

ANALYSIS OF SIMULTANEOUS CURRENT AND PRESSURE  
OBSERVATIONS IN THE BURLINGTON SHIP CANAL

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## Executive Summary

A feasibility experiment was carried out in the Burlington Ship Canal to demonstrate the capability of measuring the horizontal pressure difference between two points separated by a few hundred meters as related to the water currents. Measurements were carried out successfully of about two months. Coincident measurement by current meters gave comparative information for evaluation of the calculated values using the pressure measurements. Statistical comparison of these data indicate a high correlation and provide a validation of the experimental methods. Application of a one-dimensional model to predict the currents from lake levels and winds show some departure from the measured values.

### Résumé pour la direction

On a exécuté, dans le canal pour navires de Burlington, une expérience de faisabilité visant à démontrer notre aptitude à mesurer des différences de pression dans le plan horizontal entre deux points éloignés de quelques centaines de mètres en fonction des courants. Les mesures ont été effectuées avec succès pendant environ deux mois. Des mesures simultanées au moyen de courantomètres ont fourni des données comparatives pour l'évaluation de valeurs calculées à partir des mesures de la pression. La comparaison statistique de ces données a révélé une étroite corrélation et a permis de valider les méthodes expérimentales. À l'application, un modèle à une seule dimension de la prévision des courants à partir des niveaux d'eau et des vents, s'écarte quelque peu des valeurs mesurées.

**INTRODUCTION**

During the early months of 1983, simultaneous measurements of pressure gradients, currents and winds were made in the ship canal between Hamilton Harbour and Lake Ontario. The purpose of the program was to evaluate the feasibility of measuring pressure differences between two points on the lake bottom by transferring both pressure signals to a central measurement site via semi-rigid tubing. The incentive for developing this method of pressure gradient measurement derived from two considerations. First, statistical and numerical studies of alongshore current fluctuations established that knowledge of alongshore pressure gradients is essential to eliminate uncertainties in existing models and to determine the momentum balance in the coastal zone. Second, the pressure gradients of interest are equivalent to surface slopes of order  $10^{-7}$  with characteristic horizontal scales of less than 10 kilometers, thus requiring observations of water level differences well below 1 millimeter. This cannot be accomplished by two independent pressure sensors and hence there is a need for direct measurement of relative pressure differences. The method proposed here has been described in the NWRI report: Measuring horizontal pressure gradients in lakes: a feasibility study by F.M. Boyce, M.A. Donelan, T.J. Simons, December 1982.

**PRESSURE MEASUREMENTS**

Since the proposed method of observation is based on the principle of hydrostatic balance, the environmental pressure at the central measurement site must be regulated according to the height of this site in relation to the mean surface level of the lake. In the feasibility study, this problem was avoided by locating the central site in the basement of the CCIW building. The central measuring component consists of two stilling wells and a differential pressure transducer. This site is connected with two pressure ports in the lake by two independent runs of tubing. In the feasibility study, the pressure ports were located at opposite ends of the Burlington ship canal, the distance between the inlet ports being 703 meters.

The differential pressure transducer used for the experiment was an M.K.S. Baratron. At intervals of three hours, the ports of the transducer were "shorted" for 15 minutes to record the zero drift of the instrument. This was done by opening a valve in a secondary connecting arm between the two stilling wells. The characteristic response time of the system of tubing and stilling wells was designed to be 21 minutes, which was verified by experiment. Thus, upon closing of the valve used for the zero check, another 20 to 30 minutes of data are lost. The data during the period of zero testing and the

subsequent adjustment time must be obtained by subjective interpolation.

The pressure data were recorded in digital form and on strip charts. Since the digital data have not yet been processed, the strip charts were used for the present analysis. The accuracy of the charts is more than sufficient in view of the uncertainty of the zero level between the three-hourly checks and the errors inherent in the above interpolation. The charts were digitized at 10 minute intervals with one reading coinciding with the start of a zero check and the first reading thereafter being taken 40 minutes later, thus allowing for an adjustment time of 25 minutes. This required interpolation of three out of 18 samples in each three-hourly period, that is, one-sixth of the record length. The movement of the zero level between the three-hourly checks was estimated by drawing a smooth curve through the check points.

The pressure measurements started on 31 January, 1983 and ended on 8 March, 1983. During the first few days, various adjustments were made in the measurement system, but from 9 February onwards the data are considered reliable. On 5 March, irregularities were observed in the response and when the system was recovered on 8 March, one of the pressure ports was found to be obstructed. Thus, the data analyzed here cover the period from 9 February to 5 March, 1983.

These data are presented in Figures 1a to 1e with a positive value representing the case of the harbour level exceeding the lake level. The black dots represent the 15 minute check of zero pressure gradient and the dashed curves during the following 25 minutes show the observed response of the system after the closing of the valve used for the zero check. These dashed curves give an indication of the time variation of the actual pressure variation and thus they remove some of the uncertainty from the subjective interpolation.

To check the response time of the system, the average root-mean-square value of water level differences as a function of time between two zero checks was computed. The solid curve in Figure 2 shows the results before interpolation which appears consistent with the above-mentioned response time of 21 minutes. The dashed curve represents the interpolated data and indicates a slight bias toward the zero level.

### 3 CURRENT MEASUREMENTS

Currents were measured by pairs of current meters of fixed orientation, the first member of each pair measuring lake-ward flow through the canal, the second member recording flow in opposite direction. The two records are then to be merged into a single data

file. Three pairs of current meters were installed but only two meters produced useful data, the first one measuring lake-ward currents at a depth of 6 meters, the second one measuring bay-ward currents at a depth of 7 meters. These data were treated as if they had been recorded at the same depth and hence they were merged into a single record.

The sampling interval of the current meters was set at 20 minutes. The first current meter was operational from 21 January, 20:15 GMT to 9 March, 14:54 GMT, and collected 3368 samples, one less than expected. The second meter took observations from 21 January, 20:11 GMT to 9 March, 14:54 GMT and its record was 17 samples short. By matching the two records, the first record was found to have one sample missing on 3 February, while the 17 missing samples of the second record appeared to have been lost on 8 February. With these corrections, a consistent current meter record was obtained with sampling times at 14, 34, and 54 minutes after the hour. Since the Plessey current meters integrate current speeds over the sampling interval of 20 minutes, representative measurement times are 4, 24, and 44 minutes after the hour.

Unfortunately, after the above analysis of the current meter data was completed, visual comparison with the pressure measurements showed immediately that the whole current meter record should be



shifted forward in time by one sampling interval. This conclusion is confirmed by the following spectral analysis of both records which show that the short period fluctuations of the currents lag those of the pressure by 90 degrees if this time shift is carried out. For periods of about two hours, the sampling interval of 20 minutes represents one-sixth of a period or 60 degrees, so there is little doubt that the current meter data must be shifted as indicated. It is, however, not clear how the current meter data can be that much in error since the start and stop times have been carefully checked.

#### 4 SPECTRAL RESPONSE OF CANAL MODEL

It may be assumed that the currents in the canal are governed by the one-dimensional equation:

$$\frac{\partial u}{\partial t} = -g \frac{\partial h}{\partial x} + \frac{\tau_{sx}}{H} - \frac{C_b |u| u}{H} \quad (1)$$

where  $t$  is time,  $x$  is the coordinate along the canal,  $u$  is the mean velocity over a cross section,  $g$  is gravity,  $h$  is the surface elevation,  $H$  the mean depth,  $\tau_{sx}$  the component of the wind stress along the canal and the last term represents bottom friction.

Since the characteristic time of the wind forcing is longer than a day while the pressure fluctuations have typical periods of a few hours, the frequency response of currents to pressure variations may be estimated without regard to wind forcing. Given a mean depth of 9 meters, a distance between pressure ports of 703 meters, a bottom drag coefficient of  $2.5 \times 10^{-3}$  and a typical current speed of 25 centimeters/second, the linearized equation becomes:

$$\frac{\partial u}{\partial t} = a \Delta h - b \cdot u \quad a = 1.4 \cdot 10^{-2} \text{ sec}^{-2} \quad b = 6.9 \cdot 10^{-5} \text{ sec}^{-1} \quad (2)$$

where  $\Delta h$  is the water level difference between pressure ports.

