde{x}_{jLkv}^t + \sum_{s \in S} \sum_{p \in P_v} \rho_{vL}^p \tilde{q}_{kvs}^{pt} \leq \eta^{max}, \quad (19a)$$

$$k = 1, \dots, n_{vs}, v \in V_s, t \in T.$$

Vehicle flow constraints (between satellites and customers)

$$\sum_{j \in C_s^+} x_{cjkvs(c)}^{rt} = \sum_{j \in C_s^+} x_{jckvs(c)}^{rt}, \quad (20)$$

$$c \in C_s^+, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T.$$

$$\sum_{j \in C_s^+} x_{cjkvs(c)}^{rt} \leq 1, \quad (21)$$

$$c \in C_s^+, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T.$$

$$y_{kvs}^{prt} \leq \sum_{j \in C_s} x_{sjkvs(c)}^{rt}, \quad s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T, p \in P. \quad (21a)$$

Vehicle flow constraints (between L and satellites)

$$\sum_{j \in S^+} \tilde{x}_{sjkv}^t = \sum_{j \in S^+} \tilde{x}_{jskv}^t, \quad s \in S^+, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (22)$$

$$\sum_{j \in S^+} \tilde{x}_{sjkv}^t \leq 1, \quad s \in S^+, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (23)$$

$$\tilde{y}_{kv}^{pt} \leq \sum_{s \in S} \tilde{x}_{Lskv}^t, \quad k = 1, \dots, n_{vL}, v \in V_L, t \in T, p \in P. \quad (23a)$$

Vehicle flow constraints (between F and L)

$$\sum_{t'=t}^{t+\delta_v+\varepsilon_v} \hat{x}_{kv}^{t'} \leq 1, \quad k = 1, \dots, n_{vF}, v \in V_F, t \in T \setminus \{t | t + \delta_v + \varepsilon_v > \bar{t}\}. \quad (24)$$

$$\hat{y}_{kv}^{pt} \leq \hat{x}_{kv}^t, \quad k = 1, \dots, n_{vF}, v \in V_F, p \in P_v, t \in T. \quad (25)$$

Constraints linking variables  $q$  and  $x$  (between satellites and customers)

$$\sum_{p \in P} q_{kvcs(c)}^{prt} \leq u_v \sum_{j \in C_s^+} x_{jckvs(c)}^{rt}, \quad (20aa)$$

$$c \in C_s, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T.$$

$$\sum_{p \in P} q_{kvcs(c)}^{prt} \leq u_v \sum_{j \in C_s^+} x_{cjkvs(c)}^{rt}, \quad (20ab)$$

$$c \in C_s, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T.$$

$$c \in C_s, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T.$$

Constraints linking variables  $q$  and  $x$  (between L and satellites)

$$\sum_{p \in P} \tilde{q}_{kvs}^{pt} \leq u_v \sum_{j \in S^+} \tilde{x}_{jskv}^t, \quad s \in S, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (22aa)$$

$$\sum_{p \in P} \tilde{q}_{kvs}^{pt} \leq u_v \sum_{j \in S^+} \tilde{x}_{sjkv}^t, \quad s \in S, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (22ab)$$

Constraints linking variables  $q$  and  $x$  (between F and L)

$$\sum_{p \in P} \hat{q}_{kv}^{pt} \leq u_v \hat{x}_{kv}^t, \quad k = 1, \dots, n_{vF}, v \in V_F, t \in T. \quad (24a)$$

Subtour elimination constraints (between satellites and customers with  $M = |C_{s(c)}^+|$ )

$$w_{kvcs(c)}^{rt} \geq w_{kvc's(c')}^{rt} + (M + 1)x_{c'ckvs(c)}^{rt} - M, \quad (26)$$

$$k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, c \in C_s, c' \in C_s, c \neq c', r = 1, \dots, m_{vs(c)}, s \in S, t \in T.$$

$$w_{kvcs(c)}^{rt} \geq (M + 1)x_{s(c)ckvs(c)}^{rt} - M, \quad (27)$$

$$k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, c \in C_s, r = 1, \dots, m_{vs(c)}, s \in S, t \in T.$$

$$w_{kvcs(c)}^{rt} \leq |C_{s(c)}|, \quad k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, c \in C_s, r = 1, \dots, m_{vs(c)}, s \in S, t \in T. \quad (28)$$

$$\sum_{j \in C_s^+} x_{jckvs(c)}^{rt} \leq w_{kvcs(c)}^{rt}, \quad c \in C_s, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T. \quad (27a)$$

Subtour elimination constraints (between L and satellites with  $M' = |S^+|$ )

$$\tilde{w}_{kvs}^t \geq \tilde{w}_{kvs'}^t + (M' + 1)\tilde{x}_{s'skv}^t - M', \quad k = 1, \dots, n_{vL}, v \in V_L, s \in S, s' \in S, s \neq s', t \in T. \quad (29)$$

$$\tilde{w}_{kvs}^t \geq (M' + 1)\tilde{x}_{Lskv}^t - M', \quad k = 1, \dots, n_{vL}, v \in V_L, s \in S, t \in T. \quad (30)$$

$$\tilde{w}_{kvs}^t \leq |S|, \quad k = 1, \dots, n_{vL}, v \in V_L, s \in S, t \in T. \quad (31)$$

$$\sum_{j \in S^+} \tilde{x}_{jskv}^t \leq \tilde{w}_{kvs}^t, \quad s \in S, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (29a)$$

Self-loop elimination constraint

$$x_{cckvs(c)}^{rt} = 0, \quad c \in C_s^+, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T. \quad (20c)$$

$$\tilde{x}_{sskv}^t = 0, \quad s \in S^+, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (22c)$$

The model can be expanded by adding some strengthening constraints linking  $q$  and  $x$ :

Additional constraints linking variables  $q$  and  $x$  (between L and satellites)

$$\sum_{p \in P} q_{kvcs(c)}^{prt} + 1 - \mu \geq \sum_{j \in C_s^+} x_{jckvs(c)}^{rt}, \quad (20ba)$$

$$c \in C_s, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T.$$

$$\sum_{p \in P} q_{kvcs(c)}^{prt} + 1 - \mu \geq \sum_{j \in C_s^+} x_{cjkvs(c)}^{rt} \quad (20bb)$$

$$c \in C_s, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T.$$

Additional constraints linking variables  $q$  and  $x$  (between L and satellites)

$$\sum_{p \in P} \tilde{q}_{kvs}^{pt} + 1 - \mu \geq \sum_{j \in S^+} \tilde{x}_{jskv}^t, \quad s \in S, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (22ba)$$

$$\sum_{p \in P} \tilde{q}_{kvs}^{pt} + 1 - \mu \geq \sum_{j \in S^+} \tilde{x}_{sjkv}^t, \quad s \in S, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (22bb)$$

Additional constraints linking variables  $q$  and  $x$  (between F and L)

$$\sum_{p \in P} \hat{q}_{kv}^{pt} + 1 - \mu \geq \hat{x}_{kv}^t, \quad k = 1, \dots, n_{vF}, v \in V_F, t \in T \quad (24b)$$

To strengthen the routing part, the following inequalities could be added to the formulation similar to those in [Adulyasak et al, 2013] [Archetti et al, 2007, 2011]:

$$x_{ijkvs}^{rt} \leq \sum_{p \in P_v} y_{kvs}^{prt}, \quad i, j \in C_s^+; k = 1, \dots, n_{vs}; v \in V_s; s \in S; r = 1, \dots, m_{vs}; t \in T \quad (32)$$

$$\tilde{x}_{ijkv}^t \leq \sum_{p \in P_v} \tilde{y}_{kv}^{pt}, \quad i, j \in S^+; k = 1, \dots, n_{vL}; v \in V_L; t \in T \quad (33)$$

$$\hat{x}_{kv}^t \leq \sum_{p \in P_v} \hat{y}_{kv}^{pt}, \quad k = 1, \dots, n_{vF}; v \in V_F; t \in T \quad (34)$$

When a vehicle does not transport any product, constraints (32) - (34) force the corresponding  $x$  variables to 0. Constraints (32) are associated with the routes between a satellite and its customers, constraints (33) are associated with the routes between the close supply node L and the satellites. Constraints (34) are associated with the routes between the far close supply node F and the close supply node L.

Regarding the multi-vehicle aspect, constraints (35) – (37) are valid vehicle symmetry breaking constraints similar to the constraints proposed in [Adulyasak et al, 2013; Adulyasak et al, 2012a] that are shown to significantly improve the performance of the branch-and-bound process. Similarly, we added constraints (38) for the multi-route aspect of the satellite-customer echelon.

$$\sum_{j \in C_s} x_{sjkvs}^{rt} \geq \sum_{j \in C_s} x_{sj(k+1)vs}^{rt} , \quad s \in S; k = 1, \dots, n_{vs} - 1; v \in V_s; r = 1, \dots, m_{vs}; t \in T \quad (35)$$

$$\sum_{j \in S} \tilde{x}_{Ljkv}^t \geq \sum_{j \in S} \tilde{x}_{Lj(k+1)v}^t , \quad k = 1, \dots, n_{vL} - 1; v \in V_L; t \in T \quad (36)$$

$$\hat{x}_{kv}^t \geq \hat{x}_{(k+1)v}^t \quad k = 1, \dots, n_{vF} - 1; v \in V_s; t \in T \quad (37)$$

$$\sum_{j \in C_s} x_{sjkvs}^{rt} \geq \sum_{j \in C_s} x_{sjkvs}^{(r+1)t} , \quad s \in S; k = 1, \dots, n_{vs}; v \in V_s; r = 1, \dots, m_{vs} - 1; t \in T \quad (38)$$

Constraints (35) - (37) state that the  $k^{th}$  vehicle is used before  $(k+1)^{st}$  vehicle, for all three echelons: satellites – customers, close supply node – satellites, far supply node – close supply node, respectively. Constraints (38) state that the  $r^{th}$  route of vehicle  $k$  is used before  $(r+1)^{st}$  route, between satellite node and its customers.

Each vehicle route in Echelon 1 (between the close supply node and the satellites) and Echelon 2 (between the satellites and the customer) has to finish in the time period within which it started. This is assured by the constraints (17) and (19). Only the routes in Echelon 0 can spread over several time periods.

The travel times along the edges of the transportation network are dealt with in constraints (17), (18), (19) and (19a).

Note that the constraints (4), (5), (6) forbid supplies received during a given time period to be used to satisfy the demand at the same time period. This increases the inventories but it makes sure that the supply is there when it is needed. On the other hand this also restricts the supply that arrived in this time period to leave immediately.

These constraints can also be removed, but the danger is that the node may rely on a supply that has not arrived yet (if the demand is at the beginning of the time period and the supplies arrive closer to the end of the time period). This may be overcome by decreasing the duration of the time period. Unfortunately, this might make the constraints on the route duration (17, 19a) infeasible. Another way to resolve these problems is to include time windows on the demand, but this would substantially increase the problem size and the complexity.

Another possibility is to keep the reserve inventories at each store at the level needed to satisfy the demand for one time period. In this case the constraints (4), (5), (6) could be removed and supplies that arrive at a node can immediately be re-loaded and go further.

### **5.3 Changing the MIP Model to Encompass Other Problem Versions**

#### ***Demand at the close supply node and the satellite nodes:***

In both military and humanitarian logistics, the demand at the intermediate nodes of the network might not be negligible. For example, food and fuel consumption at intermediate nodes can be as high as 10% to 20% of the total consumption. This might be modeled by adding the corresponding demand term (parameter) to the right hand side of the constraints (5) and (6) of the model.

#### ***Safety and risks, and their dynamic changes through the time periods:***

The transportation cost along arc (i, j) can encompass the level of risk on this arc, and thus the objective function would be also used to minimize risks. We can also add an additional index  $t$  to the cost parameters to model the differences between the risks/ costs in different time periods. This may significantly change the distribution between the time periods.



## 6 Stochastic Model

Now, we consider that the demands are stochastic, and provide some of the possible stochastic models.

The mathematical programming model given in this section is derived from the deterministic model presented in Section 5.

The stochastic demand is captured by designing a number of representative demand realization scenarios. This type of approach to solving the stochastic problem is also called *sampling* [Pillac et al, 2013], and it has been proposed as a way of handling the complexities of uncertain environments.

Using sampling, a stochastic model can be designed by generating an Extensive or Explicit Stochastic Form of the problem model – from the deterministic model – in the following manner:

- Add an additional index  $\theta$  representing a scenario  $\theta$  to all scenario-dependent variables and parameters
- Extend the objective function by one more summation multiplying the terms with the probabilities of each scenario.

In order to create this explicit stochastic form, one needs to decide which variables are scenario-dependent. These scenario-dependent variables are called the second-stage variables because their exact values could be decided at the second-stage of the decision-making process when more information about the situation is known. The remaining variables are called the first-stage variables, and they will be assumed not to be scenario-dependent. Their values will determine the plan that will not be changed based on the scenario realization in the real world.

We propose three reasonable decisions for choosing subsets of the scenario-dependent and the scenario-independent variables – three Explicit Stochastic Models. The mathematical programming formulation of the first Explicit Stochastic Model ESM1 is presented in Section 6.1 and it has been implemented in our software package.

**Explicit Stochastic Model - ESM1:** One possibility is to have for:

- the first-stage variables: all routing variables, and
- the second-stage variables: all inventory and flow variables.

We implemented this version on the mathematical programming model presented in Section 0.

**Explicit Stochastic Model – ESM2:** Another possibility is to have for:

- the first-stage variables: the routing variables determining the plan for the routing between the far supply node and the close supply node, and
- the second-stage variables: all other variables.

It is hoped this version can be tested in the future.

**Explicit Stochastic Model – ESM3:** Another possibility is to have for:

- the first-stage variables: all the routing variables, and the flow variables determining the quantities moved between the far supply node and the close supply node, and
- the second-stage variables: all other variables.

It is hoped this version can be tested in the future.

In this case, the mathematical programming formulation will be as follows: all second-stage variables will get an additional index  $\theta$  and the objective function will change to account for the different scenarios and their probabilities  $b_\theta$ .

## 6.1 Mathematical Programming Formulation

The following pages present the mathematical programming formulation of the explicit form of the stochastic problem with the first-stage and the second-stage variables as described in the previous paragraph. The notation of the new parameters as well as the notation of all decision variables of the new mathematical programming formulation is also presented. The labels of the changed constraints is extended by *-sef* that stands for *Stochastic Explicit Form*.

### Notation

$\Theta$	set of scenarios that account for the probability distributions of the demands at the customer nodes
$e_\theta$	the probability of scenario $\theta$
$d_{pc}^{t\theta}$	in scenario $\theta$ , the demand for product of type $p \in P$ at customer $c \in C$ in period $t \in T$

### Decision variables

$x_{ijkvs}^{rt}$	= 1 if the $r^{\text{th}}$ route of the $k^{\text{th}}$ vehicle of type $v$ of satellite $s$ travels from $i$ to $j$ in time period $t$ for $i, j \in C_s^+$ , $i \neq j$ , $k = 1, \dots, n_{vs}$ , $v \in V_s$ , $s \in S$ , $r = 1, \dots, m_{vs}$ , $t \in T$ ; 0 otherwise.
$\tilde{x}_{ijkv}^t$	= 1 if the $k^{\text{th}}$ vehicle of type $v$ of close supply node $L$ travels from $i$ to $j$ in time period $t$ for $i, j \in S^+$ , $i \neq j$ , $k = 1, \dots, n_{vL}$ , $v \in V_L$ , $t \in T$ ; 0 otherwise.
$\hat{x}_{kv}^t$	= 1 if the $k^{\text{th}}$ vehicle of type $v$ of remote supply node $F$ is sent to close supply node $L$ in time period $t$ , $k = 1, \dots, n_{vF}$ , $v \in V_F$ , $t \in T$ ; 0 otherwise.

$y_{kvs}^{prt}$	= 1 if the $r^{\text{th}}$ route of the $k^{\text{th}}$ vehicle of type $v$ of satellite $s$ is used to transport product $p$ in time period $t$ for $k = 1, \dots, n_{vs}$ , $v \in V_s$ , $s \in S$ , $p \in P_v$ , $r = 1, \dots, m_{vs}$ , $t \in T$ ; 0 otherwise.
$\tilde{y}_{kv}^{pt}$	= 1 if the $k^{\text{th}}$ vehicle of type $v$ of close supply node $L$ is used to transport product type $p$ in time period $t$ for $k = 1, \dots, n_{vL}$ , $v \in V_L$ , $p \in P_v$ , $t \in T$ ; 0 otherwise.
$\hat{y}_{kv}^{pt}$	= 1 if the $k^{\text{th}}$ vehicle of type $v$ of remote supply node $F$ is sent to close supply node $L$ with product type $p$ in time period $t$ for $k = 1, \dots, n_{vF}$ , $v \in V_F$ , $p \in P_v$ , $t \in T$ ; 0 otherwise.
$q_{kvcs}^{prt\theta}$	in scenario $\theta$ , quantity of product type $p$ delivered to customer $c$ on the $r^{\text{th}}$ route of the $k^{\text{th}}$ vehicle of type $v$ of satellite $s(c)$ in time period $t$ , for $k = 1, \dots, n_{vs(c)}$ , $v \in V_{s(c)}$ , $c \in C$ , $p \in P_v$ , $r = 1, \dots, m_{vs(c)}$ , $t \in T$ .
$\tilde{q}_{kvs}^{pt\theta}$	in scenario $\theta$ , quantity of product type $p$ delivered to satellite $s$ by the $k^{\text{th}}$ vehicle of type $v$ of close supply node $L$ in time period $t$ , for $k = 1, \dots, n_{vL}$ , $v \in V_L$ , $s \in S$ , $p \in P_v$ , $t \in T$ .
$\hat{q}_{kv}^{pt\theta}$	in scenario $\theta$ , quantity of product type $p$ sent to close supply node $L$ by the $k^{\text{th}}$ vehicle of type $v$ of remote supply node $F$ in time period $t$ , for $k = 1, \dots, n_{vF}$ , $v \in V_F$ , $p \in P_v$ , $t \in T$ .
$h_{gcs}^{pt\theta}$	in scenario $\theta$ , inventory level of product type $p \in R_g$ in store type $g \in G_c$ at customer node $c$ (of the satellite node $s(c)$ ) at the beginning of period $t \in T$ . Note that $h_{gcs}^{p0}$ should be fixed and equal to the initial inventory levels if those are known in advance.
$\tilde{h}_{gs}^{pt\theta}$	in scenario $\theta$ , inventory level of product type $p \in R_g$ in store type $g \in G_s$ at satellite node $s$ at the beginning of period $t \in T$ . Note that $h_{gs}^{p0}$ should be fixed and equal to the initial inventory levels.
$\hat{h}_g^{pt\theta}$	in scenario $\theta$ , inventory level of product type $p \in R_g$ in store type $g \in G_L$ at the close supply node $L$ at the beginning of period $t \in T$ . Note that $h_g^{p0}$ should be fixed and equal to the initial inventory levels.
$w_{kvcs}^{rt}$	position of customer $c$ in the $r^{\text{th}}$ route of the $k^{\text{th}}$ vehicle of type $v$ of satellite $s(c)$ in time period $t$ for $k = 1, \dots, n_{vs(c)}$ , $v \in V_{s(c)}$ , $c \in C$ , $r = 1, \dots, m_{vs(c)}$ , $t \in T$ .
$\tilde{w}_{kvs}^t$	position of satellite $s$ in the route of the $k^{\text{th}}$ vehicle of type $v$ of close supply node $L$ in time period $t$ for $k = 1, \dots, n_{vL}$ , $v \in V_L$ , $s \in S$ , $t \in T$ .
$\underline{d}_{pc}^{t\theta}$	in scenario $\theta$ , amount of unsatisfied demand of product $p$ at customer $c$ in time period $t$ .

### Objective

$$\begin{aligned} \text{Min } \sum_{t \in T} \left( \omega_1 \sum_{s \in S} \sum_{v \in V_s} \sum_{k=1}^{n_{vs}} \sum_{r=1}^{m_{vs}} \sum_{i \in C_s^+} \sum_{\substack{j \in C_s^+ \\ i \neq j}} c_{ijv} x_{ijkvs}^{rt} + \omega_2 \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \sum_{s \in S^+} \sum_{\substack{s' \in S^+ \\ s \neq s'}} c_{ss'v} \tilde{x}_{ss'kv}^t \right. \\ \left. + \omega_3 \sum_{v \in V_F} \sum_{k=1}^{n_{vF}} \varphi_v \hat{x}_{kv}^t \right. \\ \left. + \sum_{\theta \in \Theta} e_{\theta} \left( \omega_4 \left( \sum_{s \in S} \sum_{c \in C_s} \sum_{g \in G_c} \sum_{p \in R_g} f_g^p h_{gcs(c)}^{pt} + \sum_{s \in S} \sum_{g \in G_s} \sum_{p \in R_g} \tilde{f}_g^p \tilde{h}_{gs}^{pt} \right. \right. \right. \\ \left. \left. + \sum_{g \in G_L} \sum_{p \in R_g} \hat{f}_g^p \hat{h}_g^{pt} \right) + \omega_5 \sum_{p \in P} \sum_{c \in C} \alpha_{pc} d_{pc}^t \right) \end{aligned}$$

### Constraints

Inventory balance at customer nodes

$$\sum_{g \in G_c} h_{gcs(c)}^{pt\theta} + \sum_{v \in V_{s(c)}} \sum_{k=1}^{n_{vs(c)}} \sum_{r=1}^{m_{vs(c)}} q_{kvcs(c)}^{prt\theta} - d_{pcs(c)}^{t\theta} = \sum_{g \in G_c} h_{gcs(c)}^{p(t+1)\theta} - d_{pcs(c)}^{t\theta}, \quad (1)$$

$$c \in C_s, s \in S, p \in P, t \in T \setminus \{\bar{t}\}, \theta \in \Theta.$$

Inventory balance at customer nodes for the last period  $\bar{t}$  in the planning horizon

$$\sum_{g \in G_c} h_{gcs(c)}^{p\bar{t}\theta} + \sum_{v \in V_{s(c)}} \sum_{k=1}^{n_{vs(c)}} \sum_{r=1}^{m_{vs(c)}} q_{kvcs(c)}^{pr\bar{t}\theta} - d_{pcs(c)}^{\bar{t}\theta} \geq -d_{pcs(c)}^{\bar{t}\theta}, \quad c \in C_s, s \in S, p \in P, \theta \in \Theta. \quad (1a\text{-sef})$$

Inventory balance at satellites

$$\sum_{g \in G_s} \tilde{h}_{gs}^{pt\theta} + \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \tilde{q}_{kvs}^{pt\theta} - \sum_{c \in C_s} \sum_{v \in V_s} \sum_{k=1}^{n_{vs}} \sum_{r=1}^{m_{vs}} q_{kvcs(c)}^{prt\theta} = \sum_{g \in G_s} \tilde{h}_{gs}^{p(t+1)\theta}, \quad (2\text{-sef})$$

$$s \in S, p \in P, t \in T \setminus \{\bar{t}\}, \theta \in \Theta.$$

Inventory balance at satellites for the last period  $\bar{t}$  in the planning horizon

$$\sum_{g \in G_s} \tilde{h}_{gs}^{p\bar{t}\theta} + \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \tilde{q}_{kvs}^{p\bar{t}\theta} - \sum_{c \in C_s} \sum_{v \in V_s} \sum_{k=1}^{n_{vs}} \sum_{r=1}^{m_{vs}} q_{kvcs(c)}^{pr\bar{t}\theta} \geq 0, \quad s \in S, p \in P, \theta \in \Theta. \quad (2a\text{-sef})$$

Inventory balance at close supply node L

In the second term when  $t - \delta_v < 0$ , assume  $\hat{q}_{kv}^{p(t-\delta_v)\theta} = 0$

$$\sum_{g \in G_L} \hat{h}_g^{pt\theta} + \sum_{v \in V_F} \sum_{k=1}^{n_{vF}} \hat{q}_{kv}^{p(t-\delta_v)\theta} - \sum_{s \in S} \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \tilde{q}_{kvs}^{pt\theta} = \sum_{g \in G_L} \hat{h}_g^{p(t+1)\theta}, \quad p \in P, t \in T \setminus \{\bar{t}\}, \theta \in \Theta. \quad (3\text{-sef})$$

Inventory balance at close supply node L for the last period  $\bar{t}$  in the planning horizon

$$\sum_{g \in G_L} \hat{h}_g^{p\bar{t}\theta} + \sum_{v \in V_F} \sum_{k=1}^{n_{vF}} \hat{q}_{kv}^{p(\bar{t}-\delta_v)\theta} - \sum_{s \in S} \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \tilde{q}_{kvs}^{p\bar{t}\theta} \geq 0, \quad p \in P, \theta \in \Theta. \quad (3a\text{-sef})$$

Demand constraints at customer nodes

$$\sum_{g \in G_c} h_{gcs(c)}^{pt\theta} \geq d_{pcs(c)}^{t\theta} - \underline{d}_{pcs(c)}^{t\theta}, \quad c \in C_s, s \in S, p \in P, t \in T, \theta \in \Theta. \quad (4\text{-sef})$$

