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Mathematical Modelling of Sediment-Laden Flows in Natural Streams

B.G. Krishnappan and N. Snider

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Abstract

In this report, a mathematical model of a stream carrying sediment has been described. This model solves the continuity equation for the sediment-water mixture and the momentum equation numerically, and corrects the solution at each time step using the continuity equation for the sediment. This model uses an implicit finite difference approximation scheme to discretize the governing equations and a Double Sweep method to solve the resulting system of algebraic equations. The roughness characteristics of the natural streams are predicted using a method proposed recently by two Japanese scientists, Kishi and Kuroki. This method considers the effects of the various bed configurations (sand waves) present in natural streams in an adequate manner and also considers the flow regime and "skin friction" characteristics. The sediment transport rate required for the model is predicted using the method of Ackers and White, which has been found to be superior to most existing methods. The model thus incorporates the most recent advances in the field of sediment transport and should be capable of yielding reliable predictions of the responses of natural streams to changes in flow and sediment inputs, and to changes in geometry due to river crossings, protection works, realignment, etc. The application of the model is indicated using a hypothetical river reach. The flow charts, the description of the input data, the listing of the computer program and the sample model output are also given.

Résumé

Dans ce rapport, nous décrivons le modèle mathématique d'un cours d'eau transportant des sédiments. Numériquement, ce modèle résoud l'équation de continuité pour le mélange sédiments-eau et l'équation de la quantité de mouvement et il rectifie la solution à chaque étape de temps d'après l'équation de continuité pour les sédiments. Ce modèle présente un schéma d'approximation implicite aux différences finies pour discrétiser les équations qui régissent le phénomène et une méthode de double balayage pour résoudre les équations algébriques qui en découlent. On peut prévoir l'irrégularité des cours d'eau naturels par la méthode que viennent de proposer deux savants japonais, Kishi et Kuroki. Cette méthode tient compte des diverses configurations de lits (ondes de sable) suffisamment présentes dans les cours d'eau naturels et du régime de l'écoulement, des caractéristiques de "frottement superficiel". On peut prévoir le transport des sédiments nécessaire pour le modèle par la méthode d'Ackers et de White, qui s'est révélée supérieure à toutes les autres méthodes existantes. Le modèle réunit donc les progrès les plus récents dans le domaine du transport des sédiments et doit fournir des prévisions sûres sur les réponses des cours d'eau naturels touchant les changements de géométrie causés par les conduites sous-fluviales, les ouvrages de protection, le redressement. Pour l'application de ce modèle il faut se servir de la plage hypothétique d'une rivière. On donne aussi les organigrammes, les données d'entrée, le listage des programmes d'ordinateur et le rendement du modèle qui sert de spécimen.

List of Symbols

x:	Cartesian coordinate along the length of the river
η:	Cartesian coordinate along the vertical, measured from the bottom of the river
y:	flow depth
z:	height of the river bottom from a datum level
B:	width of the water surface
B_w:	width of the river at the bottom
$\xi(\eta)$:	width of the river at a level η
A:	flow cross-sectional area
A_w:	flow cross-sectional area occupied by water alone
P:	wetted perimeter
R:	hydraulic radius
Q:	flow rate of water-sediment mixture (m^3/s)
Q_w:	flow rate of water alone (m^3/s)
Q_s:	volumetric sediment transport rate ($m^3/s \cdot m$)
q_ℓ:	lateral inflow of water from tributaries ($m^3/s \cdot m$)
q_s:	lateral inflow of sediments from overland flows etc. ($m^3/s \cdot m$)
ρ:	density of sediment-water mixture
ρ_w:	density of water
ρ_s:	density of sediments
g:	acceleration due to gravity
γ_s:	submerged specific weight of sediment
υ:	kinematic viscosity of sediment-water mixture
D_{35}:	size of sediment (35% (by weight) of sediment is finer than this size)
p:	porosity
X:	concentration of sediment by weight
S_x:	bottom slope of the river
S_f:	energy gradient
v:	average flow velocity = Q/A
v_*:	shear velocity = $\sqrt{gRS_f}$
C:	friction coefficient = v/v_*

Z: relative hydraulic radius = R/D_{35}

Y: mobility number = $\rho v_*^2/\gamma_s D_{35}$

D_{gr}: dimensionless grain size = $D_{35} g \gamma_s/(\gamma \nu^2)^{1/3}$

F_{gr};G_{gr}: dimensionless parameters

F_p: pressure force acting on the surfaces of the control volume

F_f: frictional force acting along the wetted perimeter

τ_0: shear stress at the bed

Mathematical Modelling of Sediment-Laden Flows in Natural Streams

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1. INTRODUCTION

Flows in natural streams invariably carry sediments either in the vicinity of the bed or over their entire cross sections. Because of this, natural streambeds are always covered with sand waves (ripples, dunes). These sand waves are not stationary, but move along with the flow, thereby introducing further unsteadiness in the basically unsteady character that is a consequence of the time-dependent discharges of the natural streams. Predicting the interaction of the sediment movements with the water flow in natural streams is a necessity in the field of water resources management, where one is often confronted with questions about the environmental effects of changes in the flow hydrograph or sediment input.