For a pressure variation of period  $T$ , the amplitude response of the current is

$$\frac{aT}{2\pi} \left[ 1 + \left( \frac{bT}{2\pi} \right)^2 \right]^{-\frac{1}{2}} \quad (3)$$

and the phase lag of the current behind the pressure is

$$\arctan \left( \frac{2\pi}{bT} \right) \quad (4)$$

For periods shorter than a day, the amplitude response is approximately equal to eight times the forcing period in hours with

frictional damping varying from 3% for six-hour periods to 10% for 12-hour periods. The phase lag in degrees is approximately equal to  $90 - 2T$  where  $T$  is the forcing period in hours.

The spectral analysis of observed currents and water level differences is based on the 24-day period from 9 February to 4 March, 1983. Figure 3a shows energy densities of currents (above) and water levels (below). Except for a peak at twelve hours, the major portion of the energy is confined to periods shorter than six hours. The similarity of the two spectra is striking. Figure 3b presents the coherence, the spectral amplitude ratio and the phase between currents and pressure gradients. The amplitude and phase are shown only for frequencies with coherence exceeding an arbitrary value of 0.7. The smooth curves represent the spectral response solutions of the linearized equations (3) and (4), while the dashed lines represent the inviscid solutions. Except for the spectral peak at twelve hours, the observed amplitude and phase agree reasonably well with the theoretical values thus confirming that the currents in this frequency range are consistent with the pressure gradients.

## 5 NUMERICAL INTEGRATION OF CANAL MODEL

The one-dimensional equation (1) was used to predict currents from observed water levels and winds. Since the error in the integration tends to build up, the calculation is restarted at regular intervals. This interval is taken to correspond with the three-hourly period between zero checks of the pressure transducer. Since the error is expected to be largest during the 40-minute period of interpolated pressures, each integration starts one hour after the start of the zero check, that is at 3, 6, 9, ... GMT, using the current observed at that time. In the first experiment the wind stress coefficient is set at  $1.2 \cdot 10^{-3}$  and the bottom drag coefficient at  $2 \cdot 10^{-3}$ . The results (solid curves) are compared with observed currents (dashed) in Figures 4a to 4e. Significant errors are seen to accumulate over some of the three-hour integration periods. At times, these errors increase during the third hour and hence they could be partly due to interpolation of the water levels. In general, however, the errors seem as likely to originate in the first two hours as in the third hour of integration.

By comparison with the encouraging results of the spectral comparison of currents and pressure, the results of the numerical integration are somewhat disappointing. It may be noted however that the spectral amplitude response of currents to water levels tends to

drop below the theoretical curve in the range of periods between three and six hours. This is consistent with the fact that the predicted currents tend to be greater than the observed ones. Part of this error can be rectified by wind and bottom stress. To illustrate this, solutions for different wind stress coefficients ( $c_d$ ) and bottom drag coefficients ( $c_b$ ) are shown in Figure 5. The example selected is the day which showed the largest effects of these parameters. Also shown is the interpolated pressure curve for this day. It is seen that only part of the error can be removed by different interpolation. The most likely explanation appears to be that the horizontal scales of currents and pressure gradients in the canal are less than the distance between the pressure ports.

#### Acknowledgements

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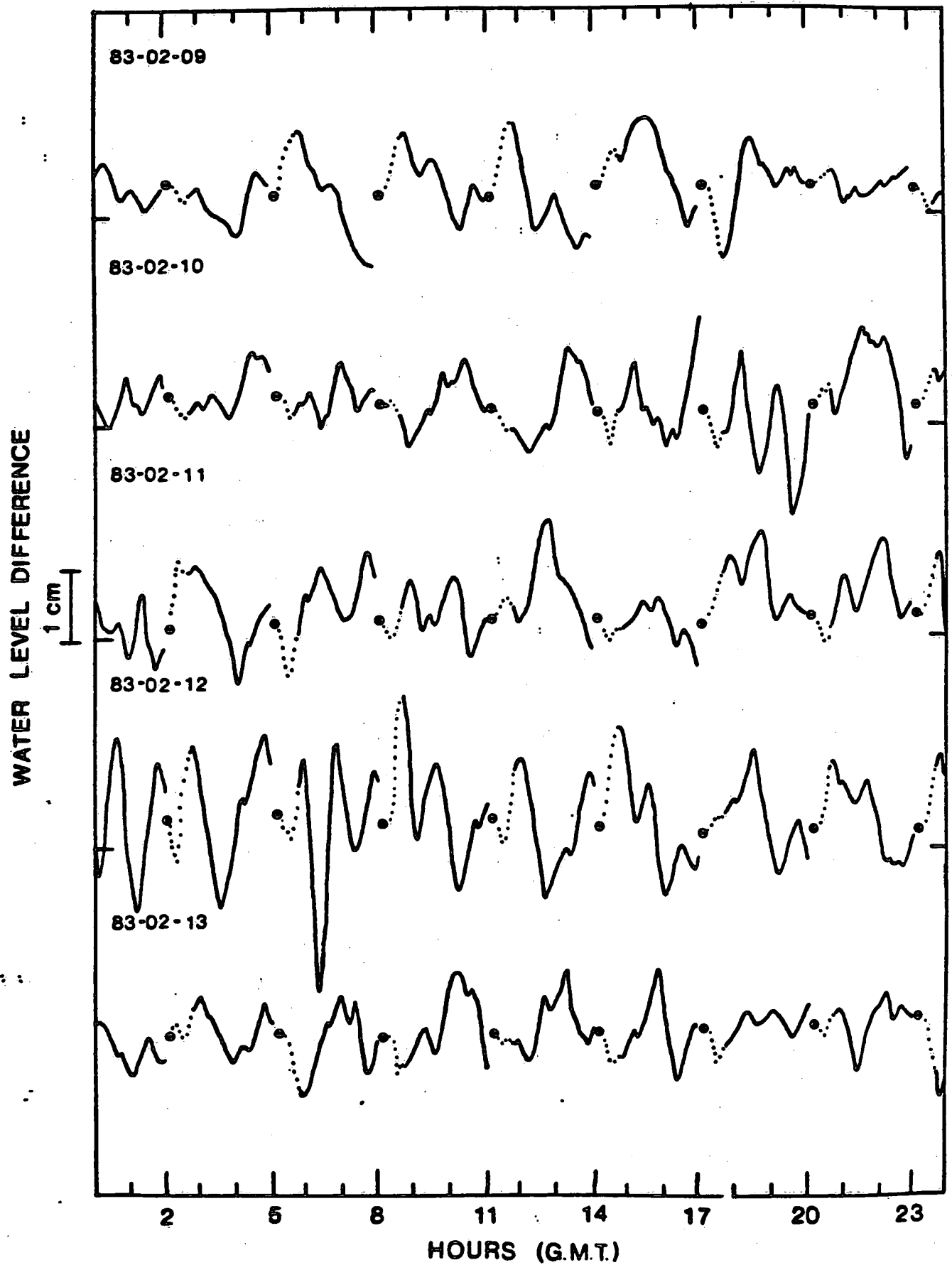


Fig. 1a. Observed water level differences.

