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Estimation of Minimum Required Thrust for Spacecraft Collision Avoidance

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Defence R&D Canada – Ottawa

Technical Memorandum
DRDC Ottawa TM 2013-113
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Abstract

The minimum thrust required for a spacecraft to perform a collision avoidance maneuver, subject to a three day warning, has been estimated. The method used is based on the concept of a proximity region violation, where a collision risk is based on whether a space object violates the proximity region of another object. As such, a collision avoidance maneuver, such that the spacecraft is outside of the proximity region at the Time of Closest Approach (TCA), was determined. Hill's equations were used as part of an optimization algorithm in order to estimate the minimum thrust required for such a maneuver. This effort validates the concept that minimalistic thrusters with very small effective Δv could be used for performing collision avoidance with sufficient waning time.

Résumé

On a évalué la poussée minimale nécessaire pour qu'un engin spatial effectue une manœuvre d'évitement de collision, avec préavis de trois jours. La technique utilisée est basée sur le concept de violation d'une région à proximité où un risque de collision est basé sur le fait qu'un objet spatial viole la région à proximité d'un autre objet. On a ainsi déterminé une manœuvre d'évitement de collision telle que l'engin spatial se trouve à l'extérieur de la région à proximité au moment du rapprochement maximal (TCA). On a utilisé les équations de Hill dans le cadre d'un algorithme d'optimisation pour évaluer la pouss'ee minimale que nécessite une telle manœuvre. Cet effort valide le concept selon lequel on peut utiliser des propulseurs minimalistes de très faible Δv efficaces pour procéder à l'évitement d'une collision avec un préavis suffisant.

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Executive summary

Estimation of Minimum Required Thrust for Spacecraft Collision Avoidance

Patrick Gavigan; DRDC Ottawa TM 2013-113; Defence Research and Development Canada – Ottawa; November 2013.

Background: The minimum thrust required for a spacecraft to perform a collision avoidance maneuver, subject to a three day warning, has been estimated. The method used is based on the concept of a proximity region violation, where a collision risk is based on whether a space object violates the proximity region of another object. A collision avoidance maneuver, such that the spacecraft is outside of the proximity region at the Time of Closest Approach (TCA), was determined using the optimization of Hill's equations. This effort validates the concept that minimalistic thrusters with very small effective Δv could be effective for performing collision avoidance with sufficient waning time.

Principal results: It was found that very small Δv thrusts, ranging in magnitude from 0.01 m/s to 0.05 m/s, can be used for successful collision avoidance if applied at least 20 hours before the TCA in Low Earth Orbit (LEO). The fuel consumption cost of waiting to apply the thrust grows dramatically in the last 12 hours prior to the TCA. In Geosynchronous Earth Orbit (GEO) a thrust magnitude ranging from 0.15 m/s to 0.4 m/s is sufficient assuming the thrust occurs at least 24 hours prior to the TCA. Similar to the LEO case, the cost of waiting to apply the thrust grows dramatically in the last 24 hours prior to the TCA. In both cases, the optimized results confirmed that in-track thrusts are the optimal collision avoidance manoeuver.

Significance of results: Cold gas propulsion systems and solid MicroelectroMechanical System (MEMS) thrusters both have favourable performance for this application. Cold gas systems require approximately 0.014 g of propellant per kg of spacecraft mass. If a spacecraft were already to have such a system installed this could potentially be used for performing this maneuver. Single use solid thrusters require 0.0034 g of propellant per kg of spacecraft mass. Their miniaturized counterparts, the MEMS thrusters, offer an attractive option for spacecraft to use as they only require surface area on an external body panel of the spacecraft as opposed to more complex systems. Their performance does, however, require the largest proportion of mass per kg of spacecraft mass, requiring 0.091 g of propellant.

Future work: This work could be extended by using spacecraft collision probability to characterize the risk of collision. Using this scheme, a successful avoidance maneuver would be defined by having the conjunction risk drop below a determined acceptable risk threshold. Additional work could also be done to investigate potential thruster systems that could provide the necessary Δv for this application while remaining minimally intrusive to the design of the spacecraft bus systems. Finally, depending on the mission parameters, the costs of adding a restitution maneuver could be considered.

Sommaire

Estimation of Minimum Required Thrust for Spacecraft Collision Avoidance

Patrick Gavigan ; DRDC Ottawa TM 2013-113 ; Recherche et développement pour la défense Canada – Ottawa ; novembre 2013.

Contexte : On a évalué la poussée minimale nécessaire pour qu'un engin spatial effectue une manœuvre d'évitement de collision, avec préavis de trois jours. La technique utilisée est basée sur le concept de violation d'une région à proximité où un risque de collision est basé sur le fait qu'un objet spatial viole la région à proximité d'un autre objet. Grâce à l'optimisation des équations de Hill, on a déterminé une manœuvre d'évitement de collision telle que l'engin spatial se trouve à l'extérieur de la région à proximité au moment du rapprochement maximal (TCA). Cet effort valide le concept selon lequel des propulseurs minimalistes de très faible Δv peuvent être efficaces pour procéder à l'évitement d'une collision avec un préavis suffisant.