The normal procedure to solve such problems is to build physical models in the laboratory; but with the recent advent of high-speed digital computers, there is great interest and incentive to replace the physical model with a mathematical one. Mathematical models have certain advantages over physical models. For example, physical models, because of the large geographical area involved are usually distorted (i.e. the vertical scale and horizontal scale are different) and are calibrated to study selected aspects of the flow. It is difficult to model more than one phenomenon, and flow processes such as dispersion of mass cannot be studied at all in distorted physical models. Such restrictions do not apply to the mathematical models and in general they can be used to study all aspects of the flow processes.

Mathematical models of physical processes are usually the numerical solutions of the governing differential equations. The validity of the mathematical model, therefore, depends on the accuracy of the numerical methods, the adequacy of the differential equations to describe the natural processes and the accuracy of the various input parameters.

In the case of free surface flows whose boundaries are rigid, the flow behaviour can be adequately described by "the de Saint-Venant partial differential equations of unsteady flow," which were developed by Barré de Saint-Venant as early as 1871. These equations are derived by considering the conservation principle for mass and for the momentum of the flows. As the derived partial differential equations are the hyperbolic type, they are usually solved by using the method of characteristics and a variety of finite difference and finite element methods. A summary of the various mathematical models developed for this case can be found in Ref. 1. One major problem, which is also common to physical models, is the selection of the parameter to describe the roughness characteristics of the flow boundaries.

For mobile boundary channels, three equations are needed to describe the sediment-water mixture. The third equation is obtained from consideration of the continuity equation of the sediments. In contrast to rigid boundary flows, only a limited amount of work has been done in building mathematical models for mobile boundaries. The first attempt was by Cunge and Perdreau (2) in 1973. Another attempt in the same year was by Chen (3) from Colorado State University. In the case of mobile boundary flow models, in addition to specifying the roughness characteristics of the channels, there is also a need to specify the amount of sediment transported per unit time as input parameter. In both of the above referenced models, neither the roughness character nor the sediment transport rate is adequately expressed. Cunge and Perdreau used a constant roughness factor in terms of a Strickler coefficient to express the roughness characteristics, and Meyer, Peter and Mueller's formula to express the sediment transport rate. Chen (3) used Manning's n to describe roughness and used Einstein's and Tofalleti's methods to estimate the sediment transport rate. Data collected in laboratory and field indicate that the roughness character of the flow changes, depending on the flow regimes and the type of bed forms present at the bottom of the mobile boundary channels, and hence cannot be adequately represented by a constant value for Strickler or Manning's roughness parameters. A recent paper by White et al. (9) reviewed the various existing theories for the sediment transport rate in light of a large number of laboratory and field data and concluded that the methods used by the above two models exhibit larger variation than some of the other existing methods. For these reasons, the existing mathematical models cannot predict the behaviour of the mobile boundary flows over a wide range of flow conditions and hence there is a need for further work in improving their predictive capabilities. In this report, a model is described which incorporates the most recent developments in the field of sediment transport in the areas of the roughness character of the mobile boundary flows and the prediction of the sediment transport rates. The derivation of the governing equations and the description of the numerical scheme are elaborated in this report to elucidate fully the underlying assumptions and consequently the extent of the applicability of the mathematical model.

2. DERIVATION OF THE GOVERNING EQUATIONS

The governing equations are derived for non-prismatic rivers with irregular cross sections. The velocity field of the river flow under consideration is assumed to be one dimensional and the pressure field varies in the vertical direction according to the hydrostatic pressure distribution. This implies that the river reach to be modelled should be reasonably straight and the vertical accelerations negligible.

The symbols and the coordinate system used in the derivation are indicated in Figure 1, which illustrates schematically the river cross section and the longitudinal profile.