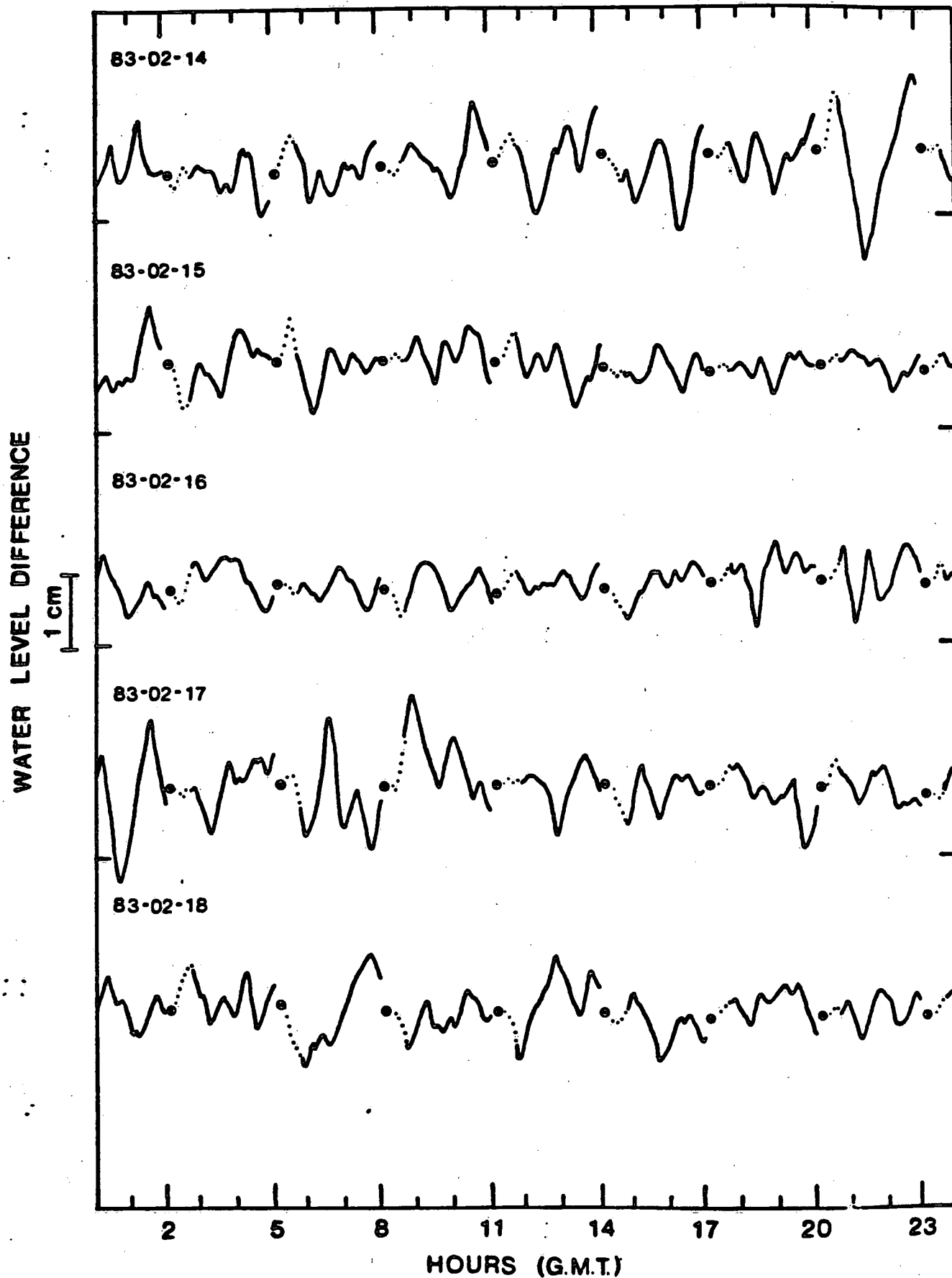


Fig. 1b. Observed water level differences.

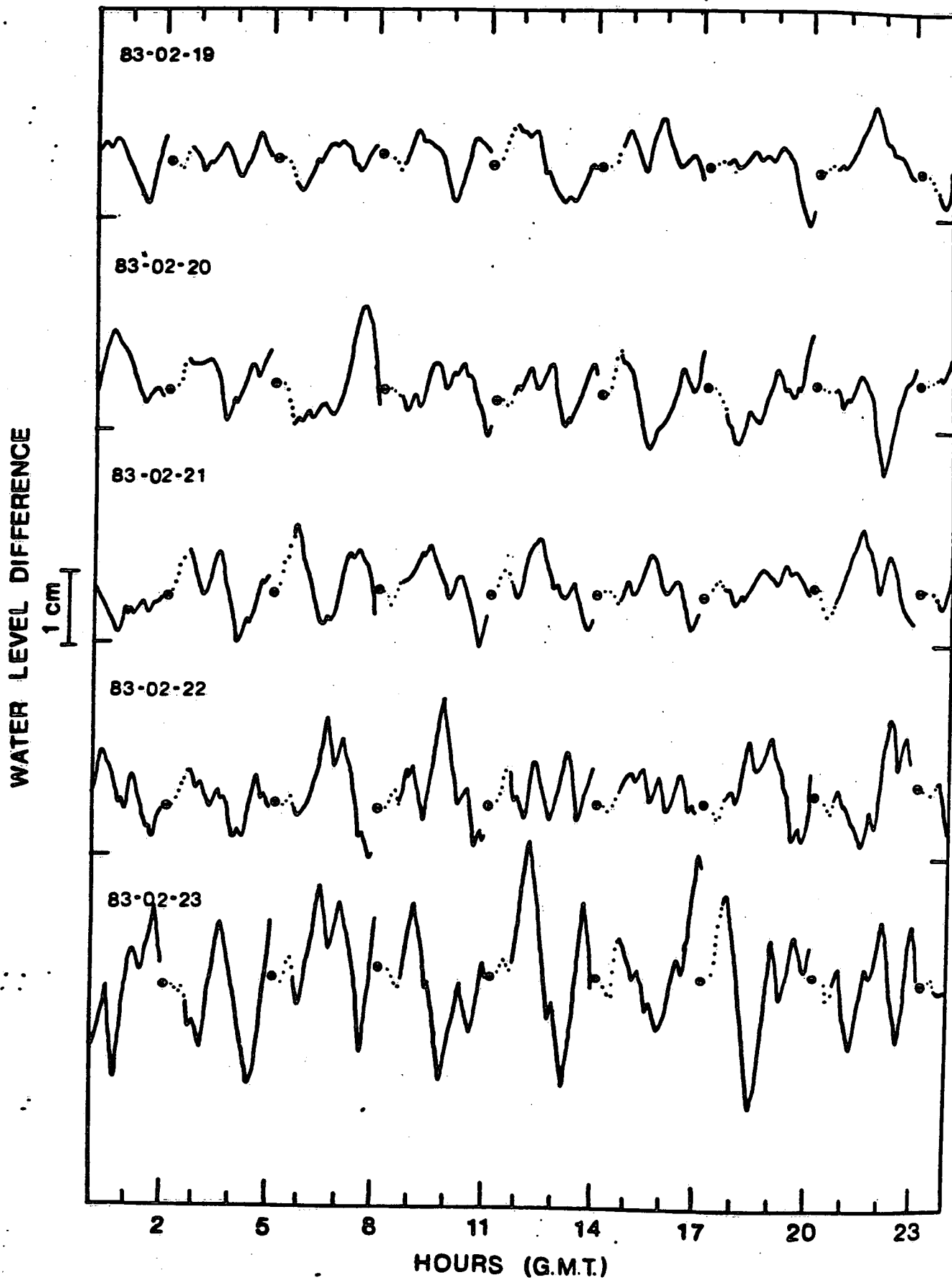


Fig. 1c. Observed water level differences.



