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INCEPTION OF SEDIMENT TRANSPORT ON STEEP SLOPES - PRELIMINARY REPORT

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NWRI Contribution No. 97-70

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MANAGEMENT PERSPECTIVE

Effective aquatic ecosystem protection requires better understanding of the sediment transport process at the bed of steep streams. Movement of gravel and the intrusion of sediment fines into the gravel bed greatly restrict transport of dissolved oxygen into and through the bed substrate. The ability of interstitial water flow to sustain substrate organisms and remove metabolic waste is thus reduced. These factors have significant implications for maintaining river habitat functions and point to the importance of continuing research on physical river processes. This report shows that present knowledge on the beginning of sediment transport in steep gravel bed streams needs to be expanded to provide design information for channel maintenance and management over a wide range of practical flow conditions.

SOMMAIRE À L'INTENTION DE LA DIRECTION

Pour protéger efficacement l'écosystème aquatique, il faudra mieux comprendre le processus de transport des sédiments au niveau du lit de cours d'eau à forte pente. Le déplacement du gravier et l'intrusion de fines de sédiments dans le lit de gravier limitent fortement le transport de l'oxygène dissous vers le substrat du lit et à l'intérieur de celui-ci. La capacité de l'écoulement d'eau interstitielle de soutenir les organismes du substrat et d'éliminer les résidus métaboliques se trouve ainsi réduite. Ces facteurs jouent un rôle significatif pour le maintien des fonctions de l'habitat de la rivière et montrent l'importance de recherches soutenues sur les processus physiques intervenant dans les cours d'eau. Le présent rapport indique que les connaissances actuelles sur le début du transport des sédiments dans des cours d'eau à lit de gravier en forte pente doivent être élargies pour l'obtention de l'information technique nécessaire à l'entretien et à la gestion d'un cours d'eau pour une vaste gamme de conditions d'écoulement pratiques.

ABSTRACT

Using dimendional and theoretical analysis together with limited available experimental data, it has been shown how flow conditions and stream bed slope should affect the beginning of sediment transport. Experimental data show good agreement with theoretical curves developed. Further experiments are recommended.

RÉSUMÉ

L'analyse dimensionnelle et théorique, de concert avec les données expérimentales limitées disponibles, ont montré comment les conditions d'écoulement et la pente du lit du cours d'eau influent sur le début du transport des sédiments. Les données expérimentales révèlent une bonne corrélation entre les courbes théoriques obtenues. D'autres expériences sont recommandées.

TABLE OF CONTENTS

INCEPTION OF SEDIMENT TRANSPORT ON STEEP SLOPES - PRELIMINARY REPORT

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INTRODUCTION

The problem of defining critical flow conditions associated with the inception of sediment transport is of fundamental importance in the study of sediment transport mechanics. It is very relevant to the specific requirements of stable channel design and protection against erosion and scour. There are basically two ways of looking at the problem: 1) by considering some average velocity such as the mean flow velocity, velocity at the bed or velocity at some given distance above the bed and 2) by considering the average shear stress exerted on the bed particles by the flow. Velocity criteria were used by early investigators. Fortier and Scobie (1926) presented results for permissible bottom velocities in canals. Hjulstrom (1935) presented a diagram of critical mean flow velocity as a function of grain diameter over a wide range of sizes. This diagram was widely used and is included in many text books of geology and engineering. A major advance in the determination of the beginning of sediment transport was provided by Shields (1936) when he introduced his critical shear stress diagram. The dimensionless shear stress was plotted as a function of the grain size Reynolds number over the smooth, transitional and fully rough turbulent flow regimes. Since the introduction of the Shields diagram the trend in research has been to abandon velocity criteria in favour of shear stress because, in contrast to velocity, it does not require flow depth as an independent variable. Nevertheless, when water surface slope cannot be determined reliably, velocity criteria are still used (Neill, 1967; Engel, 1983; Whitehouse and Hardisty, 1988). The result has been the production of a large body of data to further define the Shields curve. The most recent and complete version of this curve was presented by Yalin and Karahan (1979) using carefully scrutinized, available data and results from their own experiments (Figure 1).

