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Industrial migration in Ontario

Forecasting Aspects of Industrial Activity through Markov Chain Analysis

BY L. COLLINS

Statistics Canada Statistique Canada

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Regional Statistics Research and Integration Staff

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FORECASTING ASPECTS OF INDUSTRIAL ACTIVITY THROUGH MARKOV CHAIN ANALYSIS

by

Lyndhurst Collins

Published by Authority of

The Minister of Industry, Trade and Commerce

June 1972

1300-501

Price: $1.00

Information Canada

Ottawa

FOREWORD

Statistics Canada is pleased to publish this pilot-study on industrial migration.

The author spent the summer months of 1968 and 1969 in the employment of the Regional Statistics, Research and Integration Staff, when – with the full cooperation and support of the Manufacturing and Primary Industries Division – the data problems were solved and the computations carried out. The final text was edited for printing by J.S. Lewis and M.L. Szabo of the bureau's Regional Staff.

I wish to thank all individuals who gave assistance to the author and made the study and its publication possible.

Although the study has been supported by Statistics Canada and published by the bureau, responsibility for the analyses and conclusions is that of the author.

W.E. DUFFETT,
Chief Statistician of Canada.

PREFACE

This study examines the application of Markov chain models to the forecasting of two aspects of industrial activity: the relocations of manufacturing establishments, and their growth as measured by number of employees. The Province of Ontario was chosen as a pilot study area which would provide an adequate and manageable volume of data. The modelling is based on data, spanning the period 1961 - 65, derived from the Census of Manufactures, an annual survey made by Statistics Canada. A further purpose of the study is to explore techniques of improving the accuracy of Markov chain models which could be helpful in this and similar contexts.

The support of Statistics Canada (formerly the Dominion Bureau of Statistics) has made this study possible. In this respect, the author wishes to thank M.L. Szabo, Coordinator, Regional Statistics Staff for his sponsorship, and J.S. Lewis, Special Advisor of that staff, for valuable assistance in the mathematical aspects and the computer programming. Thanks are extended also to D.G. Campbell, Assistant Director, Manufacturing and Primary Industries Division for his assistance in the data acquisition, and to Dr. L.O. Stone, Consultant on Demographic Research, who contributed a helpful critique.

The writer is indebted to many of the staff of the Department of Geography, University of Toronto: in particular, to Professor Leslie Curry whose philosophy and guidance initiated the study's underlying theme, and to Professor Larry Bourne whose constructive comments and advice have been instrumental in the study's organization. Professors Britton, Field, Kerr, MacKinnon and Simmons also contributed valuable suggestions. Miss J.E. Wilcox, Assistant Cartographer, prepared the maps and charts.

Finally, the constructive criticism and extensive editorial assistance of Valerie Collins, the author's wife, have greatly helped to guide this work to completion.

The opinions expressed and the analyses and any errors therein are the responsibility solely of the author.

Lyndhurst Collins.

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CHAPTER I

INTRODUCTION

The prediction of changes in the spatial arrangement of the economic structure within a given system is the ultimate goal of economic geography. But economic geographers, until recently, have concentrated upon analysing locational patterns of economic activities at one point in time; relatively little attention has been given to locational and structural changes. Consequently, most studies have rendered "statistical photographic stills" rather than an understanding of either dynamic spatial processes or economic processes that have spatial consequences. Even when the dynamic aspects of the "economic landscape" have been considered, geographers have tended to focus on past processes rather than on future time paths of spatial units, the study of which is essential for ameliorative economic planning.

Aims and Objectives

An economic landscape, such as that envisaged by Lösch, 1954,[1] is scattered with peaks of urban-industrial activity, the heights and internal structures of which are continually changing. One of the most dynamic components of these urban-industrial agglomerations is secondary manufacturing activity which is the concern of this study. The initial purpose is the statistical separation of the processes affecting the spatial and structural dynamics of manufacturing activity in Ontario and, as such, is an essential prerequisite to the second aim: the formulation and adoption of a descriptive and operational forecasting model of manufacturing activity within this arbitrarily defined urban-industrial system. Although there is little formal theory to suggest which general approach would be the most appropriate, recent developments in the mathematics of stochastic processes add attraction to a probabilistic framework as a possible avenue of research.

Conceptually, the requisite operational forecasting model should accommodate not only processes of birth, growth and death but also migration. General observation indicates for example, that an important factor influencing the differential growth patterns of urban-industrial areas is their interdependency which in terms of manufacturing activity can manifest itself in the form of "industrial migration" involving the relocation of plants from one urban area to another. The simplest stochastic model, amenable to migration research, is Markov chain analysis. In a recent discussion of a wide range of industrial location models, Hamilton, 1967, suggests that although "...they are still in their infancy,

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[1] All the references are listed alphabetically in the Bibliography following the appendices.

Markov-chain models seem to have most potential in tracing trends in industrial location . . ." and that "Markov-chain analysis may also lend itself to problems of industrial migration." Markov chain models have provided good approximations to physical processes but in the social sciences, and particularly in geography, their successful application has been limited by severe data constraints, which are discussed in Chapter III. Hence, in addition to the desirability of evaluating Hamilton's comments a further reason for adopting a Markov framework is the need to test the validity of the technique for spatial analysis – given a suitable body of data.

Technically, there are several advantages to a Markov chain model though it is not within the scope of this study to demonstrate that a Markovian framework is the best approach to the study's objectives. A critical assumption of Markov models, as with other models, is that of constant parameters or "stationarity" but in Markov chain analysis statistical procedures are available for testing this and other underlying assumptions. The stochastic properties of Markov models allow a multiplicity of variables to be embraced by a random component, thereby considerably simplifying the computational procedures. Moreover, the basic matrix structure of a Markov model avoids the necessity of replicating the analysis over as many spatial units as comprise the study area so that in generalizing the processes involved the technique provides insight which may not be so readily attainable by conventional methods of analysis (Rogers, 1968, p. 5). In its ability to utilize well tested matrix algebra formulations, Markov chain analysis is attractive not only because of its predictive capability but also because of its descriptive qualities. Thus, it is a combination of factors – both conceptual and technical – which encourages the adoption of a Markov model.

Methodology

The spatial and structural dynamics of manufacturing activity can be measured and statistically analysed by using such variables as employment, type of activity, value added, number of establishments, corporate structure, sales, and assets. In this study only three variables are used: number of establishments which are grouped into size categories measured by total number of employees, type of activity, and location. The manufacturing establishment is typically a plant or factory and is defined as the "smallest unit which is a separate operating entity capable of reporting all elements of basic industrial statistics" (Statistics Canada, 1960). In the Census of Manufactures, these statistics include, among others, those on materials and supplies used, goods purchased for resale as such, fuel and power consumed, number of employees and salaries and wages, man-hours worked and paid, inventories and shipments.

The study is based on establishment data since the establishment, being the smallest geographical feature of the industrial landscape, is the most appropriate observation for analysing industrial migration. In this context, migration refers not only to the physical movement of activity from one location to another but to the spatial rearrangements created by differential growth rates resulting from a birth/death process and by differential expansion rates involving the location of locally and foreign-owned branch plants. It is recognized, that spatial trends and forecasted patterns of manufacturing establishments do not always correspond to those observed and predicted for other variables. In Metropolitan Toronto, for example, the concentration of manufacturing plants is proceeding at a significantly faster pace than the concentration of manufacturing employment, (Field and Kerr, 1969).

For computational simplicity two sets of Markovian matrices are developed. One set is aspatial in that the Markov "states" comprise establishment size categories; such structural matrices are used to predict changes in the size distributions of establishments for the Province, individual towns, and industrial categories. This model, therefore, analyses the internal structures of selected areal units on a disaggregated basis since frequency distributions are used as input parameters. These parameters are analysed within the conceptual framework of two hypotheses – the Pareto and the lognormal – which are concerned with prevailing size distributions in economic phenomena. Distinctions between the two distributions will be elaborated in detail in Chapter IV. Both hypotheses embody Gibrat's Law of Proportionate Growth which postulates that the proportional change in the size of a plant in any one time interval is independent of its absolute size (Gibrat, 1957). This implies that large and small plants have the same proportionate rates of growth. Changes in the configurations of these probability distributions are generally assumed to be generated by a simple stochastic or Markov process.

Spatial states comprise the second set of Markovian matrices. Ideally, any system of spatial states should cover completely the study area so that all movement is included in the model. A discussion of specific-order Markov models follows in Chapter II. The basic first-order Markov property, for example, provides that the future location of an observation unit at time t+1 will be dependent upon its location at time t but not on previous locations. Conceivably, then, a system of states can be adopted whereby all movement is masked because the "size" of the state, in terms of geographical area, effectively excludes movement out of that state. Thus, a ten state matrix representing the Canadian provinces may not be appropriate for predicting provincial trends in manufacturing activity measured in terms of number of establishments if interprovincial relocations do not occur; in this event the provincial increase in number of establishments would be independent of events elsewhere. It is necessary in the present context, therefore, to analyse recent trends in the spatial dynamics of Ontario's manufacturing activity in order to devise a meaningful descriptive set of spatial states. The rationale for adopting the set of spatial states used in this study is presented in Chapter V.

Perspective on Forecasting Models

Although the development of forecasting models for projecting recent trends in the economic landscape can be criticised on the basis of our insufficient knowledge of the complex causal interrelationships of known and recorded variables, the need and rationale for developing such models stem from both theoretical and practical interests. The philosophy adopted here is that of Curry, 1964, who, in an applied context comments:

> . . . all of government and private planning can, in a real sense, never be better than the ability to predict.

Some of the problems encountered in developing forecasting techniques are due to lack of disaggregated data which it seems has rendered most models of the social sciences incomplete and of a highly generalized nature. Nowhere do deficiencies of data manifest themselves more clearly than in studies of the urban-industrial environment in which heavy ". . . reliance on cross-section data and the ad hoc methods of projecting employment changes in metropolitan growth models have biased these models toward underestimating the amount of change in existing distributions . . .", (Kain and Meyer, 1968). A direct consequence of such data deficiencies is that the application of all forecasting models is contingent upon the acceptance of certain limiting assumptions, which are discussed in Chapter III. Any forecast, therefore, can be interpreted only in the framework of these assumptions.

The practical need for forecasting models of the economic environment has been recognized by both geographers and economists alike. Spatial and economic forecasting models overlap considerably though their objectives are quite distinct. Economic forecasting models, for example, focus on temporal changes in the values of certain economic phenomena and upon their effect on the economy. Spatial forecasting models, on the other hand, deal with the dynamics of variables as they are distributed across the landscape. Theoretically, these variables are regarded as being spatially and temporally continuous though their gradients or slopes can be detected only at discrete points in space. In the context of the urban-economic landscape, Bourne, 1967, has commented on the difficulty of forecasting spatial trends in the process of urban redevelopment which is usually too localized and scattered for the identification of spatial surfaces. The same difficulty exists for the analysis of manufacturing activity which, as previously mentioned, appears as peaks in the Löschian economic landscape. The convoluted tent-like surface is far more irregular, and hence future patterns tend to be less predictable than those derived from the analysis of orthogonal surfaces representing ciruclation patterns used by meteorologists for weather forecasting, (Malone, 1956).

The utility of forecasting depends partly on the length of the forecast which is referred to as short, medium, or long-range. General economic forecasting usually considers three month periods for its short range forecasts.

Short-range forecasts for the spatial economy, however, extend to three to four years after date of prediction, (e.g. Colm, 1958, p. 178 ff; Fouraker, 1957, p. 285 ff) medium range forecasts normally predict for a five- to ten-year period, and anything in excess of this is termed long-range. The present study considers only short- and medium-term forecasts.

The prediction of comprehensive changes in the economic landscape would require a dynamic model of innumerable dimensions embracing a host of variables. Such a model, theoretically, would provide forecasts of locational and structural changes of economic activity under varying degrees of technological change and general economic advancement; few assumptions would be necessary. Realistic attempts at model building must of necessity select aspects of the spatial economy and postulate certain relationships between selected variables which are intended to stand proxy for the most important characteristics of the phenomena in the real world. Since in this study the number of variables is limited to three the model adopted is simple and highly generalized.

The Data and Their Organization

Most of the data used in this study have been extracted from information collected by Statistics Canada in its annual Census of Manufactures. This information is recorded on computer tapes for the years 1961 - 66 which form the study period. Although 1961 is the first year for which the data are available on computer tapes, the main reason for using 1961 as the base year is the change in that year of the definition of the establishment. Prior to 1960, the Census of Manufactures attempted to cover the manufacturing activities of a large number of establishments which were not principally engaged in manufacturing operations. Beginning with the 1961 Census, and the redefinition of the establishment, those establishments not primarily engaged in manufacturing were no longer included in the manufacturing universe, (Statistics Canada, 1960, p. 8). At the same time, however, provision was made to collect statistics on the non-manufacturing activities of manufacturing establishments, thus providing statistics on "total activity" as well as on "manufacturing activity" of establishments classified to the manufacturing industries. The new definition, of course, resulted in a decrease in the number of recorded establishments. In Ontario, by the old definition there were 13,387 manufacturing establishments in 1960, but the new definition decreased the number by 10% to 12,090 for the same year, (Statistics Canada, 1969). Since this study concerns the analysis of individual establishments, the definitional change limits the length of the data series. One other constraint affecting the presentation of data is the confidentiality imposed by the Statistics Act. For this reason some of the statistics in the tables are presented in the form of percentages or probabilities and, where necessary, graphs have been truncated.

The study includes all manufacturing establishments in Ontario as defined by Statistics Canada, with total employees of individual establishments ranging from zero to the size of the largest plant.[2] Plants recorded with zero employment are those in which only working owners or partners are engaged or in which the paid employment amounts to less than one-half man-year. Each establishment on the computer tape is assigned a code for its province, county, municipality, industry (at the 4-digit level), and sequentially assigned establishment number. Thus a plant (013), manufacturing coffins (S.I.C. 2580), in Gravenhurst (13), Muskoka County (30) in the province of Ontario (05) is identified as 05-30-13-2580-013. Each establishment, then, can be identified as within the province by its **combined** S.I.C. code and establishment number.

Sawmills (S.I.C. 2513) are the one exception. These are specifically numbered only within counties and duplicate numbers occur within a province. Since such plants cannot be uniquely identified from year to year on a provincial basis, sawmills have been excluded from the analysis. For all other industries the establishment number is unique within a province.

The unique number enables the data to be organized into four groups which for convenience are assigned demographic terms. A **resident** or permanent plant is one which appears in the same location in the five-year period 1961 - 65. Only this period is used for the analysis. The 1966 data were processed but have been used only for testing the models' predictive accuracy. A plant is "born" when its unique number appears in the data for the first time and is described as a **birth**; similarly a plant "dies" when its number disappears for the first time and is termed **a death**. Those plants that appear with different location codes in two successive years are termed **migrants** or **relocations.**

Data Constraints

In the strictest sense, the present study deals with aspects of industrial activity in Ontario as measured by the Census of Manufactures. This survey, like any other, filters the "true world" through a particular data acquisition procedure which necessarily imposes certain conceptual and definitional constraints, along with the usual risks of errors in reporting, recording, and processing. These constraints and errors cause certain distortions of the data relative to any specific model of manufacturing activity. While the possible sources of these difficulties are readily identified, the detection and correction of actual errors is not often feasible. These problems stem partly from a situation very common in research: the study has objectives which extend beyond those for which the data were originally developed. This restricts the choice of modelling techniques to those which are not highly sensitive to data distortion; in this respect, the suitability of Markov chain models is examined in later sections.

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[2] For brevity, the term "plant" will be used synonomously with "manufacturing establishment".

The study has been limited to the province of Ontario which, therefore, is viewed as a closed system. Hence, a plant relocating from the province of Quebec to Ontario, for example, is classified as a birth. Similarly, a plant relocating from Ontario to any other province or outside the country is classified as a death. The number of births and deaths can also be distorted by production changes. A plant manufacturing electronic components for aircraft in 1961 would be assigned a S.I.C. code of 3210 (Aircraft and Parts Manufacturers), but by 1962 the plant may be geared to the production of satellite components and would then assume a S.I.C. code of 3350 (Communications Equipment Manufacturers). In such cases a birth and death is recorded in the respective industries.

Certain other conceptual qualifications apply to any measurement of births and deaths. For reasons arising from the accounting of reporting units, an establishment can be "born" or "die" because it begins or ceases to be able to report, or to report with reasonable ease and accuracy, the required minimum range of principal statistics for an establishment; that is, reporting units are split or merged from time to time for purely statistical reasons (apart from actual organizational mergers or splits of business units). In these and other cases, the components of an establishment may not always be in one location. As the *1960 Standard Industrial Classification Manual* puts it: "Theoretically, an establishment would be engaged in only one kind of activity in one location but in practice . . . the unit for which information is usually obtained on statistical surveys is engaged in a number of activities and sometimes these activities take place in different locations".

There are additional sources of possible errors. In the clerical processing of the survey information, the misallocation of establishment numbers can result in false births and deaths, and the miscoding of locations can create apparent plant movement. Apparent relocations can also occur through errors in a further processing step necessary to this study. Since the location data are specified in terms of the municipality definitions at the time of each annual survey, to make comparisons over a span of years requires the difficult conversion to a common set of boundaries. Although total movement is underestimated because relocations within municipalities cannot be distinguished, this particular deficiency has no serious implications for the present study which is mainly concerned with interurban migration patterns. Whatever their short-comings, the data still represent the best and most complete source of information for a statistical analysis of Ontario's manufacturing activity.

Study Design

The design and organization of the study mirror its twofold aim: to provide a detailed analysis of the processes underlying the structural and spatial dynamics of manufacturing activity, and to adopt and develop a probabilistic forecasting model. Markov chain analysis, the simplest stochastic approach, is selected as an appropriate methodology and the rationale for adopting a probabilistic framework is discussed in Chapter II. Chapter III provides a basic description of Markov chain models and their assumptions as they apply to industrial geography. The fourth chapter focuses on the analysis of prevailing size frequencies which act as states in the structural matrices. Changes in the configurations of these frequencies for permanent establishments are examined in the context of hypothesized stochastic growth mechanisms leading to the Pareto and lognormal distributions. Structural variations in the birth and death process, which modifies the form of these distributions for "all establishments", are also examined in Chapter IV. Spatial variations of the process as well as other processes of change are analysed in Chapter V which provides the rationale for the selected system of spatial states. Chapter VI concerns the development, improvement, and testing of the structural and spatial Markov models which are applied to provide short and medium-term forecasts of the number and size distribution of manufacturing establishments. A general evaluation of the study's aims and objectives is presented in Chapter VII.

CHAPTER II

STOCHASTIC PROCESSES AND THE SPATIAL ECONOMY

Given the rationale for developing forecasting models of manufacturing activity, the principles for adopting a probabilistic framework, as opposed to a more traditional deterministic approach, are presented in four interrelated sections. The first outlines the basic concepts and properties of stochastic processes which are then broadly classified in section two; this section also places in context Markov chain analysis. Reasons for the increasing trend to inject concepts of random processes into spatial analysis are presented in section three which leads to a discussion of the concept of "uncertainty" or randomness in section four.

A. Basic Concepts of Stochastic Processes

The term random process describes a series of events to each of which there corresponds probabilities of particular outcomes. In an independent random process each event has no dependence on other events and is thus synonymous with "pure chance". Theoretically, the terms random process and stochastic process are also synonymous but in practice the latter term is normally used when a time parameter is introduced. A probabilistic model specifies the complete joint probability distribution of different kinds of events at each point in time, and the whole process is referred to as a stochastic process. The concept of the random variable is central to the theory of stochastic processes and may be regarded as a mathematical entity arising from probabilistic mechanisms just as conventional nonstochastic (systematic) variables are associated with deterministic mechanisms, for which the outcome of an experiment is exactly predictable. A stochastic model, therefore, is one which produces results in terms of the probabilistic law or distribution governing the process and provides only the probability or likelihood associated with a set of possible future states.

B. Classification of Stochastic Processes

Stochastic process models, like other models, are subject to classification which makes them more amenable to discussion. Harvey, 1967, for example, in his review relating to models of spatial patterns, draws attention to the stochastic counterparts of various classes of deterministic models which are classified as "comparative statics", "process models", "growth models with spatial assignment", and "time-space models". Harvey also outlines three specific types of stochastic models: "quantitative ecological models" which depend upon a general understanding of the Poisson process, and have been used in plant ecology with "quadrat sampling" and "nearest neighbour analysis"; Markov chain models; and "Monte Carlo simulation techniques". A similar yet broader grouping suggested by Cox and Miller, 1965, describes stochastic models as "open" or "closed". The words are used in a direct sense; membership of a closed system does not change over time so that in a study of industrial structure the number of manufacturing plants would be assumed constant. No births or deaths would be allowed and the focus of such a study would be on short-term internal changes. Conversely, although an open system with both gains and losses would be more realistic it would be highly complex both operationally and conceptually. A variant of these models would allow one or the other growth operators to exist so that a study focused on a rapidly declining/developing area would provide insight into the rates of the processes involved.

Cox and Miller also distinguish stochastic models as being "discrete" or "continuous" depending on whether the time variable is treated in these terms. Most economic processes are continuous and in an analysis of industrial structure, since changes of plant size or location can take place at any time, a continuous time model based on an open system is therefore apposite. But since data on changes are available only at discrete time intervals, discrete or "discontinuous" time models are usually employed. Fortunately, however, discrete time models can be used profitably to approximate a system in continuous time.

Stochastic models are also classified by Cox and Miller depending on whether or not they possess the Markov property. **Markov process** models possess this property and can be regarded as generalizations of **Markov chains**; in a Markov process model a transition from one state to another can take place at any point in time but in a Markov chain the state varies only at discrete time intervals. Markov chains are described by Feller, 1968, as:

> . . . stochastic processes in which the future development depends only on the present state, but not on the past history of the process or the manner in which the present state was reached.

Thus; in a Markov chain a system of states changes, according to some probability law, with time t in such a manner that the system changing from a given state S_i at time t_{0+1} depends only on the state S_i at time t_0 and is independent of the states of the system at times prior to t_0. Formally, this is written:

$$\text{Prob.}\left\{\xi_t \leqslant x \mid \xi_{t_1} = y_1, \xi_{t_2} = y_2, \ldots \xi_{t_n} = y_n\right\} = \\ = \text{Prob.}\left\{\xi_t \leqslant x \mid \xi_{t_n} = y_n\right\}$$

which holds for all $t_1 < t_2 < \ldots < t_n < t$ and for all x where ξ_t is the random variable of the process. This means that we can predict the value of the random variable ξ_t at time t, on condition that its value at a previous point of time t_n is known. If the process is Markovian, then we cannot improve this prediction on the basis of our knowledge of the state of the system at times prior to t_n. If the state of the system at time t_{0+1} is only dependent on the state of the system at time t_0 plus some independent random component, the process is referred to as a **first-order Markov chain**. In a second-order Markov chain the state of the system at time t_2 would depend on the states of the system at both time t_0 and t_1. In this way, provided sufficient data are available, the "dependence" of a Markov chain can be extended to any length. Pattison, 1965, for example, used sixth-order Markov chains in a study of hourly rainfall rates. Thus, generally in an n-th order Markov chain the state of the system at time t_n is dependent on the n prior states. It should be noted, however, that even in a first-order Markov chain, although at time t_1, the state S_1 depends on S_0 and need not refer to $S_{0-1}, S_{0-2}, \ldots S_{0-n}$, the effects of $S_{0-1}, S_{0-2}, \ldots S_{0-n}$, are subsumed in S_0 which represents the sum of past history. To express the degree of such dependency of a given state upon previous states, Feller provides the term "memory". In using this concept Krumbein, 1965, has outlined the position occupied by Markov chain models within the broad conceptual spectrum that includes classical deterministic models at one extreme and purely random models at the other. Consider, for example, that the industrialization process in a particular area is a system comprising a set of states, then in a classical deterministic model the state of the system in time or space can be exactly predicted from knowledge of the functional relation specified by the underlying differential equations. At the other extreme, in a purely random model, the state of the system at any instant or point in time or space is wholly independent of its state at any other instant or point and is specified by **underlying fixed probabilities**. The latter notion is exemplified in a Bernouilli trials problem and Poisson process. A classical deterministic model, where the state of a system at t_0 depends upon all previous states, has a **long memory**, whereas a purely random model has a marked **lack of memory**. The first-order Markov chain model, therefore, although it embodies the sum of past history contained in state S_0, occupies a position of **partial dependence**. But as stated above, Markov models include an independent random component that precludes exact prediction of future events and in this respect the model has some resemblance to the completely random model. The terms dependency, predictability and memory clearly represent gradations rather than mutually exclusive categories.

In this study, for reasons outlined above, only time homogeneous discrete Markov processes are considered. To what extent, then, can we justifiably analyse the spatial and structural dynamics of manufacturing activity in a probabilistic or Markovian framework?

C. Stochastic Processes and Spatial Analysis

Since the probability distribution depends explicitly on time, stochastic processes represent the "dynamic aspect of statistical theory" (Bartlett, 1953) and it is this property which has prompted both Dacey, 1963, 1964, 1966, and Curry, 1964, 1967, to inject probability theory into spatial and locational analysis. The necessity for formulating **stochastic** models in spatial analysis is readily appreciated by considering the possible range of **alternatives** associated with individual events. In a study of industrial migration, for example, there are no certain means of predicting if or when a single industrial plant will expand in situ, establish a branch plant, relocate, or close down. But given a sufficient number of observations we can attach probabilities to each particular "alternative".

Demographic studies of interregional migration by Rogers, 1966, have shown that the interplay of economic and social forces is too complex to be formulated within the simple cause and effect relationship of a deterministic framework; yet it is precisely this static framework that has been employed to model the economic landscape. In settlement theory both Christaller, 1966, and Lösch, 1954, theorize about settlement patterns as being components of an essentially static framework. Neither of these models reveals the underlying random and dynamic aspects in the development of urban systems. The deterministic approach has equally dominated industrial location theory. Ever since Schaffle, Launhardt, and Weber formed their strictly deterministic models of industrial location, geographers and economists alike have endeavoured to modify and improve them. Isard, 1965, synthesized all of the earlier location models to formulate a general equilibrium model, but still deterministic and essentially static his model was not operational because of the large number of variables involved. In this respect, both Curry, 1964, and Hamilton, 1967, using evidence from studies by Koopmans and Beckman, 1957, Garrison, 1959, and Bos, 1965, have pointed to the almost insuperable computations required for the solution of a relatively simple problem of allocating growth units to locations when only a small number of variables are incorporated into a deterministic model. Such observations encourage the application of stochastic models with their greater elasticity to industrial migration especially in those areas where industrial location is not strictly and scientifically planned (Hamilton, 1967).

Under these conditions it would be relatively easy to show that economic variables like population, demand, and firm growth which by their interaction specify the course of growth of an economy, or at least some of its sectors, are probabilistic in nature. This is because the decisions underlying demand and production variations are not made in a world of complete certainty, but only in one of imperfect knowledge and uncertainty. Irregular and unpredictable fluctuations in demand and supply occur constantly so that the analysis of economic growth and of the spatial economy in a probability framework acquires a crucial role. From the purely formal and analytical standpoint a probabilistic approach to the analysis of the spatial economy serves to generalize the purely deterministic results derived from conventional location models which neglect stochastic influences. The theory of stochastic processes, for example, shows that for linear models restricted to very short periods, the solution of a deterministic model is very similar to the mean solution of the corresponding stochastic model, (Bharucha-Reid, 1960). But whenever there are a large number of factors to be considered or whenever events are highly disaggregated a stochastic model is usually more realistic than its deterministic counterpart. Deterministic models may provide a reliable estimate for the whole economy, but at the finer level of small area analysis stochastic process models may be more appropriate. Britton Harris, 1956, in his critical assessment of methods for projecting industrial growth in metropolitan areas vis a vis national projections has endorsed this notion:

. . . any metropolitan area is small relative to the national economy and there is more opportunity for random variation in development.

Within the present context, the concept of regarding the growth pattern of urban-industrial areas as a stochastic process is best illustrated by reference to a hypothetical example. In the simplest case, consider the changes in the amount of manufacturing activity measured by the number of establishments in any one town. The manufacturing activity in this town grows or decreases in irregular jumps as a result of births, deaths, and relocations; in the case of one town; however, relocations are considered as births or deaths. The growth/decline of the town's industry can be typified by a discontinuous stochastic process in which the random variable – the plant – as a function of time is the step function; the appearance or disappearance of a plant is randomly distributed along the time axis. A second random variable is the height of the step measured, in this case, by the size of the plant which is justifiably considered independent of the size of the town measured by the population of establishments. With this assumption it can be shown that the evolution or development of the population of establishments in our hypothetical town can be approximated by a simple stochastic birth and death process as has been outlined by Simon, 1955. In this process, only two kinds of transitions are considered, namely a transition to the next higher state (one birth) and a transition to the next lower state (one death). Formally, Feller, 1968, shows that in an interval of time Δt during which the state of the system $S_i \rightarrow S_{i+1}$ there is a probability $\mu\Delta t + 0(\Delta t)$ of a plant's dying and a probability $\lambda\Delta t + 0(\Delta t)$ of a plant's being born – the probability of multiple births or multiple deaths is $0(\Delta t)$. The corresponding probabilities for a population of n planst are $n\mu\Delta t + 0(\Delta t)$ and $n\lambda\Delta t + 0(\Delta t)$. Thus, a contagion effect is postulated by which new establishments are attracted to a town because of its apparent economic viability reflected in the number of manufacturing establishments; each plant contributes to the prosperity of the town which benefits and which in turn is able to provide better utilities, transportation networks and other social overheads. The process typifies that of cumulative causation, (Myrdal, 1957; Pred, 1966), in that existing plants attract new ones. For the town the birth intensity λ will depend on the generative growth structure of the new industry and on how effective the town is in using its increased capacity to improve its locational attractiveness.

Another aspect of the economic landscape which may be interpreted through the language of probability is the tendency for the size distribution of cities and towns to conform to the Pareto distribution. Both Zipf, 1949, and Madden, 1956, have provided empirical evidence for the existence of the Pareto distribution as applied to the ranked populations of urban systems, and Berry and Garrison, 1958, in their study of urban rank size relationships write: "There seems to be no doubt that the empirical regularity with which we are concerned exists." Thus, it seems pertinent to ask the question: does this empirical regularity exist also for manufacturing establishments at various disaggregated levels of spatial units? The frequent occurrence of this phenomenon among economic variables has concerned both geographers and economists for some time but like many empirical relationships it has never been satisfactorily explained. That the Pareto distribution may result from a stochastic process was first explained by Champernowne, 1953, in his analysis of income distributions. These distributions, Champernowne argued, approach a steady-state equilibrium which is described by Feller, 1968, as a state of "... macroscopic equilibrium ... maintained by a large number of transitions in opposite directions." Within a stationary human population, for example, there exists a state of continual flux in which people are born, age, and die. Individual births and deaths are random events and are not predictable but the total population and its age structure remain stable, the configuration being determined by the birth and death probabilities. Thus, the result of the process depends only on the transition probabilities from one state to another and is independent of the initial distribution. Within geography, Curry, 1964, has attempted to formalize the organizational features of systems and cities in terms of cybernetics or self-organizing systems, and concludes that if a system of cities assumes a Pareto distribution then entropy has been maximized. The available evidence, then, suggests that the explanation of the phenomenon depends ultimately upon probability theory, by which order in the mass is produced out of individual chaos by the very fact of the chaotic or random character of individual action, (Steindl, 1965). The validity of this viewpoint in relation to prevailing distributions of manufacturing establishments is examined in Chapter IV.

D. Uncertainty and Randomness

Perhaps one of the strongest forces encouraging the adoption of a probabilistic framework to the exclusion of a deterministic approach is the concept of economic uncertainty referred to above. In this context Curry, 1966, states:

> Uncertainty is a basic fact of life for both individuals and groups of men. It matters little operationally whether this uncertainty be inherent indeterminacy or simply reflects ignorance of deterministic sets of events. Uncertainty is particularly important in connection with the future so that action predicated [Sic] on future conditions may be understood as the making of decisions within the range of possible future.

Acceptance of this concept should not be misinterpreted as a step towards invoking Heisenberg's principle that because of uncertainty an exact theory cannot exist. Rather, the argument is that the deterministic model, regardless of its mathematical sophistication, can never accommodate the uncertainty prevalent in the economic landscape. A probabilistic model, however, can include the element of uncertainty by introducing probabilities in place of mathematical variables, that is by introducing random variables. These variables, unlike deterministic quantities, assume different values with different probabilities. Indeterminacy or randomness exists not only because of imperfect foresight but also because of "economic man's" incomplete information or inability to abstract from the maze of interrelationships the possible effect(s) of many small factors in reality.

Random behaviour, be it individual or collective, does not preclude a model capable of providing reliable estimates and insight which may lead to an explanation to the type of processes involved. The model builder, equipped with a detailed knowledge of the economic environment can, by comparing alternative situations, state what types of establishments or mode of behaviour will have a relatively greater probability of survival. In this respect the commercial application of probabilistic forecasting has been elaborated by meteorologists, (Crossley, 1952; Malone, 1957), in probabilistic weather statements for operational decisions in military manoeuvres, (Jacobs, 1947) and in the construction industry for pouring concrete, (Thompson, 1950; Brier, 1955). Clearly, any prediction arising from a probabilistic approach will not assert that all or even particular individual plants necessarily change their characteristics. It asserts instead that the characteristics of the new population of establishments, or possibly an existing population, will change. This population may be characterised by the "representative plant" which stands as a purely statistical concept and signifies a vector of averages with one dimension for each of the several qualities of plants. No single producer, therefore, need typify the representative establishment which is a set of statistics summarizing the "various modal characteristics of the population", (Alchian, 1950). Since the subject matter of spatial analysis is fraught with uncertainty it does seem appropriate to incorporate the notion of randomness in studies of geographic phenomena.

In spite of the many advantages offered by a stochastic approach, concepts of random processes have invaded the social sciences, especially geography, only recently and have not advanced very far. One of the distinct advantages of the probabilistic approach is that it allows the problem to be tackled in ignorance of causal deterministic relationships. This may seem a basis for criticism, but it is precisely that man is a free agent that his behaviour pattern is largely unpredictable and hence must be described in probabilistic terms.

In a stochastic model of industrial structure it would be appropriate to postulate a chance mechanism to describe the transition of plants from one size category to another. The objection in this case would be that an entrepreneur does not make his decisions to expand or decrease production by resorting to "dice or a roulette table". On the contrary, the entrepreneur carefully weighs the advantages and disadvantages of changing his productive capacity, thereby deriving a responsible and rational decision. The contention here, is not that the individual entrepreneur actually uses a chance device to make a decision but that industry as a group behaves as if separate components did use such a method. In sum, the function of probability theory in the present context is simply to describe observed variability; it carries no implications about the freedom or otherwise of human choice. "It is a fact of experience that 'choice may mimic chance'", (Bartholomew, 1967, p. 6).

CHAPTER III

MARKOV CHAINS AND INDUSTRIAL GEOGRAPHY

This chapter resumes the discussion of the concepts and properties of Markov chain analysis. The single most important objective is the examination of **Markovian assumptions** as they apply to industrial activity. This chapter terminates with a discussion of various Markov chain applications most of which have implicitly accepted the validity of these assumptions.

Transition Probabilities

The transition probabilities, the p_{ij}'s, which give the probability that the process will move from state S_i to state S_j are given for every pair of states. The "set" of probabilities or "outcome functions" describes the process as it moves through any finite number of steps. For computational reasons and notational simplicity the transition probabilities are best represented in the form of a transition matrix P.

$$
P = \begin{array}{c|cccc}
 & S_1 & S_2 & S_3 & \ldots S_n \\
\hline
S_1 & p_{11} & p_{12} & p_{13} & p_{1n} \\
S_2 & p_{21} & p_{22} & p_{23} & p_{2n} \\
\vdots & & & & \vdots \\
S_n & p_{n1} & p_{n2} & p_{n3} & p_{nn}
\end{array}
$$

where
$$\sum_{j=1}^{n} p_{ij} = 1$$

and
$$p_{ij} \geqslant 0 \text{ for all i and j}$$

The elements of P denote the probability of moving from state S_i to S_j in the next step. Since the elements of this matrix must be non-negative and the sum of the elements in any row is 1, each row is called a **probability vector** and the matrix P is a **stochastic matrix**. If for some power of the matrix P there are only positive entries the transition matrix is described as **regular.** A Markov chain is regular if, and only if, P^k has no **zero** entries for some k. Thus, regular Markoy chains are readily distinguished from **absorbing** Markov chains since the latter contain at least one state which once entered cannot be left; a state S_i is absorbing if, and only if, $p_{ii} = 1$. A Markov chain depicting the life cycle of industrial establishments and having "death" as a state would of necessity be absorbing. A brief development of some of the basic theory of regular Markov chains is given in Appendix A, along with a numerical example.

The Markov Property

The application of a Markov model is contingent on the identification of the specific-order property. Given adequate data this property can be readily determined by a substantial body of statistical theory. It should be noted, however, that any transition matrix suggests a Markovian model but the partial dependence or Markovity property of a Markov chain renders it unsuitable for the analysis of an independent series of events. A transition matrix depicting such a series is often described as **zero order**, (Billingsly, 1961). Before determining the specific-order of a stochastic matrix, therefore, it is essential to test the validity of the Markovity assumption. For this the **maximum likelihood ratio criterion**, which may be extended to determine the specific-order of the process, is an appropriate test statistic. The design of these tests involving asymptotic distribution theory and the closely related Chi-square tests of the form used in contingency tables is elaborated in two studies of Markov methods by Anderson and Goodman, 1957, and Kullback, Kupperman and Ku, 1962. Basic to all these tests is the actual number of observations for all cells represented in a "Tally Matrix". Assume that plant relocations among three towns (states) have been observed and tabulated as in Table 3.1. For Markovity, the likelihood ratio criterion tests the null hypothesis –

TABLE 3.1. Tally Matrix

	S_1	S_2	S_3	Marginal totals
S_1	120	40	40	200
S_2	60	80	60	200
S_3	120	120	360	600
	300	240	460	1,000

that the movement of plants from one location to another is statistically independent as against the alternative that the observations exhibit partial dependence. The test for Markovity is detailed in Appendix B. In this hypothetical example, $-2 \log\lambda = 165.8$ which is greatly in excess of the tabled value of Chi-square with four degrees of freedom, and the hypothesis of an independent trials process is rejected. Other methods have been used for tests of Markovity; Grant, 1957, for example, used autocorrelation techniques which are applied directly to the sequence of observed values.

A First-order Markov Chain

Alone, the dependence property is insufficient justification for adopting a specific-order Markov model. The assumption of most studies that any transition matrix typifies a first-order Markov process, is akin to that of assuming normality for the application of standard statistical procedures. Such assumptions usually arise from inadequate data. The test for a first-order Markov chain cannot be applied to a simple two dimensional tally matrix. Their test requires observations on individual movements through at least two time intervals; such observations are distinct from the aggregate observations derived by a comparison of the states at two dates. Assume, for example, that the tally matrix presented above is for the 1941 - 51 period. This indicates that during the interval, 40 plants moved from S_1 to S_3 and 120 plants moved from S_3 to S_2, and so on. But to determine that the data typify a first-order process it is necessary to observe the individual movements of these plants during the next time interval (1951 - 61). In this way we can attach a probability to a plant's moving to S_3 in the next interval **given** that it has already moved from S_1 to S_2. The notion is best illustrated by a **conditional probability tree**, which is given for the three-town example in Appendix B. Formally, this shows the probability of moving to the j-th state in the k+1 "realization" given that movement has already occurred from the i-th state to a j-th state in the k-th realization.

The probability tree shows that of the 40 plants which moved from S_1 to S_2 between 1941 and 1951, 28 remained in their locations during the 1951 - 61 period but two returned to S_1 and ten relocated to S_3. Such data are best presented in a **three way** or **cubic matrix** which takes the general form shown in Fig. 3.1. For the hypothetical example the three facets or leaves of the cubic matrix are presented also in Fig. 3.1.

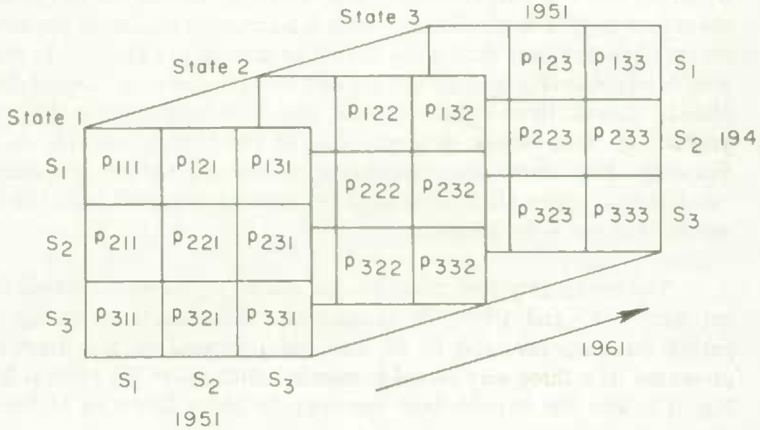
Given such data it is possible to test the null hypothesis that the chain is first-order against the alternative that it is second-order. The statistical test for a first-order Markov chain is detailed in Appendix B. Assuming that $-2 \log\lambda$ is less than the tabled value of Chi-square with $n(n-1)^2$ degrees of freedom the data are considered as typifying a first-order Markov chain. Clearly, by extending the probability tree and hence expanding to n dimensional matrices the likelihood criterion can be transformed to test the more general null hypothesis that the chain is of order r-1 against the alternative that it is of order r. So far, however, there has been no application of this test to socio-economic variables. Gale, 1969, pinpoints the main reason for this inadequacy as being a problem of ". . . obtaining permission to derive the proper parameters and of developing computer programs to do this quickly and efficiently".