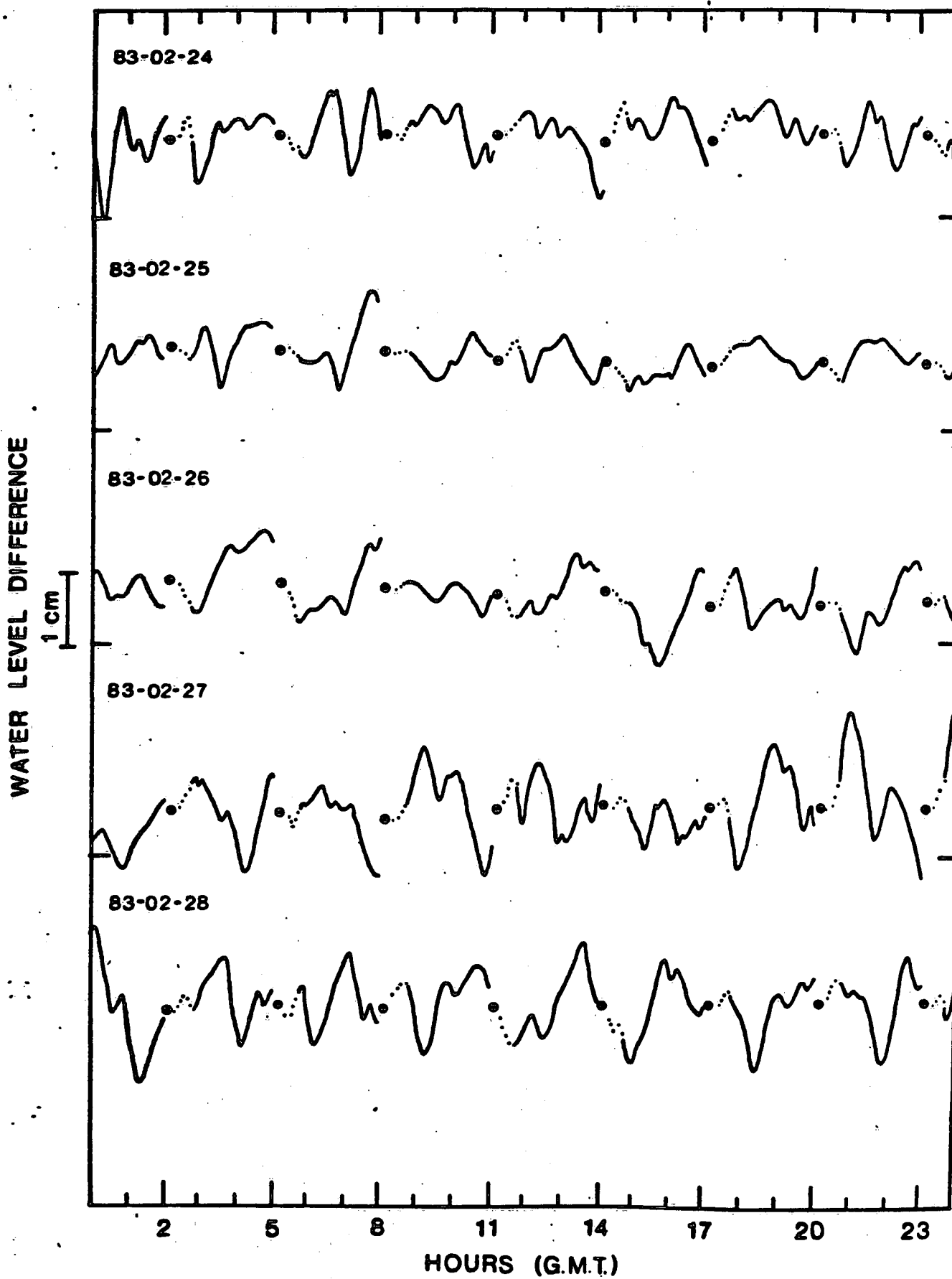


Fig. 1d. Observed water level differences.

WATER LEVEL DIFFERENCE

1 cm

83-03-01

83-03-02

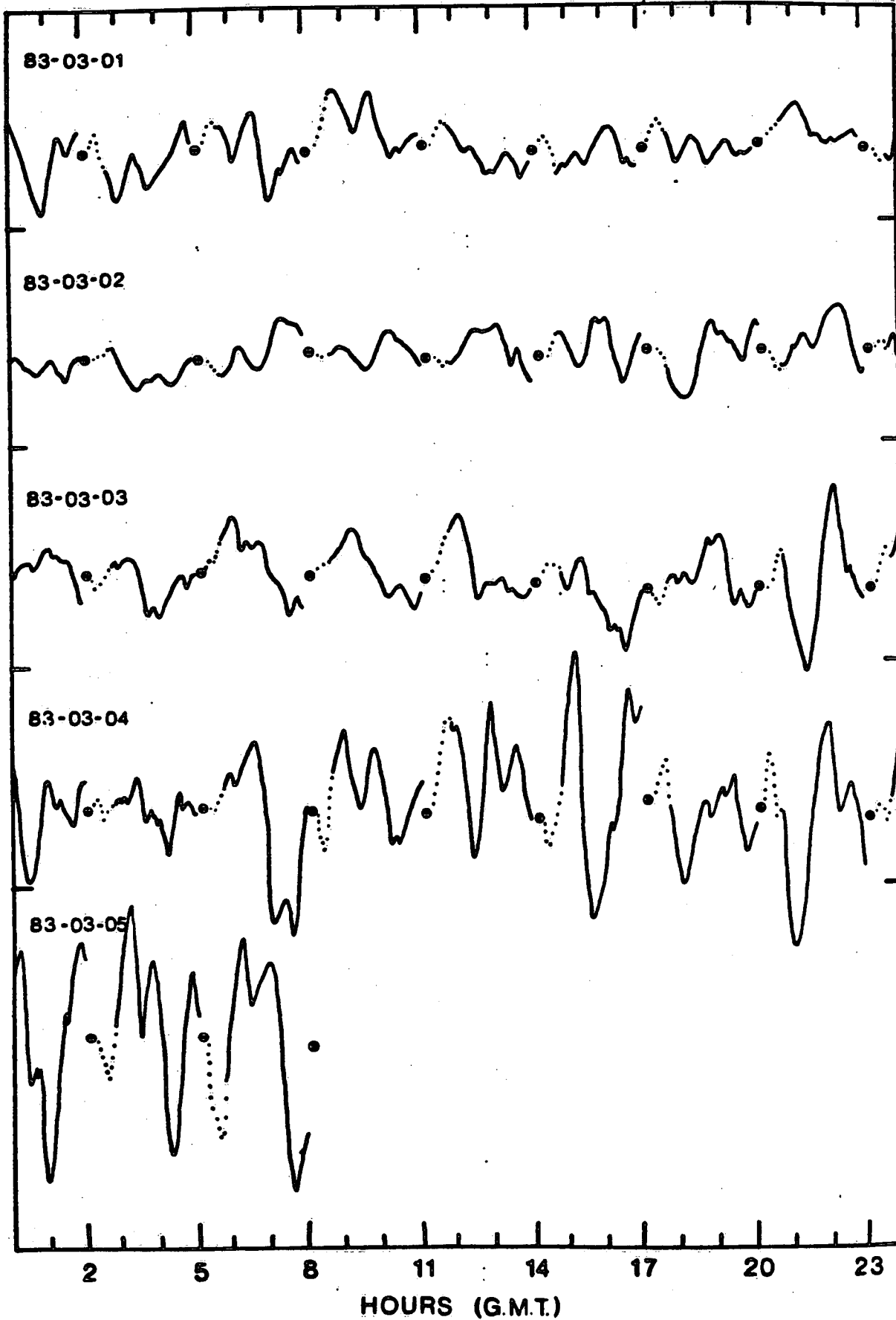
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HOURS (G.M.T.)

Fig. 1e. Observed water level differences.