Résultats principaux : Il a été établi que l'on peut utiliser des poussées de très faible Δv, d'une magnitude de $0,01$ m/s à $0,05$ m/s, pour éviter avec succès une collision si elles sont appliquées au moins 20 heures avant le TCA en orbite terrestre basse (LEO). Le coût de la consommation en carburant en attente de l'application de la poussée augmente considérablement au cours des 12 dernières heures précédant le TCA. En orbite terrestre géosynchrone (GEO), une magnitude de poussée de $0,15$ m/s à $0,4$ m/s suffit pourvu que cette poussée survienne au moins 24 heures avant le TCA. De la même façon que dans le cas de la LEO, le coût en attente de l'application de la poussée augmente considérablement au cours des 24 dernières heures précédant le TCA. Dans les deux cas, les résultats optimisés ont permis de confirmer que les pouss'ees à l'intérieur de la trajectoire constituent la manœuvre optimale d'évitement des collisions.

Portée des résultats : Les systèmes de propulsion à gaz froid et les propulseurs solides à système microélectromécanique (MEMS) ont tous reçu une bonne évaluation pour cette application. Les systèmes de propulsion à gaz froid requièrent quelque $0,014$ g d'agent propulsif par kg de masse de l'engin spatial. Si un tel système était déjà installé sur un engin spatial, on pourrait l'utiliser pour effectuer cette manœuvre. Les propulseurs solides à usage unique requièrent $0,0034$ g d'agent propulsif par kg de masse de l'engin spatial. Leurs homologues miniaturisés, les propulseurs à MEMS, constituent une option attrayante à utiliser sur les engins spatiaux, car ils ne requièrent qu'une surface sur un panneau externe de la coque de l'engin spatial au lieu de systèmes plus complexes. Cependant, leur fonctionnement requiert la plus grande proportion de la masse par kg de masse de l'engin spatial, c'est à dire $0,091$ g d'agent propulsif.

Recherches futures : On pourrait étendre ces travaux en utilisant la probabilité de collision des engins spatiaux pour caractériser le risque de collision. En utilisant cette pratique, on définirait une manœuvre d'évitement réussie en faisant chuter le risque de conjonction sous un seuil de risque acceptable déterminé. On pourrait également procéder à des travaux additionnels pour étudier des systèmes de propulseurs potentiels qui pourraient fournir le Δv nécessaire à cette application, tout

en demeurant peu intrusifs quant à la conception des systèmes de bus de l’engin spatial. Pour terminer, selon les paramètres de mission, on pourrait étudier les coûts de l’ajout d’une manœuvre de restitution.

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1 Introduction

Due to the increasing amount of space debris in Low Earth Orbit (LEO) and Geosynchronous Earth Orbit (GEO), and the resulting increasing risk of satellite collisions, this study sought to determine the minimum thrust required for a spacecraft to perform a collision avoidance maneuver in the event that an orbital conjunction risk was detected. The method used is based on the concept of a proximity region violation, where the spacecraft position uncertainty is modeled as a fixed sized orbiting box centered on the spacecraft. A successful avoidance maneuver requires the spacecraft to be outside of its nominal proximity region at the time of the predicted Time of Closest Approach (TCA). A minimum thrust profile for this maneuver was sought in an effort to validate the potential for minimalistic thrusters, with very small effective Δv, to be effective for performing emergency collision avoidance. Such a thruster would ideally not be overly intrusive on the design of a spacecraft that would otherwise not require a propulsion system to complete its mission. For example, a surface mount solid propellant thruster array, such as the MicroelectroMechanical System (MEMS) device proposed in [2] could be considered. Such a system would not require the complex plumbing and tank fixtures which can drive mission costs upwards.

2 Background

This section introduces three major concepts used in this paper: relative motion using Hill's equations, the concept of a spacecraft proximity region, as well as a brief introduction of thruster technologies. Hill's Equations are presented in Section 2.1. The concept of the proximity region, in the context of satellite conjunctions in orbit, is introduced in Section 2.2. A brief introduction to spacecraft thruster technologies is presented in Section 2.3.

2.1 Hill's Equations

Relative motion, under the assumption of a two body circular orbital regime, is defined by the Hill's Equations, provided in Equation (1) [3, pp. 372-381]. These functions are more typically used for spacecraft formation flying applications, but can also be used to determine the relative motion of a spacecraft that has applied a thrust with respect to its orbital position had the thrust not been applied.

As defined in the Nomenclature, provided in Annex A, Δv is modeled as the initial speed differential induced by the thrust in the radial, in-track, and cross-track directions, represented in Equation (1) as $\dot{x}_o$, $\dot{y}_o$, and $\dot{z}_o$ respectively (in km/s). The initial relative position of the spacecraft in radial, in-track and cross-track directions, x_o, y_o, z_o respectively, is set to 0 km for this application. The relative position of the spacecraft with respect to the position it would have had the collision avoidance thrust not been fired in radial, in-track and cross-track directions is depicted as x, y, z in km. The speed of the spacecraft relative to the orbital position the spacecraft would be located had the collision avoidance thrust not been fired in radial, in-track and cross-track directions is $\dot{x}$, $\dot{y}$, $\dot{z}$ in km/s. The time relative to the TCA is represented as t in sec. The initial semi major axis of the spacecraft's orbit is represented as a in km. The Earth's standard gravitational parameter, μ, was set to $3.986\,004\,356 \times 10^5 \frac{\mathrm{km}^3}{\mathrm{s}^2}$ [1]. Finally, the spacecraft orbital angular speed is depicted as ω in rad/sec.