The Concept of Stationarity

Fundamental Markov theory requires, in addition to the first-order property, that the parameters be stationary. This implies that the estimated transition probabilities are **fixed** or **constant** throughout the predictive period and as such is a restricting assumption of Markov theory. It is often possible, however, to estimate a series of transition matrices or a set of realizations which typify recent trends. The constancy of these trends can be determined by statistical tests.

FIGURE–3.1

CUBIC MATRIX FOR A THREE STATE MARKOV CHAIN



State 1:1961

1941 \ 1951	S_1	S_2	S_3
S_1	105	2	2
S_2	33	25	28
S_3	100	20	30

State 2:1961

1941 \ 1951			
S_1	5	28	8
S_2	2	50	2
S_3	16	96	10

State 3:1961

1941 \ 1951			
S_1	10	10	30
S_2	5	5	10
S_3	4	4	320

Anderson and Goodman, 1957, for example, provide a likelihood ratio criterion which closely approximates Chi-square, and was applied by Gale, 1969, to sets of small matrices. One alternative, desirable for the larger matrices of this study is the **minimum discrimination information statistic** (m.d.i.s.) which is equal to $-2 \log\lambda$. The advantages of this statistic are in its properties of additivity, convexity and computational facility.

The utility of using the m.d.i.s. in association with its component properties for testing the null hypothesis – that several realizations of a first-order Markov chain come from the same but unspecified matrix of transition probabilities – has been discussed in detail by Kullback, Kupperman and Ku, 1962. Assuming that we have two transition matrices for the three-town example we may postulate the null hypothesis that the two realizations, 1941 - 51, and 1951 - 61 are homogeneous, that is the parameters are constant. The set comprises s (i.e. two realizations) of a first-order Markov chain with n (i.e. three) states. For this the null hypothesis is: the probability of moving from state i to state j in the k-th realization, that is $p_k(S_jS_i)$, is the same for all k(k=1, 2, . . .s) for every possible pairing of i and j where i=1, 2, . . .n, and j=1, 2, . . .n. Under the null hypothesis the additivity property of the m.d.i.s. can be used to provide information statistics for: the homogeneity of the marginal probabilities –(i) – homogeneity; the conditional homogeneity (j/i); and the two way independence homogeneity (i,j). The formulation of these tests is given in Appendix B.

Derivation of Transition Probabilities

The transition probabilities, the p_{ij}'s, form the heart of any Markov chain model; their derivation, therefore, is of the utmost importance. Such probabilities must be estimated for models of real world situations, and alternative approaches are classified here in two main groups: conceptual and statistical estimation.

(a) Conceptual

The general problem is a lack of suitable disaggregated data showing individual temporal or spatial movements of selected economic variables. Krenz, 1964, in a study of temporal changes of farm size in North Dakota, adopted a conceptual approach by postulating rules of behaviour for the micro units (farms). Basic data were obtained from the Quinquennial Census of Agriculture which only enumerates the **total number** of farms in each of several size categories (size based on acreage); no information concerning the movement of individual farms from one size group to another was available. Krenz postulated that: if possible, farm operators would always expand their acreage; the farms most likely to expand are initially larger than average; increases in farm size are most likely to result from gradual increases in acreage; and that a farm is more likely to go out of business than to decrease its acreage. Krenz adopted a six state system (state refers to farm size) and used the assumptions to postulate three **rules of behaviour**. First, farms in the largest category S_6, remain in this category. Second, increases in number of farms in any state S_i move from the next smaller state S_{i-1}. Finally, any decrease in the number of farms in any state, other than from the second rule, results in a movement to S_0 which represents a cover all state for going out of business. An absorbing Markov model is thereby applied. Clearly, this approach is highly dependent on an intimate knowledge and conceptualization of the processes involved.

(b) Statistical Estimation

Where a detailed knowledge of the underlying processes is absent the parameters must be statistically estimated from either aggregate occurrence data of the type used by Krenz or from observations of individual movements between states.

(i) From aggregate data – Transition probabilities can be estimated from aggregate or total occurrence data by linear and quadratic programming procedures which produce **least squares estimates**. The idea was first conceived by Miller, 1952, but has since been refined and extended by Goodman, 1953, Kao, 1953, Madansky, 1959, Telser, 1963, Scott, 1965, Lee, Judge and Takayama, 1965, and Lee, Judge and Zellner, 1970. Miller's initial formulation produces **unrestricted** least squares which automatically fuffil the condition:

$$\sum_{j=1}^{n} P_{ij} = 1$$

but the **non-negativity** and not greater than unity condition, i.e. $0 \leqslant p_{ij} \leqslant 1$, may be violated and hence non-admissible estimates of the transition probability values may appear. In this event adjustments to the parameters are necessary and are usually accomplished by an iterative procedure requiring a subjective termination to the solution, (Telser, 1963).

(ii) From individual observations – Given observations on the individual movements of economic variables, Anderson and Goodman provide the **maximum likelihood** technique for estimating transition probabilities. Maximum likelihood estimates of the p_{ij} are derived by dividing the number of times micro units move from S_i to S_j by the total number of occurrences of S_i; the total number of occurrences and individual movements are obtained from empirical observation. Thus:

$$P = \begin{bmatrix} \\ P_{ij} \\ \\ \end{bmatrix} = \begin{bmatrix} \dfrac{f_{ij}}{\sum_{j=1}^{n} f_{ij}} \end{bmatrix}$$

where f_{ij} is the number of movements of the sample elements from state S_i to S_j.

The maximum likelihood criterion stands as the best alternative for estimating the **underlying fixed probabilities**. Some researchers have inferred, however, that where the objective is one of predicting the proportions in each of the states over time, because of the basis on which the estimates are derived, the unrestricted and restricted least squares estimates may be superior to the maximum likelihood estimates. But the results of Scott's analysis, the only geographic application of the former approach, do not support the inference. Intuitively, it seems that where the objective is both predictive and descriptive, then maximum likelihood estimates of the structural parameters will be more useful. Moreover, the above discussion has shown that when individual observations are available the transition matrices (tally matrices) can be subjected to rigorous statistical tests for determining the specific-order of the chain. All studies predicated on the aggregate formulation of a first-order Markov model carry the uncomfortable assumption that the frequency distributions represent vectors of state probabilities generated by a first-order Markov process. The most that can be determined for such matrices is Markovity and the ergodic nature typical of a regular chain.

Although the maximum likelihood criterion is upheld as the best estimating procedure it should be noted that its limiting property is one of restriction to large samples.

Basic Assumptions of Markov Chain Models

No matter how sophisticated are the mathematical techniques, the formulation of a predictive model depends upon the acceptance of certain restricting assumptions. Markov chain models are no exception. Some of the assumptions referred to earlier are capsuled below, prior to a detailed examination of their full implications for industrial geography. Such a discussion is necessary because each postulate assumes a different level of importance depending on the variables and processes examined.

Four basic assumptions emerge, the first being one of definition. In discontinuous Markov processes it must be assumed that the system is typified by distinctive states and that transitions occur at discrete time intervals – an assumption not unique to Markov chain analysis. Space and time are continuous variables but by virtue of their mode of tabulation all social science data are discrete. Most models, therefore, are formulated in discrete terms.

The second and most limiting assumption has been summarized by Bailey, 1964:

> The restriction that the future probability behaviour of the process is uniquely determined once the state of the system at the present stage is given is the characteristic Markov property.

This implies the ability, in the absence of information about the history of the process, to deduce its future development from knowledge of its present state. But given adequate data the memory of a Markov chain can be infinitely extended.

A third limiting restriction, the concept of stationarity, implies a constant relationship among the transition probabilities throughout the predictive period. For many studies this is not a severe restriction since its limitations can be overcome by manipulating two independent sets of transition matrices; Pattison, 1965, for example, used both first-order and sixth-order chains to traverse transition periods separating rainfall cycles. Presumably the approach could be useful for long-term studies traversing two or more business cycles.

Finally, any structural or spatial grouping of economic variables into a set of distinctive states assumes uniform probabilities for all individual components of each cell or state in the matrix. But the assumption is not always tenable; sociological studies of occupational mobility, for example, suggest a marked negative correlation between length of time in any one state and the tendency to move out of that state. Such observations have encouraged the adoption of a "Mover-Stayer" dichotomy in which one transition matrix represents occupational transients and a second matrix represents those who possess relative occupational stability.

Implications of Markov Chain Postulates for Industrial Geography

One important characteristic of these four assumptions is their interdependence evolving from the initial classification of states.

(i) A system of states – To a considerable extent the pragmatic value of Markov chain theory depends upon the distinctive classification of states. Evidence suggests that manufacturing establishments relocate once, twice, or many times during their existence, (Kerr and Spelt, 1958). Most studies of industrial relocation have recognized the tendency of plants to relocate over a considerable range of distances, (e.g. McLaughlin and Robock, 1949; Ellis, 1949; Hamilton, 1963; Keeble, 1965). Spatial states of origin and destination typified by well defined geographic areas are clearly discernible. But industrial activity is not distributed in a continuum across the landscape; rather it is concentrated into selected nodes separated by conspicuously non-industrial areas. A discontinuous system of spatial states may, therefore, be an appropriate framework for a Markov chain model of industrial activity. Within such a framework, at least in terms of existing migration theory, the probability of a plant's moving from one state to another would be a function of the characteristics of the individual plant and the characteristics of both the state of origin and the state of destination. Since these characteristics cannot be adequately evaluated the underlying fixed transition probabilities using statistical techniques elaborated earlier are most easily estimated from actual observations. The selection and delimitation of the states, as with any formal regionalization of the landscape, involves subjectivity which will in varying degrees affect the estimated transition probabilities.

The same problem arises in a dynamic analysis of industrial structure. States, in this case, refer to establishment size categories, represented by employment, value added, or some other viable measure of productive capacity. Here, the system of states must be continuous within the limits of the actual range of the size criteria examined, though the upper and lower bounds must be subjectively determined. The size of the state will affect the estimated transition probabilities. The degree of effect can be determined more easily for structural than for spatial states by examination of the transition matrices under changing limits. Generally, the smaller the states the greater is the tendency for the appearance of small, unstable elements in the off diagonals of the matrix; whereas the larger the states the more pronounced is the main diagonal. The size of the states and the method of classification do, of course, depend largely on the quality of the data. Kemeney and Snell, 1967, remark that "... if we decide that the Markov assumption is reasonable for a certain method of classification, then we cannot arbitrarily treat a coarser classification as a Markov chain . . . unless the condition for lumpability is satisfied".

Suppose that the movement of plants from one size category to another actually typifies a Markov process and that the location data are divided into as many states as is desirable. Then, given a sufficiently large number of time periods we could obtain, at least theoretically, the transition matrix for the network of movements. This network may reveal that the transition probabilities to and from several spatial states are identical, in which case the states could be grouped into a coarser classification for which the Markov matrices would still give a correct picture of the expected movement.

In practice, however, data are not available for an arbitrarily fine classification so that even if the process examined were of an exact Markov type, the classification adopted might include several states whose patterns of movements are disparate because of industrial mix and age characteristics. A fundamental problem of Markov chain analysis, therefore, is the adoption of a classification scheme which is good enough to enable a reasonably simple model to fit the data. The specific states adopted will depend ultimately on the degree of individual awareness of the problem being studied. Justification of the specific states adopted in this study is made in Chapters IV and V.

(ii) First-order assumption – The ability to verify statistically the assumption that a set of data conforms to a first-order Markov process clearly depends on the method of classification. However, data for such a short time period as is covered by this study are not adequate to test the assumptions for the spatial matrices; in five years it is not to be expected that many manufacturing establishments will change spatial states more than once – no matter what classification scheme is adopted. Nevertheless, several studies of intraurban location (e.g. Reeder, 1954; Martin, 1966) have shown that the new location of a plant will be dependent upon its existing location though not necessarily on previous locations – a familiar property of a first-order Markov chain. Thus, it is reasonable to hypothesize, that the pattern of industrial location for S_i at time t is a function of the industrial location pattern at time t-1 plus some component of change which may be defined by a set of probabilities. It has been shown in several location studies that the component of change affecting industrial migration in influenced by a multiplicity of factors which include tax incentives, rezoning, lease expiry, introduction of pollution control, and expanded markets. Given that data are not available to test the first assumption and given that plant relocation may be approximated by a first-order Markov process, the task of delimiting a meaningful set of spatial states assumes a crucial role in the research design.

The theoretical model used in this study assumes that of those economic factors – such as entrepreneurship, financial structure and position, proneness to introduce technological change, and profits – which may determine the growth pattern of manufacturing plants, **size**, measured in terms of employment, is assumed to be the most important summary criterion. By using the latter variable the first-order assumption can be statistically verified for the system of structural states adopted in this study.

(iii) Constant parameters – The assumption concerning the future stability of transition probabilities is also partially dependent on the method of classification. By definition, this postulate is dependent on the first-order property which is assumed in statistical tests for stationarity; similarly, statistical tests for the first-order property are predicated on the constancy of the parameters. Acceptance of stationarity for long-term prediction of industrial activity may not always be justifiable since technological change could have a significant impact on existing trends. On the other hand, there is no evidence to suggest that any technological innovation so far has profoundly affected the spatial distribution of manufacturing during a short-term period. Factors likely to have the greatest impact on the spatial rearrangement of manufacturing activity include the construction of new superhighways, direct government subsidies, and the construction of new airports. But the influence of such factors is only asserted gradually. Technological changes may well influence the size structure of industry to a greater degree but again there is little evidence to suggest that even this is substantial in the short-term. In this study, however, the constancy of recent trends is tested lending credence to the assumption of short-term stationarity.

(iv) Uniform probabilities – The valid assignment of spatial and structural probabilities to large numbers of plants depends upon careful grouping which again relates to the method of classification and size or content of the states. In terms of size, a plant employing 200 employees locating in a country town is unlikely to satisfy the notion of uniform probabilities but as part of a large industrial complex it might well do so. Similarly, in terms of state content a factory manufacturing pencils is more likely to relocate than a plant manufacturing locomotives; plants in urban areas are more likely to relocate than plants in rural locations or vice versa. Collins, 1966, in an earlier study supported inferences that branch plants have a higher propensity to relocate than other types of plants. Conceivably, other sub-groups within the broader aggregate categories could be distinguished as having a higher/lower propensity to migrate.

But it is unlikely that a mover-stayer dichotomy adopted in sociological studies is applicable to industrial activity as there is no evidence to suggest a correlation between the length of time a plant remains in a location and the likelihood of its relocating.

Thus, an important aspect of the Markovian assumptions is their interdependency by which the basic classification of states can influence the Markovity component as well as the assumptions of stationarity and uniform probabilities. When applied to industrial activity the severity of the constraints depends on whether the analysis is structural or spatial since the concept of stationarity is not so rigid for spatial as it is for structural patterns. Against the background of these assumptions the following section examines the various applications of Markovian analyses in geography and related disciplines.

Applications of Markov Chain Analysis

Although Markov process models have been applied in a large variety of studies, most applications of Markov chain analysis in the social sciences have been concerned with social mobility and the tendency of economic activity to concentrate among large organizations. The limited success of these studies has been summarized by McGinnis, 1968:

> Few applications of these temporal functions worked especially well when tested against data and some of them were howling failures. In each case Markov chain theory was applied but in no case did it prove to be a particularly good representation of social phenomena.

Many of the failures could be attributed to inadequate data and insufficient number of observations. Normally, the term observation denotes a micro unit in the sample but in some analyses, in an attempt to increase the apparent sample size, the term has been given a different interpretation. Recognizing the basic need of maximum likelihood techniques for large samples Judge and Swanson, 1962, basing their analysis on 83 units for a six-state model, concluded that, "Since each hog producing firm moved (or had the option to move) from one state to another 12 times during the 13-year period the transition matrix is based on 996 observations." A much smaller sample size was used by Archer and McGuire, 1965, in a seven state model in which 13 observations comprised the average sample size. The problems of using such small samples extend to the model testing mechanism. In their analysis of the market structure of food processing firms, Preston and Bell, 1961, for example, using 35 observation units for a six-state model, applied the Chi-square test to the expected distribution vectors which contained five cells with values below five, two of which were less than unity. The validity of such a test has been elaborated by Ray, 1965.

Criticism of these earlier studies focuses not only on the insufficient observations but also on the quality of the data. Clark, 1956, has commented, for example, on the limited sample used by Hart and Prais, 1956, in their seminal application of Markov chain analysis to business concentration. They used only those firms quoted on the stock exchange, and manufacturing establishments were grouped with finance companies, service industries, shipping and trade corporations. Later studies, such as those of Kaplan, 1954, Adelman, 1958, Collins and Preston, 1961, used only the 100 largest industrial firms enumerated on a voluntary basis in public reference manuals.

A common goal of many Markovian applications has been the derivation of equilibrium vectors. Hart and Prais, because of their difficulty in handling realistically the phenomena of firm entry and exit, were not able to derive an equilibrium market structure for business activity. Adelman's technique of adding an additional state comprising a reservoir acting as a source of potential entrants and as a pool for liquidated firms, partially solved the problem of entry and exit and was adopted in several later studies. For the equilibrium state, Adelman provides a proof which shows that the size of the reservoir – no matter how large – does not affect the economically relevant portion of the results.

A comparison of equilibrium vectors computed for both white and negro flows among California's S.M.A.'s provided Rogers with a mobility index for the two populations. The value of the equilibrium vectors to migration studies is summarised by Rogers, 1968:

> At more disaggregated levels, the equilibrium solutions present a detailed, quantitative picture of the spatial implications of current mobility trends. Moreover, they provide indications of temporal changes and of differentials between migrant sub-classes.

Rogers also used mean first passage time matrices to define aspatial measures of interregional "migrant distance" which when interpreted in relative terms provides a measure of interdependence among the respective states. Mean first passage time matrices were also derived by Bostwick, 1962, in a study concerned with the application of Markov chain analysis to decision making in farm management.

Markov chain models were introduced to geographic analysis by Brown, 1964, in a study of the diffusion of innovation. Clark, 1965, presented transition matrices of urban land values in selected American cities to illustrate the concept's potential. In a related study, Bourne, 1969, powered a 1952 - 62 transition matrix of urban land use change to extrapolate land use matrices for decennial periods up to the year 2002. Bourne's approach deviates from others in that he is concerned only ". . . with change data and not the total land use inventory." In an earlier exploratory paper Marble, 1964, showed that the Markov model ". . . despite its very real limitations appears to have some value in the study of certain aspects of travel behaviour."

Implicit in all these studies has been the assumption that any transition matrix can be validly manipulated in a Markovian framework; none of these studies has questioned the assumptions of Markovity, stationarity or first-orderedness. The recent study by Gale, 1969, however, is more encouraging. Gale's explicit aim is "... to test the viability of a methodology" and for this he goes as far as to test his data for stationarity. Unlike his predecessors, Gale opens up new vistas by suggesting a continuous time and discrete space state model. This, he augments with a multivariate approach which is used to test a total of eighteen hypotheses relating to social and spatial mobility of negroes in Ann Arbor 1870 - 99. All his models, Gale claims, are significant at the 0.95 level. Unfortunately, he gives no indication whether his augmented model is an improvement over the simple Markov concept. The notion of a continuous time and discrete space model has since been adopted by Drewett, 1969, in analysing the land conversion process from rural to urban use, and is very similar to the semi-Markov process model adopted by Harris, 1968. In this, the successive selections of the states of development are independent of the time it takes to go from one state to the other so that the "... wait is a random length of time sampled from the distribution."

In addition to outlining the statistical theory associated with a Markov methodology, this chapter has emphasized the implications of Markovian assumptions for industrial geography. The interdependency of the assumptions is shown to be strongly influenced by the classification of the respective states. As well, the foregoing discussion has shown that Markov chain models can be considered on three levels. First, investigation of the input parameters provides a suitable framework for analysing the structural and/or spatial dynamics of the variables involved. This **descriptive** aspect of the concept can also provide the foundation for inferential or causal statements. The second level comprises the use of Markov chain analysis as a **predictive** tool. In this respect, the approach has usually been one of estimating past events from which the parameters were derived in the first instance. In other words, the model has merely been tested within the limits of all the available data. Some studies, however, lacking adequate test data, have extended the predictions to some future date. Closely related to the second approach is the use of Markov chain models to derive an **equilibrium situation** of the variables studied. Such studies have focused on the tendencies of economic phenomena, such as firms, to concentrate in various size categories.

CHAPTER IV

STRUCTURAL CHARACTERISTICS OF MANUFACTURING ACTIVITY IN ONTARIO 1961 - 65

Two important issues emerge from Chapter III: forecasts derived from Markov chain analysis are projections of the future state of industries if the observed patterns of change continue; and the application of a Markov model is contingent upon the careful selection of an appropriate system of states. Accordingly, Chapters IV and V comprise analyses of recent trends in Ontario's manufacturing activity to determine the validity of accepting the assumption of stationarity for projected trends and to provide a framework for selecting a suitable system of Markov states. Whereas the analyses of this chapter focus on **structural** characteristics, those of Chapter V concern the **spatial** dynamics of Ontario's manufacturing activity.

Generally, **industrial structure** refers to all the component parts of industrial activity in any one area, region or system. The two most important industrial units, the firm and the establishment, may be analysed in such terms as size, age, labour force characteristics, productivity, type of activity, location, linkages, ownership, and managerial organization. The firm is an economic-legal unit with no areal bounds whereas the establishment or plant is a technical-economic unit with a specific location and as such is the more appropriate unit for spatial analysis. Whereas the population of establishments accounts for all manufacturing activity, the total number of firms does not since all manufacturing establishments are not firms. In this study, structure refers specifically to three characteristics relating to establishments: size measured in terms of total employment, type of activity, and location.

Plant size which varies widely between industrial sectors has been the focus of many studies. A large number of small plants is a characteristic feature of most manufacturing activity though exceptions exist in such industries as cement, smelting, refining, and automobiles. In the past, even these industries were characterized by small concerns; at the turn of the century the American automobile industry was typified by small assembly units. Later, economies of scale, largely initiated by the introduction of Ford's conveyer belt in 1908 encouraged many units to expand production to the detriment of less innovative concerns, and today well over 90% of the U.S.A.'s output is controlled by only three corporations. Similar characteristics in other industries have led to a general belief that productive efficiency declines below a certain minimum size. Why then are most manufacturing establishments small and what is the most appropriate measure for interindustry comparisons?

Reasons for the continued existence of a large number of small plants in some industries (e.g. metal fabricating) include market and product differentiation, the geographical dispersion of population, and demand for specialized functions. Once well established, an industry produces a multiplicity of products and tends to move towards increasing specialization in which large establishments concentrate on mass production, medium size plants manufacture non-mass producible goods at an efficient scale, and the numerous small establishments perform functions of a specialized nature such as repairs, rush orders, and personalized services. The result tends to be a rather stable distribution or structure of large and small establishments working in "complementarity", (Andrews, 1956). The most striking feature of this structure is the tremendous variation of scale which consistently ranges from small family concerns employing one or two people to multi-million dollar complexes with an employment in excess of several thousand. Thus, this chapter seeks to identify prevailing plant size distributions. Because of their skewed nature, the distributions can be appropriately analysed within the framework of two established theoretical concepts, one of which relates to the Pareto Curve or, more appropriately, to the Pareto Tail, and the other via Gibrat's Law of Proportionate Growth to the lognormal distribution. Both models may be generated from a simple stochastic or Markov process which offer a dynamic interpretation of the underlying growth mechanisms and changes in the configurations of the respective distirbutions.

Size Distributions and Structural Variations

The traditional approach to analysing spatial and temporal variations in the size distribution of industrial activity is through the comparison of ratios or Gini coefficients derived from cumulative frequencies plotted as Lorenz curves which provide a visual measure of the degree of concentration in employment. If employment were distributed equally in all size categories then the curve would be a straight line at an angle of 45°; hence, the more convex the curve the more concentrated is the employment in the largest plants. But such an approach, although highly descriptive, does not identify or give insight into the processes influencing changes in the distribution. In Ontario, for example, between 1961 and 1965 employment in the constant sample[1] of permanent establishments of the primary metals industry, and in "all industries" for the city of Toronto became more concentrated whereas no noticeable change occurred in the corresponding plants for the foods and beverage industry (Fig. 4.1). In particular, plants with more than 400 employees in 1965 accounted for approximately one third of the respective totals in foods and beverages, non-metallic mineral products,

[1] This refers to all plants with at least two employees throughout the 1961 - 65 period and which remained in the same location for that period. The term "all establishments", used elsewhere, refers to the total number of establishments employing at least two people in any one year. Similarly, the term "all industries" is an aggregate term denoting the sum of "all establishments" in 20 2-digit industries or in 37 3-digit industries. Where reference is made to the **total** number of establishments the phrase all establishments as recorded by Statistics Canada is used.

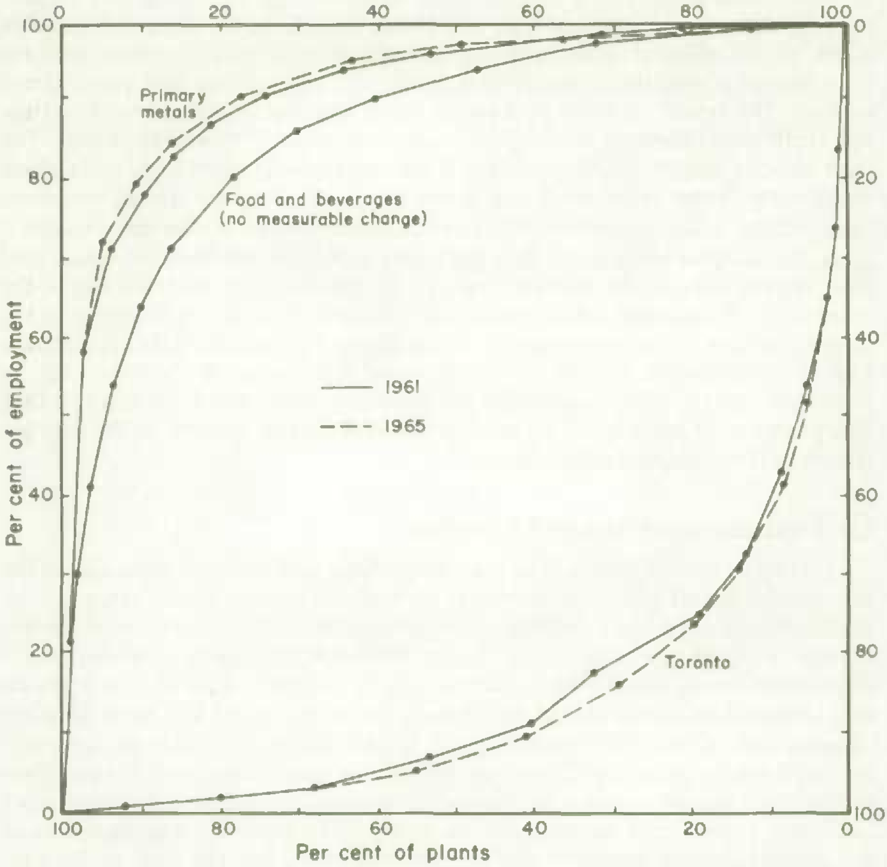


Figure 4.1: Cumulative Distributions (Lorenz Curves) of Total Employment and Number of Permanent Establishment for Selected Industries in Ontario, and for Toronto, 1961-1965

printing and publishing, metal fabricating and chemical products industries, whereas in the transportation and primary metals industries over 80% of the total employment was found in such plants; in the clothing industry only 12.5% of the total was found in plants with over 400 employees (Table 4.1). These figures emphasize the considerable variation in the size distributions of individual sectors but when plotted with the 1961 data as Lorenz curves no insight is provided concerning the underlying mechanisms generating the changes.

TABLE 4.1. Cumulative Percentages for Permanent Establishments in Selected Industries for Ontario, 1961

Size of plant by No. of employees	Foods and beverages	Transport equipment	Clothing	Non-metallic minerals	Printing and publishing	Primary metals	Metal fabricating	Chemicals
961	14.6	72.2	5.6	13.8	18.1	71.4	4.2	20.1
619	20.8	77.9	9.8	21.8	23.9	78.5	13.7	29.1
399	30.4	81.0	12.4	34.6	32.9	83.4	28.3	37.4
257	43.3	87.8	22.9	44.9	43.3	87.3	46.1	44.9
166	54.4	92.3	37.0	55.7	48.5	91.1	55.6	58.8
106	65.0	96.0	51.4	68.3	59.1	94.2	66.9	73.9
68	73.7	97.7	61.5	76.0	71.4	96.6	75.4	83.8
44	80.7	98.8	74.5	83.0	77.2	97.8	84.3	90.0
28	87.2	99.3	85.7	89.3	83.8	98.9	90.5	93.9
18	91.8	99.5	92.1	93.4	89.5	99.5	95.0	97.0
11	95.1	99.8	97.1	96.9	94.5	99.8	97.7	98.5
7	97.7	99.9	98.8	99.0	97.2	99.9	99.1	99.3
4	99.3	99.9	99.8	99.8	99.1	99.9	99.8	99.9
2	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

Some comparative measure of Ontario's manufacturing with respect to that of other countries should also be given. For Britain in 1951 and the U.S.A. in 1954, Florence, 1957, calculated that plants with more than 100 employees accounted for 74.8% and 74.3% of the respective employment totals; the corresponding figure for Ontario in 1965 was 73.7%. But whereas plants employing more than 500 in Britain and the U.S.A. accounted for 42.4% and 45.2% respectively, in Ontario similar plants accounted for only 37% of the total employment in 1965. In contrast, only 53.4% of the total Norwegian employment in 1948 was found in plants with more than 100 employees and only 26% in plants employing more than 500, (Wedervang, 1965).

Although the symmetry of the Lorenz curves (Fig. 4.1) shows that the size distribution of selected industries is skewed one of the most widely used statistics in previous studies has been the representative average (mean) size. Florence used the median in his comparison of British and American industry, but used the measure to describe the plant in which the "mid-most" worker is found. Using this measure Table 4.2 compares the Ontario statistics with those derived by Florence; the median size of plant for all Ontario establishments as recorded by Statistics Canada in 1961 was eleven.

TABLE 4.2. International Comparisons of Average Plant Size: Ontario, U.S.A., and Britain

Place	Plant Size by No. of employees	
	Mean	Median[1]
Britain, 1935	31	235
Britain, 1951	53. 2	370
U.S.A., 1954	54. 8	415
Ontario, 1961	51	281

[1] Median denotes plant size with "mid-most" worker.

At a more disaggregated level a comparison of the means and medians for individual industries provides some measure of structural variations (Table 4.3). As an example, the furniture and fixture industry, although possessing some large establishments is numerically dominated by small plants with less than four employees; whereas, by contrast, over half the tobacco product manufactories have more than 200 employees. Such differences in size distribution are portrayed graphically by frequency polygons (Fig. 4.2) where the almost symmetrical size distribution for non-metallic mineral products is replaced by a bimodal distribution for leather industries and a highly skewed distribution for foods and beverages; the latter is a more representative distribution relative to all other major groups.

Urban areas are equally characterized by structural variations which are examined for a selection of those urban centres possessing more than ten permanent establishments for the 1961 - 65 period (Table 4.4). Although there are marked variations in the mean size of plants among the various urban centres the differences among the respective median values are much smaller. In their study, Kerr and Spelt, 1965, contrasted the industrial structures of Toronto, Hamilton and Windsor according to their respective mean firm sizes; the average Toronto

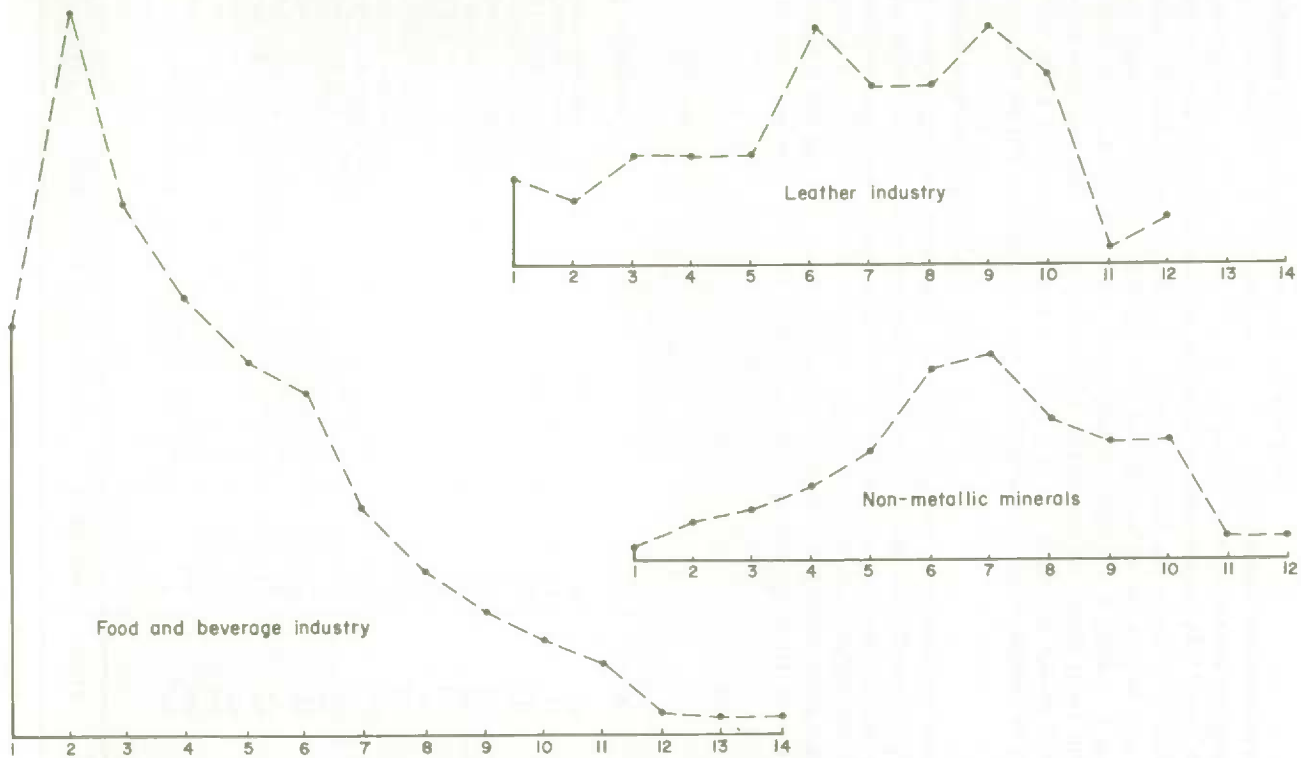


Figure 4.2: Frequency Polygons of Size Distributions for Permanent Establishments in Selected Industries in Ontario, 1961

TABLE 4.3. Size of Permanent Establishments by Employment in Twenty 2-digit Industries, Ontario, 1961

Industry	Median	Mean	Standard deviation
Foods and beverages	7	37	121
Tobacco products	206	206	130
Rubber	89	286	408
Leather	39	80	112
Textiles	18	64	136
Knitting mills	46	77	89
Clothing	19	43	84
Wood	9	28	48
Furniture and fixtures	3	22	47
Paper and allied products	45	112	166
Printing and publishing	6	30	116
Primary metals	42	311	1,191
Metal fabricating	15	46	96
Machinery	37	126	314
Transportation equipment	49	268	1,017
Electrical products	73	183	325
Non-metallic mineral products	15	46	113
Petroleum and coal products	44	222	381
Chemical products	24	74	189
Miscellaneous	8	34	100
All industries	13	63	[1]

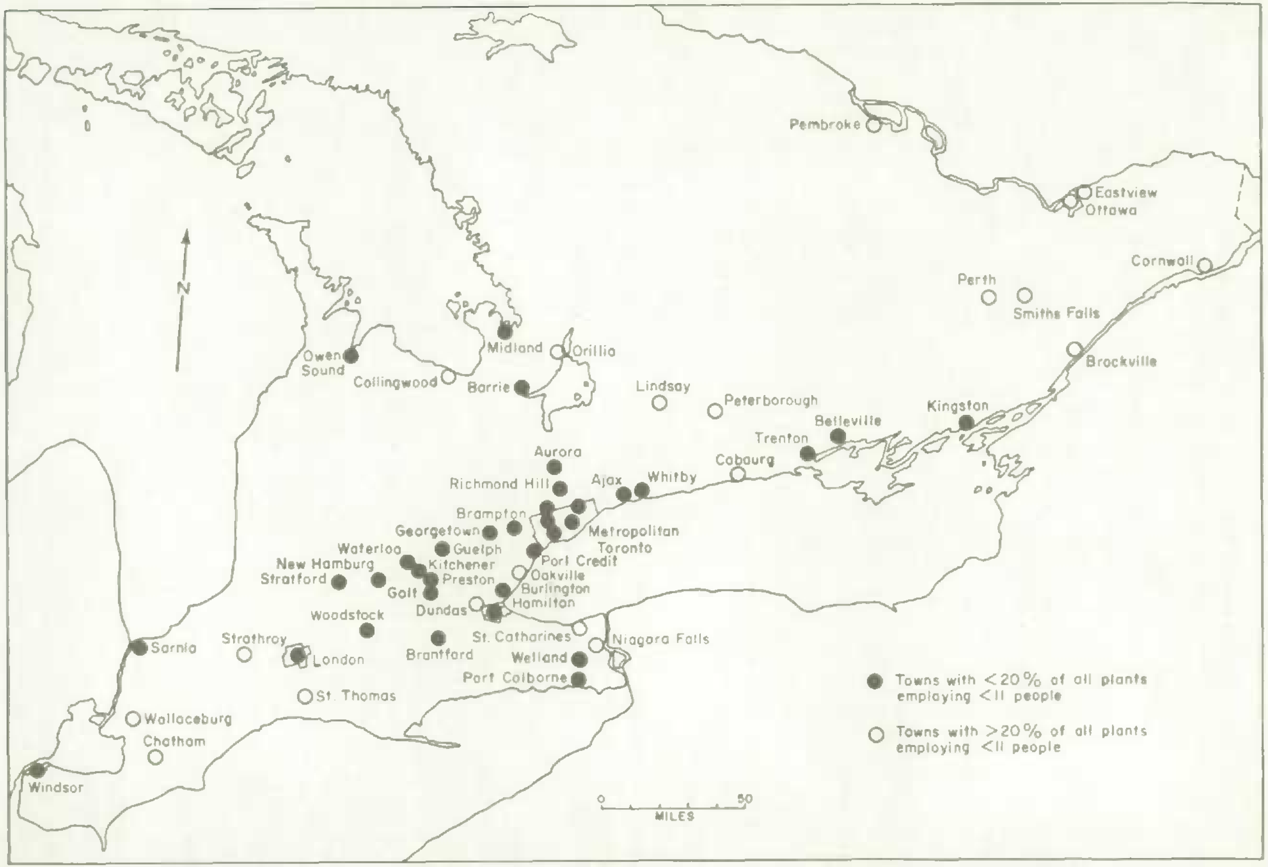
[1] Not computed.

firm in 1958 was cited as having "... 44 workers while its counterparts in Hamilton and Windsor, cities characterized by fewer and larger industries, employ 88 and 89 workers respectively". By 1961 the values for establishments in these cities were 52, 126, and 94 with respective standard deviations of 158, 597, and 454. When median plant sizes are compared, however, the values of 13, 16, and 14 indicate that the differences are much less and that there is a much closer correspondence in terms of total structure.

The median size variation is not correlated with size of urban area (for those urban areas shown in Table 4.4, $r = -0.19$), but a distinct spatial grouping of towns with less than one fifth of their establishments having fewer than eleven employees occurs in the "core" or Golden Horseshoe area (Map 4.1). Within this area only two centres – Oakville and Dundas – have more than 20% of their establishments employing less than eleven. Outside the intensely developed core area, the size distributions of most towns are characterized by a larger percentage of small establishments and are presumably not so stable as the size distributions of those towns in the Golden Horseshoe. Possible reasons for observed structural variations are examined in the following section.

