Design of Tactical Support Strategies in Military Logistics: Trade-offs Between Efficiency and Effectiveness

A Column and Cut Generation Modeling Methods

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Canadian Operational Support Command
Operational Research & Analysis

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Abstract

Military tactical logistics planning is concerned with the problem of distributing heterogeneous commodities (e.g., fuel, food, ammunition, etc.) from main operating bases to forward operating bases in a theatre of operations using a combination of heterogeneous transportation assets such as logistics trucks and tactical helicopters. Minimizing the sustainment cost while satisfying the operational demand under time and security constraints is of high importance for the Canadian Forces. In this study, a logistics planning model is developed to explore effective and efficient strategies for tactical logistics distribution. A mathematical optimization algorithm based on a column and cut generation technique, using Gomory-Chvátal rank-1 cuts, is developed to solve the problem.

This report presents details of a mathematical formulation and a solution algorithm along with an example application to demonstrate the methodology. Computational results are presented in order to measure the degree of efficiency and scalability of the proposed approach, and to study the trade-off between the efficiency and effectiveness in the resulting sustainment strategies.

Résumé

Le problème propre à la planification de la logistique tactique militaire concerne la distribution de biens hétèrogènes (p. ex., du carburant, de la nourriture, des munitions, etc.) depuis des bases d'opérations principales vers des bases d'opérations avancées dans un théâtre d'opérations, à l'aide de moyens de transport hétérogènes tels que des camions et des hélicoptères tactiques. Il est primordial pour les Forces canadiennes de réduire au minimum le coût du maintien en puissance, tout en répondant aux exigences opérationnelles malgré les contraintes qu'imposent le temps et la sécurité. Dans la présente étude, un modèle de planification logistique est mis au point pour explorer des stratégies efficientes et efficaces de distribution par les services de logistique tactiques. Un algorithme d'optimisation mathématique fondé sur le modèle de génération de colonnes et de coupes et utilisant les plans de coupe d'ordre 1 de Gomory-Chvàtal est élaboré pour régler le problème.

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Executive summary

Design of Tactical Support Strategies in Military Logistics: Trade-offs Between Efficiency and Effectiveness

S. Sebbah, A. Ghanmi, A. Boukhtouta; DRDC CORA TM 2011-211; Defence R&D Canada – CORA; December 2011.

Background: Military tactical logistics planning is concerned with the problem of distributing heterogeneous commodities (e.g., fuel, food, ammunition, etc.) from Main Operating Bases (MOBs) to Forward Operating Bases (FOBs) in a theatre of operations using a combination of heterogeneous transportation assets such as logistics trucks and tactical helicopters. Given the amount of supplies the Canadian Forces (CF) transports during each mission, optimization of the sustainment costs is becoming a primordial issue in order to effectively and efficiently continue to support the CF deployed troops.

In the design of supply chain strategies, decision-makers are facing the challenge of finding good trade-offs between efficiency and effectiveness, which are related to the cost of achieving an objective and the degree of satisfaction of that objective. The CF is continually making tactical decisions to strike good trade-offs between efficiency and effectiveness during deployment and sustainment phases. In this study, we focus on the design of tactical logistics strategies, which achieve different optimal balances between support efficiency and its effectiveness, i.e., meet the requirements of the troops with the optimal supply cost. We introduce a new concept "Quality-of-Support (QoS)" which is a combination of parameters that measure the effectiveness of the elaborated support strategies. These parameters are the lead-time, safety of routes, security in transportation of commodities, and reliability of transportation assets.

Objective and scope: The objective of this research is to develop a scalable and efficient mathematical optimization method to the military tactical logistics planning problem. Our main goal is to provide the military decision makers with a practical tactical logistics planning tool that will help in striking good trade-offs between efficiency and effectiveness in elaborating tactical support strategies. In this work, we tried to answer the question: if you are given a tactical logistics network, a set of heterogeneous transportation assets of different classes and modes (land, air) with different capacities, speed, range, etc., a set of heterogeneous commodities, a set of end-users requesting different commodities with different QoS requirements, then, what is the optimal way, in terms of cost, to load and route an optimal fleet-size

(to be determined) of transportation assets to carry the different commodities with their associated QoS requirements to their destinations?

Significance of contributions and results: In this study, a tactical logistics planning method, that is to be integrated into a logistics decision support system, is developed. We analyzed the trade-offs between cost, lead-time and safety of routes with different demand patterns of commodities. The obtained results can be used to estimate a weekly sustainment cost given the targeted QoS. Furthermore, for different distributions of demands over a given lead-time interval, we showed the distribution of the number of lifts over the classes of transportation assets. These statistics would be used to compose the optimal support fleet to respond to the terrain reality and the challenge of meeting the requirement of different demands. The analysis of the safety of land routes showed that this parameter has a significant impact on the support cost. Different levels of safety of routes were considered and the inherent cost was analyzed. The computational results obtained in this study and the proposed algorithmic approach are recommended in "what-if" tactical logistics analyses and design of large scale tactical logistics strategies, respectively. Different perspectives were explored and several alternatives were analyzed to help decision makers to strike different balances between efficiency and effectiveness in tactical support and to respond efficiently to different situations.

Future work: While the main focus of this work is on military sustainment strategies to achieve optimal trade-offs between efficiency and effectiveness, further improvement and extension of the methodology could be considered. First and foremost, the problem of the convoy formation and escorting, and how it affects the selection of transportation assets and their routing. The convoy escorting cost, which is part of the operational cost, is not considered in this study. Including this parameter in the study will probably change the modeling and solution approach and the optimal solution. Secondly, we assume different percentages of reliability of randomly selected land routes. However, if all land routes are unsafe and a limited escorting budget has to be met, then, a question arises which routes to secure in order to optimize the support cost? Or, how to form and route the convoys given the safety status of the routes?

Some other questions are still open: how similar or different are the two problems of optimizing the tactical logistics footprint (number of transportation assets) and the problem of optimizing the operational cost? Or, does optimization of the footprint implies the operational cost (and vice versa)? The answer is not straightforward and necessitates more investigation. In this study, we focused on the impact of safety of routes on the support cost, however, other facilities may fail as well as routes (e.g., MOBs, Depots). This research direction is highly promising, and our proposal of tactical logistics design strategies could be extended to survive intentional and accidental disruptions by building back-up strategies.

Sommaire

Design of Tactical Support Strategies in Military Logistics: Trade-offs Between Efficiency and Effectiveness

S. Sebbah, A. Ghanmi, A. Boukhtouta ; DRDC CORA TM 2011-211 ;
R & D pour la défense Canada – CARO ; décembre 2011.

Contexte : Le problème propre à la planification de la logistique tactique militaire concerne la distribution de biens hétérogènes (p. ex., du carburant, de la nourriture, des munitions, etc.) depuis des bases d'opérations principales (BOP) vers des bases d'opérations avancées (BOA) dans un théâtre d'opérations, à l'aide de moyens de transport hétérogènes tels que des camions et des hélicoptères tactiques. Vu la quantité d'approvisionnements que les FC transportent au cours de chaque mission, l'optimisation du maintien en puissance devient un objectif primordial pour que les Forces canadiennes puissent continuer de soutenir avec efficacité et efficience leurs troupes en déploiement. Quand les décideurs conçoivent des stratégies relatives à la chaîne d'approvisionnement, ils doivent trouver de bons compromis entre l'efficience et l'efficacité, compromis qui se rapportent au coût à subir pour atteindre un objectif et à la mesure dans laquelle ce dernier sera effectivement réalisé. Les FC prennent constamment des décisions tactiques pour en arriver à de bons compromis entre l'efficience et l'efficacité aux stades du déploiement et du maintien en puissance. Dans la présente étude, nous nous concentrons sur l'élaboration de stratégies de logistique tactique qui aboutissent à divers équilibres optimums entre l'efficience et l'efficacité du soutien (satisfaire aux besoins des troupes, tout en maintenant le coût du ravitaillement à un niveau optimal). Nous présentons un nouveau concept, soit celui de la "Qualité du Soutien" (QdS), qui résulte d'une combinaison de paramètres utilisés pour mesurer l'efficacité des stratégies de soutien élaborées. Ces paramètres sont les suivants : le délai d'exécution, la sécurité des itinéraires et du transport des biens, et la fiabilité des moyens de transport.

Objectif et portée : La présente étude vise à élaborer une méthode d'optimisation mathématique extensible et efficiente pour régler le problème posé par la planification de la logistique tactique militaire. Notre principal objectif consiste à procurer aux décideurs militaires un outil de planification pratique qui, en matière de logistique tactique, les aidera à trouver de bons compromis entre l'efficience et l'efficacité quand ils élaboreront leurs stratégies de soutien tactique. Nous avons tenté ici de répondre à la question suivante : étant donné un réseau logistique tactique, un ensemble de ressources de transport hétérogènes appartenant à diverses classes et modes (terre, air) et ayant différentes capacités, vitesses et autonomies, etc., une

gamme de biens hétérogènes (besoins exprimés) et des utilisateurs finaux qui demandent différents biens et dont les besoins en matière de QdS varient, quelle est la façon optimale, quant au coût, de charger les différents biens à bord d'une flotte de taille optimale (à définir) pour qu'elle les transporte, en suivant l'itinéraire choisi, jusqu'à leurs destinations tout en répondant aux besoins connexes relatifs à la QdS ?

Importance des contributions et des résultats : Dans la présente étude, nous élaborons une méthode de planification de la logistique tactique qui doit être intégrée dans un système de soutien du processus décisionnel relatif à la logistique. Dans l'analyse, nous opérons des compromis entre le coût, le délai d'exécution et la sécurité des itinéraires dans le cadre de différentes répartitions des besoins. Les résultats obtenus pourraient servir à estimer le coût hebdomadaire du maintien en puissance, compte tenu de la QdS voulue. En outre, pour différentes répartitions des besoins au cours d'un délai d'exécution donné, nous avons montré la répartition du nombre de voyages entre les catégories de moyens de transport. Ces statistiques serviront ensuite à composer la flotte de soutien optimale en fonction des caractéristiques réelles du terrain et du défi consistant à répondre aux besoins des différents utilisateurs. L'analyse de la sécurité des routes terrestres a montré que ce paramètre influe considérablement sur le coût du soutien. Nous avons examiné différents degrés de danger sur les routes et nous avons analysé le coût inhérent des déplacements sur ces dernières. Les résultats computationnels obtenus au cours de l'étude et la démarche algorithmique proposée sont recommandés dans des analyses hypothétiques sur la logistique tactique et dans la conception de stratégies de grande envergure en matière de logistique tactique, respectivement. Nous avons envisagé diverses perspectives et analysé plusieurs solutions de rechange pour aider les décideurs à trouver différents équilibres entre l'efficience et l'efficacité dans le soutien tactique et à faire face avec efficience à différentes situations.

Travail à venir : Notre étude porte principalement sur les stratégies militaires de maintien en puissance devant permettre d'opérer des compromis optimums entre l'efficience et l'efficacité, mais on pourrait envisager d'améliorer la méthodologie et d'en étendre encore plus la portée. Tout d'abord, songeons au problème de la formation et de l'escorte des convois et à la façon dont il influe sur le choix des moyens de transport et des itinéraires. Dans la présente étude, nous ne prenons pas en considération le coût de l'escorte des convois, qui fait partie du coût opérationnel. Si l'on inclut ce paramètre dans l'étude, cela changera sans doute l'approche de la modélisation et de la définition de la solution ainsi que la solution optimale. En second lieu, nous supposons différents pourcentages de fiabilité à l'égard d'itinéraires terrestres choisis au hasard. Cependant, si tous les itinéraires terrestres sont dangereux et qu'il faut s'en tenir à un budget limité pour l'escorte, la question se pose de savoir quels itinéraires il faut sécuriser pour optimiser le coût du soutien. Ou encore, comment former et diriger les convois, étant donné le degré de sécurité de ces itinéraires ? D'autres questions sont encore en suspens : dans quelle mesure les deux problèmes qui consistent à optimiser les ressources de transport affectées à la logistique tactique, d'une part, et, d'autre part, le coût opérationnel sont-ils semblables, ou différents ? Ou encore, l'optimisation des ressources va-t-elle de pair avec le coût opérationnel (et vice versa) ? La réponse n'est pas évidente et elle nécessite d'autres recherches. Dans la présente étude, nous avons mis l'accent sur l'effet de la sécurité des itinéraires sur le coût du soutien. Cependant, hormis les itinéraires, d'autres éléments infrastructurels (p. ex., les BOP, les dépôts) peuvent flancher. L'orientation que notre recherche offre est très prometteuse, et notre proposition d'élaborer des stratégies de logistique tactique pourrait être élargie, de manière que l'on en vienne à dresser des stratégies auxiliaires pour survivre à des bouleversements intentionnels ou accidentels.

Table of contents

List of figures

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1 Introduction

1.1 Preliminaries

Military tactical logistics planning addresses the problem of distributing heterogeneous commodities (e.g., fuel, food, ammunition, etc.) from Main Operating Bases (MOBs) to Forward Operating Bases (FOBs) in a theatre of operations using a combination of heterogeneous transportation assets such as logistics trucks and tactical helicopters. The Canadian Forces (CF) makes extensive use of a variety of tactical transportation assets to supply its oversea deployments, and sometimes uses multiple sources of supply.

Given the amount of supplies the CF transports during each mission, optimization of the sustainment costs is becoming crucial to the effective and efficient support of CF deployed troops. In the design of supply chain distribution strategies, particularly in the military, there are different trade-offs between the capital/operational cost and the achievable supply performance [1–3]. In this report, we use these two metrics to measure the efficiency and the effectiveness of military support strategies, respectively. These trade-offs are key to developing the most efficient and effective logistics and supply chain management strategy. Focusing solely on the supply cost may results in strategies that are less flexible and not effective in meeting the requirements of deployed troops. For example, using a land transportation mode, e.g., trucks, to reduce transportation costs may increase the delivery time and the vulnerability of the transported supplies. Furthermore, a flexible supply strategy is the one that can be dynamically changed to respond to a new unexpected event with less disturbance of the whole supply chain. Such a supply strategy is usually not necessarily highly optimized [4–7].

In the design of supply chain strategies, decision-makers face the challenge of finding good trade-offs between efficiency and effectiveness, which are related to the cost of achieving an objective and the degree of satisfaction reached toward that objective. In the military, the trade-off between efficiency and effectiveness appears at different levels during the deployment and sustainment phases of operations. During the deployment phase, the operational support goals are to ensure deployment speed by reducing the time as much as possible while keeping its cost at its lowest value. Similarly, during the sustainment phase, the objective is to ensure that the deployed forces are able to achieve their objectives during this period while minimizing the cost to provide the required support level. Therefore, in the design of military supply strategies, the focus should be on cost-efficient strategies that guarantee some Quality-of-Supply (QoS) parameters (e.g., sustainment lead-time, security, integrity, etc.).

Organizations struggle to achieve efficiency and effectiveness. The CF, because of its size and complexity, is continually called to elaborate strategies and take decisions at different levels to balance between these two dual concepts. In terms of logistics, although integrating QoS considerations in the design of supply strategies may increase the supply cost, their advantages in some cases largely offset their costs. The scientific literature on the trade-off between efficiency and effectiveness with real-world characteristics in logistics planning is not rich. Some references [3, 4, 8, 9] provided qualitative studies and comparative analysis of the trade-off between the two concepts.

Several inter-related problems need to be addressed in the design of military logistics networks. The design problem includes strategic decisions, such as the location of the main and forward operating bases, forward support groups, communication technology, and transportation modes. Once the logistics network physical configuration is determined, the focus shifts to the tactical and operational level's decisions, such as distribution decisions of supplies within the network. Several studies have considered the location and distribution issues in the design of industrial logistics networks [10] and some in military strategic logistics [11–13].

In this study, we focus on the design of tactical logistics strategies, which achieve different optimal balances between supply efficiency and its effectiveness, i.e., meet the QoS requirements of the troops with the optimal supply cost. From the optimization point of view, our main concern is to find the optimal loading and routing of an optimized fleet-size of heterogeneous transportation assets, given transportation capacity and security constraints, to supply the different FOBs while meeting the expressed QoS in terms of supply lead-time and security.

1.2 Analysis of the problem and literature review

Our tactical logistics planning problem can be divided into two sub-problems: loading of commodities onto transportation assets and routing of the loaded transportation assets. These two problems are solved jointly to identify the optimal fleet mix of transportation assets to meet the demands of the end-users with the required QoS. For the loading of commodities sub-problem, in addition to the classical constraints inherent to the capacity (payload and bulk capacities) of the transportation assets, there are some additional constraints related to security issues (e.g., ammunition is transported separately) and reliability of transportation assets (e.g., some truck classes are more suitable, due to their reliability, to transport some classes of commodities than others). These constraints cannot be treated separately as they directly affect the loading of commodities and the selection of the appropriate transportation assets. The routing sub-problem involves finding the optimal routing of an optimal fleet mix of transportation assets to provide QoS guarantees to the different FOBs. In planning of the transportation assets routing, constraints inherent to the range of the used transportation assets, their cruising speed, and reliability should be taken into consideration in the selection of the routing plan. Planning of these two sub-problems cannot be done separately as they are related to each other, and their constraints overlap.