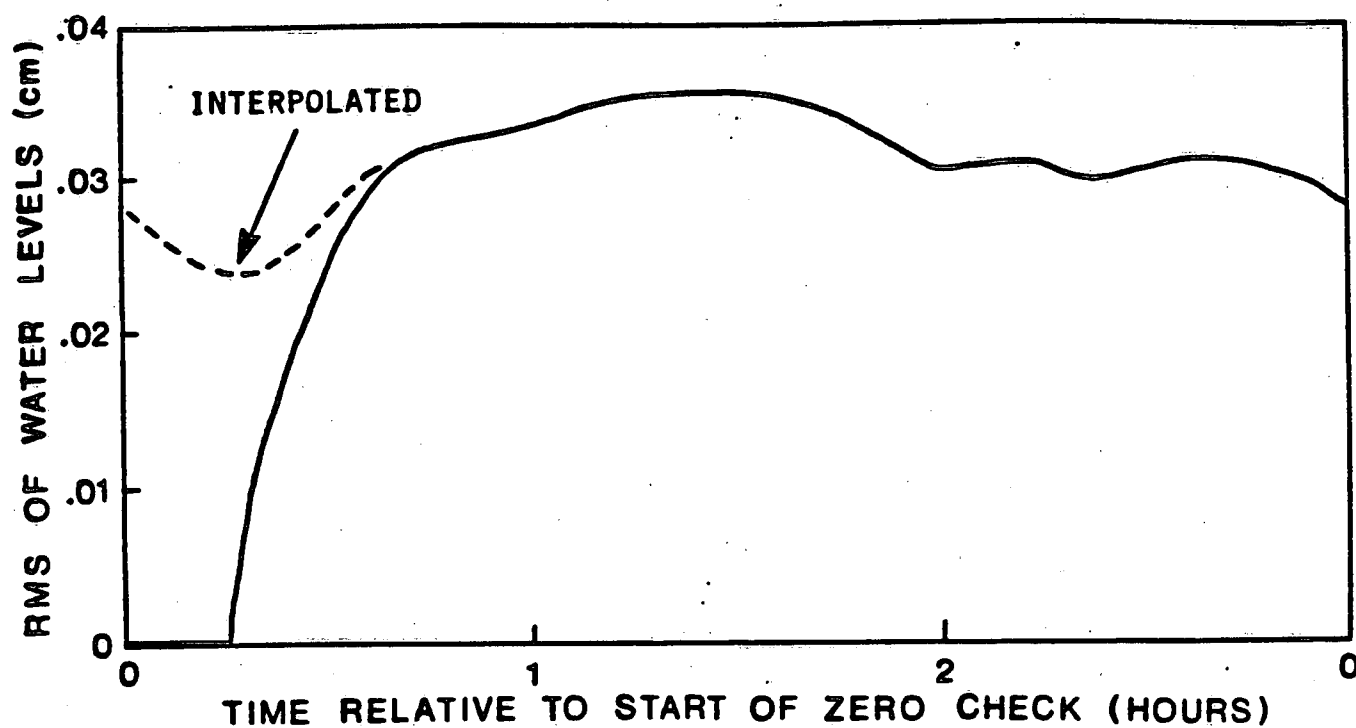


Fig. 2. Root mean square value of water level difference as a function of time between zero checks, before and after interpolation.

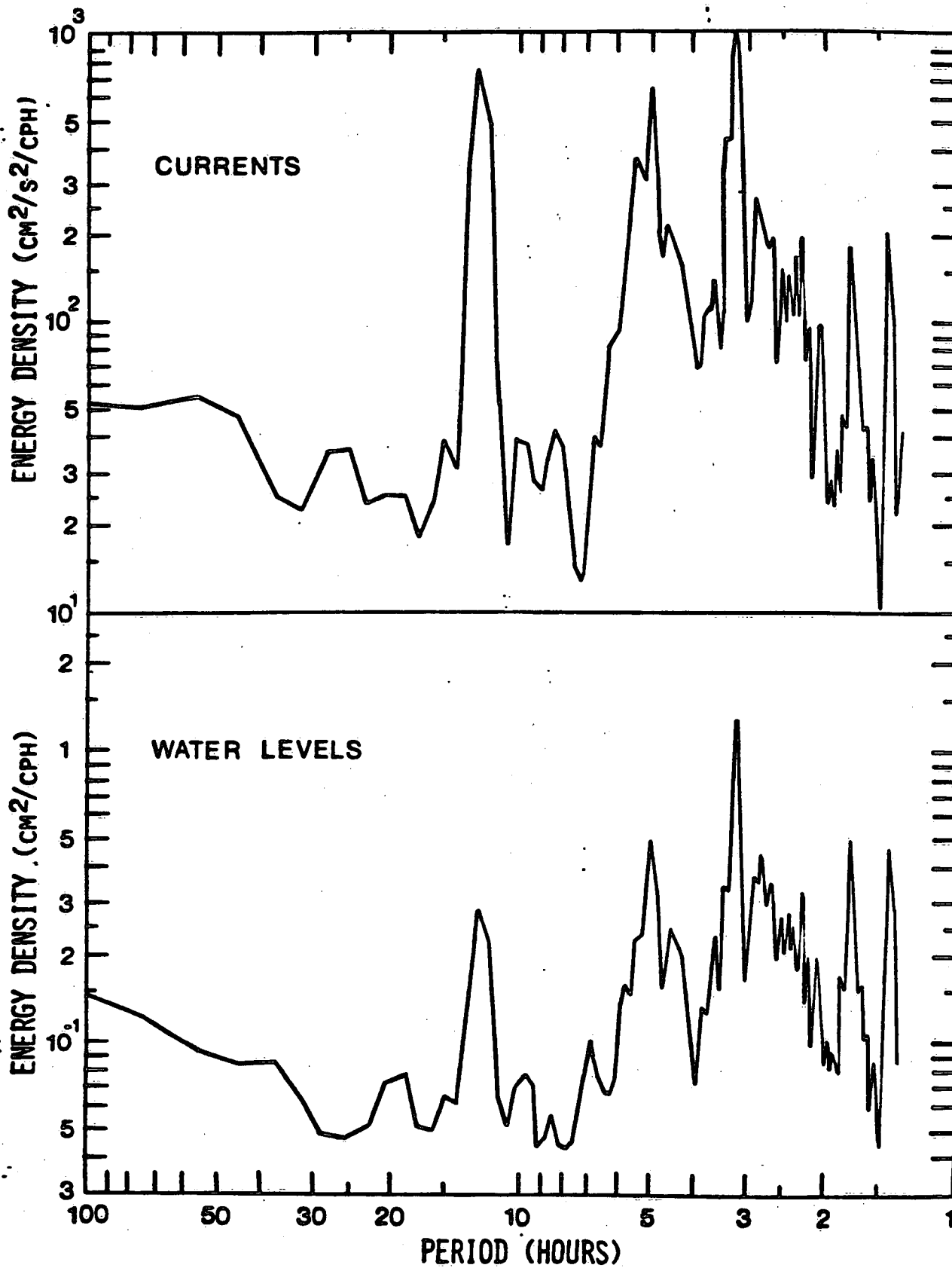


Fig. 3a. Energy spectra of currents and water levels (differences) from February 9 to March 4, 1983.

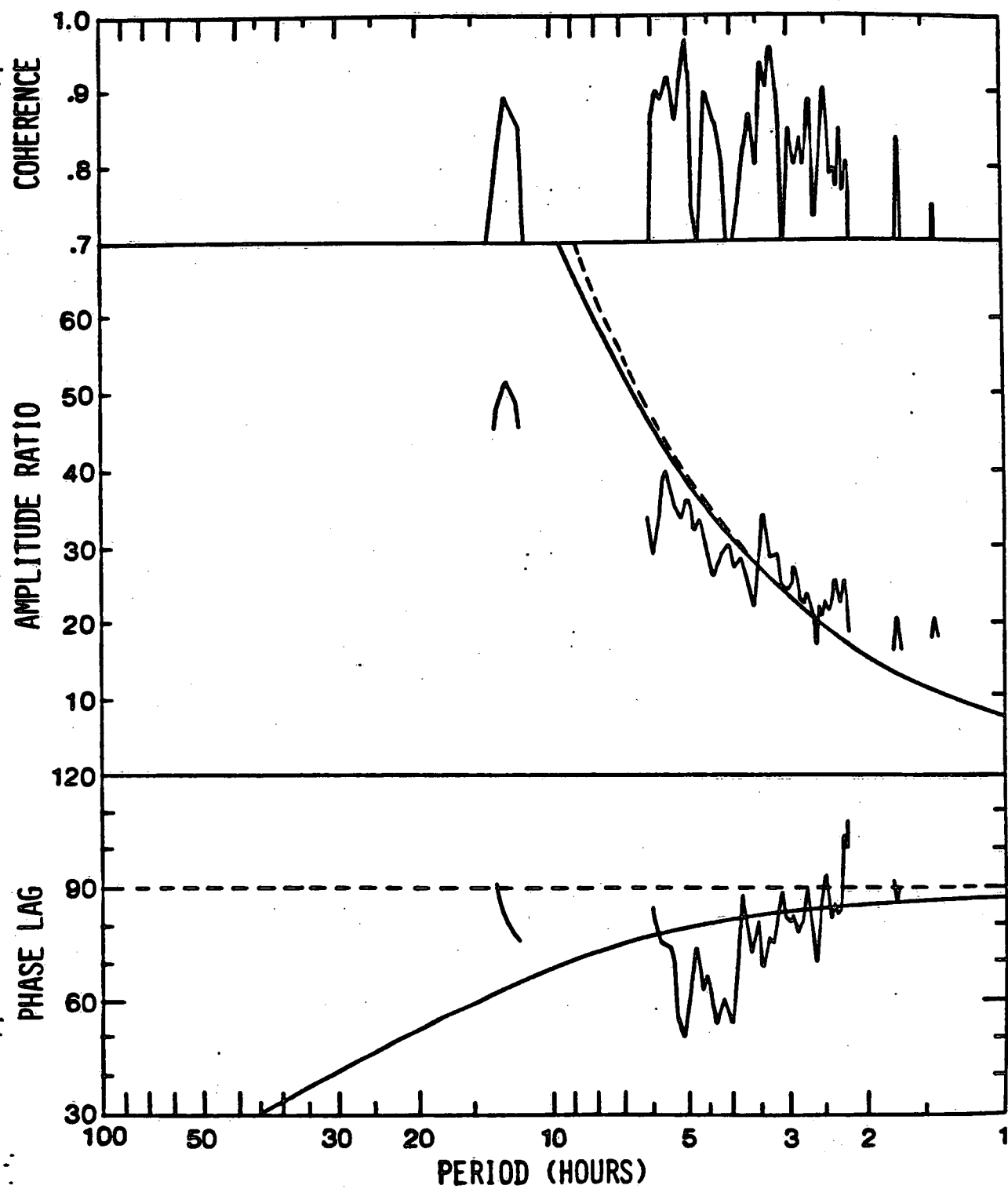


Fig. 3b. Spectral response of currents to water levels from February 9 to March 4, 1983.