Demand constraints at satellites

$$\sum_{g \in G_s} \tilde{h}_{gs}^{pt\theta} \geq \sum_{c \in C_s} \sum_{v \in V_s} \sum_{k=1}^{n_{vs}} \sum_{r=1}^{m_{vs}} q_{kvcs(c)}^{prt\theta}, \quad s \in S, p \in P, t \in T, \theta \in \Theta. \quad (5\text{-sef})$$

Demand constraints at close supply node L

$$\sum_{g \in G_L} \hat{h}_g^{pt\theta} \geq \sum_{s \in S} \sum_{v \in V_L} \sum_{k=1}^{n_{vL}} \tilde{q}_{kvs}^{pt\theta}, \quad p \in P, t \in T, \theta \in \Theta. \quad (6\text{-sef})$$

Store capacity constraints at customer nodes

$$\sum_{p \in R_g} h_{gcs(c)}^{pt\theta} \leq \bar{u}_g, \quad g \in G_c, c \in C_s, s \in S, t \in T, \theta \in \Theta. \quad (7\text{-sef})$$

Compatibility between store types and product types at customer nodes

$$h_{gcs(c)}^{pt\theta} \leq a_g^p \bar{u}_g, \quad p \in P, g \in G_c, c \in C_s, s \in S, t \in T, \theta \in \Theta. \quad (7a\text{-sef})$$

Store capacity constraints at satellite nodes

$$\sum_{p \in R_g} \tilde{h}_{gs}^{pt\theta} \leq \bar{u}_g, \quad g \in G_s, s \in S, t \in T, \theta \in \Theta. \quad (7'\text{-sef})$$

Compatibility between store types and product types at satellite nodes

$$\tilde{h}_{gs}^{pt\theta} \leq a_g^p \bar{u}_g, \quad p \in P, g \in G_s, s \in S, t \in T, \theta \in \Theta. \quad (7'a\text{-sef})$$

Store capacity constraints at the close supply node

$$\sum_{p \in R_g} \hat{h}_g^{pt\theta} \leq \bar{u}_g, \quad g \in G_L, t \in T, \theta \in \Theta. \quad (7''\text{-sef})$$

Compatibility between store types and product types at the close supply node

$$\hat{h}_g^{pt\theta} \leq a_g^p \bar{u}_g, \quad p \in P, g \in G_L, t \in T, \theta \in \Theta. \quad (7''\text{a-sef})$$

Product compatibility constraints aboard the vehicles (between satellites and customers)

$$y_{kvs}^{prt} + y_{kvs}^{p'rt} \leq 1 + \gamma_v^{pp'}, \quad (8)$$

$$k = 1, \dots, n_{vS}, v \in V_S, s \in S, p \in P_v, p' \in P_v, p \neq p', r = 1, \dots, m_{vS}, t \in T.$$

Product compatibility constraints aboard vehicles (between L and satellites)

$$\tilde{y}_{kv}^{pt} + \tilde{y}_{kv}^{p't} \leq 1 + \gamma_v^{pp'}, \quad k = 1, \dots, n_{vL}, v \in V_L, p \in P_v, p' \in P_v, p \neq p', t \in T. \quad (9)$$

Product compatibility constraints in vehicles (between F and L)

$$\hat{y}_{kv}^{pt} + \hat{y}_{kv}^{p't} \leq 1 + \gamma_v^{pp'}, \quad k = 1, \dots, n_{vF}, v \in V_F, p \in P_v, p' \in P_v, p \neq p', t \in T. \quad (10)$$

Vehicle capacity constraints (between satellites and customers)

$$\sum_{c \in C_s} q_{kvcs(c)}^{prt\theta} \leq u_v y_{kvs}^{prt}, \quad k = 1, \dots, n_{vS}, v \in V_S, s \in S, p \in P, r = 1, \dots, m_{vS}, t \in T, \theta \in \Theta. \quad (11\text{-sef})$$

$$\sum_{p \in P} \sum_{c \in C_s} q_{kvcs(c)}^{prt\theta} \leq u_v, \quad k = 1, \dots, n_{vS}, v \in V_S, s \in S, r = 1, \dots, m_{vS}, t \in T, \theta \in \Theta. \quad (12\text{-sef})$$

Other linking constraints between  $q$  and  $y$  variables ( $\mu$  is a small number smaller than  $q$ )

$$\sum_{c \in C_s} q_{kvcs(c)}^{prt\theta} + 1 - \mu \geq y_{kvs(c)}^{prt}, \quad (11\text{a-sef})$$

$$k = 1, \dots, n_{vS}, v \in V_S, s \in S, p \in P, r = 1, \dots, m_{vS}, t \in T, \theta \in \Theta.$$

Compatibility between vehicle types and product types (between satellites and customers)

$$q_{kvcs(c)}^{prt\theta} \leq b_v^p u_v, \quad k = 1, \dots, n_{vS}, v \in V_S, c \in C_s, s \in S, p \in P, r = 1, \dots, m_{vS}, t \in T, \theta \in \Theta. \quad (12\text{a-sef})$$

Vehicle capacity constraints (between L and satellites)

$$\sum_{s \in S} \tilde{q}_{kvs}^{pt\theta} \leq u_v \tilde{y}_{kv}^{pt}, \quad k = 1, \dots, n_{vL}, v \in V_L, p \in P, t \in T, \theta \in \Theta. \quad (13\text{-sef})$$

$$\sum_{p \in P} \sum_{s \in S} \tilde{q}_{kvs}^{pt\theta} \leq u_v, \quad k = 1, \dots, n_{vL}, v \in V_L, t \in T, \theta \in \Theta. \quad (14\text{-sef})$$

Other linking constraints between  $q$  and  $y$  variables ( $\mu$  is a small number smaller than  $q$ )

$$\sum_{s \in S} \tilde{q}_{kvs}^{pt\theta} + 1 - \mu \geq \tilde{y}_{kv}^{pt}, \quad k = 1, \dots, n_{vL}, v \in V_L, p \in P, t \in T, \theta \in \Theta. \quad (13a-sef)$$

Compatibility between vehicle types and product types (between L and satellites)

$$\tilde{q}_{kvs}^{pt\theta} \leq b_v^p u_v, \quad k = 1, \dots, n_{vL}, v \in V_L, s \in S, p \in P, t \in T, \theta \in \Theta. \quad (14a-sef)$$

Vehicle capacity constraints (between F and L)

$$\hat{q}_{kv}^{pt\theta} \leq u_v \hat{y}_{kv}^{pt}, \quad k = 1, \dots, n_{vF}, v \in V_F, p \in P, t \in T, \theta \in \Theta. \quad (15-sef)$$

$$\sum_{p \in P} \hat{q}_{kv}^{pt\theta} \leq u_v, \quad k = 1, \dots, n_{vF}, v \in V_F, t \in T, \theta \in \Theta. \quad (16-sef)$$

Other linking constraints between  $q$  and  $y$  variables ( $\mu$  is a small number smaller than  $q$ )

$$\hat{q}_{kv}^{pt\theta} + 1 - \mu \geq \hat{y}_{kv}^{pt}, \quad k = 1, \dots, n_{vF}, v \in V_F, p \in P, t \in T, \theta \in \Theta. \quad (15a-sef)$$

Compatibility between vehicle types and product types (between F and L)

$$\hat{q}_{kv}^{pt\theta} \leq b_v^p u_v, \quad k = 1, \dots, n_{vF}, v \in V_F, p \in P, t \in T, \theta \in \Theta. \quad (16a-sef)$$

Maximum route length constraints – to handle re-fueling, if needed (between satellites and customers)

$$\sum_{i \in C_s^+} \sum_{\substack{j \in C_s^+ \\ i \neq j}} \tau_{ijv} x_{ijkvs}^{rt} \leq \tau_v^{max}, \quad (17)$$

$$k = 1, \dots, n_{vs}, v \in V_s, s \in S, r = 1, \dots, m_{vs}, t \in T.$$

The duration of all routes of one vehicle has to be smaller than the time period duration (for the vehicles traveling between satellites and customers)

$$\sum_{r=1}^{m_{vs}} \left( \sum_{i \in C_s^+} \sum_{\substack{j \in C_s \\ i \neq j}} (\tau_{ijv} + \sigma_{jv}) x_{ijkvs}^{rt} + \sum_{j \in C_s} \tau_{jsv} x_{jskvs}^{rt} + \sum_{c \in C_s} \sum_{p \in P_v} \rho_{vs}^p q_{kvcs(c)}^{prt\theta} \right) \leq \eta^{max}, \quad (18-sef)$$

$$k = 1, \dots, n_{vs}, v \in V_s, s \in S, t \in T, \theta \in \Theta.$$

Maximum route length constraints – to handle re-fueling, if needed (between L and satellites)

$$\sum_{i \in S^+} \sum_{\substack{j \in S^+ \\ i \neq j}} \tau_{ijv} \tilde{x}_{ijkv}^t \leq \tau_v^{max}, \quad k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (19)$$

The route of each vehicle has to finish within the time period (between L and customers)

$$\sum_{i \in S^+} \sum_{j \in S} (\tau_{ijv} + \sigma_{jv}) \tilde{x}_{ijkv}^t + \sum_{j \in S} \tau_{jLv} \tilde{x}_{jLkv}^t + \sum_{s \in S} \sum_{p \in P_v} \rho_{vL}^p \tilde{q}_{kvs}^{pt\theta} \leq \eta^{max}, \quad (19a-sef)$$

$$k = 1, \dots, n_{vS}, v \in V_s, t \in T, \theta \in \Theta.$$

Vehicle flow constraints (between satellites and customers)

$$\sum_{j \in C_s^+} x_{cjkvs(c)}^{rt} = \sum_{j \in C_s^+} x_{jckvs(c)}^{rt}, \quad (20)$$

$$c \in C_s^+, s \in S, k = 1, \dots, n_{vS(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vS(c)}, t \in T.$$

$$\sum_{j \in C_s^+} x_{cjkvs(c)}^{rt} \leq 1, \quad (21)$$

$$c \in C_s^+, s \in S, k = 1, \dots, n_{vS(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vS(c)}, t \in T.$$

$$y_{kvs}^{prt} \leq \sum_{j \in C_s} x_{sjkvs(c)}^{rt}, \quad s \in S, k = 1, \dots, n_{vS(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vS(c)}, t \in T, p \in P. \quad (21a)$$

Vehicle flow constraints (between L and satellites)

$$\sum_{j \in S^+} \tilde{x}_{sjkv}^t = \sum_{j \in S^+} \tilde{x}_{jskv}^t, \quad s \in S^+, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (22)$$

$$\sum_{j \in S^+} \tilde{x}_{sjkv}^t \leq 1, \quad s \in S^+, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (23)$$

$$\tilde{y}_{kv}^{pt} \leq \sum_{s \in S} \tilde{x}_{Lskv}^t, \quad k = 1, \dots, n_{vL}, v \in V_L, t \in T, p \in P. \quad (23a)$$

Vehicle flow constraints (between F and L)

$$\sum_{t'=t}^{t+\delta_v+\varepsilon_v} \hat{x}_{kv}^{t'} \leq 1, \quad k = 1, \dots, n_{vF}, v \in V_F, t \in T \setminus \{t | t + \delta_v + \varepsilon_v > \bar{t}\}. \quad (24)$$

$$\hat{y}_{kv}^{pt} \leq \hat{x}_{kv}^t, \quad k = 1, \dots, n_{vF}, v \in V_F, p \in P_v, t \in T. \quad (25)$$

Constraints linking variables  $q$  and  $x$  (between satellites and customers)

$$\sum_{p \in P} q_{kvcs(c)}^{prt\theta} \leq u_v \sum_{j \in C_s^+} x_{jckvs(c)}^{rt}, \quad (20aa-sef)$$

$$c \in C_s, s \in S, k = 1, \dots, n_{vS(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vS(c)}, t \in T, \theta \in \Theta.$$



$$\sum_{p \in P} q_{kvcs(c)}^{prt\theta} \leq u_v \sum_{j \in C_s^+} x_{cjkvs(c)}^{rt}, \quad (20ab-sef)$$

$$c \in C_s, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T, \theta \in \Theta.$$

Constraints linking variables  $q$  and  $x$  (between L and satellites)

$$\sum_{p \in P} \tilde{q}_{kvs}^{pt\theta} \leq u_v \sum_{j \in S^+} \tilde{x}_{jskv}^t, \quad s \in S, k = 1, \dots, n_{vL}, v \in V_L, t \in T, \theta \in \Theta. \quad (22aa-sef)$$

$$\sum_{p \in P} \tilde{q}_{kvs}^{pt\theta} \leq u_v \sum_{j \in S^+} \tilde{x}_{sjkv}^t, \quad s \in S, k = 1, \dots, n_{vL}, v \in V_L, t \in T, \theta \in \Theta. \quad (22ab-sef)$$

Constraints linking variables  $q$  and  $x$  (between F and L)

$$\sum_{p \in P} \hat{q}_{kv}^{pt\theta} \leq u_v \hat{x}_{kv}^t, \quad k = 1, \dots, n_{vF}, v \in V_F, t \in T, \theta \in \Theta. \quad (24a-sef)$$

Subtour elimination constraints (between satellites and customers with  $M = |C_{s(c)}^+|$ )

$$w_{kvcs(c)}^{rt} \geq w_{kvc's(c')}^{rt} + (M + 1)x_{c'ckvs(c)}^{rt} - M, \quad (26)$$

$$k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, c \in C_s, c' \in C_s, c \neq c', r = 1, \dots, m_{vs(c)}, s \in S, t \in T.$$

$$w_{kvcs(c)}^{rt} \geq (M + 1)x_{s(c)ckvs(c)}^{rt} - M, \quad (27)$$

$$k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, c \in C_s, r = 1, \dots, m_{vs(c)}, s \in S, t \in T.$$

$$w_{kvcs(c)}^{rt} \leq |C_{s(c)}|, \quad k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, c \in C_s, r = 1, \dots, m_{vs(c)}, s \in S, t \in T. \quad (28)$$

$$\sum_{j \in C_s^+} x_{jckvs(c)}^{rt} \leq w_{kvcs(c)}^{rt}, \quad c \in C_s, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T. \quad (27a)$$

Subtour elimination constraints (between L and satellites with  $M' = |S^+|$ )

$$\tilde{w}_{kvs}^t \geq \tilde{w}_{kvs'}^t + (M' + 1)\tilde{x}_{s'skv}^t - M', \quad k = 1, \dots, n_{vL}, v \in V_L, s \in S, s' \in S, s \neq s', t \in T. \quad (29)$$

$$\tilde{w}_{kvs}^t \geq (M' + 1)\tilde{x}_{Lskv}^t - M', \quad k = 1, \dots, n_{vL}, v \in V_L, s \in S, t \in T. \quad (30)$$

$$\tilde{w}_{kvs}^t \leq |S|, \quad k = 1, \dots, n_{vL}, v \in V_L, s \in S, t \in T. \quad (31)$$

$$\sum_{j \in S^+} \tilde{x}_{jskv}^t \leq \tilde{w}_{kvs}^t, \quad s \in S, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (29a)$$

Self-loop elimination constraint

$$x_{cckvs(c)}^{rt} = 0, \quad c \in C_s^+, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T. \quad (20c)$$

$$\tilde{x}_{sskv}^t = 0, \quad s \in S^+, k = 1, \dots, n_{vL}, v \in V_L, t \in T. \quad (22c)$$

Additional strengthening constraints

$$\sum_{p \in P} q_{kvcs(c)}^{prt\theta} + 1 - \mu \geq \sum_{j \in C_s^+} x_{jckvs(c)}^{rt}, \quad (20ba-sef)$$

$$c \in C_s, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T, \theta \in \Theta.$$

$$\sum_{p \in P} q_{kvcs(c)}^{prt\theta} + 1 - \mu \geq \sum_{j \in C_s^+} x_{cjkvs(c)}^{rt} \quad (20bb-sef)$$

$$c \in C_s, s \in S, k = 1, \dots, n_{vs(c)}, v \in V_{s(c)}, r = 1, \dots, m_{vs(c)}, t \in T, \theta \in \Theta.$$

$$\sum_{p \in P} \tilde{q}_{kvs}^{pt\theta} + 1 - \mu \geq \sum_{j \in S^+} \tilde{x}_{jskv}^t, \quad s \in S, k = 1, \dots, n_{vL}, v \in V_L, t \in T, \theta \in \Theta. \quad (22ba-sef)$$

$$\sum_{p \in P} \tilde{q}_{kvs}^{pt\theta} + 1 - \mu \geq \sum_{j \in S^+} \tilde{x}_{sjkv}^t, \quad s \in S, k = 1, \dots, n_{vL}, v \in V_L, t \in T, \theta \in \Theta. \quad (22bb-sef)$$

$$\sum_{p \in P} \hat{q}_{kv}^{pt\theta} + 1 - \mu \geq \hat{x}_{kv}^t, \quad k = 1, \dots, n_{vF}, v \in V_F, t \in T, \theta \in \Theta. \quad (24b-sef)$$

$$x_{ijkvs}^{rt} \leq \sum_{p \in P_v} y_{kvs}^{prt}, \quad i, j \in C_s^+; k = 1, \dots, n_{vs}; v \in V_s; s \in S; r = 1, \dots, m_{vs}; t \in T \quad (32)$$

$$\tilde{x}_{ijkv}^t \leq \sum_{p \in P_v} \tilde{y}_{kv}^{pt}, \quad i, j \in S^+; k = 1, \dots, n_{vL}; v \in V_L; t \in T \quad (33)$$

$$\hat{x}_{kv}^t \leq \sum_{p \in P_v} \hat{y}_{kv}^{pt}, \quad k = 1, \dots, n_{vF}; v \in V_F; t \in T \quad (34)$$

$$\sum_{j \in C_s} x_{sjkvs}^{rt} \geq \sum_{j \in C_s} x_{sj(k+1)vs}^{rt}, \quad s \in S; k = 1, \dots, n_{vs} - 1; v \in V_s; r = 1, \dots, m_{vs}; t \in T \quad (35)$$

$$\sum_{j \in S} \tilde{x}_{Ljkv}^t \geq \sum_{j \in S} \tilde{x}_{Lj(k+1)v}^t, \quad k = 1, \dots, n_{vL} - 1; v \in V_L; t \in T \quad (36)$$

$$\hat{x}_{kv}^t \geq \hat{x}_{(k+1)v}^t, \quad k = 1, \dots, n_{vF} - 1; v \in V_s; t \in T \quad (37)$$

$$\sum_{j \in C_s} x_{sjkvs}^{rt} \geq \sum_{\substack{j \in C_s \\ \in T}} x_{sjkvs}^{(r+1)t}, \quad s \in S; k = 1, \dots, n_{vs}; v \in V_s; r = 1, \dots, m_{vs} - 1; t \quad (38)$$

## 7 Implemented Solution Approaches

The idea of using an Open Source general purpose MIP solver was initially considered, but had to be dropped based on a brief literature survey and on previous experiences of the team members that showed that Open Source MIP tools in general are not mature enough to provide good performance when solving complex and large MIP problems such as the one this project is dealing with. The general purpose MIP Solver Gurobi was selected instead as this is known to be more reliable.

Thus, this section describes the solution approaches using the general-purpose MIP solver Gurobi.

### 7.1 Deterministic Model and General-Purpose MIP Solver

One approach to solving a deterministic optimization problem consists of proposing a mathematical programming formulation of the problem and using a general-purpose MIP solver to solve it.

As stated above, we will use the general-purpose MIP solver Gurobi and its Java API.

Thus, the process for solving our deterministic problem includes:

- Designing a mathematical programming formulation – the model – of the problem. Our mathematical programming formulation of the e2E-SCM problem integrating the inventory management and distribution management is presented in Section 5
- Implementation (programming) of the model by using the Java API (functions) of Gurobi
- Implementation (programming) of the reader of the problem parameters stored in an input file containing the problem instance of the e2E-SCM
- Implementation (programming) of the source code that writes the details of the solution
- Run the implemented source code
- Fine tune the model and the parameters

### 7.2 Stochastic Model and General-Purpose MIP Solver

Having the explicit stochastic model presented in Section 6.1, we can solve the stochastic problem using any solution approach for solving the deterministic problem, including a general-purpose MIP solver, to solve the stochastic problem.

This approach to solving the stochastic problems is the simplest and its implementation requires the least time. Due to budget constraints in Task Authorization #1, we implemented this solution approach in order to provide a way for solving the stochastic model.

We have implemented the Explicit Stochastic Model ESM1, and solved it using Gurobi.

Such a solution approach is often called the frontal approach to solving the stochastic problem.

Although sampling approach to handling uncertainties may often require that thousands of scenarios are generated, some recent studies have shown that with careful study of the potential situations sometimes a very small number of scenarios can be sufficient to model the existing uncertainties. Thus we can start with 3, 4, 8 or 16 demand scenarios including average and extreme situations.

## 8 Other Solution Approaches

This section describes a few alternative solutions approaches for solving deterministic and stochastic versions of our problem. None of these approaches have been implemented as part of the Task Authorization #1. Some of them might be implemented as part of the Task Authorization #2.

The deterministic problem solution approaches include:

- Variable-fixing heuristic (H1 and H2) that are calling a commercial general purpose MIP solver, Gurobi, to solve the proposed sub-problems (Section 8.1)
- Tabu search framework where the problem is divided into a discrete portion and a continuous portion). The discrete portion is solved using a tabu search heuristic, and the continuous portion is a network flow problem that can be solved using an MIP solver such as Gurobi.

The stochastic programming solution approaches, apart from the stochastic programming using explicit form (Section 7.2), we propose include:

- Progressive hedging in combination with CPLEX or Gurobi (Section 8.3)
- A variation of the Heuristic H2 proposed for solving the deterministic model. The perturbations could be done using different demand scenarios. The intermediate solutions of the Alternate Heuristic could be saved and combined, or this may be done at the end of the heuristic. We will incorporate these demand scenarios in the continuous sub-problems only dealing with the network flow. The solution to this problem will determine the flow of products, and then the combinatorial optimization sub-problem will be solved to handle the needs for transportation.
- Benders' decomposition and CPLEX or Gurobi. Benders' decomposition (named after Jacques F. Benders) is an optimization method for solving exactly large linear programming problems with a special block structure. This structure often occurs in applications such as stochastic programming.

Progressive hedging is slightly simpler to implement than a Benders decomposition method and it is slightly easier to explain to a wider audience. A disadvantage of the progressive hedging approach is that it does not guarantee convergence in the case of an integer program.

The advantage of the Benders decomposition methods, like the L-shaped method, is that they guarantee optimality when solving MIP problems. However, the development of a Benders decomposition approach requires substantial effort and it cannot be done within the current contract. This approach can be considered for future research.

## 8.1 Iterated Local Search and Variable Fixing Heuristic

The variable fixing heuristic presented in this section is a combination of the alternate heuristic referred to in [Perron et al, 2010] and a guided iterated local search heuristic. The alternate heuristic is presented first since it is embedded in the iterated local search heuristic.

The alternate heuristic has its origins in the work of Coope (1964) where it was used in the context of location-allocation problems. It was proposed for solving global supply chain management problems by several authors as referred in [Perron et al, 2010].

The alternate heuristic repeats two consecutive steps each of which represent solving the sub-problem of the initial MIP model. Each sub-problem is generated by fixing a certain subset of variables. Each sub-problem is solved using the general purpose MIP solver, and the solver can run until the optimal solution of the sub-problem is found or until a pre-specified time limit is reached. The alternate heuristic can repeat those two steps until the solutions start converging or until another pre-specified time limit is reached.

The two sub-problems of the extended two-echelon heuristic are:

- The supply flow problem within the network combined with the inventory problem, when the variables defining the routes are fixed, and
- The routing problem, when the supply flow and inventory levels are fixed.

### **Heuristic H1: Alternate Heuristic**

*while (the solutions do not start converging or time limit 1)*

#### ***Step 1:***

*Step 1.1 (Initial iteration): Do either (a) or (b):*

*(a) Fix variables  $x$  and  $w$  such that they reflect one of the two following two options for the routes between the satellite nodes and the customers:*

- a. (Route  $s_1-c_{11}-c_{12}-c_{13}-s_1$ , where this is the shortest route visiting all the customers associated with this satellite node (a solution to the associated TSP problem). Repeat this route three times within a period. Have similar routes for all the satellite nodes. Later, when the risks are to be considered we will remove the risky edges from the transportation network graph.*
- b. Routes  $s_1-c_{11}-s_1$ ,  $s_1-c_{12}-s_1$ ,  $s_1-c_{13}-s_1$ . Repeat these routes three times within a period. Have the similar routes for all the satellite nodes.*

*(b) Solve the linear relaxation of the problem by CPLEX. Arrange  $x$  variables in descending order, and take one by one to fix them to 1 keeping the routes feasible. When infeasibility is reached, close the routes by connecting them to depots*

*Step 1.1 (Other iterations): In the second iteration, fix the routes –  $x$  and  $w$  variables – as per the solution achieved in Step 2.*

*Step 1.2: (Potentially) fix  $y$  variables to 1 to correspond to fixed  $x$  variables – consider constraints (8) and (11) and other relevant constraints*

*Step 1.3: Solve the remaining problem using CPLEX.*

***Step 2:***

*Step 2.1: Fix variables  $q$  and  $h$  to the values corresponding to the solution of the problem solved in Step 1.*

*Step 2.2: Solve the remaining problem with CPLEX.*

*end while*

As per [Perron et al, 2010] reference to Audet et al (2004), an Alternate Heuristic would converge to a local optimum of the problem if there is a unique solution of both steps at each iteration.