TABLE 4.4 Average Size of Permanent Establishments for Selected Cities and Towns in Ontario, 1961

Town	1961 population	Mean plant	Standard deviation	Median
Toronto	640,588	52	158	13
Hamilton	261,114	126	597	16
Ottawa	255,608	57	104	13
London	158,158	71	150	17
Windsor	116,160	94	454	14
St. Catharines	83,941	90	406	15
Kitchener	72,961	92	223	22
Brantford	53,616	75	154	16
Sarnia	49,089	190	517	17
Kingston	48,028	81	329	16
Peterborough	46,424	115	367	17
Burlington	44,709	58	82	27
Fort William	43,968	29	93	6
Cornwall	43,448	76	247	13
Port Arthur	42,581	29	51	9
Guelph	38,323	65	102	22
Welland	35,967	93	212	22
Chatham	29,271	52	109	16
Belleville	29,070	66	157	19
Galt	26,945	100	147	48
Eastview	23,750	18	19	15
Niagara Falls	22,575	59	103	18
St. Thomas	22,348	45	75	22
Barrie	21,271	56	81	16
Waterloo	20,562	67	91	26
Stratford	20,432	53	76	21
Woodstock	19,923	84	123	40
Owen Sound	17,657	65	114	16
Riverside	17,549	18	15	18
Brampton	17,385	62	87	31
Brockville	17,124	90	187	22
Mimico	16,380	34	39	16
Pembroke	16,214	64	93	9
Richmond Hill	16,095	15	18	14
Port Colborne	15,024	147	486	20
Orillia	14,515	45	87	9
Dundas	12,790	35	54	15
Trenton	12,314	56	74	35
New Toronto	11,664	242	437	41
Preston	11,338	66	131	18
Lindsay	11,052	45	81	19
Long Branch	10,783	49	75	19
Georgetown	10,015	75	104	28
Weston	9,419	86	177	19
Midland	8,615	62	65	47
Wallaceburg	8,029	61	163	14
Aurora	7,124	92	118	32



Map 4.1: Proportions of Small Establishments in Selected Urban Centres, Southern Ontario, 1961

Economic Theory and Structural Variations

Economic explanations for the observed size distributions among either firms or plants usually assume that the basic causal mechanism is the shape of the long-run average cost curve. But why should this postulated mechanism, even occasionally, produce such highly skewed distributions? General economic theory proposes that the long-run average cost curve is U-shaped; this shape supposedly determines a definite optimum size beyond which diseconomies of scale exist, and thus most plants within an industry would approximate this size so that the whole distribution would be platykurtic. Wedervang, 1965, points out that some dispersion would result from the creation of new suboptimal plants, lack of efficient competition or the fact that productivity depends upon factors that are not positively related to size. If the cost curve is U-shaped then the range of the dispersion will depend upon the slope of the curve on both sides of the optimum point. Clearly, the observed distributions in Ontario do not correspond with theoretical expectation.

Some uphold the alternative theory that long-run average cost curves are falling and that economies of scale continue to exist indefinitely (e.g. Hymer and Pashigan, 1962). The reply to this argument invokes the inevitable cost advantages gained by large establishments so that average size will increase gradually until one or a few establishments remain. However, decreasing long-term average costs would be countered by monopolistic competition developed in terms of product differentiation, services provided, and location.

A third alternative postulates that the long-run cost curve is horizontal, in which case any size is efficient and the size distribution is influenced not by costs but by other factors such as entry and growth rates. Bain's, 1956, analysis supports this notion, at least for plants above a certain minimum size. He found that for most industrial plants the long-run cost curve is "L-shaped", i.e. declines sharply and then levels off. Similarly, Johnston, 1958, in a review of statistical studies relating to cost functions concluded that the proponderance of the L-shaped pattern of long-run average cost curves stands out. Other isolated explanations for the size distributions of establishments involve the assumption that their size is dominated by one variable which has a natural distribution. Florence, 1957, for example, and later Tuck, 1954, suggested that the size of establishments reflects the distribution of managerial talents. But other factors, such as monopolistic competition, also need to be considered.

Structural Change as a Stochastic Process

Unfortunately, none of the economic explanations based solely on the shape of the cost curve has any predictive value for changes in the size distribution of industrial establishments. In this respect, Simon and Bonini, 1958, have stressed the urgency for understanding such changes so that public policy may be directed towards arresting undesirable trends as they appear in different areas. In the same context, Engwall, 1968, has remarked that "... research ought to be devoted to explain the generation of the distributions. We can in this way reach a better understanding ... and use it as a forecast implement." Towards this end alternative approaches following the pioneering works of Kapteyn, 1903, and Gibrat, 1931, postulate a process in which growth in proportion to size is a random variable with a given distribution that is considered constant in time. Two of the most common probability distributions that approximate the observed distributions, the Pareto and the lognormal, can both be generated by a simple stochastic process.

The Pareto distribution postulates that when the number of units n with a size in excess of s is plotted against s on logarithmic paper the result is a straight line of slope −1. The frequency distribution is determined by:

$$f(s) = c.s^{-a} = c.e^{-ax} \tag{4.1}$$

where $x = \log(s)$, and c and a are constants, the latter depending on the rate at which new firms enter the industry. The two-tailed lognormal distribution is obtained by the addition of a second parameter:

$$f(s) = c.e^{-ax - bx^2} \tag{4.2}$$

Such a distribution arises from a theory of elementary errors combined by addition.

The usefulness of both the Pareto and lognormal distributions as models for analysing frequency distributions with highly skewed upper tails has been demonstrated for various phenomena. In particular, the distributions of city populations have been approximated by the Pareto curve and have attracted the attention of many geographers concerned with the Rank Size Rule and its explanatory role in Central Place Theory. The importance of the lognormal probability distribution in locational analyses has been emphasized in successive studies by King, 1961, Thomas, 1962, and Curry, 1964. Kulldorf, 1955, and, later, Morrill and Pitts, 1957, have used the function in migration studies. Applications of these models have appeared in studies concerned with the unequal distribution of economic activity among large corporations or firms. Most of these studies have shown that the observed frequencies certainly look like Pareto or lognormal distributions but there is no known satisfactory mechanism to specify accurately the degree of resemblance since the problem of "fitting" skew distributions is similar to that of testing "extreme hypotheses".

Nevertheless, it is in such studies, which began with the work of Gibrat, that attention is focused on the dynamic interpretation of the lognormal distribution. To explain how this distribution arises in a population of firms Gibrat invoked the simplest kind of stochastic process which he called the Law of Proportionate Growth. In its strongest form this proposes that temporal changes in firm sizes are governed by a simple Markov process in which the probabilities of specified percentage increments are independent of a firm's absolute size. Gibrat's model bears a strong resemblance to an unrestricted random walk on a line in which the length of the steps taken at each time interval is a random variable relating to its position on the line.

Theoretically, if the law of proportionate growth is valid and the process is allowed to continue unhampered for an identical sample of firms the resulting model of diffusion will be analogous to that used in physics, the so-called Brownian movement. Osborne, 1959, has shown, that for an identical sample of common stock prices there is an ever growing dispersion since the variance of the logarithms of the quotations increases in proportion to time. Clearly, these are not the characteristics of a population of firms or establishments and more recent theorists have attempted to introduce stability conditions to offset the dispersive tendencies. Two of the most notable variants of Gibrat's law are those proposed by Kalecki, 1945, and, more recently, by Simon and Bonini, 1958. For his model, Kalecki assumed that the variance of an identical sample of units remains constant and implies that growth is negatively correlated with size. Simon and Bonini's model is more complex and is predicated on four assumptions, the first of which relies on Bain's analysis: (1) there is a minimum size of firms S_m above which unit costs are constant; (2) there exist steady states in the evolution of the size distributions; (3) the law of proportionate effect or Gibrat's law is valid; and (4) new firms are born at a constant rate in the lowest size class. It is the last assumption – that of a constant birth rate for new firms – which separates the Simon-Bonini model from others in that it leads to a Yule distribution which is given by:

$$f(s) = kB(s,p+1) \tag{4.3}$$

where $B(s,p+1)$ is the Beta function of s and $(p+1)$, k is a normalizing constant, and p is a parameter. When $s \rightarrow \infty$, that is only when very large firms are considered, equation (4.3) can be approximated by the Pareto distribution. By subsuming Gibrat's law, the Simon and Bonini model assumes that the unrestricted random walk does apply to a sample of identical firms which would exhibit Brownian movement but that the diffusion or increase in the variance for firms as a whole would be offset by a stream of new firms entering and of old firms dying. It is implied, therefore, that unlike the assumption of Kalecki's model, there is no correlation between growth and firm size.

The adequacy of these theoretical concepts to give insight into changes in the structural components of manufacturing activity depends on two considerations: the plausibility and agreement with known facts of their assumptions, and the model's "goodness of fit" with observed distributions. In this, some stress the necessity to test the validity of the assumptions while others, the so-called "instrumentalists", Puu, 1967, emphasize only the predictive value. As mentioned already, in spite of Quandt's recent attempts to test extreme hypotheses, there is still no dependable means of measuring the goodness of fit of highly skewed distributions. Nevertheless, the purpose of the following section is to approximate and analyse observed distributions within the conceptual framework outlined above and wherever possible to test the assumptions of the respective models. It should be noted that in most cases the stochastic growth models leading to the Pareto distribution have been applied to samples, less than 500 in most studies, of large firms which do not suffer from impeded growth to the same extent as establishments. Gibrat's law of proportionate growth, therefore, when applied to industrial establishments, gives rise to the lognormal distribution.

Analysis of Observed Distributions

In accordance with Gibrat's law of proportionate growth, frequency distributions should be calibrated with constant geometric size intervals. Champernowne, 1953, for example, adopted a common logarithmic scale whereas Hart and Prais, 1956, and Archer and McGuire, 1965, set the upper limit of their intervals as twice that of the lower limit (i.e., an interval progression factor of two). Most other studies have adopted an arbitrary classification. Since the goodness of fit of the observed frequencies to the lognormal distribution depends on the size of the class intervals selected, and since individual establishment size observations were available for this study, an iterative search procedure was used to derive the best interval classification scheme for approximating the lognormal distribution. The computed progression factor is 1.55 with $r = 0.9624$ and slope of -1.01 for the constant sample of permanent establishments; the derived size categories were listed previously in Table 4.1.

The remaining analyses of this and succeeding chapters are based, unless otherwise stated, on establishments with at least 2 employees. Thus the lower limit of the smallest size category is set at two for all search operations. The first search operation is given a starting interval of one so that the first size interval is 2-3 employees. The lower limits of each successive higher interval are multiplied iteratively by a series of progression factors until the range of the size distribution of "all establishments" is covered. For each set of frequencies produced by each progression factor for each starting interval a regression line is derived.

When plotted as a Pareto curve on logarithmic paper the distribution for the constant sample of Ontario establishments takes the form shown in Fig. 4.3. The curve differs from that derived by Wedervang[2] for Norwegian establishments where only the small units corresponded to a straight line with an approximate slope of -1. For the Ontario data only the large plants – those with over 400 employees – fall within the range of the Pareto tail with a slope of -1.06; the number of medium plants – those below 150 – is grossly under-predicted. These

[2] Wedervang did not use constant samples for his curves.

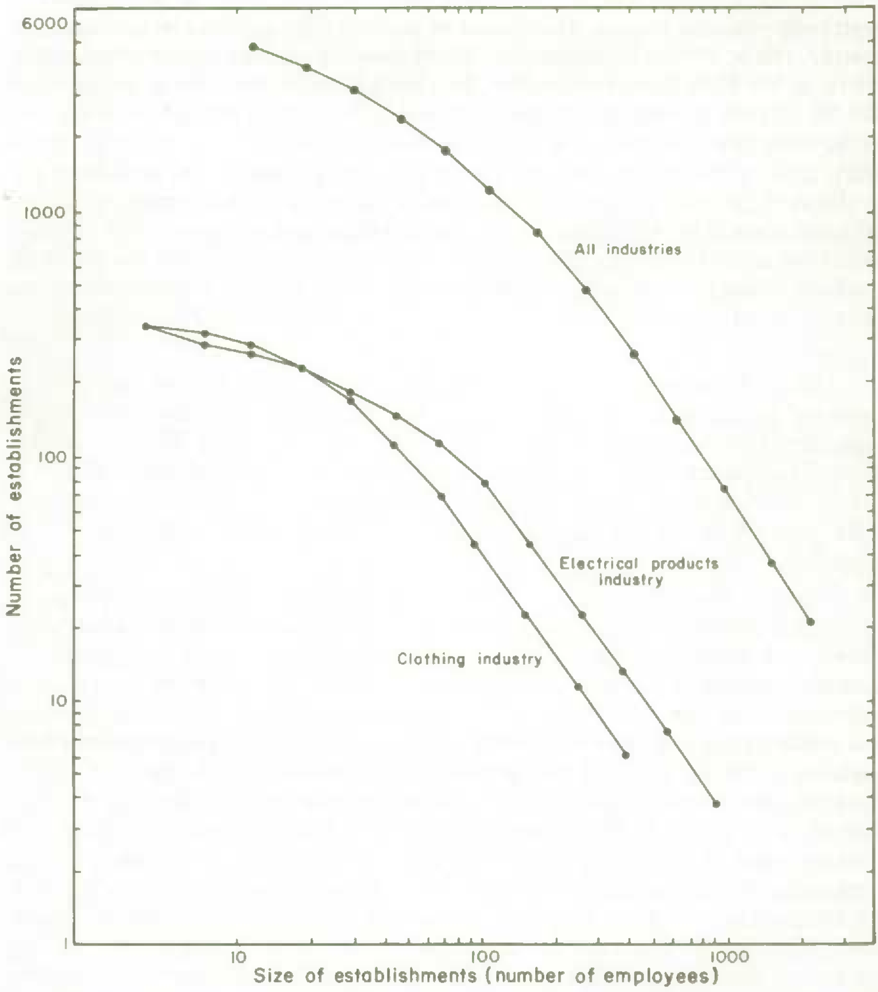


Figure 4.3: Cumulative Size Distributions (Pareto Curves) for Permanent Establishments in Selected Industries for Ontario, 1965

data, then, unlike those of Wedervang, tend to support the Simon and Bonini model which assumes a straight Pareto line with slope −1 in the range of constant costs. Below a certain size, in the Ontario case 150 employees, the slope would be drastically reduced because the chance of survival decreases as average costs rise sharply, (Bain, 1956). Consequently, the number of medium and small establishments is less than that predicted by the Pareto distribution. The same approach can be adopted to analyse sectoral variations in the critical average cost structure of manufacturing activity. The electrical products industry, for example, has a sharp kink in the Pareto curve for plants with approximately 100 employees but in the clothing industry the curve's kink is in the region of plants employing only 30 employees (Fig. 4.3). This implies that constant average costs in the clothing industries are achieved at a much smaller scale of production than in the electrical products industry. For both industries the mean and median employment values are respectively electrical products 183 and 73, clothing 43 and 19.

When the observations for the constant sample, meaning those plants which remained in the same location 1961 - 65 are plotted on logarithmic probability paper the distribution assumes an almost linear form (Fig. 4.4). Thus, as anticipated, the lognormal is a more appropriate model for industrial plants than the Pareto curve, and the almost parallel upward movement of the curve from the 1961 position to that of 1965 indicates that there is a dispersive tendency or Brownian movement.

This dispersive tendency is contrary to Kalecki's viewpoint but supports the Simon and Bonini postulate that the unrestricted random walk does apply to a constant sample of plants. For individual industries the degree of dispersion is summarized in Table 4.5. Although the time period is short, the figures indicate the tendency towards Brownian movement for 9 of the 20 2-digit categories whose variance of the logarithm of size increases proportionately with time; for "all industries" the correlation coefficient between variance and time covered is 0.989, but the best examples are furniture and fixtures, and machinery industries. The general trend is also apparent for the more disaggregated 37 3-digit industrial categories, the best examples of which are other textiles $r = 0.999$, and scientific profession equipment $r = 0.995$; at this level the main exception is grain milling $r = -0.366$. As hypothesized by Simon and Bonini, the dispersive tendency is offset for "all establishments". Although in Fig. 4.4 there is a slight tendency towards convexity, indicating a greater number of plants than expected among the middle categories, in Fig. 4.5 there is a reverse tendency for the curve to be concave upwards showing a greater number of small establishments than predicted by the lognormal model. Usually the lognormal and Pareto distributions are only fitted to aggregate industries, (Simon and Bonini, 1958). Steindl, 1965, noted that a ". . . neat division of firms, if it goes beyond the broad division of manufacturing, trade etc., is artificial, because of the arbitrary allocation of many firms, and because firms in growing spread from one line of business to another." Nevertheless, in Ontario, individual industries do conform quite closely to the lognormal distri-

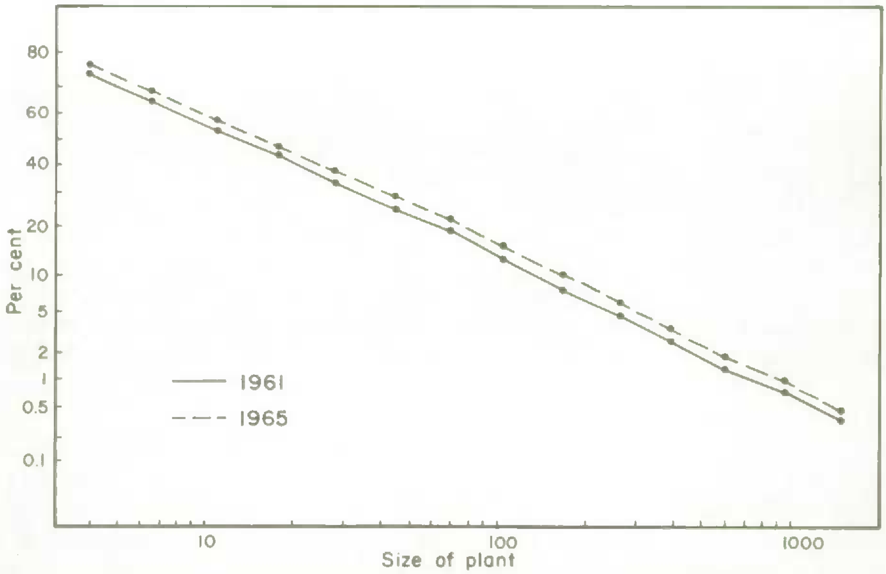


Figure 4.4: Cumulative Size Distributions (Lognormal Probability Curves) for Permanent Plants in All Industries, 1961 and 1965

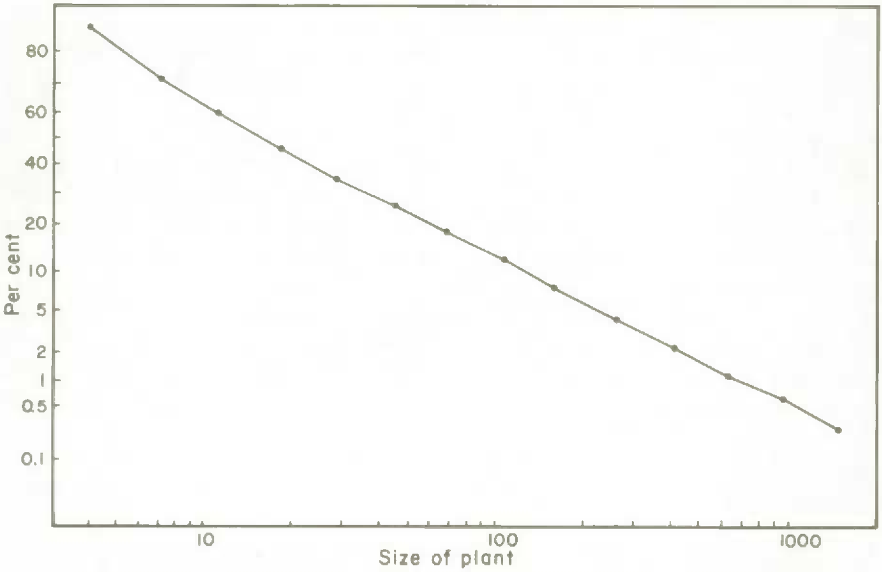


Figure 4.5: Lognormal Probability Curve for All Establishments in All Industries, 1966

TABLE 4.5. Regression of Variance of Log Employment on Time for a Constant Sample of Plants in Twenty 2-digit Industries, 1961-65

Industry	1961		1962		1963	
	Mean	Variance	Mean	Variance	Mean	Variance
1	.96	.459	.96	.469	.95	.485
2	2.13	.422	2.12	.555	2.12	.525
3	1.95	.605	1.99	.545	2.02	.587
4	1.45	.583	1.46	.610	1.46	.601
5	1.24	.547	1.25	.570	1.27	.569
6	1.56	.422	1.60	.380	1.61	.339
7	1.25	.370	1.23	.403	1.24	.414
8	.93	.524	.94	.555	.96	.564
9	.69	.542	.70	.549	.72	.558
10	1.67	.380	1.68	.380	1.67	.412
11	.84	.450	.83	.461	.84	.456
12	1.60	.690	1.63	.685	1.65	.703
13	1.16	.438	1.21	.448	1.23	.449
14	1.60	.395	1.63	.406	1.66	.414
15	1.60	.732	1.64	.776	1.69	.769
16	1.81	.463	1.86	.458	1.88	.568
17	1.18	.406	1.20	.421	1.20	.426
18	1.73	.656	1.76	.606	1.81	.625
19	1.32	.531	1.34	.529	1.34	.532
20	.94	.518	.95	.530	.97	.537
All industries	1.11	.563	1.13	.581	1.14	.590

Industry	1964		1965		Correlation coefficient	Regression coefficient
	Mean	Variance	Mean	Variance		
1	.95	.493	.96	.499	.9859	.0104
2	2.08	.510	2.03	.536	.5624	.0184
3	2.03	.502	2.07	.536	– .6949	– .0181
4	1.47	.605	1.46	.596	.3217	.0021
5	1.28	.586	1.27	.604	.9655	.0130
6	1.63	.320	1.62	.357	– .7630	– .0190
7	1.26	.412	1.25	.425	.9014	.0119
8	.97	.583	.98	.574	.8929	.0127
9	.72	.567	.73	.575	.9998	.0084
10	1.69	.411	1.72	.389	.4791	.0048
11	.85	.455	.86	.464	.6229	.0022
12	1.65	.731	1.69	.737	.9355	.0139
13	1.26	.453	1.28	.465	.9588	.0060
14	1.68	.422	1.72	.435	.9955	.0096
15	1.73	.801	1.74	.819	.9465	.0198
16	1.89	.482	1.92	.474	.7902	.0047
17	1.21	.437	1.23	.448	.9919	.0100
18	1.83	.617	1.83	.635	– .2488	– .0030
19	1.36	.527	1.37	.535	.3385	.0006
20	.99	.545	1.00	.551	.9922	.0081
All industries	1.15	.598	1.16	.609	.9890	.0109

bution (Fig. 4.6); in particular very good fits are obtained for printing and publishing, and foods and beverages. The main exception is the furniture and fixture industry in which the distribution bends downwards in the upper tail indicating the strong impediments to continued economies of scale. This may result from such factors as raw material and labour force restrictions, sharply increasing transport costs, entrepreneurship, and a highly fluctuating market.

TABLE 4.6. Mean Plant Growth Rates[1] 1961-65, by Quartile,[2] for Thirty 3-digit Industries, Ontario

Industry	First quartile	Second quartile	Third quartile	Fourth quartile
Meat products	19.5	37.5	11.6	- 3.1
Dairy products	9.2	16.2	14.6	45.4
Fruit and vegetables	14.1	17.3	6.2	6.2
Grain mills	- 2.9	7.7	15.6	23.8
Bakery products	7.1	- 11.2	- 5.0	20.1
Other food processes	3.9	6.5	38.1	- 10.2
Beverages	8.1	7.8	2.3	11.9
Leather industries	10.2	5.8	1.7	49.5
Other primary textiles	13.0	22.1	4.2	80.6
Other textiles	36.2	9.3	19.6	19.8
Hosiery mills	- 3.3	4.4	- 4.2	37.7
Other knitting mills	- 2.1	10.2	10.6	303.6
Clothing industry	6.4	16.9	16.4	12.5
Wood industries	21.6	27.2	44.2	103.6
Household furniture	18.8	11.8	35.8	9.6
Other furniture	16.0	27.3	7.8	55.6
Paper and allied industries	7.4	19.9	13.8	26.3
Commercial printing	8.8	7.6	10.5	44.0
Engraving and allied industries	.6	6.6	1.6	16.9
Printing and publishing	10.5	7.2	30.0	57.7
Primary metals	32.3	37.7	28.4	35.0
Metal fabricating	33.6	44.1	51.2	79.1
Machinery industries	29.6	65.7	38.0	57.2
Transportation equipment	60.0	71.6	45.2	87.9
Electrical products	27.7	43.2	52.5	53.3
Cement, lime + gypsum	24.3	30.3	38.3	52.5
Other non-metallic mineral products	18.9	15.6	15.3	41.0
Chemical products	8.9	18.4	37.6	37.0
Scientific professional equipment	26.9	26.3	48.7	9.3
Miscellaneous manufacturing	20.5	20.5	42.8	73.5

[1] Mean Growth Rate = $\frac{100}{N} \sum_i (x_i/y_i - 1)$

where x_i = size of plant in 1965, y_i = size of plant in 1961, N = number of plants in each quartile.

[2] Quartiles refer to plant sizes by employment, with the first quartile as the largest.

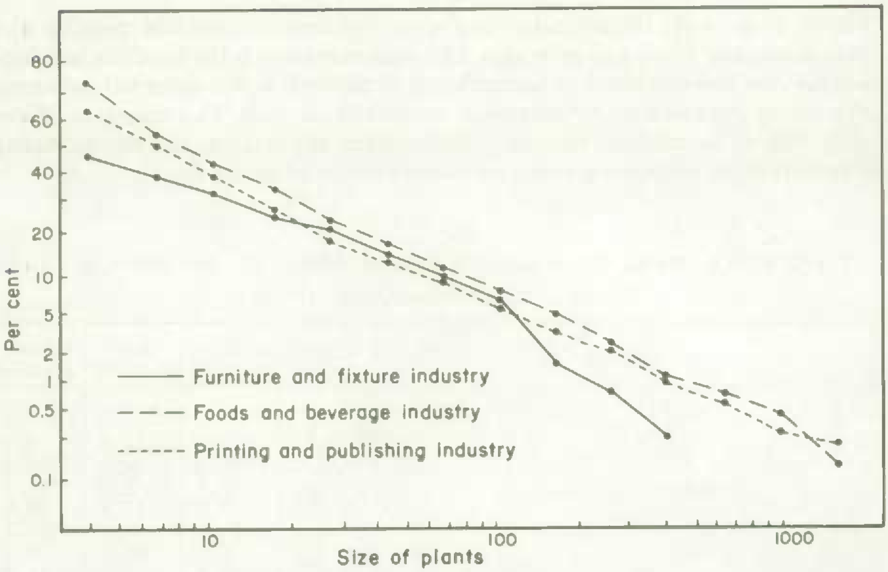


Figure 4.6: Lognormal Probability Curves for Permanent Establishments in Selected Industries, Ontario, 1965

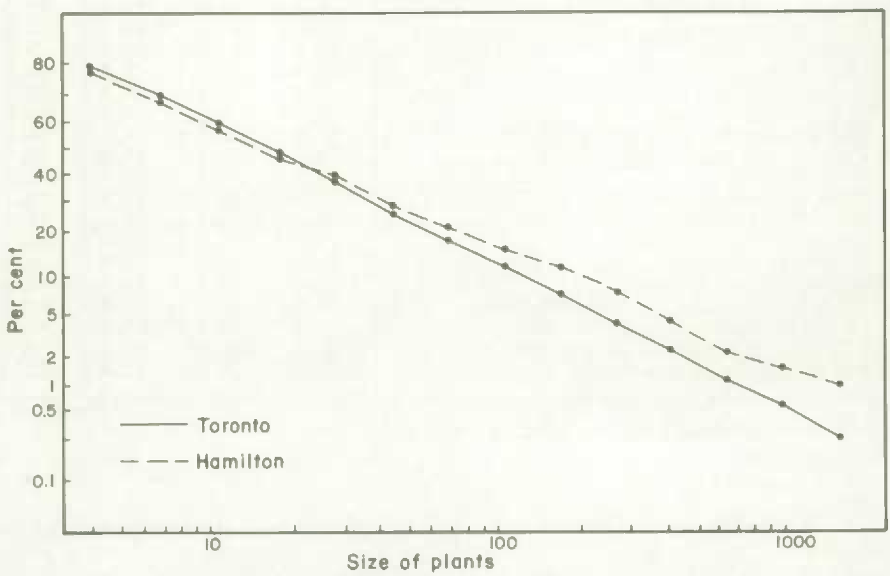


Figure 4.7: Lognormal Probability Curves for Permanent Establishments in the Cities of Toronto and Hamilton, 1965

When the frequency distributions are plotted for "all industries" in individual industrial agglomerations the resulting curves, especially those of Toronto and Hamilton, closely resemble those for the whole province (Fig. 4.7). Other major urban area distributions, e.g., those of London, Windsor and Ottawa, display strong linearity in the lower and middle ranges but in general there are fewer large establishments than are predicted by the lognormal model (Fig. 4.8).

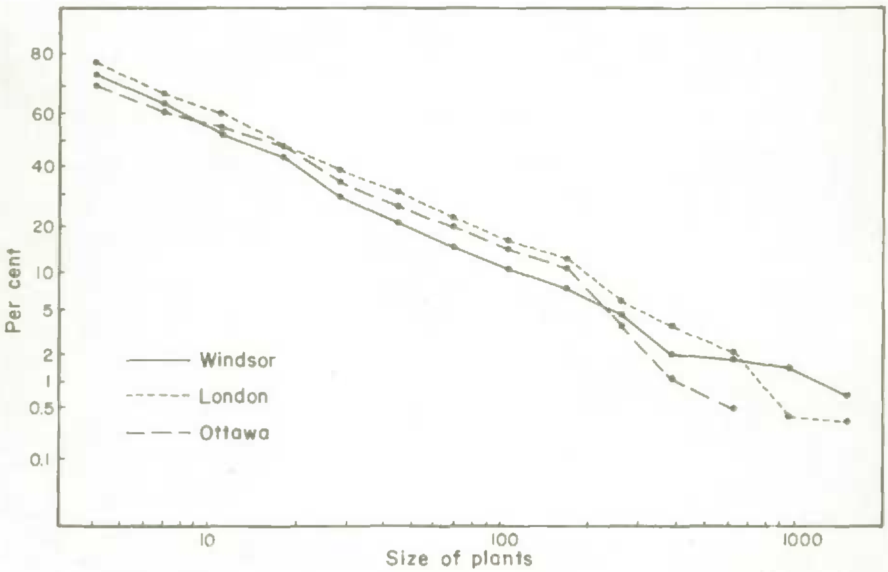


Figure 4.8: Lognormal Probabiliy Curves for Permanent Establishments in the Cities of Windsor, London, and Ottawa, 1965

The simplest way to test Gibrat's law is to plot on a logarithmic scale plant sizes at the beginning of the period against those at the end. Although the law can be accepted when the slope of the regression line is 45° and the plots are homoscedastic, much more information is gained by examining the mean growth rates for individual size categories, (Hymer and Pashigan, 1962). In general, these rates exhibit no systematic trend in the quartile values for 37 3-digit industries (Table 4.6). On the basis of the figures in Table 4.7, however, it is tempting to postulate, as Kalecki did, that growth is negatively correlated with size. But although 25 of the 37 categories show a negative correlation, only four are significant at 0.05%.

TABLE 4.7. Regression of Growth[1] Between 1961 and 1965 on Size of Plants in 1961 for Thirty-seven 3-digit Industries, Ontario

Industry	Correlation coefficient	No. of plants
Meat products	.1111	128
Dairy products	– .1178	497
Fruit and vegetable canners	.1446	111
*Grain mills	– .2474	297
Bakery products	.0347	421
Other food processors	.0170	115
Beverages	.0562	165
Tobacco products	.1865	13
*Rubber industries	– .3446	47
Leather industries	– .1054	137
Cotton and wool	– .1878	47
Synthetic textiles	– .1562	14
Other primary textiles	.1974	60
Other textiles	.1281	150
Hosiery mills	– .2881	31
*Other knitting mills	– .3159	65
Clothing industry	.0223	398
Wood industries	– .1338	168
Miscellaneous wood	.0046	33
Household furniture	– .1327	251
Other furniture	– .1027	93
Paper and allied industries	– .0850	189
Commercial printing	– .1375	516
Engraving and allied industries	– .0089	107
Printing and publishing	– .1128	313
Primary metals	– .0224	150
Metal fabricating	– .0892	932
Machinery industries	– .0067	256
Transportation equipment	.0299	196
Electrical products	– .1093	278
Cement, lime+gypsum	– .1458	176
Clay products	.2622	40
Other non-metallic mineral products	.0288	99
Petroleum and coal products	– .1833	25
Chemical products	– .1623	377
Scientific professional equipment	– .0518	161
*Miscellaneous manufacturing	– .1958	493
All industries	– .0316	7,550

[1] Both growth and size are in terms of employment which is measured logarithmically.

*Significant at .05.

Markov Matrices and the Dynamics of Size Distributions

The analyses of the observed size distributions indicate the relevance of interpreting Gibrat's law as a mechanism of change, but perhaps a more profitable way of verifying this assumption is to examine the individual observations contained in transition matrices. The **structural** matrices comprise, as states, establishment size categories derived for analysing the data in the context of the Pareto and the lognormal distributions. Thus, the rationale for the adopted system of structural states rests on an optimum classification derived for analysing the lognormal distribution which is assumed to be generated by a simple stochastic process. When arranged into Markov matrices the observed frequencies yield additional information of particular importance.

First, the 1961 - 65 tally matrix for a large sample of permanent establishments gives insight into the dynamics of the respective size distributions (Table 4.8). Among those plants employing less than 106 people in 1961, approximately 45% in each size category remained in the same category for the five-year period; over half of the plants, therefore, experienced significant proportionate changes in number of employees. Significant proportionate changes also occurred among those plants with more than 106 employees in 1961 but in each size category the number of plants moving to other categories was slightly exceeded by the number remaining; in category eleven, for example, almost 53% of the plants initially in this category remained there for the five-year period. At a more detailed level, the tally matrix shows that a greater proportion of plants in the second and third smallest size categories moved down to the next lowest category. But plants in these size categories, as well as those in categories one and four, tended to move a larger number of intervals than those initially in the higher categories.

Second, in accordance with the hypothesized L-shaped long-run average cost curves the tally matrix may be interpreted to suggest that the smallest establishments, because of high unit costs, have a greater probability of decline than the large plants; but because of the incentive to realize cost savings through increased size, they have a greater probability of a higher proportionate change. This is especially evident from the four annual stochastic matrices (Tables 4.9 - 4.12). The transition probabilities are maximum likelihood estimates for

TABLE 4.8. 1961 - 65 Structural Tally Matrix for Permanent Establishments

No. of employees	Size categories	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2 - 3	1	337	164	55	20	2	1								
4 - 6	2	179	426	202	68	17	8	1	1						
7 - 10	3	18	154	353	228	52	12	2	2						
11 - 17	4	8	27	142	402	228	64	15	3	4					
18 - 27	5		2	18	115	364	223	56	10	1					
28 - 43	6			5	15	89	343	219	51	10					
44 - 67	7			3	7	10	70	249	184	36	12	2			
68 - 105	8			1	1	5	14	63	226	137	33	8			
106 - 165	9						1	6	37	208	108	21	2	1	
166 - 256	10				1			1	3	35	159	80	14		
257 - 398	11				1		2			2	13	90	51	10	1
399 - 618	12										2	12	58	21	6
619 - 960	13												5	31	17
961 -	14													5	54

TABLE 4.9. 1961 - 62 Structural Probability Matrix for Permanent Establishments

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.7375	.2228	.0294	.0086	.0017									
2	.1475	.6530	.1674	.0244	.0055		.0011	.0011						
3	.0085	.1681	.5883	.2132	.0183	.0024	.0012							
4	.0011	.0157	.1265	.6585	.1825	.0090	.0045	.0022						
5		.0025	.0038	.1648	.6368	.1660	.0203	.0038						
6		.0014	.0027	.0096	.0984	.6940	.1883	.0123	.0014					
7					.0052	.0873	.7068	.1867	.0140					
8			.0020			.0061	.1086	.6967	.1660	.0184	.0020			
8							.0052	.1018	.7318	.1484	.0130			
10								.0034	.1092	.7201	.1638	.0034		
11								.0059	.0059	.0941	.7588	.1235	.0118	
12							.0101			.0101	.1111	.7071	.1515	.0101
13												.0755	.8491	.0755
14												.0169	.0847	.8983

TABLE 4.10. 1962 - 63 Structural Probability Matrix for Permanent Establishments

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.7641	.2095	.0211	.0053										
2	.1283	.6881	.1627	.0195	.0011	.0023								
3	.0104	.1545	.6610	.1623	.0104	.0013								
4		.0087	.1122	.7077	.1553	.0108	.0022	.0011	.0011					
5			.0062	.1324	.6894	.1586	.0144							
6				.0085	.0897	.7422	.1538	.0057						
7					.0098	.1187	.6992	.1610	.0081	.0016			.0016	
8						.0119	.1074	.7318	.1412	.0060	.0020			
9							.0050	.0693	.7748	.1485	.0025			
10							.0034	.0034	.0850	.8027	.0988	.0068		
11							.0052			.1082	.7526	.1340		
12											.1031	.7835	.1031	.0103
13												.1343	.7463	.1194
14													.0345	.9655

TABLE 4.11. 1963-64 Structural Probability Matrix for Permanent Establishments

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.7996	.1787	.0162	.0054										
2	.0898	.7423	.1560	.0106	.0012									
3	.0078	.1089	.7134	.1530	.0104	.0052	.0013							
4		.0055	.0837	.7434	.1564	.0088	.0022							
5		.0013	.0053	.1043	.7233	.1631	.0013	.0013						
6		.0027		.0041	.1076	.7262	.1553	.0041						
7				.0016	.0099	.0755	.7373	.1691	.0049	.0016				
8				.0020		.0100	.0778	.7784	.1257	.0020	.0040			
9								.0699	.7831	.1470				
10								.0031	.0748	.7913	.1308			
11									.0053	.0535	.7968	.1390	.0053	
12											.0796	.7965	.1150	.0088
13								.0159				.0476	.8413	.0952
14												.0154		.9846

TABLE 4.12. 1964-65 Structural Probability Matrix for Permanent Establishments

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.8533	.1390	.0076											
2	.1074	.7228	.1538	.0134	.0024									
3	.0052	.1310	.7004	.1543	.0078	.0013								
4	.0023	.0079	.1160	.7061	.1554	.0090	.0023	.0011						
5			.0051	.1120	.7001	.1686	.0116	.0026						
6				.0139	.0933	.7396	.1407	.0111	.0014					
7			.0033	.0050	.0099	.0924	.7310	.1535	.0050					
8					.0076	.0152	.1042	.7235	.1439	.0038				.0019
9				.0024		.0048	.0048	.0697	.7716	.1370	.0096			
10								.0031	.0948	.7768	.1254			
11						.0050		.0050	.0050	.0693	.7723	.1436		
12											.1000	.7917	.1000	.0083
13												.0746	.8060	.1194
14												.0141	.0282	.9577

observations of the permanent establishments – i.e., those plants which remained in the same location 1961 - 65. The tables show the higher probabilities of decreasing employment in the smallest size categories for the 1961 - 62 period (Table 4.9). At this time many of the smaller plants in the constant sample would be recently established plants which would not have had sufficient time to settle down to a viable operating size. This, of course, subsumes the Simon and Bonini postulate that new plants enter the smallest size categories. The validity of this assumption is examined in a later section. By the 1964 - 65 transition period all plants in the constant sample would be at least 5 years old and hence more stable so that the probabilities of decline are more equitable with those for larger plants (Table 4.12).

Third, reference to the four annual stochastic matrices indicates substantial stability in plant size changes over time. There is an almost equal probability for a plant to increase to the next higher size category as to regress to the next lower one. Moreover, the increasing value of the main diagonal elements with increasing size in all matrices suggests a systematic increase in the dispersion of proportionate growth with decreasing size of plant. These general trends are illustrated in Fig. 4.9 in which are plotted three probability row vectors of the 1961 - 62 matrix contained in Table 4.9. This tendency is summarized at a more disaggregated level for 37 3-digit industries in Table 4.13 which shows that 16 industries exhibit a continuous increase in the dispersion of mean plant growth rates with decreasing size. The general trend also prevails among the other industries since the standard deviation increases in 90 cases and only in 21 cases does it decrease.

Stochastic matrices for individual industries and spatial units exhibit different probabilities of proportionate change among the various size categories. A comparison of the 1961 - 65 probability matrices for foods and beverages (Table 4.14) and metal fabricating (Table 4.15) reveals the greater tendency among plants in the former industry to remain in the same size class whereas the much lower main diagonal values of the latter indicate a much more dynamic process of change. In almost all categories of the metal fabricating industry there has been a greater probability of expansion than of remaining in the same size category. Matrices for other industries show similar trends; in general, plants manufacturing electrical products had four times the chance of increasing to the next higher size category as decreasing to the next lower one. Industries conforming to the equal probability distribution of proportionate growth exhibited by "all industries" are foods and beverages, knitting mills, and printing and publishing.