The Shields curve is limited to bed slopes which are very close to zero. In the case of mountain gravel bed streams, the slopes are much larger and for these the Shields curve is not valid. When non-cohesive sediments are being transported along a sloped surface by fluid motion, gravity will have an increased effect upon the magnitude of the sediment transport (Damgaard et al, 1996). Limited experiments by Luque and Van Beek (1976) have shown that the critical shear stress required for the initiation of sediment movement decreases as the bed slope increases. Similar results were found by Chiew and Parker (1994) and Damgaard et al (1996). As the bed slope tends towards the natural angle of repose of the sediment, the critical shear stress tends toward zero.

In this report the physical conditions required to define the inception of sediment transport on sloping beds are examined using dimensional and theoretical analysis together with limited experimental data from the literature. The results will be used to develop a plan for an experimental investigation to determine the critical shear stress over a wider range of bed slopes and Reynolds numbers.

DIMENSIONAL ANALYSIS

The present analysis is restricted to two dimensional, clear, uniform tranquil flow over a plane bed composed of uniformly sized, granular, cohesionless particles of similar shape. For such a flow the sediment discharge should be expressed as

$$q_s = f(h, D_{50}, S, \rho, \rho_s, \nu, g) \quad (1)$$

where q_s = the sediment discharge per unit width, f denotes a function, h = the depth of flow, D_{50} = the median diameter of the bed particles, S = the water surface slope of the uniform flow, ρ = the density of the fluid, ρ_s = the material density of the bed particles, ν = the kinematic viscosity of the fluid and g = the acceleration due to gravity. The gravitational acceleration can be replaced by u_* because $u_* = \sqrt{ghS}$ and ρ_s can be replaced by γ_s because $\gamma_s = g(\rho_s - \rho)$. Substituting these replacements into equation (1) results in

$$q_s = f_1(h, D_{50}, S, \rho, \gamma_s, \nu, u_*) \tag{2}$$

where f_1 denotes a function, γ_s = the submerged specific weight of the particles and u_* = the shear velocity. Using the Buckingham Π theorem of dimensions and taking ρ, D_{50} and u_* as repeating variables, one obtains the dimensionless equation

$$\frac{q_s}{\rho u_*^3} = f_2(Y, X, Z, S) \tag{3}$$

where f_2 denotes a function, $Y = \frac{\rho u_*{}^2}{\gamma_s D_{50}}$, $X = \frac{u_* D_{50}}{\nu}$, $Z = \frac{h}{D_{50}}$. At the point of inception of transport, $q_s \approx 0$ and for this condition one has

$$f_2(Y_{cr}, X_{cr}, Z_{cr}, S) = 0 \tag{4}$$

which after rearranging becomes

$$Y_{cr} = f_3(X_{cr}, Z_{cr}, S_{cr}) \tag{5}$$

where f_3 denotes another function. Bettes (1984) found that for values of $Z_{cr} > 4$, Y_{cr} is independent of Z_{cr} and therefore for such conditions Z_{cr} can be removed from equation (5). As a result, equation (5) is reduced to

$$Y_{cr} = f_4(X_{cr}, S_{cr}) \tag{6}$$

where f_4 denotes another function. Equation (6) is the general form of condition for the inception of sediment transport on sloped beds. The Shields curve in Figure 1 represents the special case of zero or very near zero slope S_{cr} in equation (6). For values of $X_{cr} > 40$, the effect of viscosity is negligible and for such cases, $Y_{cr} \approx 0.05$. Clearly, for rough turbulent flows and values of $S_{cr} > 0$, the inception of sediment transport depends only on the slope S_{cr} as long as $Z_{cr} > 4$.

Experimental results from Luque and van Beek (1976) show the effect of slope on the Y_{cr}-X_{cr} plot shown in Figure 2. The results were obtained using walnut grains, sand, gravel and magnetite. The plot shows that the Shields curve ($S_{cr} = 0$) agrees quite well with the measurements for $X_{cr} > 40$. In the region of $X_{cr} \approx 15$ the data plot slightly above the Shields curve. As values of S_{cr} increase (angle α in Figure 2) Y_{cr} decreases with all curves being parallel to the Shields curve and thus to one another. The data for $X_{cr} < 30$ are less conclusive.