Our loading and routing sub-problems are closely related to the Multi-Bin Packing Problem (MBPP) [14] and the Capacitated Vehicle Routing Problem with Time Window (CVRPTW) [15], respectively. The MBPP requires the determination of a packing pattern of a given set of items into a minimum number of identical or different bins. While in the CVRPTW, the objective is to determine the optimal routing of a fleet of capacitated vehicles in order to supply clients who have time windows within which the deliveries must be made. By analogy, our loading problem involves loading of standard units of commodities (pallets) of different weights onto trucks and helicopters of different payload and bulk capacities. Furthermore, incompatibility constraints between goods and between goods and transportation classes are added in our case. The commodities-clustering constraints state that some classes of commodities are not mixed with others, and the reliability of transportation assets forbids the loading of some classes of commodities on some classes of transportation assets. In our tactical logistics problem, the transportation model involves routing of trucks and helicopters of different capacities to serve FOBs with time windows (lead time) supporting a split-delivery strategy. Indeed, as most of the demands cannot be loaded onto a single transportation asset, several trucks and/or helicopters would be needed to satisfy the demands of any FOB.

It is clear that our problem is $\mathcal{NP}-$Hard [1] since it generalizes the CVPR and the BPP. Different solutions to handle the loading and routing of transportation assets have been proposed to solve several logistics problems where the objective was to optimize the operational cost and meet different loading and routing constraints. In [15], Laporte presented an exhaustive review of the several extensions proposed for the VRP, and a classification of the different extensions was made according to some characteristics: (i) topology: demand locations to be satisfied, their number, number of depots, etc.; (ii) vehicles: homogeneous/heterogeneous fleet, fixed or variable fleet size, similar or different capacities, time windows for vehicle availability; (iii) supply strategy: split delivery, mixed delivery, time windows; (iv) objective: minimize the supply footprint, penalties in case where the service requirements are not met, penalties implying fleet size. Most of the vehicle routing problems are at least $\mathcal{NP}$-hard. A variety of solutions based on heuristics, and exact methods have been developed to the different versions of the problem [15–19].

Some previous studies of the VRP have considered different versions of the vehicles

1. Non-deterministic polynomial-time hard. The optimization problem, "what is the optimal solution of the loading and routing problem in our tactical logistics problem?", is NP-hard, since there is no easy way (polynomial-time algorithm) to determine if a solution is the optimal one.

loading problem. In [20], Rebeiro and Soumis treated vehicles as commodities. In their non-split VRP model, trips are pre-determined and the problem is to allocate one vehicle to each trip. The authors proposed a Column Generation (CG) solution method, and showed that the optimal Linear Problem (LP) relaxation gives a good bound for the Integer Linear Problem (ILP). In [21], Fisher *et al*. also treated vehicles as commodities. The authors proposed a mixed-delivery VRP where each vehicle is restricted to carry one order at a time. Approximation algorithms have been proposed to effectively approach feasible good solutions. In [22], Malapert *et al*. proposed a constrained programming based approach to the two-dimensional pickup and delivery routing problem with loading constraints. Therein, the set of items requested by each client can be loaded onto a single vehicle, though split deliveries are not allowed. The objective was to find a partition of the clients into routes of a minimal total cost, such that for each route there exists a feasible loading of the items onto the vehicle loading surface that satisfies the capacity constraints. In [23], Iori *et al*. proposed an exact approach based on branch-and-cut for the vehicle routing problem with two-dimensional loading constraints. The considered case is a symmetric capacitated vehicle routing problem in which a fleet of K identical vehicles are used to serve customers. All vehicles are identical and have a known lift capacity and a single rectangular loading surface. All items of a given customer must be assigned to a single vehicle (called item clustering constraints). Furthermore, sequential loading constraints are added: items of a customer must not be blocked by items of customers to be visited later along the route.

The vehicle fleet mix optimization problem has been studied by some scientists at the Defence Research and Development Canada (DRDC). Kaluzny and Erkelens [24], in support of the project office management Medium Support Vehicle System (MSVS), studied the optimal mix of MSVS required for the replenishment of first line units. The study computed the minimum number of vehicles needed to transport the lift requirements using an optimization approach based on a classical ILP approach. Different configurations of transportation trucks with different transportation capabilities were considered and recommendations of a fleet of vehicles that minimizes the logistics footprint (number of trucks) of deployed combat service support units were formulated. Asiedu [25] has extended the work and presented a study to assess the impact of using 8-tonne variants of trucks on the optimal vehicle fleet mix. Asiedu and Hill [26] have proposed a new approach to tactical logistics support fleet mix planning, which includes some dynamic aspects of the logistics problem. The authors proposed an integrated simulation and classical ILP-based optimization method to optimize the vehicle fleet mix given a set of dynamic operational factors. Given a number of vehicles and containers, the proposed method selects the optimal fleet mix that optimizes a multi-term objective function.

Although the tactical logistics planning problem considered in this study involves vehicle routing and loading, it is quite different from the classical CVRPTW with loading constraints. Indeed, we have: (i) heterogeneous transportation assets of two modes (land, air) with different characteristics, i.e., capacity, speed, range, operational cost, reliability, which can transport different classes of commodities but not all; (ii) multiple classes of commodities with different weights which could be transported on some classes of transportation assets but not necessarily on others depending on the reliability of the transportation asset and the sensitivity of the transported commodity; (iii) demands for commodities are expressed with QoS parameters including time and reliability, although, for each destination and commodity, there are transportation and loading constraints to be observed; (iv) depending on the range of the transportation assets and the traveled distance, multiple transportation assets of different classes would be necessary to convey a given commodity to its destination. This configuration also happens when different lead-times have to be met; (v) the objective is to minimize an objective function of the operating cost, and at the same time meet the lead-time of the different FOBs, and guarantee reliable delivery of the commodities. Based on these new practical aspects of the problem, the routing and loading problems have to be re-designed in order to meet the different new requirements. To the best of our knowledge this problem has not been dealt with in the literature, and this study is the first to include the different new practical aspects of the problem.

1.3 Objective and scope

The objective of this research is to develop a scalable and efficient mathematical optimization method to the military tactical logistics planning problem. Our main goal is to provide the military decision makers with a practical tactical logistics planning tool that will help in striking good trade-offs between efficiency and effectiveness in elaborating tactical support strategies. This study tries to answer this question: Given a tactical logistics network, a set of heterogeneous transportation assets of different classes and modes (land, air) with different capacities, speed, range, etc., a set of heterogeneous commodities, a set of end-users requesting different commodities with different QoS requirements, then, what is the optimal way, in terms of cost, to load and route an optimal fleet-size (to be determined) of transportation assets to carry the different commodities with their associated QoS requirements to their destinations?

This work is a major extension and enhancement over existing work in the literature. The following characteristics of the military tactical support model, constraints, and characteristics of the solution method are considered:

- Characteristics of the military tactical support model:
  - two modes of transportation including air and land, and heterogeneous transportation assets of different classes;

- heterogenous commodities of different classes;
- demands for commodities are associated with QoS parameters;
- each land route is characterized by a binary safety status (safe, not safe);
- each pair of transportation class and land route is characterized by a binary status (practicable or not) stating whether transportation assets of the class can travel on the specified land route.
- similarly, each pair of transportation class and class of commodity is characterized by a binary status (can be transported or not) stating whether transportation assets of the class can transport commodities of the specified class.
- furthermore, each pair of two classes of commodities is characterized by a binary status (can be clustered or not) stating whether commodities of the two classes can be clustered on the same transportation asset.
- demands of any destination cannot be conveyed using a single lift.

- constraints in the military tactical support model:
  - lead-time: demands for commodities issued at FOBs are required within different lead-times. Priorities are often associated with demands of commodities in military logistics. The lead-time of demands decreases as their priorities increase.
  - reliability of transportation assets and transportation safety: transportation assets of different classes have different levels of reliability, which could forbid some loading patterns, e.g., loading of ammunition on light support trucks. Furthermore, some commodities-clustering patterns are forbidden, because of safety reasons during transportation. For example, ammunition is never clustered with other commodities of other classes.
  - land routes safety: land routes are not necessarily safe. In this case, in order to ensure safe delivery of supplies, land routes known to be not safe are not used. Furthermore, depending on the nature of the terrain e.g., stony routes, some land route may be forbidden for some transportation assets of some specific classes.
- The main characteristics of the proposed methodology and solution method are summarized as follows:
  - mathematical decomposition method based on CG is developed to obtain an optimal solution to the Linear Problem (LP);
  - the CG approach is augmented with a cut generation strategy, based on Gomory-Chvátal rank-1 cuts, to cut off continuous solutions and strengthen the linear bound in the derivation process of the integer solutions;
  - three versions of the pricing model are developed to generate different transportation strategies in order to meet the requirements in terms of lead-time and safety;
  - a column and cut generation algorithm is developed to use the different pieces of the optimization models.
  - the optimization model is a generic one, and can have as input different other

classes of transportation assets and classes of commodities.

– the optimal combinations of trucks and helicopters are found for several values of lead-time and different demand patterns.

This study focuses on cost efficient support strategies that are effective in achieving a set of parameters. The main parameters we address regarding the support effectiveness are the on-time delivery of supplies, reliability of transportation assets, security in transportation of commodities, and safety of routes.

1.4 Organization

This report is organized as follows. Section 2 gives an overview of our tactical logistics network model. Section 3 provides mathematical optimization models and solution algorithms for the considered problem. Section 4 presents the experimental results and discusses the trade-off between efficiency and effectiveness. Section 5 concludes the report. Annex A contains the detailed Mixed Integer Linear Problems (MILP), and Annex B contains the computational measuring of the performance of the optimization algorithm.

2 Tactical support network model

Canadian Operational Support Command (CANOSCOM) has recently proposed a global military doctrine to provide enhanced operational support capability to a Joint Task Force (JTF). The Joint Task Force Support Component Concept (JTFSC) is defined as "... an integrated operational support formation of health services support, general support engineering, communication & information systems support, military police, personnel support, logistics and land equipment maintenance units and elements ..." [27]. The JTFSC gathers all operational support elements deployed with a JTF under a single JTFSC command. The JTFSC focus is general support to JTF. It comprises a small JTFSC command and control element along with a broad range of operational support functional capabilities. The component structure within which the JTFSC functions is depicted in Figure 1 [27].

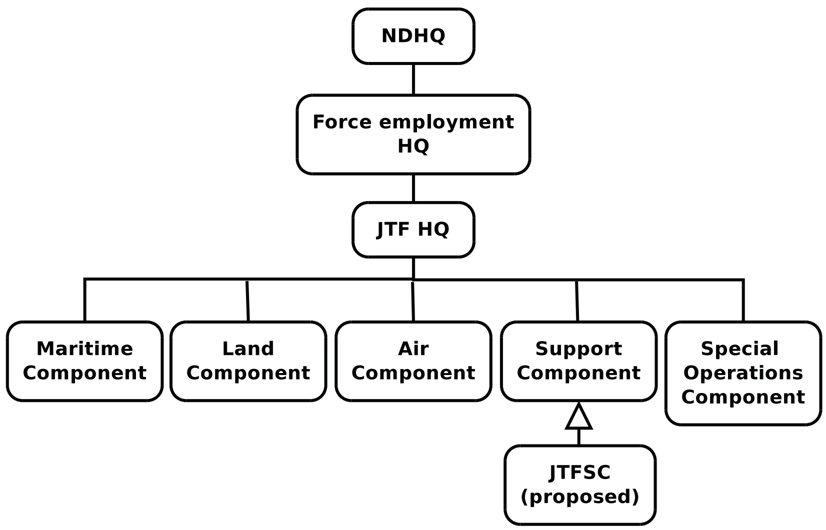


Figure 1: *Generic JTFSC Joint Task Force*

Based on the JTFSC schematic lay-out given in [27], we elaborated a more detailed logistics support network (Figure 2) illustrating the JTFSC support components, supported forces, and the flow of supplies. In a theatre of operations, the JTFSC main component is located near the Sea or Air Port Of Disembarkation ((S/A)SPOD), and the detachment and forward components are located at the entry point of the air forces and maritime forces, and at the MOBs and FOBs of the land forces. These components serve as the in-theatre delivery formation that enhances and sustains the JTF combat components.

In terms of supply flows, the JTFSC receives materiel and forces deploying at the in-theatre APOD and SPOD and delivers these forward to FOBs to meet tactical requirements. In addition to the APOD and SPOD, operations could also be externally supported by local sources (e.g., contractors), or established operational support hubs. In Figure 2, the JTFSC support components are interconnected by arrows, which define the supply network topology, showing the directions of the supply flows. Two supply network topologies, overlapping on some arrows, are shown to distinguish between land and air routes. These routes are established based on the range of the trucks and helicopters used in transportation. If any of the

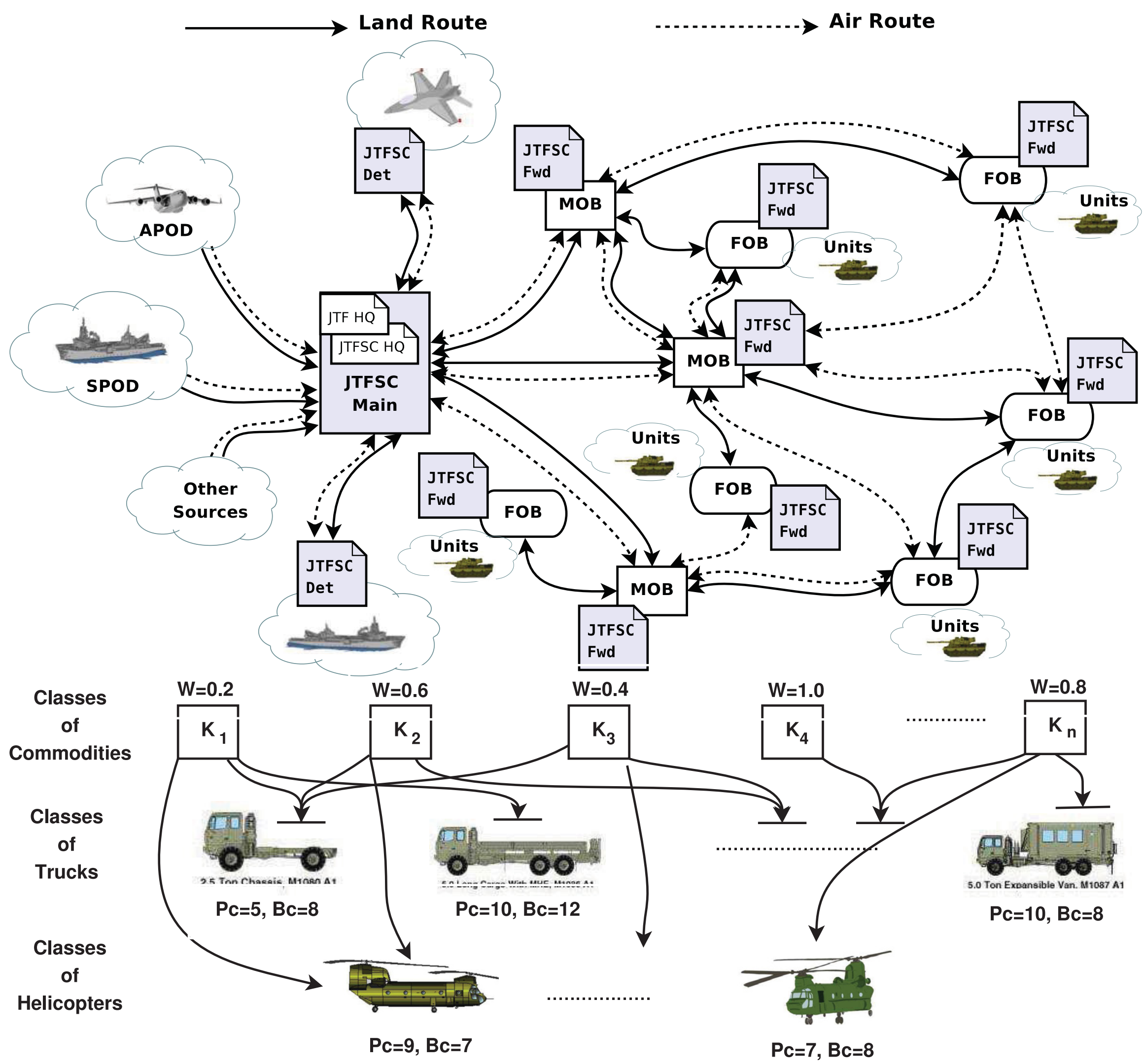
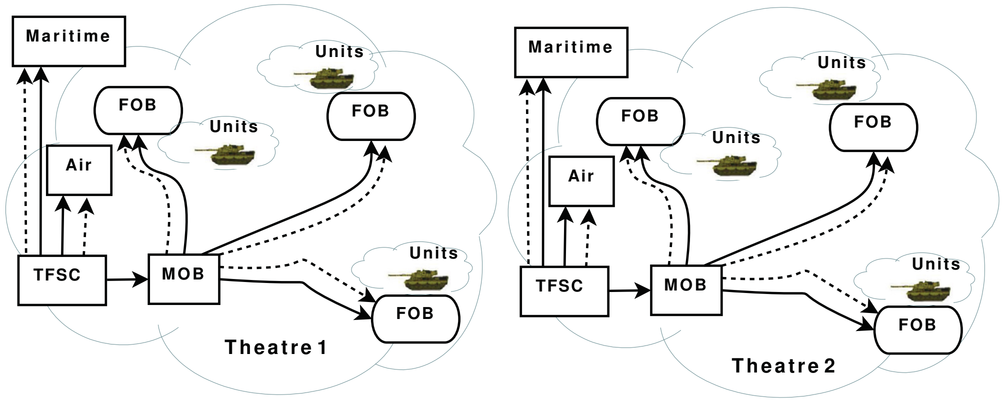


__Figure 2:__ JTFSC schematic theatre lay-out

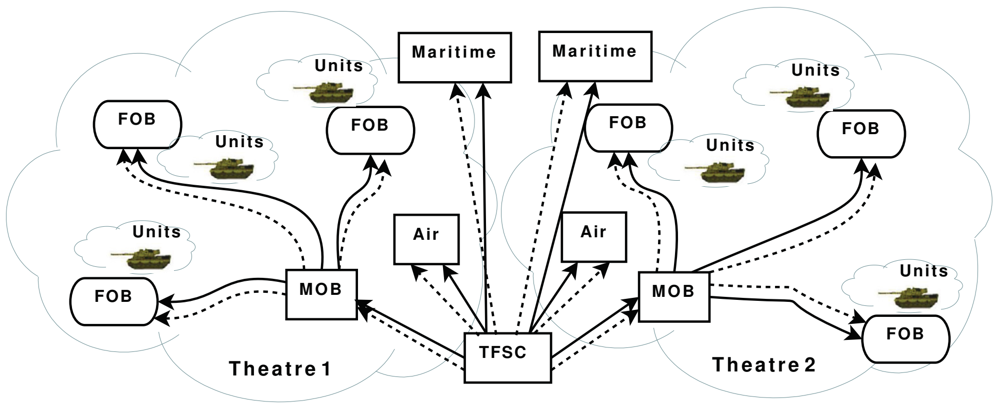
transportation assets, a truck or helicopter, can reach a point *B* from point *A*, then a corresponding route (air or land) is added to illustrate that in the related supply network topology. The arrows in the supply network topology give the reach-ability of the different network components using the different transportation assets.