Similar comparisons can be made for spatial units. Plants employing less than 106 people in the twelve suburban municipalities of Metropolitan Toronto have experienced a highly dynamic growth process but the comparatively high diagonal values of plants employing more than 106 indicate greater stability at this level (Table 4.16). On the other hand, in the city of Toronto plants in the higher categories have exhibited greater expansionary tendencies than their counterparts

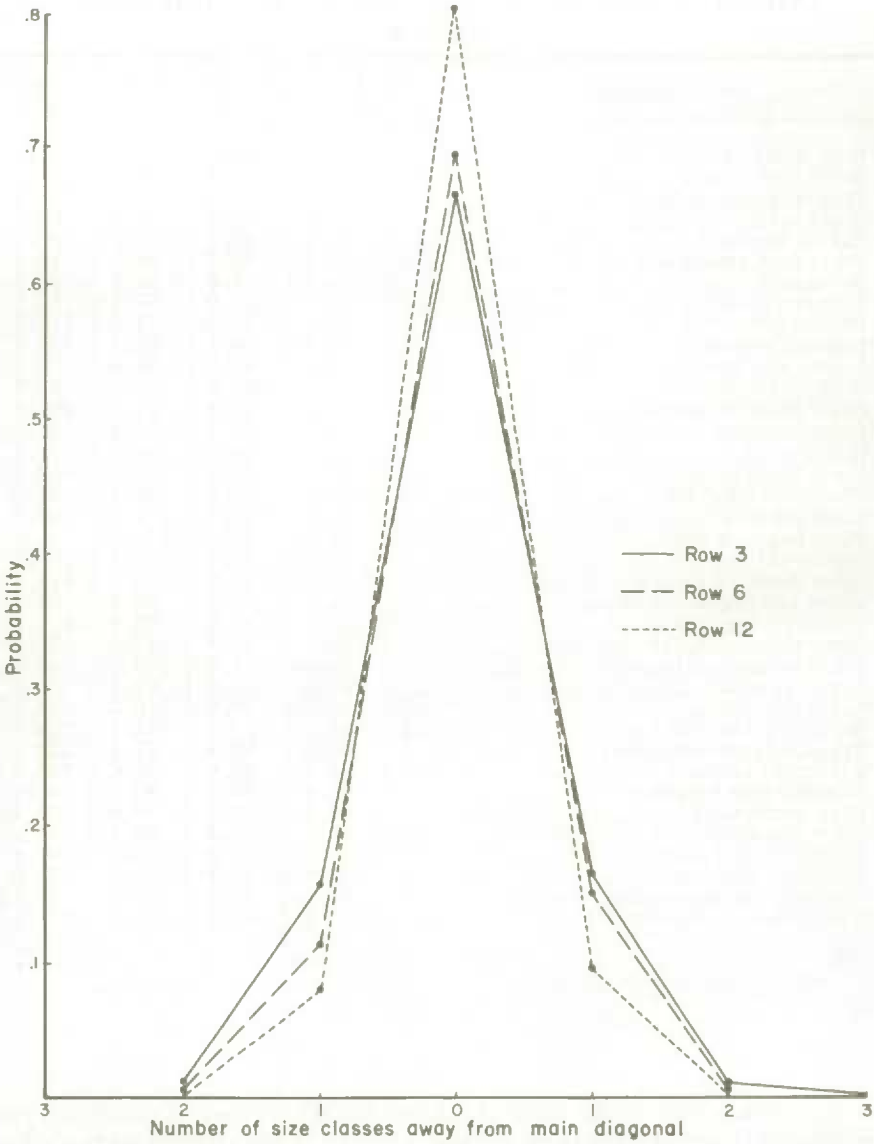


Figure 4.9: Probability Distributions Acoss Three Rows of 1961-1962 Structural Matrix for Permanent Plants

TABLE 4.13. Standard Deviation of Mean Plant Growth Rates 1961-65, by Quartile, for Thirty-seven 3-digit Industries, Ontario

Industry	First quartile	Second quartile	Third quartile	Fourth quartile
Meat products	42.7	76.6	52.4	70.3
Dairy products	31.4	68.4	45.0	115.0
Fruit and vegetables	43.6	37.9	39.6	71.2
*Grain mills	33.6	35.5	63.6	67.6
*Bakery products	32.3	44.1	59.5	62.5
Other food processors	26.5	39.7	94.1	37.7
Beverages	23.8	28.8	28.6	59.2
Tobacco products	27.9	25.0	64.2	55.5
*Rubber industries	19.2	36.2	74.2	159.2
*Leather industries	32.0	39.4	66.7	135.7
*Cotton and wool	20.6	41.5	43.6	73.9
Synthetic textiles	35.3	67.5	59.2	67.9
Other primary textiles	26.4	76.9	33.3	202.3
*Other textiles	54.7	55.1	66.0	96.5
Hosiery mills	15.7	49.5	18.2	71.2
*Other knitting mills	25.2	48.7	56.5	599.2
*Clothing industries	43.7	56.9	58.4	94.0
*Wood industries	60.6	77.9	103.6	589.9
Miscellaneous wood	27.3	45.2	146.3	41.1
Household furniture	41.6	76.0	88.8	38.3
Other furniture	29.9	50.9	50.6	114.5
*Paper and allied industries	24.4	33.4	38.8	64.5
*Commercial printing	33.5	43.6	79.9	132.0
Engraving and allied industries	26.4	42.3	41.4	77.9
Printing and publishing	28.1	67.3	180.7	142.5
Primary metals	61.8	42.2	48.2	69.0
*Metal fabricating	47.5	70.6	116.3	170.3
Machinery industries	42.1	112.5	73.1	138.5
Transportation equipment	66.0	99.4	87.3	222.8
*Electrical products	50.7	60.6	73.6	122.9
Cement, lime + gypsum	70.0	65.4	92.1	81.7
Clay products	41.4	36.0	30.9	63.2
*Other non-metallic mineral products	37.6	44.4	82.1	136.4
Petroleum and coal products	70.3	58.8	28.4	97.6
Chemical products	36.0	42.1	139.5	74.6
Scientific professional equipment	43.0	76.7	115.7	44.6
*Miscellaneous manufacturing	48.4	61.0	99.4	211.1

*Continuous increase.

in the suburbs, but in the lower size categories the number of employees per establishment remained rather more stable (Table 4.17). The matrices for the group of four other large cities resemble those of the city of Toronto but in general there were much lower probabilities of plants employing more than 100 people to decrease in size than in Toronto.

TABLE 4.14. 1961 - 65 Structural Probability Matrix for Permanent Establishments in the Food and Beverage Industry

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.6789	.2064	.0917	.0229										
2	.2254	.5108	.2206	.0360	.0048	.0024								
3	.0238	.2054	.4554	.2351	.0625	.0149		.0030						
4	.0120	.0359	.1766	.4701	.2395	.0449	.0210							
5			.0236	.1655	.5000	.2500	.0439	.0169						
6			.0125	.0188	.1285	.4984	.2915	.0408	.0094					
7			.0136	.0227	.0136	.1545	.4909	.2591	.0409	.0045				
8			.0059		.0176	.0235	.1706	.4529	.2471	.0647	.0178			
9							.0213	.1277	.5745	.2553	.0213			
10				.0079			.0079	.0079	.1102	.6378	.1969	.0315		
11				.0154		.0154			.0154	.1077	.5692	.2769		
12											.2121	.6970	.0606	.0303
13												.1250	.6250	.2500
14													.1667	.8333

TABLE 4.15. 1961 - 65 Structural Probability Matrix for Permanent Establishments in the Metal Fabricating Industry

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.3896	.3896	.1558	.0519		.0130								
2	.1212	.3485	.2652	.1667	.0682	.8227	.0076							
3	.0132	.1258	.3907	.3377	.0927	.0199	.0132	.0066						
4		.0101	.1307	.3719	.2965	.1558	.0201	.0050	.0101					
5			.0287	.0805	.3966	.3333	.1437	.0172						
6				.0136	.1156	.3605	.3673	.1224	.0204					
7					.0167	.0667	.3833	.3417	.1167	.0583	.0167			
8					.0100	.0100	.1099	.3626	.3956	.0879	.0220			
9							.0120	.0602	.4217	.3976	.0964	.0120		
10									.0500	.3167	.5333	.1000		
11										.0488	.5366	.2439	.1707	
12										.0690	.0690	.5172	.2069	.1379
13												.0769	.3077	.6154
14														1.0000

TABLE 4.16. 1961 - 65 Structural Probability Matrix for Permanent Establishments Located in the Inner Suburbs

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.4697	.3030	.1667	.0455										
2	.1443	.3093	.2990	.2268	.0206									
3		.0980	.3824	.3725	.1176	.0294								
4	.0078	.0391	.1250	.3750	.2656	.1406	.0313	.0078	.0078					
5			.0290	.1014	.3623	.3768	.1087	.0217						
6			.0074		.0515	.3750	.4338	.0809	.0515					
7				.0092	.0092	.0550	.3945	.4404	.0642	.0275				
8					.0109	.0326	.0652	.4130	.3261	.1304	.0217			
9						.0128	.0128	.0513	.5128	.3462	.0641			
10							.0233	.0233	.0930	.5581	.3023			
11						.0357			.0357	.1071	.6429	.1786		
12											.1000	.7000	.1500	.0500
13												.1250	.6250	.2500
14													.1000	.9000

TABLE 4.17. 1961 - 65 Structural Probability Matrix for Permanent Establishments Located in the City of Toronto

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.5887	.3145	.0565	.0323	.0081									
2	.2332	.4456	.2176	.0674	.0155	.0155		.0052						
3	.0303	.2677	.3586	.2525	.0608	.0101								
4	.0166	.0415	.1826	.4813	.2282	.0373	.0124							
5		.0047	.0237	.2038	.5166	.2133	.0237	.0142						
6			.0096	.0192	.1635	.4952	.2740	.0240	.0144					
7			.0079	.0079	.0315	.1890	.5276	.2047	.0236	.0079				
8					.0194	.0388	.2427	.4466	.2136	.0388				
9								.1688	.6104	.1948	.0260			
10								.0370	.2037	.5185	.2407			
11				.0278					.0278	.0833	.5556	.2222	.0833	
12											.2000	.5500	.2500	
13												.0833	.5833	.3333
14													.2727	.7273

The growth mechanism postulated by Gibrat requires only that the probability of moving to the next higher interval is the same for all intervals. If Gibrat's law is valid we can expect a similar distribution of proportionate growth among size classes in the off-diagonals of each stochastic matrix. It is reasonable also to hypothesize that such similarities will be greater at the end of the 1961 - 65 period than at the beginning because, as has been stated, many of the constant sample will be those recently established before 1961, thus exhibiting greater probabilities of decline or growth at the beginning than at the end of the period. The null hypothesis – that the proportions of growth along the first off-diagonal do not deviate from some average value – is tested by means of Chi-square, and the computed statistics for twelve degrees of freedom are presented in Table 4.18. Thus, the longer the period covered by the constant sample the greater our confidence in accepting Gibrat's law of proportionate growth.

TABLE 4.18. Chi-square Statistics for Hypothesis of Equidistribution

Transition period	Chi-square statistics (12 D.F.)
1961 - 62	10.23
1962 - 63	6.76
1963 - 64	4.40
1964 - 65	2.71
1961 - 65	4.46

In accordance with the requirements of the law of proportionate growth the analyses, so far, have focused on the constant sample of permanent establishments which have been shown to exhibit the so-called Brownian movement. Presumably, this dispersive tendency is offset by the constant influx of new establishments and the outflow of old establishments. Simon and Bonini, in their model, postulated that this influx of new establishments would be concentrated in the smallest size category. To what extent is this assumption valid? More pertinent perhaps is the question relating to the "rate of entry and exit" of manufacturing establishments. In his study, Michael Beesley, 1955, uses the terms "entry" and "exit" in preference to births and deaths because ". . . transfers of establishments from and to other areas could not be identified." In this study, however, such transfers have been identified as "relocations", and the terms "entry" and "exit" do refer to actual births and deaths. If the size distribution of industrial plants is accepted as being determined by a stochastic process, then the appropriate way to think about public policy, hitherto based on static equilibrium analysis, is to consider means by which the stochastic process can be altered and the consequences evaluated. Within this frame of reference, to what extent can we expect entry and exit rates to be stable and equal among industries?

The Entry and Exit of Manufacturing Establishments

Variations in the rates of entry or birth and exit or death of manufacturing establishments may be viewed in two ways: by size and by industrial sector. The exit rate may be considered also in terms of age. The analyses in the following sections are based on 4,283 new plants which were established in Ontario between 1961 and 1965, and on 3,690 plants which went out of business or ceased to be reporting units, for some reason, during the same period. These figures are for total births and deaths and include plants "being born" or "dying" with less than two employees. Two points should be noted: first, both these figures are less than the true numbers because (a) sawmills are excluded from the analysis, and (b) data are recorded at annual intervals so that plants opening and closing within an interval are not included; second, approximately 500 of the births were foreign-owned branch plants which tend to be significantly larger than locally-owned establishments. The 500 new foreign-owned branch plants in Ontario form approximately 60% of the Canadian total of new foreign-owned plants for this period. Presumably, the net effect of these foreign-owned branch plants is to distort the average size of the births which, therefore, may be larger on average than in areas not dominated by foreign-owned plants.

(i) Entry and exit by size – The range in size of births for Ontario between 1961 - 65 is quite large (Table 4.19). For the four intervals there is a marked stability in the distributions of both births and deaths but in comparison to the distribution of "all establishments" and especially to that of permanent establishments there is a much greater percentage in the lower frequencies for incoming and outgoing plants; whereas 34% of the permanent establishments and 42% of all establishments had less than eleven employees in 1961, 63% of the births and 58% of the deaths between 1961 and 1965 were in this category. When plotted on a logarithmic probability chart the distribution of both births and deaths tends to be concave, indicating a larger number of small establishments than expected by the lognormal model (Fig. 4.10). Presumably, the skewed nature of these distributions is dependent on and closely correlated with the distribution of wealth among entrepreneurs. The size of new establishments, for example, is limited by the availability of equity capital which will in turn determine the size of potential loans. In general, the average size of births is slightly smaller than that of deaths which in turn is markedly smaller than the average size of the permanent establishments. For the four observation intervals shown in Table 4.19 the births had median values of 5, 8, 7, and 8 respectively whereas the respective values of deaths were 7, 9, 9, and 8; in 1961 the median was 14 for all establishments and 19 for the constant sample.

TABLE 4.19. Percentage Frequencies of Births and Deaths by Size Category, 1961-65

Size of plant	1961		Births					Deaths				
	Permanent establishments	All-establishments	1961-62	1962-63	1963-64	1964-65	Average	1961-62	1962-63	1963-64	1964-65	Average
2- 3	8.5	14.8	28.5	20.7	28.4	21.2	24.9	26.3	21.7	22.2	24.8	23.8
4- 6	13.3	14.2	23.7	23.8	20.4	21.2	22.2	21.3	19.8	18.0	18.4	19.4
7- 10	12.0	12.6	14.3	19.6	14.6	15.5	15.9	12.3	17.8	14.5	14.1	14.8
11- 17	13.1	12.8	11.4	15.6	11.3	15.5	13.4	12.3	15.2	15.2	14.9	14.4
18- 27	11.5	10.6	8.3	7.9	9.1	10.8	9.1	8.7	7.2	9.2	8.9	8.5
28- 43	10.7	9.4	5.5	5.8	5.5	6.1	5.6	5.4	7.3	7.9	7.5	7.0
44- 67	8.4	7.4	4.2	3.2	4.9	3.5	3.9	4.5	4.9	6.4	4.5	5.0
68-105	7.1	6.2	1.9	1.5	2.2	2.6	2.1	5.0	3.2	3.3	2.7	3.5
106-165	5.6	4.6	1.6	1.2	1.9	1.8	1.6	3.0	1.5	1.5	3.4	2.3
166 +	9.8	7.4	0.6	0.7	1.7	1.8	1.3	1.2	1.4	1.8	0.8	1.3
Totals	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0

The respective rates of entry and exit into the various categories are shown in Table 4.20. Although the birth rates for small plants (< eleven employees) are clearly higher than those for large plants the range is not so great as one would expect from the frequency size distributions (Table 4.19). The death rate also declines with increasing size but not to the same extent as that for births; this reflects, in part, the availability of equity capital for new establishments.

One additional difference which appears between the two distributions is that for plants employing between 28 and 43 people the birth and death rates are equal; for plants with less than 28 employees the birth rate is considerably higher than the death rate but the reverse is true among those plants employing more than 44 employees. The exception is the birth rate for plants employing more than 166 people – this may be attributed to the large number of new foreign-owned branch plants.

(ii) Entry by sector – For convenience, the rest of this chapter analyses all births and deaths which include plants with less than two employees. Since both birth and death rates have a close inverse correlation with size it is reasonable to hypothesize that any systematic variation in the entry rates between industrial categories or sectors will be related to the size structures of those industries. Variations in the percentage distribution of all births and deaths for 20 2-digit industries are shown in Table 4.21 where they are compared with the average of all plants for the years 1961 and 1966. It is clear that the number of births and deaths in an industry is not simply related to the total number of establishments already in that industry. Between 1961 and 1966 the foods and beverages sector accounted for 20% of all establishments but only received an average of 13% of all births. During the same period it had the largest number of deaths resulting in a net loss of 11.5% from its 1961 total. On the other hand, metal fabricating, with only 13% of the establishments, accounted for 18% of all births. This extremely high percentage of births – one and a half times that of its deaths – gave rise to

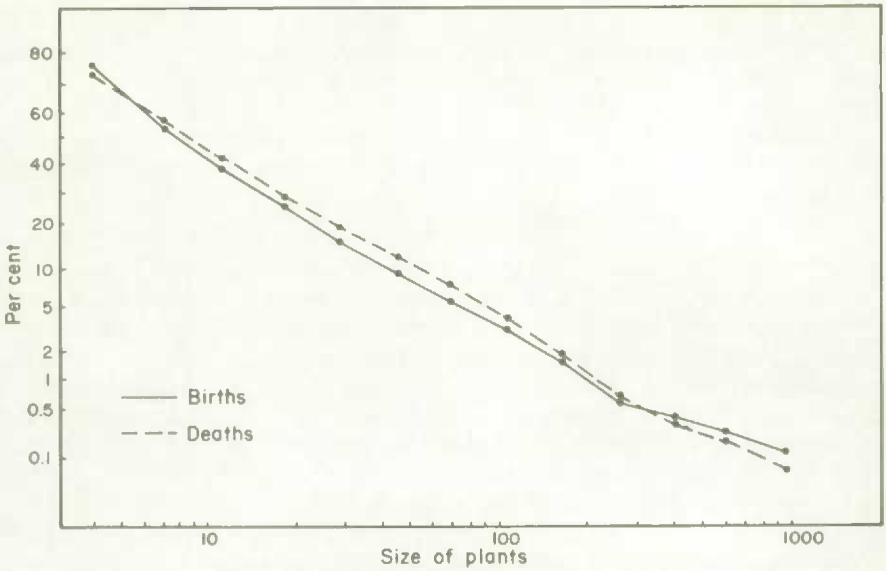


Figure 4.10: Lognormal Probability Curves for All Births and Deaths 1961-1965

TABLE 4.20. Average Birth and Death Rates[1] by Size Category, 1961-65

Size by No. of employees	Birth rate	Death rate
2- 3	12.4	10.1
4- 6	10.7	8.0
7- 10	8.7	6.9
11- 17	7.1	6.6
18- 27	5.6	4.6
28- 43	4.1	4.1
44- 67	3.6	3.8
68-105	2.2	3.2
106-165	2.4	3.1
166-	1.0	0.9
Totals	7.0	6.1

[1] Birth and death rates are calculated by expressing the total number of births and deaths between 1961 and 1965 as a percentage of the total number of plants in each size category for that period.

an exceptionally rapid growth in number of establishments which increased at the rate of 6.2% per annum; in comparison all industries expanded at less than 1% per annum. The largest proportionate net increase occurred in the machinery industry with 3.1% of all establishments and an average of 5.2% of total births. In terms of rank the machinery industry had the sixth largest number of births and the tenth largest number of deaths. The only industries which experienced a net loss in number of establishments were those of foods and beverages, knitting, clothing, and wood.

Since the number of births and deaths is not simply related to total number of plants in each sector it gives rise to several interesting hypotheses. Such hypotheses have counterparts in demographic studies which reveal significant differential growth rates among the various components of the population; biological causes for variations in the fertility rates, religious beliefs and differences in social class are but three common examples. In the first instance we can consider the hypothesis that all manufacturing sectors are homogeneous in respect to their opportunities for profits and loss. For this we would postulate that the sectoral variations of the entry rates shown in Table 4.22 are caused by random or chance fluctuations about a common or expected value. In the 1961 - 65 period this value was 7.0% and the probability would be .95 that the observed entry rate for a sector of 100 establishments (approximated by the knitting industry) would lie between 7.5% and 6.5%, and that the rates for a sector with 500 establishments (approximated by the clothing and chemical industries) would lie between 7.2% and 6.8%. Clearly, random fluctuations do not explain most of the sectoral variations in entry rates ranging from 4.1% to 11.8%. As one alternative we may hypothesize that within every industry each employee, since he already possesses experience and first-hand information concerning that form of economic activity, is a potential entrepreneur in that industry. Under this assumption the number of births in any sector would be directly proportional to the total employment of that sector. A second alternative is the hypothesis that sectoral variations in the entry rates are a function of the typical size or prevailing size distributions of the various sectors. Having shown in the preceding section that entry rates decline sharply with increasing size of plant it is reasonable to hypothesize that industries with the largest proportion of "small" establishments will exhibit the largest entry rates and vice versa. Since there is not a perfect correlation between average size (mean or median) and proportion of small plants ($r = .88$ for medians and proportions of plants with less than eleven employees) the latter hypothesis may be modified to – industries with the smallest average size will exhibit the highest entry rates. Perhaps the most obvious hypothesis relates to the influence of differential development stages of the industrial economy. The desire of entrepreneurs to establish new plants, for example, may be strongly influenced by the expansionary tendency of various sectors reflecting, in part, changes in consumer demand and product differentiation. These hypotheses, for which data are available, can be easily and quickly tested by Kendall's rank correlation coefficient. The respective rankings are listed in Table 4.23. In each case the rankings of columns 2-6 are correlated with those of column 1. The highest correlation is given for expansion rate but even this is not highly significant.

TABLE 4.21. Percentage Frequencies of All Births and Deaths for Twenty 2-digit Industries,[1] 1961 - 65

Industry	All establishments 1961-65	% change	Percentage all births				
			1961-62	1962-63	1963-64	1964-65	1961-65
1	20.7	– 11.4	16.1	20.2	10.4	88.5	13.6
2	0.1	13.3	0.2	0.1	0.1	0.0	0.1
3	0.5	26.9	0.1	0.4	0.4	0.4	0.4
4	1.6	4.6	1.2	1.2	1.6	1.1	1.3
5	3.0	3.3	2.4	2.7	2.4	2.7	2.6
6	1.0	– 8.9	0.9	1.2	0.7	1.1	1.0
7	4.4	– 9.2	2.8	4.9	3.3	3.1	3.5
8	5.7	– 1.9	4.3	5.9	4.5	4.7	4.8
9	6.9	14.0	10.8	8.8	9.6	11.6	10.2
10	2.1	5.4	1.3	1.2	2.4	1.4	1.6
11	12.2	2.3	8.9	7.8	8.1	8.6	8.4
12	1.7	9.1	1.7	1.5	1.9	0.9	1.5
13	12.9	32.0	18.2	13.2	20.7	19.7	18.1
14	3.1	41.8	3.4	4.6	6.4	6.0	5.2
15	2.4	30.6	4.0	4.2	3.6	4.7	4.1
16	3.3	16.7	2.4	2.6	2.8	3.5	2.9
17	4.2	3.3	5.9	3.7	4.2	3.9	4.5
18	0.2	3.2	0.3	0.1	0.2	0.2	0.2
19	4.5	1.5	2.8	3.8	4.7	4.6	4.0
20	9.5	18.0	11.6	11.7	11.4	12.7	11.8
Totals	**100.0**	**4.6**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**

Industry	Percentage all deaths				
	1961-62	1962-63	1963-64	1964-65	1961-65
1	23.2	28.0	21.9	22.9	24.1
2	0.0	0.1	0.1	0.0	0.1
3	0.2	0.2	0.5	0.8	0.4
4	0.5	1.5	1.4	1.3	1.2
5	3.4	1.8	2.4	2.5	2.5
6	1.5	1.5	1.3	1.7	1.5
7	5.8	4.3	4.5	4.6	4.8
8	6.2	7.7	7.0	5.2	6.6
9	8.2	9.7	8.1	9.0	8.8
10	1.2	1.3	1.8	1.1	1.3
11	6.7	9.6	12.4	7.7	9.1
12	1.2	0.6	0.7	1.1	0.9
13	13.4	12.2	12.1	11.9	12.4
14	2.8	2.8	3.2	4.1	3.2
15	3.3	2.0	2.4	3.3	2.7
16	3.1	1.3	2.0	2.9	2.3
17	5.1	3.5	5.6	4.7	4.7
18	0.1	0.2	0.3	0.1	0.1
19	2.6	3.3	3.2	4.5	3.4
20	11.6	8.3	8.8	10.4	9.6
Totals	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**

[1] Sawmills excluded.

TABLE 4.22. Average Birth and Death Rates for Twenty 2-digit Industries, 1961 - 65

Industry	Birth rate	Rank	Death rate	Rank
Foods and beverages	4.78	16	6.75	3
Tobacco products	4.69	18	3.12	20
Rubber	6.35	9	5.08	12
Leather	5.93	10	4.95	13
Textiles	5.16	15	4.94	14
Knitting mills	4.70	17	6.62	4
Clothing	5.22	14	7.39	2
Wood	5.47	13	5.55	10
Furniture and fixtures	9.35	4	8.27	1
Paper and allied products	5.64	11	4.13	18
Printing and publishing	4.29	19	4.22	17
Primary metals	4.10	20	3.59	19
Metal fabricating	9.75	3	5.38	11
Machinery	11.79	1	6.60	5
Transportation equipment	10.33	2	6.27	7
Electrical products	6.76	7	4.89	15
Non-metallic mineral products	6.60	8	6.34	6
Petroleum and coal products	5.63	12	5.63	9
Chemical products	6.85	6	4.37	16
Miscellaneous manufacturing	8.60	5	5.81	8
All industries	7.0		6.1	

It is apparent that other factors for which data are either not available or not quantifiable, may have more important influences. General economic conditions such as cartelization and trade associations requiring certain qualifications or financial resources, and legal restrictions to entry imposed by public authorities are among the factors that may be influential. In more specific terms, Bain has pointed to the different capital requirements among industries to establish a plant of minimum efficient size, and to the influence of patent laws and economies of large scale production. Other factors which may be peculiar to various industries include accessibility to raw materials, power, water supply and buildings. Likewise, sectors typified by low average operating costs may be expected to have relatively high entry rates just as those sectors earning high profits.

On a regional basis, Beesley, 1955, attempted to test the hypothesis: "... about the effect of differences in the structure of the industries of zones upon the emergence of new establishments, therein, and the latter's effect, in the long run, on employment in the zones". For this purpose, Beesley defined structure as the "specialization of each zone to various stages in the production sequence of metal industries." He distinguished four main stages: (1) early manufacturing and forming processes (e.g., iron and steel and non-ferrous metal rolling and drawing), (2) later forming processes (e.g., stamping, piercing and forging semi-finished metal to make finished goods and parts for further assembly), (3) sub assembly (bringing together parts to go forward for assembly), and (4) assembly (of complete articles). The two zones were delimited from the Birmingham conurbation as the S.W. and N.W.. Beesley attributed the higher incidence of entrants in the latter to its more integrated or "virtuous circle" form of structure. Geographic location is yet another factor which may indirectly influence entry rates but treatment of this aspect is retained for Chapter V.

TABLE 4.23. Rank Correlation for Hypotheses Relating to Sectoral Variations of Birth Rates

Industry	Entry rates 1961-65	No. of plants 1961	Median[1] values 1961	Prop.[2] small 1961	Expansion[3] rate 1961-65	No. of employees 1961
Foods and beverages	16	1	3	2	20	1
Tobacco	18	20	20	20	8	20
Rubber	9	18	19	19	4	14
Leather	10	16	12	13	12	15
Textiles	15	11	8	8	14	11
Knitting	17	16	16	18	19	17
Clothing	14	5	9	11	9	12
Wood	13	6	5	7	18	18
Furniture	4	7	1	3	7	16
Paper	11	13	15	17	11	10
Printing	19	3	2	1	16	6
Primary metals	20	15	13	14	10	4
Metal fabrication	3	2	6	5	2	5
Machinery	1	12	11	12	1	7
Transport	2	14	17	16	3	2
Electrical	7	10	18	15	6	3
Non-metallic	8	9	6	6	13	13
Petroleum and coal	12	19	14	10	15	19
Chemicals	6	8	10	9	17	8
Miscellaneous	5	4	4	4	5	9
Rank correlation coefficients		0.08	0.03	0.07	0.46	0.16

[1] Median value refers to size of "mid-most" plant in each industry – size measured by number of employees.

[2] Small refers to plants with less than eleven employees.

[3] Expansion rate refers to expansion in the employment in the constant sample of permanent establishments. If the expansion rate of all establishments were used the expansion rate would be dependent, in part, on the entry rate.

(iii) Exit by sector and age – The average exit rate for the 1961 - 65 period was 6.1% comapred to the average entry rate of 7.0%. Around this average death rate, however, there was a considerable sectoral variation ranging from 3.1% for the tobacco industry to 8.3% for the furniture and fixture industry (Table 4.22). Death rates decline not only with increasing size but also with increasing age. The survival patterns for selected industries born in Ontario between 1961 and 1962 are shown in Fig. 4.11 which also shows the pattern for all manufacturing firms in American industries born in 1944.[3] After seven years, approximately 45% of all the new establishments which began production in Ontario between 1961 and 1962 were still operating, but the sectoral variations ranged from 20% for textile industries to 72% for machinery industries.

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[3] The graph for U.S.A. manufacturing is constructed from data presented by Betty Churchill in "Age and Life Expectancy of Business Firms", *Survey of Current Business* (Dec., 1955). The data are based on the full record of U.S. firms available from the centrally administered Old Age Insurance (Bureau of Old Age and Survivors' Insurance) since 1944. The data, however, unlike the Ontario data, include transfer as well as death; when a firm is passed from a father to son or changed from partnership to corporation it is regarded as a death. Note also that the first annual interval for the American industries is six months after birth.

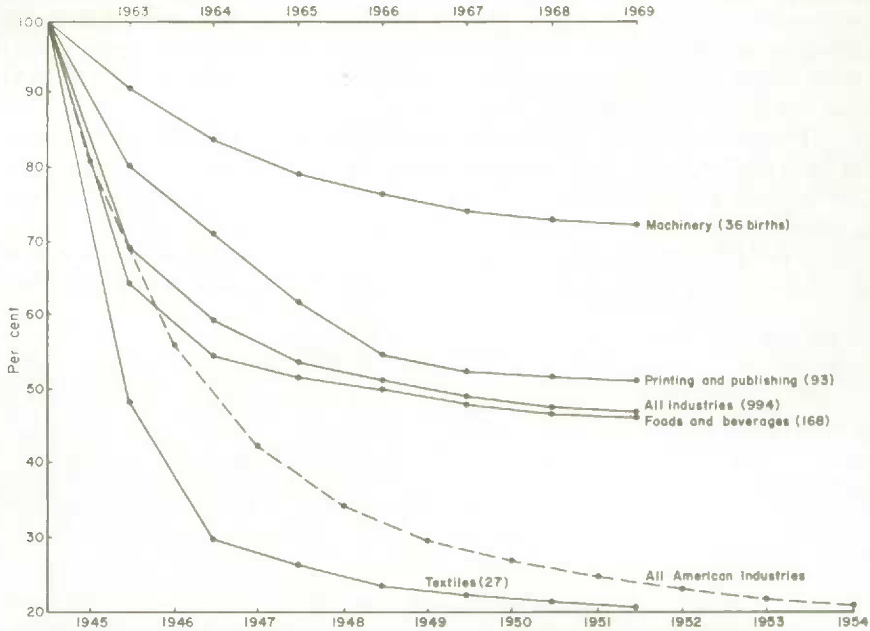

Figure 4.11: Survival Patterns for Plants Born in Ontario Between 1961 and 1962 in Selected Industries

The general survival pattern for Ontario establishments resembles that for all American manufacturing firms born in 1944 but the relatively sharper decrease among the American firms in the first three years may be attributed, in part, to the limitations of the American data which include "apparent deaths" in the form of transfers, and, in part, to the time differential; in 1944, for example, the birth rate for all American manufacturing industries was 10.75% but by 1958 it had systematically declined to 7.45% which compares more favourably with that of Ontario for the 1961 - 65 period.

After the first two or three years the exit rate gradually decreases with increasing age though the rate of decrease, especially in the formative years, varies from industry to industry. Age, it seems, is of independent importance to young plants because difficulties of adjustment and inadequate experience may be a considerable handicap to them. Once a plant has survived its "youth" and has adapted to the economic environment, age assumes lesser importance in comparison to other factors. This notion is consistent with the almost constant level of mortality ratios for plants still operating after five years. At this stage, size is probably the most important factor but risk differences between industries are also influential. In general, however, because of their greater resources to resist crises, and better qualified management than is common to small establishments, large plants are able to survive longer. Once well established, plants continue to grow despite the tendency of the average cost curve to remain constant above a certain minimum size. Continued growth is not just directed towards ever-increasing economies of scale, and hence greater profit margins, but also towards the accumulation of reserves for survival in times of economic stress. Large organizations, therefore, use much of their power to ensure survival rather than to maximize profits. (Wedervang, 1965).

The continued decline in the survival ratios by approximately 2% per annum can be related partly to the owner's business life which normally ranges between 20 - 50 years. In larger partner-controlled establishments the activity is inherited by several generations. The ability to control the relative weights of either age or size would simplify the analysis of the dynamics of a population of plants. Added to this complexity is the spatial component which is the focus of the next chapter.

CHAPTER V

SPATIAL DYNAMICS OF MANUFACTURING ACTIVITY IN ONTARIO 1961 - 65

Changes in the locational patterns of manufacturing activity usually occur very slowly but the processes leading to such changes are continuous. This chapter attempts to identify and analyse these processes and to evaluate their relative impact on recent spatial trends in Ontario's manufacturing activity. Such analyses offer a framework for the classification of a system of spatial states for the Markov model.

Alternative Systems of States

Ideally, the most appropriate system would be one comprising as states the smallest areal units for which data are available, in this case the individual municipalities (townships and urban centres), but they are too numerous for computational feasibility. Operationally, therefore, it is necessary to aggregate the municipalities into spatial groups. Conceptually, for the analysis of manufacturing activity, several alternative methods of grouping can be proposed though the combinations of the number and size of states within any one system are innumerable. One alternative is the "regionalization" of contiguous municipalities on the basis of their economic viability measured in terms of a variety of factors, (e.g., Ray, 1967; Ray and Berry, 1966; Amedeo, 1969). A second alternative, and one that is implicitly suggested by studies stressing the increasing tendency towards decentralization (Slater, 1961; Hay, 1965), is a system of states based on a series of concentric distance bands radiating outwards from the core area of industrial activity. Another alternative is the grouping of locations according to their industrial attractiveness or "similarities" in which case the system need not be spatially continuous as it would be in the first two alternatives. But as noted in Chapter III, the system adopted should be one in which the transition probabilities are as homogeneous as possible. The concept of uniform probabilities, however, depends, in part, on the underlying processes and patterns of change but our knowledge of these is limited since little effort ". . . has been made to explain the existing pattern of industry location. Even less attention has been devoted to explaining changes in this pattern", (Kain and Meyer, 1968, p. 177).

Processes of Distributional Change

Most studies which have been concerned with the spatial dynamics of manufacturing activity are confused by terminology. Usually the focus has been on the identification of resulting regional patterns of "distributional change", "industrial migration", or "locational shifts". In those few studies which have examined the underlying mechanisms of change, terms such as decentralization are used to describe the **processes** as well as their **results**. Given its broadest meaning as used by both Creamer, 1935, and Woodbury, 1953, decentralization refers to the process whereby there is an apparent decrease of manufacturing activity within a particular area as contrasted to a larger region to which it is being compared.

When the process of decentralization is considered **within** the boundaries of a metropolitan region the terms used are "suburbanization" and "diffusion" (Linge, 1963). In an attempt to be more precise, Linge used the term "deconcentration" to describe ". . . the process whereby the productive capacity of firms already established in the inner zone is partly or wholly shifted to the outer zone." When decentralization occurs **from** a metropolitan region to areas adjoining it – i.e. to smaller surrounding cities – the term "intraregional dispersion" is used, and if the process occurs between major regions then "interregional dispersion" is adopted.

Most of the resulting patterns, except that of "deconcentration" defined by Linge, require only simple identification through a comparison of locational maps drawn at two widely separated points in time. Given adequate data, however, even for a relatively short time period, it is possible to assess the relative importance of the underlying mechanisms of change. Consider, for example, the most familiar form of decentralization – that of suburbanization within a single metropolitan area – which may be defined as any of the following (Gilmour, 1965): (1) appearance of manufacturing in the suburbs, (2) greater relative growth of manufacturing in the suburbs as compared to the central city, or (3) movement of plants from the central city to the suburbs. Several processes, singly or together, can produce the condition which the definitions describe. For instance, according to the second definition suburbanization can evolve in the following ways: (1) outward movement of plants from the central city to the suburbs, (2) inward movement of plants from areas outside the city to the suburbs, (3) decline in the expansion of industries in the central city, (4) greater expansion of manufacturing plants in the suburbs, (5) location in the suburbs of local branch plants from other areas, and (6) birth of new enterprises in the suburbs.

Once the analysis of these locational changes is extended beyond the boundaries of a single metropolitan area the resulting patterns become increasingly complex and lead to difficulties of interpretation. Forty years ago, Thorp, 1929, p. 218, interpreted the movements of certain patterns in the U.S. as foretelling ". . . great **decentralization** of industry", whereas a contemporary, H.H. McCarty, 1930, p. 26, concluded from his analysis that the observed movements simply represented changes in the areas of **concentration**. In the Canadian context, both Slater, 1961, and Hay, 1965, described and measured the increasing trend towards **decentralization** of industry in southern Ontario but in 1969 Field and Kerr, 1969, analysed the same data to illustrate the trend to increasing **centralization** in Metropolitan Toronto.

In studies which have considered the processes of change a variety of specific generalizations emerge. In one of the earliest and most comprehensive surveys (Metropolitan Life Insurance Company, 1929), 2,084 communities in the United States and Canada reported that of the 10,000 plants gained between 1926 - 27, relocations accounted for 9.4%, branch plants for 8.8%, and new establishments for 81.8%; at the same time they lost 5,903 plants of which 18% moved away and 82% discontinued operations. In his study, 1961, Slater reported that "except for manufacturing for local markets, it appears that most manufacturing is dispersed by means of branch plants, or at least by decisions and arrangements made outside the community where the plant is located." Kerr and Spelt, 1958, however, found that the development of manufacturing in suburban Toronto resulted largely from the relocation of existing "Toronto city firms", whereas the plants comprising the remaining third were entirely new enterprises or branch plants. On the other hand, Hamilton's opinion, 1967, p. 410, is that usually ". . . "migration" results from differential rates of industrial growth which is accentuated if stagnant or declining and expanding industries are localized in separate areas". A somewhat negative appraisal was given by T.R. Smith, 1968, p. 49, supported later by Keeble, 1968, p. 1, that the ". . . actual movement of firms generally accounts for but a small part of the overall change in manufacturing importance of an area." Clearly, there is uncertainty concerning the respective processes influencing locational patterns of manufacturing activity, and further examination of these processes is required.

Within this framework the following sections focus on the rates and relative weights of distributional change influencing the spatial patterns of manufacturing activity in Ontario. The analyses are arranged to provide chronologically a survey of the general pattern in 1961, the net changes which occurred between 1961 and 1966, spatial variations in the birth/death differentials, expansion in the relative rates of permanent establishments, impact of foreign-owned branch plants, and the patterns of relocations between 1961 and 1965.