The data in Figure 2 provide significant insight into the effect of bed slope on the inception of sediment transport. The data are limited to a narrow range of Reynolds number, covering mostly the transitional regimes of turbulent flow. The difference in the shape of the Shields curves and lower limit of the turbulent flow regime ($X_{cr} \approx 40$ in Figure 1 and $X_{cr} \approx 200$ in Figure 2) is mainly due to interpretation of experimental data. It is the writers' opinion that Figure 1 is more rigorous for the case of zero or near zero slope. More data are required to cover the turbulent flow regime which is most important for engineering applications. It is also necessary to determine the shape of the curve for each slope from the turbulent through the transitional into the smooth turbulent flow regimes.

THEORETICAL CONSIDERATIONS

Water flowing over a bed of sediment exerts forces on the grains which tend to move or entrain them. The forces that resist the entraining action of the flowing water differ depending on the properties of the bed material. For coarse sediments such as sands and gravels, the resisting forces mainly relate to the weights of the particles. When the hydrodynamic forces acting on a grain of sediment have reached a value such that the grain will move if the forces are increased even slightly, then critical or threshold conditions are said to have been reached. Under such critical conditions the hydrodynamic forces acting on a grain are just balanced by the resisting force of the particle.

The forces acting on a particle of bed material in a turbulent flow are the submerged weight of the particle, the lift force and the drag force, as shown in Figure 3. In this figure, F_L = the lift force, F_d = the drag force, α = the angle of the sloped bed and W_s = the submerged weight of the particle.

The critical condition at which a sediment particle is just at the point of moving can be expressed as a safety factor which is equal to the ratio of the overturning moment and restoring moment. This can be expressed by writing

$$S_F = \frac{e_2 W_s \cos\alpha}{e_1 W_s \sin\alpha + e_3 F_d + e_4 F_L} \qquad (7)$$

where S_F = the safety factor, and e_1, e_2, e_3 and e_4 are the moment arms as shown in Figure 3. Simplifying and rearranging terms in equation (7) results in

$$S_F = \frac{\cos\alpha}{\frac{e_1}{e_2}\sin\alpha + \left(\frac{e_3 F_d}{e_2 W_s} + \frac{e_4 F_L}{e_2 W_s}\right)} \qquad (8)$$

Equation (8) can be simplified by writing

$$\eta = \left(\frac{e_3 F_d}{e_2 W_s} + \frac{e_4 F_L}{e_2 W_s}\right) \qquad (9)$$

which, after substitution in equation (8) results in

$$S_F = \frac{\cos\alpha}{\frac{e_1}{e_2}\sin\alpha + \eta} \tag{10}$$

The ratio $\frac{e_1}{e_2}$ can be determined by using the boundary conditions of zero flow for which $\eta = 0$. At the critical condition, $S_F = 1$, and for zero flow, the angle of the slope α is equal to the natural angle of repose of the bed material. Denoting the angle of repose as ϕ, one obtains

$$\frac{e_2}{e_1} = \frac{\sin\phi}{\cos\phi} = \tan\phi \tag{11}$$

Substituting equation (11) in equation (10) results in

$$S_F = \frac{\cos\alpha}{\frac{\sin\alpha}{\tan\phi} + \eta} \tag{12}$$

If one now assumes that $F_d = C_1 D_{50}^2 \tau$ and $F_L = C_2 D_{50}^2 \tau$, where τ = the bed shear stress, then upon substituting in equation (8) one obtains

$$S_F = \frac{\cos\alpha}{\frac{e_1}{e_2}\sin\alpha + \tau\left(\frac{e_3 C_1 D_{50}^2}{e_2 W_s} + \frac{e_4 C_2 D_{50}^2}{e_2 W_s}\right)} \tag{13}$$

which after writing

$$K = \tau\left(\frac{e_3 C_1 D_{50}^2}{e_2 W_s} + \frac{e_4 C_2 D_{50}^2}{e_2 W_s}\right) \tag{14}$$

becomes

$$S_F = \frac{\cos\alpha}{\frac{\sin\alpha}{\tan\phi} + \tau K} \tag{15}$$

Finally, after some rearranging of terms in equation (15) one arrives at

$$\tau = \frac{1}{K}\left(\frac{\cos\alpha}{S_F} - \frac{\sin\alpha}{\tan\phi}\right) \tag{16}$$