$$
\begin{aligned}
x(t) &= \frac{\dot{x}_o}{\omega}\sin(\omega t) - \left(3x_o + \frac{2\dot{y}_o}{\omega}\right)\cos(\omega t) + \left(4x_o + \frac{2\dot{y}_o}{\omega}\right) \\
y(t) &= \left(6x_o + \frac{4\dot{y}_o}{\omega}\right)\sin(\omega t) + \frac{2\dot{x}_o}{\omega}\cos(\omega t) - (6\omega x_o + 3\dot{y}_o)\,t + \left(y_o - \frac{2\dot{x}_o}{\omega}\right) \\
z(t) &= z_o\cos(\omega t) + \frac{\dot{z}_o}{\omega}\sin(\omega t) \\
\dot{x}(t) &= \dot{x}_o\cos(\omega t) + (3\omega x_o + 2\dot{y}_o)\sin(\omega t) \\
\dot{y}(t) &= (6\omega x_o + 4\dot{y}_o)\cos(\omega t) - 2\dot{x}_o\sin(\omega t) - (6\omega x_o + 3\dot{y}_o) \\
\dot{z}(t) &= -z_o\omega\sin(\omega t) + \dot{z}_o\cos(\omega t) \\
\omega &= \sqrt{\frac{\mu}{a^3}}
\end{aligned}
\tag{1}
$$

2.2 Proximity Region

A proximity region is an imaginary geometric box surrounding a satellite which defines a volume which another satellite should not traverse. A violation of this box is considered to be an elevated risk of satellite collision. Its dimensions are primarily defined based on measurement error from observations by the Space Surveillance Network (SSN). Table 1 provides the half dimensions of the proximity region for the two main Earth orbital regimes, LEO and GEO. A conjunction threat is considered to exist if another spacecraft is projected to enter the proximity region. Therefore, it can be assumed that if the spacecraft is not in its proximity region during the TCA that the risk of collision is effectively avoided. Therefore, a successful collision avoidance maneuver is one which enables the spacecraft to ensure that no other object is inside its proximity region at the TCA. For the purposes of this paper a restitution maneuver, a maneuver which returns the satellite to its original orbit is not considered.

Table 1: *Typical Control Box Half Dimensions.*

Orbit Type	**Radial** (x_p)	**In-Track** (y_p)	**Cross-Track** (z_p)
LEO	0.2 km	1 km	1 km
GEO	10 km	10 km	10 km

2.3 Thruster Technologies

A high level description of various thruster technologies is presented in Table 2. Typical specific impulse (I_{sp}) values from [1, pp. 533] are shown and related to the propellant exhaust velocity, v_e, using Equation (2). Parameters for the MEMS thruster technology were obtained from [2]. A more substantive comparison of these thruster technologies in the context of this application is presented in Section 4.2.

$$I_{sp} = \frac{v_e}{g_o} \tag{2}$$

Table 2: *Typical Spacecraft Thruster Technologies [1, pp.529-541], [2].*

Thruster Type	**Typical I_{sp} (s)**	**Typical v_e (m/s^2)**	**Qualitative Description**
Cold Gas	45–73	441.45–716.13	Expulsion of compressed gas, no combustion, requires plumbing, valves and pressure vessels.
Solid	290–304	2844.9–2982.24	Simple, single use, no plumbing for pressurized fuels, may include nozzles for flow and direction control. Cannot be stopped once fired.
MEMS	11.2	110	Array or very small single use solid thrusters surface mounted on spacecraft body. Thrusters can be fired individually or in groups.
Liquid Monopropellant	200–235	1962–2305.35	Single liquid fuel combustion in the presence of a catalyst to generate high pressure gases for thrust. Requires plumbing, valves, pressure vessels and thermal control of the working fluid.
Liquid Bipropellant	274–467	2687.94–4581.27	Fuel and oxidizer combustion to generate high pressure gases for thrust. Highly complex.
Electric	500–3000	4905–29430	1) Electrothermal thrusters heat propellant material and expel particles to generate thrust. 2) Electrostatic thrusters use electric fields to accelerate charged particles to generate thrust. Large power requirement for low Δv.

3 Methodology

To determine the minimum thrust required for a collision avoidance maneuver the Hill equations, discussed in Section 2.1, were optimized using Matlab's global optimization tools with the "fmincon" solver using "GlobalSearch" [4, 5]. A flow chart of the software routine is provided in Figure 1. The software routine's required input parameters include Δv_{max} which represents the maximum thrust that the spacecraft can generate (in km/s), the semi major axis of the orbit (a, in km), nearest acceptable distance between the spacecraft's nominal position other resident space objects before a conjunction warning is issued (in km for radial, in-track and cross-track directions). Also required is the time window during which an avoidance maneuver can be performed relative to the conjunction time, defined by the time interval t_{min} - t_{max} (in seconds). Ultimately, the optimization is seeking the optimal thrust time, t in seconds and values of $\dot{x}_o$, $\dot{y}_o$, and $\dot{z}_o$, representing the Δv required in radial, in-track and cross-track directions to perform a successful conjunction avoidance maneuver with minimum Δv consumed. This result is based on the assumption that the spacecraft must exit