Structure and Change 1961 - 66

Ontario's manufacturing activity in terms of both employment and number of establishments as tabulated by Statistics Canada is completely dominated by Metropolitan Toronto[1] (Map 5.1). In 1961 this urban complex of over 1.6 million people – 26% of the provincial total – accounted for 4,584 or 37% of

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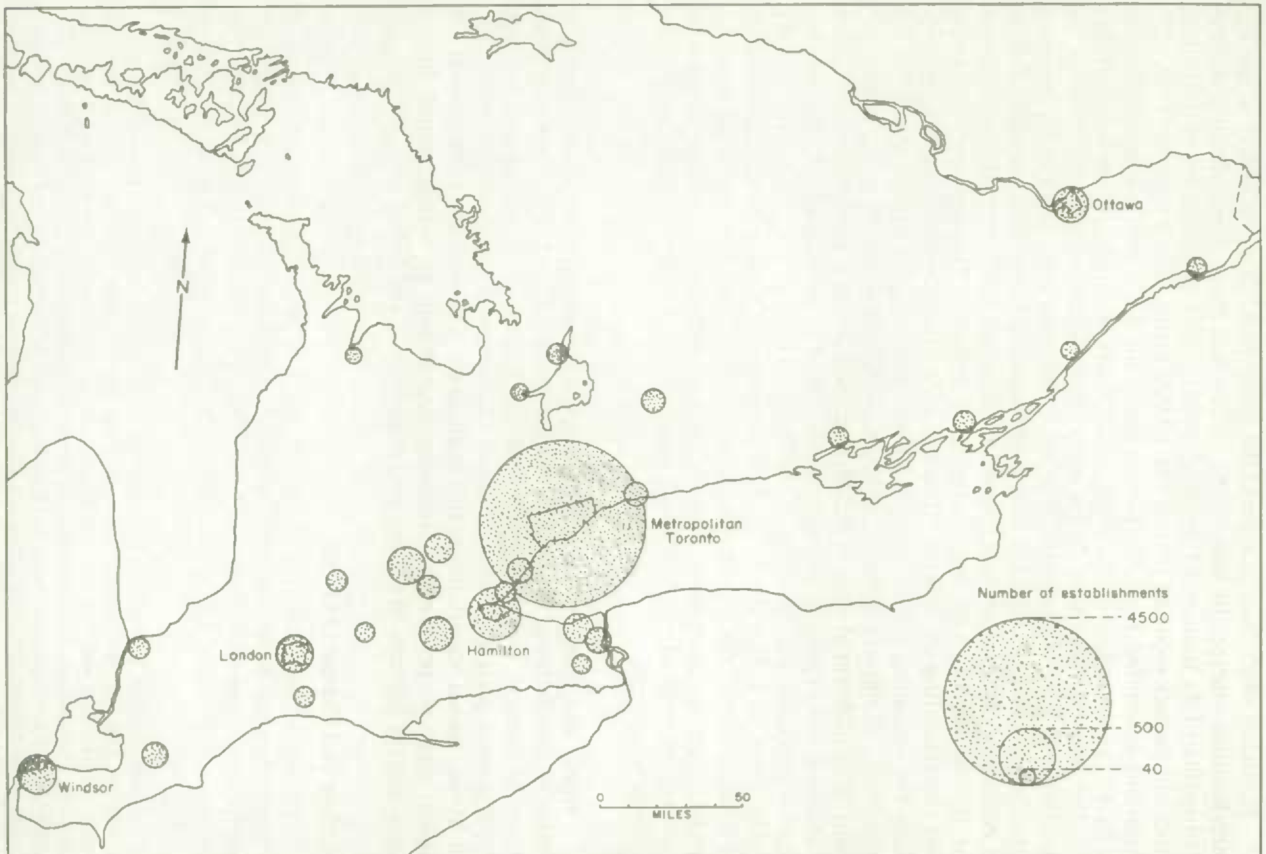
[1] Metropolitan Toronto (or Metro) refers to *Political Metropolitan Toronto* comprising the city of Toronto, the municipalities of Long Branch, New Toronto, Mimico, Swansea, Weston, Forest Hill, and Leaside, and the townships of Etobicoke, North York, East York, York, and Scarborough. For convenience, in this study the city of Toronto is referred to as *Toronto*, the townships and municipalities of Metropolitan Toronto are referred to as the *Toronto Suburbs* and the satellite towns and their respective townships of Toronto, Vaughan, Markham, Chinguacousy, and Pickering are referred to as the *Fringe areas*. The number and boundaries of the suburbs have subsequently been changed but this need not affect this study.



Map 5.1: Total Number of Establishments for Selected Urban Centres, Southern Ontario, 1961

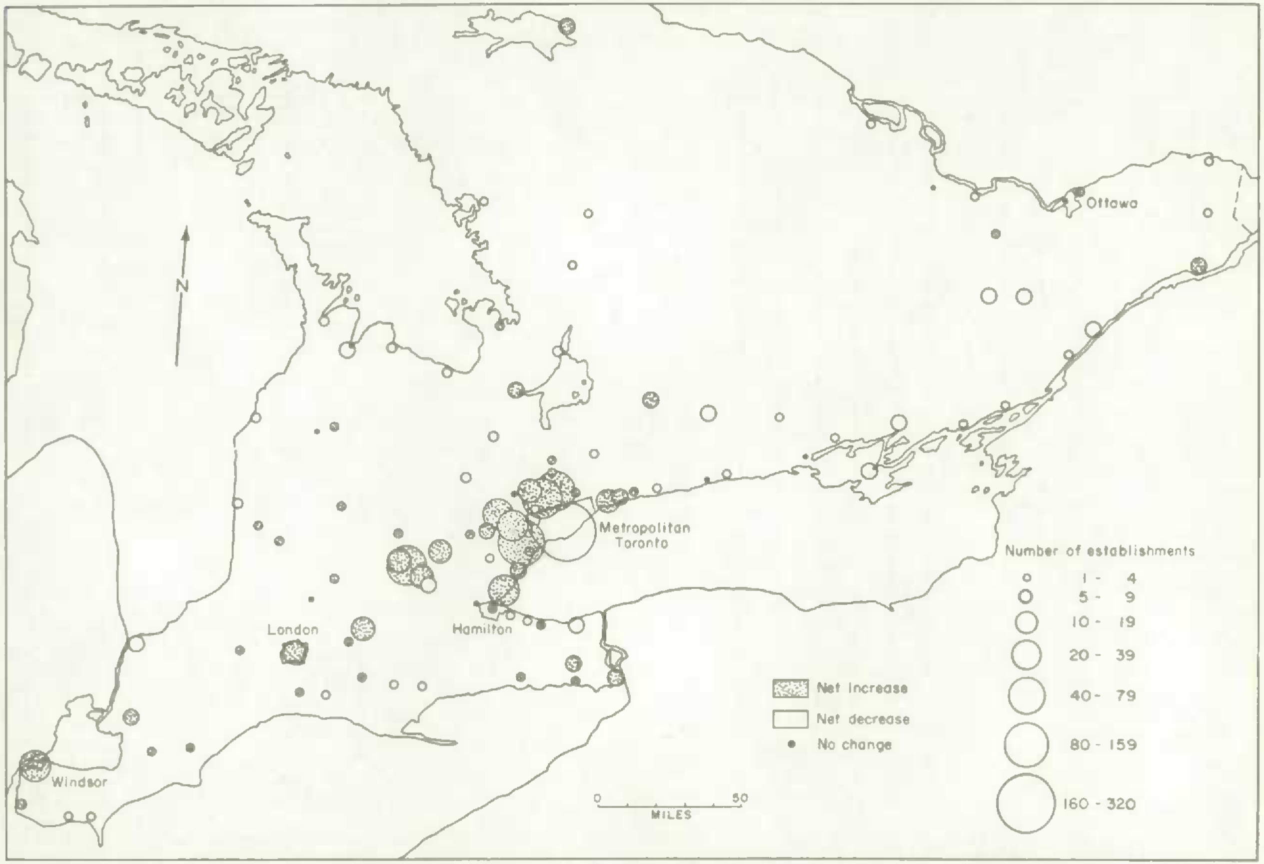
Ontario's manufacturing establishments; in terms of employment Metro's share was 33%. Within the metropolitan area over 60% of the plants with 53% of the jobs were located in the city of Toronto which, at the same time, accounted for 41% of the metropolitan population. Outside Metropolitan Toronto another 1,877 or 15% of Ontario's establishments were concentrated into eight main cities: Hamilton(520), Windsor(279), London(281), Kitchener-Waterloo(258), Ottawa(219), Brantford(168), and St. Catharines(152); together these eight centres accounted for 16% of the provincial population. Most of the remaining 48% of the establishments were distributed in clusters of 30-50 in south western Ontario.

During the 1961 - 66 period Ontario increased its number of establishments by 4.5% but the dominant trend was towards the development of an "industrial doughnut" around Toronto (Map 5.2). Between 1961 and 1966 the city of Toronto suffered a net loss of 314 establishments representing a decrease of 11%. In five years, therefore, Toronto lost more establishments than were located in any other centre except Hamilton. Although less marked, the 8% decrease in employment was greater than the total 1965 manufacturing employment of Welland(8,300) and almost equalled that of Guelph(8,600).

But far more spectacular were the gains that occurred in Toronto's suburbs and Fringe areas which more than offset the loss in the city. Metro increased its number of establishments by 10% and its share of Ontario's total by 1.6%. Altogether, the twelve suburbs had a net increase of 769 plants or 42% of their 1961 total. The largest increases occurred in North York with a gain of 383 (74%), Scarborough with 210 (62%), and Etobicoke with 200 (44%). The greatest relative increase, however, took place in the Fringe areas, between 1961 and 1966, there was a net increase of 65%.

Beyond the limits of this ring of intense industrial development very little change occurred except for the increasing concentration in the Kitchener-Waterloo-Preston-Guelph complex of urban centres which gained a total of 81 plants (20%). Similar evidence concerning the growing strength of this complex was noted in L.S. Bourne and A.M. Baker, 1968, pp. 12 - 17. Hamilton had a net gain of only three, though London and Windsor had more substantial increases of 6% and 11% respectively. No change was recorded for Ottawa but net losses were experienced by Niagara Falls, St. Catharines, Sarnia, Owen Sound, and Peterborough. In Eastern Ontario over 75% of the urban centres (greater than 10,000 population in 1961) had a net loss of establishments for the five-year period.

Changes in the size distributions of manufacturing establishments also vary systematically for urban areas of similar size. This relationship is graphically presented in Figs. 5.1 - 5.3. The net loss which occurred in Toronto is distributed almost proportionately in each size category so that the 1966 distribution resembles very closely that of 1961 (Fig. 5.4). There was, however, a slight



Map 5.2: Net Changes in Total Number of Establishments for Selected Urban Areas in Southern Ontario, 1961-1966

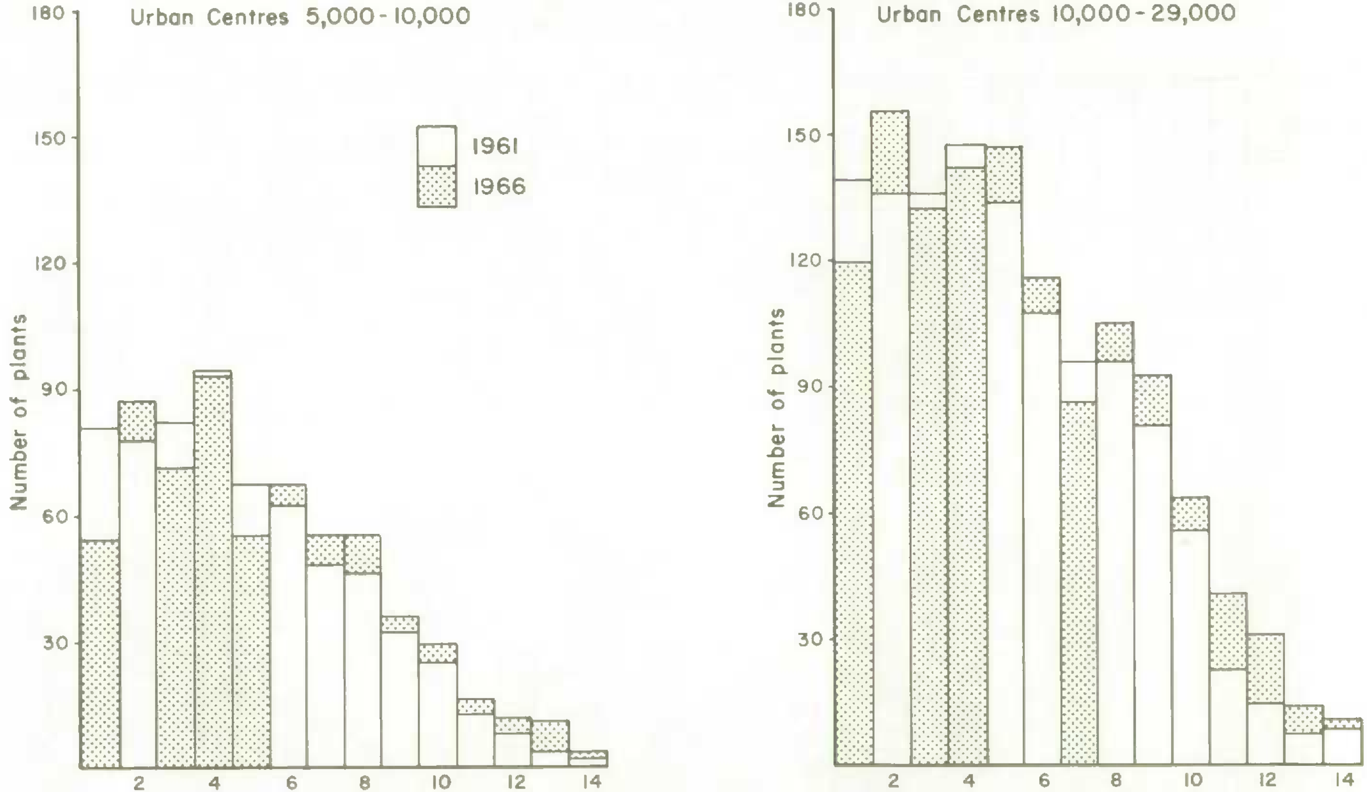


Figure 5.1: Net Changes in the Frequency Distributions of All Establishments in Urban Size Groups, Ontario, 1961-1966

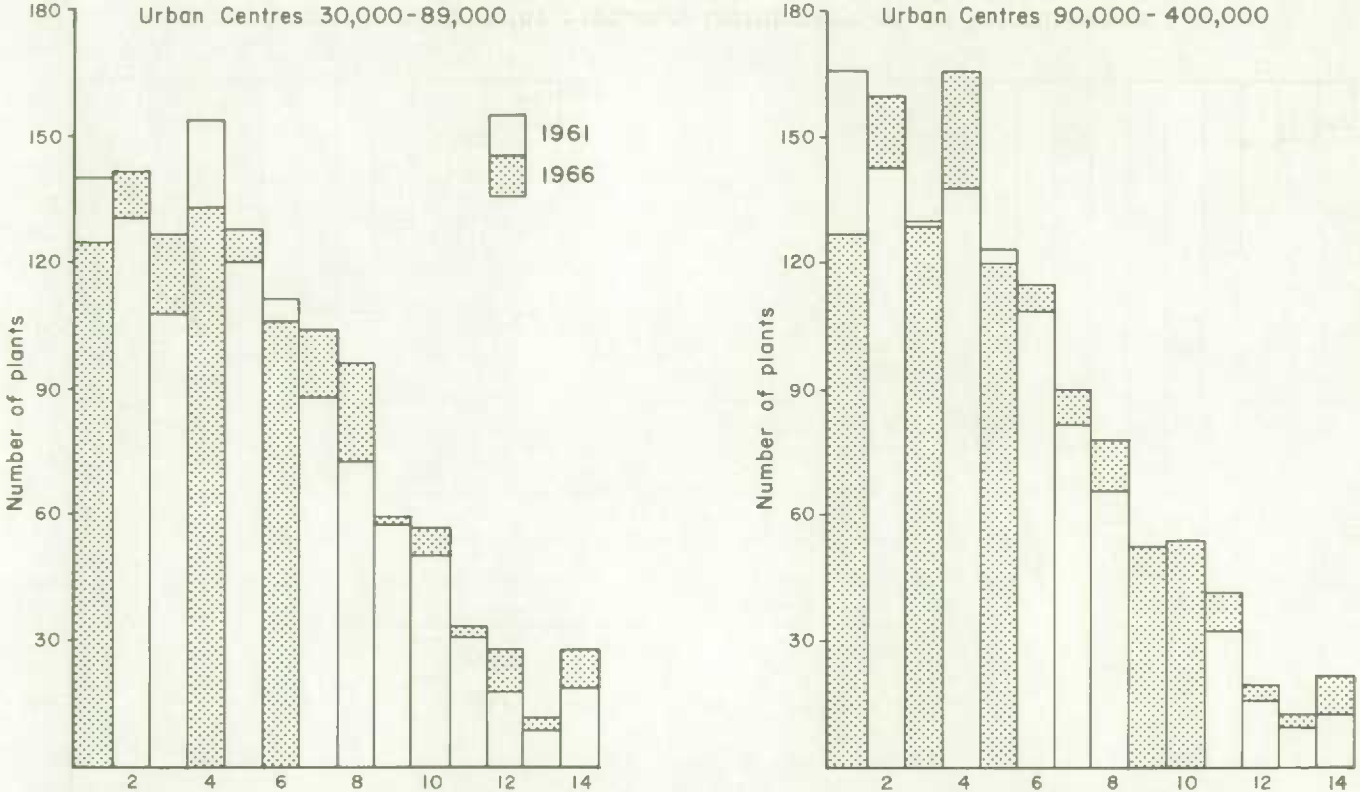


Figure 5.2: Net Changes in the Frequency Distributions of All Establishments in Urban Size Groups, Ontario, 1961-1966

increase in three of the four largest size categories. Much different is the picture presented by the respective groups of all other urban areas which recorded a net loss among the smallest size categories but a net increase in the number of plants with more than 40 employees. The main exception is the group of four large urban cities (90,000 - 400,000 in Fig. 5.2) which experienced a slight decrease in the number of plants employing between 100 and 250 people: this led to a slight change in the form of the upper tail of the distribution (Fig. 5.5).

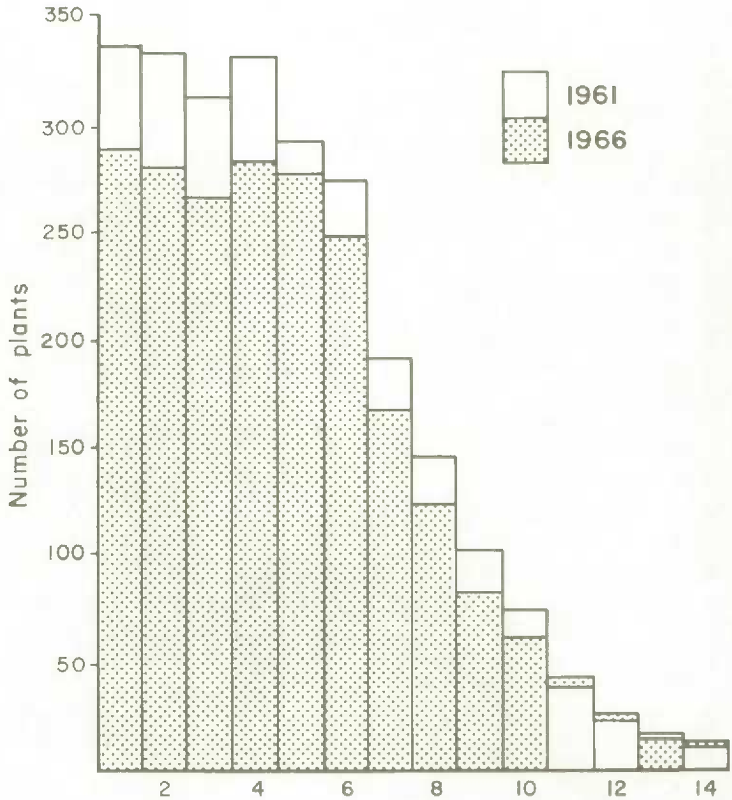


Figure 5.3: Net Changes in the Frequency Distributions of All Establishments in Toronto, 1961-1966

Spatial Variations in Birth/death Differentials

Net changes in the number of establishments is a function of two processes: a birth/death differential which may result in either a "natural increase" or "decrease"; and a "migration" process. Recalling the definitions outlined in Chapter 1, births are those plants, including foreign-owned and locally-owned branch plants, which appear in Ontario for the first time. Thus, plants relocating to an Ontario site from outside the province are termed births; likewise, plants relocating out of Ontario are grouped with those plants that "ceased operations" and are termed deaths. Plants which move from one municipality to another are termed relocations.

The impact of the birth/death differential between 1961 and 1965 had marked spatial variations in terms of both the number of establishments and employment. Some measure of the degree to which the birth/death process affected the distribution pattern of Ontario's manufacturing establishments is summarized in Table 5.1. During the 1961 - 65 period Toronto lost 300 establishments but the average annual death rate was only 0.2% higher than the birth rate; between 1961 and 1965 Toronto gave birth to 801 new establishments and 823 died. Toronto's natural decrease, therefore, was only 22 or 7.5% of its total loss; hence 92.5% of Toronto's loss for this period can be attributed to the migration process which is treated in a later section. Despite Toronto's loss the Metropolitan area gained 276 plants – an increase of 6% for the 1961 - 65 period. Most of Metro's gains were concentrated in the three suburbs of Etobicoke, Scarborough, and North York where the respective natural increases accounted for 80%, 64%, and 43% of the total gains.

Thus the spatial pattern of development in terms of the birth/death process in and around Toronto's industrial doughnut is extremely varied. The centre is characterized by an almost stable state of births and deaths so that if no migration occurred a very gradual decrease in the number of establishments would prevail. Around this area in the most industrialised suburbs there is a concentric band of lower than provincial average death rates which are greatly exceeded by higher than average birth rates. The result has been a substantial natural increase that has accounted for 63% of the suburban gains. Circling this zone is a peripheral area with an exceptionally high birth rate which is double the death rate. In this area, between 1961 and 1965, the natural increase contributed almost 80% of the total gains.

Beyond the outer periphery of Metro the only centres with birth rates noticeably higher than the provincial average of 7% were Waterloo, Barrie, Burlington, and Oakville. Even these are not independent centres: Waterloo is part of the Kitchener/Waterloo Metropolitan Area; Burlington is a "suburb" of both Hamilton and Toronto and was part of the former's Census Metropolitan Area; Oakville is a part of the Toronto Census Metropolitan Area; and Barrie, though not part of a metropolitan area is strongly affected by Toronto's "spill over"

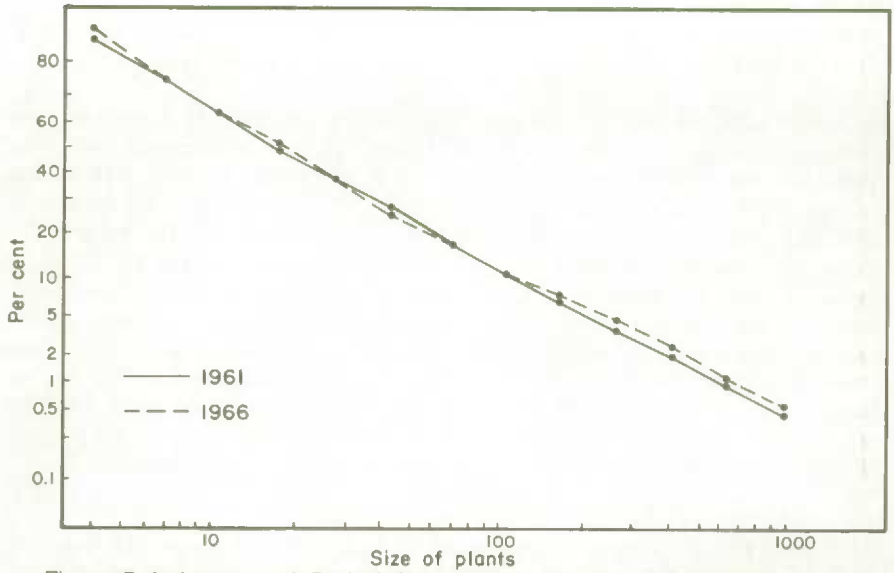


Figure 5.4: Lognormal Probability Curves for All Establishments in the City of Toronto, 1961 and 1966

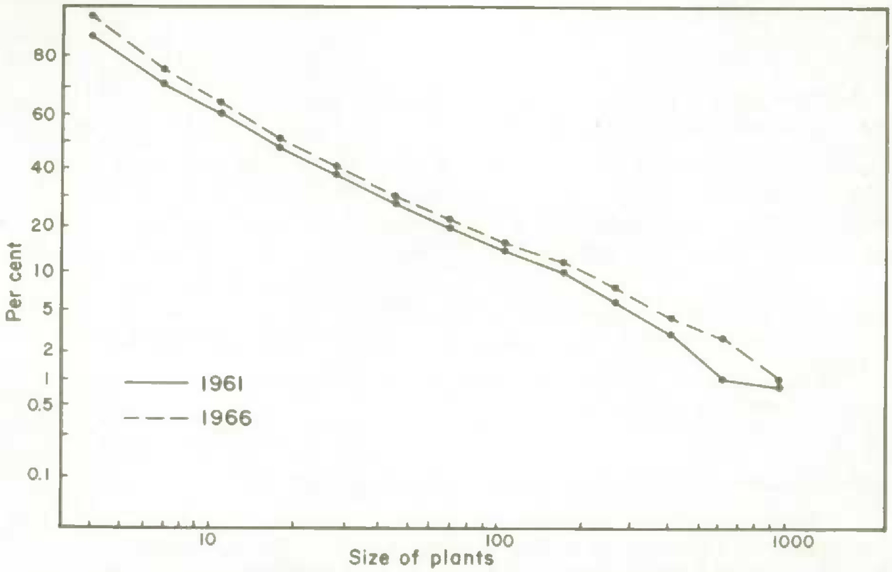


Figure 5.5: Lognormal Probability Curves for All Establishments in the Cities of Hamilton, Windsor, London, and Ottawa, 1961 and 1966

effects. Except for Oakville all these centres also had death rates lower than the provincial average. Altogether, 13 of the 28 towns and cities with more than 40 plants in 1961 had death rates higher than their respective birth rates.

Although the city of Toronto's birth rate is considerably lower than the provincial average, indicating that it is no longer Ontario's predominant nursery of manufacturing establishments, it is clear that Metropolitan Toronto is increasing its role as the focus of industrial development. On the basis of its total number of establishments for the 1961 - 65 period Metro attracted almost 10% more of the births than one would expect; during this period Metro accounted for 37.4% of Ontario's manufacturing establishments but recorded 41.4% of all births. Similarly, the suburbs attracted 26.5% more of the births than expected on the basis of total number of establishments. Obviously, the suburbs are no longer just "reception areas" (Kerr and Spelt, 1958), for plants relocating from the city of Toronto but are, in fact, self generating. However, this does not imply that the suburbs are "self contained" and "independent" of the city which is still the hub of industrial development and acts as a central focus for the surrounding activity.

TABLE 5.1. Average Annual Birth and Death Rates[1] and Net Migration Figures for Selected Urban Areas, 1961 - 65

Urban area	No. of plants		Net change	1961 - 65		Birth rate	Death rate	Net migration
	1961	1965		Births	Deaths			
All Ontario[2]	**11,966**	**12,559**	**593**	**4,283**	**3,690**	**7.0**	**6.1**	
Metro[3] Toronto	4,579	4,855	276	1,775	1,438	7.5	6.1	– 61
City of Toronto....	2,762	2,462	– 300	801	823	6.1	6.3	– 278
Toronto suburbs	1,817	2,393	576	974	615	9.3	5.9	217
Fringe areas	374	551	177	280	140	12.4	6.2	37
Other urban areas[4]................	3,196	3,263	67	912	840	6.3	5.9	– 10

[1] Birth and death rates are calculated by expressing total number of births and deaths for 1961 - 65 as a percentage of total number of plants for 1961 - 65.

[2] Totals differ from those published in the Census because sawmills are excluded.

[3] Metro Toronto includes City of Toronto and its suburbs.

[4] Those with at least 40 plants in 1961.

Spatial Variations in Size Distributions of Births and Deaths

Spatial variations in employment change are due in part to differences in the respective birth and death size distributions. As noted by Field and Kerr, if "mini establishments" are defined as those plants with fewer than four employees, then Metropolitan Toronto is the dominant breeding ground of these plants

(Table 5.2). Within this area, however, most of the mini establishments were born in the suburbs; over 26% of the total suburban births (two or more employees) between 1961 and 1965 were in the lowest size category compared with 16% for the central city. The variation is reduced, however, when plants with less than eleven employees are considered; these plants account for 64.6% of all suburban births and 57% of those in the city.

TABLE 5.2. Percentage Frequencies of Births and Deaths for Urban Size Groups in Ontario, 1961 - 65

Size of plant by No. of employees	5,000-9,999	10,000-29,999	30,000-89,999	90,000-399,999	City of Toronto	Toronto suburbs	Metro Toronto	Fringe areas
	Births							
2	8.0	12.3	14.1	19.4	15.8	26.8	22.3	21.5
4	25.6	25.4	23.5	29.5	27.1	21.5	23.8	21.0
7	25.6	22.3	21.7	15.8	16.0	16.3	16.2	16.6
11	14.2	15.6	16.5	14.3	15.8	14.4	14.9	14.5
18	10.2	6.4	8.2	8.1	8.4	10.3	9.4	11.4
28	7.4	7.6	6.6	5.8	7.9	4.4	5.8	6.2
44	4.5	5.1	1.8	2.7	4.1	4.4	4.2	3.5
68	1.1	3.0	3.2	1.6	2.8	1.4	1.9	2.2
106	3.4	2.3	4.4	2.8	2.1	0.5	1.5	3.1
Totals	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**
	Deaths							
2	12.7	9.3	9.7	15.2	11.6	21.0	16.0	22.7
4	23.4	25.0	20.2	25.5	25.2	17.4	21.4	18.2
7	21.0	15.0	15.5	21.6	14.2	13.4	13.9	14.5
11	17.7	19.0	22.6	13.0	15.1	15.7	15.5	16.4
18	7.8	11.6	9.7	8.1	9.8	10.6	10.2	10.9
28	5.5	8.0	8.8	7.6	9.7	7.4	8.4	10.0
44	4.3	4.4	7.3	4.0	5.1	6.5	5.8	1.8
68	3.1	3.6	2.9	2.3	5.2	3.6	4.5	0.9
106	3.6	4.1	3.3	3.7	4.1	4.4	4.4	4.6
Totals	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**

Although the birth rate in Toronto's suburbs exceeded the death rate by 3.4%, a differential which accounted for 63% of the new suburban plants, the structure of the birth and death size distributions did not create a similar proportional increase in the suburban employment. In the suburbs, for example, only 6% of the births had more than 44 employees; the death rates for these categories were 14.5% and 4.5% respectively (Table 5.2). If we ignore the expansion of the new plants during the 1961 - 65 period the birth/death differential added 1,549 new jobs in the suburbs, or 3.7% of the 40,500 new jobs created between 1961 and 1965. On the other hand, in Toronto where the employment decreased by 8,300 the structural imbalance within the almost equal birth and death rates accounted for 51.5% of the net loss in employment; over 14% of the deaths in Toronto had more than 44 employees as compared with 9% of the births.

Only in the fringe areas did plants employing more than 44 employees record a greater percentage of births than deaths. Here the percentages among the births and deaths were 8.8% and 7.3% respectively. The direct contribution of the birth/death differential to the total employment which increased in the Fringe area from 15,900 in 1961 to 25,500 in 1965 was 3,350 or 33.6% of the new jobs. The subsequent expansion of births is not taken into account in the calculation.

In the rest of southern Ontario between 1961 and 1965 towns with less than 10,000 people (in 1961) increased their total employment by 9,369 from 34,448 to 43,819. Because the death rate in these towns was slightly higher than the birth rate and because the respective birth and death distributions were similar (Table 5.2) the birth/death differential resulted in a net loss of only 66 employees. In towns with between 10,000 and 30,000 people the slightly higher birth rate provided 3.5% of the net increase of 19,000 jobs which increased from 52,500 to 71,500; in the group of large towns (30,000 - 90,000) the excess of births over deaths, despite the significantly larger percentage of deaths in relation to births with over 44 employees, accounted for 5.5% of the net gain of 33,700 employees. The highest percentage contribution of the birth/death process outside Toronto's industrial doughnut occurred in the cities of Hamilton, Windsor, Ottawa, and London where the introduction of a few large plants, each employing more than 400 employees, helped to create 11% of the 23,700 new jobs generated in the city size group between 1961 and 1965.

Differential Growth Rates

Although the spatial rearrangement of manufacturing activity in terms of the number of establishments is not directly influenced by the expansion in situ of plants the attractiveness of certain areas over others as potential sites for new and relocating establishments is greatly influenced by rapidly growing and developing plants. Approximately 85% of Ontario's 28% increase in employment between 1961 and 1965 can be attributed to expansions of the permanent establishments. The mean rate of growth for the permanent plants in each of the most important areas in southern Ontario is shown in Map 5.3. Again, the dominant trend has been one of increasing concentration in job opportunities in the peripheral area around Metropolitan Toronto. In the city of Toronto the permanent establishments had a relatively low mean growth rate of 19% as compared with 28% for all of Metro; the mean growth rate for plants in the suburbs was 40%. Such differences are to be expected since the suburbs, in general, are less confined than the more highly developed downtown areas where physical constraints often limit expansion. Similar differences, however, did not occur for the suburban areas bordering three of the other four large metropolitan centres. Establishments located in the cities of Hamilton, London, and Ottawa had mean growth rates of 24.8%, 22%, and 17% respectively; when their respective suburban growth rates are added the metropolitan rate of Hamilton decreased to 23.9%, and those of London and Ottawa increased only to 23.9% and 18.7% respectively. The exceptions were those plants in Windsor's suburbs which had a mean growth rate of 76% compared to 33% in the city; for Metropolitan Windsor the mean growth rate of its permanent establishments was 42.5%.

In the Fringe townships of Chinguacousy, Markham, Pickering, Toronto, and Vaughan the mean growth rate was 47%. The growth rates of these suburbs were matched elsewhere in Ontario by those of Cornwall(44%), Woodstock(47%), and Stratford(48%), and were exceeded by those of Wallaceburg(86%), Preston(98%), and Aurora(120%).

The development of branch plants is also part of the "expansion process" and probably the most widely referred to characteristic in Canadian manufacturing, particularly in Ontario, is the foreign-owned branch plant. Unfortunately, for geographical analysis, less is known about this component of manufacturing activity than any other. The most useful available data are the annual numbers of "planned births", above a minimum size category, by type of activity for municipal locations.[2] No separate employment figures, other than those derived by private questionnaire, are available. The locations of all new foreign-owned branch plants which were established in Ontario between 1961 and 1966 are shown in Map 5.4. It may be assumed that these branch plants accounted for a greater net share of the development, both in terms of number of establishments and employment, than the same number of indigenous enterprises. Foreign-owned branch plants tend to be more viable in their formative years since their continued existence is not always contingent on their own profitability; parent companies with large financial reserves often continue inefficient or unprofitable branch plant operations until the offspring develop into self sufficiency. It may be expected, therefore, that foreign-owned branch plants have a different survival pattern from that which was shown for all establishments in Fig. 4.12. Under such conditions a disproportionately larger share of local or indigenous enterprises will contribute to the total number of deaths.

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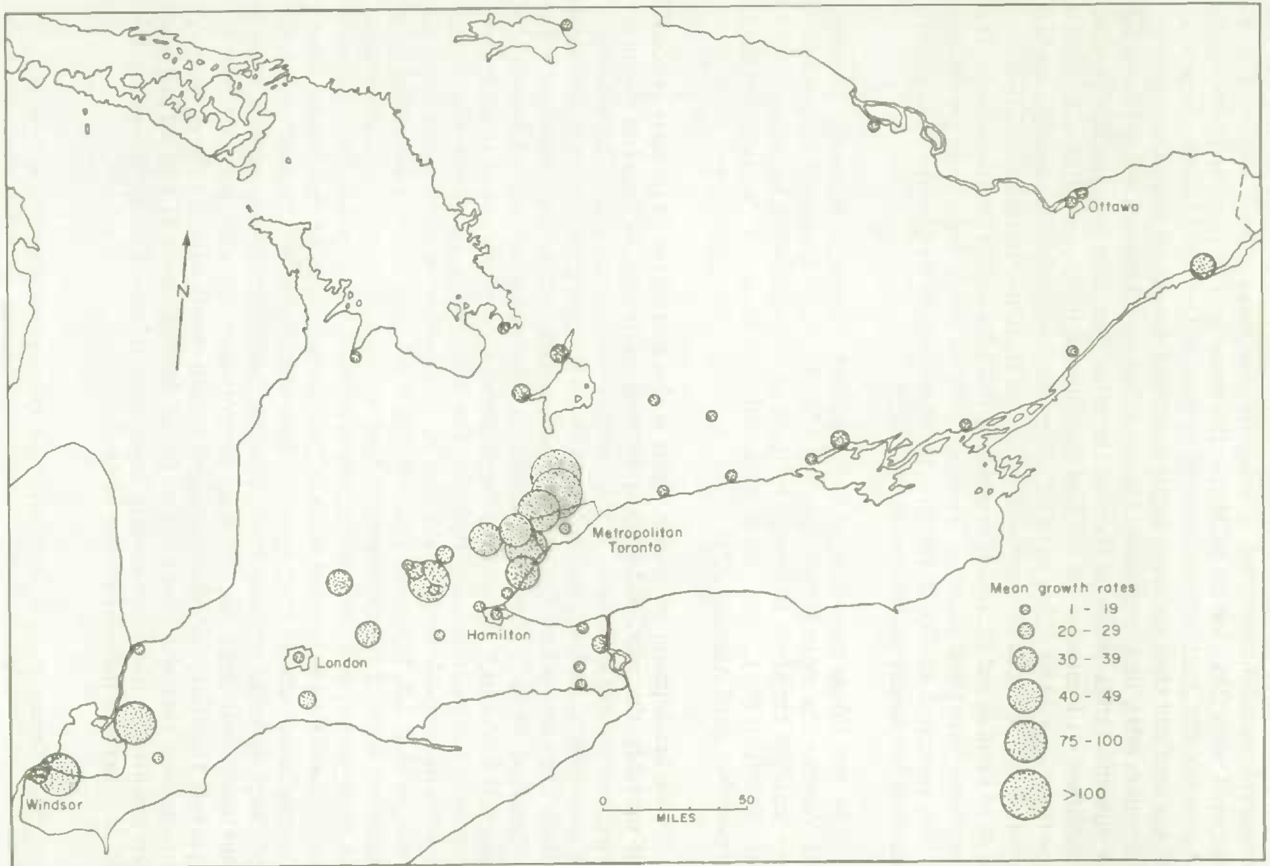
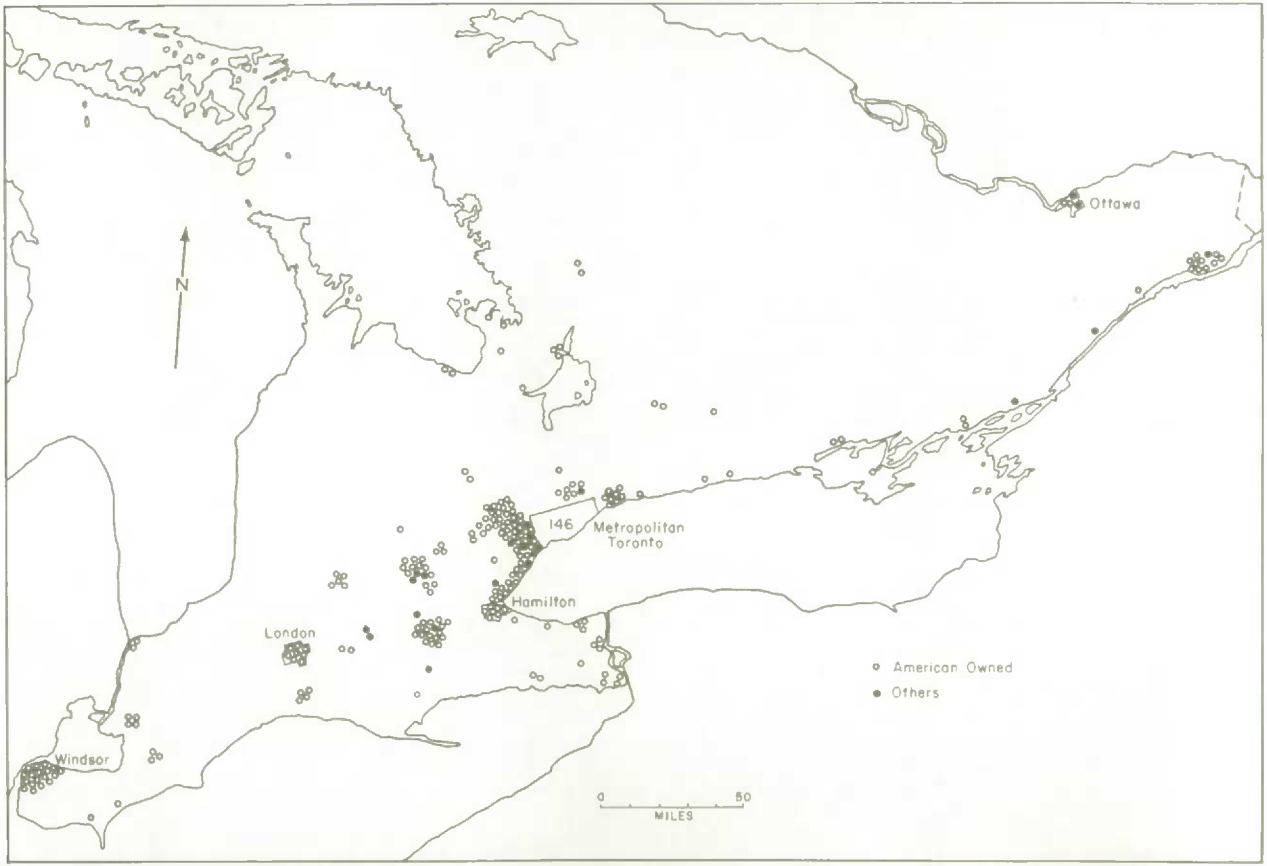
[2] The Ontario Department of Trade and Development (formerly the Department of Economics and Development) publishes annual lists of foreign firms planning to open a new establishment which will either "employ 10 persons, occupy 5,000 sq. ft. of manufacture or assembly space, or have sales in excess of $100,000 annually."