Equation (16) is the general equation for the bed shear stress that can be sustained for a given safety factor. Clearly, the bed is stable for any value of safety factor S_F greater than 1. Alternatively, when the safety factor is less than 1, the bed particles will move freely and the bed is said to fail. At the critical condition, when the particles are just on the verge of moving, $S_F = 1$. For this case equation (16) becomes

$$\tau_{cr} = \frac{\cos\alpha}{K_{cr}}\left(1 - \frac{\tan\alpha}{\tan\phi}\right) \tag{17}$$

Equation (17) can be compared with equation (6) if one makes the critical shear stress dimensionless so as to be identical to Y_{cr}. This results in

$$Y_{cr} = \frac{\tau_{cr}}{\gamma_s D_{50}} = \frac{\cos\alpha}{\gamma_s D_{50} K_{cr}}\left(1 - \frac{\tan\alpha}{\tan\phi}\right) \tag{18}$$

Most data are available for beds with horizontal or virtually horizontal bed slopes and these data have been consolidated into the Shields curve as shown in Figure 1. For this condition $\alpha \approx 0$ and equation (18) is reduced to

$$Y_{cr} = \frac{1}{\gamma_s D_{50} K_{cr}} \tag{19}$$

It can be seen from equation (6) that Y_{cr} is a function of the roughness Reynolds number X_{cr} and therefore K_{cr} must also be a function of X_{cr}. For the rough turbulent flow regime, $Y_{cr} = 0.05$ and for this case

$$K_{cr} = \frac{20}{\gamma D_{50}} \tag{20}$$

When the bed slope is greater than zero, equation (18) represents the theoretical equivalent of equation (6) with K_{cr} representing the effect of X_{cr} and the terms containing α representing the effect of the bed slope.

Determining the critical shear stress with equation (17) is not very convenient because it requires knowledge of the relationship between K_{cr} and X_{cr}. Although this relationship can be determined from the Shields curve, it is much simpler to take the ratio of the critical shear stress for the sloped bed and the critical shear stress from the Shields curve for a particular value of X_{cr}. This ratio can be obtained from equations (18) and (19) which takes the form

$$\frac{Y_{cr\alpha}}{Y_{crh}} = \cos\alpha\left(1 - \frac{\tan\alpha}{\tan\phi}\right) \tag{21}$$

where $Y_{cr\alpha}$ = the critical dimensionless shear stress when the bed is sloped at angle α and Y_{crh} = the critical dimensionless shear stress for horizontal bed (obtained from Shields curve). A similar equation was derived by Chiew and Parker (1994). Equation (21) can be further simplified with some algebraic manipulation by using basic trigonometric relationships. This results in the final relationship

$$\frac{Y_{cr\alpha}}{Y_{crh}} = \frac{\sin(\alpha - \phi)}{\sin\phi} = C_B \tag{22}$$

where C_B = the bed slope coefficient. Equation (22) shows that for a given bed material, represented by the angle of repose ϕ, the Y_{cr} ratio decreases as the slope, represented by α increases. This is compatible with intuitive expectations because, as the slope increases, the particle stability decreases and less shear stress is required to move the particle. Values of $Y_{cr\alpha}$ are easily computed because for a given condition, α, ϕ are given and Y_{crh} is determined from the Shields curve for the particular value of X_{cr}.

Data from experiments by Chiew and Parker (1994) are plotted in Figure 4 as $\frac{Y_{cr\alpha}}{Y_{crh}}$ versus $\frac{\alpha}{\phi}$. The curve for equation (21) is superimposed on the plot. It can be seen that equation (21) fits the data quite well, thus confirming its validity. Values of $Y_{cr\alpha}$ and Y_{crh} were taken from the curves prepared by Luque and van Beek (1976) in Figure 2 and their ratios also plotted on Figure 4 for the corresponding ratio $\frac{\alpha}{\phi}$. The good agreement shows that determination of $Y_{cr\alpha}$, by using the Shields curve, is valid thereby making use of a large amount of already available information.