The lower part of Figure 2 shows different classes of commodities, trucks, and helicopters. Each commodity class is associated with a weight per pallet, and each truck and helicopter class is associated with a payload (P_c) and bulk (B_c) capacity. The arrows departing from the commodities to the transportation assets show some potential transportation relationships. In our supply model, depending on the classes of the commodities (perishable, security sensitive, ammunition, etc.) a transportation asset of a given class could be used because of its adaptability (e.g., safety and reliability), but others may not be used. In addition, because of the danger of mixing some classes of supplies, some commodities are never transported together. What is not illustrated in the supply network topology of Figure 2 is the potential allocations of transportation assets to routes. The illustrated arrows are added whenever the range of any given class of transportation assets is greater or equal to the distance of the route separating the two end-locations.

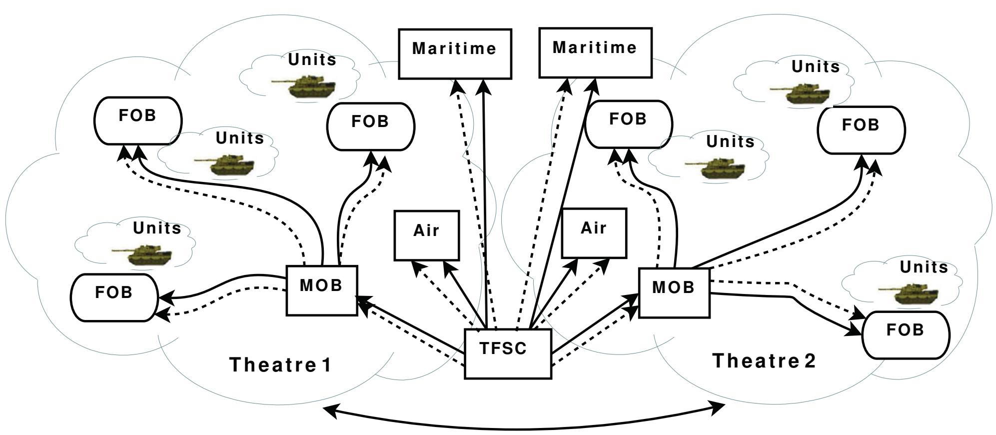
Figure 3 illustrates three more potential configurations of the JTFSC within different theatres: (a) Two independent JTFSCs provide support to two deployed JTFs; (b) a single JTFSC provides support to two deployed JTFs where supply does go through a theatre to another; (c) a single JTFSC provides support to two deployed JTFs where supply goes through one theatre to another (inter-theatre supply). These configurations are potential JTFSC deployments. The main JTFSC coordinates its efforts with its forward and detachment JTFSCs in the the tactical support at the different locations to transport commodities to their destinations along the most efficient ways.



(a) Different JTFSC to supply different MOBs in different theatres



(b) MOBs in different theatres supplied by the same JTFSC



(c) MOBs in different theatres supplied by the same JTFSC + inter-theatre supply

Figure 3: *Different JTFSC configurations*

2.1 Support efficiency

The support efficiency in our model is measured by the cost of the established transportation plan, i.e., the total capital expenditure to operate the selected transportation assets. The cost model is set as a combination of different cost components [28], including the operating cost, crew cost, spare-part cost, amortized capital cost, etc. The operating cost of a transportation asset depends on its hourly cost rate, its cruising speed, and the length of the traveled distance (or time). Equation (1) gives the mathematical expression of the operating cost.

$$Operating\,cost\,(\$) = \frac{\text{Hourly Cost Rate}(\$/h) \times \text{Distance}\,(km)}{\text{Cruising Speed}\,(km/h)} \tag{1}$$

In the selection process of transportation assets to optimize the operating cost, the advantage is given to transportation assets of classes which offer a good ratio between their hourly cost and cruising speed to achieve the targeted tactical support. Recall that the two support network topologies, air and land, although overlap, contain routes joining similar end-locations of different distances. This parameter is relevant as it enters in the computation of the operating cost.

2.2 Support effectiveness

Beyond war-fighting, the CF has been involved in a variety of critical-time and security-sensitive missions with regard to domestic security, disaster relief, peacekeeping, and humanitarian assistance, which require high support effectiveness (e,g., operation CADENCE: security to the 2010 G8 and G20 , operation HESTIA: response to 2010 Haiti earthquake) [29]. In general, the success and failure of such missions greatly depend on the effectiveness of the elaborated support strategy. The main issues we address regarding the support effectiveness are the on-time delivery of supplies, reliability of transportation assets, security in transportation of commodities, and safety of routes. In this research study, an effective support strategy is the one that takes into consideration these issues.

Demands for commodities are required within different lead-times depending on assigned priorities. For the JTFSC, it is of high importance to elaborate support strategies that manage the different priorities without incurring a high support cost. Our study of the security of supplies, reliability of transportation assets, and safety of routes is motivated by the fact that these issues have a direct impact on the on-time delivery of supplies.

The impact of safety of the network's assets on the supply chain cost has been considered by some authors in military and commercial logistics applications [30–

32]. In this study, our goal is to provide support taking into account the safety status of the supply routes. For example, if the support network topology is not 100% safe, then the model should update the support strategy to consider the new situation.

3 Mathematical modeling and solution methods

In logistics planning, two techniques have generally been used to solve the logistics problem: simulation and mathematical programming models.

Simulation is perceived by logistics planners as an essential tool for supply chain processes. Given the high costs associated with the implementation of some supply chain decisions, simulation provides a means for detailed evaluation of these decisions before their physical implementation. However, simulation models proposed in the literature are usually not aimed at solving the supply chain problem explicitly [33]. Simulation models are developed as a general descriptive representation of the supply chain activities resulting from any other mathematical model, for example, scheduling and capacity planning. Simulation can be perceived as a test-bed for implementing, testing, and evaluating decisions made by mathematical models. It is very useful in evaluating some factors, particularly those representing supply chain dynamic, stochastic and uncertainty factors. However, simulation is also characterized by some limitations. One of the main limitations of simulation, especially in supply chain optimization, is primarily the fact that it is a descriptive tool, which requires alternative scenarios to construct and explore by simulation. While in some case it is possible to construct such alternatives, but not possible in the general case because the number of alternatives is large.

The second class of solution models in logistics planning is mathematical programming. The most common type of mathematical programming models is linear and integer programming models, commonly known as LP and ILP. These models have the objective function and all constraints expressed as a linear function in variables. The objective function represents the objectives to optimize, and the constraints limit the values that decision variables can assume, i.e., the solution space of the model. Mathematical programming models are used to optimize decisions regarding certain activities subject to resource and budget constraints. Their main advantages are that they provide an accurate approximation of complex decision-making problems, an ability to efficiently explore even large solution spaces, and effectively supporting analysis of decisions made. There are also some limitations of mathematical programming models. They have difficulties representing some dynamic and stochastic aspects of the optimization problem. Additionally, solving large-scale problems, where the solution space is large, is computationally challenging and requires efficient tools to solve the problem.

In order to simultaneously optimize the fleet-size and the routing and loading of the selected transportation assets, we adopt a CG decomposition approach. CG is an efficient optimization method for solving large scale linear programs and its performance unfolds in solving integer linear programming problems [15, 20]. The main idea behind CG is that as most variables in many large programs are set to zero in the optimal solution, then, considering only a subset of the most promising variables would be an interesting approach to reduce the size of the problem and increase the scalability of the solution method.

In our tactical logistics problem, the number of combinations of loading and routing of an optimal set of transportation assets of given transportation classes is large. This number corresponds to the number of arrangements of the commodities onto the different transportation assets times the number of ways to route the optimal set of transportation assets, given their ranges and transportation capabilities, and the characteristics of the support network topology. Furthermore, the symmetry of the model resulting from the similitude of transportation assets and commodities would make any pure ILP solution method intractable. The motivation in using CG is both to reduce the size of the resulting ILP model and increase the scalability of the optimization algorithm.

As illustrated in Figure 2, our tactical logistics network model is composed of a set of locations, including MOBs, FOBs, and main JTFSC which is generally co-located within a MOB, and a set of air and land routes. We suppose that demands for tactical support are issued by end-users at FOBs, and the source of tactical support is the main JTFSC.

Following CG modeling, the whole tactical logistics problem is split into two problems: the master problem and the pricing problem. The master problem is a restricted version of the problem with only a subset of variables being considered. The pricing problem is a new problem created to identify a new promising variable, which would improve the linear objective function of the master problem. These two problems are executed concurrently until the optimal objective of the restricted master problem is achieved. In the next section we present our decomposition approach, and define the master and pricing problems.

3.1 Column generation decomposition based on support plans

Our CG decomposition approach is based on the separation of the optimization of the tactical support plans and their design. A support plan $p \in \mathcal{P}$, is a combination of heterogeneous transportation assets distributed along the support network routes to transport different amounts of commodities to different destinations. The distribution is performed in a way that sends at most one transportation asset on each route. Figure 4 illustrates a support plan where a set of heterogeneous trucks and helicopters are used to supply a set of FOBs with commodities of classes $k_1, k_2, \ldots k_n$.

This illustrative supply plan partially responds to the needs of the supplied FOBs, and does not supply all FOBs. In order to satisfy all demands, a combination of these support plans will be needed.

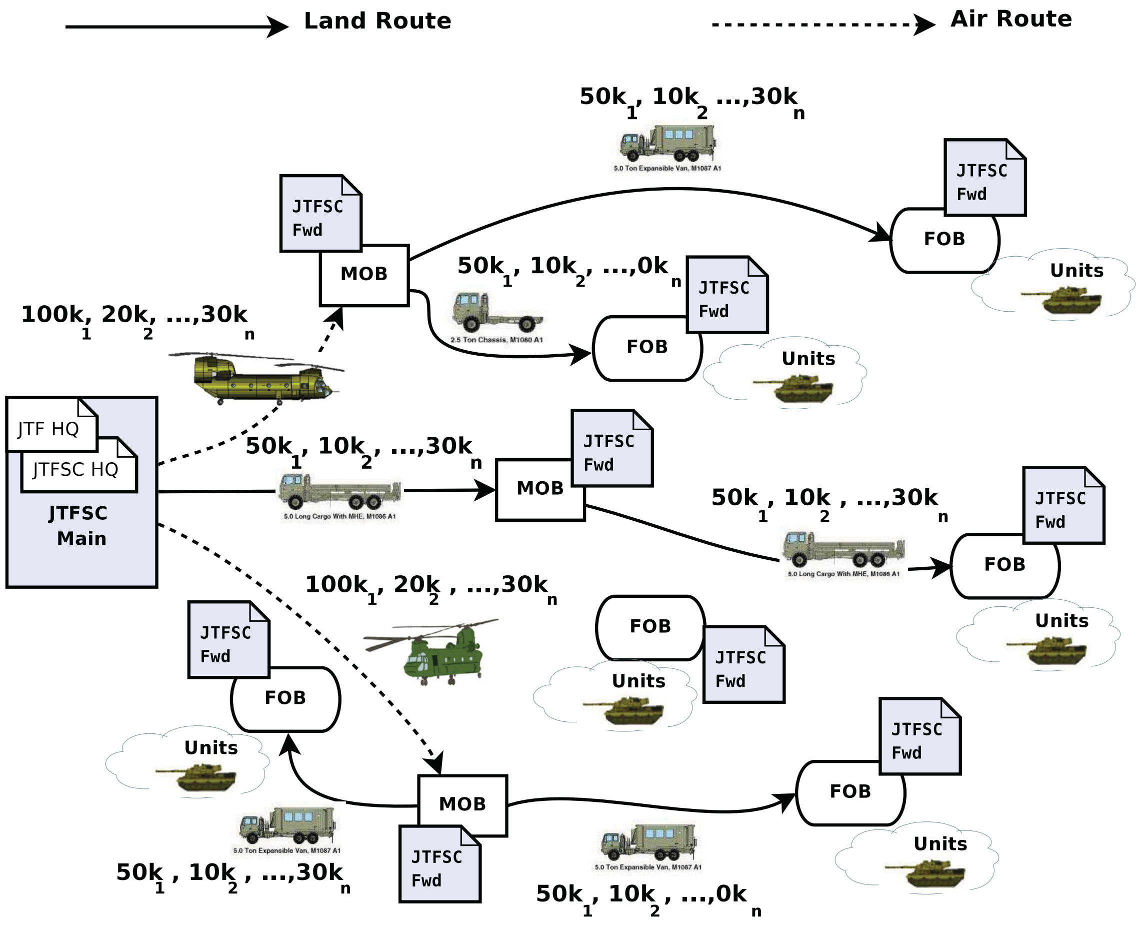


Figure 4: *A support plan*

We define these sets and parameters:

Sets:

$\mathcal{P}$	set of support plans, indexed by p,
$\mathcal{M}$	set of locations, including JTFSC, MOBs, and FOBs, indexed by m,
$\mathcal{N}$	set of destinations, indexed by n, $\mathcal{N} \subset \mathcal{M}$,
$\mathcal{R}$	set of routes, including land and air routes, indexed by r, recall that, to each route is associated two oriented arcs,
$\mathcal{V}$	set of classes of transportation assets, e.g., Heavy Logistics Vehicle Wheeled (HLVW) cargo trucks, CH-147D Chinook helicopters, indexed by v,
$\mathcal{K}$	set of classes of commodities, indexed by k,
$\mathcal{D}$	set of demands, indexed by $d_{n,k} \in \mathbb{N}$ which is the amount of commodities (number of pallets) of class k required at destination n,
$\omega^+(m)$	set of outgoing routes from location $m \in \mathcal{M}$, similarly $\omega^+(n)$ for $n \in \mathcal{N}$,
$\omega^-(m)$	set of incoming routes to location $m \in \mathcal{M}$, similarly $\omega^-(n)$ for $n \in \mathcal{N}$.

Parameters:

D_r	distance of route r (km),
Q_v	payload capacity of transportation assets of class v (Ton),
B_v	bulk capacity of transportation assets of class v (number of pallets),
G_v	range of transportation assets of class v (km),
S_v	cruising speed of transportation assets of class v (km/h),
W_k	weight of one unit (pallet) of commodity of class k (Ton),
$C_{v,r}$	cost of a transportation asset of class v on route r (\$) (see Equation 1),
C_p	cost of supply plan p. It is equal to the sum over the cost of each transportation asset used on the different routes within the supply plan p (\$),
$\left(a_{n,k}\right)_p$	amount of commodities of class k transported from the main JTFSC to destination n within the supply plan $p \in \mathcal{P}$ (number of pallets),
$T_{n,k}$	lead-time within which destination n requires a commodity of class k $(sec.)$,
$T_{v,r}$	travel time for transportation assets of class v on route r $(sec.)$,
$T_{v,m}$	servicing time (re-fuelling, transshipment) for transportation assets of class v at location m $(sec.)$.