Map 5.3: Mean Growth Rates 1961-1965 of Permanent Establishments in Selected Urban Areas, Southern Ontario



Map 5.4: Locations of Foreign-Owned Branch Plants Established in Southern Ontario Between 1961 and 1966

Spatial Patterns of Relocation

Almost without exception, studies of industrial relocation have treated this phenomenon as "unidirectional" in that either the source area or the reception area for the migrant plants has been analysed. But as this analysis shows, industrial migration is not a simple one way process. This study identifies five main processes:

(i) **Suburbanization** – involves the movement of plants from the city of Toronto to its suburbs and the movement of plants from the cities of Hamilton, London, Windsor, and Ottawa to their respective suburbs bounded by the limits of their Census Metropolitan Areas.

(ii) **Suburban dispersion** – summarizes the movement of plants lacking a marked spatial pattern which, in this context, refers to the interchange of plants among Toronto's suburbs.

(iii) **Decentralization** – describes the process whereby plants move out of Metropolitan Toronto to other areas of Ontario including the Fringe areas. The latter movement, it may be argued, is part of the suburbanization process of Metropolitan Toronto, but here it is treated as decentralization.

(iv) **Centralization** – is the counter movement of plants from the rest of Ontario into Metropolitan Toronto and includes also those plants which moved inwards from the suburbs to the city of Toronto. Centralization as defined here, therefore, excludes those plants relocating from Ontario into the Fringe areas.

(v) **Dispersion** – as defined above, relates to the residual interurban relocations which have no well defined spatial pattern.

These definitions emerge from the analysis of plants with at least two employees which relocated from one municipality to another between 1961 and 1965 (Table 5.3). Although it was shown in Chapter IV that the size structure of Ontario's manufacturing activity closely resembles that of Britain, the size structure of plants which have relocated in Ontario is markedly different to that observed by Keeble, 1968, for London. Keeble analysed only those plants with more than ten employees and found that relocations to the Metropolitan and Provincial zones[3] averaged 365 and 728 employees respectively. In contrast, only four plants which relocated within Ontario between 1961 and 1965 had more than 365 employees. Moreover, whereas the relocations analysed by Keeble were much larger on average than all establishments the mean value of 38 for relocations in Ontario is significantly lower than the 1961 provincial average of 62 for those plants with more than two employees.

(i) Suburbanization – By far the largest component of change in Ontario involved the relocation of plants from the city of Toronto to its suburbs. These plants with an average of 49 employees, at time of relocations from Toronto, were responsible for 7,600 or 18.7% of the new job opportunities generated in

[3] The Metropolitan zone was defined as that area between 10 and 100 miles from north-west London. The Provincial zone extended outwards from the Metropolitan zone.

the suburbs between 1961 and 1965. In moving, 67% of the relocations exhibited an "immediate increase" in number of employees, 12% maintained the same number and 27% had a decrease. "Immediate increase/decrease" refers to the difference in employment recorded by a plant at its original location and that first recorded in its new location. The net result, in total employment among the relocations was an immediate increase of 3.2%.

The suburbanization process was rather less well marked in the other four Census Metropolitan Areas of Windsor, Hamilton, London and Ottawa. The plants were much smaller on average (18 employees) than their counterparts in Toronto and only accounted for 500 jobs in their original sites; after relocating, only 18% decreased their employment and the net increase in employment among the relocations was 24%.

For all plants the most "mobile" were those in the machinery industry (Table 5.4) with a mobility index of 1.82 but among the suburbanizing plants in Metropolitan Toronto furniture and fixture plants accounted for one quarter – twice the percentage for this group among all relocations.

(ii) Suburban dispersion – The main reception areas were Etobicoke, Scarborough and North York, all of which received most of their plants from adjacent older suburbs closer to the city centre. The average size of the relocating plants changed from 36 to 43 employees during relocation; over 80% added more employees after relocating and the total employment of these plants immediately increased by 20% from 2,142 to 2,569. Metal fabricating plants accounted for 27% of the moves and miscellaneous industries 12%.

TABLE 5.3 Percentage Frequencies of Relocations by Size Category, 1961-65

Size by No. of employees	All relocations	Decentralization	Centralization	Dispersion	Suburbanization Toronto	Suburban dispersion	Suburbanization large urban
2- 3	8.3	9.8	9.1	10.8	5.9	5.0	21.4
4- 6	11.1	11.7	12.7	21.0	11.2	3.3	10.7
7- 10	14.5	9.8	12.7	8.7	15.9	13.3	28.6
11- 17	12.7	13.7	16.3	10.3	11.3	15.0	10.7
18- 27	15.0	11.7	10.9	17.4	13.2	26.0	10.7
28- 43	12.0	15.7	14.5	7.9	11.3	15.0	7.1
44- 67	11.2	13.7	3.6	13.0	14.6	10.0	3.6
68-105	8.2	7.8	14.5	8.7	7.3	6.6	3.6
106-165	2.7	3.9	3.6		2.6	3.3	3.6
166-256	2.0		1.8	2.2	3.9		
257-398	2.0	1.9			1.3	1.6	
399+	0.5				1.3		
Totals	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**	**100.0**

TABLE 5.4. Percentage Frequencies of Relocations for Twenty 2-digit Industries, 1961 - 65

Industry	Mobility[1] Index	All relocations	Decentralization	Centralization
Food and beverages	0.52	10.7	7.2	5.8
Tobacco				
Rubber	0.40	0.2	1.8	
Leather goods	0.75	1.2	1.8	1.9
Textiles	0.86	2.6	1.8	
Knitting	1.50	1.5	1.8	1.9
Clothing	1.04	4.6	1.8	3.9
Wood	0.49	2.8		3.9
Furniture and fixtures	1.81	12.5	3.6	5.9
Paper and allied industries	0.86	1.8	1.8	1.9
Printing and publishing	0.56	6.9	10.9	5.9
Primary metals	1.65	2.8	8.5	
Metal fabricating	1.74	22.5	12.7	21.5
Machinery	1.82	5.6	5.4	11.7
Transportation equipment	0.95	2.3		2.5
Electrical products	1.78	5.9	12.7	9.8
Non-metallic minerals	0.61	2.6	4.6	1.9
Petroleum and coal products	1.00	0.2	1.8	
Chemical products	1.02	4.6	10.9	9.8
Miscellaneous	0.88	8.4	10.9	11.7
Totals		**100.0**	**100.0**	**100.0**

Industry	Dispersion	Suburbanization Toronto	Suburban dispersion	Suburbanization large urban
Food and beverages	17.4	13.2	6.6	10.7
Tobacco				
Rubber				
Leather goods	2.2	1.3		
Textiles	2.2	3.3	5.0	
Knitting		2.6		
Clothing	4.4	5.9	6.6	
Wood	2.2	2.0	6.6	3.5
Furniture and fixtures	2.2	23.2	8.3	7.1
Paper and allied industries		2.6	1.6	
Printing and publishing	4.4	7.3	5.0	7.1
Primary metals	6.5	0.6	5.0	
Metal fabricating	26.0	20.5	26.6	39.3
Machinery	6.5	2.6	6.6	7.1
Transportation equipment	8.7	0.6		10.7
Electrical products	4.4	4.6	3.3	
Non-metallic minerals	6.5	1.8	3.3	3.5
Petroleum and coal products				
Chemical products	2.2	1.3	3.3	3.5
Miscellaneous	4.4	6.6	12.2	7.1
Totals	**100.0**	**100.0**	**100.0**	**100.0**

[1] Mobility Index calculated by dividing percentage of relocations in each industry by the percentage of all establishments in each industry.

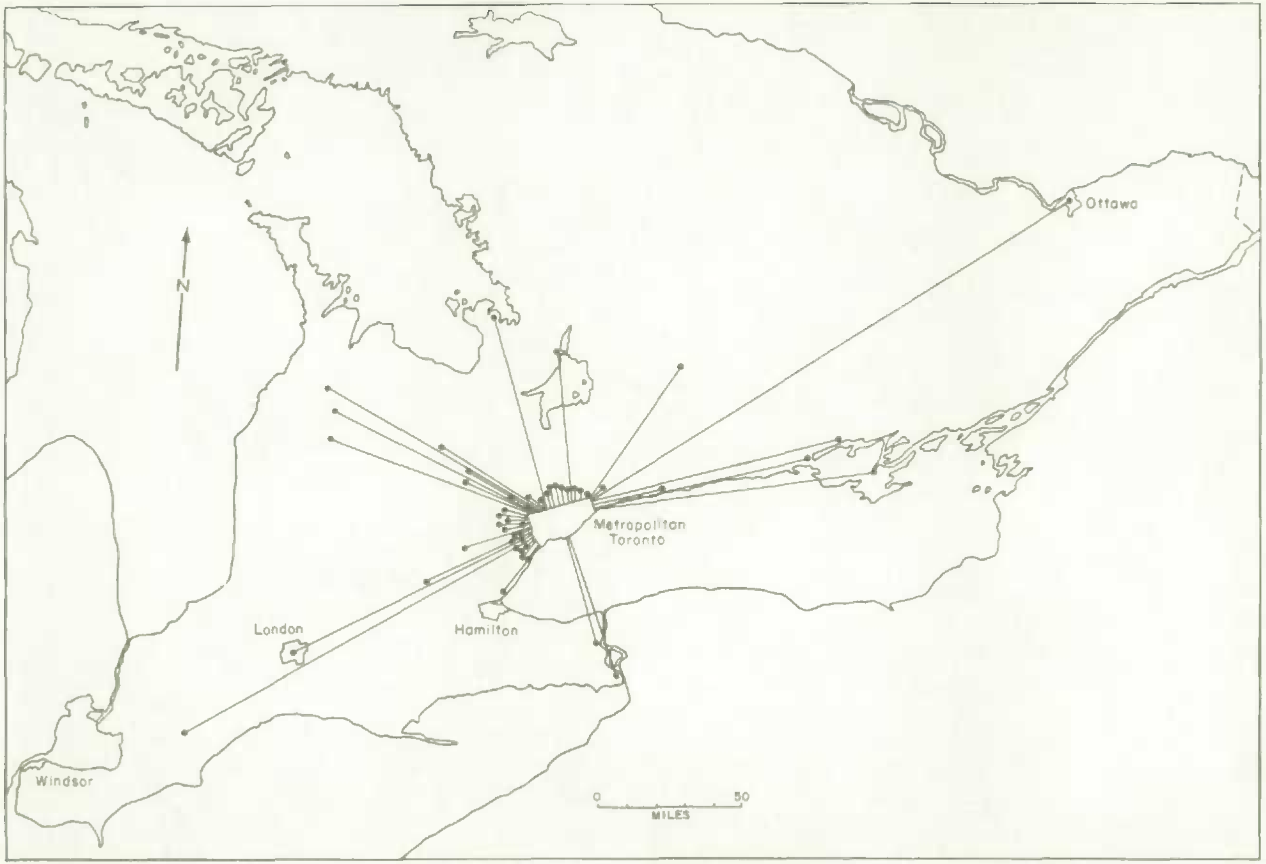
(iii) Decentralization – Most of the decentralizing plants (69%) moved to locations within a 50 mile radius of downtown Toronto and thereby reinforced the outer shell of the doughnut's periphery (Map 5.5). Fifty-one plants with a total of 2,242 jobs in their new locations were involved in the process. Over 80% of these plants increased their size which on average changed from 38 employees to 44 employees and generated 293 new jobs – an increase of 23%. By far the greatest proportion of the plants were metal fabricating units, and miscellaneous industries comprised a poor second.

(iv) Centralization – In contrast to the process of decentralization, of the plants which moved inwards to Metropolitan Toronto almost 80% relocated from outside a 50 mile radius around the city (Map 5.6). These plants, with 1,205 employees in their original locations reduced their average number of employees slightly and suffered a total employment loss of 5%; more than half reduced their size.

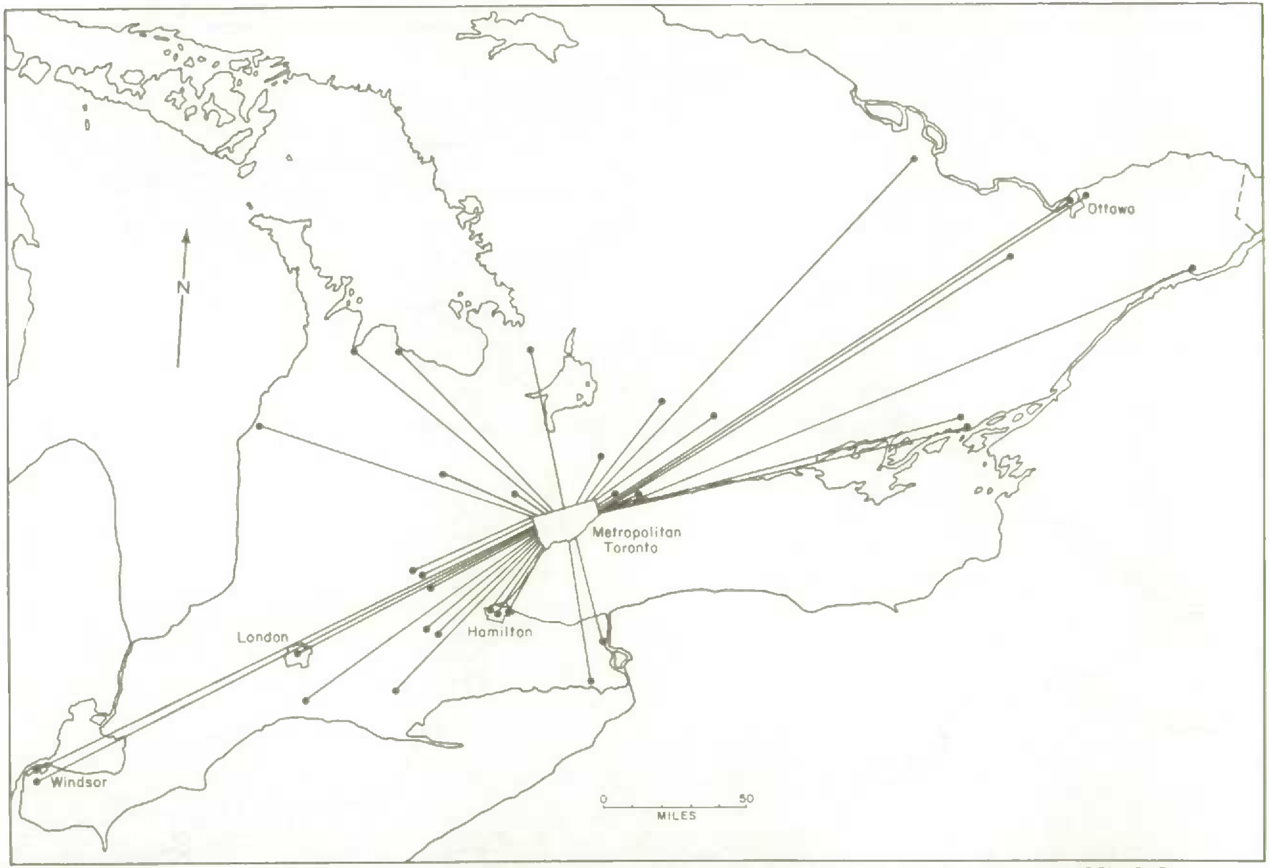
Added to this general inward movement of plants to Metropolitan Toronto was the relocation of plants which moved further inwards from the suburbs to the central city. These plants were considerably smaller on average (29 employees) and carried only a total of 629 jobs to their new locations, but in so doing 60% experienced a loss of employment. Almost half the plants which moved inwards to the city belonged to the electrical products and printing industries whose principal locations are in the core area.

(v) Dispersion – The complexity of movements in interurban relocations is shown in Map 5.7. Rural relocations are not marked because of uncertainty of exact location within the respective townships. Of the 46 plants involved 28 or 61% moved from a smaller to a larger urban centre. If there is a pattern the dominant characteristic of the movements is one of concentration into selected metropolitan nodes such as Windsor. This city seems to be playing a similar role, but on a much smaller scale, to that of Toronto in that it is attracting plants from less viable centres in the extreme south-western region. The two most important nodes are Hamilton-Burlington, and Kitchener/Waterloo, both of which have attracted plants over a considerable range of distance. The average size of all these plants was 29, and only one had more than 100 employees. In relocating 60% increased their size, 26% decreased and 14% showed no change.

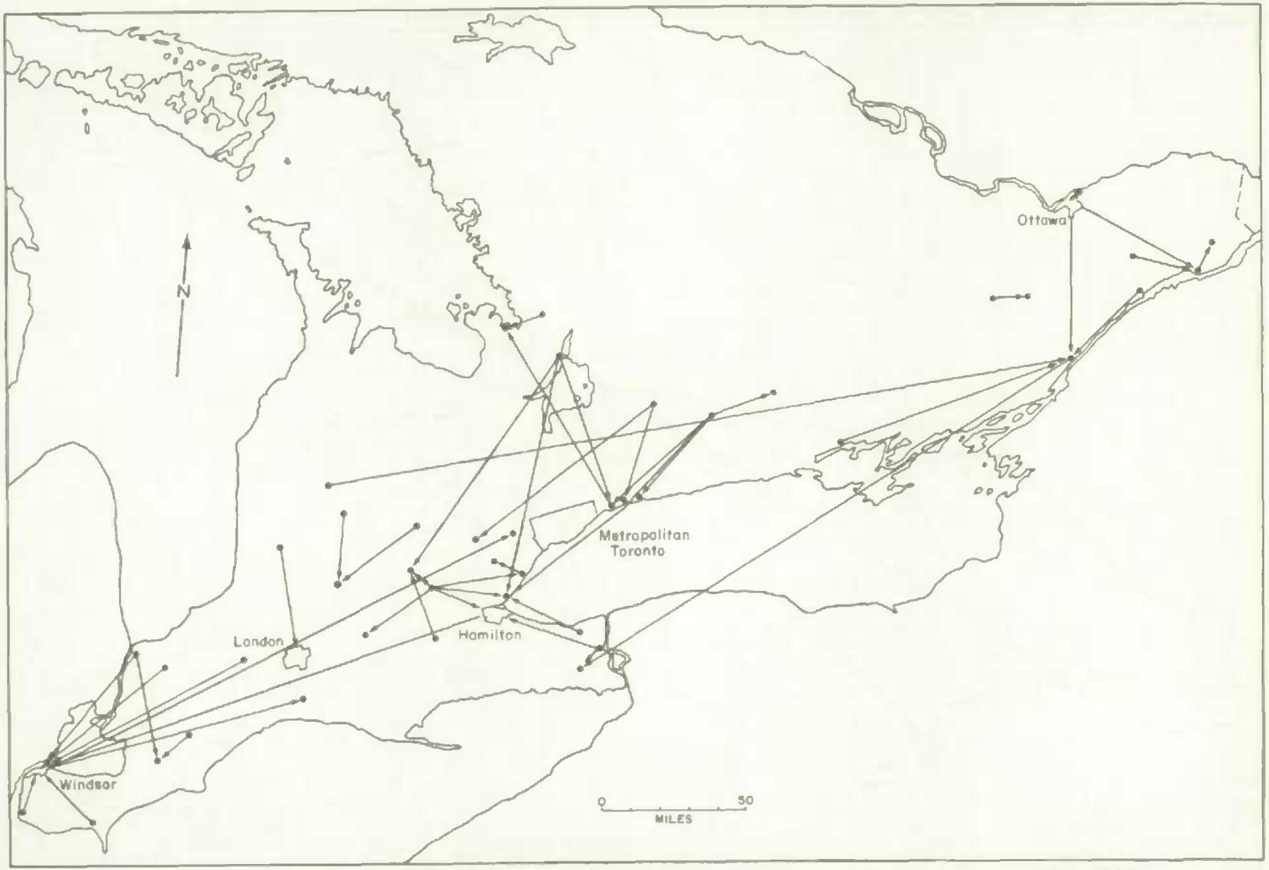
Generalizing, it is seen that the dominant trend in Ontario's manufacturing activity between 1961 and 1965 was towards the development of an industrial doughnut around the traditional centre of manufacturing activity in the city of Toronto. This development was undoubtedly initiated, as observed by Kerr and Spelt in 1958, by the relocation of plants to the suburbs from downtown Toronto where physical limitations to site expansion encourage some firms to establish suburban branch plants. These "migrant" plants helped to "pioneer" suburban locations. This aspect of the suburbanization process was accelerated by the influx of foreign-owned branch plants which tend to favour locations with easy access to



Map 5.5: Destinations of Fifty One Establishments Relocating from Metropolitan Toronto, 1961-1965



Map 5.6: Origins of Thirty Three Establishments Relocating to Metropolitan Toronto from Southern Ontario, 1961-1965



Map 5.7: Origins and Destinations of Forty Six Establishments Relocating in Southern Ontario, 1961-1965

the international airport. The gradual amelioration of the uncertain economic environment in the suburbs subsequently encouraged a rapid rise in the birth rate of indigenous mini-establishments hitherto incubated in the safe and inexpensive atmosphere of the central city. Such plants, having attained a viable operating level and requiring more room for expansion, tend to relocate still further outwards, thereby extending the outer margin of the industrial doughnut. Thus, the "hollowing out" process at the centre of the doughnut has proceeded in both absolute and relative terms.

Around the city the increasing gravitational force of the suburbs has not only attracted plants from the central city but has "pulled in" plants from as far away as Kingston, Ottawa, and Windsor. Although Metropolitan Toronto has also "spun off" plants to other towns in Ontario less than one-third relocated beyond a radius of 50 miles; in contrast, almost 80% of those attracted to Toronto originated beyond this 50 mile zone. In general, those plants which decentralized, and especially those which relocated within the suburbs, were larger than those which centralized and whereas the greater majority of those plants which moved outwards increased their size upon relocating over half of those which moved inwards reduced their number of employees.

The Classification of a System of Spatial States

Recalling the alternative systems of spatial states outlined earlier in this chapter the analyses tend to support the notion of increasing concentration in and around Metropolitan Toronto rather than that of widespread decentralization in southern Ontario. Therefore, a system of states based solely on the concept of "distance bands" radiating outwards from Toronto may not be the most appropriate. Likewise, the analyses indicate no well defined interregional character in the migration process. Clearly discernible, however, has been the tendency for plants to relocate from the city of Toronto to its suburbs and other centres; from the larger cities to their respective suburbs; and from smaller to larger urban centres. Such observations encourage the adoption of a system of states that gives emphasis to the varying degrees of industrial attractiveness exhibited by urban areas rather than to that associated with distance bands or economic regions. Perhaps the best system, therefore, would be one with each state comprising an urban area or its suburbs. But since the number of "migration observations" is comparatively small it is apposite in the present context to limit the size of the system to a small number of states. This requires grouping and the particular system adopted is presented in Table 5.5. The directional probabilities generalize the migration process of the manufacturing establishments which relocated in Ontario between 1961 and 1965. The relatively high diagonal values for the "Toronto suburbs" and the "Rest of Ontario" could be reduced but this would involve disaggregation of the respective states and hence would considerably enlarge the system. When "all establishments" are considered the respective annual transition probabilities for the spatial states assume the values shown in Tables 5.6 - 5.9.

TABLE 5.5. Directional Probabilities of Relocations

	Toronto 1	Toronto suburbs 2	Large urban 3	Large urban Suburbs 4	Small urban 5	Rest of Ontario 6
Toronto 1		0.8631	0.0038		0.0418	0.0912
Toronto suburbs 2	0.0940	0.7351	0.0069		0.0278	0.1358
Large urban 3	0.0434	0.1304	0.0217	0.5652	0.1739	0.0652
Large urban suburbs 4			0.6296	0.2407	0.0740	0.0555
Small urban 5	0.0245	0.1311	0.0245		0.3442	0.4754
Rest of Ontario 6	0.0067	0.0536	0.0100	0.0100	0.3926	0.5268

Note: (Large urban refers to the four cities of Hamilton, Windsor, London, and Ottawa. Small urban refers to all other towns over 10,000 in 1961).

TABLE 5.6. Spatial Matrix, 1961 and 1962

	1	2	3	4	5	6
1	0.9567	0.0385			0.0017	0.0029
2	0.0115	0.9805	0.0008		0.0026	0.0044
3	0.0012		0.9830	0.0072	0.0048	0.0003
4			0.1063	0.8865		0.0070
5	0.0004	0.0019			0.9866	0.0108
6		0.0022		0.0007	0.0477	0.9492

TABLE 5.7. Spatial Matrix, 1962 and 1963

	1	2	3	4	5	6
1	0.9776	0.0193			0.0012	0.0018
2	0.0027	0.9916			0.0009	0.0046
3		0.0012	0.9902	0.0085		
4			0.0387	0.9457	0.0155	
5		0.0024	0.0005		0.9945	0.0024
6	0.0007	0.0023	0.0015		0.0054	0.9899

TABLE 5.8. Spatial Matrix 1963 and 1964

	1	2	3	4	5	6
1	0.9556	0.0366			0.0035	0.0115
2	0.0044	0.9805			0.0035	0.0115
3	0.0012	0.0012	0.9842	0.0084	0.0048	
4			0.0384	0.9384	0.0076	0.01538
5					0.9916	0.00741
6		0.0023		0.0007	0.0162	0.98000

TABLE 5.9. Spatial Matrix, 1964 and 1965

	1	2	3	4	5	6
1	0.9539	0.0401	0.0005		0.0017	0.0035
2	0.0051	0.9801	0.0008			0.0137
3		0.0012	0.9902	0.0073		0.0012
4			0.0671	0.9253	0.0074	
5	0.0009	0.0029	0.0004		0.9876	0.0079
6	0.0007	0.0052	0.0076	0.0007	0.0181	0.9743

All these matrices, as did those for the size distributions developed in Chapter IV, show only the transition probabilities for a constant sample of establishments. The analyses of this chapter, however, have shown that the locational patterns of manufacturing activity are changed not only by a process of relocation but also by a birth/death differential. The Markov models developed in the next chapter combine both processes to estimate changes in the total pattern of Ontario's future manufacturing activity.

CHAPTER VI

THE MARKOV MODEL: DESIGN AND APPLICATION

This chapter has three objectives: first, to **implement** a methodology for adopting a specific-order Markov model; second, to **test** the accuracy of the model in its ability to describe temporal changes in Ontario's manufacturing previously documented; and third, to **improve** the model. These procedures rely in the first instance on the **constant population** of permanent establishments which, it is generally assumed, provides the best "Markov laboratory". Ostensibly, a model's inability to predict changes for a homogeneous sample would render it inadequate as a predictive tool for a fluctuating population characterized by birth and death processes. Thus, the birth and death vectors are added only to the "improved" model.

The methodology corresponds to that outlined in Chapter III and relies on the statistical tests for specific-order properties and for stationarity outlined in Appendix B. These tests, at least for the structural matrices, provide the rationale for adopting a first-order Markov model. Non-parametric tests are used in this chapter to test the accuracy of the model by comparing its **estimates** of the annual distribution vectors with those observed. The frequency estimates are obtained by successively powering the relevant matrices and by post multiplying the initial state probabilities. Since much of the variance of the 1961 - 65 expected vectors is explained by the data from which the initial parameters are estimated, per se, these expected values are not regarded as **predictions**. It must be emphasized that predictions in this case would require "true" Markov transition probabilities contained in a mathematical model and derived theoretically from a probability distribution. The improvement of the model – the chapter's third objective – relies on three concepts: **fractional powers** based on the Binomial theorem, **matrix surfaces**, and the **average matrix**. Birth and death vectors are added to the improved model to provide estimates for 1966 which acts as a test year, and forecasts are projected for annual periods up to 1975.

Rationale for a First-order Markov Model

A null hypothesis is tested to establish whether or not the four independent annual structural matrices possess the Markov property. Formally, it is postulated that the movement of plants from one size category to another is statistically independent against the alternative that the observations are Markovian. The likelihood criterion provides the $-2 \log_e \lambda$ values shown in Table 6.1. Thus clearly, the null hypothesis is rejected and we can consider the structural matrices as realizations of a Markov chain. Using the same procedure the likelihood ratio criterion for the null hypothesis – that the **relocation** of plants from one location to another is statistically independent as against the alternative that the observations are from a Markov chain – provides the following values of the test statistic (Table 6.2). Again, rejection of the null hypothesis justifies the acceptance of the spatial matrices as realizations of a Markov chain.

TABLE 6.1. Test of Markovity for Structural Matrices

Realization	$-2 \log_e \lambda$	D.F. $(n-1)^2$
1961-62	20,444	169
1962-63	21,803	169
1963-64	22,959	169
1964-65	22,520	169

TABLE 6.2 Test of Markovity for Spatial Matrices

Realization	$-2 \log_e \lambda$	D.F. $(n-1)^2$
1961-62	980	25
1962-63	954	25
1963-64	1,026	25
1964-65	973	25

Markovity alone, however, provides no indication of the specific-order of the chain. Such additional information is obtainable only from cubic or three-way tally matrices which are necessary for testing the null hypothesis that the chain is first-order against the alternative that it is second-order. For the structural data four cubic matrices are developed:

(1) 1961 - 63
(2) 1962 - 64
(3) 1963 - 65
(4) 1961 - 65

Tables 6.3 - 6.6 show three "facets" or "leaves" of the first matrix. Table 6.4 (Leaf 6), for example, shows that of the 763 plants recorded for category six in 1963, one plant regressed from category nine in 1961 to category seven in 1962 and continued its decline to category six in 1963. Substitution of all these observations in equation A.2 provides the test statistics of Table 6.7. For all realizations the values of $-2 \log_e \lambda$ are less than the appropriate degrees of freedom. The null hypothesis is not rejected and the change in a plant's employment structure is considered to typify a first-order Markov process. For the spatial matrices, however, the study's relatively short time period precludes the possibility of developing cubic matrices. However, on the basis of the empirical analysis of Chapter V, the first-order property is assumed.

TABLE 6.3. Leaf 2 of 1961 - 63 Cubic Tally Matrix for Permanent Establishments

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	64	75	6											
2	53	428	43	2										
3	2	88	59	3										
4		3	8	3										
5			3	1										
6														
7														
8														
9														
10														
11														
12														
13														
14														

TABLE 6.4. Leaf 6 of 1961 - 63 Cubic Tally Matrix for Permanent Establishments

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1														
2														
3														
4				1	2	2								
5		1	1	4	22	4								
6				3	73	91	5	1						
7		1		2	21	389	24							
8					3	35	42	1						
9							1	4						
10							1							
11														
12														
13														
14														

TABLE 6.5. Leaf 9 of 1961-63 Cubic Tally Matrix for Permanent Establishments

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1														
2														
3														
4				1				1						
5														
6														
7							3	9	6					
8							2	44	59	2				
9								17	229	7				
10									18	15				
11									1	1				
12														
13														
14														

TABLE 6.6. Leaf 12 of 1961-63 Cubic Tally Matrix for Permanent Establishments

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1														
2														
3														
4														
5														
6														
7														
8														
9											1			
10										1	5	1		
11										1	17	16		
12											3	57	6	
13												2	3	
14														

TABLE 6.7. Test of First-order Property for Structural Matrices

Realization	$-2 \log_e \lambda$	D.F. $n(n-1)^2$
1961-63	697.62	2,366
1962-64	631.21	2,366
1963-65	607.78	2,366
1961-65	610.16	2,366

Chapter III emphasized the relevance of determining the homogeneity among serially independent Markov realizations designed for estimating future states of the system. Two series are obtainable for both structural and spatial matrices:

(1) 1961 - 62, 1962 - 63, 1963 - 64, 1964 - 65

(2) 1961 - 63, 1963 - 65

Whether or not the components of each set are from the same but unspecified matrix of transition probabilities is testable by the **minimum discrimination information statistic** – the m.d.i.s. Formally, a null hypothesis is postulated that the four annual and two biennial matrices are realizations from the same sets of transition probabilities. The components due to each information statistic for the structural matrices are shown in Tables 6.8 and 6.9. The (i) component measures the lack of homogeneity among the initial distributions (1961 - 64), and the (j/i) component measures that of the transitions to the final distributions (1962 - 65) from the corresponding initial distributions. The (i,j) component is the sum of these two.

TABLE 6.8. Test of Homogeneity for Four Annual Structural Matrices

Component due to	Information	D.F.
(i) homogeneity	31.36	39
(j/i) conditional homogeneity	384.43	546
(i,j) homogeneity	415.79	585

In both tables the three information components are not significant; this implies that the differences between the various independent realizations within each set are small enough to be attributed to random or chance fluctuations. By statistical inference the two sets of independent realizations are accepted as homogeneous and representing the same first-order Markov chain. A similar conclusion is reached for the spatial matrices, the "information components" of which are shown in Tables 6.10 and 6.11. In these, however, the validity of the results is predicated on the first-order assumption.

TABLE 6.9. Test of Homogeneity for Two Biennial Structural Matrices

Component due to	Information	D.F.
(i) homogeneity	11.77	13
(j/i) conditional homogeneity	143.61	182
(i,j) homogeneity	155.38	195

TABLE 6.10. Test of Homogeneity for Four Annual Spatial Matrices

Component due to	Information	D.F.
(i) homogeneity	12.6	15
(j/i) conditional homogeneity	74.2	90
(j,i) homogeneity	86.8	105

TABLE 6.11. Test of Homogeneity for Two Biennial Spatial Matrices

Component due to	Information	D.F.
(i) homogeneity	3.8	5
(j/i) conditional homogeneity	24.3	30
(i,j) homogeneity	28.1	35

The chapter's first objective is fulfilled; the applications of statistical tests have indicated that the use of a first-order Markov model for predicting future states of manufacturing activity is justified. Such a model is particularly relevant for describing temporal changes among the structural components since their highly transient nature is amenable to tests for specific-order properties as well as the more general properties of Markovity and stationarity. The latter properties are equally tenable for spatial relocations, but the relative "spatial immobility" of manufacturing establishments in the short-term yields insufficient observations for specific-order tests, in which case the first-order property is assumed.

Evaluation of the model's predictive power – the second objective – is based initially on the 1961 - 62 structural matrix. The actual change in the shape of the size distribution for the constant sample that occurred between 1961 and 1965 is shown in Fig. 6.1. In estimating this change, because of random effects, we must expect, as stated by Blumen, Kogan and McCarthy, 1955, p. 54, that ". . . the fit of the model must therefore be determined by examining the magnitude of observed deviations in the light of what one would expect on a chance basis."

Multiplication of the initial distribution vector (1961) with successive powers of the 1961 - 62 matrix yields the estimated distribution vectors shown in Table 6.12 where they are compared with observed frequencies; the respective configurations of the estimated and observed distributions are shown in Fig. 6.2. The degree of conformity of the powered transition probabilities contained in the 1961 - 62 matrix with the observed probabilities for the 1961 - 65 period are shown in Table 6.13.

The initial results indicate the limited ability of transition probabilities estimated from a one-year interval to describe future annual changes. Even for this short time period the computed Chi-square statistic – total 109.15 – used as a measure of the accuracy of the model indicates that the "goodness of fit" is poor. Still more disturbing for predictive purposes is the tendency of the goodness of fit to deteriorate with increasing time. Chi-square statistics for the other three annual matrices show the same trends except for the fact that the diagonal element is lowest in each case (Table 6.14).

Although these results illustrate the short-comings of applying a Markov model to one specific temporal matrix the information contained in Table 6.13 indicates that the patterns of origin and destination do seem to maintain approximately the same relative positions in both observed and expected matrices and suggests that "something regular" is occurring in the observed structural changes. Accordingly, alternatives for improving the predictive capacity of the model are now examined.

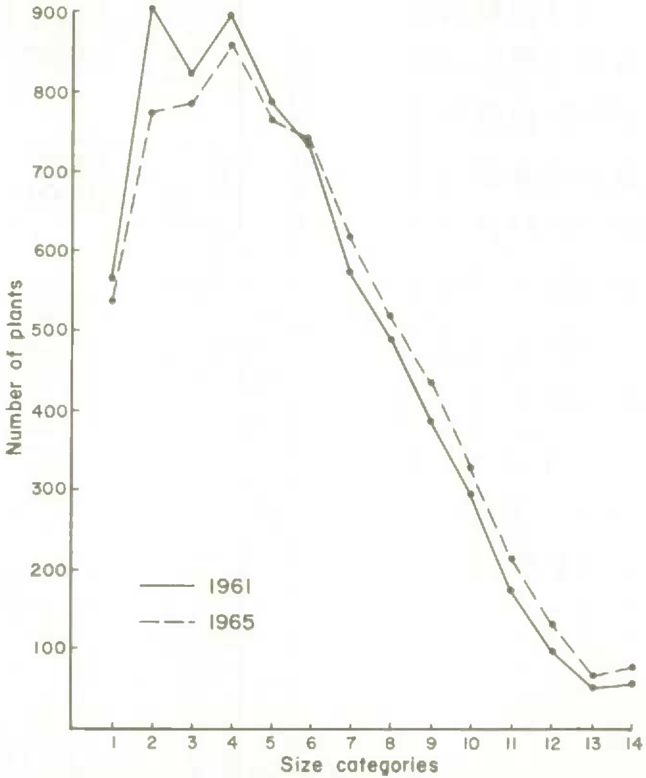


Figure 6.1: Observed Distributions for Permanent Establishments, 1961 and 1965

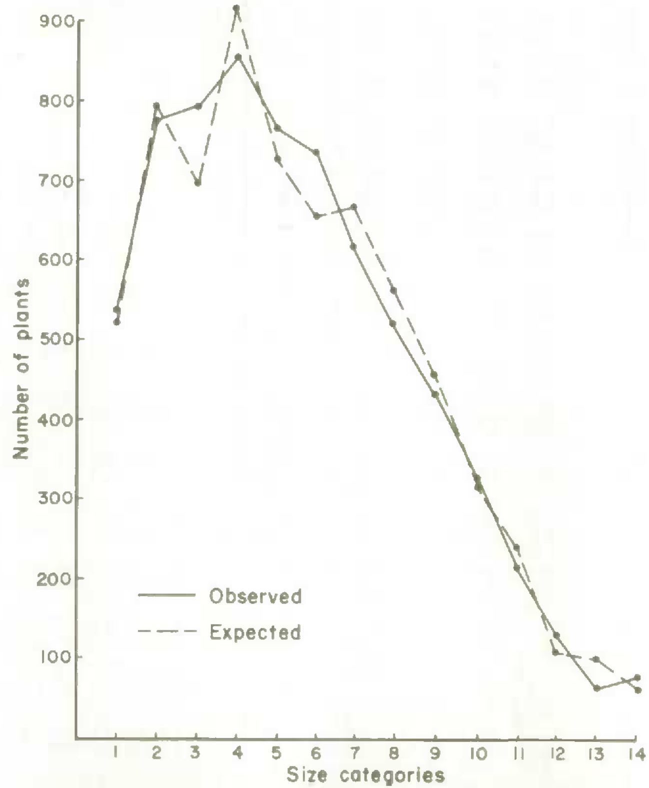


Figure 6.2: Estimated Distribution from 1961-1962 Structural Matrix and Observed Distribution (1965) for Permanent Establishments

TABLE 6.12. Estimated Distribution Vectors 1962 - 65 from 1961 - 62 Structural Probability Matrix for Permanent Establishments

	1	2	3	4	5	8	7	8	9	10	11	12	13	14
1962:														
Observed....	568.0	873.0	770.0	927.0	763.0	702.0	615.0	503.0	404.0	294.0	194.0	97.0	67.0	58.0
Expected....	568.0	873.0	770.0	927.0	763.0	702.0	615.0	503.0	404.0	294.0	194.0	97.0	67.0	58.0
	Chi - square = 0.00													
1963:														
Observed....	554.0	846.0	771.0	908.0	748.0	734.0	609.0	501.0	415.0	321.0	187.0	113.0	63.0	65.0
Expected....	555.2	843.5	739.0	933.0	748.8	680.8	640.5	523.1	421.9	300.2	212.4	99.6	78.8	58.1
	Chi - square = 19.09													
1964:														
Observed....	525.0	819.0	771.0	888.0	777.0	718.0	608.0	528.0	416.0	327.0	202.0	120.0	67.0	71.0
Expecled....	541.2	816.2	716.1	927.2	738.1	666.1	656.7	543.4	439.5	309.5	228.0	104.6	89.4	59.2
	Chi - square = 32.27													
1965:														
Observed....	542.0	773.0	779.0	858.0	787.0	738.0	612.0	517.0	433.0	327.0	213.0	130.0	68.0	78.0
Expected....	526.6	791.2	696.9	915.7	728.2	655.5	687.6	562.2	457.0	320.8	242.2	110.9	99.5	61.0
	Chi - square = 57.79													

TABLE 6.13. Fourth Power of 1961 - 62 Matrix (Expected) and 1961 - 65 Matrix (Observed)

		1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	Observed....	.5820	.2832	.0950	.0345	.0034	.0017								
	Expected....	.4009	.3529	.1475	.0700	.0213	.0040	.0021	.0012	.0002					
2	Observed....	.1984	.4723	.2239	.0754	.0188	.0089	.0011	.0011						
	Expected....	.2280	.3521	.2191	.1354	.0457	.0102	.0053	.0032	.0008	.0002				
3	Observed....	.0219	.1876	.4300	.2777	.0633	.0146	.0024	.0024						
	Expected....	.0811	.2138	.2558	.2732	.1236	.0341	.0129	.0047	.0007	.0001				
4	Observed....	.0090	.0302	.1590	.4502	.2553	.0717	.0168	.0034	.0045					
	Expected....	.0201	.0794	.1600	.3415	.2438	.1002	.0386	.0131	.0028	.0005	.0001			
5	Observed....		.0025	.0228	.1457	.4613	.2826	.0710	.0127	.0013					
	Expected....	.0041	.0220	.0602	.2189	.2913	.2345	.1197	.0392	.0084	.0013	.0002			
6	Observed....			.0068	.0205	.1216	.4686	.2992	.0697	.0137					
	Expected....	.0012	.0081	.0141	.0609	.1412	.3282	.2849	.1236	.0327	.0059	.0010			
7	Observed....			.0052	.0122	.0174	.1222	.4345	.3211	.0628	.0209	.0035			
	Expected....	.0001	.0009	.0027	.0091	.0329	.1368	.3616	.2942	.1219	.0308	.0064	.0004		
8	Observed....			.0020	.0020	.0102	.0287	.1291	.4631	.2807	.0676	.0184			
	Expected....	.0002	.0010	.0029	.0023	.0063	.0385	.1727	.3513	.2790	.1093	.0324	.0036	.0006	
9	Observed....						.0026	.0156	.0963	.5417	.2813	.0547	.0052	.0026	
	Expected....		.0001	.0008	.0002	.0007	.0061	.0437	.1691	.3956	.2585	.1065	.0159	.0029	.0002
10	Observed....				.0034			.0034	.0102	.1194	.5427	.2730	.0478		
	Expected....			.0001			.0007	.0075	.0442	.1882	.3739	.2933	.0721	.0183	.0016
11	Observed....				.0059		.0116			.0118	.0765	.5294	.3000	.0586	.0059
	Expected....					.0001	.0006	.0068	.0170	.0489	.1728	.4319	.2142	.0946	.0131
12	Observed....										.0202	.1212	.5859	.2121	.0606
	Expected....				.0001	.0004	.0028	.0163	.0088	.0107	.0508	.1942	.3356	.3112	.0689
13	Observed....												.0943	.5849	.3207
	Expected....						.0002	.0027	.0006	.0006	.0051	.0312	.1561	.5943	.2092
14	Observed....													.0847	.9152
	Expected....						.0001	.0008	.0002	.0001	.0015	.0095	.0632	.2419	.6827

TABLE 6.14. Chi-square Statistics for Four Annual Structural Matrices, 1961 - 65

Transition matrix	Chi-square values for 13 D.F.			
	1962	1963	1964	1965
1961 - 62	0.0	19.09	32.27	57.79
1962 - 63	20.14	7.88	20.04	21.12
1963 - 64	14.11	14.52	6.60	18.77
1964 - 65	20.79	18.25	29.58	15.50

The iterative procedure used to delimit the initial size categories in Chapter IV indicates the potential for rearranging the frequency limits and subsequent redefinition of the states. Clark, for example, has suggested that "... experimentation should yield the most useful classes of states." This suggestion, however, carries the undesirable implication that the results justify the means, though it may be the only alternative where the data are so inadequate that the underlying Markov assumptions cannot be tested. But since these assumptions for the structural matrices have been statistically verified the central problem is not one of selecting the best classification scheme to enable a reasonably simple model to fit the data, but is one of developing an adequate model for the accepted set of frequencies. Most other Markov applications which have recorded satisfactory results have used smaller matrices than those adopted for this study. The Cornell model (Blumen, Kogan, and McCarthy, 1955) – the most comprehensive – used ten states but generally the number has been smaller than seven. Gale, for example, used three and five states. A tempting alternative, therefore, for improving the existing model's accuracy is the reduction of the number of states through "lumping" adjacent states into a 7 x 7 matrix. This is analogous to the procedure outlined in Chapter III.