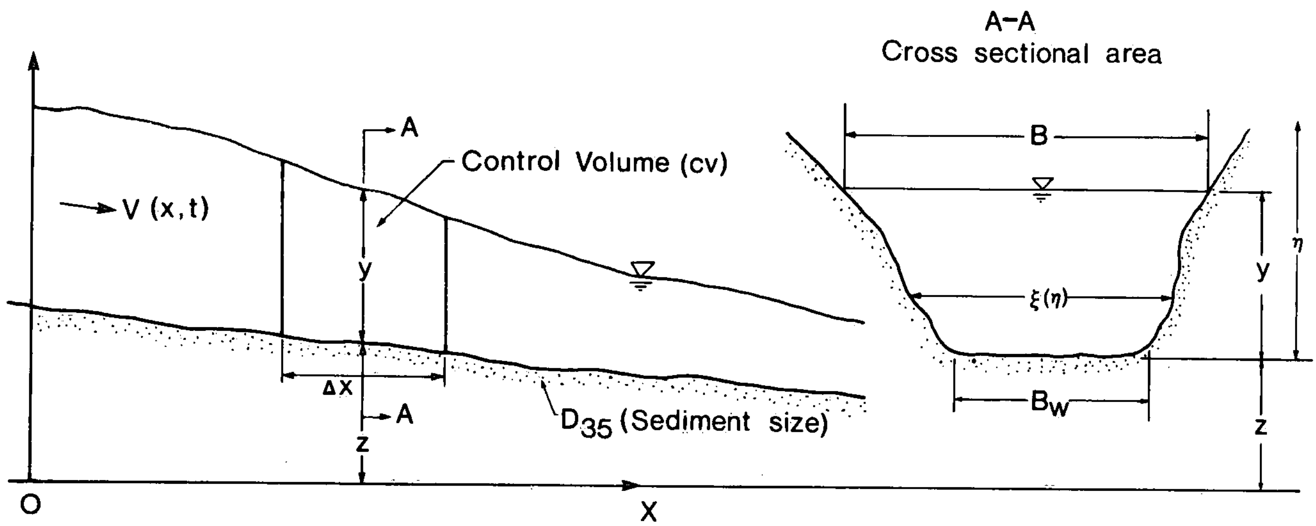

Figure 1. Schematic representation of the longitudinal profile and a flow cross section in a river.

Sediment Continuity Equation

Let Q_s be the total volume of sediment transported by the river flow per unit time. Q_s, in general, is a function of both x and t. Let q_s be the total volume of sediment entering the river because of the overland flow, etc. and it is expressed in volume per unit length and unit time. Considering the control volume (cv) separately, as shown in Figure 2, and considering a time interval of Δt, the mass of sediment entering the control volume is given by $(\rho_s Q_s \Delta t + \rho_s q_s \Delta x \Delta t)$. ρ_s is the density of the sediment particles, and

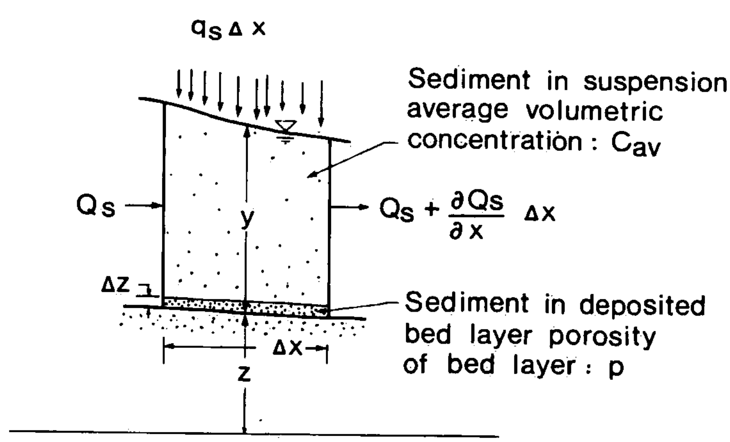

Figure 2. Control volume to derive the sediment continuity equation.

the mass of sediment leaving the control volume is given by $\rho_s[Q_s + (\partial Q_s/\partial x)\Delta x]\Delta t$. The difference between the two, which is

$$(\rho_s Q_s \Delta t + \rho_s q_s \Delta x \Delta t) - \rho_s[Q_s + (\partial Q_s/\partial x)\Delta x]\Delta t = \rho_s q_s \Delta x \Delta t - \rho_s(\partial Q_s/\partial x)\Delta x \Delta t \tag{1}$$

should be equal to the change in the mass of sediment stored within the control volume during the time interval Δt.