The selected transportation assets within each support plan should respond to the QoS expressed by the end-users. Given that a support plan cannot meet all the demands, it is a combination of support plans, which are QoS compliant, that are used to construct an effective global support strategy. In our CG approach, the optimization and design of support plans are modelled by the master and pricing problems, respectively. These two models are presented below.

3.2 Master model

In the master model, we optimize the selection of support plans $p \in \mathcal{P}$. We define the following variables:

$u_p \in \mathbb{N}$ is the number of copies of support plan p. These variables allow to build up global support strategies by combining similar support plans. For example, within a support plan p, if a truck of class v is used on route r and $u_p = n\,(\in \mathbb{N})$, then, n similar trucks of class v will be used in the global support strategy on route r by plan p.

The ILP Model: the master ILP model is given as follows:

Minimize:

$$z^{\text{MASTER}} = \sum_{p \in \mathcal{P}} C_p \, u_p$$

subject to:

$$\sum_{p \in \mathcal{P}} \left(a_{n,k}\right)_p u_p = d_{k,n} \qquad n \in \mathcal{N}, k \in \mathcal{K} \tag{2}$$

$$u_p \in \mathbb{N} \qquad p \in \mathcal{P}. \tag{3}$$

The objective function measures the cost of the candidate support plans $p \in P$. Constraints (2) are the demand constraints to ensure that the requested commodities (number of pallets) of each class $k \in \mathcal{K}$ by each destination $n \in \mathcal{N}$ are met. Constraints (3) define the domain of variables u_p.

If we know all the potential support plans with their loaded vehicles, then a solution to this problem would be to construct all of them and provide them to the master model to select the most promising ones. However, this approach is intractable and impractical as the possible number of support plans is large, and we do not know how to explicitly enumerate all of them. Instead, we use a pricing problem to construct only a subset of the most promising support plans. The master model is a restricted version of the above model obtained by replacing $u_p \in \mathbb{N}$ in Constraints (3) by $u_p \in \mathbb{R}_+$ for all $p \in \mathcal{P}$.

3.3 Pricing model

The pricing problem, which is used to generate a promising support plan when it is executed, corresponds to the minimization of the reduced cost of the master problem subject to a set of design constraints. In this model, we do not assume any specific number of transportation assets, rather, the only available input information is the classes of transportation assets.

The reduced cost is expressed as the difference between the support plan cost and the number of pallets it transports multiplied by the values of the dual variables $\gamma_{n,k}$ associated with constraints (2). It is written as follows:

$$\overline{C}_p = C_p - \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} a_{n,k}\,\gamma_{n,k}{}^{2}$$

We define the following variables and parameters of the pricing problem for designing a single support plan:

Variables

$x_{v,r} \in \{0,1\}$ encodes the information whether a transportation asset of class $v \in \mathcal{V}$ is used on route $r \in \mathcal{R}$, or not. It is equal to

$$x_{v,r} = \begin{cases} 1 & \text{if a transportation asset of class } v \text{ is used on route } r, \\ 0 & \text{otherwise.} \end{cases}$$

$y_{n,k} \in \mathbb{N}$ for each destination $n \in \mathcal{N}$ and class of commodities $k \in \mathcal{K}$, it is equal to the amount of commodities k transported to destination n.

$y_{n,k,r} \in \mathbb{N}$ for each destination $n \in \mathcal{N}$, commodity $k \in \mathcal{K}$, and route $r \in \mathcal{R}$, it is equal to the amount of commodities of class k transported to destination n along route r.

Parameters

$\alpha_{v,k} \in \{0,1\}$ encodes the information whether transportation assets of class $v \in \mathcal{V}$ can transport commodities of class $k \in \mathcal{K}$, or not. It is equal to

$$\alpha_{v,k} = \begin{cases} 1 & \text{if transportation assets of class } v \text{ can transport commodities of class } k, \\ 0 & \text{otherwise.} \end{cases}$$

The expression of the reduced cost is re-written as a function of the variables of the pricing problem as follows:

$$\overline{C}_p = \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}} C_{v,r}\,x_{v,r} - \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} y_{n,k}\,\gamma_{n,k}.$$

The ILP Pricing Model:

Minimize:

$$\overline{C}_p = \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}} C_{v,r}\,x_{v,r} - \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} y_{n,k}\,\gamma_{n,k}$$

2. $a_{n,k}$ for the current support plan p is similar to $(a_{n,k})_p$ in the master model

subject to:

$$\sum_{r \in \omega^+(m)} y_{n,k,r} - \sum_{r \in \omega^-(m)} y_{n,k,r} = \begin{cases} y_{n,k} & \text{if } m = JTFSC \\ -y_{n,k} & \text{if } m \in FOBs \\ 0 & \text{otherwise} \end{cases} \qquad k \in \mathcal{K}, n \in \mathcal{N} \tag{4}$$

$$\sum_{\substack{v \in \mathcal{V} \\ G_v \leq D_r}} x_{v,r} \leq 1 \qquad r \in \mathcal{R} \tag{5}$$

$$y_{n,k,r} \leq \sum_{v \in \mathcal{V}} \alpha_{v,k} B_v x_{v,r} \qquad n \in \mathcal{N}, k \in \mathcal{K}, r \in \mathcal{R} \tag{6}$$

$$\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} W_k y_{n,k,r} \leq \sum_{v \in \mathcal{V}} Q_v x_{v,r} \qquad r \in \mathcal{R} \tag{7}$$

$$\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} y_{n,k,r} \leq \sum_{v \in \mathcal{V}} B_v x_{v,r} \qquad r \in \mathcal{R} \tag{8}$$

$$x_{v,r} \in \{0, 1\} \qquad r \in \mathcal{R}, v \in \mathcal{V} \tag{9}$$

$$y_{n,k} \in \mathbb{N} \qquad k \in \mathcal{K}, n \in \mathcal{N}. \tag{10}$$

$$y_{n,k,r} \in \mathbb{N} \qquad k \in \mathcal{K}, n \in \mathcal{N}, r \in \mathcal{R}. \tag{11}$$

The objective function of the pricing problem, which corresponds to the minimization of the reduced cost of the restricted master problem, minimizes the supply cost that maximizes the transportation capability of the current support plan according to the values of the dual variables ($\gamma_{n,k}$) associated with constraints (2). These dual values are used as the communication vector between the master and pricing models. Using these values, the master problem guides the pricing in the building of the most efficient and effective support plan. Based on the obtained information from the master problem, the pricing problem builds a QoS compliant support plan to carry commodities to the different destinations waiting to be delivered commodities. The master problem updates these values at each iteration to reflect the new solution obtained by adding the previously generated support plan by the pricing problem.

Constraints (4) are flow conservation constraints. They are used to route flow of commodities from the main JTFSC (source) to the destinations (FOBs). These constraints are used for all adjacent routes to each location in the network topology, including air and land routes. Constraints (5) are used for each route $r \in \mathcal{R}$ to set the maximum number of transportation assets of all classes to at most one. In this selection, only transportation assets v with a range greater or equal to the distance of the selected route r are considered. Constraints (6) are used to select the appropriate transportation assets to transport the different classes of commodities. Depending on different criteria, e.g., sensitivity of transported commodities and reliability of transportation assets, some transportation assets could be more suitable to transport some commodities than others. Constraints (7) and (8) are payload and bulk transportation capacity constraints, respectively. These constraints are used to limit the total amount of transported units of all commodities by the payload or bulk capacity of the selected transportation asset. Finally, domain constraints (9) to (11) define the domains of the optimization variables.

The output of the pricing problem is a supply plan. Within a supply plan, on each route $r \in \mathcal{R}$ is scheduled at most one vehicle (truck or helicopter) of a given class $v \in \mathcal{V}$ carrying a set of commodities each of a class $k \in \mathcal{K}$ to a set of destinations each indexed by $n \in \mathcal{N}$. The flow of commodities of class $k \in \mathcal{K}$ that reaches a destination n is given by the variable $y_{n,k}$, and on each route r by variable $y_{n,k,r}$.

3.4 Lead-time constraints

In this study, the lead time, which is the time within a commodity of class k is required at a destination n, is one of the criteria used to measure the effectiveness of the whole support strategy. In order to meet the lead-time of the different demands expressed at the different destinations, we propose the following extension of the pricing problem to generated support plans, which are lead-time compliant. For this purpose, we define the following sets of variables:

Variables:

$h_{n,k,r} \in \{0,1\}$ for each destination $n \in \mathcal{N}$, commodities of class $k \in \mathcal{K}$, and route $r \in \mathcal{R}$, it encodes whether route r is used by any transportation asset to carry commodities of class k to destination n. It is equal to:

$$h_{n,k,r} = \begin{cases} 0 & \text{if route } r \text{ is used in carrying commodities of class } k \text{ to destination } n \\ 1 & \text{otherwise} \end{cases}$$

$T_{n,k,m} \in \mathbb{R}_+$: used for each destination $n \in \mathcal{N}$, commodities of class $k \in \mathcal{K}$, and location $m \in \mathcal{M}$. It is the time to get commodities of class k destined to n at location m from all its adjacent locations. In Figure 5, the amount of time to get commodities of class k_1 of destination FOB to location m is equal to the amount of time to get the commodities to m from all adjacent end-nodes $m'_i(i = 1,\ldots,4)$, i.e., the amount of time to get the last pallet of k.

The lead-time constraints are expressed as follows:

$$\lambda_{n,k} - \psi_{n,k}\ y_{n,k,r} \leq h_{n,k,r} \leq 1 - \psi_{n,k}\ y_{n,k,r} \qquad k \in \mathcal{K}, n \in \mathcal{N}, r \in \mathcal{R} \tag{12}$$

$$T_{n,k,m} \geq \max_{\substack{r:\omega^-(r)=m \\ m':\omega^+(m')=r}} \left[\overbrace{T_{v,m'}\ \left(x_{v,r} - h_{n,k,r}\right)}^{Servicing-Time} + \overbrace{T_{v,r}\ \left(x_{v,r} - h_{n,k,r}\right)}^{Transportation-Time} \right]$$

$$n \in \mathcal{N}, k \in \mathcal{K}, m \in \mathcal{M}, v \in \mathcal{V} \tag{13}$$

$$\sum_{m \in \mathcal{M}} T_{n,k,m} \leq T_{n,k} \qquad k \in \mathcal{K}, n \in \mathcal{N} \tag{14}$$

$$h_{n,k,r} \in \{0,1\} \qquad n \in \mathcal{N}, k \in \mathcal{K}, r \in \mathcal{R} \tag{15}$$

$$T_{n,k,m} \in \{0,1\} \qquad n \in \mathcal{N}, k \in \mathcal{K}, m \in \mathcal{M} \tag{16}$$

where $\psi_{n,k} \ll 1\,(e.g., 1/d_{n,k})$ and $\lambda_{n,k} \ll \psi_{n,k}\,(e.g., 1/(2 \times d_{n,k}))$.

Constraints (12) are used to set up variables $h_{n,k,r}$ to their appropriate values. If a given route r is used by any transportation asset to carry commodities of class k to destination n, then, $h_{n,k,r}$ will be set to zero, otherwise one. Constraints (13) are used, for each class of commodities $k \in \mathcal{K}$ and destination $n \in \mathcal{N}$, to capture the time required to get commodities of class k at location m from all its neighbors m'. This time includes a servicing time at adjacent locations $T_{v,m'}$ (also depends on the class of transportation assets) and transportation time $T_{v,r}$ (which also depends on the class of transportation assets) along the travelled routes r between the adjacent locations m and m' for all $m' : \omega^+(m') = \omega^-(m)$. Although we allow split delivery, i.e., pallets (even of the same class of commodity) of a given destination can be transported on different transportation assets, we compute the delivery-time as the time to get all the pallets of the considered class of commodity to their destination. Over all adjacent locations m' and travelled routes r to the current location m, the time required to get commodities of any class and of any destination to m is equal to the time that the last pallet going through m' arrives at m.

Figure 5, which illustrates the total time, helps to demonstrate how the time to get commodities of class k_1 of destination FOB at a given location m is computed. At location m, the illustrated arriving transportation assets are all transporting pallets of class k_1. The demand of a given class of commodities is considered as met when all its pallets are received at their destination. The required time to get these pallets to the illustrated location m, from all its neighbours $m'_i(i \in 1 \ldots 4)$ is equal to the time to get the last pallet from any neighbour. It is equal to the transportation time along the route used by the last transportation asset (helicopter or truck) of class v to reach m, plus, the servicing time at the departure location. Constraints (14) sum over the time required to get all the pallets of any class and destination to any location in the network, and set up an upper bound on the sum by the lead time

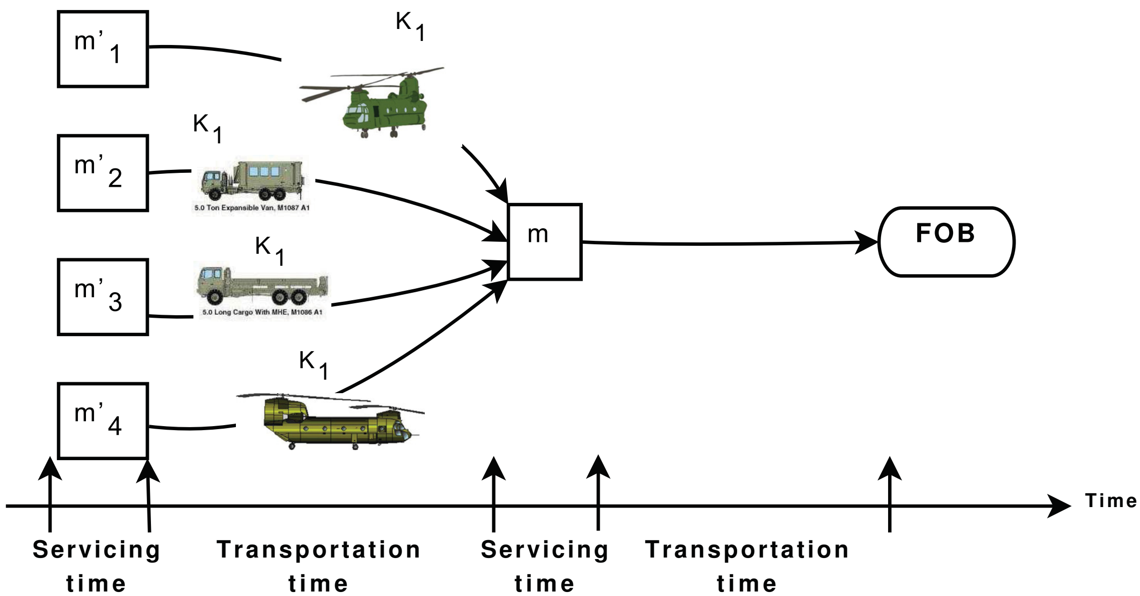


Figure 5: *Computation approach of lead-time*

expressed by the destination for the class of commodity. Constraints (15) and (16) define the domains of the added variables.

3.5 Support reliability constraints

In this section we add some constraints to guarantee reliability of the provided support. The parameters that define a reliable support are: security and safety of transportation assets, security in transporting commodities, and safety of land routes. These constraints are detailed below.

3.5.1 Security and suitability of transportation assets

Some security-sensitive commodities (e.g., ammunition) may require a specific transportation asset of a specific mode (e.g., air). In order to set the incompatibility constraints between the transportation assets and commodities, we use parameters $\alpha_{v,k}$ (defined in Section 3.3). If a class of transportation assets $v \in \mathcal{V}$ is reliable and suitable to transport a class of commodity $k \in \mathcal{K}$, then, $\alpha_{v,k}$ will be equal to one, otherwise to zero.

3.5.2 Commodities clustering

In order to simulate the commodities-clustering constraints, we use as many similar transportation classes that could transport commodities, which cannot be transported together. For example, if a class v of transportation assets can transport commodities of these two sets of classes, $k_1, \ldots, k_i$ and $k_{i+1}, \ldots k_{i+\tau}$ which cannot be clustered together, then, two similar classes of transportation assets v_1 and v_2, identical to v, are added to replace v, and parameters of $\alpha_{v_1,k}$ and $\alpha_{v_2,k}$ for each k are set up to avoid clustering of commodities of the two classes. Table 1 shows the values of the parameters according to the stated clustering strategy.

Table 1: *Commodities clustering*

		k_1	k_2	$\ldots$	k_i	k_{i+1}	k_{i+2}	$\ldots$	$k_{i+\tau}$
v	v_1	1	1	1	1	0	0	0	0
	v_2	0	0	0	0	1	1	1	1

3.5.3 Safety of routes

Unsafe routes can be forbidden in the model by simply adding the following constraints for unsafe routes.

$$x_{v,r} \leq \rho_r \qquad r \in \mathcal{R}, v \in \mathcal{V} \tag{17}$$

where

$$\rho_r = \begin{cases} 1 & \text{if route } r \text{ is safe} \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

Constraints 17 state that if a route is not safe ($\rho_r = 0$), then, no transportation asset of any class can use it. These parameters are used only for land routes as we suppose that air routes are safe.