A metaheuristic can be built by embedding this algorithm in another heuristic framework such as the Variable Neighborhood search heuristic as proposed in [Perron et al, 2010].

This additional heuristic will create perturbations, so that more local optima are explored. The perturbations proposed in [Perron et al, 2010] are not applicable to our problem since they use the direct explicit change to the values of a subset of transfer price variables within the achieved feasible solution. The advantage of such a change is that any arbitrary change to the values of the transfer prices (within their explicitly known bounds) will not create an infeasible solution.

Thus, we propose expanding the alternate heuristic – in order to explore other local optima – by the following Iterated Local Search that is based on the ideas from Guided Local Search heuristic [Voudouris et al, 2010].

### **Heuristics H2: Iterated Local Search**

*Start*

*while (Time Limit 2)*

#### ***Alternate Heuristic***

***Perturbations:*** *use one of the following*

- (a) release all the variables, and run CPLEX providing the local optimum solution from the Alternate Heuristic as initial. Run CPLEX until Time Limit 3 is reached, or until a better solution is reached, or until another feasible solution is found*
- (b) change cost or demand and run CPLEX for several minutes or run several iterations of the Alternate Heuristic. Switch the demand/ cost back to the initial demands/ costs*
- (c) Use probabilities of demand scenarios*

*use this feasible solution to re-start Alternate Heuristic*

*end while*

*end*

The probabilities for the demand scenarios are determined based on the statistical analysis of historical data. If historical data is not available, consultation with SMEs and decision makers are required to decide which probabilities will be assigned to each scenario. The scenarios will also be proposed by the SMEs and decision makers in the case that historical data is not available.

## **8.2 Tabu Search (TS) Heuristics**

The two proposed tabu search heuristic differ in the way the two sub-problems – the VRP-type problem and the network-flow-type problem – relate to each other and interact.

### **TS and Network Flow Solver Alternate Heuristic – Heuristic H3**

Since dealing with continuous variables within a tabu search algorithm should be avoided, we decompose the problem in the following manner:

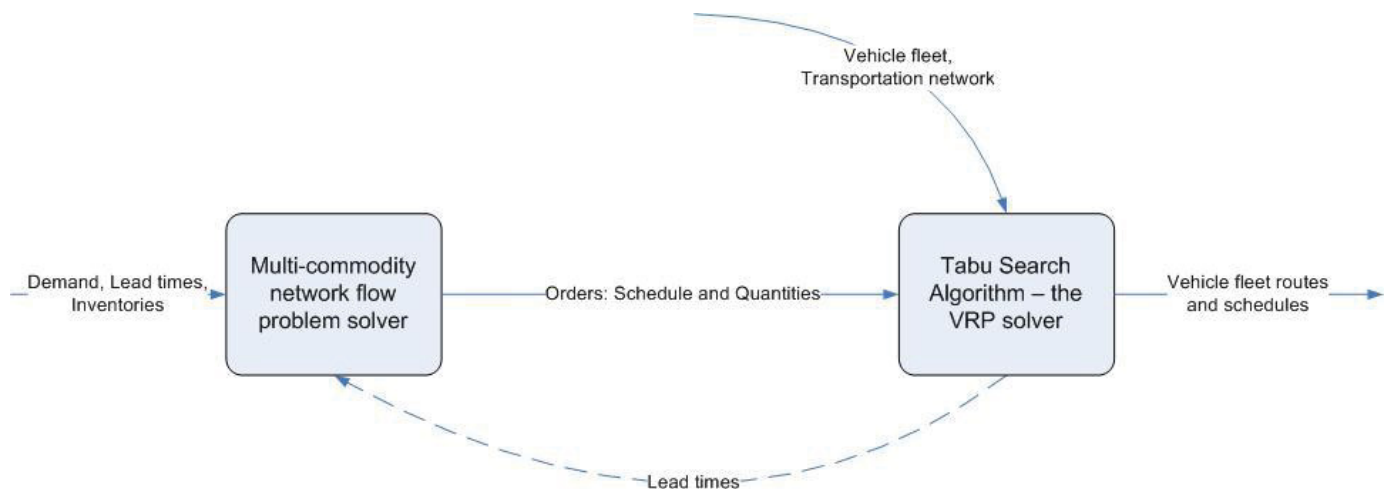
1. the portion of the problem containing continuous variables and dealing with the flow of products through the network and with the inventory management.
2. the combinatorial optimization portion of the problem – that will be a problem of the VRP type.



The first sub-problem – the min-cost network flow problem – will be solved using an algorithm for multi-commodity network flow, and the second sub-problem – the VRP – will be solved with a tabu search heuristic.

Thus, our solution approach consists of the following two algorithms (Figure 8-1):

- Tabu search for solving the VRP. Known and successful neighborhoods from the VRP literature will be used. Also, we can consider a split delivery vehicle routing sub-problem for which exact solutions exist in the literature.
- Min-cost network flow type of problem: for the flow of materials and inventory management.



**Figure 8-1 Tabu Search Framework**

Initial values for the lead times are randomly generated, and we solve the multi-commodity network flow problem to determine the orders with the quantities and schedules. These orders are fed to the tabu search algorithm for solving the VRP problem that generates a solution that fulfills these orders. This solution provides potentially different lead times that can be fed back to the network flow solver. The iterations may continue until the lead times start converging.

Orders can also be based on the target inventory formula often used, or its revised version: Target inventory levels = Maximum Forecasted consumption within the maximum replenishment time.

Within the tabu search algorithm, simple neighborhoods could be used: swapping nodes within a route of one vehicle, swapping the nodes between the routes of two vehicles, or swapping the compatible products between the vehicles. Or, even swapping the products between the periods could be considered.

## **TS Heuristic with Embedded Network Flow Solver – Heuristic *H4***

Another version of the tabu search heuristic may start by solving the VRP problem and solving the network flow problem within the inner loop of the tabu search algorithm, for evaluating each intermediate VRP solution by evaluating the associated min-cost multi-commodity network flow.

Still, since dealing with continuous variables within a tabu search algorithm must be avoided, we decompose the problem in a similar manner as in the previous section:

1. the combinatorial portion of the problem – the VRP problem type.
2. the portion of the problem containing continuous variables and dealing with the flow of products through the network and with the inventory management.

The first sub-problem will be solved with a tabu search heuristic while the second sub-problem will be solved using a network flow algorithm. Thus, our solution approach will consist of the following two algorithms:

- Tabu search for solving the combinatorial optimization problem – the VRP-like problem.
- Min-cost network flow type of problem: for the flow of materials and inventory management.

Initial orders of the products will be assumed at each node, at each time period, and we will solve the VRP problem using tabu search algorithm to fulfill these orders.

The second problem – a min-cost network flow problem – will be solved to get an evaluation of the VRP solution produced by the tabu search in the neighborhood of the current solution. Given that this evaluation might be costly, only a fraction of the neighborhood will be explored.

In similar problems that authors from CIRRELT tackled in the past, they had to be quite parsimonious with the min-cost flow calculations. The idea then is to be able to derive an easily computed approximation of the min-cost flow objective (or rather, of changes of this objective), which is used to identify promising candidates in the neighborhood. Only a handful of the most promising candidates (say, 5) are fully evaluated at each iteration of the tabu search heuristic. An efficient min-cost network flow algorithm is needed for this solution approach.

## 8.3 Progressive Hedging

### 8.3.1 Two-Stage Model Decomposition

This section presents the two-stage model that could be used in progressive hedging heuristic. For the simplicity, we assume that there is only one product. The model can be easily transformed to the multi-product model using the deterministic and stochastic models presented in Sections 5 and 6.

The First-stage model is a network flow (NF) problem dealing with the flow of supplies through the entire network and with inventory management ( $h$  and  $q$  variables). The objective function is extended by adding the estimation of the transportation costs.

The second-stage problem is the vehicle routing problem (VRP).

**First-stage problem: NF – Network flow of supplies from the close supply node L to customers**

$\zeta_{s(c),c}$  is the estimated transportation cost per unit (tonne) of the product from satellite to the customer node (the estimate of  $c_{ij}$ )

$\zeta_{Ls}$  is the estimated transportation cost per unit (tonne) of the product from the close supply node L to satellite s

$\zeta_{FLv}$  is the estimated transportation cost per unit (tonne) of the product from the far supply node F to the close supply node L by vehicle of type v (the estimate of  $\varphi_v$ )

***Objective***

$$\text{Min} \sum_{t \in T} \left( \sum_{i \in N} \sum_{g \in G_i} f_g h_g^{t\theta} + \sum_{c \in C} q_c^{t\theta} * \zeta_{s(c),c} + \sum_{s \in S} \tilde{q}_s^{t\theta} * \zeta_{Ls} + \sum_{v \in V_F} \hat{q}_v^{t\theta} * \zeta_{FLv} \right. \\ \left. + \text{penalties due to supply shortages} \right)$$

**Second-stage problem**

The Second-stage model is a VRP used to generate routes that implement the flow of supplies defined in the first model. We decomposed the second-level problem VRP in two sub-problems:

- *The Second-stage Low-Level VRP- LL-VRP* dealing with the routes from the close supply node to the customers, and

- *The Second-stage High-Level VRP – HL-VRP* dealing with the routes between the far supply node and the close supply node.

Some of the  $q$  variables in this model are parameters taken from the solutions of the first-stage model. To distinguish them from the other  $q$  variables, they are shown in bold.

### **Second-stage problem: Low-Level VRP – LL-VRP**

#### **Objective**

$$\text{Min} \sum_{t \in T} \left( \omega_1 \sum_{s \in S} \sum_{k=1}^{n_s} \sum_{r=1}^{m_s} \sum_{i \in C_s^+} \sum_{\substack{j \in C_s^+ \\ i \neq j}} c_{ij} x_{ijks}^{rt\theta} + \omega_2 \sum_{k=1}^{n_L} \sum_{s \in S^+} \sum_{\substack{s' \in S^+ \\ s \neq s'}} c_{ss'} \tilde{x}_{ss'k}^{t\theta} \right)$$

### **Second-stage problem: High-Level VRP – HL-VRP**

#### **Objective**

$$\text{Min} \sum_{t \in T} \left( \sum_{v \in V_F} \sum_{k=1}^{n_{vF}} \varphi_v \hat{x}_{kv}^t \right)$$

## **8.3.2 Progressive Hedging Heuristic**

The *progressive hedging* algorithm, as initially proposed in [Rockafellar and Wets, 1991], deals with stochastic characteristics of a continuous problem by handling the set of scenarios that capture the uncertainty. Subsequent studies used the progressive hedging for solving IP and MIP problems. For example, in [Crainic et al, 2013] a progressive hedging-based meta-heuristics is developed for solving a stochastic network problem involving 0-1 decision variables. A two-stage stochastic programming formulation is proposed, where design decisions make up the first stage, while recourse decisions are made in the second stage to distribute the commodities according to the observed demands. A meta-heuristic framework inspired by the progressive hedging algorithm is proposed for solving the problem. Following this strategy, scenario decomposition is used to separate the stochastic problem following the possible outcomes or scenarios of the random event. Each scenario sub-problem then becomes a deterministic problem to be solved.

This section describes our progressive hedging heuristic that consists of decomposing the problem in the network flow problem and the vehicle routing problem, and solving them iteratively. The network flow problem is solved for the set of scenarios using the

straight progressive hedging algorithm and the VRP problems are solved as deterministic problems.

### **Progressive Hedging Heuristics - PHH**

*Estimate the transportation costs of one unit of the supply for each edge of the SCN based on the costs to transport full-truck load or 80% of the truck-load on the edge and back. Take the full price, or the percentage of it assuming that sometimes several delivery locations are visited on one route. The simple cost estimate could be*  
$$\text{Cost} = \text{edge\_length} * \text{fuel\_consumption} * \text{fuel\_cost} / \text{truck\_capacity}$$

*while (the transportation costs are not converging or time limit TL-PHH is not reached)*

*Solve the first-stage network flow FH problem with the straight progressive hedging algorithm (sPHA) aiming at the quantities of the supply that are moved from the far supply node to the close supply node to converge.*

*Use values of  $q$  variables as linking variables between the first-stage model and the second-stage model*

*Solve HL-VRP problem with the converged values for  $q$*

*For each scenario, solve the LL-VRP with the scenario dependent values for  $q$*

*Re-Estimate the transportation costs of one unit of the supply for each edge of the SCN based on the solutions of the VRP problems*

*end while*

The HL-VRP is scenario independent because the sPHA was run until the quantities transferred on F-L edge converged.

The LL-VRP problems are to be solved for each scenario, and the values used could be either the values achieved in the Step 1 of the sPHA or at the end of sPHA.

### **Straight Progressive Hedging Algorithm - sPHA**

Straight progressive hedging algorithm (sPHA) for the network flow problem based on the algorithms introduced in [Rockafeller and Wets, 1991], [Haugen et al, 2001]:

Step 1: For each scenario  $\theta$ , solve the Network Flow (NF) problem over the entire SCN. Use the deterministic solution approach.

Step 2: Find a mean of the solution values at each node in the scenario tree for the time index of the node for all scenarios at the node. These are used to update estimates for the dual variables associated with the (implicit) non-anticipativity constraints.

Step 3: For each scenario, solve the problem using objective function with additional terms that penalize anticipativity. See the objective function at the end of this section.

Step 4: If the quantity transferred between the far supply node F and the close supply node S converge “sufficiently” then stop. Otherwise, go to Step 2.

The objective function of the problems to be solved in Step 1 is:

$$\text{Min} \sum_{t \in T} \left( \sum_{i \in N} \sum_{g \in G_i} f_g h_g^{t\theta} + \sum_{c \in C} q_c^{t\theta} * \zeta_{s(c)c} + \sum_{s \in S} \tilde{q}_s^{t\theta} * \zeta_{Ls} + \sum_{v \in V_F} \hat{q}_v^{t\theta} * \zeta_{FLv} \right)$$

We introduce dual variables for the inventory levels  $h_g^t$  as  $\beta_g^{ht}$ , and for the quantities of supply that flow  $q_c^{t\theta}$ ,  $\tilde{q}_s^{t\theta}$ ,  $\hat{q}_v^{t\theta}$  as  $\beta_c^{qt}$ ,  $\tilde{\beta}_s^{qt}$ ,  $\hat{\beta}_v^{qt}$  respectively. The objective function to be used in Step 3 of the Straight Progressive Hedging algorithm could be:

$$\begin{aligned} \text{Min} \sum_{t \in T} \left( \sum_{i \in N} \sum_{g \in G_i} (f_g h_g^{t\theta} + \beta_g^{ht} h_g^{t\theta} + \frac{1}{2} \alpha (h_g^{t\theta} - \bar{h}_g^t)^2) + \sum_{c \in C} q_c^{t\theta} * \zeta_{s(c)c} + \beta_c^{qt} q_c^{t\theta} \right. \\ \left. + \frac{1}{2} \alpha (q_c^{t\theta} - \bar{q}_c^t)^2 + \sum_{s \in S} \tilde{q}_s^{t\theta} * \zeta_{Ls} + \tilde{\beta}_s^{qt} \tilde{q}_s^{t\theta} + \frac{1}{2} \alpha (\tilde{q}_s^{t\theta} - \bar{\tilde{q}}_s^t)^2 \right. \\ \left. + \sum_{v \in V_F} \hat{q}_v^{t\theta} * \zeta_{FLv} + \hat{\beta}_v^{qt} \hat{q}_v^{t\theta} + \frac{1}{2} \alpha (\hat{q}_v^{t\theta} - \bar{\hat{q}}_v^t)^2 \right) \end{aligned}$$

Where  $\alpha$  is a sPHA parameter.

The given objective function performs “hedging” at each level of the network. Alternatively, we could perform “hedging” only on the first level – F-L link (last term in the summation). This will model the situation when all flows and inventory levels are scenario-dependent, except the flow on the F-L link.

Note also that the approximate transportation costs are only defined later.

## 8.4 Traveling in Convoys

We propose one possible heuristic that could provide solutions that satisfy the requirement for the transportation using convoys of vehicles. In addition, there is a possibility that the solutions will provide variability to the routes and thus increase safety in hostile environments.

**Convoys Heuristic – Heuristic H5**

**Step 1:** Solve the problem with homogenous fleet of vehicles and only one supply product – the one that has the most fluctuating demand. Solve the problem on the entire extended two-echelon network using a general purpose MIP solver or heuristics H1 or H2.

**Step 2:** Solve the problem for all the products by increasing the number of vehicles and by using the same routes (as the solution in Step1). We can fix  $x$ ,  $y$  and  $w$  variables, add vehicle of each type to each convoy, release the capacity constraints on vehicles, and solve the problem for  $q$  and  $h$ . Vehicle capacity constraints will be resolved by adding as many vehicles as needed to satisfy them.

This approach might create a variability in the routing due to the high variability in demand.

Ref: RX-RP-53-5714  
Issue/Revision: 1/1  
Date: OCT. 30, 2013



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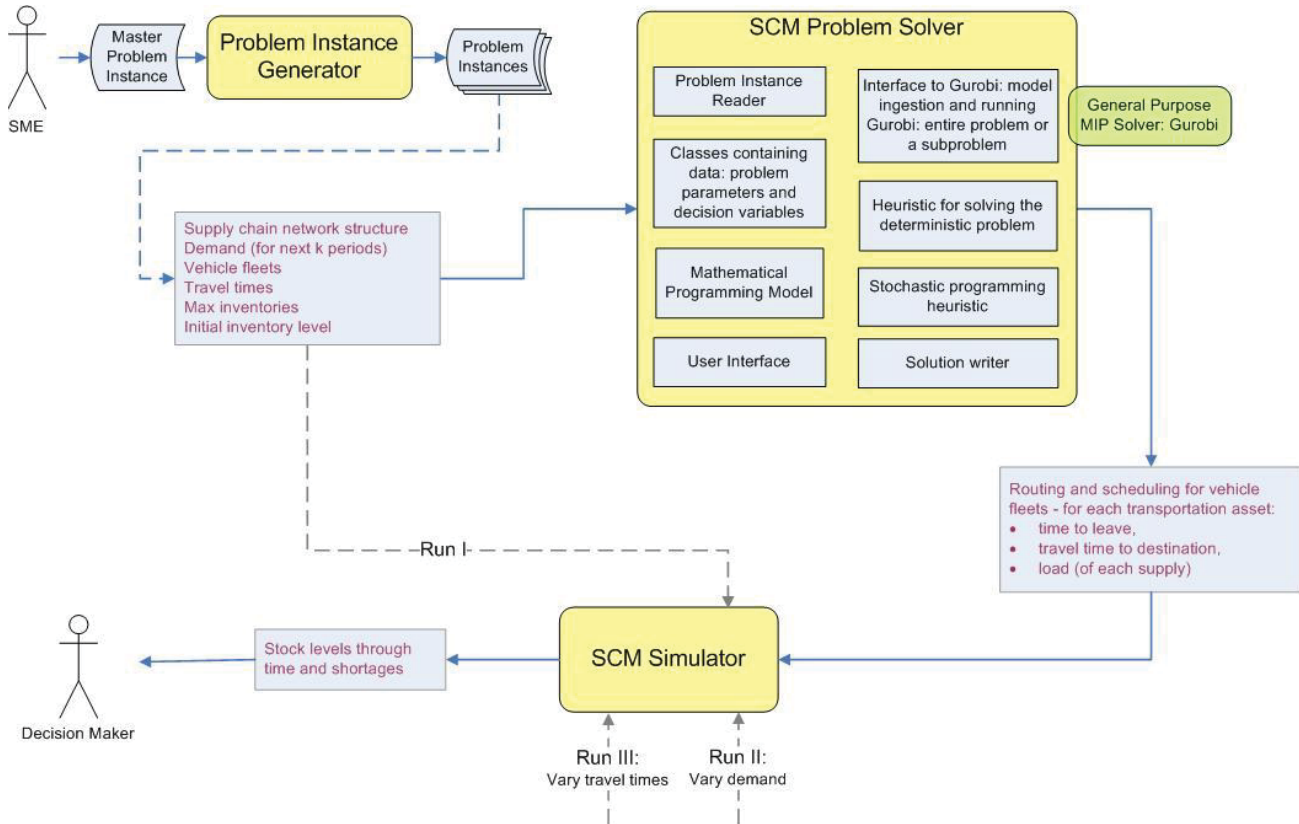
## **9 e2E-SCM Software Package**

### **9.1 High-Level Software Architecture**

The Software Package implemented consists of the three subsystems:

- The extended two-echelon supply chain management (e2E-SCM) optimization problem solver
  - An algorithm for solving the deterministic problem
  - An algorithm for solving the stochastic problem using explicit form
- The Supply Chain Management (SCM) simulator
- User interface (UI) – simple user interface to select the problem instance file, select some parameters of the problem and start the problem solving
- Problem instance generator

The Solver solves the inventory and distribution (transportation) problem. The problem instance generator generates the problem instances according to data provided by an analyst or Subject Matter Expert (SME) and stored in a master data file called Master Problem Instance. The simulator can be used for testing and comparing the solutions generated by solver. The simulator executes an SCM solution and generates the inventory level diagrams. The simulator is a relatively simple program without animations.



**Figure 9-1 High-Level Architecture for 2E-SCM Decision Support System**

The high-level process for using the software is shown in Figure 9-1. Additional details about the SCM solver and simulator follow.

The SCM Solver:

- Inputs for SCM solver:
  - Mixed-Integer Programming (MIP) formulation
  - Demand
  - Fleets of vehicles
  - Travel times
  - Maximal inventories
- Output:
  - Minimal inventory levels
  - Orders
  - Routes and schedules

- Interface
  - For changing deterministic and stochastic problem features
  - For changing constraints

The Supply Chain Network (SCN) simulator:

- Input
  - Routes and schedules for the fleet of vehicles
  - Demand
  - Travel times
  - Maximal inventories
- Output
  - Stock levels
  - Shortages
  - Other metrics (lead times, etc.)
- Interface
  - For changing demand
  - For changing travel times

## 9.2 Source Code

The software tools developed include:

- The deterministic model (Section 5) and the stochastic model (Section 6) and the solution approach (Section 7) are implemented in Java. The implementation consists of more than 8,300 lines of source code: 7627 (Model.java) + 440 (scm.data.reader.model) + 77 (scm.data.reader) + 213 (SCM.java) lines of source code.
- Problem Instance Generator with more than 700 lines of Python code.
- Discrete Event Simulator with more than 900 lines of Python code.

### 9.3 e2E-SCM Solver

The Optimization Problem solver component (subsystem) implements the model and the algorithm for solving the distribution and inventory management problems of the extended two-echelon supply chain management.

The SCM Problem Solver implements the designed mathematical programming formulation models of the e2E-SCM transportation (distribution) and inventory problem and solves the problem using the general-purpose MIP solver Gurobi.

The source code is written in Java using Gurobi's Java API, and consists of about 8000 lines of code. The Gurobi package `gurobi.jar` must be available to the solver in order to run. This file is included with the Gurobi solver package. Additionally, the freely-available GSON package must be installed.

The Solver implements all the classes needed for storing the problem instance parameters and decision variables. The class diagram showing some of the classes implemented is shown in Figure 9-2, Figure 9-3, and Figure 9-4.

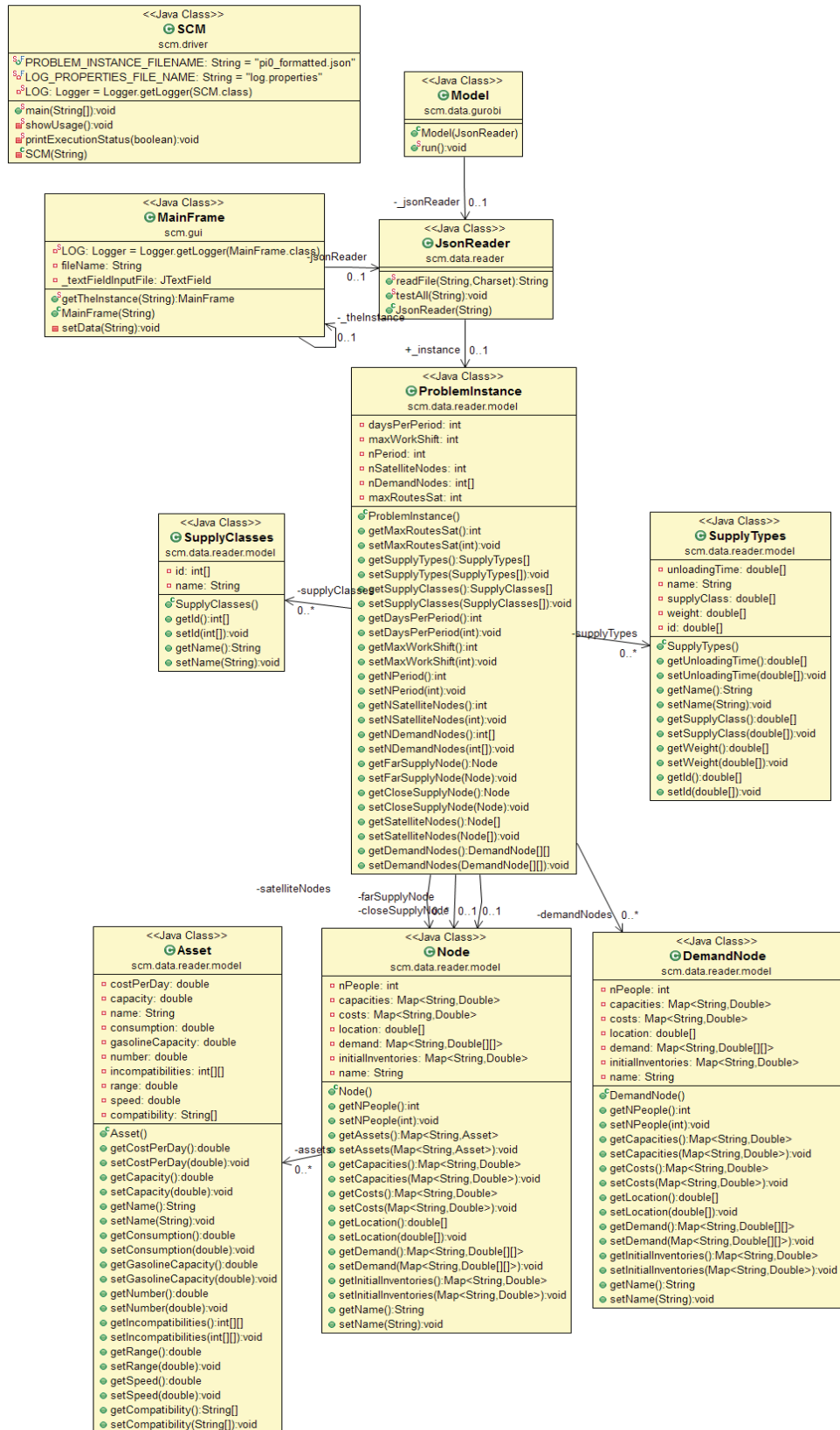


Figure 9-2 Initial Class diagram with some of the implemented classes.