The change in the sediment storage within the control volume is effected in two ways: firstly, by the deposition or the scour on the bed of the river, which alters the elevation of the river bed by an amount Δz, and secondly, by the change in the average concentration C_{av} of the sediment in suspension. Assuming that the deposition or scour occurs uniformly over the whole bed area, the mass of sediment in a bed layer of thickness Δz is given by $\rho_s(P\Delta z\ \Delta x)p$, where P is the wetted perimeter at the section where the control volume is located and p is the volume of sediment per unit volume of the bed layer. If Δz is expressed as

$$\Delta z = (\partial z/\partial t)\Delta t, \tag{2}$$

the change in the mass of sediment storage due to deposition or scour is given by

$$p\rho_s P(\partial z/\partial t)\Delta t\Delta x. \tag{3}$$

The change in the mass of sediment due to the change in the average concentration of sediment in suspension can be expressed as

$$\rho_s(\partial/\partial t)(A\Delta x\ C_{av})\Delta t \tag{4}$$

where A is the area of the flow cross section at the section where the control volume is located and C_{av} is the average volumetric concentration of the sediment at that cross section.

The total change in the mass of sediment storage within the control volume during Δt is given by

$$\rho_s\left[P(\partial z/\partial t)p + (\partial/\partial t)(AC_{av})\right]\Delta x\Delta t. \tag{5}$$

By equating (4) and (5) as

$$\rho_s\left[q_s - (\partial Q_s/\partial x)\right]\Delta x\Delta t = \rho_s\left[P(\partial z/\partial t)p + (\partial/\partial t)(AC_{av})\right]\Delta x\Delta t \tag{6}$$

the equation for the sediment continuity is obtained:

$$(\partial Q_s/\partial x) + P(\partial z/\partial t)p + (\partial/\partial t)(AC_{av}) = q_s. \tag{7}$$

Continuity Equation for the Sediment-laden Flow

In this case, both the mass of water and the mass of sediment are considered together. During an increment of time Δt, the mass of inflow to the control volume is

$$(\rho_w Q_w + \rho_s Q_s + \rho_w q_w \Delta x + \rho_s q_s \Delta x)\Delta t \tag{8}$$

where ρ_w is the density of water, Q_w is the water flow rate and q_w is the lateral inflow of water from tributaries, etc. The value q_w is expressed in volume per unit length of the river per unit time. The mass flow out of the control volume is

$$\left\{\rho_w[Q_w + (\partial Q_w/\partial x)\ \Delta x] + \rho_s[Q_s + (\partial Q_s/\partial x)\ \Delta x]\right\}\Delta t \tag{9}$$

and hence the difference becomes

$$[\rho_w q_w \Delta x\Delta t + \rho_s q_s \Delta x\Delta t - \rho_w(\partial Q_w/\partial x)\Delta x\Delta t - \rho_s(\partial Q_s/\partial x)\Delta x\Delta t]. \tag{10}$$

According to the principle of conservation of mass, the difference expressed by Equation 10 should be equal to the change of storage of mass of sediment-water mixture within the control volume during the interval of time Δt.

The change of storage of water within the control volume during Δt can be expressed as

$$(\partial/\partial t)(\rho_w A_w \Delta x)\Delta t + \rho_w(1 - p)P\Delta z\Delta x \tag{11}$$

where A_w is the flow cross-sectional area occupied by the fluid only, while the change of storage of sediment is given by expression 5. Therefore, the change in the storage of the sediment-water mixture is given by

$$\rho_w\left(\frac{\partial A_w}{\partial t}\right)\Delta x\Delta t + \rho_w(1 - p)\ P\left(\frac{\partial z}{\partial t}\right)\Delta x\Delta t + \rho_s pP\left(\frac{\partial z}{\partial t}\right)\Delta x\Delta t + \rho_s\left(\frac{\partial}{\partial t}\right)(AC_{av})\Delta x\Delta t. \tag{12}$$

Equating (10) and (12), we get

$$\rho_w\left[\frac{\partial Q_w}{\partial x} + \frac{\partial A_w}{\partial t} + (1 - p)P\left(\frac{\partial z}{\partial t}\right)\right] + \left[\rho_s\ \frac{\partial Q_s}{\partial x} + \frac{\partial}{\partial t}(AC_{av}) + P\left(\frac{\partial z}{\partial t}\right)p\right] = \rho_w q_w + \rho_s q_s. \tag{13}$$

Substituting Equation 7 into Equation 13, we can simplify the latter as

$$(\partial Q_w/\partial x) + (\partial A_w/\partial t) + (1 - p)\ P(\partial z/\partial t) = q_w. \tag{14}$$