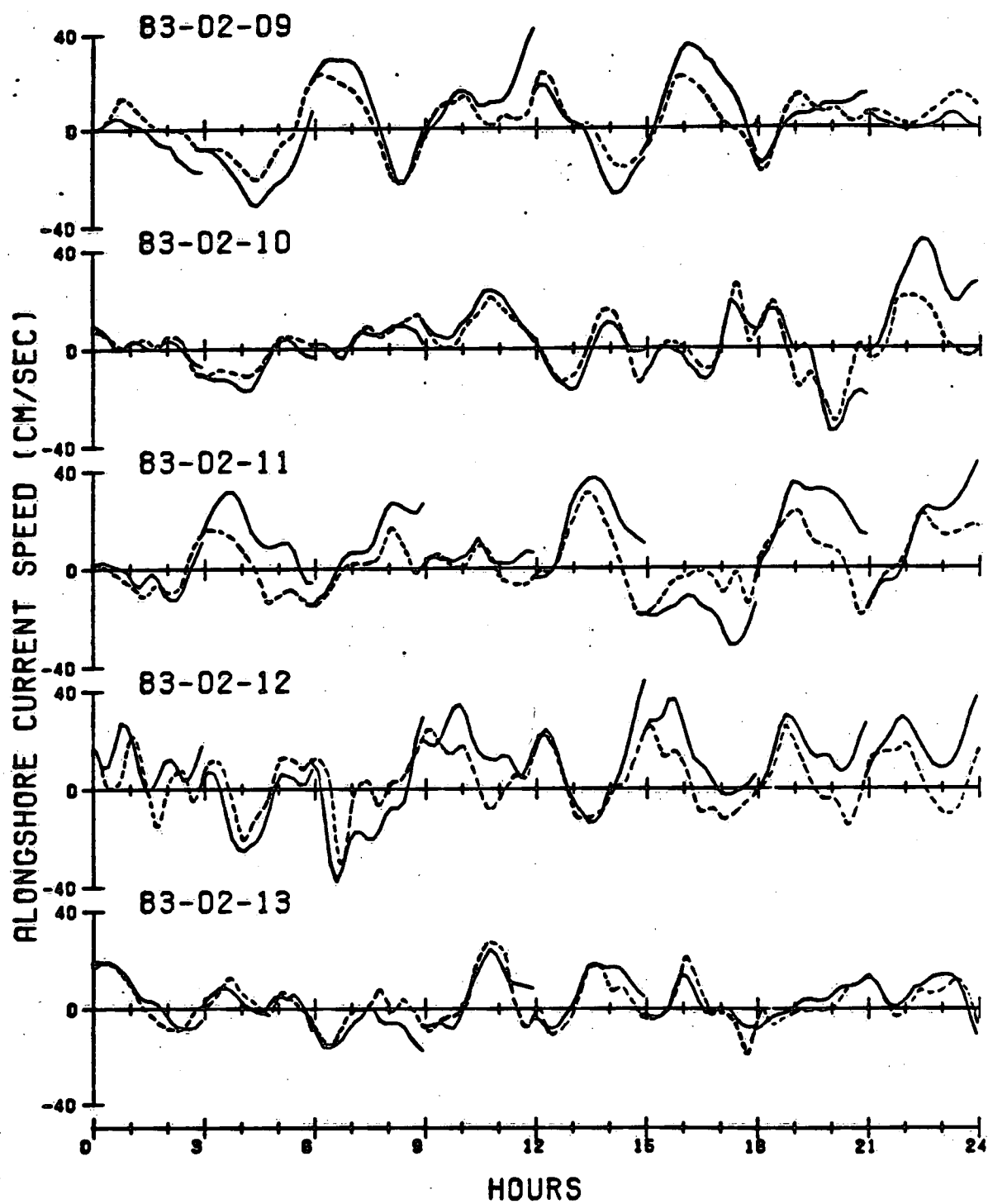


Fig. 4a. Observed (dashed) and computed (solid) currents.

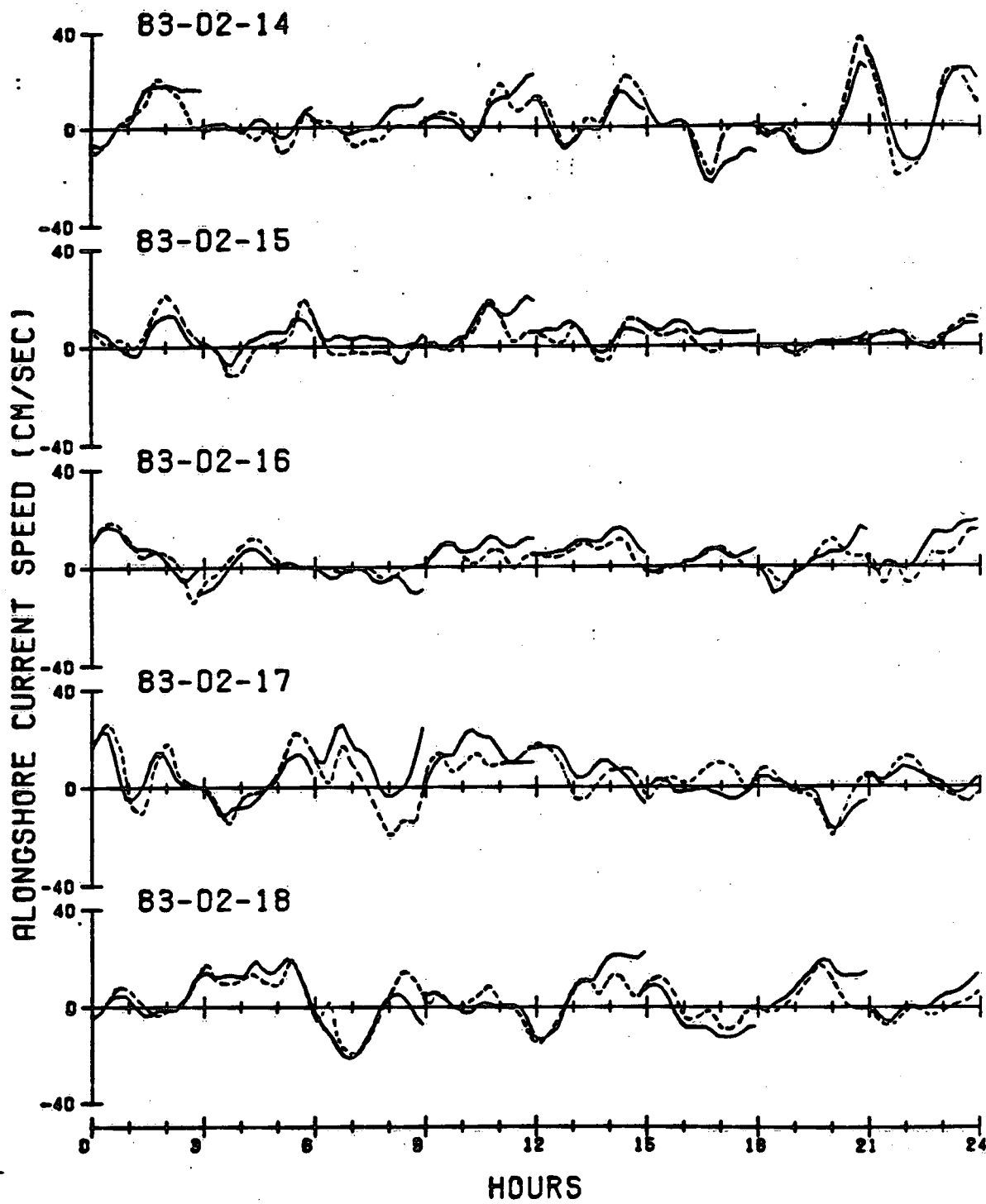


Fig. 4b. Observed (dashed) and computed (solid) currents.