The "lumped" seven state probability matrix for the 1964 - 65 transition matrix is given in Table 6.15. As expected, the increased "width" of the frequency intervals creates a "heavier" diagonal. Successive powering and multiplication of the matrix by the initial vector of state probabilities provide the estimated distribution vectors shown in Table 6.16.

The model's increased capacity to estimate structural changes is clearly evident and the Chi-square statistics suggest that the model is "acceptable". Clearly, the smaller the matrix the more accurate is its predictive capacity. Weighed against the improvement, however, is the marked loss of valuable information which, in the present study, is particularly important because of the large variance within the size intervals. Since the much lower "within group" variance of the 14 x 14 matrix provides more accurate information, alternatives for improving the larger model are examined below.

TABLE 6.15. Lumped Seven State Structural Probability Matrix 1964 - 65 for Permanent Establishments

	1	2	3	4	5	6	7
1	.8936	.1049	.0015				
2	.0687	.8373	.0922	.0018			
3		.0676	.8515	.0803	.0007		
4		.0044	.0653	.8580	.0714		
5		.0013	.0027	.0431	.8923	.0606	
6			.0031	.0031	.0466	.9068	.0404
7						.0435	.9565

TABLE 6.16. Estimated Vectors from Seven State 1964 - 65 Matrix and Observed Distributions, 1961 - 65

1962:							
Observed	1,441.0	1,697.0	1,405.0	1,118.0	698.0	291.0	125.0
Expected	1,441.2	1,698.8	1,527.3	1,065.5	693.4	289.8	118.9
	Chi-square = 5.47						
1963:							
Observed	1,400.0	1,679.0	1,482.0	1,110.0	736.0	300.0	128.0
Expected	1,404.6	1,682.3	1,531.6	1,070.7	709.4	310.0	126.4
	Chi-square = 4.41						
1964:							
Observed	1,344.0	1,669.0	1,495.0	1,134.0	743.0	322.0	138.0
Expected	1,370.8	1,665.0	1,534.2	1,076.2	725.0	329.6	134.4
	Chi-square = 5.35						
1965:							
Observed	1,315.0	1,637.0	1,505.0	1,129.0	760.0	343.0	146.0
Expected	1,339.3	1,647.2	1,535.2	1,081.8	740.2	348.6	142.8
	Chi-square = 3.85						

Smoothing Surfaces of Probability Matrices

Whereas the 7 x 7 matrix possesses a continuously trending probability distribution among the off-diagonals the 14 x 14 annual matrices do not. The discontinuities appear as small, unstable elements in the matrix representing an incomplete estimation of the underlying fixed probabilities. Given a system of size categories and given that some plants will move one, two, or four size categories in any one time period, it is reasonable to assume that there is an underlying probability of a plant's moving three categories. Acceptance of this assumption underlies the general assumption of a continuous growth process basic to economic theory. Using the movement of plants from category two in the 1961 - 62 matrix (Table 4.9) as an example, there are probabilities of 0.0055 for a plant's moving to state five, 0.0011 for transition to state seven, and 0.0011 for transition to state eight; but there is an estimated probability of 0.0 for movement to state six. Such discontinuities in the initial transition probabilities bounded by empirically derived upper and lower limits are not consistent with the theoretical expectations of the structural mobility of manufacturing establishments.

The existence of such limits can be given a theoretical interpretation. Consider, for example, the mechanism of structural mobility. The size of a plant at any point in time represents its position on a continuum extending from "zero employment" to some fixed upper limit. Theoretically, a plant can increase its size infinitely but operationally an upper limit does exist. Since transition matrices illustrate the varying ability of plants to move along the continuum it is reasonable to assume, in view of the short time intervals, that the extreme upper "off-diagonals" typify the furthest extent to which plants can expand. But is it reasonable to make a similar assumption for the lower diagonals? On the lower end of the structural spectrum the death of a plant is the limiting category. A plant may reduce its employment for either of two main reasons: increased mechanization and subsequent reduction of the labour coefficient, or a decline in demand resulting in lower production. Thus, the extreme lower diagonals represent limits to which a plant will reduce its employment for either of these reasons. However, there is a minimal point to which a plant, no matter what its size, can reduce its operations before going out of business. A plant employing 200 people, for example, forced to reduce its payroll would perhaps require an absolute minimum of 50 employees to continue its operations. These minimums represented by the lower diagonals explain the lack of a continuous probability distribution to "zero employment" from size categories above a certain size (see Tables 4.9 - 4.12).

The provision of a continuous probability distribution around the main diagonal suggests a "smoothing" process which can be accommodated with a matrix surface. This notion of blanketing a transition matrix with a smoothing surface is not entirely new. Tobler, 1967, p. 275, for example, has suggested that the contouring of a matrix ". . . might lead to a conventional isarithmic map." To date there has been no application of the technique to a Markovian framework and its usefulness is explored here as one alternative for improving the fit of the Markov model.

Although any polynomial surface can be fitted in theory the specific form of the surface will depend on the underlying processes. It should be noted, however, that the concept is only appropriate under conditions of theoretically continuous distributions – as exist for the structural transition matrices. The fitting of a surface, for example, will smooth out irregularities and will "fill" gaps in the distribution. The analyses of Chapter IV and particularly the graphic results of Fig. 4.10 demonstrated that the transition probabilities across the rows assume an almost normal distribution, the variance of which decreases systematically with increasing size. In this case, a normal surface, but one that is modified to accommodate the "taper" effect along the diagonal, seems appropriate; to incorporate this "shifting mean" a three parameter model is minimal. The normal probability density function, using only the mean and variance is given by:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left[-\tfrac{1}{2}(x\text{-}m)^2/\sigma^2\right] \quad -\infty < X < \infty \qquad (6.1)$$

where m is the mean of the distribution and the variance is σ^2. Modifying this the required probabilities – the p_{ij}'s – are given by:

$$p_{ij} = C_i \exp\left[(j - i - T_1)^2 / (T_2 - iT_3)\right] \qquad (6.2)$$

where the C_i are normalizing constants, T_1, T_2 and T_3 are fitted parameters. The search procedure for fitting the surface is applied iteratively using several starting points and the solution terminates with the "best fit" measured in terms of the minimum average absolute deviation (m.a.a.d.). This procedure is analogous to that used for deriving least squares estimates of the transition probabilities, (Ashar and Wallace, 1963). The average absolute error is used to give greater weights to the off-diagonal probabilities; the alternative of a "**weighted average**" in which the squares of the errors are summed, would lessen the importance of the smaller probabilities in influencing the slope of the surface which would tend to "fall off" more readily from the "crest" along the main diagonal. A normal surface fitted to the 1961 - 62 matrix with a m.a.a.d. of 0.0062 is shown in Table 6.17. It should be noted that this procedure involves the fitting of a **surface** to the whole matrix and is not merely a process of fitting a series of normal distribution curves across individual rows. This is apparent from the smoothness of the main diagonal as opposed to the irregular diagonals of the annual matrices. Each annual matrix was fitted with a surface which was successively powered and multiplied by the initial vector of state probabilities. In each case the total value of Chi-square for the four estimated vectors was approximately half of the respective totals derived for the observed annual probabilities; the total Chi-square value for the 1961 - 62 matrix, for example,

TABLE 6.17. Matrix Surface of 1961 - 62 Transition Probabilities

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.7765	.2200	.0035											
2	.1323	.6773	.1876	.0028										
3	.0013	.1294	.6821	.1647	.0026									
4		.0011	.1266	.6881	.1819	.0023								
5			.0010	.1237	.6942	.1790	.0021							
6				.0009	.1208	.7005	.1760	.0019						
7					.0008	.1178	.7069	.1729	.0017					
8						.0007	.1147	.7134	.1696	.0015				
9							.0006	.1116	.7201	.1663	.0014			
10								.0005	.1084	.7270	.1629	.0012		
11									.0005	.1051	.7340	.1594	.0011	
12										.0004	.1018	.7412	.1557	.0009
13											.0003	.0985	.7491	.1521
14												.0003	.1115	.8881

was reduced from 109.15 to 63.6. The four estimated distribution vectors derived from the 1961 - 62 surface matrix are shown in Table 6.18 and the 1965 estimated distribution is compared graphically with the observed 1965 distribution in Fig. 6.3. The estimated surface configuration is clearly superior to that derived from the observed 1961 - 62 probabilities. But the most significant result of this approach – as evidenced by a comparison of Table 6.12 and the respective Chi-square values of Table 6.18 – is the model's **increased stability.**

TABLE 6.18. Estimated Distribution Vectors 1962 - 65 from 1961 - 62 Structural Surface Matrix for Permanent Establishments

	1	2	3	4	5	6	7
1962:							
Observed	568.0	873.0	770.0	927.0	763.0	702.0	615.0
Expected	569.9	845.5	845.2	866.9	801.1	723.9	591.7
	Chi-square = 17.73						
1963:							
Observed	554.0	846.0	771.0	908.0	748.0	734.0	609.0
Expected	555.4	808.3	847.7	854.7	803.9	722.5	604.0
	Chi-square = 19.23						
1964:							
Observed	525.0	819.0	771.0	888.0	777.0	718.0	606.0
Expected	539.2	780.2	840.9	847.0	803.4	723.4	613.1
	Chi-square = 15.52						
1965:							
Observed	542.0	773.0	779.0	858.0	767.0	738.0	612.0
Expected	522.9	756.8	830.0	840.4	801.8	725.1	620.7
	Chi-square = 12.18						

	8	9	10	11	12	13	14
1962:							
Observed	503.0	404.0	294.0	194.0	97.0	67.0	58.0
Expected	491.6	392.1	295.5	183.1	106.1	61.9	60.6
1963:							
Observed	501.0	415.0	321.0	187.0	113.0	63.0	65.0
Expected	498.3	398.9	300.1	193.9	114.3	69.8	63.3
1964:							
Observed	528.0	416.0	327.0	202.0	120.0	67.0	71.0
Expected	505.9	405.4	305.7	203.4	122.9	77.4	66.9
1965:							
Observed	517.0	433.0	327.0	213.0	130.0	68.0	78.0
Expected	513.7	412.1	311.9	212.2	131.5	84.8	71.3

Fractional Disaggregation of Long-period Matrices

A second alternative for improving the accuracy of the model arises from the notion of generalizing into an "average annual matrix" the compounded information contained in the 1961 - 65 matrix (Table 4.8). The most obvious way involves computation of the appropriate **fractional power** of the matrix to provide representative mean annual transition probabilities which, when subjected to the usual Markov procedure, would provide annual estimates of the distribution vectors.

One approach to this problem is to calculate the appropriate root of the matrix by Newtonian approximation.

Let
$$(X_i + D_i)^n = P \tag{6.3}$$
where X_i is the i^{th} approximation to the n^{th} root of P and D_i is unknown.

Then
$$(X_i + D_i)^n \cong X_i^n + nX_i^{n-1}D_i \cong P \tag{6.4}$$
and
$$D_i \cong \frac{1}{n}\left[(X_i^{n-1})^{-1} P - X_i\right] = D_i^* \tag{6.5}$$
leading to
$$X_{i+1} = X_i + D_i^* = \frac{1}{n}(X_i^{n-1})^{-1}P + \frac{n-1}{n}X_i \tag{6.6}$$
Applied to matrices, this technique has uncertain convergence behavior. Preliminary experimentation with this solution gave satisfactory results for small matrices but for the larger 14 x 14 probability matrices too many non-admissible estimates of the transition probabilities were obtained.

Waugh and Abel, 1967, suggest a different approach, involving less computational effort, which is particularly suitable for transition matrices having dominant diagonal terms.

Let $B = cP - I$ and $r = 1/n$, with c an arbitrary scalar.

Then
$$P^r = c^{-r}(cP)^r = c^{-r}[I + (cP\text{-}I)]^r = c^{-r}(I + B)^r \tag{6.7}$$
By formal expansion
$$[I + B]^r = I + rB + \frac{r(r-1)}{2!}B^2 + \frac{r(r-1)(r-2)}{3!}B^3 + \ldots \tag{6.8}$$
which holds if the series converges; the series will converge if $B^k \to 0$ as k increases, and this will necessarily occur if the dominant characteristic root of the matrix is less than unity. To improve the convergence behavior, Waugh and Abel suggest taking
$$c = \sum_i b_{ii}/\sum_i\sum_j b_{ij}^2. \tag{6.9}$$

Good results were obtained when the formulation was tested with a 7 x 7 lumped matrix. But when applied to the 1961 - 65 matrix, even under programmed conditions of double precision, a few small negative values appeared. These were set to zero and the adjusted fourth root of the 1961 - 65 matrix derived from 17

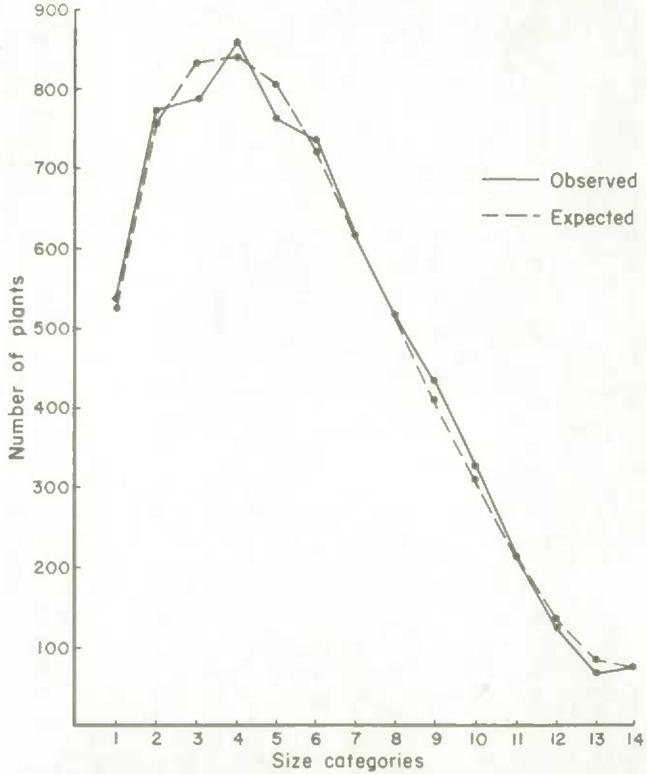


Figure 6.3: Estimated Distribution from 1961-1962 Structural Surface Matrix and Observed Distribution (1965) for Permanent Establishments

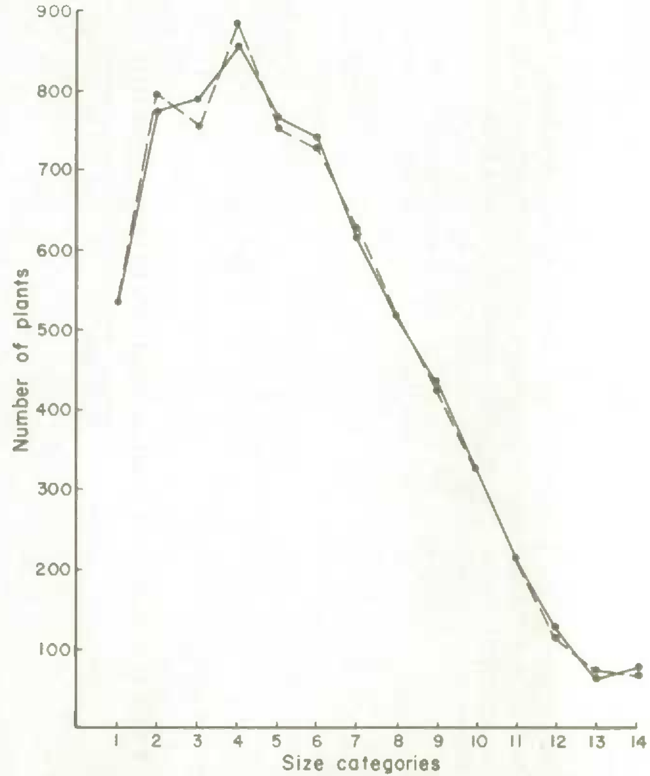


Figure 6.4: Estimated Distribution from Average Structural Matrix and Observed Distribution (1965) for Permanent Establishments

iterations is shown in Table 6.19. Estimated distribution vectors, derived from multiplication of successive powers of the fourth-root matrix, are shown in Table 6.20. The discrepancy of the 1965 Chi-square value which should be zero is attributable to the slight inaccuracy of the root. Even so, the total Chi-square value (29.3) of the four annual estimates produced by the fourth root matrix is a further improvement on the observed annual matrices and the surfaces fitted to these matrices.

The fractional disaggregation of "long-period" matrices has particular importance for future applications of Markov chain analysis especially where serial realizations are not available and where it is desirable to provide short- or medium-term forecasts. If the matrix is for a ten-year interval, for example, it can provide only decennial estimates, the value of which, in view of the "stretched" assumption of stationarity, is questionable. But decomposition of the ten-year span into some average shorter period matrix would yield, for some processes, more credible predictions for three, four, or five-year intervals.

TABLE 6.19. Fourth Root of 1961 - 65 Probability Matrix

	1	2	3	4	5	6	7
1	.8566	.1196	.0187	.0051	.0002	.0002	
2	.0868	.7916	.1071	.0076	.0046	.0022	
3		.0946	.7704	.1391		.0026	
4	.0038	.0002	.0803	.7829	.1242	.0032	.0042
5			.0002	.0707	.7946	.1342	
6	.0001		.0023	.0004	.0575	.7985	.1379
7			.0014	.0052		.0577	.7778
8			.0008		.0040	.0061	.0612
9						.0005	.0020
10				.0010	.0001		.0004
11				.0029		.0025	
12			.0001		.0002		.0002
13						.0001	
14							

	8	9	10	11	12	13	14
1							
2							
3	.0013		.0002				
4		.0010		.0001			
5	.0005		.0007				
6		.0028		.0006			
7	.1597		.0012		.0002		
8	.7978	.1245	.0027	.0026			
9	.0419	.8393	.1157		.0010	.0003	
10	.0004	.0465	.8425	.1112			
11	.0001	.0024	.0300	.8366	.1206	.0050	
12			.0041	.0481	.8600	.0806	.0066
13		.0001			.0339	.86 3	.1018
14						.0255	.9742

TABLE 6.20. Estimated Distribution Vectors 1962 - 65 from Fourth Root of 1961 - 65 Structural Probability Matrix for Permanent Establishments

	1	2	3	4	5	6	7
1962:							
Observed	568.0	873.0	770.0	927.0	763.0	702.0	615.0
Expected	577.7	861.1	814.8	883.0	786.2	734.2	581.2
	Chi-square = 13.73						
1963:							
Observed	554.0	846.0	771.0	908.0	748.0	734.0	609.0
Expected	573.0	828.0	804.8	873.9	782.8	736.1	588.6
	Chi-square = 7.35						
1964:							
Observed	525.0	819.0	771.0	888.0	777.0	718.0	606.0
Expected	566.1	800.2	792.8	865.0	778.9	737.5	595.2
	Chi-square = 6.85						
1965:							
Observed	542.0	773.0	779.0	858.0	767.0	738.0	612.0
Expected	557.7	776.3	779.7	855.8	774.7	738.4	601.1
	Chi-square = 1.52						

	8	9	10	11	12	13	14
1962:							
Observed	503.0	404.0	294.0	194.0	97.0	67.0	58.0
Expected	498.5	400.1	299.6	181.4	108.0	56.3	63.5
1963:							
Observed	501.0	415.0	321.0	187.0	113.0	63.0	65.0
Expected	508.9	415.2	307.4	192.1	117.2	60.0	68.3
1964:							
Observed	528.0	416.0	327.0	202.0	120.0	67.0	71.0
Expected	519.0	429.6	316.1	202.4	126.5	64.2	73.5
1965:							
Observed	517.0	433.0	327.0	213.0	130.0	68.0	78.0
Expected	528.7	443.3	325.5-	212.4	135.9	68.7	78.9

The Average Matrix

Another alternative for improving the accuracy of the model is to determine the "average" matrix. Where several realizations of a Markov chain are available an average matrix can be computed by summing the elements of the original tally matrices and re-estimating the transition probabilities. Such a process, Anderson, 1954, shows, would **compound** the information of several matrices. In this respect the authors of the Cornell study (Blumen, Kogan and McCarthy, 1955, p. 156) have commented that Anderson's proofs involve:

> . . . more than a trivial application of the usual pooling of observations, since we are dealing with dependent observations . . . and there is some justifiable doubt that we are adding much information with each new matrix of observations.

Anderson's proofs show that the traditional statistical techniques of averaging are appropriate to Markov processes. Using this procedure, the average transition probabilities are given by:

$$p_{ij} = \sum_k f_{kij} / \sum_k \sum_j f_{kij} \tag{6.10}$$

and the average matrix for the 1961 - 65 period shown in Table 6.21. Application of the usual Markov procedure provides the estimated distribution vectors in Table 6.22 and the 1965 estimated vector is compared with the observed vector in Fig. 6.4. Clearly, the accuracy of the model is considerably improved not only in terms of total Chi-square (15.7) but also with respect to its marked stability.

TABLE 6.21. Average Structural Matrix for Permanent Establishments, 1961 - 65

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	.7871	.1887	.0189	.0049	.0005									
2	.1189	.7000	.1602	.0172	.0026	.0006	.0003	.0003						
3	.0080	.1411	.6645	.1714	.0118	.0025	.0006							
4	.0008	.0097	.1095	.7041	.1623	.0094	.0027	.0011	.0003					
5		.0009	.0049	.1287	.6874	.1641	.0120	.0019						
6		.0010	.0007	.0090	.0974	.7252	.1577	.0063	.0007					
7			.0008	.0017	.0087	.0936	.7187	.1672	.0079	.0008			.0004	
8			.0005	.0005	.0020	.0109	.0995	.7327	.1441	.0074	.0020			.0005
9				.0006		.0012	.0037	.0772	.7659	.1452	.0062			
10							.0008	.0032	.0907	.7733	.1296	.0024		
11						.0013	.0013	.0027	.0040	.0810	.7703	.1355	.0040	
12							.0023			.0023	.0979	.7716	.1166	.0093
13								.0040				.0840	.8080	.1040
14												.0119	.035	.9526

TABLE 6.22. Estimated Distribution Vectors 1962-65 from Average Structural Probability Matrix for Permanent Establishments

	1	2	3	4	5	6	7
1962:							
Observed	568.0	873.0	770.0	927.0	763.0	702.0	615.0
Expected	570.2	866.6	803.8	897.4	776.9	731.0	590.6
	Chi-square = 8.75						
1963:							
Observed	554.0	846.0	771.0	908.0	748.0	734.0	609.0
Expected	559.0	837.9	787.1	895.3	769.0	730.1	603.6
	Chi-square = 2.10						
1964:							
Observed	525.0	819.0	771.0	888.0	777.0	718.0	606.0
Expected	546.6	813.2	770.8	889.5	763.0	729.3	613.6
	Chi-square = 2.29						
1965:							
Observed	542.0	773.0	779.0	858.0	767.0	738.0	612.0
Expected	533.8	791.2	755.2	881.3	757.7	728.8	621.5
	Chi-square = 2.58						

	8	9	10	11	12	13	14
1962:							
Observed	503.0	404.0	294.0	194.0	97.0	67.0	58.0
Expected	493.5	396.9	300.4	181.9	105.3	57.4	62.9
1963:							
Observed	501.0	415.0	321.0	187.0	113.0	63.0	65.0
Expected	501.6	408.5	309.1	192.8	112.2	61.8	67.1
1964:							
Observed	528.0	416.0	327.0	202.0	120.0	67.0	71.0
Expected	510.6	419.5	318.4	203.0	119.4	66.4	71.6
1965:							
Observed	517.0	433.0	327.0	213.0	130.0	68.0	78.0
Expected	519.7	430.1	328.1	212.9	126.8	71.2	76.5

Examination of both the fourth-root matrix and the average matrix shows that in each case there is still evidence of small, unstable elements in the off-diagonals. By combining the notions of matrix surfaces and average matrices it was hoped that the predictive accuracy of the model could be improved still further. A surface fitted to the fourth-root matrix considerably improved this model and gave a total Chi-square value of 18.4, but the surface fitted to the average matrix with a m.a.a.d. of 0.002 gave a total Chi-square value of 16.3. Two additional parameters were added to the surface equation 6.2 in an attempt to accommodate the "shallower" slope of the off-diagonal values in the average matrix. Although this detracts from the elegant simplicity of the three parameter model, the closer fitting surface – m.a.a.d. of 0.001 – made no further improvements to the model's accuracy.

A Markov Model with Birth and Death Processes

The above findings encourage the adoption of the average matrix concept for further experimentation with the model which is designed to provide short and medium term forecasts for a fluctuating population of establishments. Such a model must allow for entry into and exit from the system. In his study Gale accommodated the birth and death process by using two diagonal matrices: an attrition matrix which removes people from the system and a birth matrix which adds people to the system. Adelman suggests a much simpler approach: the addition of an extra state S_0, so that the row S_0 represents deaths.

In the structural matrix for those plants which ceased operations between 1961 and 1965 the average probabilities of their deaths occurring in the respective 14 size categories can be represented by the following column vector of death probabilities:

$$d = (.27386,\ .19417,\ .14486,\ .14401,\ .08576,\ .06634,\ .05016,\ .03559,\ .02427,\ .00647,\ .00323,\ .00161,\ .00161,\ .00080)$$

In order to combine this vector with the average transition matrix a new average tally matrix for "all establishments" is computed. This is achieved by reconverting the average probability matrix into a tally matrix on the basis of the average number of all establishments in each state. The numerical values of the death vector are then arranged alongside the S_1 column of the new 14 x 14 average tally matrix for all establishments and the row probabilities of the 14 x 15 transition matrix are re-estimated.

The initial row vector of 14 average birth probabilities is:

$$b = (.24930,\ .22160,\ .15927,\ .13296,\ .09002,\ .05678,\ .04016,\ .02077,\ .01662,\ .00554,\ .00277,\ .00104,\ .00138,\ .00173)$$

The main difficulty in this case is one of assigning a value to the element S_{00} which acts as an initial reservoir of potential entrants who may or may not enter the system through S_0. Once assigned, the value of this reservoir is summed algebraically with the elements of the "tally vector" for births and a new row vector of birth probabilities is computed. Clearly, the size of the reservoir will affect the values of the birth probabilities. For the reservoir in her study, Adelman, 1958, p. 899, uses an arbitrarily large number – 100,000 – which she argues does "... not affect the economically relevant portion of our results." But as indicated in Chapter III Adelman's, 1958, p. 901, proof of this statement applies only to the equilibrium state. Reservoirs ranging from 10,000 to 900,000 were tested and noticeable differences were recorded among the lower size categories for the first four estimated annual distribution vectors; after five years the differences became less noticeable and after ten years were very small. The best results for the first five annual distributions were obtained from a reservoir of 900,000 which, for the short period estimates, stands as an **empirically derived parameter** and assumes an analogous role to the empirically derived exponent of the gravity concept. (For a recent discussion see Houston, 1969.) For the longer period estimates any "large" reservoir provides essentially the same results. The combined average birth and death Markov matrix is shown in Table 6.23 and the annual estimated distribution vectors are presented in Table 6.24 which gives also the results for the test prediction year of 1966.

Although the accuracy of the model in terms of Chi-square is not as good for the fluctuating population as it is for the constant sample, the general fit for 1966 (Fig. 6.5) is far better than that derived from any one annual matrix for the constant sample. Undoubtedly, if the transition probabilities had been estimated directly from all establishments instead of being interpolated from the constant sample the accuracy of the model for this experiment would be enhanced. There can be no doubt, however, that the Markov procedure is a viable mechanism for analysing and estimating the structural dynamics of a population of establishments.

TABLE 6.23. Average Structural Matrix for All Establishements with Birth and Death Vectors

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	.9988	.0003	.0002	.0002	.0002	.0001									
1	.2089	.6226	.1493	.0149	.0039	.0004									
2	.1225	.1043	.6143	.1405	.0151	.0023	.0005	.0003	.0003						
3	.1051	.0071	.1263	.5947	.1534	.0106	.0023	.0006							
4	.0896	.0008	.0088	.0997	.6409	.1478	.0086	.0025	.0010	.0003					
5	.0645		.0009	.0045	.1204	.6431	.1535	.0113	.0018						
6	.0575		.0010	.0007	.0085	.0918	.6835	.1486	.0078	.0007					
7	.0491			.0008	.0016	.0083	.0890	.6834	.1591	.0075	.0008				
8	.0418			.0005	.0005	.0019	.0104	.0954	.7021	.1381	.0071	.0019			.0005
9	.0334				.0006		.0012	.0036	.0746	.7403	.1403	.0060			
10	.0128							.0008	.0032	.0895	.7634	.1279	.0024		
11	.0105						.0013	.0013	.0026	.0039	.0802	.7622	.1340	.0039	
12	.0092							.0023			.0023	.0970	.7644	.1155	.0092
13	.0158								.0039				.0827	.7953	.1024
14													.0119	.0356	.9526

TABLE 6.24. Estimated Distribution Vectors 1962 - 66 from Average Structural Probability Matrix with Birth and Death Vectors for All Establishments with Two or More Employees

	1	2	3	4	5	6	7
1962:							
Expected	1,373	1,539	1,300	1,346	1,096	972	781
	Chi-square = 38.9						
1963:							
Expected	1,296	1,571	1,322	1,377	1,118	988	803
	Chi-square = 52.2						
1964:							
Expected	1,251	1,582	1,341	1,403	1,138	1,005	822
	Chi-square = 51.4						
1965:							
Expected	1,224	1,585	1,356	1,425	1,157	1,021	839
	Chi-square = 56.6						
1966:							
Expected	1,208	1,587	1,365	1,444	1,173	1,037	854
	Chi-square = 45.5						

	8	9	10	11	12	13	14
1962:							
Expected	629	486	346	199	114	64	69
						Total = 10,313	
1963:							
Expected	642	504	353	211	121	67	74
						Total = 10,446	
1964:							
Expected	657	519	363	222	128	71	79
						Total = 10,580	
1965:							
Expected	671	534	373	232	135	75	84
						Total = 10,625	
1966:							
Expected	686	547	384	242	143	79	89
						Total = 10,836	

As in the structural matrix, the "average spatial matrix" (Table 6.25) is derived by compounding the information contained in Tables 5.6 - 5.9 and by adding the appropriate birth and death vectors. The initial column vector of death probabilities is:

	Toronto	Toronto suburbs	Large urban	Large urban suburbs	Small urban	Rest of Ontario
sd =	(.24527,	.20144,	.08966,	.01930,	.24366,	.20064)

and the initial row vector of birth probabilities is:

sb = (.20326, .27124, .08602, .02185, .23100, .18661).

The annual estimates of the spatial matrix (Table 6.26), in terms of goodness of fit, are far superior to those of the average structural matrix and endorse the notion of applying a first-order Markov model to the analysis of the spatial dynamics of industrial activity. Reasons for the greater accuracy of the spatial model relative to the structural model may be due, in part, to its fewer states and, in part, to the greater constancy of the spatial probabilities.

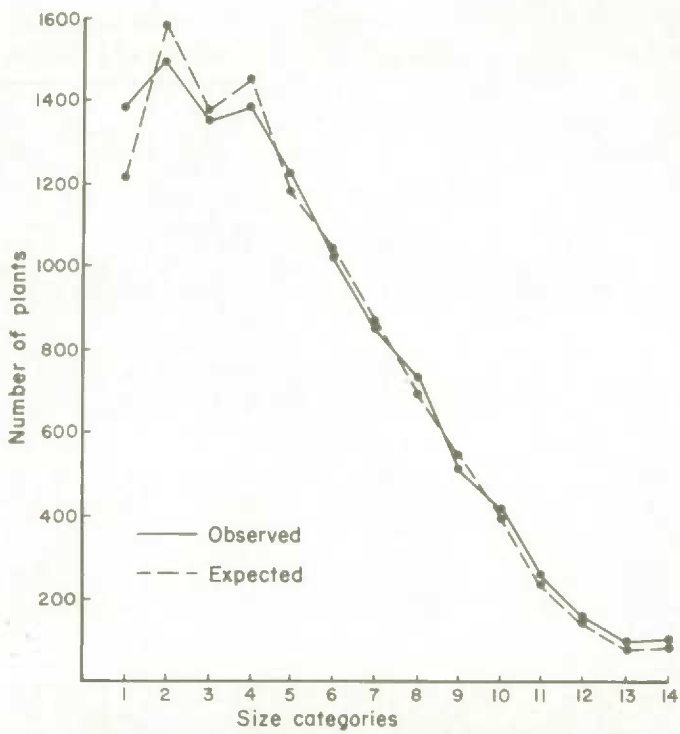


Figure 6.5: Estimated Distribution from Average Structural Matrix with Birth and Death Vectors and Observed Distribution (1966) for All Establishments

TABLE 6.25. Average Spatial Matrix with Birth and Death Vectors for All Establishments with Two or More Employees

	Deaths	Toronto	Toronto suburbs	Large urban	Large urban suburbs	Small urban	Rest of Ontario
Births	.99920	.00016	.00022	.00007	.00002	.00019	.00015
Toronto	.06076	.91306	.02259	.00012		.00108	.00239
Toronto suburbs ..	.06424	.00343	.92602	.00026		.00103	.00502
Large urban	.05176	.00046	.00074	.93822	.00603	.00186	.00093
Large urban suburbs	.08086			.05728	.81604	.01348	.03235
Small urban	.05670	.00030	.00150	.00300		.93578	.00543
Rest of Ontario ..	.05265	.00021	.00169	.00034	.00034	.01232	.93246

TABLE 6.26. Estimated Vectors from Average Spatial Matrix for All Establishments and Observed Distributions, 1961 - 66

	Toronto	Toronto suburbs	Large urban	Large urban suburbs	Small urban	Rest of Ontario
1962:						
Observed	2,450.0	1,648.0	1,124.0	184.0	2,730.0	2,295.0
Expected	2,401.5	1,603.3	1,111.7	190.3	2,757.3	2,313.3
	Chi-square = 2.93					
1963:						
Observed	2,398.0	1,798.0	1,135.0	180.0	2,764.0	2,354.0
Expected	2,344.0	1,745.8	1,119.2	180.8	2,788.6	2,328.0
	Chi-square = 3.55					
1964:						
Observed	2,316.0	1,944.0	1,152.0	166.0	2,799.0	2,392.0
Expected	2,292.0	1,876.5	1,125.8	173.0	2,818.0	2,342.1
	Chi-square = 5.49					
1965:						
Observed	2,234.0	2,075.0	1,174.0	153.0	2,838.0	2,409.0
Expected	2,245.0	1,996.4	1,131.5	166.8	2,845.6	2,355.7
	Chi-square = 6.87					
1966:						
Observed	2,171.0	2,130.0	1,186.0	146.0	2,880.0	2,423.0
Expected	2,202.4	2,106.4	1,136.6	161.7	2,871.7	2,368.8
	Chi-square = 5.65					

The results suggest that the Markov model is a viable mechanism for estimating **future** trends of manufacturing activity. Within the framework of the analysis two sets of forecasts can be generated: (1) the **spatial rearrangement** of manufacturing establishments for the six state system, and (2) the internal **structural dynamics** for the Province and the various states. Conceptually, individual industry forecasts for the whole province as well as for the selected states can also be projected. Operationally, however, this would be too time consuming and is beyond the scope of the present study. Instead, forecasts are projected only for selected spatial states and for selected industries of the whole province for which the respective stochastic matrices are estimated from a **large number** of observations.

Forecasts for Number of Establishments

The total number of establishments with at least two employees, sawmills excluded, for the Province of Ontario can be projected by successive powering of either the total average structural matrix or the average spatial matrix. Successive powering of the spatial matrix provides the 1962 - 75 estimated trend line shown in Fig. 6.6, and the successive outcomes of each iteration for 1967 - 75 with the provincial totals are shown in Table 6.27.

Application of the Markov procedure to the total structural matrix provides estimated size distribution vectors for each point on the curvilinear trend line in Fig. 6.6. Theoretically, successive powering of the spatial and structural matrices should provide identical totals but, in practice, differences of the estimated parameters resulting from approximated Markov processes contingent upon the classification of states create inequalities. By 1970, for example, the structural matrix projects a total of 11,308 plants and for 1975, 11,716. The respective estimated size distributions of these establishments are shown in Table 6.28.

TABLE 6.27. Expected Distribution Vectors 1967 - 75 for the Spatial Matrix

Year	Toronto	Toronto suburbs	Large urban	Large urban suburbs	Small urban	Rest of Ontario	Total
1967	2,164	2,207	1,141	157	2,896	2,381	10.946
1968	2,122	2,300	1,145	154	2,919	2,393	11,040
1969	2,097	2,385	1,148	152	2,941	2,405	11,127
1970	2,069	2,463	1,152	149	2,961	2,416	11,209
1971	2,043	2,534	1,155	147	2,980	2,427	11,285
1972	2,020	2,600	1,157	146	2,999	2,438	11,360
1973	1,999	2,661	1,160	145	3,016	2,448	11,428
1974	1,980	2,716	1,162	144	3,032	2,458	11,492
1975	1,963	2,767	1,164	143	3,048	2,467	11,552

TABLE 6.28. Expected Size Distribution Vectors 1970 and 1975

Year	1	2	3	4	5	6	7
1970	1,185	1,583	1,391	1,494	1,227	1,094	909
1975	1,182	1,584	1,402	1,521	1,260	1,135	953
	8	9	10	11	12	13	14
1970	738	598	430	281	172	98	113
1975	781	644	474	320	203	118	141

The elements of these vectors show clearly that although the total number of establishments will continue to increase at a significant pace – a predicted increase of almost 1,000 from 1961 - 75 – the actual size distribution will change very slightly. Most of the change will occur in the higher size categories but even by 1975 there will still be a greater proportion of small establishments than is predicted by the lognormal distribution (see Fig. 6.7).