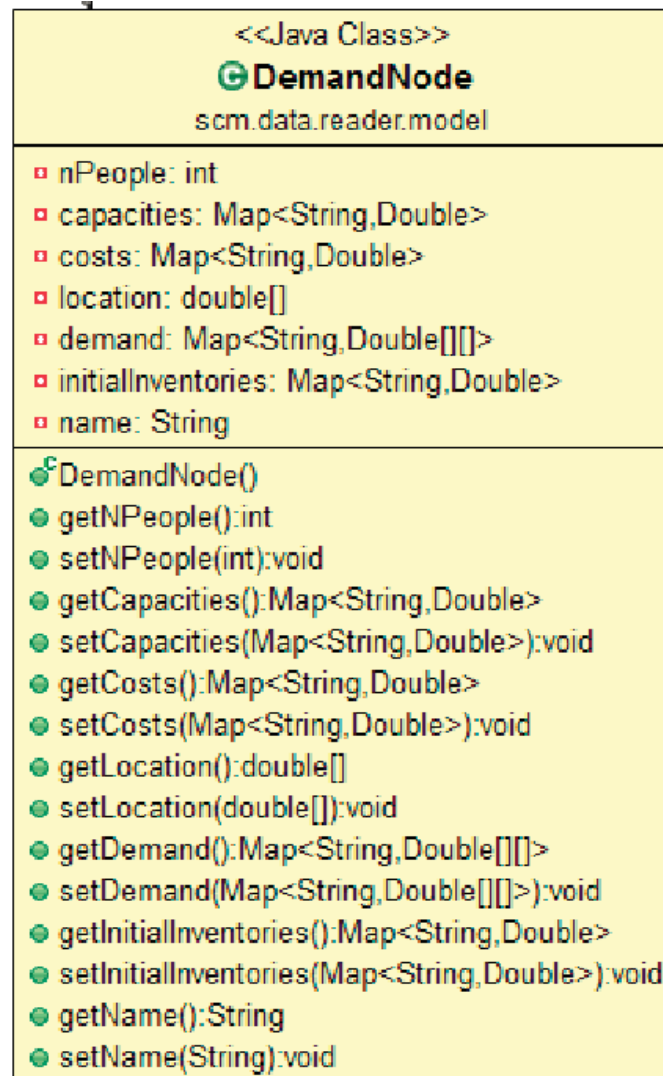


Figure 9-3 Class diagram for Demand Node

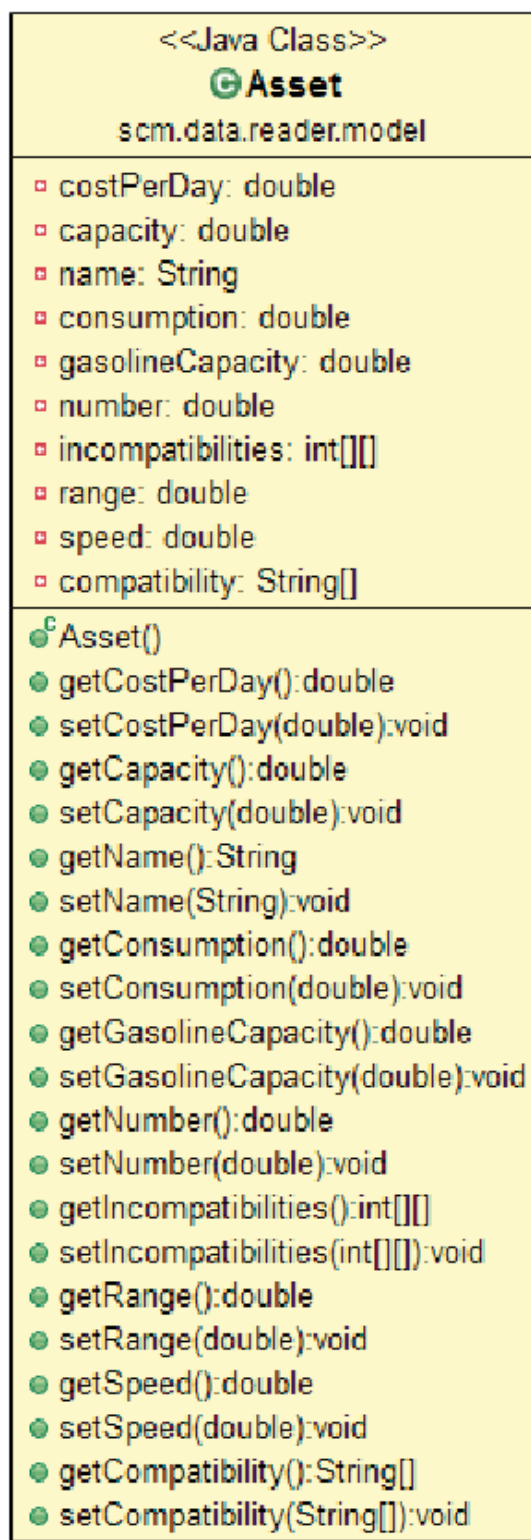
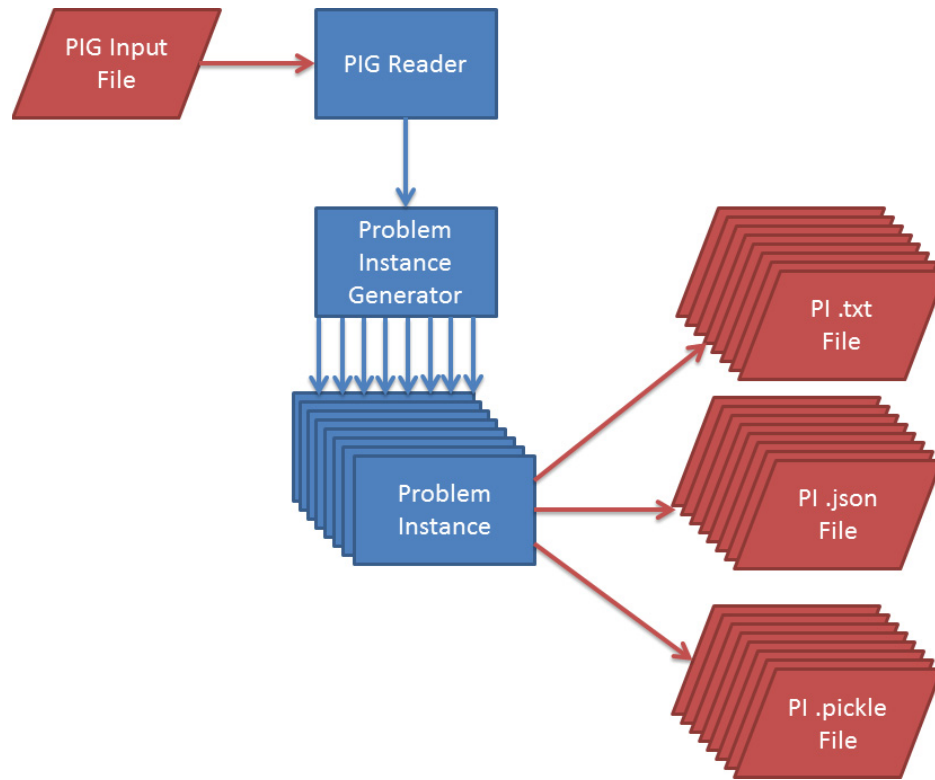


Figure 9-4 Class diagram for the transportation asset

## 9.4 Problem Instance Generator

The problem instance generator is designed to generate a number of different problem instances to use as inputs to the solver, in order to evaluate the effectiveness of the solver across a variety of specific situations within ranges of values. It is written in Python, and supports several output formats (including formats optimized for human and machine reading) for the problem instances generated. Additional output formats could easily be added.



**Figure 9-5 Problem Instance Generation Process**

The problem instance generation process is illustrated in Figure 9-5. It takes a single text file as input. This file is ingested by an instance of the PIGReader class. This class parses the text and constructs a ProblemInstanceGenerator object. This object is capable of continually generating new ProblemInstance objects. Parameters with ranges of acceptable values are selected using a quasi-random number generator with a uniform distribution between the given minimum and maximum values. In order to support stochastic problems, stochastic parameters can be generated randomly by having uniform distributions for both the mean and standard deviation of the stochastic parameter.



Each ProblemInstance object represents a problem instance that can be used as input to the solver, and for other purposes. A text file format is output in a format that is designed to be human-readable, using descriptive tags for the parameters. The problem instance is also output in a Javascript Serialized Object Notation (JSON) format. This format uses a hierarchical structure of key-value pairs and ordered lists. JSON is a well-recognized format that can easily be read using existing libraries in many languages, including Java and Python. Finally, a “pickle” file output is created. This is a Python-specific format that fully preserves an object in Python-native format. This format allows problem instances to be reloaded later in case additional formats or format changes are needed. Samples of the outputs are available in electronic format.

The Problem Instance Generator randomly generates the locations of all the nodes based on the distances provided by the user. The travel times are generated within the solver based on the length of the edge and the speed each vehicle type has stated in the input file.

The problem instance generator can be run from the command line using the `generate_problem_instances.py` script. This script takes the input file as a required option, and optional parameters to select the number of instances to generate, the destination folder for the problem instance files, and the base name for the output files.

The input file to the Problem Instance Generator is called the “master file” or “PIG input file”. It follows a specific format, a sample of which is available in electronic format. This file allows the user to set the demand per-person for each product type, and the demand per transport asset for each product type. It also allows the user to set various parameters that determine the structure of the supply chain, such as the number of satellite and demand nodes, the distances between nodes, the number of people at each node, and various properties of products and transportation assets. Many values in the file, such as demand, can be given as single values, or as ranges (i.e. “100-200”). When a range is given, each problem instance generated will use a quasi-random number generator to pick a value within that range according to a uniform probability distribution.

## 9.5 SCN Simulator

The SCN Simulator is being developed in order to provide a testing environment that can be used to evaluate the solutions produced by the solver under conditions that can be configured in a variety of ways. It also includes a simple visualization to review simulation results.

This subsection includes a high-level overview of the SCN Simulator package, some details on the implementation, a description of the inputs that it requires, and a basic explanation of the visualization component and its usage.

The SCN simulator:

- Input
  - Routes and schedules for the fleet of vehicles
  - Demand
  - Travel times
  - Maximal inventories
- Output
  - Stock levels
  - Shortages
  - Other metrics (lead times, etc.)
- Interface (optional)
  - For changing demand
  - For changing travel times

### **9.5.1 Overview**

The SCN simulator is being developed in Python using SimPy, an open source discrete event simulation package. Some of the simulator code is based on a simpler simulator that was developed under a previous project for DRDC-Valcartier [Open Source, DS-IAI Final Report, 2012].

In addition to working in standard Python, the entire package can also be run under Jython, a Python interpreter written for the Java Virtual Machine (JVM). Jython can be completely integrated in a Java software environment, allowing Jython code to call Java code, and Java code to call Jython code. The former requires no extra work, while the latter requires only minimal extra code that is well-explained in the Jython documentation.

The SCN simulator has been tested primarily under Jython 2.5.3 with SimPy 2.2. It has also been tested under Python 2.7.3 with SimPy 2.3. No differences should be expected between these two environments, and none have been found thus far.

### **9.5.2 Implementation**

The simulator models both the nodes in the supply chain network and the arcs that connect them. It operates one discrete time step at a time, simulating a time step for every object in the simulation.

The simulator allows an arbitrary number of types of items to be used. Items can be grouped into categories that can be stored and transported interchangeably. Separate categories of items will have separate capacities in warehouses and on vehicles that cannot be used to store or transport other categories of items. The simulation currently assumes that all nodes stock (or attempt to stock) all types of items.

The simulator creates one object for each node in the supply chain network. Three types of nodes are supported: Suppliers, Warehouses, and Consumers. Suppliers are modeled as having essentially limitless supplies of and capacity for items, but with a limited capacity of vehicle assets that can be used to fulfill orders from other nodes. Warehouses have limited capacities for the different item categories, and varying stock levels of every item type. Warehouses will consume some of their stock of every item type at every time step. Like Suppliers, Warehouses have a limited capacity of vehicle assets that can be used to fulfill orders. Consumers are nearly identical to Warehouses, but they do not have vehicle assets and cannot fulfill orders from other nodes.

Arcs are simulated by modeling the vehicles that are used to transport items between nodes. Every individual vehicle is modeled as a Transport object. Transport objects have a limited capacity to carry one or more item categories. Every delivery is modeled as an Order object, which describes a source node, one or more destination nodes with a defined route, and amounts of items to deliver to the destination(s). An Order will extract items from a Warehouse or Supplier, get a suitable Transport, load the Transport, and dispatch it to its destination(s). The Transport will travel to its first destination and unload contents into that Warehouse or Consumer. If it has more than one destination, it will then travel directly to that destination, again unloading contents. When all deliveries are complete, the Transport will return to its origin.

Many of the behaviors of the various objects in the simulator are configurable.

- Node capacities and initial levels
- Transport capacities
- Transports available at each node

In the current simulator many values can be configured as deterministic or as random variables. Moreover, the code is structured in such a way that any other function could be written to compute values during the simulation execution. Values that can be configured this way include:

- Travel times for each transport type on each edge
- Consumption of each item type at each node
- Fulfillment time at Warehouses and Suppliers (time between receiving Order and dispatching it)

Lastly, while the main purpose of the simulator is to demonstrate the behavior of the solution produced by the solver, it may be desirable for nodes in the simulator to make

their own decision on when and how much to order. This feature can be turned on or off, and the logic for sending orders is configurable by setting thresholds. More advanced logic could be added in the future.

### 9.5.3 Inputs

The simulator takes two input files. The first one fully describes the supply chain network and its behaviour, and the second one lists the orders that should be simulated. Both files are in YAML (“YAML Ain’t Markup Language”) format, which for these purposes is essentially an extension of the common JSON (Javascript Serialized Object Notation) data format that allows comments. The format is human-readable with a hierarchical structure.

There is a specific structure expected by the simulator for both files that can best be understood by following sample files that will be provided with the simulator.

## 9.6 User Interface

User Interface consists of:

- Tools for preparing the SCM problem instances and providing them to the solver (including the problem instance generator – Section 9.4)
- GUI of the solver that accepts the time limit for solving the problem and the name of the input file, as well as a command-line interface to execute the solver directly on an input file
- Tools for viewing the solution to the SCM problem (including the simulator – Section 9.5)

### 9.6.1 Solver GUI

The solver GUI consists of fields that allow the user to provide:

- The name of the SCM problem instance JSON file
- The time limit to be given to the solution algorithm. After this time limit expires, the solver will return the best solution found so far.

The GUI can be bypassed by calling with a command-line call such as:

```
java scm.driver.SCM pi.json
```

Note that the solver, as well as several dependent packages (Gurobi, log4j, swing, gson) must be in the classpath in order to run correctly.

Changing demand can be done by user by changing the master data file. It is also possible to edit the JSON file of the specific problem instance, but this file is more

challenging to edit compared with the master file, which is intended to be as concise as possible.

Changing the travel times cannot be done explicitly, but it can be done indirectly by changing the speed of a transportation asset type. The travel time is calculated in the solver based on the distance between the two nodes and based on the speed of the transportation asset type. Reducing the speed in the master problem instance file will increase the travel time of the transportation asset. Also, note that the cost of traversing the edge depends right now on the fuel consumption of the transportation asset type. This way by changing the fuel consumption (in the master problem instance file) one can influence the cost of the route.

The majority of the changes that can be done by the user are the changes that can be done on the Master Problem Instance file. Please see the file sample in Appendix E.

The changes to the constraints of the problem by a user could be done by changing the constants in the Java file `StochasticModel.java`:

`PRODUCT_COMPATIBILITY_CONSTRAINTS` – excluding or including constraints (7a), (8), (9), (10), (12a), (14a), and (16a)

`WAIT_NEXT_TIME_PERIOD_TO_LEAVE_CONSTRAINTS` - excluding or including constraints (4), (5), and (6)

`REFUELING_CONSTRAINTS` - excluding or including constraints (17) and (19)

## 9.6.2 Solution Viewer

For each SCM solution the software package prints:

- Objective function value for the best solution found (before the time limit given to Gurobi has expired)
- Transportation costs
  - The transportation costs for each echelon, including:
    - Total distance travelled
    - Total transportation costs
    - Weighted transportation cost (portion of the objective function)
- Inventory
  - The changes in inventories at each echelon for each period, including
    - The inventory level
    - The quantities arrived at the node
    - The quantities left the node (for each satellite node and for the close supply node)

- Demand (for each customer node)
- Unmet demand (for each customer node)
- o The inventory costs at each echelon , including:
  - Total distance travelled
  - Total transportation costs
  - Weighted transportation cost (portion of the objective function)
- o The inventory costs at each node for each time period
- Shortages
  - o The total cost of unmet demand
  - o Summary of the output results and the solver running:
- Routes for each used transportation asset in each time period
  - o Total distance travelled
  - o The number of stops (locations) in the route
  - o The quantities delivered to each location in the route

### 9.6.3 Solution Review

The SCN solver produces a solution file that can be used for subsequent interactive review and to generate order data for the SCN simulator. Scripts in Python were developed to assist with this. These scripts can also be used interactively with other Python tools for tasks such as plotting inventories at different nodes in order to visualize the evolution of the supply chain for a given solution. Similarly, the outputs of the simulation can be visualized interactively using Python.

## 9.7 Using the Software

The following would be steps a user would take to execute the full process depicted in Figure 9-1:

### 9.7.1 Generate Problem Instances

First, prepare a master problem instance file. Starting from an existing sample is strongly recommended in order to ensure that the proper format is used. To generate problem instances from this file, run:

```
python generate_problem_instances.py -n 1 --path sample --  
basename sample_ sample.pig
```

This will create one problem instance file called “sample\_0” in a folder called “sample”. If more than one problem instance is desired, switch the “-n” parameter in the command. Additional instances with \_1, \_2, etc., will be generated.

### 9.7.2 Run Solver

The next step is to run the solver. This can be done with the following command:

```
java scm.driver.SCM
```

The folder where the SCM solver is located must be in the Java classpath. Alternatively, this command can be run from the SCM solver folder.

The SCM solver will open a GUI. There is a file dialog box that can be used to select the problem instance file to use. Navigate to the .json version of the problem instance generated in the previous step, and select it. The maximum time for Gurobi to run its optimization can be chosen here as well. Other GUI options can be changed if desired, but defaults will suffice in most circumstances.

Clicking the Run button will execute the solver. Updates from the solver code and from Gurobi will be printed to the console as the execution progresses and eventually finishes. The time to find a solution is highly dependent on various input parameters as well as the capability of the system. In the end, a solution text file will be produced in the Gurobi .sol format. This can be examined in a text editor or parsed electronically to examine results.

### 9.7.3 Results Analysis

Solution data can be loaded in to Python using the SCMSolution.py module. Assuming the solution file generated is called solution.sol, the following commands in Python will load the results:

```
from SCMSolution import SCMSolution  
solution = SCMSolution('solution.sol')  
result = solution.result
```

This will produce a data structure with multi-dimensional arrays representing the data. Each solution variable has its own array. The result structure is a Python dictionary where the keys are the variable names, which match the mathematical programming formulation and the variable names in the SCM solver. Results can be plotted, sliced, and otherwise analyzed using Python’s various tools which are beyond the scope of this document.

The solution file could be imported into other environments as well – it is purely a text file and is not specific to any environment.

## 9.7.4 Simulator

The simulator takes two input files. The first file fully describes the supply chain network. The second file lists all of the "orders" to be executed. These orders describe how much of various products should be delivered to a particular node at a given time from a given source. It is highly recommended to start from these examples in order to build simulation inputs.

The orders data can be built from the solution data structure described in the previous subsection. The SCMSolution class can return a data structure that is compatible with the simulator's input. The following steps could be used to produce an orders file called orders.json:

```
from SCMSolution import SCMSolution
import json
solution = SCMSolution('solution.sol')
orders = solution.get_orders()
json.dump(orders, file('orders.json'))
```

There is no automated way to create the supply chain network description for the simulator. This JSON file must be manually edited to match the inputs used in the problem instance. Following the sample file will be necessary. In most cases, the structure of the input file will not need to be changed – only the values in certain fields. Fields have been given descriptive names to ease understanding.

Once the input files are prepared, if the simulator is in the Python path (or if the user is in the simulator folder), simply run:

```
python SupplyChainSimulator.py
```

The code in SupplyChainSimulator.py can be edited to provide a filename where simulation outputs will be stored. Alternatively, the simulator can be used interactively in Python, and results can be analyzed afterwards. This analysis is beyond the scope of this document.



## 10 Computational Study

This section describes the preliminary computational study. The main results are provided in Section 10.4 that reports on the solutions of three deterministic problem instances, and provide the outcome of solving one stochastic problem instance with three scenarios (at end of the section).

### 10.1 Experimental Setup

Computational experiments were conducted on a system with the following basic specifications:

- RedHat Linux x64 version 4.4.7.3
- 4x Intel(R) Xeon(R) CPU E5-1607 0 @ 3.00GHz
- 32GB RAM
- Java Development Kit version 1.7.0\_25
- Gurobi version 5.5.0

As detailed in Section 9.2, the solver code is developed in Java using the Gurobi Java API. Java and Gurobi are configured following standard installation guides provided. No specific configuration changes were made to either one in order to support this computational study.

Gurobi is able to leverage the multi-core capabilities of the system internally. No configuration on the part of the developer or user is required. Gurobi prints information to the console noting how many cores it is using.

The principal limitation faced during this study was that only a single Gurobi license was available. As a result, it was not possible to run multiple test cases simultaneously, nor was it possible to test any changes to the code while a test case was processing.

### 10.2 Initial Experiments

We started the experimental study with simple problem instances. We give some results to provide insights in the problem complexity and the times needed for solving larger real-world problems.

The initial problem instance has the following SCN structure:

- 1 satellite node
- 1 demand node
- 1000 people at the demand node

There is 1 product and 1 transportation asset type at each echelon:

- Cargo plane at the far supply node
- Trucks at the close supply node and the satellites

There are 15 time periods, each time period is 7 days long.

Initial inventories are 40, 0, 0, 0 at F, L, satellite, customer, respectively.

The demand at a node is equal to the number of people at the node multiplied by the demand per person given in the master problem instance file.

Optimal solution is found in 2.948 seconds.

The constraints not included in the model are: the product compatibility constraints, the re-fueling constraints, and 'waiting next period before leaving' constraints. The last group of constraints not being present allows the supplies to leave to the next echelon in the same time period when they arrived. In the same way, the supply that arrives to the consumer node can be used in the same time period when arrived. The advantages and disadvantages of these constraints being present or excluded are listed at the end of Section 5.2.

```

Optimize a model with 7725 rows, 5685 columns and 14501 nonzeros
Found heuristic solution: objective 1040
Presolve removed 6116 rows and 4360 columns
Presolve time: 0.03s
Presolved: 1609 rows, 1325 columns, 4485 nonzeros
Variable types: 684 continuous, 641 integer (641 binary)

Root relaxation: objective 1.224000e+02, 927 iterations, 0.01 seconds

   Nodes      |   Current Node   |   Objective Bounds      |   Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

    0       0   122.40000    0   43 1040.00000   122.40000   88.2%    -    0s
H    0       0           555.0000000   122.40000   77.9%    -    0s
    0       0   253.59050    0  255   555.00000   253.59050   54.3%    -    0s
H    0       0           447.0000000   253.59050   43.3%    -    0s
    0       0   259.09802    0  370   447.00000   259.09802   42.0%    -    0s
H    0       0           414.0000000   259.09802   37.4%    -    0s
    0       0   265.78151    0  455   414.00000   265.78151   35.8%    -    0s
    0       0   267.67619    0  454   414.00000   267.67619   35.3%    -    0s
    0       0   269.86933    0  299   414.00000   269.86933   34.8%    -    0s
    0       0   271.49998    0  379   414.00000   271.49998   34.4%    -    0s
    0       0   271.49998    0  305   414.00000   271.49998   34.4%    -    0s
    0       4   271.49998    0  305   414.00000   271.49998   34.4%    -    0s
H   107     76           412.0000000   309.62052   24.8%   56.8    0s
H   291    100           402.0000000   332.24512   17.4%   43.8    1s
H   321     99           399.0000000   340.46111   14.7%   41.7    1s
H   560    141           396.0000000   346.18699   12.6%   41.3    1s

Cutting planes:
  Gomory: 10
  Implied bound: 22
  Flow cover: 83
  Flow path: 89
  Network: 9

Explored 1427 nodes (51696 simplex iterations) in 2.81 seconds
Thread count was 4 (of 4 available processors)

Optimal solution found (tolerance 1.00e-04)
Best objective 3.960000000000e+02, best bound 3.960000000000e+02, gap 0.0%

```

Figure 10-1 Gurobi log of the run and the optimal solution value. A deterministic model has been solved.