SUMMARY AND CONCLUSIONS

It has been shown through dimensional and theoretical analysis and limited experimental data that the inception of sediment transport is dependent on the slope of the bed in addition to the boundary Reynolds number. As the bed slope increases, the critical shear stress decreases because the increase in slope increases the gravitational component of the forces tending to move the sediment particles. A theoretical equation has been developed, which makes it possible to use the Shields curve from Yalin and Karahan (1979), together with a bed slope coefficient, to compute the critical shear stress for steep gravel bed streams. Computed values agree well with experimental data.

Experimental data on critical shear stress at slopes greater than zero and approaching the natural angle of repose of the bed material is very limited. More experiments for slopes over the range $0 \leq \alpha \leq \phi$ and over a wider range of Reynolds number X_{cr} should be conducted to confirm the validity of the theoretical equation over a wider range of flow conditions. A simple, cost-effective, tilting pipe duct, similar to that used by Chiew and Parker (1994), can be used to obtain the desired results.

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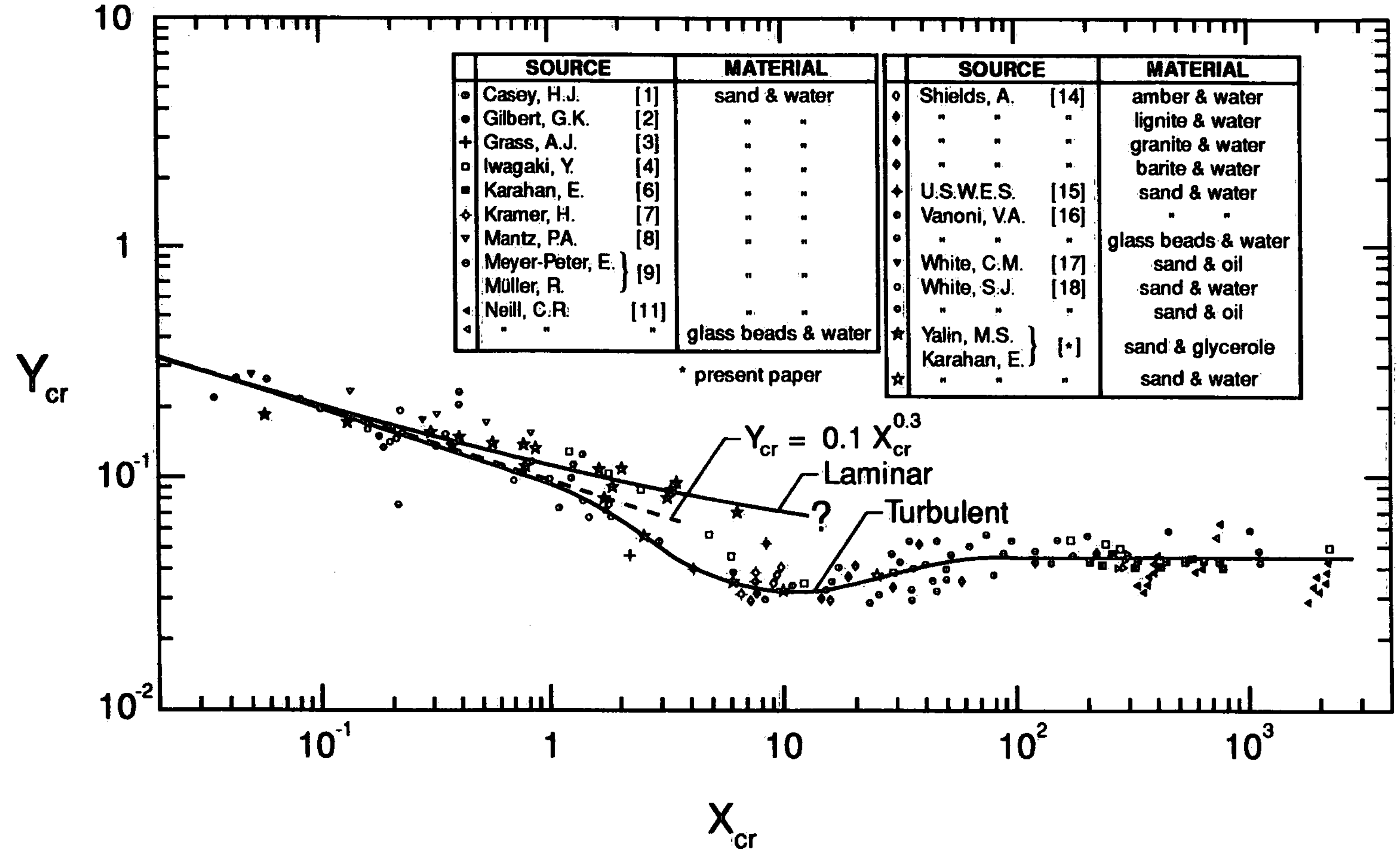