3.6 Gomory-Chvátal rank-1 cutting planes

In addition to CG, cut generation is a solution approach used in ILP to improve the efficiency of the B&B algorithm. Cuts are constraints added to cut away (become infeasible) non integer solutions that would otherwise be solutions of the continuous relaxation. The addition of cuts usually reduces the feasible solution space by cutting continuous feasible solutions. In ILP, valid cuts are those violated by the LP solution but not by the ILP. Figure 6 shows a graphical illustration of an ILP solution method using valid cuts. The optimal solution of the LP (relaxation) offers a higher objective (maximization problem) value than the ILP solution obtained by adding the illustrated two cutting planes. However, in ILP this LP solution is not feasible and needs to be cut away. In Figure 6, the addition of the two cutting planes improved the upper bound of the LP which is now equal to the ILP solution.

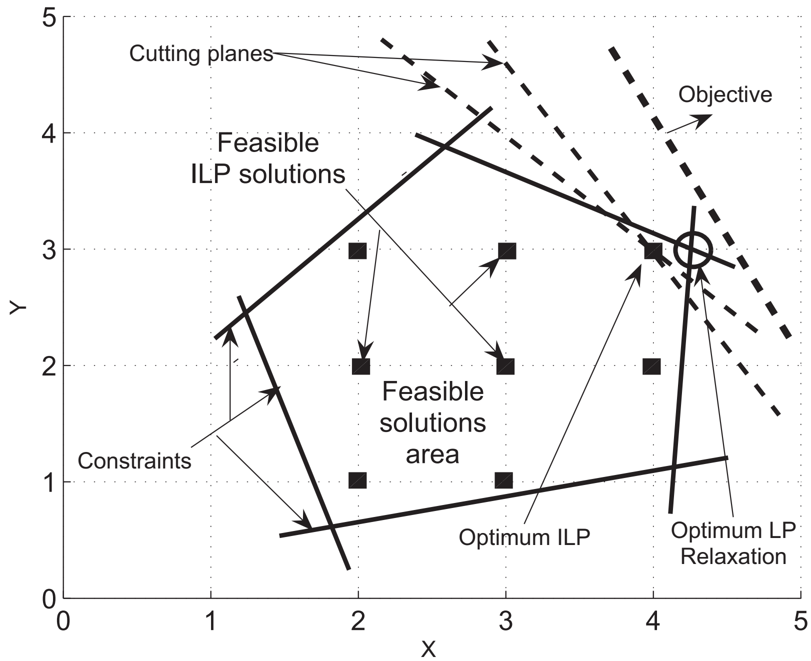


Figure 6: *ILP and cuts*

In this study, we use CG to solve the LP-relaxed master problem. The solution we obtain is used as a lower bound in the B&B algorithm to reach integrality. In order to strengthen the CG formulation, we derive general Gomory-Chvátal (GC) rank-1 cuts [34, 35] based on the restricted master problem. The GC cutting planes are valid inequalities for the integer hull of our polyhedron space but not necessarily valid for the whole polyhedron space, i.e., continuous solutions may be cut off but not integer solutions (see Figure 6).

The GC rank-1 cut for our LP is defined as follows:

$$\sum_{p\in\mathcal{P}}\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}}\left\lfloor\left(\delta_{n,k}\,a_{n,k}\right)_p\right\rfloor u_p \leq \left\lfloor\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}}\delta_{n,k}\,d_{n,k}\right\rfloor \tag{19}$$

where $\delta_{n,k}$ are called the GC multipliers.

These inequalities are used to cut off a fractional solution u_p ($p \in \mathcal{P}$) of the LP by choosing adequate multipliers $\delta_{n,k}$. These multipliers can be obtained by solving a separation problem [36], which consists of finding a GC cut that is violated by the fractional u_p solution, i.e, find $\delta_{n,k} \in \mathbb{R}_+$ such that

$$\sum_{p\in\mathcal{P}}\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} \Big\lfloor \left(\delta_{n,k}\, a_{n,k}\right)_p \Big\rfloor u_p > \Big\lfloor \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} \delta_{n,k}\, d_{n,k} \Big\rfloor \tag{20}$$

or prove that no such $\delta_{n,k}$ exist.

The GC rank-1 separation problem is also shown to be $\mathcal{NP}$-hard [37]. In [36], Fischetti and Lodi used the GC cutting planes in solving an integer programming problem. The authors showed how the separation problem can be formulated as a mixed integer problem. Interesting results were obtained showing the improvement of the lower bounds and the optimal solution when adding the violated GC rank-1 cuts.

Adding the GC cuts to the master problem implies that the pricing problem is updated for each new cut in the master problem. Each cut in the master problem results in a new resource constraint in the pricing problem. Annex *A* shows how the pricing model is updated, the extended master model, and the mixed integer separation model.

3.7 Solution algorithm

The flowchart in Figure 7 illustrates our column and cut generation based algorithm to the tactical logistics problem. The CG solution algorithm consists to execute the pair master-pricing problems until no improving variable (support plan) could be found. Each time a promising support plan (improve the value of the objective) is found it is added to the master problem, and the process re-starts again. If no improving plan is available, i.e., the reduced cost of the pricing problem is greater or equal to zero, then, the GC rank-1 separation problem is initiated. The separation problem will find, if any, a cutting plane to cut-off the fractional solution or return a certificate that no such cut is available. In case a cutting plane is found, it is added to the master problem, and the whole process re-starts again, otherwise, the process ends and a B&B algorithm is started to derive an integer solution. Both the column and cut generation processes are performed only at the root node of the tree not during the branch and bound algorithm. During the branch and bound algorithm the integer solution is derived using only the so far generated columns (i.e., support plans) and cuts.

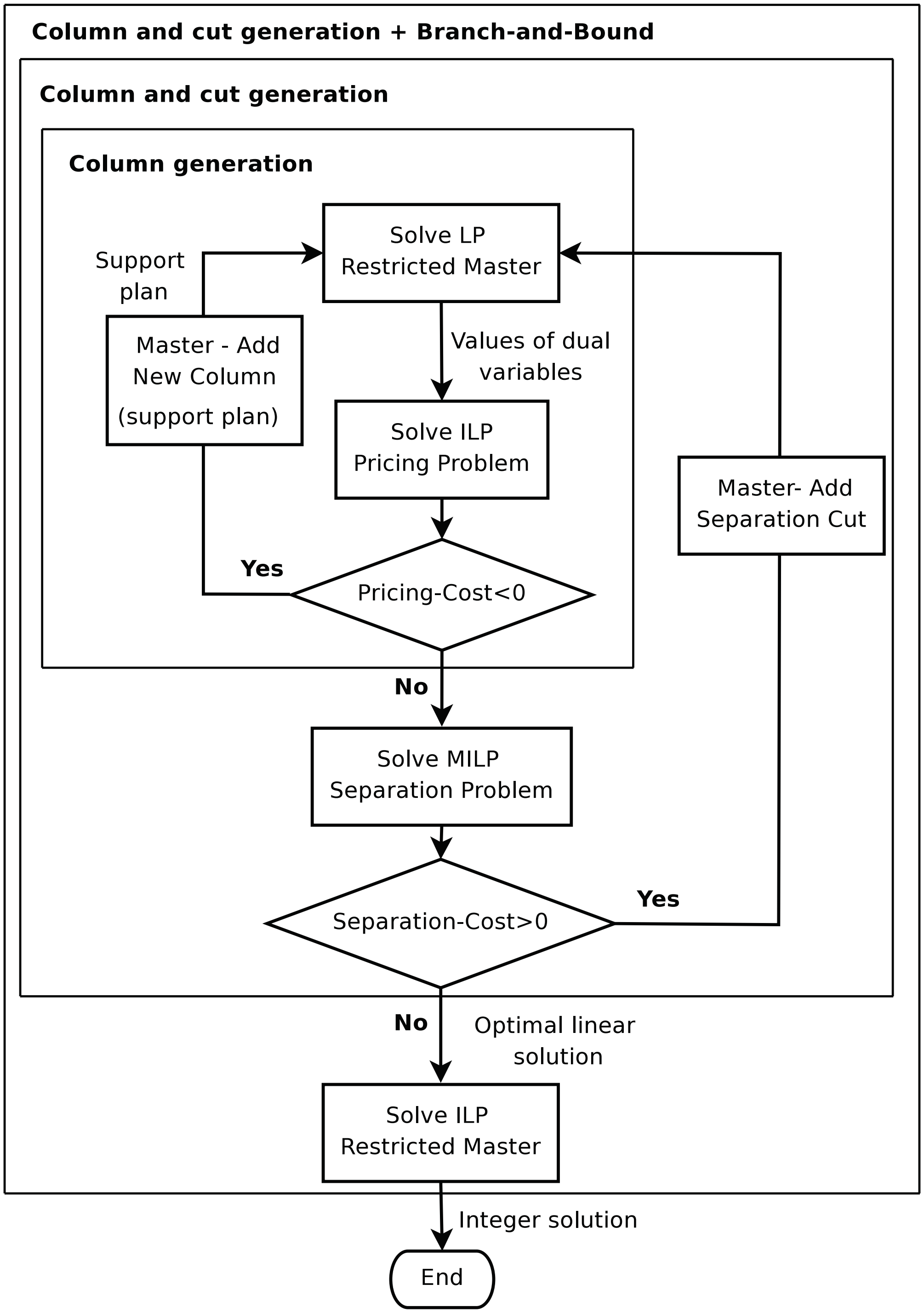


Figure 7: *The solution algorithm*

4 Computational results

This section presents our computational results obtained when solving the tactical logistics support problem using the proposed algorithm. The column and cut generation based optimization algorithm has been coded in C++, and run on a Linux laptop equipped with an Intel Centrino 2.53 GHz CPU and a 4 GB RAM. We used CPLEX 12.0 optimization tools [38] to solve the different mathematical programming models.

The main contributions of this work are twofold. First, we propose an optimization methodology to be integrated in a decision support system, in order to design large scale tactical support strategies. Secondly, we study and analyze the relationships between the different parameters involved in the design process of efficient and effective tactical support strategies. Two subsections, each focusing on one of the following aspects, are elaborated:

- We focus on the efficiency of the optimization method through the study of the quality of the obtained solutions and the running time of the proposed algorithm (see Section 4.3);
- We investigate the trade-off between efficiency and effectiveness, i.e, the impact of the lead-time, safety of the routes and the reliability of the transportation assets on the tactical support cost. Also, we investigate how the lead-time shapes the optimal fleet mix of transportation assets to provide different QoS guarantees.

4.1 The tactical support model

In this section, we present the characteristics of the different classes of tactical support components including the transportation assets and commodities. In the whole study, we consider a network topology of 7 nodes and 24 routes composed as follows: 1 JTFSC, 3 MOBs, 3 FOBs, 12 land routes, and 12 air routes (see Figure 2 for a comparable topology). The land and air routes, which are of different distances, are established based on the reach-ability of the different locations. Tables 2 and 3 and Section 4.2 present the data we used in our illustrative examples to demonstrate our methodology. The accuracy of this hypothetical example is discussed with the client and validated before further investigations.

From a transportation requirement point of view, our classes of commodities are classified into three categories: general, refrigerated, and ammunitions. Table 2 presents the types of commodities of each category and the weight per standard pallet (transportation unit) of each category. In this study we assume that standard pallets of equal size are used, and their weights are defined by the contained class

Table 2: *Characteristics of classes of commodities*

	Categories	Type	Weight per pallet (Ton)
K_1	Refrigerated	Consumable (e.g., fresh food, rations, medical materiel)	1.0
K_2	General	Durable (e.g., fortification and barrier materiels)	1.5
		Consumable (e.g., food, clothing, personal demands, medical materiel)	
		Repairable (e.g., parts required for maintenance support)	
K_3	Ammunition	Consumable (chemical and special weapons, bombs, explosives, mines, fuses, detonators, and others)	2.0

of commodity. For an exhaustive list of classes of commodities with their classification into different categories see [39].

In Table 3 are presented the characteristics of the different classes of transportation assets used in this study. As illustrated, two classes of helicopters: CH-147D Chinook and CH-146 Griffon, and three classes of trucks: LSVW (Light Support Vehicle Wheeled), MLVW (Medium Logistic Vehicle Wheeled), and HLVW (Heavy Logistic Vehicle Wheeled), are used. In addition to their physical (payload and bulk capacities) and operational (hourly cost, range, and cruising-speed) characteristics, each transportation class is characterized by the set of classes of commodities that it can transport as described in the last column. This last characteristic is elaborated based on the reliability of the transportation classes and the sensitivity of the classes of commodities.

Regarding the commodities clustering constraints, we used the approach described in Section 3.5.2 in order to avoid clustering of ammunitions with any of the two others. Recall that the cost per hour is composed of several costs [28], including fuel consumption, spare parts, crew, etc.

4.2 The benchmark data inputs

In order to test the performance of the proposed algorithm and study the trade-off between the efficiency and effectiveness of the resulting tactical support strategies, we defined a set of 75 instances by estimating the needs of a set of deployed troops. Given the size of the deployed troops and the usage rates for each class of supplies

Table 3: Characteristics of classes of transportation assets

Helicopters		Capacity: Payload (Ton)	Capacity: Bulk (#Pallet)	Range (km)	Speed (km/h)	Cost per hour ($)	Commodities
CH-147D Chinook		12.2	8	800	220	8000	K_1, K_2, K_3
CH-146 Griffon		1.9	1	550	200	5000	K_2
Trucks	LSVW-Cargo	2.0	2	550	98	352	K_1, K_2
Trucks	MLVW-Cargo	5.0	4	536	90	442	K_1, K_2, K_3
Trucks	HLVW-Cargo	10.0	8	732	85	517	K_1, K_2, K_3

[40], the number of standard pallets of a commodity required to be delivered each week is calculated as [24]:

$$\text{\#of pallets per week} = \left\lceil \frac{\text{\# personnel} \times \text{usage rate(kg/person/week)}}{\text{weight per pallet (kg/pallet)}} \right\rceil \tag{21}$$

Depending on the type of the supported mission, three different support models, where demands for some commodities of a given category are higher compared to the two other categories, are considered. The motivation behind this orientation is that the needed amount of commodities of a given class depends on the supported missions, e.g., peacekeeping missions need more K_2, fighting missions need more K_3. Following this approach, two intervals of values are used to randomly generate the number of pallets needed of each class of commodities at each FOB. These intervals are $[20,30]$ for mission-specific commodities (high demand) and $[10,20]$ for the two others.

4.3 Performance assessment

In this section, we focus on the performance of the optimization algorithms. We study the performance of the column generation algorithm alone and the combined column and cut generation one. Table 4 presents the performance variation of our algorithms using 15 different input demands. The first five input demands are generated for support of missions where demands for K_1 are higher (generated in $[20,30]$)

than K_2 and K_3 (generated in $[10,20]$). In the next five and last five instances, demands for K_2 and K_3 are respectively higher than the two others.

In Table 4, we measure the cost of the optimal linear solution Z_{LP}, the integer solution Z_{ILP} (both in dollars), the integrality gap between them (%), and the running time (in seconds) until the final integer solution (Z_{ILP}) of both the column generation and the column and cut generation algorithms. The integer solution in both the column generation and the column and cut generation algorithms is derived by applying a classical branch-and-bound algorithm when the optimal continuous solution is obtained at the root node. In this first set of 15 instances, we relaxed the lead-time constraints by setting a large bound on the lead-time of all commodities at all destinations. Annex B presents further results based on input instances with different lead-time requirements and different levels of safety of land routes.

The CG algorithm obtains integer solutions with an integrality gap in [5.2%, 17.3%] within a running time in $[1.4, 3.6]$ seconds. However, looking closely at the integer optimal solutions obtained by combining the column and cut generation (gap = 0%), we realize that only a few instances of integer solutions have been improved, i.e., the optimal solution is obtained by the column generation algorithm. The addition of the GC cuts has helped in cutting the continuous solutions and improved the current integer solution in instances 2, 7, 15. The column and cut generation algorithm obtained the optimal solution in 12 out of 15 instances and provided a good integrality gap in the remaining 3 scenarios (gap ≤ 2.7), while the column generation alone obtained the optimal solution in 9 out of 15 scenarios. The difference in running time between the two algorithms is due to the time spent in solving the separation problem (see Annex B) to obtain cutting planes. The difference in running time is of the order of 2% to 154%. Other distributions of commodities are considered and the obtained results are almost similar to the ones here (see Annex B).