Projected trend lines for individual industries will, of course, assume different configurations depending on the initial stochastic matrices. As indicated in Chapter V the two largest industries, in terms of number of establishments, with contrasting birth and death rates resulting in differential expansions, are metal fabricating, and foods and beverages. Successive powering of the respective average transition matrices yields the trend lines shown in Fig. 6.6. It is expected that the sharply increasing trend line for the metal fabricating establishments will continue to climb, at least to 1975, but the slightly decreasing trend line for foods and beverages is expected to level off after 1971 and maintain approximately 2,000 establishments. If total population continues to increase at its existing rate then we may expect a greater concentration of food and beverage output among a slightly greater proportion of medium size and large scale production units. Such a trend probably results from recent technological innovations in the manufacture of less perishable pre-packed food products which are not so market oriented as they have been in the past. Moreover, improvements in transportation facilities have also encouraged food and beverage plants to take advantage of scale-economies in large, well integrated establishments.

Predicted changes in the annual size distribution for foods and beverages are shown in Table 6.29. The significant decrease in the predicted number of establishments with less than 18 employees and the marked increase in the predicted number employing between 28 and 960 people has the effect of straightening the upper tail of the distribution drawn on logarithmic probability

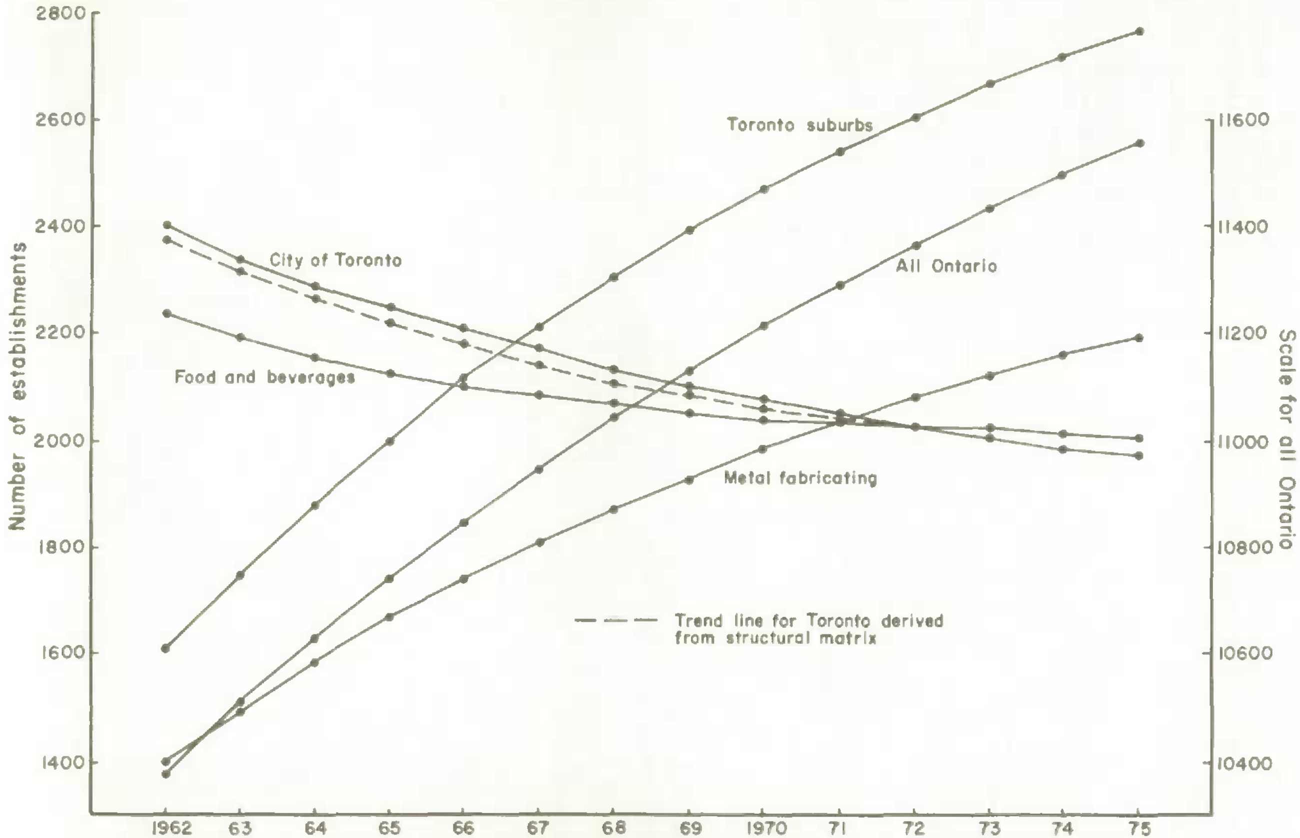


Figure 6.6: Estimated Trend Lines 1962-1975 for Structural and Spatial Matrices

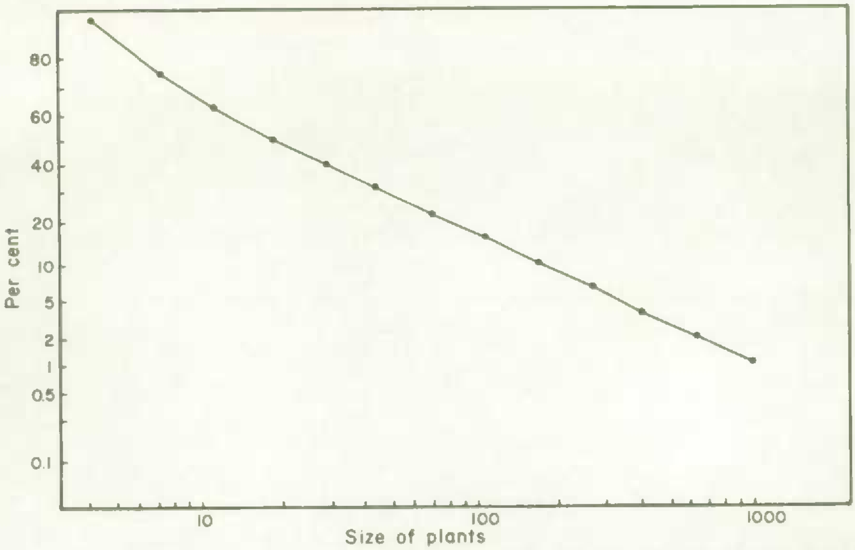


Figure 6.7: Estimated Lognormal Probability Curve for All Establishments in 1975

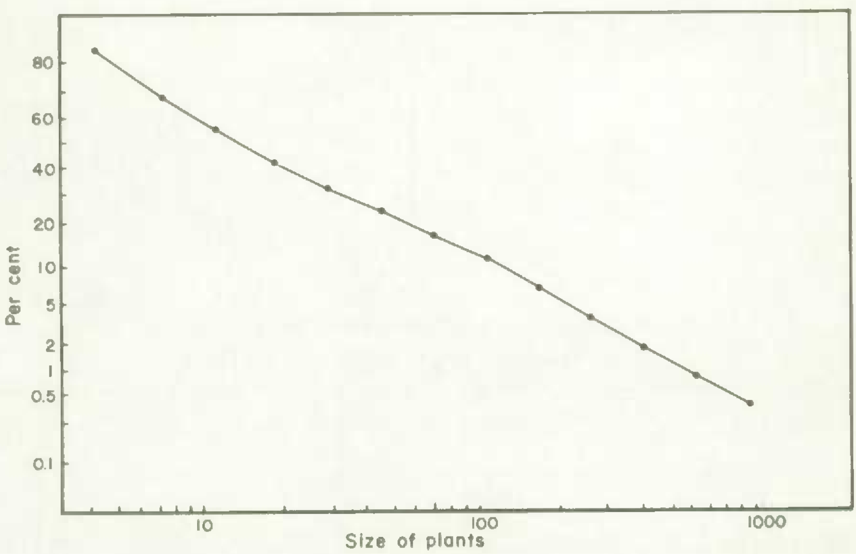


Figure 6.8: Estimated Lognormal Probability Curve for All Establishments in the Foods and Beverage Industry, 1975

paper (Fig. 4.7 and Fig. 6.8) and by 1975 it is predicted that there will be fewer "small" establishments in the food and beverage industry than one might expect from the lognormal distribution. In contrast, each size category in the metal fabricating industry is expected to increase its number of establishments (Table 6.30). These changes are proportionately equal in most size categories so that the form of the expected distribution in 1975 will be very similar to that estimated for 1962 (Fig. 6.9).

TABLE 6.29. Expected Size Distribution Vectors for Food and Beverages, 1962 - 75

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1962	428	421	350	284	207	162	123	90	71	48	31	12	6	8
1963	404	394	336	282	213	164	125	93	75	48	31	13	7	8
1964	383	375	324	279	216	166	126	95	79	49	31	14	7	8
1965	366	360	314	276	218	168	128	98	82	51	32	15	7	8
1966	353	348	306	273	218	170	130	100	85	52	32	16	8	8
1967	342	339	299	270	218	171	132	102	87	54	33	17	8	8
1968	332	331	293	267	218	172	134	103	89	55	34	17	8	8
1969	325	324	288	264	217	173	135	105	92	57	34	18	9	8
1970	319	319	284	261	216	173	136	107	94	58	35	19	9	8
1971	314	315	281	259	215	174	137	108	95	60	36	19	9	8
1972	310	311	278	257	214	174	138	109	97	61	37	20	9	8
1973	307	308	275	256	213	174	139	110	99	63	38	21	10	8
1974	304	306	273	254	212	174	140	111	101	64	39	21	10	8
1975	302	304	272	253	211	174	140	112	102	65	40	22	10	9

TABLE 6.30. Expected Size Distribution Vectors for Metal Fabricating, 1962, 1970 and 1975

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1962	156	197	199	213	181	144	99	69	50	35	27	15	6	2
1970	172	247	269	297	266	207	169	122	76	59	52	31	15	7
1975	180	260	280	320	300	232	186	149	90	64	58	35	17	9

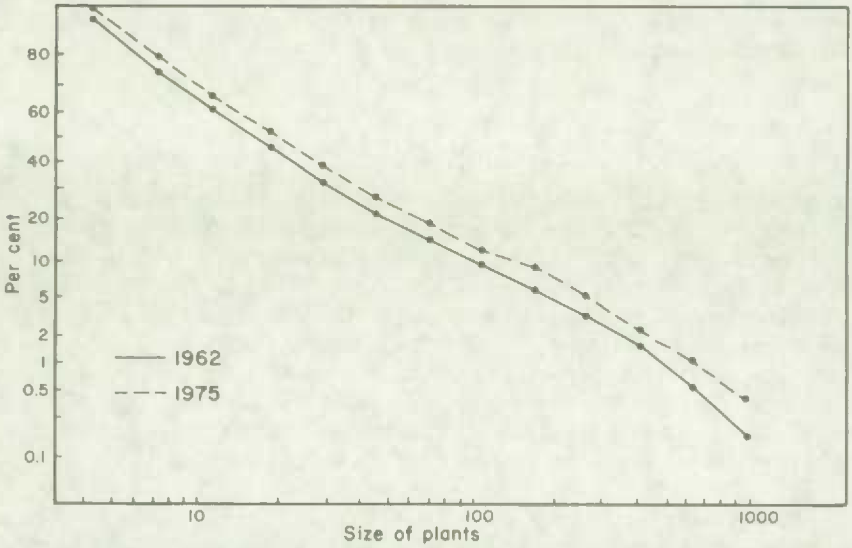


Figure 6.9: Estimated Lognormal Probability Curves for All Establishments in the Metal Fabricating Industry 1962 and 1975

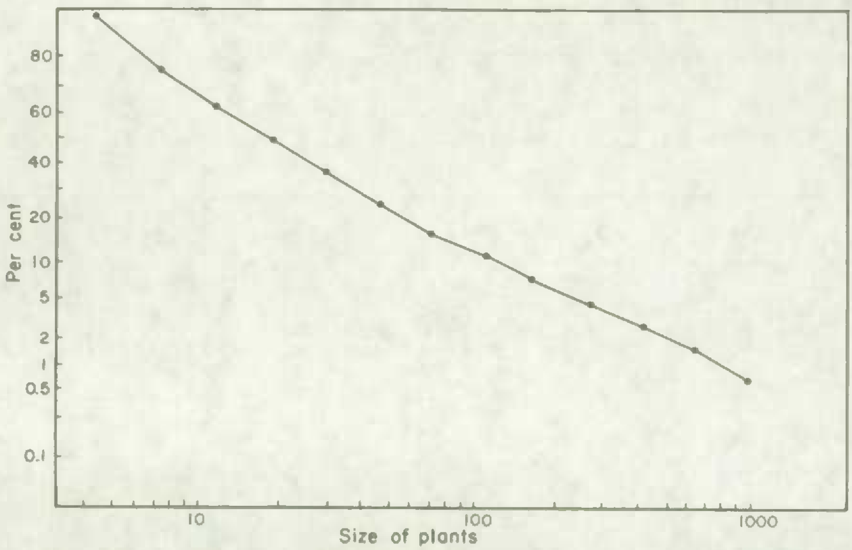


Figure 6.10: Estimated Lognormal Probability Curve for All Establishments in the City of Toronto, 1970

The long-term spatial trends of Ontario's manufacturing activity are observable from the equilibrium vector of the average spatial matrix. When normalized this vector takes the form:

(.153, .275, .094, .013, .256, .209)

and is very similar to the initial distribution of

1961: (.240, .141, .107, .020, .266, .224)

except that in 1961, 24% of Ontario's manufacturing was located in Toronto and 14.1% in the suburbs; in the limiting equilibrium state we would expect to find 15.3% in Toronto and 27.5% in the suburbs. Separate trend lines for these two areas up to 1975 were shown in Fig. 6.6. For Toronto two trend lines are presented: one is derived from powering the spatial matrix for all establishments in Ontario and the other is given by the structural matrix for Toronto. In the latter matrix relocations out of the city are added to the death vector and relocations into the city are added to the birth vector. Although differences exist between the two trend lines the maximum deviation is 1.5% for 1965 and is not considered significant enough to detract from the value of the forecasts.

Changes in the internal structure of Ontario's manufacturing activity are provided in Table 6.31; by 1970 Toronto will have far more small and medium size establishments than is predicted by the lognormal distribution (Fig. 6.10). As indicated in Chapter V, this trend is a result of the increasing tendency of large establishments to relocate to the suburbs rather than as a result of an increasingly high birth rate among the smaller establishments. The total effect as indicated in Chapter V is to emphasize the growing phenomena of an industrial doughnut centred around the city of Toronto.

TABLE 6.31. Expected Size Distribution Vectors for Toronto, 1962 - 75

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1962......	302	325	291	328	291	262	189	136	102	68	38	23	15	11
1963......	278	318	279	324	289	255	187	130	102	65	39	23	15	11
1964......	262	309	272	320	287	249	184	127	100	63	39	24	16	11
1965......	250	300	265	315	284	245	181	124	99	61	39	24	16	12
1966......	241	293	260	311	280	241	178	121	97	60	39	24	16	12
1967......	235	287	256	307	277	237	175	119	95	59	38	24	16	12
1968......	229	282	253	304	274	235	172	117	94	57	38	25	17	12
1969......	226	278	250	301	272	232	170	115	93	56	38	25	17	12
1970......	223	275	248	298	270	230	168	114	91	55	37	25	17	13
1971......	220	273	246	296	267	228	166	112	90	55	37	25	17	13
1972......	219	271	244	294	265	226	164	111	89	54	37	25	17	13
1973......	217	269	243	292	263	224	163	110	88	53	36	25	18	13
1974......	216	268	242	291	262	223	162	109	87	52	36	25	18	13
1975......	215	267	241	290	260	221	160	108	86	52	36	25	18	14

Generalizations concerning the spatial interdependence of Ontario's manufacturing establishments among the various states can be abstracted from information contained in the matrix of mean first passage times. Following the procedure outlined in Appendix A the mean first passage times for the average spatial matrix are given in Table 6.32. A measure of their reliability is given by the matrix of standard deviations in Table 6.33.

This matrix shows that the means cannot be taken as typical values so that they are best discussed in relative terms. In general, the "migrant distance" from Toronto to other larger urban areas is five times greater than that from Toronto to its suburbs; on the other hand, the migrant distance from the suburbs to Toronto is almost as great as any from any other state. Perhaps the most surprising aspect is the exceptionally high migrant distance from the four larger urban centres to their respective suburbs in relation to the migrant distances from these centres to other areas, especially other small urban centres and suburban Toronto. Although these analyses indicate that the Markov procedure as used in this study has a limited ability for long-term forecasts the technique remains attractive for analysing differential movements within a given system of states. One of the main attractions is that such a system can be enlarged to any size so that generalizations concerning migration differentials for a large number of areas can be readily obtained from the same simple matrix operations used for a small system of states.

TABLE 6.32. Matrix of Mean First Passage Times

	1 Toronto	2 Toronto suburbs	3 Large urban	4 Large urban suburbs	5 Small urban	6 Rest of Ontario
1	57. 2	320. 3	1, 547. 0	4, 009. 7	503. 8	595. 1
2	635. 1	31. 5	1, 543. 2	4, 008. 0	501. 1	579. 6
3	662. 8	428. 8	93. 0	3, 624. 0	495. 5	663. 7
4	663. 3	427. 5	1, 064. 3	715. 0	454. 2	502. 9
5	663. 7	422. 8	1, 543. 9	4, 009. 2	32. 7	571. 9
6	665. 2	422. 1	1, 542. 4	3, 993. 4	423. 7	41. 5

TABLE 6.33. Matrix of Standard Deviations

	1 Toronto	2 Toronto suburbs	3 Large urban	4 Large urban suburbs	5 Small urban	6 Rest of Ontario
1	264. 6	402. 5	1, 535. 9	3, 993. 2	499. 4	606. 3
2	650. 7	156. 6	1, 535. 9	3, 993. 2	499. 3	605. 5
3	651. 1	416. 8	523. 7	3, 974. 6	499. 2	606. 5
4	651. 4	416. 7	1, 459. 9	2, 278. 5	496. 4	596. 7
5	651. 5	416. 7	1, 535. 9	3, 993. 2	175. 0	604. 8
6	651. 5	416. 7	1, 535. 9	3, 993. 1	491. 4	217. 9

CHAPTER VII

SUMMARY AND CONCLUSIONS

The study's twofold aim, to analyse recent trends of selected aspects in Ontario's manufacturing activity and to project these trends through an operational forecasting model, has been approached in a Markovian framework. Several recent studies have suggested that Markov chain analysis may be a viable mechanism for analysing changes in the locational patterns of industrial activity; this study has attempted to demonstrate that it can be a valuable analytic concept possessing considerable flexibility for more extensive and detailed applications.

The spatial and structural dynamics of manufacturing activity have been examined in terms of the establishment which is considered to be the most appropriate variable for analysing industrial migration patterns. Analysis of the structural dynamics has focused on frequency size distributions and is accomplished within the conceptual framework of Gibrat's law of proportionate growth embodied in the Pareto and lognormal distributions which can be generated by a simple stochastic or Markov process. The distribution of Ontario's manufacturing establishments is closely approximated by the lognormal model but only large plants with more than 400 employees fall within the range of the Pareto tail. Among the permanent establishments which exhibit Brownian movement Gibrat's law describes very well the changes in the configurations of the respective frequency distributions; the Markovian matrices indicate that although there is a slight tendency towards a systematic decrease in the variance of proportionate growth with increasing size of plant, all plants, regardless of size, have an almost equal probability of increasing to the next higher size category as of decreasing to the next smaller size category.

The dispersive tendency of the permanent establishments to change the form of the prevailing distributions for all establishments is offset by a birth and death process which has marked sectoral and spatial variations. Our knowledge of the causes for these variations is still limited and if we are to concern ourselves with influencing or planning future industry patterns much more detailed and extensive research is required.

The successful application of a Markov chain model to the analysis of any social science data depends in part on a meaningful system of states, the selection of which is a fundamental problem of Markov chain analysis. In many cases the adopted system of states is defined by the available data but in this study the system of states for the structural matrices is calibrated in accordance with the size frequencies computed for the evaluation of Gibrat's law of proportionate growth. The system of spatial states, however, requires more subjectivity and emerges from the analysis of recent spatial trends in Ontario's manufacturing activity.

In analysing these trends no attempt has been made to evaluate or provide detailed or causal relationships of the observed spatial patterns since such relationships, especially those concerned with spatial variations in industrial opportunities for new plants, branch plants, and relocations, would be more appropriately analysed with the aid of questionnaire data drawn from a wide range of plants distributed across Ontario. Such data would facilitate the analysis of spatial linkages, site factors, and entrepreneurial perception of locational attractiveness. Nevertheless, the statistical analyses of this study have shown that between 1961 and 1965 the dominant trend in Ontario's manufacturing activity was towards the development of an industrial doughnut centred on the city of Toronto. Toronto has declined both relatively, due largely to a negative birth/death differential, and absolutely, as a result of the significant number of relocations to the surrounding suburbs and to smaller towns and cities elsewhere in Ontario. On the other hand, the ring of intensive development in suburban Metropolitan Toronto as well as in the fringe areas of Chinguacousy, Toronto, Vaughan, Markham, and Pickering townships, is being accentuated by a highly favourable birth/death differential, the relocation of plants outwards from the city of Toronto as well as the relocation inwards from the rest of Ontario, and the establishment of a large number of foreign-owned branch plants. The net result is one of increasing concentration of manufacturing establishments in and around Metropolitan Toronto. Outside this area, the Kitchener-Waterloo-Preston complex is seen as an important growth centre and if present trends continue it should firmly establish itself as a dominant secondary area of manufacturing activity. As Ontario's second major "Growth Pole" it is favourably situated to Hamilton, the major producer of unfinished steel products used as input to the "light industries" which typify the industrial structures of both Metropolitan Toronto and the Kitchener-Waterloo-Preston complex. It is suggested that, if in the future, industry is to be "steered away" from Metropolitan Toronto, the Kitchener-Waterloo-Preston complex, where external economies are apparent, should be considered the prime reception area.

The analysis of these trends substantiates their projection in the framework of two interrelated Markovian models. Simple Markov models using one year transition matrices do not provide adequate test predictions for the study period, but the refinement of the matrices involving the fitting of smoothing surfaces and the fractional disaggregation of long period matrices increases the stability of the annual estimates. The introduction of these two concepts to Markovian analysis provides scope for further research; both techniques are particularly useful for investigations lacking the detailed data sources available for this study. The concept of fitting matrix surfaces is easily extended to fitting surfaces to the observed state probability distributions, and where the underlying processes are identifiable the technique may be more viable than least squares estimates based on linear and quadratic programming procedures. Together, these refinements give Markov chain models an advantage over conventional data analysis techniques in that they provide a means for estimating a growth process, in the absence of interregional establishment movements, using only historical data on interregional establishment distributions. The concept of fractional disaggregation of long-period matrices, in the absence of short-term transition matrices, is particularly attractive for deriving short-term estimates. Where adequate data are available the best results are obtained from average matrices which compound all the information contained in the parameters of a series of stochastic matrices. Successive powering of both average spatial and structural matrices provides insight into the short- and medium-term trends of Ontario's manufacturing activity.

Projections of the 1961 - 65 trends indicate a continued increase in the total number of establishments for the province, but in the city of Toronto where there was a sharp decrease in the number of establishments between 1961 and 1965 the downward trend is anticipated to level off between 1970 and 1975. One independent interpretation that may be attached to such a levelling off is that the continued growth of Metropolitan Toronto is contingent on a solid core of industrial activity characterized by types of operations that are highly dependent on agglomeration economies. On the other hand, the projected continued upward swing in the number of establishments in the suburbs almost parallels the estimated provincial trend line. In the short term this seems most likely, though a physical saturation point must inevitably be reached. Such constraints are not embodied in the existing model but future research might consider further refinements. Similar configurations of the estimated trend lines for spatial units are also evident for industrial groups. All these trend lines, however, have relevance only in the framework of the simplifying and highly generalized Markov assumptions, especially that of stationarity which is dependent on the first-order assumption and adopted classification of states.

This study's exploratory experiments represent but one step in what might be envisioned as the beginning of a much broader research design oriented towards a deeper understanding of the structural and spatial dynamics of manufacturing activity. The analyses have used three variables only: size of establishment measured by total employment, type of activity, and location, but future research could augment them with other variables such as value added, sales or cost of materials. Moreover, changes in the pattern of manufacturing establishments are not necessarily the same as those changes exhibited by other variables such as total employment; thus a possible related avenue of research might be the application of Markov chain analysis to small area labour force estimation. It is hoped that this study will motivate further research in this and other directions.

APPENDIX A

For regular Markov chains two important theorems relating to the existence and uniqueness of an equilibrium solution are provided by Kemeny and Snell, 1967, p. 70.

Theorem I

If P is a transition matrix for a regular Markov chain then:

(1) the powers of P approach a matrix A

(2) each row of A is the same probability vector α

(3) the elements of α are all positive.

Theorem II

If P is a regular transition matrix for a regular Markov chain and A and α are as in Theorem I, then the vector α is the unique probability vector such that $\alpha P = \alpha$. The matrix A is defined as the **limiting matrix.**

These theorems are best exemplified by specific reference to a hypothetical example which extends the simplified single town example of Chapter II. Consider a constant sample of manufacturing establishments distributed in three towns possessing varying degrees of industrial attractiveness. Assume that at time t_0, 20% of the total number of plants of the three towns are located in town A, 20% are in town B, and 60% are in town C. Thus, the **initial state** of the system can be represented by the **initial distribution vector** – $p^{(0)}$ – which can be expressed as:

$$p^{(0)} = (.2, .2, .6)$$

Assume also that the probability of a manufacturing establishment relocating from one state (town) to any other state during a specified time period is described by the following transition matrix:

	S_1 (town A)	S_2 (town B)	S_3 (town C)
S_1 (town A)	.6	.2	.2
P = S_2 (town B)	.3	.4	.3
S_3 (town C)	.2	.2	.6

The matrix shows that the probability of a plant's remaining in town A during a given time period is .6, whereas the probability of a plant's moving from town A to town B is .2, and so forth. Given this initial transition matrix it is now possible to compute the transition probabilities after 1,2,3, . . .n, stages by calculating the relevant power of the matrix. Thus after two stages:

	S_1	S_2	S_3
S_1	.46	.24	.30
P^2 = S_2	.36	.28	.36
S_3	.30	.24	.46

and after four stages:

		S_1	S_2	S_3
	S_1	.3880	.2496	.3624
$P^4 =$	S_2	.3744	.2512	.3744
	S_3	.3624	.2496	.3880

This displays a rapid convergence towards some **average state** of the system, which is represented by the limiting or A matrix:

		S_1	S_2	S_3
	S_1	.3750	.2500	.3750
$A =$	S_2	.3750	.2500	.3750
	S_3	.3750	.2500	.3750

and the probability vector $\alpha = (.3750, .2500, .3750)$ holds the system in equilibrium. In the present context, the notion of equilibrium can be defined as that distribution for which the average number of plants entering a given town per period equals the average number of plants leaving it. The concept of equilibrium is thus statistical in nature for the industry or system and dynamic for the individual plant (Adelman, 1958, p. 896). In Markov chain analysis the equilibrium distribution is of interest not as a **forecast** of the future state of the industry but as a projection of what it would be if the observed pattern of movement continued (Padberg, 1962, p. 192). Thus the proportion of plants in any one town at the end of each time period is derived by multiplying the P^N transition matrix by the initial distribution vector $p^{(0)}$. After two stages, for example, the proportion of plants in each town is given by:

$$p^{(0)}P^2$$

The limiting probability a_j of being in state S_j is independent of the starting state and represents the fraction of the time that the process can be expected to be in state S_j during a large number of transitions and after a large number of steps from $p^{(0)}$. This arises from the law of large numbers for regular Markov chains. Applying this theorem to the example above, after a large number of time periods 37.5% of the plants will be in town A, 25% will be in town B and 37.5% will be in town C.

The limiting matrix is but one important property of a regular Markov chain. Most of the other interesting descriptive quantities for the behaviour of these chains are computed from the Z or **fundamental** matrix (see Kemeny and Snell, 1967, pp. 75 - 84). They are reproduced below in a simplified format, again in the context of the hypothetical three town example. In matrix notation:

$$Z = (I-(P-A))^{-1}$$

where I is an identity matrix
P is a regular matrix
A is the limiting matrix of P.

For our example,

		S_1	S_2	S_3
	S_1	1.36458	−0.06250	−0.30208
Z =	S_2	−0.09375	1.18750	−0.09375
	S_3	−0.30208	−0.06250	1.36459

The Z matrix describes how the system approaches equilibrium from a given initial distribution: from any initial state, the expected percentage of time the system will spend in state j approaches a_j as the number of time periods n becomes large; however, starting from a given state i, this expected percentage differs from a_j by approximately $(Z_{ij}-a_j)/n$.

The Z matrix can be employed also to provide a descriptive measure of an area's industrial stability. Several studies (e.g. Kerr and Spelt, 1957; Moses and Williamson, 1967), have shown, for example, that core areas of large cities act as incubator areas for numerous small establishments which, once nurtured, relocate to less congested areas where expansion is more practical. Once born, therefore, **how long** does it take an average plant to move from S_i to S_j for the first time? The distribution describing this random variable is called the first passage time distribution, and the expected value is called mean first passage time. The matrix of mean first passage times is denoted by M and the entries, the m_{ij}'s, give the expected time to move from S_i to S_j for the first time. For a regular Markov chain the mean first passage time matrix is given by:

$$M = (I - Z + EZ_{dg})D$$

where
- I is an identity matrix
- Z is the fundamental matrix
- E is a matrix with all entries 1
- Z_{dg} results from Z by setting off-diagonal entries equal to 0
- D is the diagonal matrix with j-th entry $1/a_j$

For our three town example:

		S_1	S_2	S_3
	S_1	2.667	5.000	4.444
M =	S_2	3.889	4.000	3.889
	S_3	4.444	5.000	2.667

Since the entries $m_{ij} = M_i [f_j]$ represent the mean number of time periods – in our case one year intervals – to arrive in any given state, the average plant would take 5.0 years to relocate from town A to town B, and 3.9 years to relocate from town B to town C.

Computation of the variance of the first passage times proceeds as a simple matrix operation:

$$Var_i[f_j] = M_i[f^2{}_j] - M_i[f_j]^2$$

Kemeny and Snell denote $M_i[f^2{}_j]$ by W which represents the matrix of second moments of the first passage times. The matrix W satisfies the equation:

$$W = M(2Z_{dg}D - I) + 2(ZM - E(ZM)_{dg})$$

where $(ZM)_{dg}$ results from the product of the fundamental and mean first passage matrices by setting off-diagonal entries equal to 0. The entries of the matrix $M_i[f_j]^2$ are the squares of the first moments for the M matrix. The hadamard product of M is best denoted by M_{sq}. A simple matrix operation gives:

$$\left\{Var_i[f_j]\right\} = V = W - M_{sq}$$

(Matrix notation in this equation differs from that used by Kemeny and Snell. They denote $Var_i[f_j]$ by M_2.)

so that

V =

	S_1	S_2	S_3
S_1	9.630	20.000	14.074
S_2	13.087	18.000	13.087
S_3	14.074	20.000	9.630

Since, in the present example, the standard deviations of the first passage times – the V_{ij}'s – are of the same order of magnitude as the means – the m_{ij}'s – the means are not to be taken as typical values.

The most important descriptive measure is derived from the Central Limit Theorem for Markov chains. The computation of this property requires the elements of the limiting covariance matrix. The entries, the c_{ij}'s – of the C matrix are given by:

$$c_{ij} = a_i Z_{ij} + a_j Z_{ij} - a_i \delta_{ij} - a_i a_j$$

where a_i are the limiting probabilities

Z_{ij} the entries of the fundamental matrix

$$\delta_{ij} = 1 \quad i = j$$
$$= 0 \quad i \neq j$$

In the illustrative example the limiting covariance matrix is:

C =

	S_1	S_2	S_3
S_1	0.50781	-0.14062	-0.36718
S_2	-0.14062	0.28125	-0.14062
S_3	-0.36718	-0.14062	0.50781

The limiting variances, the diagonal entries of C, are denoted by:

$$\beta = [b_j] = [c_{jj}]$$

and in the example the vector

$$\beta = (0.50781, 0.28125, 0.50781)$$

These quantities appear in the Central Limit Theorem defined in the following manner by Kemeny and Snell, 1967, p. 89. For any regular Markov chain, let $y^{(n)}_j$ be the number of times in state S_j in the first n steps and let $\alpha = a_j$ and $\beta = b_j$ respectively be the fixed vector and the vector of limiting variances. Then if $b_j \neq 0$ for any numbers $r < s$

$$Pr_k \left[r < \frac{y^{(n)}_j - na_j}{\sqrt{nb_j}} < s \right] \longrightarrow \frac{1}{\sqrt{2\pi}} \int_r^s e^{-x^2/2} dx$$

as $n \to \infty$, for any choice of starting state k.

Using the alpha and beta values the Central Limit Theorem gives for town B (S_2) in the illustrative example:

$$\frac{y^{(n)}_2 - .25n}{\sqrt{.28125\ n}}$$

which would for large n have approximately a normal distribution. The high limiting variances, however, suggest the low predictive value for the long term of this illustrative model. For example, after approximately 100 time intervals (years) the percentage of plants in town B would, with probability 0.68, not deviate from 25% by more than

$$\sqrt{100 \times .2812} = 5.3\%$$

Thus the attractiveness of Markovian theory emanates from both its operational simplicity in the form of well tested matrix techniques and its ability to provide a probabilistic measure of the reliability of its forecasts.

APPENDIX B

MAXIMUM LIKELIHOOD RATIO CRITERION TEST FOR MARKOVITY

This tests the null hypothesis that a stationary transition matrix is of "zero" order, that is $p_{ij} = p_j$ for all i, against the alternative of a first-order chain.

The ratio criterion is:

$$\lambda = \prod_{i,j} (\hat{p}_j \; / \; \hat{p}_{ij})^{f_{ij}} \qquad (1)$$

where the marginal probability

$$\hat{p}_j = \sum_i f_{ij} \; / \; \sum_i \sum_j f_{ij} = f_{.j} \; / \; f_{..}$$

and

$$\hat{p}_{ij} = f_{ij} \; / \; \sum_j f_{ij} = f_{ij} \; / \; f_{i.}$$

and f_{ij} is the number of observations in each cell. The required statistic is $-2 \log_e \lambda$ which under the null hypothesis has an asymptotic Chi-square distribution with $(n-1)^2$ degrees of freedom. Equation (1) may be written as:

$$-2 \log \lambda = 2 \sum_{i=1}^{n} \sum_{j=1}^{n} f_{ij} \log \frac{f_{ij} \, f_{..}}{f_{i.} \, f_{.j}}$$

The logarithms are Naperian.

Reference: Anderson and Goodman, 1957.

MAXIMUM LIKELIHOOD RATIO CRITERION TEST FOR A FIRST-ORDER MARKOV CHAIN

This tests the null hypothesis that the chain is first-order against the alternative that it is second-order. The null hypothesis is that $p_{1jk} = p_{2jk} = \ldots = p_{njk} = p_{jk}$, for j, k=l, . . . n. The likelihood ratio criterion for testing this hypothesis is:

$$\lambda = \prod_{i,j,k=1}^{n} (\hat{p}_{jk} \; / \; \hat{p}_{ijk})^{f_{ijk}}$$

where

$$\hat{p}_{jk} = \sum_i f_{ijk} \; / \; \sum_i \sum_k f_{ijk} = f_{.jk} \; / \; f_{.j.}$$

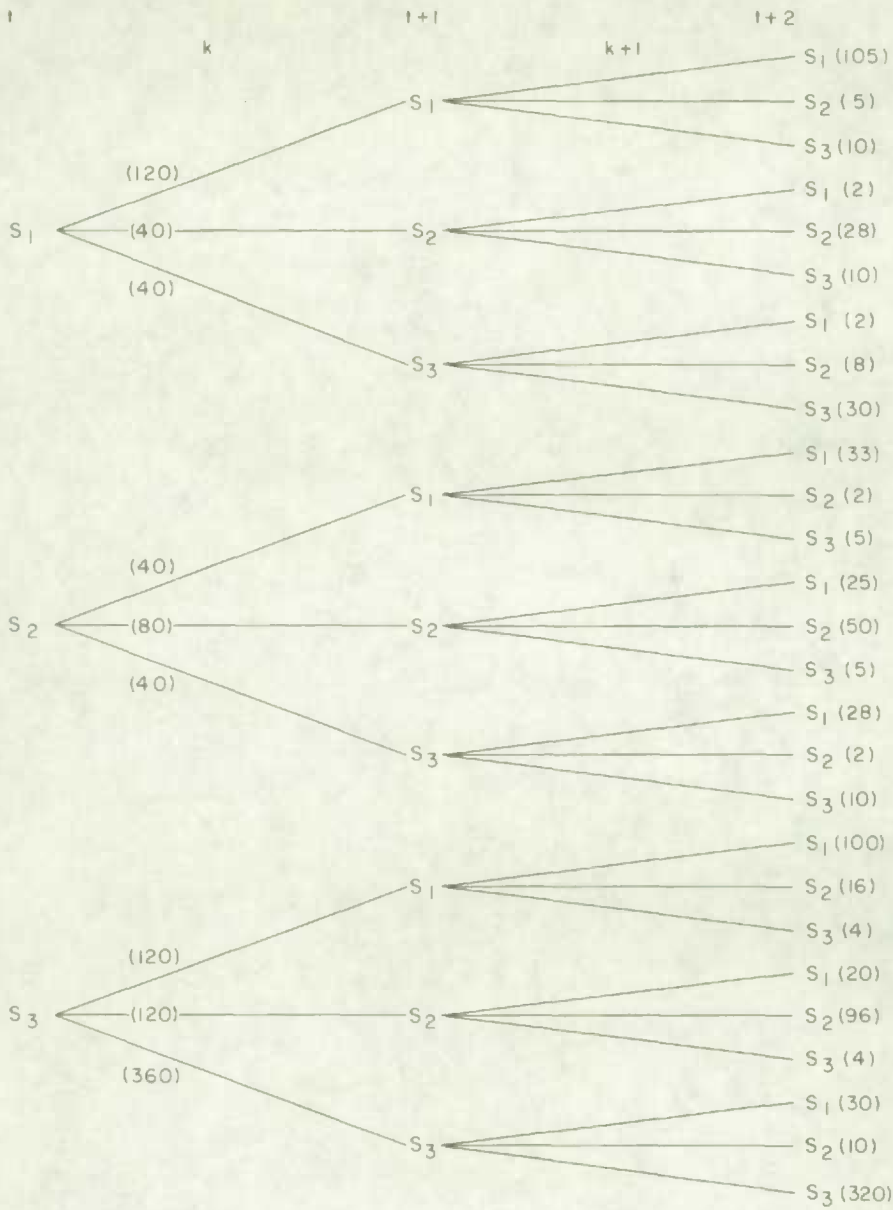
and

$$\hat{p}_{ijk} = f_{ijk} \; / \; \sum_k f_{ijk} = f_{ijk} \; / \; f_{ij.}$$

Under the null hypothesis, $-2 \log \lambda$ is asymptotically χ^2 with $n(n-1)^2$ degrees of freedom.

Reference: Anderson and Goodman, 1957.

3. PROBABILITY TREE FOR A FIRST-ORDER MARKOV CHAIN



INFORMATION TABLE FOR STATISTICAL TESTS OF HOMOGENEITY

The conditional homogeneity component provides a test of the null hypothesis that r realizations come from the same (unspecified) matrix of transition probabilities of a Markov chain of order 1. The indices used here differ from those of the reference.

Component due to	Information	D.F.
(i) homogeneity	$2 \sum_{k=1}^{r} \sum_{i=1}^{n} f_{ki.} \log \frac{f_{...} f_{ki.}}{f_{k..} f_{.i.}}$	(r-1)(n-1)
(j/i) conditional homogeneity	$2 \sum_{k=1}^{r} \sum_{i=1}^{n} \sum_{j=1}^{n} f_{kij} \log \frac{f_{kij} f_{.i.}}{f_{ki.} f_{.ij}}$	n(r-1)(n-1)
(i,j) homogeneity	$2 \sum_{k=1}^{r} \sum_{i=1}^{n} \sum_{i=1}^{n} f_{kij} \log \frac{f_{...} f_{kij}}{f_{k..} f_{.ij}}$	$(r-1)(n^2-1)$

Reference: Kullback, Kupperman and Ku, 1962.

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