If Q is the total discharge, A is the total cross-sectional area and q_ℓ is the total lateral inflow, then Equation 14 can be expressed in terms of Q, A and q_ℓ. If

$$\begin{aligned} Q &= Q_w + Q_s \\ A &= A_w + AC_{av} \\ q_\ell &= q_w + q_s \end{aligned} \tag{15}$$

and if we substitute into Equation 14, we get

$$\left[\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} + P\left(\frac{\partial z}{\partial t}\right)\right] - \left[\frac{\partial Q_s}{\partial x} + \frac{\partial (AC_{av})}{\partial t} + P\left(\frac{\partial z}{\partial t}\right)p\right] = q_\ell - q_s. \tag{16}$$

Again, using Equation 7 in Equation 16, the continuity equation for the sediment-laden flow can be expressed as

$$(\partial Q/\partial x) + (\partial A/\partial t) + P(\partial z/\partial t) = q_\ell. \tag{17}$$

Momentum Equation for the Sediment-laden Flow

Using the principle of conservation of momentum, which states: "the net rate of momentum flux into the control volume plus the sum of the forces acting on the control volume is equal to the rate of accumulation of momentum within the control volume," the momentum equation can be derived as follows.

$$\text{Momentum entering the control volume} = (\rho Q^2/A) + \rho q_\ell U_q \Delta x \tag{18}$$

where q_ℓ is the lateral inflow, U_q is the velocity of the lateral inflow in the direction of the main flow, and ρ is the density of the sediment-water mixture.

$$\text{Momentum leaving the control volume} = \frac{\rho Q^2}{A} + \frac{\partial}{\partial x}\left(\frac{\rho Q^2}{A}\right)\Delta x + \rho q_\ell\left(\frac{Q}{A}\right)\Delta x \tag{19}$$

The net rate of momentum flux entering the control volume =

$$-\frac{\partial}{\partial x}\left(\rho\,\frac{Q^2}{A}\right)\Delta x + \rho q_\ell\left(U_q - \frac{Q}{A}\right)\Delta x. \tag{20}$$

The forces acting on the control volume are gravity, pressure and frictional resistance, which will be considered one by one.

1. Gravity: The force due to gravity is the weight of the fluid within the control volume. If S_x is the slope of the bottom of the control volume with the horizontal, then the component of this weight along the flow direction can be expressed as

$$\rho g \; A \; \Delta x \; S_x. \tag{21}$$

It is assumed here that within the segment Δx the flow is uniform.

2. Pressure force: The pressure force along the direction of the flow can be divided into two parts: (1) the difference in the pressure forces acting on the two ends of the control volume, and (2) the difference in pressure force in the direction of the flow on the banks of the control volume due to widening or narrowing along the length of the non-prismatic channels. Assuming a hydrostatic pressure distribution, the first part of the net pressure force acting on the end surfaces of the control volume in the direction of this flow can be evaluated.

The pressure force acting on the left side of the control volume is

$$F_p = \int_0^y \rho g (y - \eta) \xi(\eta) \; d\eta \tag{22}$$

where $\xi(\eta)$ is the width of the channel at a height of η from the bottom of the channel (see Fig. 1).

The pressure force acting on the right side of the control volume is

$$F_p + (\partial F_p/\partial x)\Delta x. \tag{23}$$

Therefore, the net pressure force acting on the sides of the control volume is

$$-\left(\frac{\partial F_p}{\partial x}\right)\Delta x = -\left(\frac{\partial}{\partial x}\right) \int_0^y \rho g (y - \eta) \xi(\eta) \; d\eta \; \Delta x. \tag{24}$$

Changing the order of differentiation and integration using the Leibnitz's rule, we can express the above equation as

$$-\left(\frac{\partial F_p}{\partial x}\right)\Delta x = \left[-\rho g \left(\frac{\partial y}{\partial x}\right) A - \rho g \int_0^y (y - \eta) \left(\frac{\partial \xi(\eta)}{\partial x}\right) d\eta \right] \Delta x. \tag{25}$$

The pressure force acting on the banks of the channel as a result of its widening or narrowing can be calculated as follows. Consider a volume element within the control volume at a height of η from the bottom with a thickness of $d\eta$. The pressure force per unit length acting at any point within the volume is

$$\rho g(y - \eta)d\eta. \tag{26}$$

This normal force cancels itself out at all points within the volume except on those located on the banks of the channel. The unbalanced pressure force along the flow direction for a change in width of $\Delta\xi$ is given by

$$\rho g(y - \eta)d\eta \; \Delta\xi(\eta). \tag{27}$$

Expressing $\Delta\xi$ as $(\partial\xi/\partial x)\Delta x$, and integrating over the whole depth of the flow, we can calculate the pressure force acting on the banks in the direction of the flow due to widening or narrowing of the channel as

$$\rho g \int_0^y (y - \eta) \left(\frac{\partial\xi(\eta)}{\partial x}\right) d\eta \; \Delta x. \tag{28}$$