Table 4: Performance assessment of the solution algorithm

instances	Column Generation				Column and Cut Generation			
	z_{LP} ($)	z_{ILP} ($)	Gap (%)	Time (sec)	z_{LP} ($)	z_{ILP} ($)	Gap (%)	Time (sec)
1	166504.4	182470.5	9.5	2.7	177514.5	182470.5	2.7	413.6
2	132208.2	152058.8	15.0	3.0	145976.4	145976.4	0.0	275.6
3	143543.5	152058.8	5.9	2.7	152058.8	152058.8	0.0	3.2
4	152572.0	164223.5	7.6	2.8	164223.5	164223.5	0.0	294.9
5	139285.0	152058.8	9.1	3.6	152058.8	152058.8	0.0	5.5
6	139894.1	164223.5	17.3	3.2	164223.5	164223.5	0.0	5.3
7	154491.7	167264.7	8.2	2.3	164223.5	164223.5	0.0	5.1
8	173973.1	194635.2	11.8	1.4	194635.2	194635.2	0.0	284.4
9	152777.6	170305.8	11.4	3.0	165744.1	170305.8	2.7	269.2
10	160574.1	176388.2	9.8	1.4	173347.0	176388.2	1.7	310.1
11	179125.2	188552.9	5.2	2.7	188552.9	188552.9	0.0	4.5
12	178823.3	188552.9	5.4	2.1	188552.9	188552.9	0.0	281.7
13	198436.7	209841.1	5.7	2.3	209841.1	209841.1	0.0	270.2
14	181345.3	194635.2	7.3	2.9	194635.2	194635.2	0.0	275.4
15	190377.6	206800.0	8.6	2.0	200717.6	200717.6	0.0	266.1

4.4 Trade-off between lead-time and cost

In this section, we study the trade-off between the lead-time expressed with the different commodities and the incurred transportation cost. For each destination and class of commodities, we define a time interval $[T_{SP}, \infty[$ containing the possible values of the lead-time within the commodities are required at their destinations. T_{SP} corresponds to the travel-time along the shortest path (loading plus transportation) to reach a given destination using the fastest transportation asset, and ∞ means no lead-time constraint is associated with the class of commodities at the destination.

In Figure 8, we show the variation of the normalized transportation cost as a function of the normalized lead-time of different distributions of commodities in the defined lead-time interval. The normalized values of the transportation cost (y-

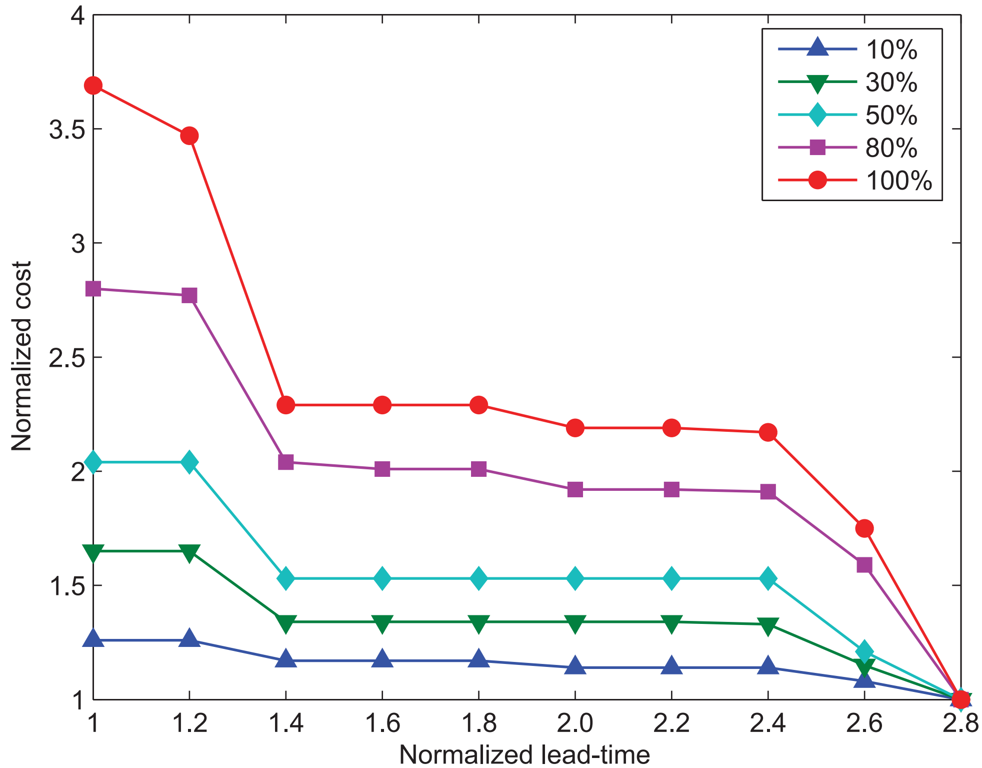


Figure 8: *Trade-off between cost and lead-time*

axis) and lead-time (x-axis) are obtained by dividing the different values of the cost and lead-time by the lowest cost value and lowest lead-time value T_{SP}, respectively. We consider five distributions of commodities for each value of lead-time taken from the lead-time interval above as follows: 10%, 30%, 50%, 80%, and 100% of commodities require at least the selected lead-time (an upper bound equals to the selected lead-time) value and the others are not constrained in terms of lead-time. Note that the curve tagged 10% is associated with the distribution consisting of 10% of commodities constrained by the given lead-time on the x-axis, and 90% of commodities not constrained (with respect to lead-time) at all.

In Figure 8, two noticeable lead-time values appear at the two ends of our interval. The points corresponding to the normalized lead-time equal to 1 and 2.8 (lead-time equals to T_{SP} and $2.8 * T_{SP}$, respectively). These two values are particular because the support cost is at its highest and lowest values, respectively. Inside the interval, we notice that as the lead-time constraint is relaxed, i.e., lead-time increases, the transportation cost drops down to reach its lowest value when the lead-time is $2.8 * T_{SP}$ for all distributions of commodities. The high drop in cost is mainly due to the large difference in speed between the helicopters and trucks, and also to the difference in length of the land and air routes.

The different curves associated with the distributions of commodities follow the same pattern. The transportation cost is directly proportional to the percentage of commodities requiring low lead-time, and inversely proportional to the lead-time. However, it costs more to provide tactical support when a high level of demands are required within a shorter lead-time.

Three main levels of transportation cost appear in Figure 8. These levels correspond to normalized lead-time values in the intervals $[1, 1.4[$, $[1.4, 2.4[$, $[2.4, \infty[$, respectively. In the next section, we analyze the inherent solutions in these intervals to find out about the number of lifts performed and the used classes of transportation assets.

4.5 Trade-off between lead-time and types of lifts

Figures 9 to 14 show the number of lifts realized by each class of transportation assets. We define a lift as a transportation from one location to another without stops for refueling or transshipment.

In Figure 9, 10%, 30%, 50%, 80%, and 100% of demands are required within a lead-time $\leq T_{SP}$. First, we see that the SLVW Cargo, MLVW Cargo trucks, and the CH-146 Griffon are not part of the optimal set of lifts of any distributions. This is mainly due to their limited capacity, range, and suitability to transport some classes of commodities. Secondly, The optimal sets of lifts, of all configurations, are composed solely of HLVW Cargo trucks and CH-147D Chinook helicopters. The set of used transportation assets is dominated by MLVW Cargo trucks as the percentage of commodities requiring a lead-time $\leq T_{SP}$ is under 50%. As the percentage of commodities with tight lead-time $(\leq T_{SP})$ increases, the number of helicopters to meet the new requirements increases and the number of trucks decreases. When 100% of commodities are required within the shortest lead-time, then, the only feasible solution is by using transportation assets of the fastest class, i.e., CH-147 Chinook helicopters. Note that the total number of lifts is almost the same for the different distributions of commodities.

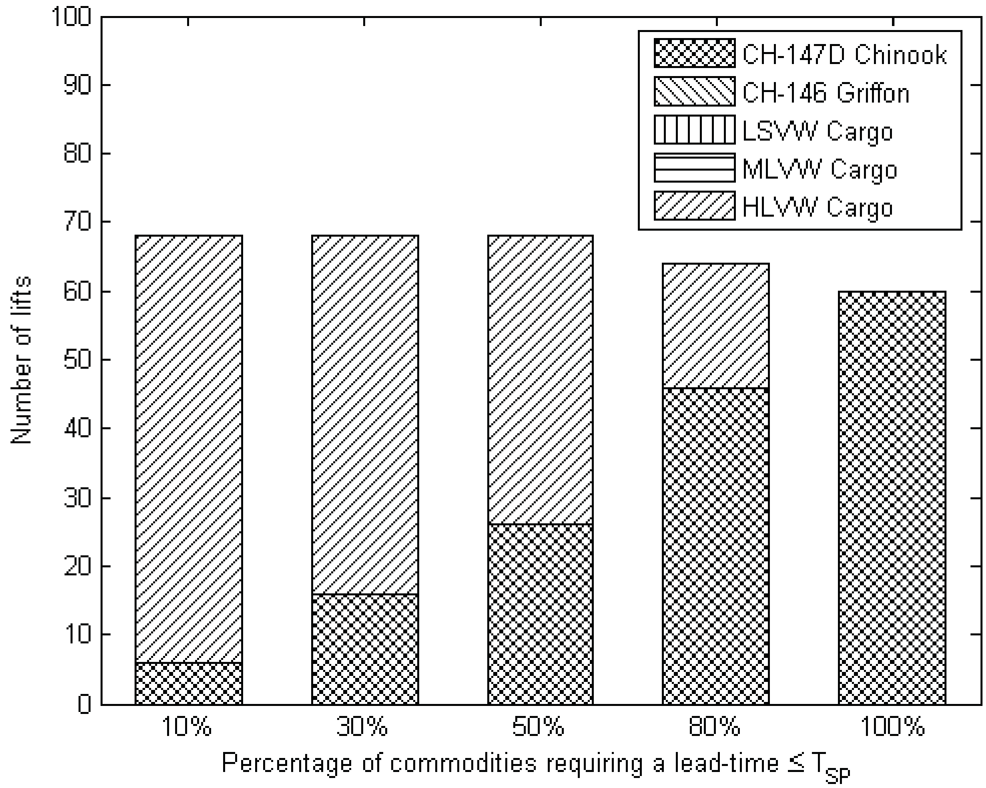


Figure 9: *Number of lifts vs. constrained lead-time* $\leq T_{SP}$

The next two Figures 10 to 11 illustrate the distributions of the performed lifts over the different classes of transportation assets when the lead-time value is in the interval $[1.4, 2.4]$. In Figure 8, the variation of the transportation cost within this interval of lead-time is not as high as the other parts of the whole lead-time interval. Figures 10 and 11 show the same pattern of lifts over the different classes of transportation assets. When the number of commodities requiring a lead-time in $[1.4, 2.4]$ is equal to 10%, the number of lifts realized by the CH-147 Chinook helicopters is equal to 3 and those by the HLVW Cargo trucks is 65. The number of lifts realized by the CH-147 Chinook helicopters and HLVW Cargo trucks increases and decreases respectively as the percentage of commodities requiring a lead-time in $[1.4, 2.4]$ increases. For all values of lead-time in $[1.4, 2.4]$, when the percentage of commodities requiring a lead-time in this interval is equal to 100%, the number of lifts realized by CH-147D Chinook helicopters is equal to the number of lifts realized by the HLVW Cargo trucks.

In Figure 12 we focus on the lead-time interval $[2.4, 2.8]$. The variations of the cost within this interval is high. In this interval the distributions of the realized lifts over the transportation assets contain another class of transportation assets: the MLVW