Inventories at customers)

Inventory at Customer Nodes:

Scenario	Time period	Satellite	Customer	INV	QNTY_Arr	demand	unmet_demand
0	0	0	0	0.00	5.00	2.50	0.00
0	1	0	0	2.50	0.00	2.50	0.00
0	2	0	0	0.00	5.00	2.50	0.00
0	3	0	0	2.50	0.00	2.50	0.00
0	4	0	0	0.00	2.50	2.50	0.00
0	5	0	0	0.00	5.00	2.50	0.00
0	6	0	0	2.50	0.00	2.50	0.00
0	7	0	0	0.00	5.00	2.50	0.00
0	8	0	0	2.50	0.00	2.50	0.00
0	9	0	0	0.00	5.00	2.50	0.00
0	10	0	0	2.50	0.00	2.50	0.00
0	11	0	0	0.00	5.00	2.50	0.00
0	12	0	0	2.50	0.00	2.50	0.00
0	13	0	0	0.00	5.00	2.50	0.00
0	14	0	0	2.50	0.00	2.50	0.00

Inventory at Satellite Nodes:

Scenario	Time period	Satellite	INV	QNTY_Arr	QNTY_Left
0	0	0	0.00	10.00	5.00
0	1	0	5.00	0.00	0.00
0	2	0	5.00	0.00	5.00
0	3	0	0.00	0.00	0.00
0	4	0	0.00	7.50	2.50
0	5	0	5.00	0.00	5.00
0	6	0	0.00	0.00	0.00
0	7	0	0.00	10.00	5.00
0	8	0	5.00	0.00	0.00
0	9	0	5.00	0.00	5.00
0	10	0	0.00	0.00	0.00
0	11	0	0.00	10.00	5.00
0	12	0	5.00	0.00	0.00
0	13	0	5.00	0.00	5.00
0	14	0	0.00	0.00	0.00

Inventory at Close Supply Node:

Scenario	Time period	INV	QNTY_Arr	QNTY_Left
0	0	0.00	10.00	10.00
0	1	0.00	0.00	0.00
0	2	0.00	0.00	0.00
0	3	0.00	0.00	0.00
0	4	0.00	7.50	7.50
0	5	0.00	0.00	0.00
0	6	0.00	0.00	0.00
0	7	0.00	10.00	10.00
0	8	0.00	0.00	0.00
0	9	0.00	0.00	0.00
0	10	0.00	0.00	0.00
0	11	0.00	10.00	10.00
0	12	0.00	0.00	0.00
0	13	0.00	0.00	0.00
0	14	0.00	0.00	0.00

**Figure 10-2 Inventory level changes at the echelons. . A deterministic problem has been solved.**

For a similar problem instance – with 1 satellite node, 1 demand node, 1 product, 1 transportation asset type, 15 time periods – and 10,000 people at the demand node, an optimal solution is found in 0.334 seconds.

For the problem instance with 1 satellite node, 3 demand nodes, and 1,000 people at each demand node, the optimal solution is found in 112.048 seconds. There were 1 product, 1 transportation asset type, and 15 time periods.

For the problem instance with 3 satellite nodes, 3 demand nodes per satellite node, 1,000 people at each demand node, 1 product, 1 transportation asset type and 15 time periods, an optimal solution is found in 286.372 seconds – almost 5 minutes.

This problem instance used the same number of demand nodes at each satellite node, but it is possible to have different numbers of demand nodes at each satellite node. This can be configured through the master file.

In Figure 10-2, the columns are as follows: INV is the inventory level of the product in each time period for each node, QNTY\_Arr is the amount of supplies arrived at that node at that time period, and QNT\_Left is the amount of supplies left the node at this time period to be delivered to the nodes further. The amount of supplies left cannot be smaller than the amount in inventories (when the constraints (4), (5) and (6) are included in the model).

The ‘weighted’ costs (inventory or transportation) are the costs multiplied by the weight used in the objective function of the problem. The weights used in our experiments are (0.2, 0.2, 0.2, 0.2, 10,000), where the first four weights are the multipliers for the transportation costs between F and L, the transportation costs between L and the satellite nodes, the transportation costs between the satellite nodes and the demand nodes, and the inventory costs, respectively. The last weight is the multiplier for the unmet demand. The reasoning behind these values is given in Section 5.2. We started with (0.2, 0.2, 0.2, 0.2, 0.2), the sum of all weights was equal to 1, but after few runs increased the value of the last weight to 10,000 in order to avoid unmet demand.

The optimality gap is the gap between the best MIP solution found and the best lower bound found.

## 10.3 Solution Details

The SCM solver prints the solutions details on the computer terminal. It contains information as presented in Section 9.6.2. The summary of the solution is printed at the end. An example is shown below.

-----  
Transportation costs, inventory costs, and unmet demand  
-----

Transport (distances, full cost, weighted costs)

To Close Supply Node: 600.0, 900000.0, 180000.0

To Satellite Nodes: 2397.642041673445, 119882.10208367204, 23976.420416734458

To Customer Nodes: 4001.799595182156, 200089.97975910763, 40017.995951821445  
-----

Inventories (amounts, full costs, weighted costs)  
-----

Close Supply Node: 558.0000000009896, 5580.0000000009898, 1116.0000000019793

Satellite Nodes: 624.000000000411, 6240.000000004109, 1248.000000000822

Customer Nodes: 1795.9999999989623, 35919.999999979336, 7183.999999995849  
-----

Unmet Demand (amount, full cost, weighted cost)  
-----

Customer Nodes: 107.9999999935449, 107.9999999935449, 1079999.999993545  
-----

Input file name: /home/dathomson/pig/sDRel\_1p/sDRel\_1p\_min\_1.json

Output file name: /home/dathomson/solutions/sDRel\_1p\_min\_1.sol

Number of scenarios: 1

Number of time periods: 15

Number of satellites: 3

Number of demand nodes: 6

Number of products: 1  
-----

Execution Time: 244.905 seconds Time limit set to Gurobi by user: 240.0 seconds

Objective value: 1333542.4163620998; Gap: 2.5134635096738753%; Runtime: 240.97220611572266  
-----

Number of constraints : 673635

Number of variables: 238335; Num of binary variables 187740; Num of integer variables 214740  
-----

The transportation cost is the product of the distance traveled, the asset's fuel consumption rate, and the cost of gasoline per dollar. This produces a cost value in dollars.

The weighted cost is, *e.g.*, the transportation cost multiplied by the weight, *e.g.*, 0.2. For the appropriate values of the weights see Section 5.2. The weights actually used when solving the specific problem instance, are printed on the terminal as part of the program log.

## 10.4 Computational Study Results

The comparison study compares the solutions to the deterministic problem with different levels of the demand.

Table 1 presents:

- an optimal solution or the best solution found for expected (average) demand
- an optimal solution or the best solution found for the max demand
- an optimal solution or the best solution found for the min demand

The following experimental study is done by imposing a time limit of 5 minutes to Gurobi.

Disaster relief problem instances are randomly generated based on the earthquake data provided by the SME, John Conrad.

The deterministic model does not include the product compatibility constraints and the one route duration constraints (dealing with the re-fueling issues). The other constraints assuring that the routes finish within a time period have been included. Also the constraints that assure that the supply is in the store before it is being used are included. The constraints that have not been considered have been commented out, in this process of testing the model, source code, and the solver.

The problem instances are characterized by the following:

The SCN structure is: 3 satellite nodes, 6 demand nodes per each satellite (18 demand nodes in total), 1,000 people per demand node = 18,000 people to serve in these 18 demand nodes.

Number of time periods is 15 = 15 weeks ~ 4 months.

Number of products is 1.

Objective function weights: (0.2, 0.2, 0.2, 0.2, 10,000).

See discussion on the reasonable values for weights in Section 5.2.



Table 10-1 Deterministic Problem Instances

Problem instance	CPU time	Opt. soln	Gap	Best solution value	Transportation cost total and weighted (weight, cost)	Distances travelled per each echelon and total	Transp. costs per each echelon and total	Inventory costs total and weighted (weight, cost)	Inventory amounts per each echelon	Inventory costs per each echelon	Unmet demand
sDReL_1p_avg_1 d = 6 inv at F 4000 # t.assets (3, 30, 10)	5 min	No	1.87%	<b>3,973,590.11</b>	0.2 717,974.09	1,800.00 7,392.73 10,404.68 <b>19,597.41</b>	2,700,000.00 369,636.48 520,233.95 <b>3,589,870.43</b>	0.2 15,616.02	1,462.00 1,581.99 2,382.01 <b>5,426.00</b>	14,620.00 15,819.90 47,640.20 <b>78,080.10</b>	20.0%
sDReL_1p_max_1 d = 10 inv at F 4000 # t.assets (3, 30, 10)	5 min	No	2.47%	<b>6,662,191.07</b>	0.2 1,137,949.51	3,000.00 10,989.19 12,805.76 <b>26,794.95</b>	4,500,000.00 549,459.63 640,287.94 <b>5,689,747.57</b>	0.2 23,341.56	2,589.83 2,520.17 3,280.39 <b>8,390.39</b>	25,898.30 25,201.70 65,607.80 <b>116,707.80</b>	20.4%
sDReL_1p_min_1 d = 2 inv at F 4000 # t.assets (3, 30, 10)	5 min	No	2.51%	<b>1,333,542.42</b>	0.2 243,994.42	600.00 2,397.64 4,001.80 <b>6,999.44</b>	900,000.00 119,882.10 200,089.98 <b>1,219,972.08</b>	0.2 9,548.00	558.00 624.00 1,796.00 <b>2,978.00</b>	5,580.00 6,240.00 35,920.00 <b>47,740.00</b>	20.0%

Number of variables (total, binary, integer) for the problems in the previous tables are 238,335; 187,740; and 214,740, respectively. The number of constraints is 673,305.

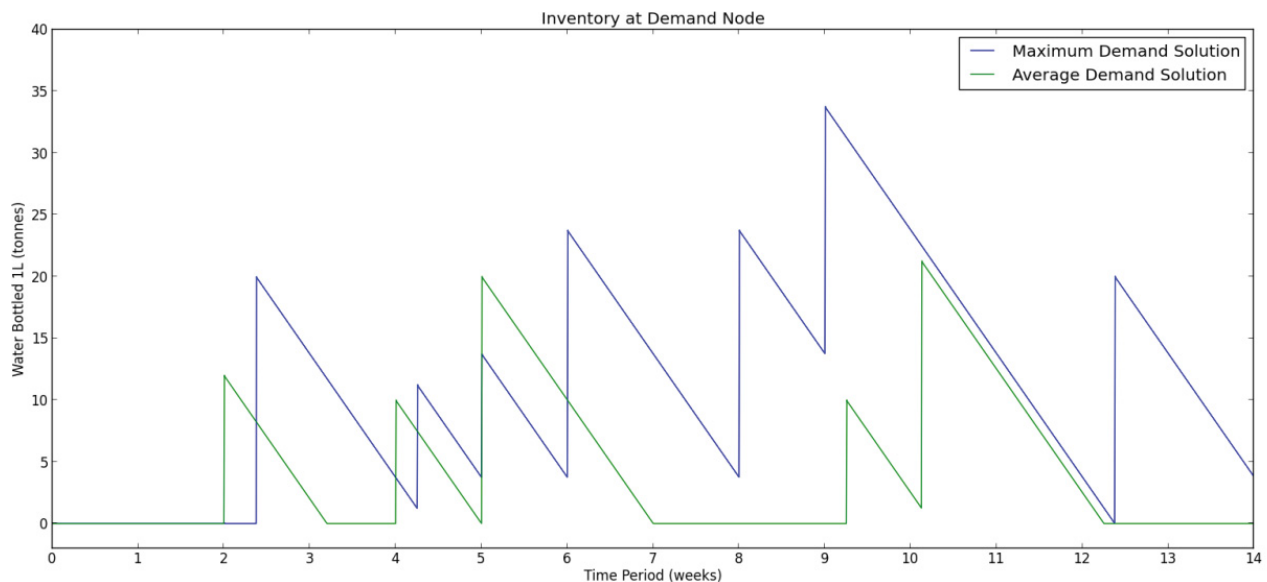
The unmet demand rates of 20% are the result of the initial condition where all inventory is located at the far supply node, with no inventory anywhere else. As a result, demand in the first three periods can never be met. When looking at 15 time periods, this means 20% is the minimum value for unmet demand.



We have succeeded to design and develop an SCM simulator using a Discrete Event Simulation (DES) open source tool SimPy. The simulator can execute solutions generated by SCM problem solver, and record the changes in the inventory levels.

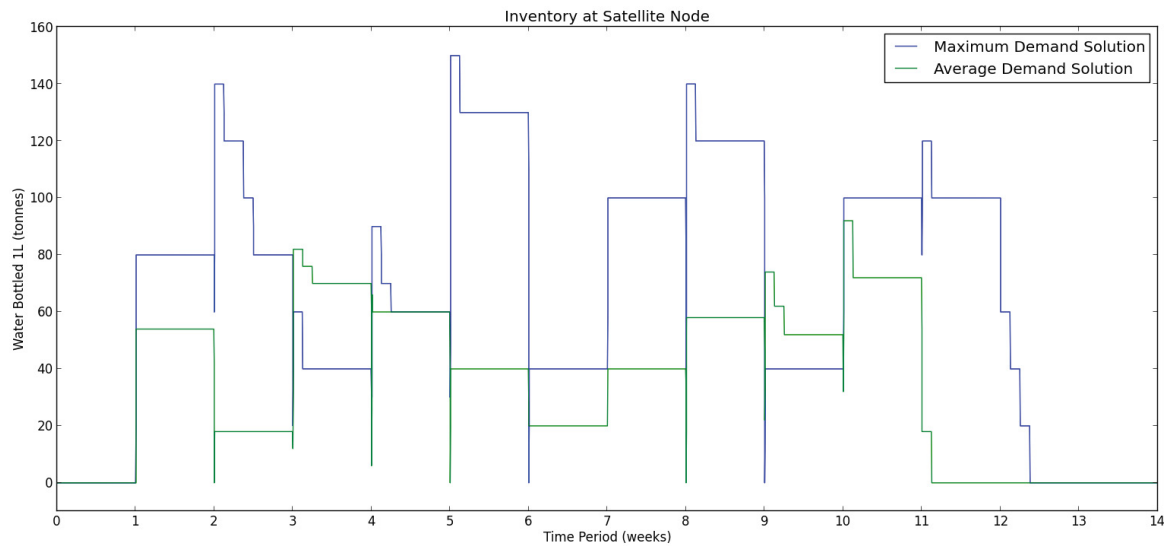
The SCM simulator uses the supply chain structure together with the delivery amounts and routes generated by the solver. To validate the outputs of our computational study, the solutions generated from the average and maximum demand cases were used. In both cases, the consumption levels at maximum demand were used.

The inventories observed at a demand node, a satellite node and the close supply node are shown in Figure 10-3, Figure 10-3, Figure 10-5 respectively. At the demand node, there are shortages resulting in unmet demand through time periods 0, 1, 3, 7, 8, 9, 12, and 13 when using the average demand solution. On the other hand, there are only shortages during periods 0, 1, and 2 when using the maximum demand solution, and these are purely the result of the initial inventory being 0 throughout the supply chain, meaning that it takes at least 2 full time periods before any inventory can possibly arrive at the demand nodes.

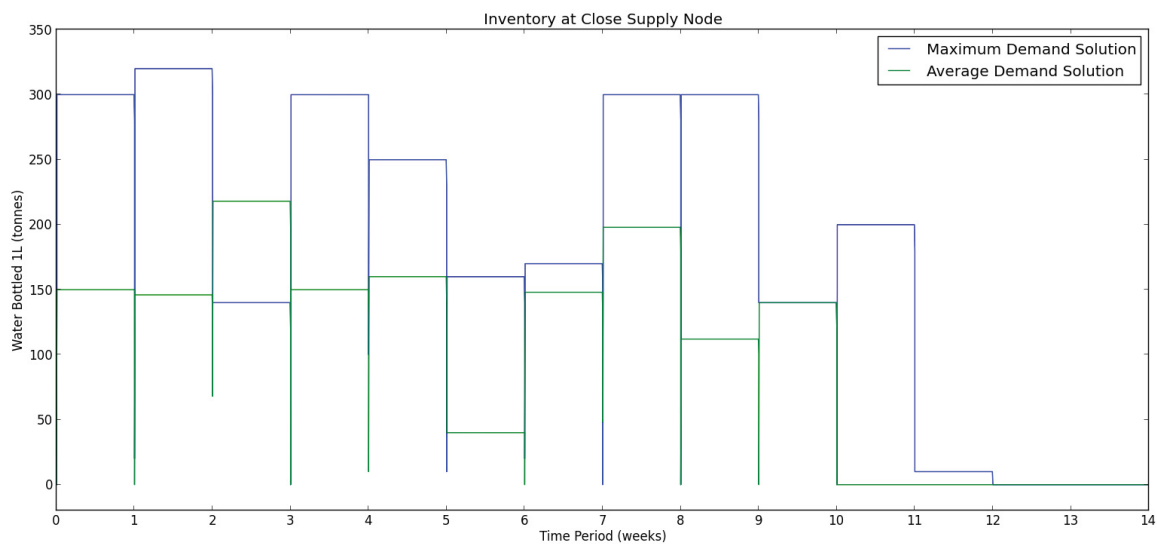


**Figure 10-3 Inventory at Demand Node**

The impacts of the demand can be seen throughout the supply chain, as both the satellite and close supply nodes maintain higher inventories (and thus higher costs) through the time period studied.



**Figure 10-4 Inventory at Satellite Node**



**Figure 10-5 Inventory at Close Supply Node**

Unfortunately, we have not been able to perform sensitivity analysis either by varying demands nor by varying travel times and to evaluate solution robustness. Right now the weights for the objective functions are hard coded in the SCM solver.

Also, at this stage, the demands are equal for each time period. This is true for the stochastic problem instances as well in this preliminary computational study. The recommendation is that more comprehensive computational study needs to be done, but extensive consultation between SMEs, decision makers, and DRDC TAs is needed to set all the parameters and generate appropriate problem instances thus also limiting the variations to a reasonable number, so that the study is feasible and extensive enough.

In terms of the computational results, we have not been able to compare the results generated with the current distribution policies used in DND.

We have been able to implement the explicit form of the stochastic model, and we succeeded running the problem instance similar to those we reported on in the table. The stochastic problem has 3 scenarios. The probabilities of the scenarios are 0.33. The number of variables was 805545, 619740, 668340 for total, binary, and integer, respectively. The number of constraints was 2,162,265. The stochastic problem was run for 100 minutes and generated the feasible solution with the optimality gap of 0.057%. Unfortunately, the results showed that further testing and potentially debugging is needed for the stochastic problem solver.

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## 11 Conclusion and Future Work

### 11.1 Conclusion

We developed a model and a solver for the extended two-echelon supply chain management problem. We started from the deterministic version of the problem since it is needed before the stochastic version of the problem can be tackled. The aim was also to have a workable model and a solution approach at the end of the Task Authorization #1 of the project. This has been achieved.

The accomplishments of this Task Authorization are as follows:

- Started from a disaster relief scenario because:
  - This scenario is slightly simpler than the military scenario
  - All disaster relief scenario characteristics and relevant problem characteristics are present in a military scenario
  - Literature is available on supply chain management in emergency management and disaster relief
- Performed a literature survey of humanitarian disaster relief operations and two-echelon SCM problem models, including the following characteristics: multi-product, heterogeneous fleets, inventory capacities, uncertainty, and risks.
- Created a mathematical programming formulation of the deterministic version of the Supply Chain Management (SCM) problem – integrating the distribution (transportation) and inventory management problems. Characteristics include: extended two-echelon, multi-products, multi-periods, heterogeneous fleets, heterogeneous stores, inventory capacities.
- We modeled the uncertainty to demand using scenarios
- Proposed solution approaches for deterministic and stochastic versions of the problem
- Briefly analyzed general-purpose MIP solvers, including open source tools
- Selected a commercial general purpose MIP solver for solving the problem
- Implemented the mathematical programming formulation in Gurobi API and a solver for the problem
- Created an SCM simulator for testing the solutions generated by the solver
- Created a problem instance generator
- Created tools exploring the SCM solutions used for debugging and testing
- Performed an initial computational study

- Refined the IP model

The novelties of the proposed models compared to the published research include:

- Integration of inventory management in a multi-echelon vehicle routing problem
- Multiple realistic problem characteristics – multiple products, heterogeneous fleet, multiple periods, compatibility constraints – that have been considered separately in various multi-echelon VRP or SCM models, but never all of these sources of complexity simultaneously in the same problem.
- Stochastic demand with combination with all characteristics and features listed in the previous bullet.
- Extension of the two-echelon network with an additional level – made of a unidirectional link between a far supply node and a close supply node – that must be handled in a particular way, when compared to the rest of the network, due to a different time scale.
- Multi-echelon routing problems often involve different types of vehicles, but fleets are homogeneous by level; our problem is different because it considers heterogeneous fleets at each level.

## **11.2 Future Work**

### **11.2.1 Potential Tasks**

Potential directions for future research include:

1. Perform a more comprehensive computational study for both deterministic and stochastic problems. The recommendation is to do this in close collaboration with SME(s), DRDC TA, and, potentially, decision makers, in order to properly select between the numerous combinations of the SCN structures, travel times, time periods, scenarios, constraints, parameters, etc. The consultations need to be relatively detailed and extensive, in order to set all the parameters and generate appropriate problem instances, designing the experimental study that is extensive enough, but also feasible and doable based on setting the limitations the variations to a reasonable level.
2. Incorporate uncertainty in travel times. Initially, stochastic travel times could be considered between F and L
3. Implement the iterative local search and alternate heuristic.
4. Implement the progressive hedging stochastic programming approach
5. Modeling of risks and safety issues for the latest level of the SCN: security risks at certain edges, or unavailability of certain edges or portions the transportation network.. Model some arcs in a transportation network that are less safe than the others, i.e., some parts of the transportation network are damaged or unavailable.

This can be assumed to be a relatively static situation – random and without intelligent adversaries.

6. Order priorities: urgent, essential, routine, replenishment
7. Graphical User Interface (GUI) enhancement:
  - The number of product types. Or, the list of product types to choose from (out of the product types listed in the input file).
  - Removing/ inserting the constraints that allow the supplies to arrive and leave a satellite node (or the close supply node) within the same time period. Right now, this can be changed by changing the constants in the SCM Solver.
  - Removing/ inserting the constraints that deal with re-fueling. Right now, this can be changed by changing the constants in the SCM Solver.
  - Removing/ inserting the constraints that deal with product compatibilities. Right now, this can be changed by changing the constants in the SCM Solver.
  - The number of time periods (up to a maximum listed in the input file). The GUI will allow user to select the maximal number of time periods or to decide to run the solver just for the first  $k$  number of periods.
  - The number of scenarios (for the stochastic case). Or, the list of scenarios to choose from out of the scenarios listed in the input file.
  - Safety issues/ Risks associated with an edge
  - Consumption within the Distribution Network: The information on whether the products are also consumed at the Close Supply Node and at the Satellite Nodes.
  - Changes in the cost of inventories: this will allow increasing the cost of inventories in the satellite nodes and in the demand nodes relative to the Close Supply Node. The costs would be multiples of the cost at the Close Supply Node. The costs of the inventories for the Close Supply Node will still be taken from the input file containing the problem instance, whereas the other inventory costs will be overwritten with the values generated from the user input.
  - Risks: Security risks along certain edges. In this situation, the costs in the objective function will not be a monetary cost, but the level of security risk. The risk could be proportional to the edge length, and it could be calculated by multiplying the edge length by the security risk level. For example, the security risk level should be an integer between 1 and 10.
8. Travel times – being more stochastic going from left to right in the SCN
9. Safety/risk – risk being higher when going from left to right in the SCN
10. Introduce the notion that some supplies could get lost in transport and warehouses (see Section A5)

11. Refine modeling of stochastic demand, consider dependencies.
12. Consider modeling other more complex safety issues. This could include a method for considering the safety aspect by allowing multiple arcs with different travel times and different costs that can include expected losses (this is one way to keep it linear). Another aspects is modeling traveling in convoys.