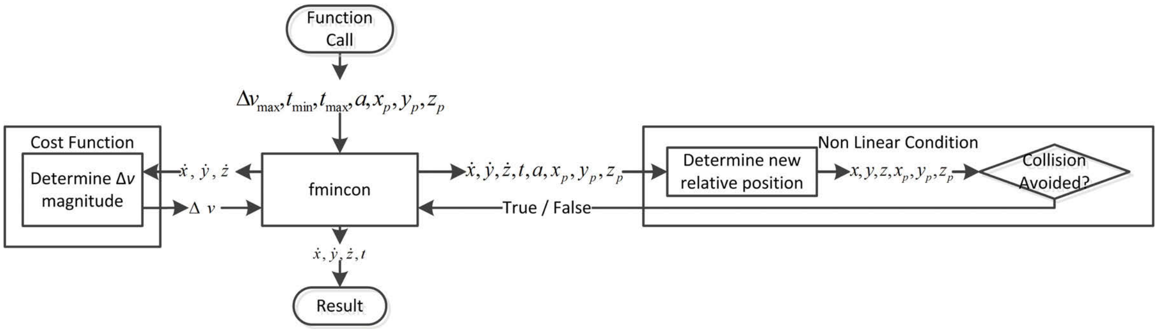

Figure 1: *Software Flow Chart.*

its proximity region due to a conjunction threat. Therefore, the optimizer must seek values of these parameters such that Δv is minimized (discussed in Section 3.1) and that the thrust is sufficient for the spacecraft to leave its proximity region (discussed in Section 3.2). The results from the optimization are also compared against the theoretical in-track collision avoidance thrust, discussed in Section 3.3.

3.1 Cost Function

The cost function is simply the magnitude of the thrusts in all three dimensions, as defined in Equation (3). This cost function must be used in conjunction with the nonlinear condition for a successful maneuver to be found, as discussed in Section 3.2, to determine if the spacecraft has indeed been pushed outside of its proximity region, otherwise the optimizer would never find a non-zero thrust magnitude.

$$\Delta v = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} \tag{3}$$

3.2 Non Linear Optimization Condition

In addition to the cost function, defined in Section 3.1, a nonlinear condition defined as C in Equation (4), is used for validating whether or not the thrust is sufficient for moving the spacecraft out of its proximity region or not. The optimization routine rejects solutions that do not satisfy the nonlinear condition. This condition requires that the spacecraft be outside of its proximity region at time t (in seconds), defined by x_p, y_p, z_p for the radial, in-track and cross-track directions respectively and measured in km. The spacecraft's relative position from the center of the proximity region at time t is defined by $x(t)$, $y(t)$, and $z(t)$ in the radial, in-track and cross-track directions respectively and measured in km and determine by calculating the Hill equations presented in Section 2.1.

$$C = (|x(t)| \geq x_p) \vee (|y(t)| \geq y_p) \vee (|z(t)| \geq z_p) \tag{4}$$

3.3 Theoretical In-Track Manoeuver

It is anticipated that the optimal collision avoidance manoeuver is a thrust in the in-track direction where Δv is given by Equation (5), where $\dot{x}_o$ and $\dot{z}_o$ are each set to 0.

$$\Delta v = \begin{bmatrix} 0 \\ \dot{y}_o \\ 0 \end{bmatrix} \tag{5}$$

The theoretical in-track collision avoidance thrust can be calculated by substituting Equation (5) into Equation (1) and calculating the required $\dot{y}_o$ which results in the spacecraft having a position of $y(t) = y_p$ at time t, where y_p is defines the distance to the edge of the proximity region in the in-track direction. Equation (6) defines the theoretical in-track collision avoidance thrust at time t based on Hill's Equations.

$$\Delta v = \frac{y_p}{\frac{4}{w}\sin(\omega t) - 3t} \tag{6}$$

4 Results

Results were generated for two common orbital regimes: LEO, assuming an altitude of 800 km, and GEO, assuming an altitude of 35,786 km. Section 4.1 provides the optimization results for both the LEO and GEO cases. Specifically, the minimum required Δv for each scenario is presented. Section 4.2 explores the implications of these results with respect to the thruster technologies discussed in Section 2.3.

4.1 Optimization Results

Results generated for this study found that the optimal collision-avoidance thrust was a thrust in the in-track direction only. Therefore, the results in this section focus on comparing the optimization results with the theoretical expected collision avoidance thrust in the in-track direction. It was noted, as a result, that smaller thrusts in this direction were required in order for the spacecraft to leave its proximity region relative to the other thrust directions. Section 4.1.1 discusses the results for the LEO case and Section 4.1.2 discusses the results for the GEO case.

4.1.1 LEO Case

The optimized results for the LEO case are in Figure 2. This figure shows the minimum Δv that a thrust would have to generate, with respect to thrust time relative to the TCA, in order to successfully perform a collision avoidance maneuver. Due to the complexity of the optimization problem, the optimizer would occasionally converge on non-optimal solutions or not converge at all. These