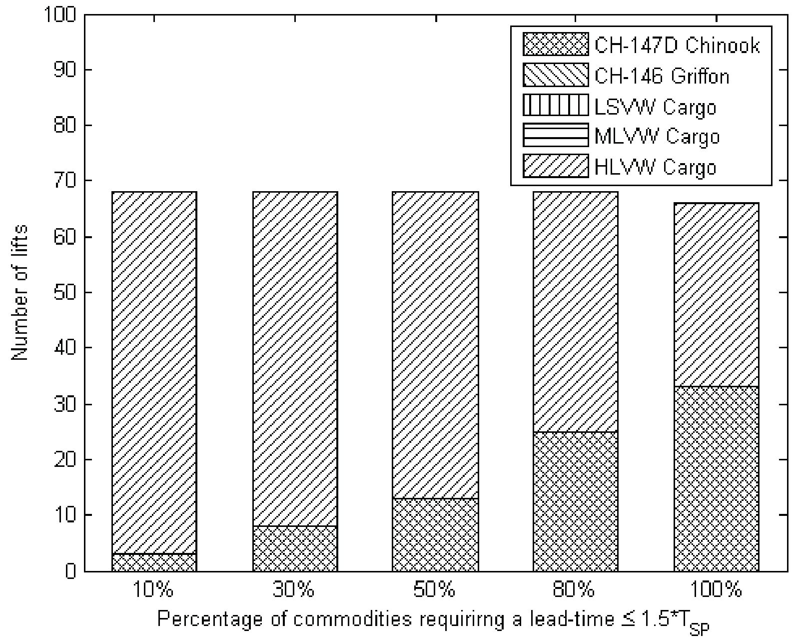


Figure 10: *Number of lifts vs. constrained lead-time* $\leq 1.5 * T_{SP}$

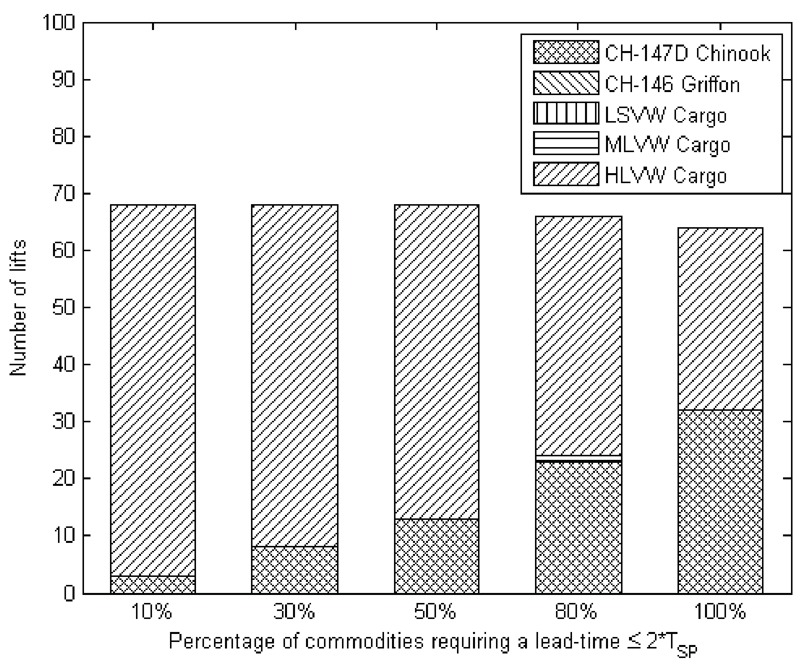


Figure 11: *Number of lifts vs. constrained lead-time* $\leq 2 * T_{SP}$

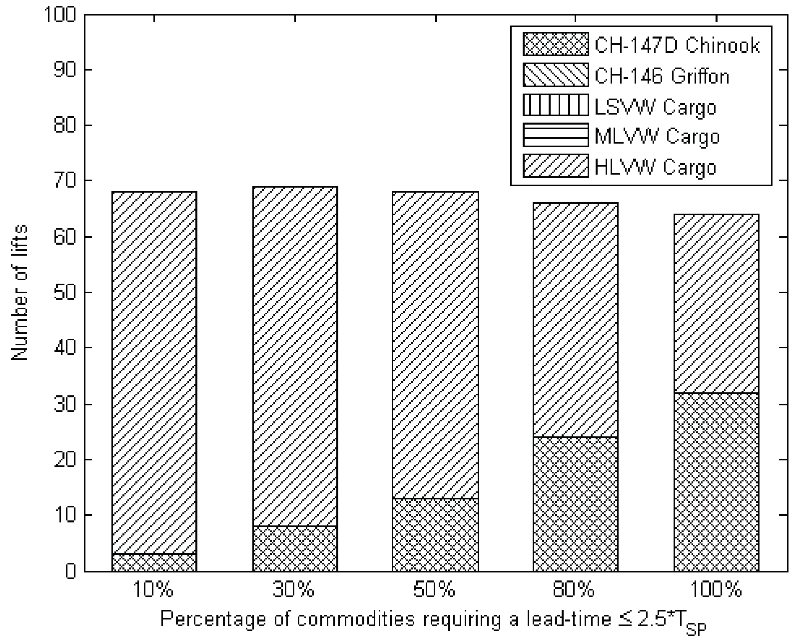


Figure 12: *Number of lifts vs. constrained lead-time* $\leq 2.5 * T_{SP}$

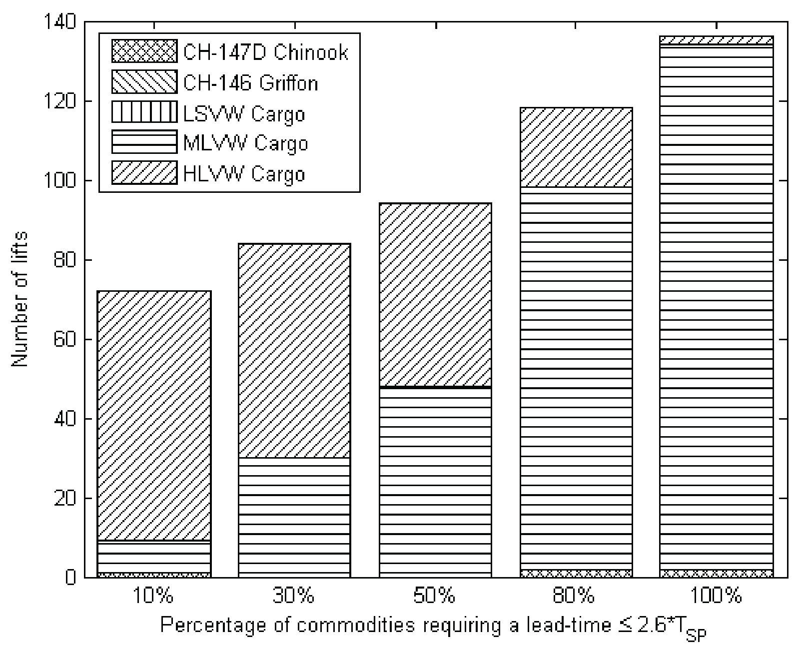


Figure 13: *Number of lifts vs. constrained lead-time* $\leq 2.6 * T_{SP}$

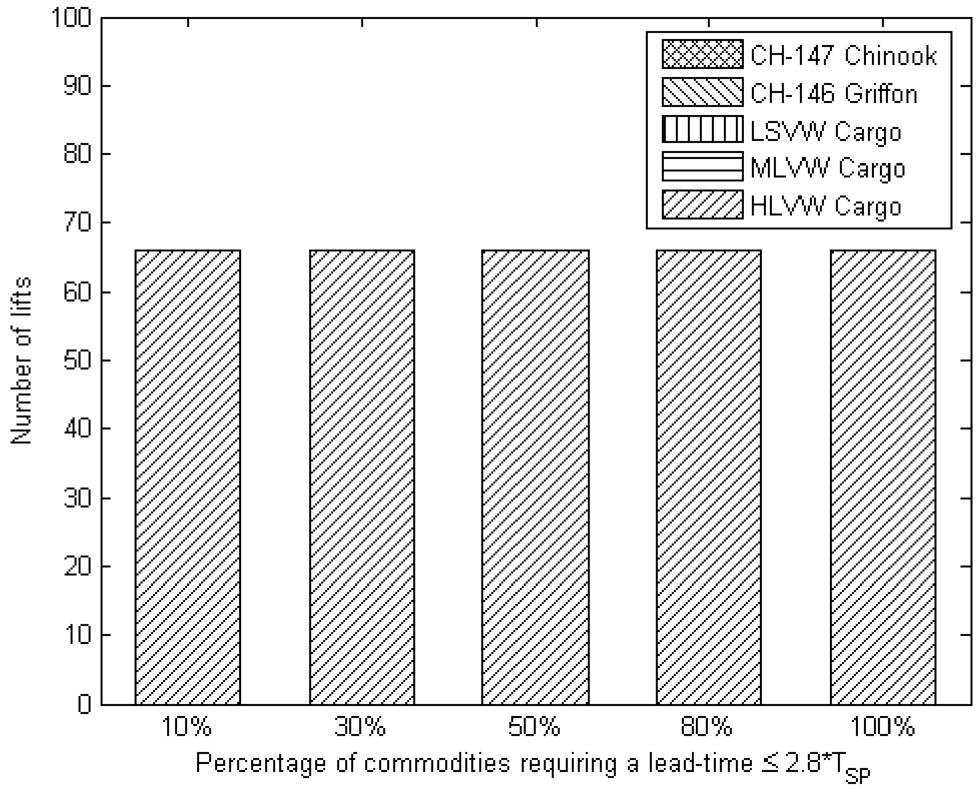


Figure 14: *Number of lifts vs. constrained lead-time* $\leq 2.8 * T_{SP}$

Cargo trucks. The number of lifts realized by trucks of the MLVW class increases as the percentage of demands requiring shorter lead-time increases. Another noticeable fact in this interval is the large number of lifts. This number is larger than the number of lifts in all other intervals of lead-time and distributions of demands. The number of lifts when 100% of demands require a lead-time $\leq 2.6 * T_{SP}$ is twice the number of lifts performed in other lead-time intervals. Therefore, we conclude that optimization of the transportation cost is not necessarily equivalent to optimization of the support footprint. This observation could be explained by the cruising speed of this class, which is a bit higher compared to the two other classes of trucks, and much less expensive compared to the two other classes of helicopters. However, these patterns of distributions appear only in this short interval. Indeed, as illustrated in Figure 14, in the next interval $]2.6, \infty[$ the distributions do not contain any truck of the MLVW Cargo class. The distributions are formed exclusively of the HLVW Cargo trucks. The difference in the cruising-speed between these two classes of trucks is very low to justify the advantage of the MLVW Cargo trucks over HLVW Cargo trucks over a long lead-time interval.

4.6 Trade-off between safety of routes and cost

In Figure 15, we show the variation of the normalized support cost as a function of the vulnerability of the land routes. In these experiments, we suppose that all air routes are reliable but not land routes. Land routes are of two categories: front and end routes. Front routes are at the front of the support network topology, i.e., close to the JTFSC, and end routes are those at the end of the network topology, i.e, close to the end-nodes (FOBs).

In this section, we study the effect of the unsafety of the routes on the support cost. In this model, an unsafe land route is not used by transportation assets. We study how the distribution of the safety over the land routes affects the cost by considering scenarios where front and end land routes are unsafe.

In Figure 15 are recorded on the x-axis the percentages of unsafe front and end land routes. Recall that in this set of experiments we do not impose any tight bound on the lead-time. As the percentage of unsafe land routes increases, the supply cost increases. A cost increase of the order of $\sim 230\%$ results when 50% of the land routes are unsafe. For 50% or over of unsafe land routes, the transportation cost stays the same. This is because the 50% remaining routes become not useful to reduce the transportation cost.

An interesting finding in Figure 15 is that the unsafety of the end routes costs more than the front routes. The explanation of this difference resides in the loading of the transportation assets. Indeed, front routes are used by assets to push as many commodities as possible to the intermediate operating bases, where they are dispatched to their destinations. In this case, when helicopters are used instead of trucks on the front routes, they are loaded at their maximum capacity, by mixing commodities of different destinations, to optimize the transportation cost. However, the loadings of helicopters when used on end routes depend on their capacity and the demands of the end FOBs. In this case, if the demand of a given FOB is lower than the capacity of the used helicopters, then, more helicopters will be used to dispatch small quantities of commodities to the FOBs.

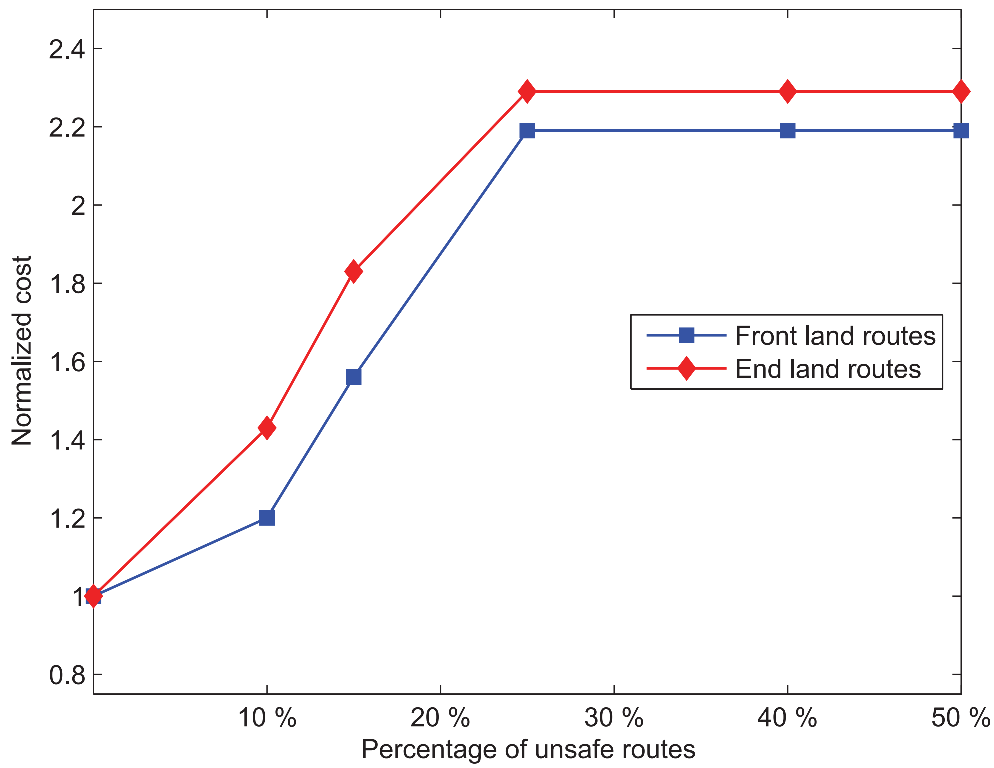


Figure 15: *Trade-off between cost and safety of land routes*

5 Conclusions and future work

5.1 Summary of principal results

In this study, a tactical logistics planning method, that is to be integrated into a logistics decision support system, is developed. We proposed an optimization method to find an optimal mix of transportation assets with their loading and routing given a set of constraints, including capacity of transportation assets, reliability and suitability of transportation assets, security of commodities, safety of land routes, and lead-time constraints associated with each commodity and destination. The proposed optimization approach is based on a large scale optimization method named Column Generation (CG). The CG algorithm was extended by the addition of a cut generation algorithm based on Gomory-Chvátal rank-1 cutting planes to cut continuous solutions and approach the optimal integer solutions.

We focused on the trade-offs between efficiency and effectiveness, and proposed an extensive study of the resulting balances of optimizing different parameters of the tactical support model. We investigated the lead-time and security effects on the supply cost through experiments with different classes of commodities and transportation assets of air and land modes.

The experimental results showed that the proposed algorithmic approach is efficient. The optimal solution was obtained within a reasonable time in most of the studied instances. The CG alone succeeded to find the integer optimal solution of a large number of instances, and the computational results showed that the addition of the Gomory-Chvátal rank-1 cuts improves the lower bound significantly and helps to solve a majority of the instances in the root node of the branch-and-bound tree without adding any elaborated branching strategy.

In the analysis of the trade-off between cost and lead-time, and between cost and safety of routes we constructed several plots to show the variations of the support cost *vs*. lead-time requirement; and support cost *vs*. safety of land routes of different distributions of demands. For a similar network topology and support models, these plots can be used to estimate weekly sustainment costs given the targeted QoS (lead-time, safety, reliability). Furthermore, for different distributions of demands over different lead-time intervals, we showed different distributions of the number of lifts over the classes of transportation assets. Such statistics can be used to compose the optimal support fleet to respond to the terrain reality and the challenge of meeting the requirements of different demands. The analysis of the safety of land routes showed that this parameter has an impact on the support cost. Different levels of safety of routes were explored and the inherent cost analyzed.

The computational results obtained in this study, and the proposed algorithmic approach are recommended for "what-if" tactical logistics analyses. Different perspectives were explored and several alternatives were analyzed to help decision makers to strike different balances between efficiency and effectiveness in tactical support and to respond efficiently to different situations.

5.2 Future work

The obtained results open a number of interesting research directions that we intend to pursue in the future. First, we suggest that the problem of convoy formation and escorting, and how it affects the selection of transportation assets and their routing is worth exploring. In this study, selected transportation assets are routed independently of each other. In convoy formation and routing, the speed of a transportation asset is defined by the speed of the convoy not its own cruising-speed. The convoy escorting, which is part of the operational cost, was not considered in this study. Including this parameter would change the modeling and solution approaches and the optimal solution. Second, we assumed different percentages of reliability of randomly selected land routes. However, if all land routes are unsafe and a limited escorting budget has to be met, then an obvious question that arises: Which routes should be secured in order to optimize the support cost? Also, how should the convoys be formed?

Some other questions are still open: how similar or different are the two problems of optimizing the tactical logistics footprints and the problem of optimizing the operational cost? Or, does optimization of the footprints imply necessarily the operational cost (and vice versa)? One of the instances in the computational results above showed that optimizing the operational cost does not necessarily imply optimizing the footprints. However, further research is required along this direction to clearly establish this relationship. The answer is not straightforward and necessitates more investigation.

In this study, we focused on the impact of safety of routes on the support cost. However, other facilities may also fail (e.g., MOBs, Depots). Some critical infrastructures which are those components of infrastructure that, if lost, could pose a serious threat to needed supplies such as food, energy, and water need to be highly protected, or backup infrastructure be pre-planned in case of any service interruption of those critical components. This research direction is highly promising, and our proposal of tactical logistics design strategies could be extended to survive intentional and accidental disruptions by building back-up strategies.

From the optimization perspective, as most of the solution time of the column and cut generation algorithm is spent in solving the separation problem, we are considering the benefits of developing heuristic-based approaches to enhance this part of the solution algorithm and reduce the computational time.

References

[1] Mongold, M.B. and Johnson, A.W. (2006), Pure pallets: effectiveness and efficiency impacts on the defense transportation system, *Journal of Transportation Management*, pp. 18–36.

[2] Beamon, B.M. (1998), Supply chain design and analysis: Models and methods, *International journal of production economics*, 55(3), 281–294.

[3] Mentzer, J.T. and Konrad, B.P. (1991), An efficiency/effectiveness approach to logistics performance analysis, *Journal of business logistics*, 12(1), 33–62.

[4] Fearne, A. and Fowler, N. (2006), Efficiency versus effectiveness in construction supply chains: the dangers of lean thinking in isolation, *Supply chain management: An international journal*, 11(4), 283–287.

[5] Gunasekaran, A., Patel, C., and Tirtiroglu, E. (2001), Performance measures and metrics in a supply chain environment, *International journal of operations & production Management*, 21(1/2), 71–87.

[6] Chan, F.T.S. (2003), Performance measurement in a supply chain, *The International Journal of Advanced Manufacturing Technology*, 21(7), 534–548.

[7] Mouzas, S. (2006), Efficiency versus effectiveness in business networks, *Journal of Business Research*, 59(10-11), 1124–1132.

[8] Caplice, C. and Sheffi, Y. (1994), A review and evaluation of logistics metrics, *The International Journal of Logistics Management*, 5(2), 11–28.

[9] Meepetchdee, Y. and Shah, N. (2007), Logistical network design with robustness and complexity considerations, *International Journal of Physical Distribution & Logistics Management*, 37(3), 201–222.

[10] Drezner, Z. and Hamacher, W.H (2004), Facility location: application and theory, Springer, Verlag.

[11] Ghanmi, A. and Shaw, R. H. A. D. (2006), Modeling and simulation of Canadian Forces strategic lift strategies, In *Proceedings of the 38th conference on Winter simulation*, WSC '06, pp. 1340–1348, Winter Simulation Conference.

[12] Ghanmi, A. and Shaw, R. (2007), Modelling and analysis of Canadian Forces strategic lift and pre-positioning options, *Journal of the Operational Research Society*, 59(12), 1591–1602.

[13] Ghanmi, A. (2010), Canadian forces global reach support hubs: facility location and aircraft routing models, *Journal of the Operational Research Society*.

[14] Karmarkar, N. and Karp, R.M. (1982), An efficient approximation scheme for the one-dimensional bin-packing problem, In *Foundations of Computer Science, 1982. SFCS'08. 23rd Annual Symposium on*, pp. 312–320, IEEE.

[15] Laporte, G. (1992), The vehicle routing problem: an overview of exact and approximate algorithms, *European Journal of Operational Research*, 59(3), 345–358.

[16] Desrochers, M., Desrosiers, J., and Solomon, M. (1992), A new optimization algorithm for the vehicle routing problem with time windows, *Operations Research*, 40(2), 342–354.

[17] Fisher, M. (1995), Vehicle routing, *Handbooks in operations research and management science*, 8, 1–33.

[18] Jimenez, F. and Verdegay, JL (1999), Solving fuzzy solid transportation problems by an evolutionary algorithm based parametric approach, *European Journal of Operational Research*, 117(3), 485–510.

[19] Ziliaskopoulos, A. and Wardell, W. (2000), An intermodal optimum path algorithm for multimodal networks with dynamic arc travel times and switching delays, *European Journal of Operational Research*, 125(3), 486–502.

[20] Ribeiro, C. and Soumis, F. (1994), A column generation approach to the multi-depot vehicle scheduling problem, *Operations Research*, 42, 41–52.

[21] Fisher, M.L., Tang, B., and Zheng, Z. (1995), A network flow based heuristic for bulk pickup and delivery routing, *Transportation Science*, 29(1), 45.

[22] Malapert, A., Guéret, C., Jussien, N., Langevin, A., and Rousseau, L-M (2008), Two-dimensional pickup and delivery routing problem with loading constraints, Technical Report CIRRELT.

[23] Iori, M., González, J.J.S., and Vigo, D. (2007), An exact approach for the vehicle routing problem with two-dimensional loading constraints, *Transportation Science*, 41(2), 253–264.

[24] Kaluzny, B.L. and Erkelens, A.J. (2008), The optimal MSVS fleet for first-line replenishment, Technical Report Materiel Group Operational Research, DRDC CORA, TR 2006-026.

[25] Asiedu, Y. (2010), Assessment of the impact of 8-tonne cargo vehicles on the optimal MSVS fleet mix, Technical Report Materiel Group Operational Research, DRDC CORA, TR 2010-005.