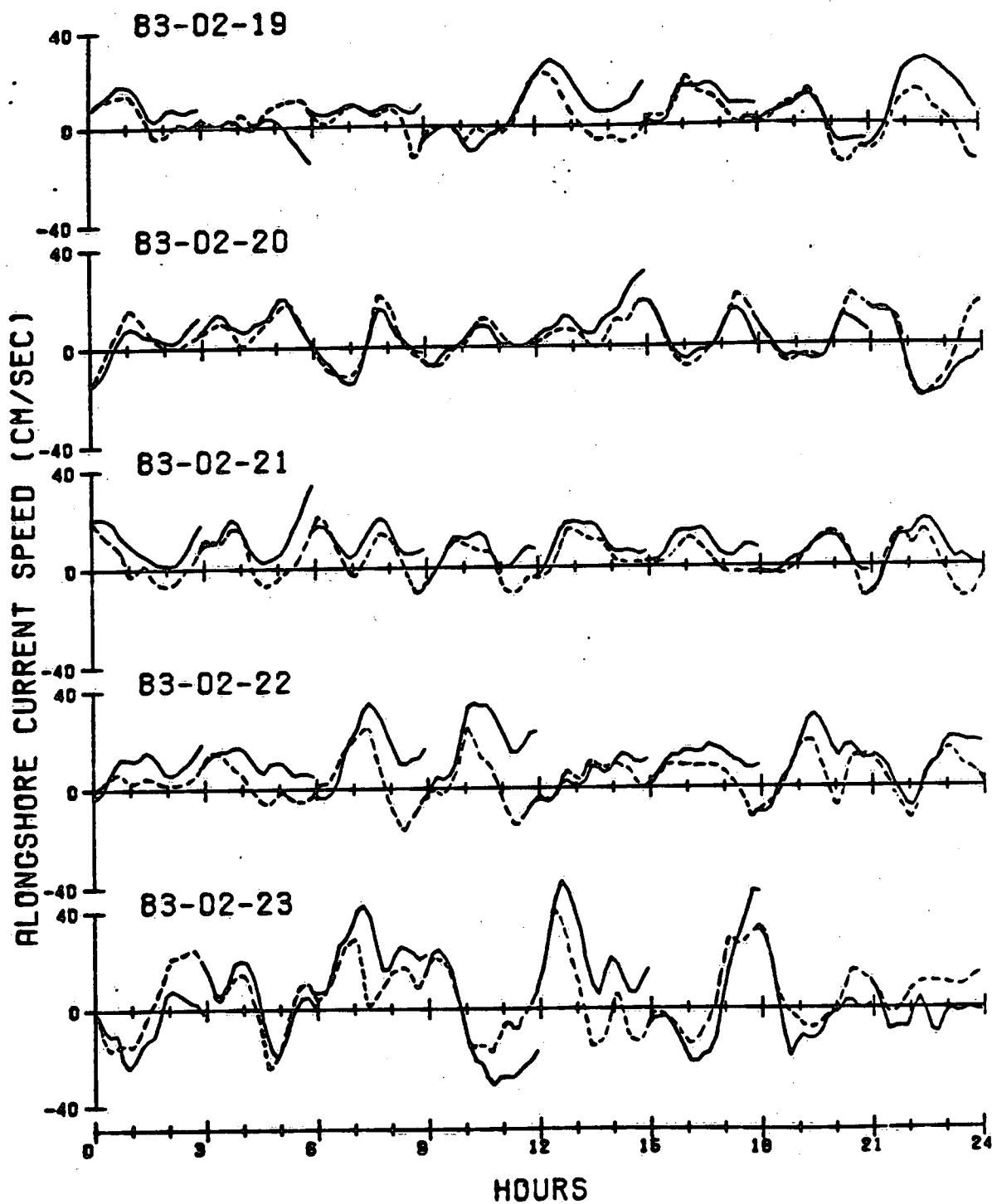


Fig. 4c. Observed (dashed) and computed (solid) currents.



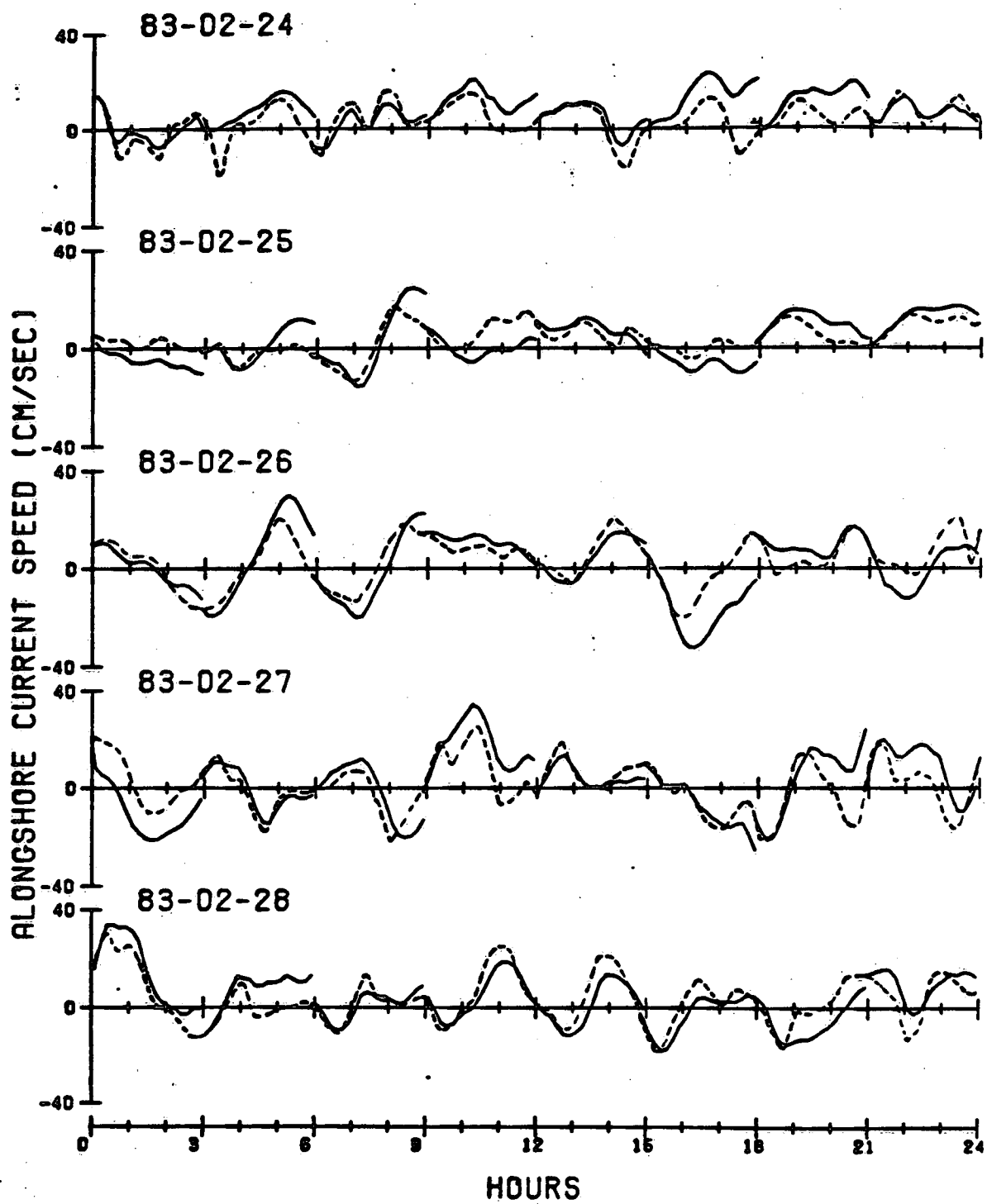


Fig. 4d. Observed (dashed) and computed (solid) currents.