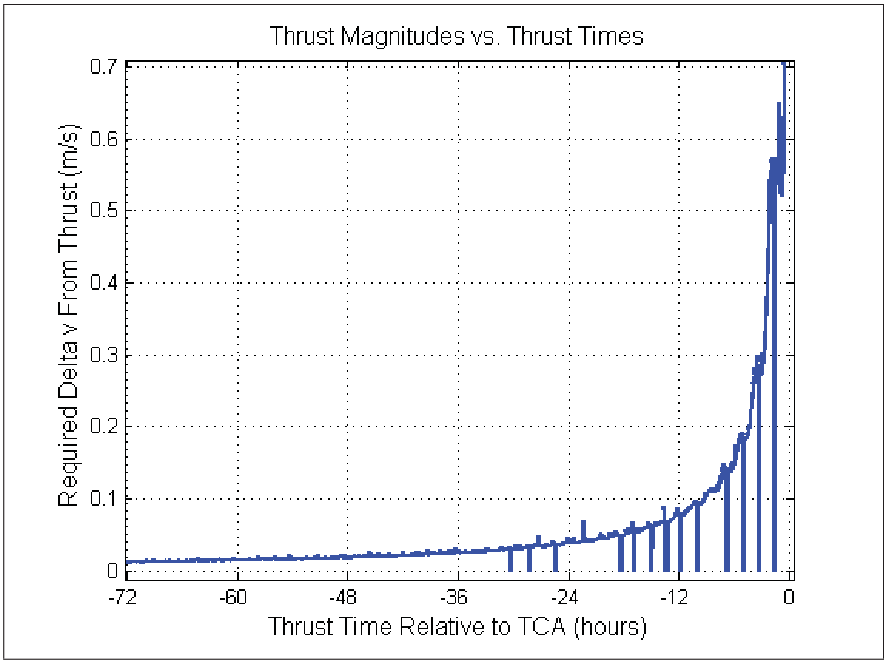

Figure 2: *Optimization Result for Minimum Required Collision Avoidance* Δv *for LEO.*

instances are visible in the curve as discontinuities in the data. That noted, these optimisation results closely match the theoretical required in-track collision avoidance thrust, shown in Figure 3. A numerical derivative of this in-track thrust, showing the rate of change of the cost of delaying the collision avoidance maneuver, is provided in Figure 4.

These results demonstrate that, in the time frame from 72 hours to approximately 20 hours prior to the TCA a very small Δv, ranging in magnitude from 0.01 m/s, to 0.05 m/s can be used for successful collision avoidance. The numerical derivative also shows that the cost of delay in action in terms of Δv, possibly due to lack of warning or reliable data, grows relatively linearly until the final 24 hours before TCA. By contrast, there is significant growth in the rate of change of the cost, in terms of Δv, for applying any mitigation thrust in the final 24 hours before the TCA. The rate of change in the final 12 hours is extremely high. Therefore, any decision to act should be made as early as possible, ideally before the final 24 hour period before the TCA. Finally, these results demonstrate that the optimal collision avoidance thrust is in the in-track direction and not in the cross-track or radial directions.

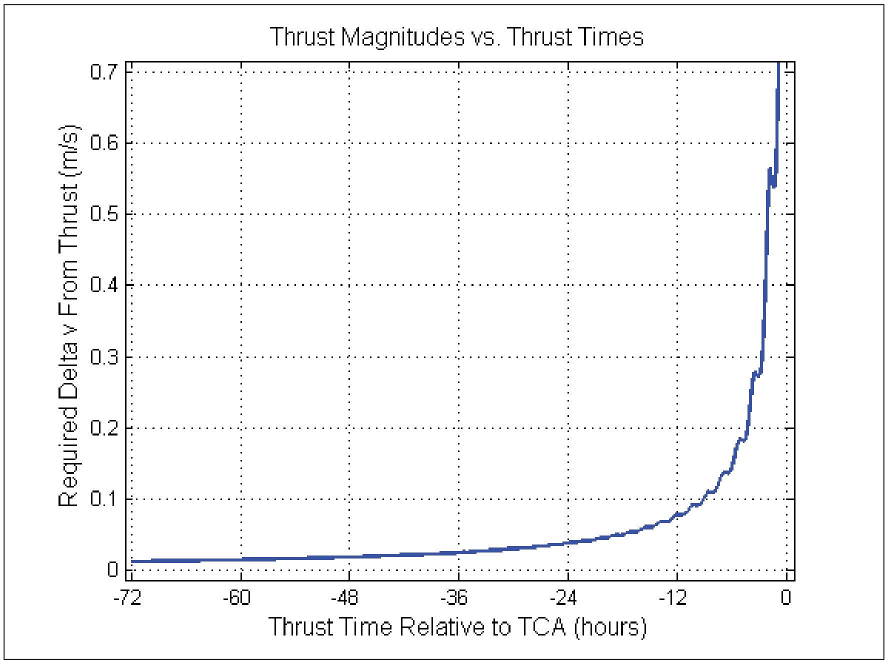

Figure 3: *Theoretical Minimum Required In-Track Collision Avoidance Δv for LEO.*