### **11.2.2 Other Relevant Problems**

- Plan unpredictable routes so that the safety of the transportation convoy is not compromised. It will be true in both chaotic and hostile environments. This problem seems to be related to the money collecting truck routing problem, where the trucks also need to follow routes that are not easy to predict.
- The vehicle routing problem would be more prevailing in urban than in rural environments. In an urban environment, the city network can first be converted to a graph whose nodes are the pickup and destination nodes. Each edge in the graph represents the shortest or the safest path. The edge weight may change through time. We can also decide to have multiple edges with separate weights between two nodes to model different criteria.
- Safety and presence of an intelligent planning adversary.



## **A Military Scenario**

The majority of the data and information provided by the team SME John Conrad is based on John's experiences as DND Logistician and on references [Cdn ADO], [Conrad, 2009], and [DND, 1998]

### **A1 Distances, Travel Times, and Frequency of Transport**

Approximate desirable distances between APOD and FOBs are the distances that could be travelled back and forth without re-fueling and within 5-10 hours. In military operations, it would be desirable that the trip back and forth could be done during the night.

The most common distances are 40-50km, and sometimes up to 100km. Maximum distance was 300-350km.

Travel times APOD-FOB1-FOB2...-APOD: 3-4 days. These trips would repeat approximately every 7 days.

Travel times can deviate very much from average. By how much it can deviate from average?

Frequency of transport between APOD and FOBs should be daily. The only reason why it is sometimes less frequent is to increase transport safety in hostile environments.

JTF = can consist of three echelons known as A2, A1 and F, in which case the frequency of transport between these echelons would be as follows:

- Daily transport between FOBs and A2 echelon
- Hourly transport between A2 and A1 echelons
- From moment to moment transport between A1 and F echelons

A basic load of 3 days of consumable supplies is spread over F, A1, and A2 echelons, and 15 days of clothes, boots, and spare parts.

### **A2 Transportation Assets**

Transportation assets used in CF include:

- Existing fleet of HLVWs (Heavy Logistic Vehicle Wheeled) vehicles
- New fleet of AHSVSs (Armoured Heavy Support Vehicle System) vehicles
- LAVs (Light Armoured Vehicle)

**Table A-1 CF Logistics Vehicle Capacities**

Vehicle type	Capacity
HLVW	16 tonnes
AHSVS	23 tonnes
LAV	

According to the latest missions, in-theatre CF commonly uses around 120 – 150 vehicles, in total.

Newest additions to the CF fleet of vehicles are 82 AHSVSs out of which 25 are cargo vehicles with material handling crane. Approximately 70 vehicles are used for logistics activities with the following roles:

- 5 vehicles are recovery vehicles
- 5 vehicles are for bulk water transport
- 10 vehicles are for bulk POLs transport
- 40 vehicles have Palletized Loading System (PLS) with container handling unit
- 12 vehicles are heavy tank transporters

Out of total 120-150 vehicles in-theatre, Task Force uses approximately 70-80 vehicles: 45 LAVs, 12 HLVW or AHSVS vehicles, and 15-20 vehicles equipped for protecting the supply delivery convoys.

**NSE Fleet of Vehicles:** NSE (Tactical Logistics) uses around 30-50 vehicles, mostly AHSVSs and HLVWs:

- 20-25 for replenishment,
- 3 recovery vehicles,
- few communication vehicles,
- few maintenance vehicles, etc.

Out of 20-25 vehicles used for replenishment, the roles are the following:

- 5 vehicles are for bulk water transport
- 8-10 vehicles (HLVW or AHSVS) are for bulk POL transport. Some of these fuel vehicles would have large fuel pods, and some would be configured to carry packaged POL products: oil, grease, brake fluids, other miscellaneous products and distillates that vehicles require.
- 6 vehicles for water and rations (food)
- 5 vehicles for general and technical stores
- 6 vehicles for equipment and the military operation consumable materiel

Note that with the palletized loading system (PLS) using easily droppable containers, the vehicles used for rations and other types of supplies are somewhat interchangeable. The only exception is the truck for the military operation consumable materiel that has additional safety features like fire extinguishers and static straps to neutralize electro static charge.

**Fuel consumption** example: Canadian LAV III fuel container capacity is 300 litres, and the vehicle can travel with this amount of fuel approximately 450 km.

**Loading and unloading times:** “During the cold war, a ‘cross load’ would take approximately 30 minutes. In the last several decades, the use of PLS Sea containers has resulted in much faster loading /unloading times.” We can assume that 5-15 minutes are needed for container unload, while a container load may require 20-40 minutes? “However, pumping fuel takes 30 minutes to an hour.” (John Conrad).

- Vehicle fleets:
  - Fleet F2 consists of 20-25 vehicles. There are three types of vehicles:
    - POL trucks, capacity 23t
    - Regular trucks, capacity 23t
    - Specialized truck (refrigerators), capacity 23tFor these 20-25 vehicles, their roles are the following:
    - 8 vehicles are for bulk POL transport
    - 6 vehicles are regular trucks
    - 6 vehicles are equipped with refrigerators
  - Fleet F3 at each satellite node  $s_i$  there are 7 vehicles. There are three types of vehicles:
    - POL trucks, capacity 2.5t
    - Regular trucks, capacity 2.5t
    - Specialized truck (refrigerators), capacity 2.5 tWe can assume the following roles:
    - 3 vehicles (Heavy Logistic Vehicle Wheeled (HLVW) or Armoured Heavy Support Vehicle System (AHSVS)) are for bulk POL transport
    - 2 vehicles are regular trucks
    - 3 vehicles are equipped with refrigerators

### **A3 Supplies Variety and its Origin**

There are 36,000 ‘line items’ – types of supplies/ goods/ materiel.

Their grouping and location of origin are given in Section 1.

### **A4 Demand**

Most of the demand during military operations is for POLs, water, food, and military operation consumable materiel. Compared to this, transportation of people seems to be negligible (as per John Conrad).

Stated logistics fleet size can support around 2,500 soldiers during a military operation and needs around 450 logistic staff (including the staff not involved in inventory and transportation activities).

**Table A-2 Demand Through History**

<b>Time</b>	<b>Soldiers</b>	<b>NSE Staff</b>	<b>Supply Demand per day</b>	<b>Number of Loads/Trips per day (20t per vehicle)</b>
WWI	20,000	5,000-7,000 <sup>1</sup>	150 tonnes	Approximately 7 trips with AHSVS, or 10 trips with HLVW
WWII	18,000-20,000	9,000 <sup>2</sup>	650 tonnes	Approximately 29 loads of AHSVS or 41 loads of HLVW
Recent	2,500 <sup>3</sup>	450 <sup>4</sup>	20 – 40 tonnes	4-5 trucks (due to vehicle specialization and load incompatibilities)

Daily demand per person is as follows:

- 10-15 litres of water
- 3 meals

One FOB supports 1 Infantry Company which is around 150 soldiers, and there are usually 3 FOBs.

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<sup>1</sup> “The sustainment system was different in the Great War having just collapsed/failed at the Battle of the Somme in 1916. I am extrapolating the number of support soldiers for the Canadian Division of 1917-18 with this figure.” John Conrad

<sup>2</sup> The Canadian Division in World War II required 9,000 army-level British support soldiers for logistic support. The source for this data is Canadian Forces College Lecture, C/JC/CPT 303/LE-30. The army of 1939 became mechanized which increased the demand for logistics support and its support base.

<sup>3</sup> “The modern battlefield is partially defined by the absence of machinery, long and widely dispersed operations. The deployment of a battle group or task force would be almost a negligible mention in a WW II context but in the 21<sup>st</sup> Century a Task force of 2,500 with a modern mechanized battle group has analogous strategic impact to a full up division of the World Wars” John Conrad

<sup>4</sup> The NSE grew from 281 in 2006 (with no contractors on the APOD) to more than 450 and 150 contractors in 2008.

Fuel is needed for moving and air-conditioning. Each Infantry Company has around 15 LAV vehicles.

Note that out of the fuel that arrives at APOD, around 20% is spent at APOD and 80% beyond APOD.

In a transportation convoy traveling from APOD to a FOB, 4-5 replenishment vehicles would consist of:

- 2 bulk fuel vehicles: If all LAVs in a company were completely empty, the company would need (15 LAVs x 300 litres=4500 litres) for fuel replenishment. If there are four companies, this would come to around 18 tonnes of fuel. For transporting this, two HLVW 16 tonne trucks would be required.
- 3 PLS vehicles:
  - 1-2 for water, food, and technical stores
  - 1-2 for the military operation consumable materiel: “The consumption rate is much more variable depending on what it going on tactically. I would say two trucks would be assigned.”

Demand/ consumption:

- Total demand load in a military scenario is 20-40 t per day. This is transported in 4-5 trucks. This assumes 2,500 soldiers and 450 National Support Element (NSE) staff (at L)
- Fuel is needed for transportation and air-conditioning. Each Infantry Company has around 15 Light Armoured Vehicle (LAV) vehicles. For each satellite s\_i and beyond, there could be 15 LAV vehicles, and thus consumption per day or for several days could be 15 x 300 litres (full tank) = 4,500 litres. This would come to around 18 tonnes of fuel for three or four satellites.

## **A5 RFID and Lost Goods**

It would be possible to introduce percentage of supplies lost in transport and warehouses as follows:

- Without RFID, this percentage could be set to 10% or to a random number between 10% and 15%.
- With partial introduction of RFID, this percentage could be set to 5%, or a random percentage between 3% and 10%.
- With fully introduced RFID system, this percentage can be 3%, or a random number between 0% and 5%.

## **B More on Military Transportation Assets**

*Prepared by John Conrad*

**Question 6.** What is the fleet size (number of vehicles) at APOD? What is the capacity of each vehicle for each load type? In kilograms and in cubic meters, or is there any other load unit being used?

It is understood that we need a fleet size number to work with for our scenario. I have had a couple of thoughts as I work on a relevant number for our use. In Adaptive Dispersed Operations (ADO), the military envisions less force-on-force set-piece fights and smaller, section-level combat engagements involving irregular, highly adaptive and technologically sophisticated adversaries. This requires some thinking with respect to how the basic load of combat supply in a unit is dispersed forward of the A2 echelon. In ADO, perhaps it no longer makes sense to send supply through a single point (Unit Quartermaster)?

Furthermore, a number of factors affect the size of the Standard Military Pattern (SMP)<sup>i</sup> fleet in a theatre of operations (or at the APOD as you have asked). The geography, strength of the economy in the country of operation, the availability of commercial trucking, weather and tactical environment.<sup>ii</sup> Therefore the fleet size at the APOD depends largely on the use of commercial trucking, availability of air assets (helicopters) and APODs for aerial delivery, rail and water ways for moving material. In our work we need to remember that ground fleets are not the only or necessarily even the best way of providing logistics in an ADO environment.

In terms of vehicle fleet size of a land task force at an APOD, let us assume for our purposes that the theatre is extremely austere and the ability to contract civilian transport is very limited. As such we have two vehicles in inventory at present --the Armoured Heavy Support Vehicle System (AHSVS) and the Heavy Logistic Vehicle Wheeled (HLVW). The AHSVS was introduced to the fleet after my tour in Kandahar with HLVW.

### **The HLVW**

The 1,200 HLVW (16 tonne lift capacity) is the far older of these two logistics haulers. It was introduced to service in the summer of 1990 and by Afghanistan War was entering the twilight of its useful life. The new vehicle was acquired between the fall of 2007 and March of 2008 and was delivered direct to Afghanistan. The HLVW was originally procured as a 10 tonne capacity vehicle. Beginning in 1995, a program to retrofit the HLVW 10 ton vehicle to carry 16 tonne with a PLS was started. This program ran as money was available up until about year 2000. Although it is recognized that the HLVW is no longer fit for operational duty overseas, it is still the CF's intent to use them domestically. I can see the truck being used for a while yet.

*The in-service fleets of Land logistics vehicles, designed in the Cold War for contiguous, linear operations, are not suited to adaptive dispersed operations, in which non-contiguous, non-linear operations have been and are expected to continue as the norm. In-service fleets are simply classed by their payload capability as light, medium, heavy and super-heavy vehicles. Delivered to the Land Force in the 1980s and 1990s, in-service logistics vehicles face obsolescence and rust out.*

See photo I found below at right for an example of the HLVW 16 tonne with sea container PLS system. This type of delivery tool was used all the time by my NSE to deliver to forward FOBs.



*Lessons learned in the Balkans and Afghanistan highlighted deficiencies in payload, functionality, mobility, protection and firepower. **The worldwide introduction of intermodal containers has allowed pre-configured loads to be seamlessly trans-shipped across any mode of transportation including by road, rail, sea and air, without the requirement to repackage loads.***<sup>iii</sup> Canadian and allied forces use intermodal containers for strategic cargo

Armoured Heavy Support Vehicle System (AHSVS)  
DaimlerChrysler (Mercedes-Benz) Actros



*shipment. While most allied forces have container capable logistics vehicles for tactical level shipment, Canada does (or at least did not until recently) not. Because our in-service logistics vehicles are not designed for such loads, significant effort is expended to manhandle and re-package loads from containers to cargo trucks for tactical level shipment.*

*Additionally, because in-service trucks have less payload capacity, more trucks and crew are potentially required to move the same load over the same time period. Because of their weight and size, intermodal containers require logistics vehicles with appropriate payloads as well as load handling functionality. The payload and functionality of logistics vehicles must therefore be optimized for transporting intermodal containers. (from DND public webpage)*

### **The AHSVS (23 Tonne)**

The new AHSVS (pictured above) will be employed in the conduct of combat service support and combat tasks. This includes towing the M777 Lightweight Towed Howitzer and its basic ammunition load, transporting general cargo, such as humanitarian supplies, recovering all vehicles up to the LAVIII, transporting tanks and delivering bulk water and fuel.<sup>iv</sup> The new truck has a payload of 23 tonne.



**Question 7, 8 and 9.**

7. *What is the fleet size (number of vehicles) at main base in Canada used by CANOSCOM? What is the capacity of each vehicle for each load type? In kilograms and in cubic meters, or is there any other load unit being used?*
8. *What is the fleet size (number of vehicles) at other bases in Canada used by CANOSCOM? What is the capacity of each vehicle for each load type? In kilograms and in cubic meters, or is there any other load unit being used?*
9. *How many combat troops can be supported by this fleet size stated in 8.? What is the number of NSE staff involved in inventory/ warehouse and transportation tasks? Please provide one or two examples.*

It is easier to determine our fleet numbers and also our daily demand numbers by working backwards from the fighting unit. The further from tactical danger, the more mixing of commercial pattern vehicles can and will occur. CFJOSG's (formerly CANOSCOM) vehicle fleet in Canada is very limited in terms of SMP vehicles. The units of CFJOSG will never have a capacity problem to move material at home in Canada. These domestic units have access to commercial carriers.

I submit we use a 2,500 soldier Task Force with a battle group as its main combat element. This Task Force is very similar to the model used for part of the NATO effort in the Balkans (where the Canadians deployed two battle groups for a time and then returned to just one deployed under SFOR) and more recently, Task Force Afghanistan. The Task Force would have:

- a. A slice of National Command Headquarters Element, staff officers, signalers and support soldiers for this HQ function.
- b. The task force's main fighting arm is an infantry BG with an artillery battery, engineer troop and additional combat support and service support elements. Based on the organization of an infantry unit, there could be as many as 45 LAVs (14 per company), 12 echelon (A1 and A2 vehicles for replenishment purposes like the HLVW or AHSVS described above) and another 15 to 20 vehicles for the attached combat arms in the Battle group. Call it **77 to 80 vehicles** 15 of which would be logistics or echelon vehicles like the AHSVS or HLVW.
- c. A National Support Element (NSE) or logistics battalion. This logistics unit would have a good **30 to 50 vehicles** of the AHSVS and HLVW type and capacity for the NSE in theatre. The NSE would have smaller, soft skinned vehicles at the APOD that do not leave the defended location such as medium to light transport trucks and material handling equipment.
- d. Total task Force Vehicles—**120 to 150**.

The number and capacity of trucks at the **APOD** is not really required as the APOD does not move material forward. The NSE does this. With a fleet size of 50 vehicles, the NSE could comfortably sustain a Canadian Task Force of 2,500 soldiers.

**Question 10.** *How often there are vehicles traveling between APOD and each FOB? How many FOBs are there (provide an example)?*

Vehicles travel between the APOD, any forward NSE location and the FOBs of the battle group daily in accordance with our doctrine and replenishment cycle. Tonnage per day required by our task force? This is

a difficult one to estimate. In the First World War, the needs of an infantry Division of 15,000 to 20,000 soldiers equated to 150 tonnes per day. A World War II Infantry Division would consume 12,000 tonnes per day. Put another way, a WW I Division's maintenance load would need 6.5 (7) loads of our brand new AHSVS truck. The WW II Division would need 522 truckloads of AHSVS material. I would estimate the needs of the modern, information-driven infantry battle group would be closer to 25,000 to 40,000 tonnes per day—particularly in the ADO context. The greatest bulk of this is liquid—POL and water. Very few spare parts and other items clog up the supply chain. The bulk of the material that flows daily is the combat supplies: fuel, water, ammunition and rations. Personnel being moved by the supply system—either going on leave or personnel replacements for casualties are small, almost negligible in number compared with the need for diesel and water. Only 20 to 25 % of the force can go on leave at any one time.

Question 12. Average travel times vary from the average when impacted by severe weather (sandstorm) or enemy interference (IED ambush).

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<sup>i</sup> SMP vehicles are specially engineered army trucks versus civilian pattern trucks. Civilian pattern vehicles are heavily used to do military hauling in Canada.

<sup>ii</sup> In most cases we can reasonably assume the APOD is secure/tactically safe. If it is not it soon becomes the number one priority for the Canadian Commander on the ground to make it safe.

<sup>iii</sup> We may wish to address containerization of loads in our modeling. Container shipment is prized among logisticians as it eliminates the need for double and triple handling of material as it moves across different modes of transport. PLS delivery of containers occurred often at the FOBs in Kandahar province.

<sup>iv</sup> In total, the project will provide the CF with 82 vehicles. The AHSVS will be broken down into four main variants:

- \* 25 cargo with material handling crane variant vehicles (of which eight will be gun tractors for the M777);
- \* Five recovery variant vehicles;
- \* 12 heavy tank transporter tractor variant vehicles; and
- \* 40 palletized loading system (PLS) with container handling unit variant vehicles (10 petroleum, oils, and lubricants variants and five water variants).

## **C Capacities and/or Stock Levels for in Theatre Modeling**

*Prepared by John Conrad*

The various holding capacities or stocking levels at different echelons of the military supply chain can vary with any number of factors. For example, proximity to Canada or to industry re-supplies possibilities. A large distance and hence long re-supply time would suggest to military planners that they should indent for larger holding capacity. As mentioned earlier in our work, Canadian Army doctrine sets the holding levels at a fighting unit as a Basic Load of 3 days' worth of combat supply and a Maintenance Load of 1 day of supply on the logistics trucks of the supporting logistics unit or NSE. Theatre stock levels have been set on replenishment times dictated by sea and air lines of communication but typically have been set at 30 to 45 day of supply in theatre.

NATO and potentially any combined joint formation in which a Canadian Task Force would operate will also establish holding levels at various fobs/ units. These levels need to be considered in the planning of our missions and resupply procedures.

### **Petroleum, Oil and Lubricants (POL)**

#### **Historical Data for 1 Canadian Mechanized Brigade Group in the mid-1990s**

I dug up these historical figures from my own notes and material from my ten years in the logistics battalion of the Army of the West—1 Service Battalion. The fuel carrying capacity of each combat arm is recorded here. This equates to the unit's own Basic Load of POL. I thought it might be a useful frame of reference. Keep in mind that a service battalion at this time provided daily replenishment to 16 (sixteen) units in the Brigade. In our NSE modelling we are talking about the logistics unit sustaining one unit—an infantry battle group.

### **Basic Load holdings of Various Arms of 1 CMBG**

Artillery (M109 105 Howitzers, self propelled):	3300 litres
Armor (Leopard I Tanks):	11,300 litres
Combat Engineers:	7,040 litres
Mechanized Infantry (PPCLI):	4,800 litres (times 2 units)
Light Infantry PPCLI (non mechanized):	2,700 litres
Service Battalion (logistics):	4,800 litres
Field Ambulance:	2,700 litres
Aviation:	2,700 litres (non aviation fuel)

### **Maintenance load for 1 CMBG**

Standard Military Pattern 5 Ton POD trucks*:	111,000 litres
Plus three, 16,000 commercial style litre tankers:	48,000 litres

**Total Maintenance load of Fuel:** 160,000 litres

\*The old dual POD 5 Ton trucks carried 4,546 litres of fuel per truck. They were replaced by the HLVW 10 ton truck beginning in the late summer of 1990.

Finally, I kept these replenishment figures for 1 PPCLI, a mechanized infantry unit that was preparing to deploy to Kandahar. These stats are more in line with our model as we were an NSE supporting one composite battle group. This was in Wainwright Alberta and as we know the location of the operations affects amounts. Still a very useful model.

### *POL Delivery to 1PPCLI. September 2005—Kandahar Workups*

14 Sep	3687 litres
16 Sep	1530 litres
17 Sep	4073 litres
18 Sep	2923 litres
19 Sep	2903 litres
20 Sep	2433 litres
21 Sep	3264 litres
22 Sep	3495 litres

The average daily delivery to the unit was 3,038.5 litres. Afghanistan fuel demand was higher given the heat, and terrain. We could use these Canada-based figures for Basic Load of 9,000 to 10,000 litres and a daily Maintenance Load of 3,100 litres and add a 10 % factor to indent for the Afghanistan factor or we could use these numbers as they exist. Theatre level POL holdings could be established in the 30 day realm (which would be in the 100,000 litre range). Note that at the APOD in Kandahar, we drew our fuel from a Coalition Fuel point. The capacity of this theatre level asset was set in the millions of litres.

## **Ammunition**

The location of the Canadian units (like Afghanistan) has an impact on holding levels. Afghanistan is surrounded by other countries (i.e., no sea access); thus, ammunition has to be flown in. The preferred method for shipping ammunition is by sea. Because of the nature of explosives and numerous international civil aviation regulations and over flight permission requirements of sovereign countries much preplanning is required to move ammunition by air ( it took 45 days to receive ammunition from Canada in Kandahar under normal replenishment circumstances). Doctrine states that a combat unit will hold a Basic Load that equates to 3 days of supply of ammunition and be topped up with a daily Maintenance Load of one day of Supply by the logistics unit (NSE).

I need to do more work on ammunition to get into the right natures of ammunition that would be fast movers in 1,200-soldier battle group or FOB. Ammo like the artillery 155 shell, 5.56 rifle ammunition for the service rifle and the 25mm chain gun ammunition would make the most sense for our model. An infantry company or FOB basic load of ammunition will equate to approximately three sea containers worth of ammunition natures. I will sort out how much makes sense for our purposes.

## **Water**

The projection of water in the military supply chain pertains to potable water delivered by water trailer or more typically, commercially bottled water on pallets. The other factor in play with this combat supply is the establishment of potable (drinkable) water points by our combat engineers. These can be established by tying in to artesian wells/existing infrastructure and employing a military Reverse Osmosis Water Processing Unit (ROWPU) if required to make even foul or radioactive contaminated water perfectly drinkable. The establishment of water points in FOBs and forward battle group locations will ease the burden of delivery of potable/bottled water by the NSE.

In the case of bottled water holding capacity, let us assume there are no (0) water points at the various FOBs of a 1,200-soldier battle group. All water must be delivered. A pallet will hold normally 50 to 60 cases of bottled water and each case has 24 bottles. If we assume for our modelling purposes, 55 cases on a pallet this equates to 1320 bottles on a pallet.

The metric for bottle/litres per day is 8 bottles under normal conditions and more like 10 for a unit engaged in close combat. Therefore the unit would need between 9,600 to 12,000 litres of bottled water per day or 8 to 10 pallets of bottled water per day. A Basic Load of water for a 1,200 soldier battle group is therefore 24 to 30 pallets of water. The Maintenance Load would be 8 to 10 Pallets and the Theatre APOD warehouse would store a large amount depending on the responsiveness of the contractors/national supply chain. We could assume 10 to 30 days worth of bottled water where 10 days would equate to 1,000 pallets of bottled water.

## Rations

The delivery of rations to a FOB or a 1,200-soldier battle group, similar to water delivery will depend on how much feeding of fresh rations (meat, dairy, fruit, vegetables) or hard rations (dried, ready-meal packs that are non perishable and prepared by the individual soldier—see below). In some cases local procurement of fresh rations will ease the burden of delivery on the tactical supply chain, however the procurement of local produce depends very heavily on the mission location, standards, availability of suppliers, etc.

Let us assume for our model that the theatre location is austere and all meals are hard rations. We were forced to use hard rations exclusively for 3 PPCLI in southern Afghanistan in 2002 and again for 1 PPCLI in 2006 so it is not too much of a stretch. There are 12 Meals Ready to Eat (MREs)\* in a case and 48 cases on a pallet--therefore, 576 meals per pallet. In a 1,200-soldier battle group the Basic Load would be: 3 meals per day X 3 days of supply or 10,800 meals or 19 pallets of rations. The Maintenance Load of rations would be a third of this at 3,600 meals or 7 pallets of rations. The theatre holdings at the warehouse would almost certainly be 30 days worth of these bulky non-perishable items—somewhere in the region of 190 to 200 pallets of hard rations. Rations are much easier for logisticians to predict because unlike the mercurial, highly variable ammunition consumption rate, as the consumption rate for soldier meals is an age-old constant.