Combining these two parts of pressure force, we obtain the net pressure force acting on the control volume as

$$-\left[\rho g \; A \; (\partial y/\partial x)\Delta x\right] \tag{29}$$

3. Frictional resistance: The frictional force which resists the motion of the fluid in the channel acts along the solid boundaries of the channel and can be expressed as

$$F_f = -(\tau_o \; P \; \Delta x) \tag{30}$$

where τ_o is the shear stress at its boundary and P is the wetted perimeter. In the case of a steady flow, the boundary shear stress τ_o is expressed in terms of the hydraulic radius R and the free surface slope S_f as

$$\tau_o = \rho g R S_f. \tag{31}$$

If we assume that the boundary shear stress in an unsteady flow can also be expressed using Equation 31, the frictional force of the control volume becomes

$$F_f = -(\rho g \; A S_f \; \Delta x). \tag{32}$$

The rate of accumulation of momentum within the control volume can be expressed as

$$(\partial/\partial t)(\rho Q)\Delta x \tag{33}$$

and therefore the momentum equation becomes

$$\frac{\partial Q}{\partial t} + \left(\frac{\partial}{\partial x}\right)\left(\frac{Q^2}{A}\right) + gA\left(\frac{\partial y}{\partial x}\right) = gA(S_x - S_f) + q_\ell\left(U_q - \frac{Q}{A}\right) \tag{34}$$

assuming that the bulk density is a constant with respect to time and space.

When the derivative of the flow cross-sectional area A with respect to t in Equation 17 and with respect to x in Equation 34 is evaluated, the Leibnitz rule for the differentiation of the integrals should be used. With reference to Figure 1, the flow cross-sectional area A is given by

$$A = \int_0^y \xi(x;\eta)\, d\eta. \tag{35}$$

Therefore $\quad \partial A/\partial t = \partial/\partial t \int_0^y \xi\,(x;\eta)\, d\eta \qquad (36)$

and $\quad \partial A/\partial x = \partial/\partial x \int_0^y \xi\,(x;\eta)\, d\eta. \qquad (37)$

Using the Leibnitz rule, we can express Equations 36 and 37 as

$$\frac{\partial A}{\partial t} = \int_0^y \left(\frac{\partial}{\partial t}\right)\xi(x;\eta)\, d\eta + \xi(x;y)\left(\frac{\partial y}{\partial t}\right) = B\left(\frac{\partial y}{\partial t}\right) \tag{38}$$

and $\quad \frac{\partial A}{\partial x} = \int_0^y \left(\frac{\partial}{\partial x}\right)\xi(x;\eta)\, d\eta + \xi(x;y)\left(\frac{\partial y}{\partial x}\right) = A_x^y + B\left(\frac{\partial y}{\partial x}\right) \qquad (39)$

where B is the top width of the channel, while A_x^y stands for the term under the integral sign in Equation 39, which is the rate of change of area with respect to x with depth y held constant. With these expressions for $\partial A/\partial t$ and $\partial A/\partial x$, the governing equations become

$$\frac{\partial Q_s}{\partial x} + P\left(\frac{\partial z}{\partial t}\right)p + BC_{av}\left(\frac{\partial y}{\partial t}\right) + A\left(\frac{\partial C_{av}}{\partial t}\right) - q_s = 0$$

$$\frac{\partial Q}{\partial x} + B\left(\frac{\partial y}{\partial t}\right) + P\left(\frac{\partial z}{\partial t}\right) - q_\ell = 0$$

$$2\frac{Q}{A}\left(\frac{\partial Q}{\partial t}\right)+\frac{\partial Q}{\partial t}-B\left(\frac{Q^2}{A^2}\right)\left(\frac{\partial y}{\partial x}\right)+gA\left(\frac{\partial y}{\partial x}\right)=gA(S_x-S_f)+q_\ell\left(U_q-\frac{Q}{A}\right)+\left(\frac{Q^2}{A^2}\right)A_x^y \tag{40}$$