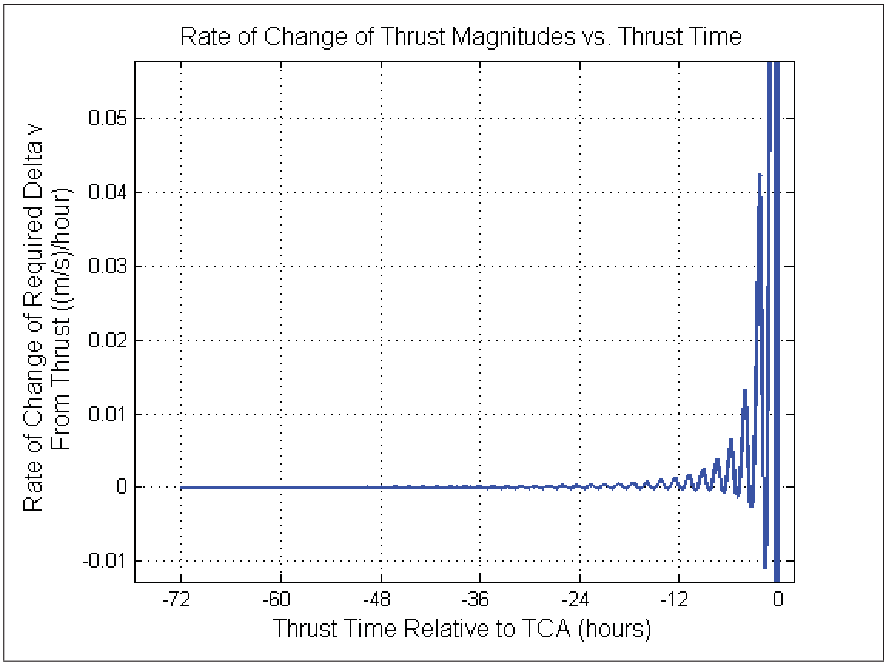

Figure 4: *Derivative of Minimum Required In-Track Collision Avoidance Δv for LEO.*

4.1.2 GEO Case

The optimized results for the GEO case are in Figure 5. This figure shows the optimization result for finding the minimum Δv that a thrust would have to generate with respect to thrust time relative to the TCA in order to successfully perform a collision avoidance maneuver. Similar to the LEO case, due to the complexity of the optimization problem, the optimizer was noted to occasionally converge on non-optimal solutions or not converge at all. These instances are visible in the curve as discontinuities in the data. Also, as in the LEO case, the results were noted to match the theoretical require in-track collision avoidance thrust, shown in Figure 6. A numerical derivative of this in-track thrust, showing the rate of change of the cost of delaying the collision avoidance maneuver, is provided in Figure 7.

These results demonstrate that, in the time frame ranging from 72 hours to approximately 24 hours prior to the TCA a small Δv, ranging in magnitude from approximately 0.15 m/s to 0.4 m/s, can be used for performing successful collision avoidance manoeuvres. The numerical derivative also shows that the cost of delay in action in terms of Δv, possibly due to lack of warning or reliable data, grows relatively slowly until the final 40 hours before the TCA. Although there is a slight dip in the rate of growth at 24 hours, there is significant growth in the rate of change of the cost, in terms of Δv, for applying any mitigation thrust in the final 24 hours before the TCA.

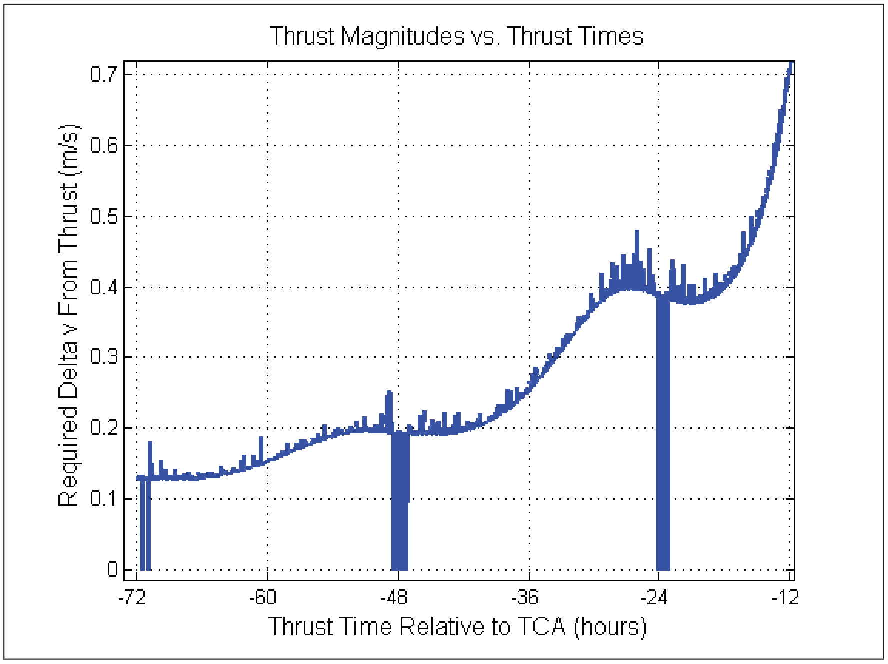

Figure 5: *Optimization Result for Minimum Required Collision Avoidance* Δv *for GEO.*