Please note that for all of these figures above , the Basic Load could be mathematical divided into different FOB locations—e.g. three FOBs would equate to a distributed Basic Load of POL of 3,300 litres per FOB site.

\*I have used the MRE instead of the Canadian equivalent Individual Meal Pack or IMP as the data was readily available in open source. IMPs and MREs are fairly similar although I believe the Canadian IMP is better. I have had a hard time getting the Canadian packing numbers but these could be obtained for our modelling if desired. Here are some points on what an MRE is:

The MRE or Meal-Ready-To-Eat is a self-contained complete meal designed to sustain an individual when normal food service is not available. One MRE = one meal. Except for the beverages, which require hydration (water), the entire meal is ready to eat.

The MRE is nutritionally balanced and provides around 1,200 calories (13% protein, 36% fat, and 51% carbohydrates) and 1/3 of the United States Military Recommended Daily Allowance of vitamins and minerals. A full day's worth of meals would consist of three MREs.

The MRE contents can and do vary, however depending upon the quantity ordered, MREs can be specifically tailored to meet your individual tastes and requirements. A typical MRE/meal will contain:

- Entree Main course Chicken, beef, pork, eggs, or beans
- Side Dish Vegetable, soup, or starch
- Snack Item Trail mix, snack bar, candy or pretzels
- Dessert Cookie, cake, fruit or chocolate bar
- Beverages Gatorade-like drink mixes, coffee, or tea
- Accessories Utensils, creamer, sugar, salt, pepper and a wet napkin

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## D Current Distribution Policies

This section contains the outcomes of the discussions on the current DND distribution policies. The sections contains: discussions were between

- the questions generated by the members of the team (in black), and
- the SME responses (in blue).

To summarize the basic rules used to manage inventory (when to issue reordering, or a 'heuristic' replenishment policy) additional discussions and effort is needed. Thus, the raw material is presented below.

John Conrad (SME): In a military operation, the currently used doctrinal precepts relevant to the distribution policies are:

- APOD:
  - Inventories: has 30-45-day supply of each product for all the personnel in the theatre? Is this the min inventories, or the average inventories over time or the level of the inventories at the time when the last delivery from Canada arrives?
  - The figure of 30 days of supply is the average inventory that the Canadian Task Force works to maintain in theatre. This can be expanded beyond 30 days in an austere environment where procuring supply locally is difficult or where the lines of communication from Canada are too long; too complicated and it takes an inordinate amount of time to get material from Canada.
  - Frequency of replenishment from Canada (for food, water, etc.): the transportation leaves every 7 days on average, and the quantities are for the demand (supply) for one week?
  - Theatre replenishment from Canada is normally more frequent than every seven days for a fighting task force and even for a Chapter 6 peacekeeping force in the UN days. Sustainment flights tend to be more like two or in some cases three in a seven day period. Additionally, sustainment by ship is another mode of transportation that is used for heavier top up—like major ammunition replenishment or hard rations (IMPs), vehicle fleet rotations, etc.
  - In routine sustainment, fresh rations (food) and water are probably poor examples for measuring/illustrating our sustainment system. The Task Force will doctrinally establish water points in theatre with engineer resources for bathing ablutions and even drinking. The transportation of water in the supply chain is more focused on the provision of bottled water for human consumption. Fresh rations and bottled water companies are almost always available much closer to the APOD and contracts for these combat supplies are established at the coalition level (lead nation provider) much closer to the APOD than Canada. Lastly, sustainment from Canada is not aimed at a

“week’s worth” of supply for the task force. I think it is easier to think of Canadian sustainment aircraft and ships as working to maintain a 30 day level of stores and equipment in theatre (at the APOD).

- Additional orders: Is there any ordering for any product that is coming from NSE apart from the regular replenishment? Except for equipment and ammunition? Are these additional orders made when a level of an inventory falls below a certain threshold? How much time does it take between the time the order is initiated at APOD and the time the delivery leaves Canada, or the time the order is ready to be delivered and waits for the next transportation asset to be available?
- Yes. Definitely yes. How much product and services beyond normal replenishment depends on what you consider regular replenishment. Repair services, repair parts for example, fall beyond the definition of replenishment but are certainly a large part of the provision of logistics and these parts and services do come from the NSE. All Canadian unique items in the supply system inventory must be demanded through the NSE as well. These might include combat clothing, ballistic glasses, new helmets, etc to replace lost or damaged items. These general and technical (G&T) stores fall outside what I think you are referring to as regular replenishment.
- Furthermore the procurement of items outside of the Canadian inventory (contracting/local procurement) are demanded and sourced through the NSE as well. These can range from unique gear to the provision of laundry and sewage services.
- These orders can be made when a unit falls below a specific threshold or they can be generated out of unique requirement. Units used to hold a small percentage of combat clothing inside their quartermaster stores( about 10% of unit strength in combat clothing)
- The time for delivery from Canada can vary depending on the operational priority of the item. A replacement engine for a critical weapon system is going to move quickly—possibly on a contracted aircraft that can be dispatched immediately. Other items might be able to wait for the next scheduled sustainment flight. One can make an assumption that an item of high priority will be at the APOD within 2-3 days. Medium priority items more like 7 to 10 days.
- FOB and beyond:
  - Inventories: each FOB has 3 day supply of each product for all the personnel at FOB and all the locations beyond the FOB that this FOB serves? Is this (3-day supply) the min inventories, or the average inventories over time, or the level of the inventories at the time when the last delivery from APOD arrives?
  - These FOB holdings are the average stock levels of the unit over time. Doctrinally, replenishment of a unit or battle group deployed happens every day and this delivery is intended to keep a unit at three days of supply.

- Frequency of replenishment: APOD to FOB: every 3-4 days?
- As mentioned above, the frequency of replenishment is to occur once every day (24 hour period). This frequency has been varied somewhat in theatres like southern Afghanistan where convoy movement that becomes rhythmic (predictable) presents a greater security threat. In these cases, larger quantities can be delivered to the FOB at more irregular intervals to ensure the units operating out of the FOB stay at 3 Days of supply. Use of helicopters for replenishment helps a great deal with the convoy problem. Another way to solve the frequency challenge is to arrange for some combat supplies to be delivered by contractors. An example of this is the delivery of diesel fuel right to a POL point on the FOB by a local contractor (like the Jingle Trucks of Afghanistan).
- You can assume an APOD to main FOB delivery every 2 days on the average.
- Locations beyond FOB:
  - Inventories: has 1-3-day supply of each product?
  - Yes, the inventory of sub-units operating beyond the FOB are less than 3 days of supply. The best way to think of it is as follows: from the furthest flung LAV III fighting vehicle operating in a section from the FOB back to the holdings on the FOB itself (POL, IMPs and ammunition) all 3 days are stretched between them. A sub-unit takes out what it needs from the FOB for its operation 1 to 2 days of supply.
  - Frequency of replenishment: FOBs to locations beyond: daily?
  - The frequency of replenishment of satellite or sub units beyond the FOB is moment to moment as required by the fighting unit. If I had to give an average perhaps every 12 hours? Assume a Main Fob to FOB 2 delivery every 12 hours or twice in a 24 hour period.

In a military operation, the routes are planned and generated as follows:

- APOD to FOBs:
  - APOD to FOB1 and back to APOD – under which circumstances?
  - APOD to FOB1 and back speaks to the routine sustainment, the topping up of 3 days of supply inside a given unit's Administration Company or Main FOB.
  - APOD to FOB1, FOB2, FOB 3, and back to FOB – under which circumstances?
  - Distribution to subsequent FOBs is handled by the unit itself using its own A2 echelon or Administration Company. It would not require the unit echelon trucks to return to the APOD at any point. Sustainment between the APOD and the Main FOB of the unit is the NSE's job. In exceptional circumstances, the NSE can be requested to deliver direct to any FOB if the tactical situation warrants it. The doctrinal intent is that the NSE replenishes the unit A2

echelon at its main FOB. Distribution internal to the unit is the responsibility of the unit Administration Company.

- There is neither strictly regular replenishment nor routing due to the need to be uncertain due to security issues.

## **E Data Sets**

The data sets provided in this section have been used in our computational study.

*Prepared by John Conrad, MSM, CD*

The classes of supply for the sets provided are set as outlined below. What varies between the military example and the civilian disaster example are commodities coded as Class 20. We may wish to designate additional items for monitoring. For example in the military example, we may wish to designate Class 20 to be ammunition. With the recent flood disaster in Alberta, the sorts of material being sought for delivery and distribution varied from dumpsters and rubble removal equipment (material handling equipment, dump trucks) as well as composite teams for teams for building/home damage assessment.

### **The Military Example.**

The best example for examining a distribution system in an unpredictable environment remains the military one. In civilian disasters, assets and medical supplies will travel to satellite nodes direct from commercial vendors to a much higher degree and not necessarily from a distribution system that sees the commodity go from a Far Supply Node to a close one and then on to a demand user. In the data set provided for a civilian disaster, I have done my best to extrapolate the data for a major earthquake disaster in the Lower Mainland of British Columbia. We will need to discuss the reality factor of Town X or Town Y getting needed equipment direct from a commercial provider, or, assume that no commercial solution exists and only items delivered in the supply chain can get to a given satellite node.

The various holding capacities or stocking levels at different echelons of the military supply chain can vary with any number of factors. For example, proximity to Canada or to industry/commercial resupply possibilities. A large distance (and hence long re-supply time) would suggest larger holding capacity. As mentioned earlier in our work, Canadian Army doctrine sets the holding levels at a fighting unit as a Basic Load of 3 days worth of combat supply and a Maintenance Load of one days supply on the logistics trucks of the supporting logistics unit or NSE. Theatre stock levels have been set on replenishment times dictated by sea and air lines of communication but typically have been set at 30 to 45 days of supply holdings in theatre.

NATO and potentially any joint, combined formation in which a Canadian task force would operate will also establish holding levels at various Forward Operating Bases (FOBs) and units.

	<b>Classes of Supply*</b>
1	General
2	Bulk Gasoline/Petroleum Product packaged and bulk liquid (Petroleum, Oil and lubricants (POL)
3	Supplies requiring refrigeration
20	Supply type unique to the specific scenario/disaster

### Types of Supply

	<b>Name</b>	<b>Class</b>	<b>Weight in Kg</b>
1	Water (1 litre)	1-General	1
2	Hard Food ration	1-General	1
3	1 Doses of Medication—non refrigerated	1-General	0.01
4	Small Equipment	1-General	50
5	Large Equipment	1-General	5000
6	Unleaded Gasoline (1 litre)	2-POL	1.15
7	Fresh (perishable) food	3-Supplies requiring refrigeration	1
8	1 Dose Medication requiring refrigeration	3-Supplies requiring refrigeration	0.02

\*Note. These are not the NATO classes of supply but rather the classes established for this study.

## E1 Military Scenario Data

Loading/ unloading time of one tonne of product type p - make the same for all the transportation assets	Structure	Number of time periods	Duration of time period (in days)	Maximum work shift duration (h)
15 min		5	7	48

Distance between Far Supply Node and Close Supply Node (km)	800-4000
Number of people at Close Supply Node	100-300
Number of satellite nodes	2-3
Distance to satellite nodes (km)	20-50
Distance between satellite nodes (km)	25-150
Number of people per satellite node	60-150
Number of demand nodes per satellite	2-4
Number of people per demand node	50-60
Distance to demand nodes (km)	15-30
Distance between demand nodes (km)	5-40

The figures in this model are predicated on the Task Force Afghanistan model in place in 2006—2011.

## *Consumption by People*

### **Demand per person, per day, at Close Supply Node**

Supply Type Id	Mean demand	Std dev demand	% of people Needing it
1	6-8*	0.2-0.5	100
2	3-4	0	100
3	0.1-0.9	0.001-0.003	100
4	0.5-1	0.1-0.5	10-40
5	0	0	0
6	0	0	0
7	0	0	0
8	0	0	0

### **Demand per person, per day, at Satellite Node**

Supply Type id	Mean demand	Std dev demand	% of people Needing it
1	8-10*	0.2-0.5	100
2	3-4	0.3-0.8	100
3	0.1-0.9	0.001-0.003	100
4	1-2	0.01-0.5	10-40
5	1-2	0.5	5-60
6	0	0	0
7			
8			

### **Demand per person, per day, at Demand Node**

Supply Type ID	Mean demand	Std Dev Demand	
1	8-10*	0.2-0.5*	100
2	3-4	0.3-0.8	100
3	0.1-0.9	0.001-0.003	100
4	1-3	0.01-0.5	10-40
5	1-2	0.5	0-60
6	0	0	0
7			
8			

\*8 to 10 litres per day per soldier does not include water for ablutions (bathing/hygiene).

\*\*This model assumes that all feeding of soldiers forward of the far Supply Node is hard ration and not fresh. Refrigeration truck still required for the storage of water and medical supply.

## ***Consumption by Assets***

### **Demand per asset, per period, at Far Supply Node**

Supply Type	Demand Per Period	Mean Demand	St Dev Demand
1	20-30*	25	
2	0	0	
3	0	0	
4	0.5-1	0.75	
5	0	0	
6	80,000—90,000**	85,000	0.2-0.5
7	0	0	
8	0	0	

\*Water associated with vehicle maintenance purposes/coolant

\*\*the daily demand at this Far Supply Node might not be replenished every day. Experience in Coalition operations in Afghanistan saw the Far Supply Node bulk fuel resources managed at maximum and minimum acceptable fuel levels to be held at the Node. To provide some context, the minimum acceptable level of bulk fuel at Kandahar Airfield (The Far Supply Node) was measured in x million litres, where X is a classified number not for public release.

### Actual Historical Data for Diesel Holding 1 Canadian Mechanized Brigade (1 CMBG) units in the mid 1990s

Artillery 3300 litre capacity

Armour (Tanks and Armour Reconnaissance Squadron)--11,300 litres

Combat Engineers--7040 litres

Infantry 1 PPCLI--4800 litres

Light Infantry PPCLI—non-mechanized unit (2700 litres)

The Service Battalion (logistics battalion) total Maintenance Load for 1 CMBG: 80,000 Plus 31,000 Total is 111,000 litres of Standard Military Pattern (SMP) vehicle diesel. The logistics battalion is a good model for the Close Supply Node—even in the contemporary operational theatre construct where the National Support Element (NSE) fills the logistics battalion function.

The Close Supply Node in this scenario is considered to be the NSE

### **Demand per asset, per period, at Close Supply Node**

Supply Type	Demand Per Period	Mean Demand	St Dev Demand
1	20-30	25	0.2-0.5
2	0	0	0
3	0	0	0
4	0.8-1.5	1.15	0.2-0.5
5	0	0	0
6	30,000—45,000	37,500	0.2-0.5
7	0	0	0
8	0	0	0



**Demand per asset, per period, at Satellite Node**

Supply Type	Demand Per Period	Mean Demand	St Dev Demand
1	20-30	25	0.2-0.5
2	0	0	0
3	0	0	0
4	0.9-2.0	1.45	0.2-0.5
5	0	0	0
6	23,300-42,500	32,900	0.2-0.5
7	0	0	0
8	0	0	0

## ***Transportation Assets***

### **Far Supply Node Transportation Assets**

		Number	Cost	Speed kph	Range (loaded)	Fuel capacity	Payload in tonnes	Product compatibility	Non compatibility
1	Boeing 737	1					17 (37,500 lbs)		
2	CF Airbus 320	1					16.6 (36,590 lbs)		
3	IL-76 Transport Aircraft	1					40 (88,000 lbs)		
4	Hercules Tactical Aircraft C 130J	1		600 kph	2,900 kms		15.4 (34,000 lbs)		
5	Train	-	TBC	80 kph			unlimited		
6	Tanker Ship			24 knots					
7	Container ship			24 knots					

### **Close Supply Node Transportation Assets**

		Number	Cost	Speed kph	Range	Fuel capacity	Payload	Product compatibility	Non compatibility
1	General cargo	10		60-80	400		16		
2	Bulk Fuel	6		60-80	350- 400		16		
3	Refrigerator	5		60-80	350- 400		42.4 cubic metres*		
4	Ch-47 Chinook Helicopter	2		130 knots/ 149 mph					
5	Sikorsky UH 60 Blackhawk	2		160 knots/ 182 mph	500		1 (internal) 4 (external)		
6	Hercules tactical Aircraft C 130J	1		414 mph	2900 kms		15.4		
7									

\*The refrigerator truck capacity is given in cubic metres vice tonnes. In most cases, the maximum amount of lift in a refrigerator truck is decided by volume and not mass. The figure cited here is for a commercial standard 26 foot reefer trailer.

### Satellite Node Assets

		Number	Cost	Speed kph	Consumption/100 km	Fuel capacity	Payload	Product compatibility	Non compatibility
1	General SMP Cargo Truck	10		90		300			
2	Bulk Fuel SMP Truck	6		90		300	10,000		
3	Refrigerator	3		90		250			
4	LAV III Fighting Vehicle								

Maximum number of routes of a transportation asset from satellite node per period=2

Warehouses (Note: unit weight is a tonne)

Far Supply Node	800000	900000	200000
Close Supply Node	75000	80000	18000
Satellite Node	5000	4000	1800
Demand Node	1000	1200	400

Snezana added:

Mode	Max cargo (tonnes)	Fuel consumption (liters per 100km)	Speed
Ship	150,000	15,000	35
Train	15,000	5,000	80
Cargo plane	150	1,500	750
Truck	20	50	60

## E2 BC Earthquake Scenario Data

Loading/ unloading time of one tonne of product type p - make the same for all the transportation assets	Structure	Number of time periods	Duration of time period (in days)	Maximum work shift duration (h)
15 min		5	7	48

Distance between Far Supply Node and Close Supply Node (km)	800-4000
Number of people at Close Supply Node	8000-10000
Number of satellite nodes	12-15
Distance to satellite nodes (km)	20-100
Distance between satellite nodes (km)	25-150
Number of people per satellite node	5-150
Number of demand nodes per satellite	3 to 15
Number of people per demand node	100-1000
Distance to demand nodes (km)	1-15
Distance between demand nodes (km)	5-20

The figures in this model are predicated on a model that sees the Far Supply Node as the APODs at Calgary and Edmonton, the Close Supply Node as the Emergency Operations Centre (EOC) Logistics Section of an incorporated Town or Municipality and the Satellite node as either a community within the Municipality. Demand nodes are either individual home owners or farm owners in the Municipality/Community that make their needs known to first responders who in turn roll the demand up to the satellite. In many cases demands can and will go direct from a demand unit (home/business/farm owner) to the Close Supply Node (EOC).

## Consumption by People

### Demand per person, per day, at Close Supply Node

Supply Type Id	Mean demand	Std dev demand	% of people Needing it
1	6-8*	0.2-0.5	100
2	0	0	100
3	0.9-2	0.001-0.003	100
4	0.9-2	0.1-0.5	10-40
5	1-2	0.5-0.9	0
6	0	0	0
7	3-4	0.3-0.8	0
8	1-2	0.01-0.05	100

### Demand per person, per day, at Satellite Node

Supply Type id	Mean demand	Std dev demand	% of people Needing it
1	6-8	0.2-0.5	100
2	0	0	100
3	0.9-2	0.01-0.03	100
4	0.9-2	0.1-0.5	10-40
5	2-4	0.5-0.9	0-60
6	0	0	0
7	3-4	0.3-0.8	0
8	1-2	0.01-0.05	100

### Demand per person, per day, at Demand Node

Supply Type ID	Mean demand	Std Dev Demand
1	6-10	0.2-0.5
2	0	0
3	5-10	0.001-0.003
4	0.9-2.0	0.01-0.5
5	0.5-0.9**	0.5
6	0	0
7	3-4	0.3-0.8
8	5-6	0.01-0.05

\*This model assumes that all feeding of people affected by the earthquake will be fresh food. Refrigeration truck(s) required for the storage of water and medical supply depending on infrastructure damage at a given node in the Lower Mainland.

\*\*There will be a spike for major/large equipment at the beginning of the event that should taper of to a small number over the recovery phase

## ***Consumption by Assets***

### **Demand per asset, per period, at Far Supply Node**

Supply Type	Demand Per Period	Mean Demand	St Dev Demand
1	20-30*	25	
2	0	0	
3	0	0	
4	1.5-2	1.75	
5	0	0	
6	30,000—50,000**	40,000	0.2-0.5
7	0	0	
8	0	0	

\*Water associated with vehicle maintenance purposes/coolant

\*\*the daily demand at this Far Supply Node might not be replenished every day. To get the demand per asset we will need to divide by the number of machines and generators etc if demand per asset truly required for the model.

### **Demand per asset, per period, at Close Supply Node**

Supply Type	Demand Per Period	Mean Demand	St Dev Demand
1	20-30	25	0.2-0.5
2	0	0	0
3	0	0	0
4	1.8-3.0	2.4	0.2-0.5
5	0	0	0
6	20,000—40,000	30,000	0.2-0.5
7	0	0	0
8	0	0	0

### **Demand per asset, per period, at Satellite Node**

Supply Type	Demand Per Period	Mean Demand	St Dev Demand
1	20-30	25	0.2-0.5
2	0	0	0
3	0	0	0
4	1.5-3.0	2.25	0.2-0.5
5	0	0	0
6	10-15,000	12,500	0.2-0.5
7	0	0	0
8	0	0	0

## *Transportation Assets*

### **Far Supply Node Transportation Assets**

		Number	Cost	Speed kph	Range (loaded)	Fuel capacity	Payload in tonnes	Product compatibility	Non compatibility
1	Boeing 737	1					17 (37,500 lbs)		
2	CF Airbus 320*	1					16.6 (36,590 lbs)		
3	IL-76 Transport Aircraft	1					40 (88,000 lbs)		
4	Hercules Tactical Aircraft* C 130J	1		600 kph	2,900 kms		15.4 (34,000 lbs)		
5	Train	-	TBC	80 kph			unlimited		

### **Close Supply Node Transportation Assets**

		Number	Cost	Speed kph	Range	Fuel capacity	Payload	Product compatibility	Non compatibility
1	General cargo	10		60-80	400		10 to 16		
2	Bulk Fuel	6		60-80	350-400		10to 16		
3	Refrigerator	3		60-80	350-400		42.4 cubic metres**		
4	Civilian Utility Helicopter	2		130 knots/ 149 mph					

\*The Hercules and Airbus are left in this Lower Main land Disaster as we can logically assume the support of the Canadian Forces will be requested by the Government of B.C. through Public Safety Canada

\*\*The refrigerator truck capacity is given in cubic metres vice tonnes. In most cases, the maximum amount of lift in a refrigerator truck is decided by volume and not mass. The figure cited here is for a commercial standard 26 foot reefer trailer.