Figure 1. Shield's curve from Yalin and Karahan (1979)

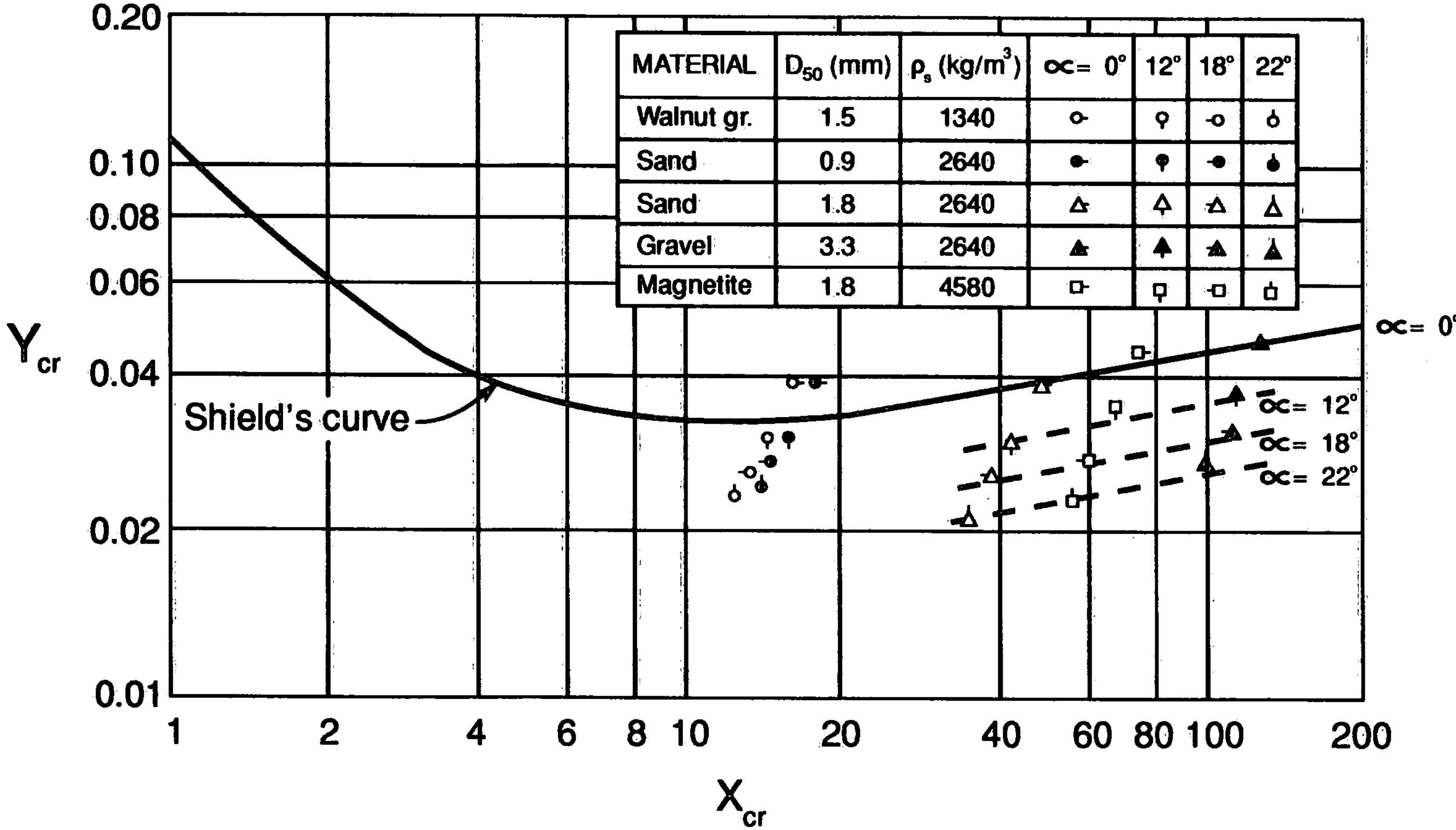

Figure 2. Experimental results from Luque and Van Beek (1976)