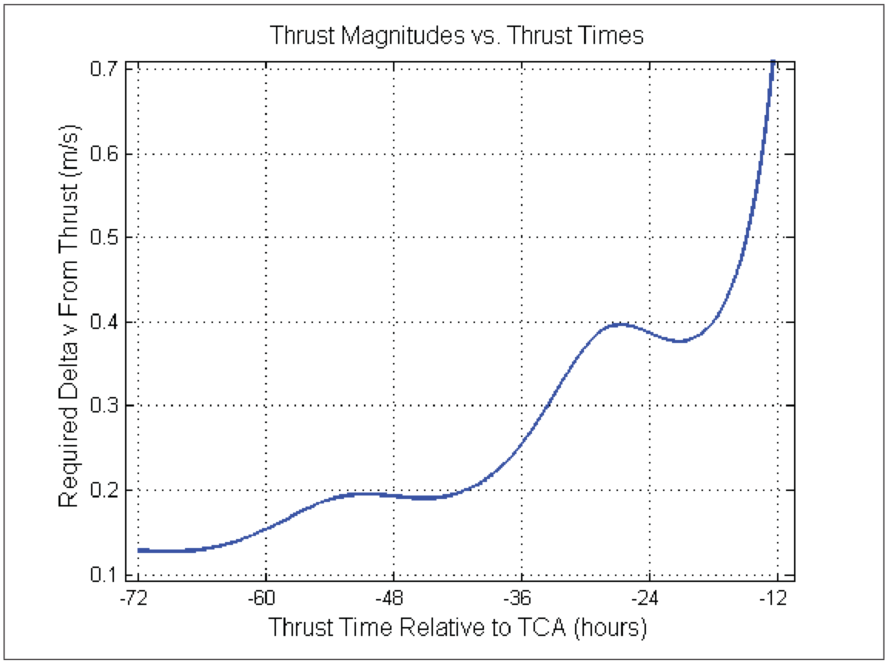

Figure 6: *Theoretical Minimum Required In-Track Collision Avoidance* Δv *for GEO.*

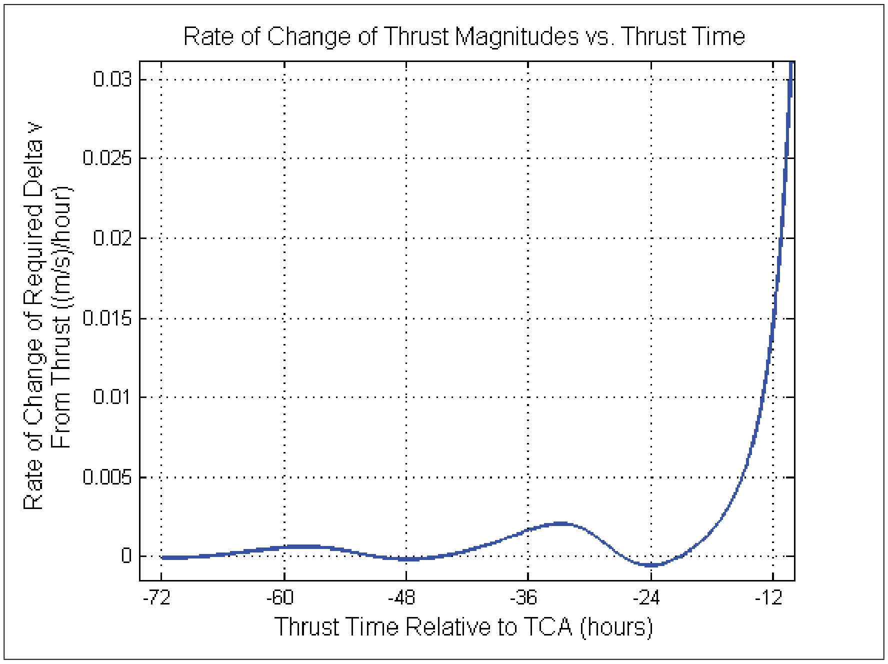

Figure 7: *Derivative of Minimum Required In-Track Collision Avoidance* Δv *for GEO.*

4.2 Thruster Performance Implications

This section compares the effectiveness of different thrusters introduced in Section 2.3 based on the results in Section 4.1. Using the rocket equation, provided in Equation (7), as per [1, pp. 531], and assuming a post thrust spacecraft mass of m_f, propellant mass of m_p and an initial spacecraft mass of m_o, and their intrinsic relationship provided in Equation (8), the rocket equation was rearranged in order to determine the required mass of the propellant required for thrusts, provided in Equation (9).

$$\Delta v = v_e \ln \frac{m_o}{m_f} \tag{7}$$

$$m_o = m_f + m_p \tag{8}$$

$$m_p = m_f \left(e^{\frac{\Delta v}{v_e}} - 1 \right) \tag{9}$$

The required propellant mass, m_p in g, for performing a thrust with an effective Δv of 0.01 m/s per kg of spacecraft mass was calculated using Equation (9). This was calculated by setting m_f to 1000g, Δv to 0.01 m/s, and v_e to the values is Table 2 in m/s. The results from these calculations are presented in Table 3. Thrusts are assumed to have instantaneous effect.

Table 3: *Required Propellant Masses for Generating Δv of 0.01 m/s per kg of Spacecraft Mass.*

Thruster	**Minimum m_p, for Thrust Resulting in Δv of 0.01 m/s, per kg of Spacecraft Mass (g)**
Electric	0.002
Liquid Bipropellant	0.0022
Solid	0.0034
Liquid Monopropellant	0.0043
Cold Gas	0.014
MEMS	0.091

The thruster results in Table 3 are listed in order from smaller required propellant mass to highest required propellant mass. The electric propulsion system requires the least amount of propellant, at 0.002 g of propellant per kg of spacecraft mass. Liquid bipropellant and monopropellant require very little propellant as well, requiring 0.0022 g and 0.0043 g respectively. These systems, although they offer attractive theoretical performance, would be very difficult to justify as these are typically much larger systems and typically not used for low impulse maneuvers on this order of magnitude. Cold gas propulsion systems may be effective for this application, requiring 0.014 g of propellant per kg of spacecraft mass, however like its liquid counterparts it requires the use of compressed gas tanks, plumbing and valves. If a spacecraft were already to have such a system installed this could potentially be used for performing this maneuver. Single use solid thrusters require 0.0034 g of propellant per kg of spacecraft mass. Their miniaturized counterparts, the MEMS thrusters, offer an attractive option for spacecraft to use as they only require surface area on an external body panel of the spacecraft as opposed to more complex systems. Their performance does require the largest proportion of mass per kg of spacecraft mass, requiring 0.091 g of propellant.