### Satellite Node Assets

		Number	Cost	Speed kph	Consumption/ 100 km	Fuel capacity	Payload	Product compatibility	Non compatibility
1	General SMP Cargo Truck	10		90		300			
2	Bulk Fuel SMP Truck	6		90		300	10,000		
3	Refrigerator	3		90		250			

Maximum number of routes of a transportation asset from satellite node per period=2





## F SCM Solver: Master Input File

-----  
Structure  
-----  
Number of explicit cases  
1  
Number of time periods  
15  
Duration of time period (in days)  
7  
Maximum work shift duration (h)  
48  
  
Distance between Far Supply Node and Close Supply Node (km)  
200  
#800-4000  
Number of people at Close Supply Node  
8000-10000  
  
Number of satellite nodes  
3  
#2-5  
Distance to satellite nodes (km)  
50  
#20-100  
Distance between satellite nodes (km)  
10  
#25-150  
Number of people per satellite node  
100  
#50-150  
  
Number of demand nodes per satellite  
12  
#3-15  
Number of people per demand node  
1000  
#100-1000  
Distance to demand nodes (km)  
5  
#1-15

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Distance between demand nodes (km)

3  
#5-20

-----

Supplies

-----

Classes of supplies (id, name)

1 'general'  
#2 'bulk gasoline'  
#3 'refrigerator'

Types of supplies (id, name, class, weight (kg), unloading time per tonne (hr) )

1 'water bottled 1l' 1 0.25  
#2 'food ration' 0.5 1 0.25  
#3 'perishable food 1kg' 1 3 0.5  
#4 'medications 1 dose' 0.01 1 0.5  
#5 'medications refrigerator 1 dose' 0.02 3 1  
#6 'gasoline 1l' 1.15 2 0.5  
#7 'small equipment' 50 1 2  
#8 'large equipment' 5000 1 3

-----

Consumption by People

-----

Demand per person, per day, at Close Supply Node (supply type id, mean demand, stddev demand, percent of people that needs it)

1 (7) (100)  
#1 (7,8,7.5,6.5,6) (100,100,100,100,100)  
#2 (3.5,4,3.75,3.25,3) (100,100,100,100,100)  
#3 (0.5,0.9,0.7,0.3,0.1) (100,100,100,100,100)  
#4 (0.75,1,0.8,0.6,0.5) (25,40,35,15,10)  
#5 (0,0,0,0,0) (100,100,100,100,100)  
#6 (0,0,0,0,0) (0,0,0,0,0)  
#7 (0,0,0,0,0) (0,0,0,0,0)  
#8 (0,0,0,0,0) (0,0,0,0,0)

Demand per person, per day, at Satellite Node (supply type id, mean demand, stddev demand)

1 (9) (100)  
#1 (9,10,9.5,8.5,8) (100,100,100,100,100)  
#2 (3.5,4,3.75,3.25,3) (100,100,100,100,100)  
#3 (0.5,0.9,0.7,0.3,0.1) (100,100,100,100,100)  
#4 (1.5,2,1.75,1.25,1) (25,40,35,15,10)  
#5 (1.5,2,1.75,1.25,1) (30,60,45,15,5)  
#6 (0,0,0,0,0) (0,0,0,0,0)  
#7 (0,0,0,0,0) (0,0,0,0,0)  
#8 (0,0,0,0,0) (0,0,0,0,0)



#1	(6)	(100)	
#1	(10)	(100)	
#1	(2)	(100)	
1	(2,6,10)	(100,100,100)	
#1	(8)	(100)	
#1	(4)	(100)	
#1	(7,10,4,8,5,5,5)	(100,100,100,100,100)	
#2	(3,5,4,3,75,3,25,3)	(100,100,100,100,100)	
#3	(0,5,0,9,0,7,0,3,0,1)	(100,100,100,100,100)	
#4	(2,3,2,5,1,5,1)	(25,40,35,15,10)	
#5	(1,5,2,1,75,1,25,1)	(30,60,45,15,5)	
#6	(0,0,0,0,0)	(0,0,0,0,0)	
#7	(0,0,0,0,0)	(0,0,0,0,0)	
#8	(0,0,0,0,0)	(0,0,0,0,0)	

Demand per asset, per period, at Far Supply Node (rows: supply type id, columns: asset type id, values: (demand at each scenario))

Demand per asset, per period, at Close Supply Node (supply type id, mean demand, stddev demand)

F-3

[illegible]

## Far Supply Node Transportation Assets

Close Supply Node Transportation Assets

Satellite Node Transportation Assets (id, name, number, mean speed - km/h, stddev speed - km/h)									
#(id, name, capacity (tonnes), asset-product compatibility, product incompatibility)	n	c	s	consump	gas cap	cap	a-p compat	p-p comp)	gasoline consumption (per 100km), gasoline tank capacity (l),
1 'truck'	10	200	40	50	150	20	(1)	(1)	
#1 'truck'	4	200	40	50	150	20	(1,2,3,4,7,8)	(1)	
#2 'oil tank truck'	5	200	40	50	150	10	(6)	(1)	
#3 'refrigerator truck'	3	300	40	75	150	10	(3,5)	((3,5))	



Warehouse Inventory Costs per tonne per time period at a class store (rows: nodes, columns: product)

Warehouse Initial Inventories - Tonnes (rows: nodes, columns: supplies)

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# To be calculated and written to the problem instance file  
# - Maximum route duration for vehicle of type v: calculate from the gasoline tank capacity and gasoline consumption  
# - Number of time periods to deliver between F and L  
# - Cost to deliver from F to L = cost per day + cost of fuel  
# - Number of periods of unavailability of a transportation asset of type v after delivery from F to L = time needed to come back

## G SCM Solver: Output

### G1 Gurobi output .sol file

```
# Solution for model Supply Chain Model
# Objective value = 1.3335424163620998e+06
x.0.0.0.0.0.0.0 0
x.0.1.0.0.0.0.0 0
x.0.2.0.0.0.0.0 0
x.0.3.0.0.0.0.0 0
x.0.4.0.0.0.0.0 0
x.0.5.0.0.0.0.0 0
x.0.6.0.0.0.0.0 0
x.1.0.0.0.0.0.0 0
x.1.1.0.0.0.0.0 0
x.1.2.0.0.0.0.0 0
x.1.3.0.0.0.0.0 0
x.1.4.0.0.0.0.0 0
x.1.5.0.0.0.0.0 0
x.1.6.0.0.0.0.0 0
...
x.6.1.0.0.1.0.10 1
x.6.2.0.0.1.0.10 0
x.6.3.0.0.1.0.10 0
x.6.4.0.0.1.0.10 0
x.6.5.0.0.1.0.10 0
...
q.0.0.0.2.0.0.3.7 1.40000000000000348e+01
q.0.0.0.2.0.0.3.8 0
q.0.0.0.2.0.0.3.9 0
q.0.0.0.2.0.0.3.10 0
...
d.0.0.2.0.2 2
d.0.0.3.0.2 2.00000000000000018e+00
d.0.0.4.0.2 2.00000000000000004e+00
d.0.0.5.0.2 1.9999999999999998e+00
d.0.0.0.1.2 2
d.0.0.1.1.2 2
```

## G2 SCM Solver Log and Solution Details

Optimize a model with 673635 rows, 238335 columns and 3801893 nonzeros  
Found heuristic solution: objective 1.62031e+07  
Presolve removed 337395 rows and 58186 columns (presolve time = 5s) ...  
Presolve removed 337400 rows and 58188 columns  
Presolve time: 9.17s  
Presolved: 336235 rows, 180147 columns, 2422688 nonzeros  
Variable types: 20505 continuous, 159642 integer (139662 binary)

Root simplex log...

Iteration	Objective	Primal Inf.	Dual Inf.	Time
37005	-2.7322577e+06	1.126078e+04	0.000000e+00	5s
56114	-2.7049967e+06	2.111952e+04	0.000000e+00	11s
68464	-2.6903224e+06	1.888799e+04	0.000000e+00	16s
79200	-2.6730016e+06	3.656076e+04	0.000000e+00	22s
86264	-2.6641307e+06	2.139600e+04	0.000000e+00	26s
93494	-2.6586469e+06	2.066231e+04	0.000000e+00	31s
104060	-2.6525473e+06	2.548280e+04	0.000000e+00	37s
111525	-2.6462822e+06	2.135130e+04	0.000000e+00	41s
118664	-2.6397954e+06	2.021240e+07	0.000000e+00	45s
127741	-2.6328587e+06	2.140497e+04	0.000000e+00	50s
139897	-2.6206688e+06	3.903966e+04	0.000000e+00	56s
149034	-2.6167596e+06	1.215524e+04	0.000000e+00	60s
159931	-2.6159428e+06	1.027347e+04	0.000000e+00	66s
169429	-2.6011270e+06	9.525906e+03	0.000000e+00	71s
176330	3.8970855e+06	1.110272e+04	0.000000e+00	75s
183149	3.8983335e+06	7.265027e+02	0.000000e+00	80s
184901	3.8983335e+06	0.000000e+00	0.000000e+00	82s

Root relaxation: objective 3.898333e+06, 184901 iterations, 82.11 seconds  
Total elapsed time = 111.55s  
Total elapsed time = 118.76s

Nodes		Current Node		Objective Bounds		Work	
Expl	Unexpl	Obj	Depth	IntInf	Incumbent	BestBd	Gap   It/Node Time
	0	0	3898333.49	0	2207	1.6203e+07	3898333.49 75.9% - 122s
H	0	0				4024659.4763	3898333.49 3.14% - 154s
	0	0	3899171.22	0	3745	4024659.48	3899171.22 3.12% - 163s
	0	0	3899171.22	0	2898	4024659.48	3899171.22 3.12% - 209s
H	0	0				3999799.1156	3899171.22 2.52% - 228s
	0	0	3899171.22	0	3568	3999799.12	3899171.22 2.52% - 235s
H	0	0				3973590.1058	3899171.22 1.87% - 262s
	0	0	3899171.22	0	1594	3973590.11	3899171.22 1.87% - 299s
	0	0	3899171.22	0	1501	3973590.11	3899171.22 1.87% - 321s
	0	0	3899171.22	0	1265	3973590.11	3899171.22 1.87% - 339s
	0	0	3899171.22	0	1304	3973590.11	3899171.22 1.87% - 356s
	0	0	3899171.22	0	1034	3973590.11	3899171.22 1.87% - 371s
	0	0	3899171.22	0	912	3973590.11	3899171.22 1.87% - 383s
	0	0	3899171.22	0	886	3973590.11	3899171.22 1.87% - 395s
	0	0	3899171.22	0	925	3973590.11	3899171.22 1.87% - 405s
	0	0	3899171.22	0	677	3973590.11	3899171.22 1.87% - 415s
	0	0	3899171.22	0	789	3973590.11	3899171.22 1.87% - 425s
	0	0	3899171.22	0	906	3973590.11	3899171.22 1.87% - 469s
	0	0	3899171.22	0	635	3973590.11	3899171.22 1.87% - 476s
	0	2	3899171.22	0	625	3973590.11	3899171.22 1.87% - 490s
	10	12	3899581.49	7	926	3973590.11	3899266.25 1.87% 1210 495s
	30	29	3900133.49	10	933	3973590.11	3899355.64 1.87% 603 550s
	62	53	3901969.49	15	906	3973590.11	3899355.64 1.87% 315 555s





90	77	3922529.49	22	887	3973590.11	3899355.64	1.87%	228	560s
102	93	3922537.49	25	878	3973590.11	3899355.64	1.87%	203	582s
327	317	3923537.85	79	675	3973590.11	3899355.64	1.87%	76.4	590s
429	412	3923841.94	101	561	3973590.11	3899355.64	1.87%	62.9	598s
508	489	3923897.94	117	529	3973590.11	3899355.64	1.87%	55.9	600s

Cutting planes:

Learned: 15  
Gomory: 13  
Implied bound: 625  
Clique: 2  
MIR: 147  
Flow cover: 165  
Zero half: 9

Explored 528 nodes (775920 simplex iterations) in 600.21 seconds  
Thread count was 4 (of 4 available processors)

Time limit reached

Best objective 3.973590105760e+06, best bound 3.899355640253e+06, gap 1.8682%

-----  
Inventories at customers)

Inventory at Customer Nodes for each scenario and for each time period:

Scenario	Satellite	Customer	Time period	INV	QNTY_Arr	demand	unmet_demand
0	0	0	0	0.00	0.00	6.00	6.00
0	0	0	1	0.00	0.00	6.00	6.00
0	0	0	2	0.00	12.00	6.00	6.00
0	0	0	3	12.00	0.00	6.00	0.00
0	0	0	4	6.00	10.00	6.00	0.00
0	0	0	5	10.00	20.00	6.00	0.00
0	0	0	6	24.00	0.00	6.00	0.00
0	0	0	7	18.00	0.00	6.00	0.00
0	0	0	8	12.00	0.00	6.00	0.00
0	0	0	9	6.00	10.00	6.00	0.00
0	0	0	10	10.00	20.00	6.00	0.00
0	0	0	11	24.00	0.00	6.00	0.00
0	0	0	12	18.00	0.00	6.00	0.00
0	0	0	13	12.00	0.00	6.00	0.00

...

0	2	4	12	18.00	0.00	6.00	0.00
0	2	4	13	12.00	0.00	6.00	0.00
0	2	4	14	6.00	0.00	6.00	0.00
0	2	5	0	0.00	0.00	6.00	6.00
0	2	5	1	0.00	0.00	6.00	6.00
0	2	5	2	0.00	6.00	6.00	6.00
0	2	5	3	6.00	6.00	6.00	0.00
0	2	5	4	6.00	8.00	6.00	0.00
0	2	5	5	8.00	10.00	6.00	0.00
0	2	5	6	12.00	0.00	6.00	0.00
0	2	5	7	6.00	18.00	6.00	0.00
0	2	5	8	18.00	0.00	6.00	0.00
0	2	5	9	12.00	0.00	6.00	0.00
0	2	5	10	6.00	6.00	6.00	0.00
0	2	5	11	6.00	18.00	6.00	0.00
0	2	5	12	18.00	0.00	6.00	0.00
0	2	5	13	12.00	0.00	6.00	0.00
0	2	5	14	6.00	0.00	6.00	0.00

-----

Inventory at Satellite Nodes:

Scenario	Satellite	Time period	INV	QNTY_Arr	QNTY_Left
0	0	0	0.00	0.00	0.00
0	0	1	0.00	54.00	0.00
0	0	2	54.00	18.00	54.00
0	0	3	18.00	70.00	18.00
0	0	4	70.00	60.00	70.00
0	0	5	60.00	40.00	60.00
0	0	6	40.00	0.00	20.00
0	0	7	20.00	20.00	0.00
0	0	8	40.00	58.00	40.00
0	0	9	58.00	52.00	58.00
0	0	10	52.00	60.00	40.00
0	0	11	72.00	0.00	72.00
0	0	12	0.00	0.00	0.00
0	0	13	0.00	0.00	0.00
0	0	14	0.00	0.00	0.00
...					
0	2	9	60.01	30.01	20.00
0	2	10	70.02	39.98	56.00
0	2	11	54.00	0.00	54.00
0	2	12	0.00	0.00	0.00
0	2	13	0.00	0.00	0.00
0	2	14	0.00	0.00	0.00

-----

Inventory at Close Supply Node:

Scenario	Time period	INV	QNTY_Arr	QNTY_Left
0	0	0.00	150.00	0.00
0	1	150.00	146.00	150.00
0	2	146.00	150.00	78.00
0	3	218.00	150.00	218.00
0	4	150.00	150.00	140.00
0	5	160.00	0.00	120.00
0	6	40.00	148.00	40.00
0	7	148.00	150.00	100.00
0	8	198.00	112.00	198.00
0	9	112.00	140.00	112.00
0	10	140.00	0.00	140.00
0	11	0.00	0.00	0.00
0	12	0.00	0.00	0.00
0	13	0.00	0.00	0.00
0	14	0.00	0.00	0.00

-----

Total inventories over all periods

Customer Nodes

Scenario	Satellite	Customer	INV	QNTY_Arr	demand	unmet_demand
0	0	0	158.00	72.00	90.00	18.00
0	0	1	115.97	72.00	90.00	18.00
0	0	2	114.00	72.00	90.00	18.00
0	0	3	121.98	72.00	90.00	18.00
0	0	4	134.05	72.00	90.00	18.00
0	0	5	132.00	72.00	90.00	18.00
0	1	0	178.00	72.00	90.00	18.00
0	1	1	116.03	72.00	90.00	18.00
0	1	2	146.00	72.00	90.00	18.00
0	1	3	123.98	72.00	90.00	18.00
0	1	4	113.99	72.00	90.00	18.00
0	1	5	104.01	72.00	90.00	18.00
0	2	0	134.00	72.00	90.00	18.00



0	2	1	112.00	72.00	90.00	18.00
0	2	2	146.00	72.00	90.00	18.00
0	2	3	148.00	72.00	90.00	18.00
0	2	4	168.00	72.00	90.00	18.00
0	2	5	116.00	72.00	90.00	18.00

Satellite Nodes

Scenario	Satellite	INV	QNTY_Arr
0	0	484.00	432.00
0	1	573.95	432.00
0	2	524.04	432.00

Close Supply Node

Scenario	INV	QNTY_Arr
0	1462.00	1296.00

-----  
Customer visits

Customer Node Visits for each time period:

Time period	Satellite	Customer	Number of visits
0	0	0	0
0	0	1	0
0	0	2	0
0	0	3	0
...			
2	1	2	2
2	1	3	2
2	1	4	12
2	1	5	4
2	2	0	2
2	2	1	2
2	2	2	2
...			

Customer Visits for all time periods:

Satellite	Customer	Number of visits
0	0	12
0	1	22
0	2	14
0	3	22
0	4	20
0	5	22
1	0	10
1	1	18
1	2	12
1	3	14
1	4	28
1	5	20
2	0	14
2	1	16
2	2	12
2	3	16
2	4	14
2	5	14

-----  
Satellite: visits and routes

For each time period:

Time period	Satellite	Number-of-visits	Number-of-routes
0	0	0.0	0.0
0	1	0.0	0.0

0	2	0.0	0.0
1	0	14.0	0.0
1	1	10.0	0.0
1	2	6.0	0.0

...

Total for all time periods:

Satellite	Number-of-visits	Number-of-routes
0	54	32
1	58	36
2	56	36

-----  
Routes: close supply node and far supply node

For each time period:

Time period	Close: Number-of-routes	Far: Number-of-routes
0	0.0	1.0
1	120.0	1.0
2	32.0	1.0
3	96.0	1.0
4	56.0	1.0
5	48.0	0.0
6	16.0	1.0
7	40.0	1.0
8	80.0	1.0
9	48.0	1.0
10	56.0	0.0
11	0.0	0.0
12	0.0	0.0
13	0.0	0.0
14	0.0	0.0

Total for all time periods:

Close Supply Node: Number of routes is 592

Far Supply Node: Number of routes is 9

-----  
Routes: close supply node --> satellites

Time period 0

Vehicle type 0

Time period 1

Vehicle type 0

Vehicle 0 ((num nodes, dist, y) (2.0, 99.90175173639349, 1.0) sat 0(q  
0.010000000000000009, (3,0)=1(0,3)=1) sat 1(q 0.0, ) sat 2(q 0.0, ) (0,3)=1(3,0)=1)  
Route: (3,0,3, )

Vehicle 1 ((num nodes, dist, y) (2.0, 99.90175173639346, 1.0) sat 0(q 0.0, ) sat  
1(q 0.0, ) sat 2(q 4.000000000214619, (3,2)=1(2,3)=1) (2,3)=1(3,2)=1) Route:  
(3,2,3, )

Vehicle 2 ((num nodes, dist, y) (2.0, 99.90175173639346, 1.0) sat 0(q 0.0, ) sat  
1(q 20.000000000000014, (3,1)=1(1,3)=1) sat 2(q 0.0, ) (1,3)=1(3,1)=1) Route:  
(3,1,3, )

Vehicle 3 ((num nodes, dist, y) (2.0, 99.90175173639349, 1.0) sat 0(q  
0.010000000000000009, (3,0)=1(0,3)=1) sat 1(q 0.0, ) sat 2(q 0.0, ) (0,3)=1(3,0)=1)  
Route: (3,0,3, )

Vehicle 4 ((num nodes, dist, y) (2.0, 99.90175173639346, 1.0) sat 0(q 0.0, ) sat  
1(q 11.980000000214844, (3,1)=1(1,3)=1) sat 2(q 0.0, ) (1,3)=1(3,1)=1) Route:  
(3,1,3, )

Vehicle 5 ((num nodes, dist, y) (2.0, 99.90175173639349, 1.0) sat 0(q  
0.010000000000000009, (3,0)=1(0,3)=1) sat 1(q 0.0, ) sat 2(q 0.0, ) (0,3)=1(3,0)=1)  
Route: (3,0,3, )

Time period 11

Vehicle type 0

Time period 12

Vehicle type 0



Time period 13  
Vehicle type 0  
Time period 14  
Vehicle type 0

-----  
Routes: satellites --> demand nodes

Time period 0  
Satellite 0  
Vehicle type 0  
Vehicle 0  
Vehicle 1

...

Time period 2  
Satellite 0  
Vehicle type 0  
Vehicle 0

Route 0 ((num nodes, dist, y) (5.0, 100.04498987955368, 1.0) cust 0(q  
0.0, ) cust 1(q 0.0100000000000001563, (4,1)=1(1,5)=1) cust 2(q 0.010000000000000009,  
(2,4)=1(6,2)=1) cust 3(q 0.0, ) cust 4(q 11.999999999999433, (4,1)=1(2,4)=1) cust 5(q  
5.999999999355126, (1,5)=1(5,6)=1) (6,2)=1(5,6)=1) Route: (6,2,4,1,5,) )

Vehicle 1

Route 0 ((num nodes, dist, y) (2.0, 100.04498987955368, 1.0) cust 0(q  
11.999999999999432, (6,0)=1(0,6)=1) cust 1(q 0.0, ) cust 2(q 0.0, ) cust 3(q 0.0, )  
cust 4(q 0.0, ) cust 5(q 0.0, ) (0,6)=1(6,0)=1) Route: (6,0,6,) )

Vehicle 2

Route 0 ((num nodes, dist, y) (4.0, 100.04498987955368, 1.0) cust 0(q  
0.0, ) cust 1(q 0.0100000000000001563, (1,2)=1(3,1)=1) cust 2(q 11.989999999999432,  
(1,2)=1(2,6)=1) cust 3(q 5.9999999999994, (3,1)=1(6,3)=1) cust 4(q 0.0, ) cust 5(q  
0.0, ) (2,6)=1(6,3)=1) Route: (6,3,1,2,6,) )

Vehicle 3

Route 0 ((num nodes, dist, y) (2.0, 100.04498987955368, 1.0) cust 0(q  
0.0, ) cust 1(q 5.9799999999994204, (6,1)=1(1,6)=1) cust 2(q 0.0, ) cust 3(q 0.0, )  
cust 4(q 0.0, ) cust 5(q 0.0, ) (1,6)=1(6,1)=1) Route: (6,1,6,) )

Vehicle 4

...

-----  
Transport (distances, full cost, weighted costs)

To Close Supply Node: 1800.0, 2700000.0, 540000.0  
To Satellite Nodes: 7392.729628493126, 369636.48142465507, 73927.2962849311  
To Customer Nodes: 10404.67894747358, 520233.9473736771, 104046.78947473594

-----  
Inventories (amounts, full costs, weighted costs)

-----  
Close Supply Node: 1462.000000000629, 14620.00000000629, 2924.000000001258  
Satellite Nodes: 1581.989999999275, 15819.899999999272, 3163.979999999855  
Customer Nodes: 2382.0099999998706, 47640.199999999742, 9528.039999999482

-----  
Unmet Demand (amount, full cost, weighted cost)

-----  
999  
-----

```
-----  
Input file name: /home/dathomson/pig/sDRel_lp/sDRel_lp_min_1.json  
Output file name: /home/dathomson/solutions/sDRel_lp_min_1.sol  
Number of scenarios: 1  
Number of time periods: 15  
Number of satellites: 3  
Number of demand nodes: 6  
Number of products: 1  
Objective function weights (transp.cost: F-L, L-s, s-c; Inventory costs; Unmet  
demand: 0.2, 0.2, 0.2, 0.2, 10000.0,  
Product compatibility constraints: false  
Wait next time period to leave: true  
Re-fueling constraints: false  
-----  
-----
```

```
Execution Time: 244.905 seconds Time limit set to Gurobi by user: 240.0 seconds  
Objective value: 1333542.4163620998; Gap: 2.5134635096738753%; Runtime:  
240.97220611572266  
-----  
-----
```

```
Number of constraints : 673635  
Number of variables: 238335; Num of binary variables 187740; Num of integer  
variables 214740  
-----  
-----
```

An example for the stochastic problem solving with  
3 scenarios:

```
-----  
Inventories and unmet demand per scenario  
-----
```

Scenario 0

```
-----  
Inventories (amounts, full costs)
```

```
Close Supply Node: 5473.99, 54739.90  
Satellite Nodes: 5442.01, 54420.10  
Customer Nodes: 5726.91, 114538.20  
-----
```

Unmet Demand (amount, full cost)

```
Customer Nodes: 2162.00, 2162.00  
-----
```

Scenario 1

```
-----  
Inventories (amounts, full costs)
```

```
Close Supply Node: 5441.96, 54419.60  
Satellite Nodes: 5424.04, 54240.40
```



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Customer Nodes: 5404.05, 108081.00

-----  
Unmet Demand (amount, full cost)

Customer Nodes: 17280.00, 17280.00

-----  
Scenario 2

-----  
Inventories (amounts, full costs)

Close Supply Node: 5442.00, 54420.00

Satellite Nodes: 5424.00, 54240.00

Customer Nodes: 5410.12, 108202.40

-----  
Unmet Demand (amount, full cost)

Customer Nodes: 32400.00, 32400.00

-----  
Transportation costs, inventory costs, and unmet demand

-----  
Transport (distances, full cost, weighted costs)

To Close Supply Node: 7200.0, 1.08E7, 2160000.0

To Satellite Nodes: 27273.178224035066, 1363658.9112017674, 272731.7822403553

To Customer Nodes: 27612.417206756985, 1380620.8603378353, 276124.17206756625

-----  
Inventories (amounts, full costs, weighted costs) - amounts and full costs averaged over scenarios

-----  
Close Supply Node: 5452.6499999999996, 54526.499999999996, 32715.8999999999976

Satellite Nodes: 5430.0166666665356, 54300.166666665355, 32580.099999992137

Customer Nodes: 5513.693333332166, 110273.866666664285, 66164.3199999986

-----  
Unmet Demand (amount, full cost, weighted cost) - amounts and full costs averaged over scenarios

-----  
Customer Nodes: 17280.666666666686, 17280.666666666686, 5.184199999999999E8

-----  
Input file name: /home/dathomson/pig/sDRel\_lp/sDRel\_lp\_MinAvgMax\_1.json

Output file name (.sol): /home/dathomson/solutions/sDRel\_lp\_MinAvgMax\_1\_wnT.sol

Number of scenarios: 3

Number of time periods: 15

Number of satellites: 3

Number of demand nodes: 12

Number of products: 1

Objective function weights (transp.cost: F-L, L-s, s-c; Inventory costs; Unmet demand: 0.2, 0.2, 0.2, 0.2, 10000.0,

Product compatibility constraints: false

Wait next time period to leave: true

Re-fueling constraints: false

-----  
Execution Time in secondt (tot, set by user, Gurobi runtime): 7579.434, 50400.0, 7567.536954879761

Objective value: 5.2126031627430755E8; Gap: 0.005920280324420959%

-----  
Number of constraints: 2162265

Number of variables (total, binary, integer): 805545, 619740, 668340

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Date: OCT. 30, 2013



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