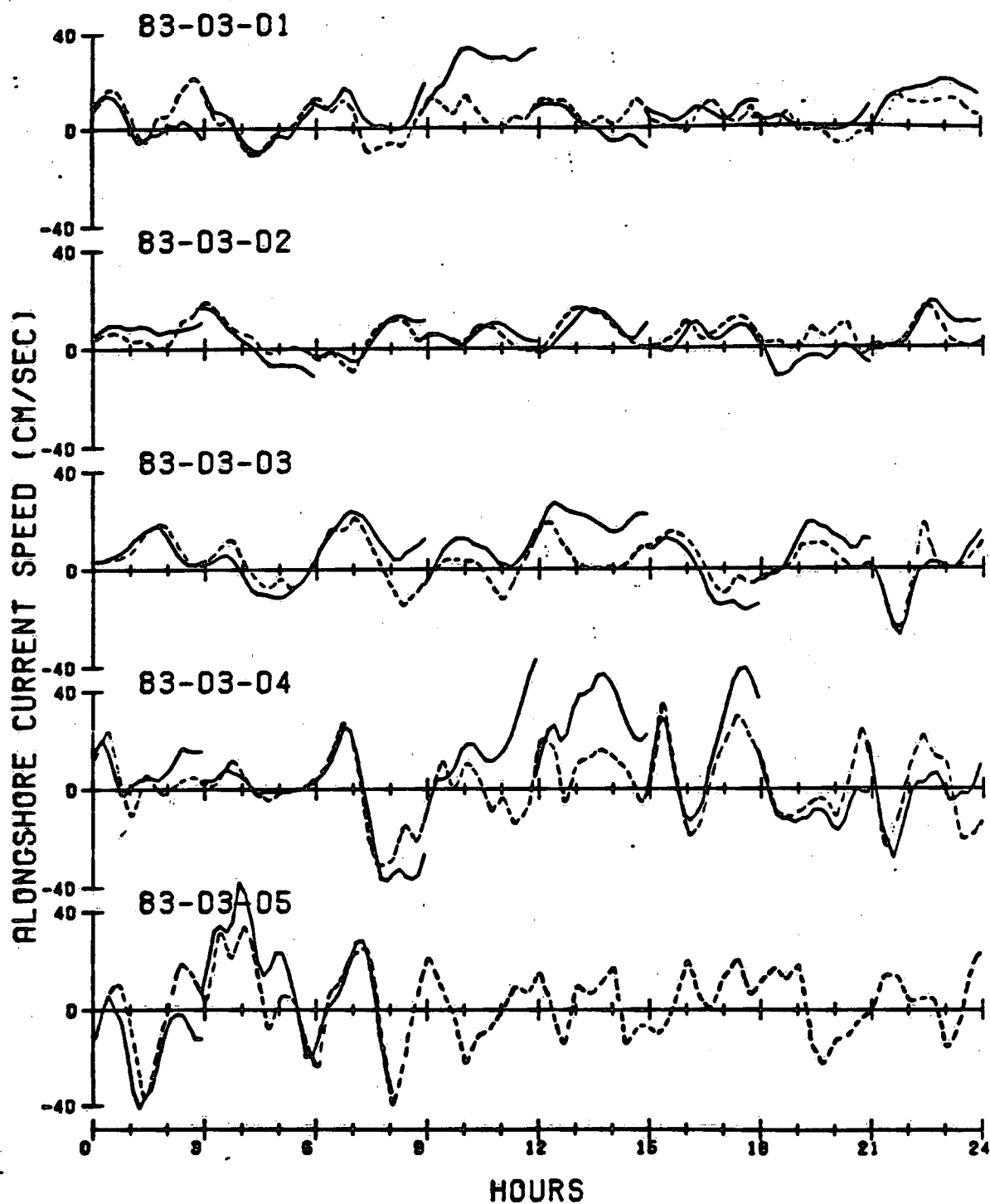


Fig. 4e. Observed (dashed) and computed (solid) currents.

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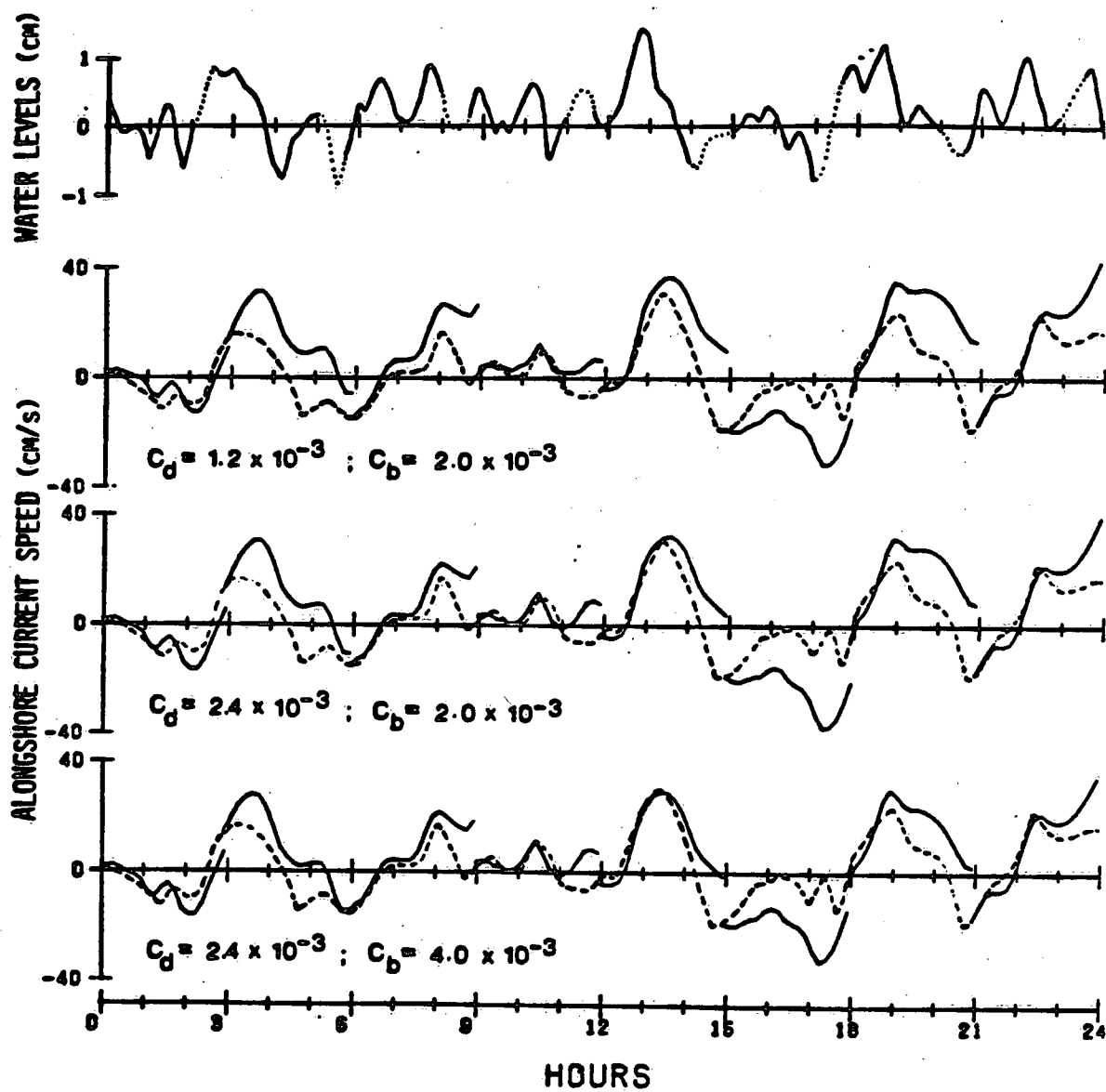


Fig. 5. Interpolated water levels and observed (dashed) and computed (solid) currents for different wind and bottom stress components.

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