5 Conclusion

This study sought to estimate the minimum required Δv needed for a successful conjunction avoidance maneuver. This work was based on the presumption of a proximity region around the spacecraft, the violation of which indicated a conjunction event. It was found that with sufficient warning and planning, a very small Δv can be used for performing this maneuver. With sufficient warning, very small Δv thrusts, ranging in magnitude from 0.01 m/s to 0.05 m/s, can be used for successful collision avoidance if applied at least 20 hours before the TCA in LEO. The cost of waiting to apply the thrust grows dramatically in the last 12 hours prior to the TCA. In GEO a thrust magnitude ranging from 0.15 m/s to 0.4 m/s is sufficient, assuming the thrust occurs at least 24 hours prior to the TCA. Similar to the LEO case, the cost of waiting to apply the thrust grows dramatically in the last 24 hours prior to the TCA. In both cases, the optimized results confirmed that in-track thrusts are the optimal collision avoidance manoeuver.

Cold gas propulsion systems and solid MEMS thrusters were both noted to have favourable performance for this application. Cold gas systems require approximately 0.014 g of propellant per kg of spacecraft mass. If a spacecraft were already to have such a system installed this could potentially be used for performing this maneuver. Single use solid thrusters require 0.0034 g of propellant per kg of spacecraft mass. Their miniaturized counterparts, the MEMS thrusters offer an attractive option for spacecraft to use as they only require surface area on an external body panel of the spacecraft. Their performance does require the largest proportion of mass per kg of spacecraft mass, requiring 0.091 g of propellant.

6 Future Work

An extension of this work would revisit this study using spacecraft position covariance to characterize the risk of collision. Using this scheme, a successful avoidance maneuver would be defined by having the conjunction risk drop below a determined acceptable probability of collision. Additional work could also be done to investigate potential thruster systems that could provide the necessary Δv for this application while remaining minimally intrusive to the design of the spacecraft bus systems. Finally, depending on the mission parameters, analysis of the costs of adding a restitution maneuver could be considered.

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Annex A: Nomenclature

a = Spacecraft semi major axis [km]

g_o = Gravity constant for Earth: $9.81\mathrm{m/s^2}$

I_{sp} = Specific Impulse [sec]

m_f = Spacecraft mass after firing thrust [kg]

m_o = Spacecraft mass prior to firing thrust [kg]

m_p = Propellant mass [kg]

t = Time [sec]

t_{min} = Minimum time before TCA for performing thrust [sec]

t_{max} = Maximum time before TCA for performing thrust [sec]

v_e = Propellant exhaust velocity from thruster [m/s]

Δv = Change in velocity of spacecraft due to thrust [km/s]

Δv_{max} = Maximum acceptable Δv [km/s]

μ = Standard gravitational parameter $[\frac{\mathrm{km}^3}{\mathrm{s}^2}]$. For Earth: $3.986\,004\,356 \times 10^5 \frac{\mathrm{km}^3}{\mathrm{s}^2}$ [1]

x, y, z = Relative position of the spacecraft with respect to the position it would have had the collision avoidance thrust not been fired in radial, in-track and cross-track directions respectively [km]

x_o, y_o, z_o = Initial relative position of spacecraft in radial, in-track and cross-track directions respectively; nominally 0 for this application [km]

$\dot{x}, \dot{y}, \dot{z}$ = Speed of spacecraft relative to the orbital position the spacecraft would be located had the collision avoidance thrust not been fired in radial, in-track and cross-track directions respectively [km/s]

$\dot{x}_o, \dot{y}_o, \dot{z}_o$ = Effective Δv of the collision avoidance thrust in the radial, in-track, and cross-track directions respectively [km/s]

x_p, y_p, z_p = Half dimensions of the spacecraft proximity region in radial, in-track and cross-track directions respectively [km]

ω = Spacecraft orbital angular speed [rad/sec]

Acronyms

DRDC	Defence Research and Development Canada
GEO	Geosynchronous Earth Orbit
LEO	Low Earth Orbit
MEMS	MicroelectroMechanical System
SSN	Space Surveillance Network
TCA	Time of Closest Approach

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The minimum thrust required for a spacecraft to perform a collision avoidance maneuver, subject to a three day warning, has been estimated. The method used is based on the concept of a proximity region violation, where a collision risk is based on whether a space object violates the proximity region of another object. As such, a collision avoidance maneuver, such that the spacecraft is outside of the proximity region at the TCA, was determined. Hill's equations were used as part of an optimization algorithm in order to estimate the minimum thrust required for such a maneuver. This effort validates the concept that minimalistic thrusters with very small effective Δv could be used for performing collision avoidance with sufficient waning time.

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