The above set of equations governs the sediment-laden flows in reaches of natural streams that are reasonably straight. These equations involve five unknowns, namely the flow rate Q, the flow cross-sectional area A, the bottom elevation z, the sediment transport rate Q_s and the frictional slope S_f. (The lateral inflows, q_ℓ and q_s, the lateral inflow velocity U_q and the porosity p are expected to be known, and C_{av} and Q_s are related.) Therefore, in addition to these governing equations, two more independent relations are required to achieve closure of the system of equations. These additional relations are provided by the sediment transport formulae, which give Q_s in terms of flow and sediment characteristics, and the equations for the friction factor in natural streams, which express the energy slope S_f in terms of the flow and of the bottom topography of the channels resulting from the movement of the sediments. There are a number of sediment transport and friction factor formulae in the literature, but each of them is limited and there is as yet no theory that is capable of predicting the above parameters for the whole of the flow regimes. For the present work, the sediment transport formula of Ackers and White (4) and the friction factor relations of Kishi (5) are adopted, which can be considered the best among the currently available theories. The details of those relationships will be taken up later after the description of the numerical scheme to solve the system of governing equations. The construction of the present mathematical model is such that as new and more complete theories on sediment transport and friction factors become available, they can be easily incorporated into the model.

3. NUMERICAL SCHEME TO SOLVE THE SYSTEM OF GOVERNING EQUATIONS

The governing equations of the sediment-laden flow can be uncoupled if the term $P(\partial z/\partial t)$ in the flow continuity equation is considered negligible in comparison to the term $B(\partial y/\partial t)$. Indeed, since the top width B and the wetted perimeter P are nearly equal for wide channels and since the water level changes are more rapid than the bed level changes, it is possible to drop the term $P(\partial z/\partial t)$ from the flow continuity equation without losing accuracy. By doing so, it is now possible to solve the flow continuity equation and the momentum equation simultaneously for one time-step independent of the sediment continuity equation and then to use the sediment continuity equation to correct the solution. Such a technique, which simplifies the solution procedure considerably, is adopted for the present model.

Solution of Continuity and Momentum Equations

An implicit finite difference scheme first developed by Preissmann (6) in 1960 is used to solve the flow continuity and the momentum equations simultaneously. According to this scheme, a variable, say f, and its derivatives are discretized as follows:

$$f(x;t) = \frac{\theta}{2}\left[f_{i+1}^{j+1} + f_i^{j+1}\right] + \frac{1-\theta}{2}\left[f_{i+1}^{j} + f_i^{j}\right]$$

$$\frac{\partial f}{\partial x} = \theta\left[\frac{f_{i+1}^{j+1} - f_i^{j+i}}{\Delta x}\right] + (1-\theta)\left[\frac{f_{i+1}^{j} - f_i^{j}}{\Delta x}\right]$$

$$\frac{\partial f}{\partial t} = \frac{1}{2}\left[\frac{f_{i+1}^{j+1} - f_{i+1}^{j}}{\Delta t} + \frac{f_i^{j+1} - f_i^{j}}{\Delta t}\right] \qquad (41)$$

where i and j, Δx and Δt are as shown in Figure 3 and θ is a weighting coefficient that can take values between 0 and 1. When $\theta = 0$, the scheme becomes fully explicit and if $\theta = 1$ it is fully implicit. Cunge (7) has analyzed this scheme fully for numerical stability and accuracy and has shown that the scheme is unconditionally stable for values of θ between 1/2 and 1 and the accuracy is first order with respect to Δx for arbitrary values of θ and second order with respect to Δx when $\theta = 0.5$. Cunge also indicated that for $\theta = 0.5$, "parasitic" oscillations are found in the solution that resemble the phenomenon of numerical instability for small values of the friction factor, and he suggested a practical range for θ of 0.6 to 1.0.