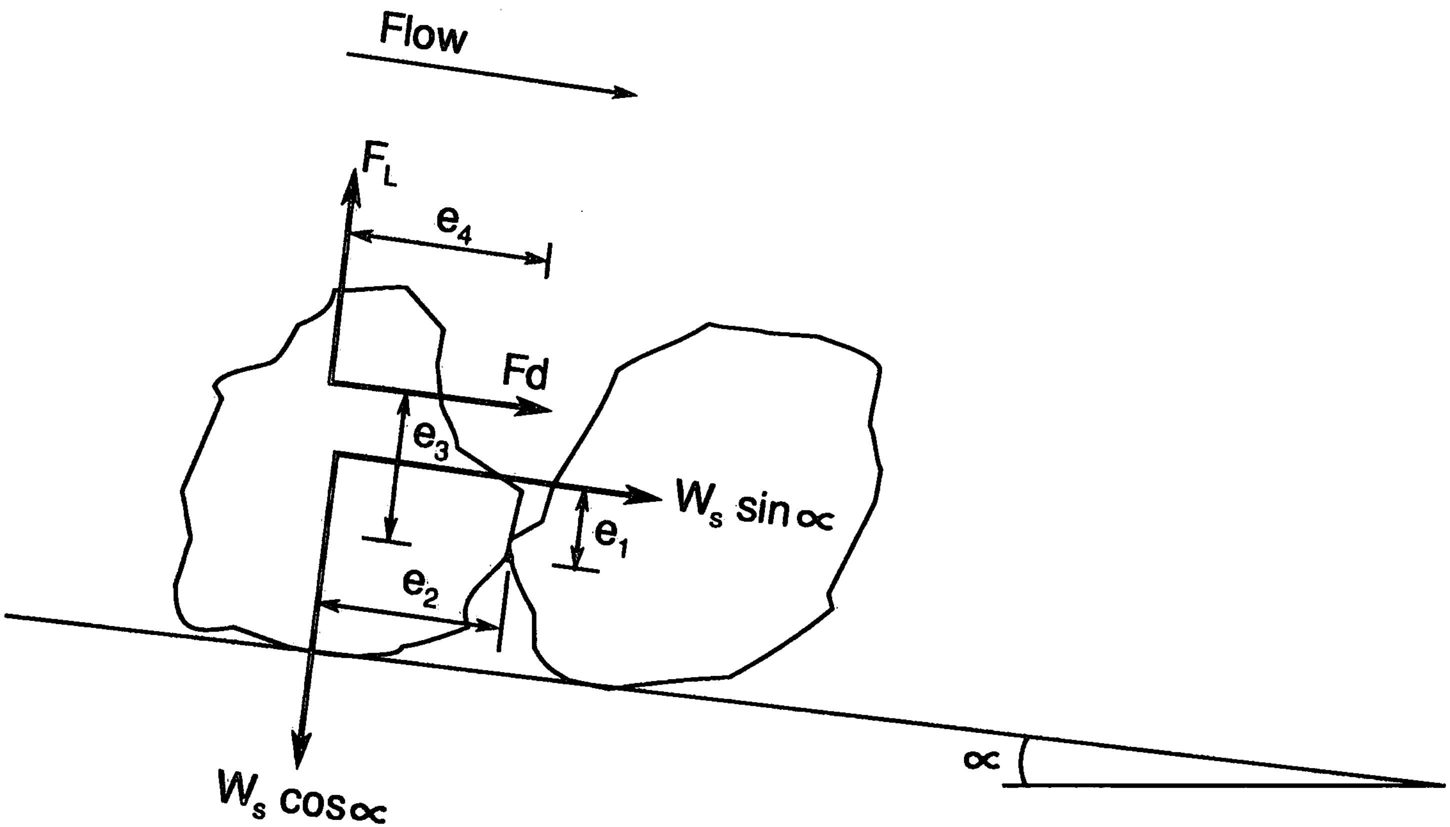

Figure 3. Schematic of forces acting on particles

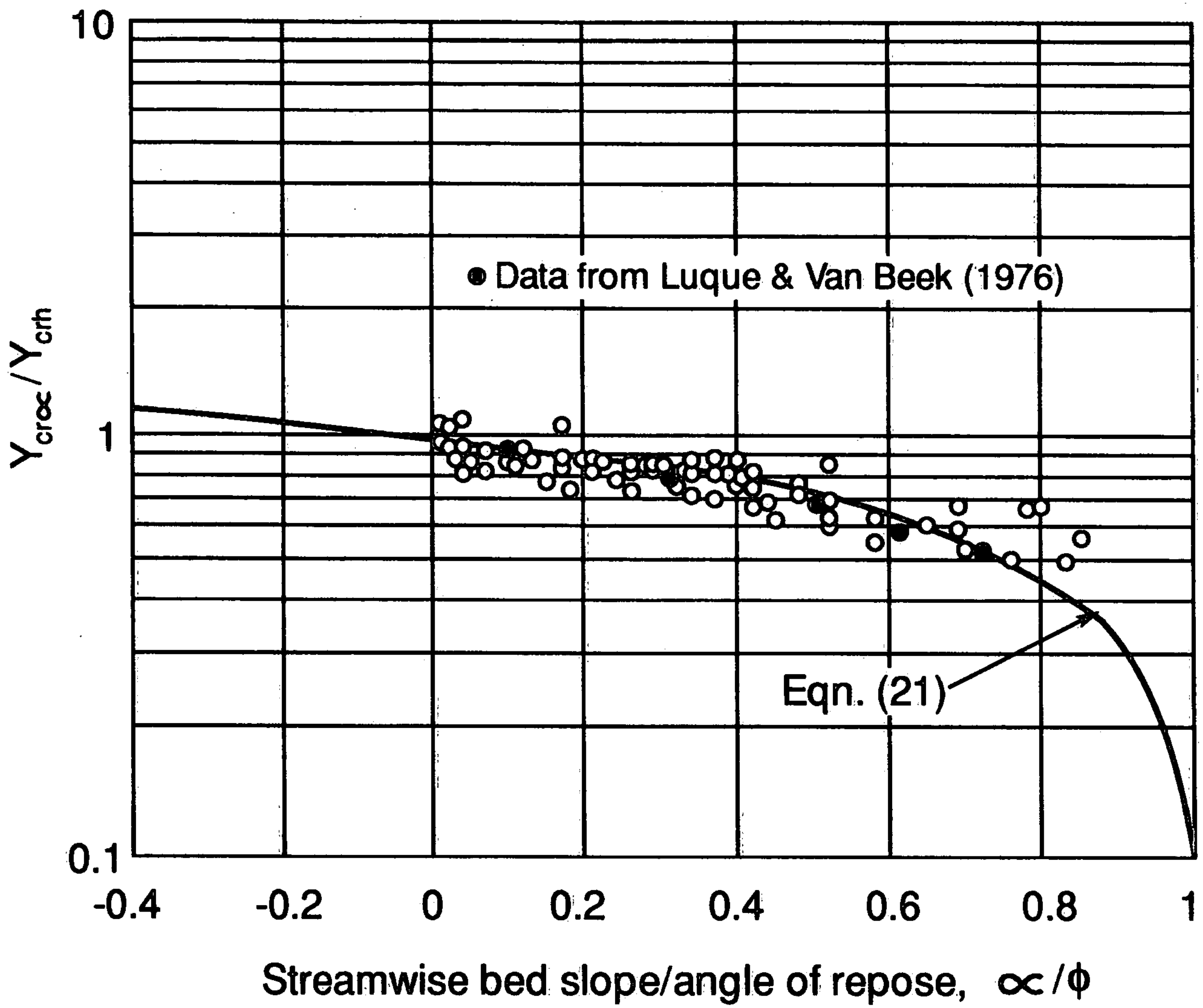

Figure 4. Experimental results from Chiew and Parker (1994)

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