[26] Asiedu, Y. and Hill, A (2011), A dynamic approach to logistics support fleet mix planning, Technical Report Directorate Materiel group operational research/acquisition support team, DRDC CORA, TM 2011-00.

[27] Gibbons, T.M. (2010), Joint Task Force Support Component Concept, Technical Report Canadian Forces. Draft 3.

[28] National Defence of Canada (2010), Cost Factors Manual.

[29] National Defence of Canada (Last access: Nov 2011). `www.forces.gc.ca/site/operations/index-enf.asp`.

[30] Church, R.L., Scaparra, M.P., and Middleton, R.S. (2004), Identifying critical infrastructure: the median and covering facility interdiction problems, *Annals of the Association of American Geographers*, 94(3), 491–502.

[31] Church, R.L. and Scaparra, M.P. (2007), Protecting critical assets: the r-interdiction median problem with fortification, *Geographical Analysis*, 39(2), 129–146.

[32] Whiteman, P.S. (1999), Improving single strike effectiveness for network interdiction, *Military Operations Research*, 4, 15–30.

[33] Terzi, S. and Cavalieri, S. (2004), Simulation in the supply chain context: a survey, *Computers in industry*, 53(1), 3–16.

[34] Chvátal, V. (1973), Edmonds polytopes and a hierarchy of combinatorial problems, *Discrete Mathematics*, 4(4), 305–337.

[35] Gomory, R.E. (1960), Solving linear programming problems in integers, *American Mathematical Society*, pp. 211–216.

[36] Fischetti, M. and Lodi, A. (2007), Optimizing over the first Chvátal closure, *Mathematical Programming*, 110(1), 3–20.

[37] Eisenbrand, F. (1999), NOTE–On the membership problem for the elementary closure of a polyhedron, *Combinatorica*, 19(2), 297–300.

[38] IBM (2010), CPLEX 12 user's manual.

[39] Girard, S., Martel, A., Berger, A., J. Boukhtouta, Chouinard, M., Ghanmi, A., and Guitouni, A (2008), Canadian Forces Overseas Supply Network: Strategic Need and Design Methodology, Technical Report CIRRELT.

[40] Department National Defence, Staff Data Handbook, B-GL-303-004/AF-001.

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List of Acronyms

APOD	Air Port Of Disembarkation
B&B	Branch-and-Bound
CANOSCOM	Canadian Operational Support Command
CF	Canadian Forces
CG	Column Generation
CH-147	Chinook
CVRPTW	Capacitated Vehicle Routing Problem with Time Windows
FOB	Forward Operating Base
JTF	Joint Task Force
JTFSC	Joint Task Force Support Component
HLVW	Heavy Logistics Vehicle Wheeled
HQ	Headquarters
ILP	Integer Linear Program
GC	ComoryChvátal
LP	Linear Program
LSVW	Light Support Vehicle Wheeled
MBPP	Multi Bin Packing Problem
MILP	Mixed Integer Linear Program
MLVW	Medium Light Vehicle Wheeled
MOB	Main Operating Base
NDHQ	National Defence Headquarters
QoS	Quality-of-Support
SPOD	Sea Port Of Disembarkation
VRP	Vehicle Routing Problem

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Annex A: Price and cut models

Master Model

Minimize:

$$z^{\textsc{Master}} = \sum_{p \in \mathcal{P}} C_p \, u_p$$

subject to:

$$\sum_{p \in \mathcal{P}} \left(a_{n,k}\right)_p u_p = d_{k,n} \qquad n \in \mathcal{N}, k \in \mathcal{K} \tag{A.1}$$

$$\sum_{p \in \mathcal{P}} \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \left\lfloor \left(\delta_{n,k} \, a_{n,k}\right)_p \right\rfloor u_p \leq \left\lfloor \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} \delta_{n,k} \, d_{n,k} \right\rfloor \tag{A.2}$$

$$u_p \in \mathbb{N} \qquad p \in \mathcal{P}. \tag{A.3}$$

Pricing Model

Minimize:

$$\begin{aligned}
\overline{C}_p &= C_p - \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} a_{n,k} \, \gamma_{n,k} - \sum_{i} \beta_i w_i \\
&= \sum_{r \in \mathcal{R}} \sum_{v \in \mathcal{V}} C_{v,r} \, x_{v,r} - \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} a_{n,k} \, \gamma_{n,k} - \sum_{i} \beta_i w_i
\end{aligned} \tag{A.4}$$

subject to:

$$\sum_{r \in \omega^+(m)} y_{n,k,r} - \sum_{r \in \omega^-(m)} y_{n,k,r} = \begin{cases} y_{n,k} & \text{if } m = JTFSC \\ -y_{n,k} & \text{if } m \in FOBs \\ 0 & \text{otherwise} \end{cases} \qquad k \in \mathcal{K}, n \in \mathcal{N} \tag{A.5}$$

$$\sum_{\substack{v \in \mathcal{V} \\ G_v \leq D_r}} x_{v,r} \leq 1 \qquad r \in \mathcal{R} \tag{A.6}$$

$$y_{n,k,r} \leq \sum_{v \in \mathcal{V}} \alpha_{v,k} B_v x_{v,r} \qquad n \in \mathcal{N}, k \in \mathcal{K}, r \in \mathcal{R} \tag{A.7}$$

$$\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} W_k y_{n,k,r} \leq \sum_{v \in \mathcal{V}} Q_v x_{v,r} \qquad r \in \mathcal{R} \tag{A.8}$$

$$\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} y_{n,k,r} \leq \sum_{v \in \mathcal{V}} B_v x_{v,r} \qquad r \in \mathcal{R} \tag{A.9}$$

$$w_i \leq \sum_{d \in \mathcal{V}} \sum_{c \in \mathcal{C}} \delta_{n,k} a_{n,k} \qquad i \in \{1, \ldots, |A.1|\} \tag{A.10}$$

$$x_{v,r} \in \{0,1\} \qquad r \in \mathcal{R}, v \in \mathcal{V} \tag{A.11}$$

$$y_{n,k} \in \mathbb{N} \qquad k \in \mathcal{K}, n \in \mathcal{N}. \tag{A.12}$$

$$y_{n,k,r} \in \mathbb{N} \qquad k \in \mathcal{K}, n \in \mathcal{N}, r \in \mathcal{R}. \tag{A.13}$$

$$w_i \in \mathbb{N} \qquad i \in \{1, \ldots, |A.1|\} \tag{A.14}$$

Separation Model

Minimize:

$$z^{\text{SEPARATION}} = \sum_{p \in P} \alpha_p u_p^* - \beta$$

subject to:

$$\alpha_p > \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} a_{n,k}\,\delta_{n,k} - 1 \qquad p \in \mathcal{P} \tag{A.15}$$

$$\beta \leq \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} d_{n,k}\,\delta_{n,k} \tag{A.16}$$

$$\delta_{n,k} \in \mathbb{R}_+ \qquad k \in \mathcal{K}, n \in \mathcal{N} \tag{A.17}$$

$$\alpha_p \in \mathbb{R}_+ \qquad p \in P \tag{A.18}$$

$$\beta \in \mathbb{R}_+ \tag{A.19}$$

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Annex B: Further computational results

Table B.1: *10% of commodities are required within a lead-time* $[T_{SP}, \ldots[$

lead-time	Column Generation				Column and Cut Generation			
	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)
$1.0 * T_{SP}$	239166.1	254007.4	6.2	2.0	254007.4	254007.4	0.0	3.2
$1.2 * T_{SP}$	238710.0	260089.8	8.9	1.4	254007.4	254007.4	0.0	188.3
$1.4 * T_{SP}$	216726.3	236486.0	9.1	4.4	230403.7	236486.0	2.6	230.2
$1.6 * T_{SP}$	217833.5	236486.0	8.5	6.1	231620.2	236486.0	2.1	291.4
$1.8 * T_{SP}$	216726.3	236486.0	9.1	2.9	231924.3	236486.0	1.9	249.5
$2.0 * T_{SP}$	216550.7	230403.7	6.3	3.6	230403.7	230403.7	0.0	276.5
$2.2 * T_{SP}$	217833.5	230403.7	5.7	6.2	230403.7	230403.7	0.0	7.4
$2.4 * T_{SP}$	217255.5	230403.7	6.0	4.9	230403.7	230403.7	0.0	271.2
$2.6 * T_{SP}$	205666.1	218019.6	6.0	6.4	218019.6	218019.6	0.0	7.3
$2.8 * T_{SP}$	188400.8	200717.6	6.5	2.2	200717.6	200717.6	0.0	255.8

Table B.2: *30% of commodities are required within a lead-time* $[T_{SP},\ldots[$

lead-time	Column Generation				Column and Cut Generation			
	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)
$1.0 * T_{SP}$	307365.3	332686.6	8.2	2.2	332686.6	332686.6	0.0	5.9
$1.2 * T_{SP}$	306287.1	332686.6	8.6	2.7	332686.6	332686.6	0.0	546.8
$1.4 * T_{SP}$	250727.3	269743.3	7.5	23.4	269743.3	269743.3	0.0	298.3
$1.6 * T_{SP}$	250988.5	269743.3	7.4	16.9	269743.3	269743.3	0.0	87.3
$1.8 * T_{SP}$	251362.3	269757.6	7.6	31.6	269757.6	269757.6	0.0	35.4
$2.0 * T_{SP}$	250639.7	269743.3	7.6	27.7	268917.6	269743.3	0.3	617.0
$2.2 * T_{SP}$	249983.3	269743.3	7.9	29.3	269729.9	269743.3	0.6	572.9
$2.4 * T_{SP}$	249244.7	268572.0	7.7	24.2	268086.0	268572.0	0.1	627.6
$2.6 * T_{SP}$	219958.4	231807.8	5.3	27.2	231807.8	231807.8	0.0	605.0
$2.8 * T_{SP}$	189389.2	200717.6	5.9	3.4	200717.6	200717.6	0.0	5.4

Table B.3: *50% of commodities are required within a lead-time in* $[T_{SP}, \ldots[$

instances	Column Generation				Column and Cut Generation			
	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)
$1.0 * T_{SP}$	377402.4	411365.7	8.9	2.3	411365.7	411365.7	0.0	3.9
$1.2 * T_{SP}$	375461.9	419233.6	11.6	2.7	416192.5	419233.6	0.7	618.9
$1.4 * T_{SP}$	285225.7	309082.8	8.3	55.6	307946.5	309082.8	0.3	656.3
$1.6 * T_{SP}$	286200.0	309082.8	7.9	205.8	309082.8	309082.8	0.0	323.8
$1.8 * T_{SP}$	287395.7	309082.8	7.5	275.9	309082.8	309082.8	0.0	320.9
$2.0 * T_{SP}$	283210.6	309082.8	9.1	241.9	308455.8	309082.1	0.2	561.8
$2.2 * T_{SP}$	284007.8	309082.8	8.8	410.6	307771.5	309082.8	0.4	782.9
$2.4 * T_{SP}$	283515.8	308497.2	8.8	339.2	307269.4	307911.6	0.2	634.0
$2.6 * T_{SP}$	236566.2	245596.0	3.8	147.6	244276.7	244276.7	0.0	153.5
$2.8 * T_{SP}$	188518.8	200717.6	6.4	7.4	200717.6	200717.6	0.0	809.9

Table B.4: *80% of commodities are required within a lead-time in* $[T_{SP},\ldots[$

instances	Column Generation				Column and Cut Generation			
	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)
$1.0*T_{SP}$	521630.6	562641.6	7.8	1.8	562641.7	562641.7	0.0	3.88
$1.2*T_{SP}$	520176.1	556559.3	6.9	1.8	556559.3	556559.3	0.0	4.4
$1.4*T_{SP}$	378842.1	409580.2	8.1	153.1	409580.2	409580.2	0.0	462.5
$1.6*T_{SP}$	403497.8	378842.1	6.5	399.7	403497.8	403497.8	0.0	442.4
$1.8*T_{SP}$	379888.3	403497.8	6.2	989	403497.8	403497.8	0.0	1302.7
$2.0*T_{SP}$	361087.2	386506.4	7.0	1441	383800	386506.4	0.7	1461.6
$2.2*T_{SP}$	363241.3	385920.7	6.2	2294.7	384567.7	385920.7	0.3	2948.4
$2.4*T_{SP}$	362269.9	389547.5	7.5	2099.6	378052.8	384135.2	1.6	2662.9
$2.6*T_{SP}$	309555.9	319941.1	3.3	706.4	319941.1	319941.1	0.0	729.3
$2.8*T_{SP}$	189059	200717.6	6.6	10.5	200717.6	200717.6	0.0	555.2

Table B.5: *100% of commodities are required within a lead-time in* $[T_{SP},...[$

instances	Column Generation				Column and Cut Generation			
	z_{LP} ($)	z_{ILP} ($)	Gap (%)	Time (sec)	z_{LP} ($)	z_{ILP} ($)	Gap (%)	Time (sec)
$1.0 * T_{SP}$	631363.6	741818.1	17.4	1.3	741818.1	741818.1	0.0	2.41
$1.2 * T_{SP}$	615340.9	720000.0	17.0	1.5	698181.8	698181.8	0.0	3.3
$1.4 * T_{SP}$	435248.3	460358.8	5.7	488.5	460358.8	460358.8	0.0	497.9
$1.6 * T_{SP}$	436643.3	460358.8	5.4	2906	60358.8	460358.8	0.0	2932.7
$1.8 * T_{SP}$	438212.7	460358.8	5.0	4820	460358.8	460358.8	0.0	5332.1
$2.0 * T_{SP}$	415356.2	440326.1	6.0	13662.8	440326.2	440326.2	0.0	3696.2
$2.2 * T_{SP}$	416066.7	440326.1	5.8	20174.0	438402.8	440326.1	0.4	20880.5
$2.4 * T_{SP}$	415236.0	437369.4	5.3	18660.8	437184.4	437369.4	0.0	19563.5
$2.6 * T_{SP}$	347987.2	352033.8	1.1	1444.0	352033.8	352033.8	0.0	1471.9
$2.8 * T_{SP}$	188578.2	200717.6	6.4	17.7	200717.6	200717.6	0.0	240.6
$3.0 * T_{SP}$	188705.0	200717.6	6.3	12.2	200717.6	200717.6	0.0	290.3

Table B.6: *Percentage of unsafe front-routes in* $[10\%, 100\%]$ *to be changed*

	Column Generation				Column and Cut Generation			
	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)
20%	227115.0	240861.1	6.0	3.6	240861.1	240861.1	0.0	5.0
30%	296326.2	314457	6.1	5.7	314457.6	314457.6	0.0	7.8
50%	427223.2	447974	4.8	11.5	443045.0	447974.6	1.1	921.8
80%	428373.2	447974.6	4.5	5.6	441843.0	447974.6	1.3	662.1
100%	427223	447974	4.8	3.4	447974	447974.6	0.0	28.2

Table B.7: *Percentage of unsafe end-routes in* $[20\%, 100\%]$ *to be changed*

	Column Generation				Column and Cut Generation			
	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)	z_{LP} (\$)	z_{ILP} (\$)	Gap (%)	Time (sec)
20%	269247.4	287264.7	6.6	4.1	287264.7	287264.7	0.0	242.0
30%	345260.1	365943.8	5.9	6.2	365943.8	365943.8	0.0	9.9
50%	433582.0	460358.8	6.1	4.3	460358.8	460358.8	0.0	273.7
80%	433758.1	460358.8	6.1	3.9	460358.8	460358.8	0.0	5.9
100%	433853.3	460358.8	6.1	3.2	460358.8	460358.8	0.0	264.3

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Military tactical logistics planning is concerned with the problem of distributing heterogeneous commodities (e.g., fuel, food, ammunition, etc.) from main operating bases to forward operating bases in a theatre of operations using a combination of heterogeneous transportation assets such as logistics trucks and tactical helicopters. Minimizing the sustainment cost while satisfying the operational demand under time and security constraints is of high importance for the Canadian Forces. In this study, a logistics planning model is developed to explore effective and efficient strategies for tactical logistics distribution. A mathematical optimization algorithm based on a column and cut generation technique, using Gomory-Chvátal rank-1 cuts, is developed to solve the problem.

This report presents details of a mathematical formulation and a solution algorithm along with an example application to demonstrate the methodology. Computational results are presented in order to measure the degree of efficiency and scalability of the proposed approach, and to study the trade-off between the efficiency and effectiveness in the resulting sustainment strategies.

14. KEYWORDS, DESCRIPTORS or IDENTIFIERS (Technically meaningful terms or short phrases that characterize a document and could be helpful in cataloguing the document. They should be selected so that no security classification is required. Identifiers, such as equipment model designation, trade name, military project code name, geographic location may also be included. If possible keywords should be selected from a published thesaurus. e.g. Thesaurus of Engineering and Scientific Terms (TEST) and that thesaurus identified. If it is not possible to select indexing terms which are Unclassified, the classification of each should be indicated as with the title.)

Tactical logistics
Tactical support efficiency
Tactical support effectiveness
Vehicle loading and routing
Optimal fleet mix
Quality-of-Support
Large scale optimization
Column and cut generation
Gomory-Chvátal rank-1 cutting planes

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