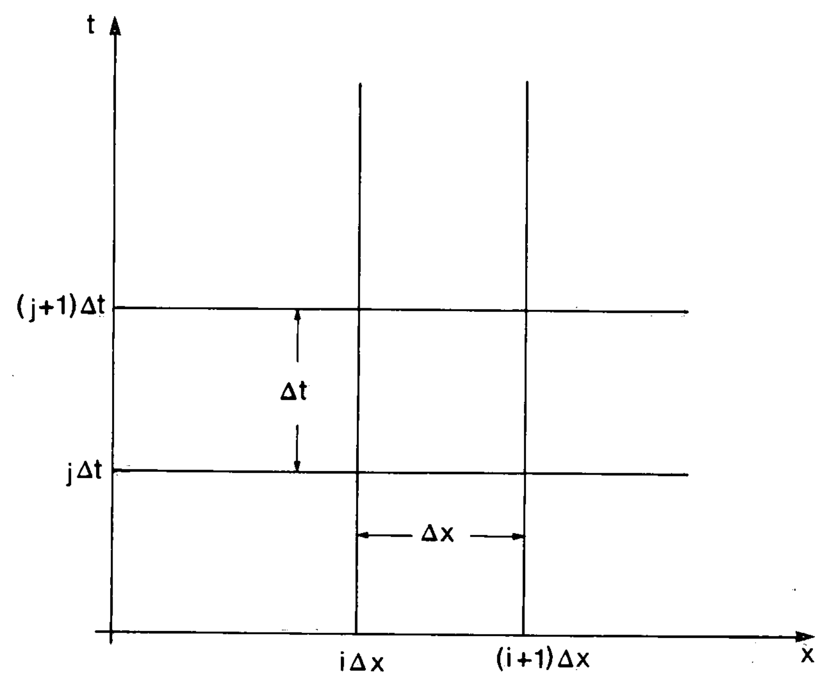

Figure 3. Discretization scheme (finite difference scheme) of Preissmann.

If we express the value of the variable, say f, at $(j+1)^{th}$ time interval, f^{j+1}, as a sum of the value of f at the j^{th} interval, f^j, and a difference Δf between these two, i.e.

$$f^{j+1} = f^j + \Delta f, \qquad (42)$$

the relationships in Equation 41 can be rewritten as

$$\left.\begin{aligned} f(x;t) &= \frac{1}{2}\left[\theta(\Delta f_{i+1} + \Delta f_i) + (f_{i+1} + f_i)\right] \\ \frac{\partial f}{\partial x} &= \frac{1}{\Delta x}\left[\theta(\Delta f_{i+1} - \Delta f_i) + (f_{i+1} - f_i)\right] \\ \frac{\partial f}{\partial t} &= \frac{1}{2\Delta t}(\Delta f_{i+1} + \Delta f_i) \end{aligned}\right\} \tag{43}$$

The superscript for f is dropped with the understanding that f without superscript corresponds to the value of f at the j^{th} time step. When these approximations are substituted for the terms of the flow continuity equation, it becomes

$$\frac{1}{\Delta x}\left[\theta(\Delta Q_{i+1} + \Delta Q_i) + (Q_{i+1} + Q_i)\right] + \left\{\frac{1}{2}\left[\theta(\Delta B_{i+1} + \Delta B_i) + (B_{i+1} + B_i)\right]\frac{1}{2\Delta t}\left[\Delta y_{i+1} + \Delta y_i\right]\right.$$

$$\left. - \frac{1}{2}\left[\theta(\Delta q_{i+1} + \Delta q_i) + (q_{i+1} + q_i)\right]^{1}\right\} = 0. \tag{44}$$

Rearranging the equation and neglecting second-order terms like $(\Delta f/f)^2$, $\Delta f\ \Delta g$, we can write Equation 44 as

$$a_i \Delta y_{i+1} + b_i \Delta Q_{i+1} = c_i \Delta y_i + d_i \Delta Q_i + e_i \tag{45}$$

where

$$\left.\begin{aligned} a_i &= \left[\frac{B_{i+1} + B_i}{2\Delta t}\right] - \frac{2\theta}{\Delta x}\left[\frac{Q_{i+1} - Q_i}{B_{i+1} + B_i}\right]\frac{dB}{dy}\bigg|_{i+1} + \theta\left[\frac{q_{i+1} + q_i}{B_{i+1} + B_i}\right]\frac{dB}{dy}\bigg|_{c+1} \\ b_i &= \frac{2\theta}{\Delta x} \\ c_i &= -\left[\frac{B_{i+1} + B_i}{2\Delta t}\right] + \frac{2\theta}{\Delta x}\left[\frac{Q_{i+1} - Q_i}{B_{i+1} + B_i}\right]\frac{dB}{dy}\bigg|_i - \theta\left[\frac{q_{i+1} + q_i}{B_{i+1} + B_i}\right]\frac{dB}{dy}\bigg|_i \\ d_i &= \frac{2\theta}{\Delta x} \\ e_i &= -\frac{2}{\Delta x}(Q_{i+1} - Q_i) + (q_{i+1} + q_i) + \theta(\Delta q_{i+1} + \Delta q_i) \end{aligned}\right. \tag{46}$$

[1]Subscript ℓ of q will be dropped henceforth and q without subscript will stand for the lateral inflow of water and sediment mixture.

The derivative (dB/dy) appearing in Equation 46 can be evaluated if the steepness of the banks of the stream is known.

In a similar fashion, the approximations expressed by Equation 43 can be substituted in the momentum equation and after lengthy algebraic manipulations we can arrive at an equation similar to Equation 45

$$a_i' \Delta y_{i+1} + b' \Delta Q_{i+1} = c_i' \Delta y_i + d_i' \Delta Q_i + e_i' \tag{47}$$

where

$$a_i' = \frac{\theta}{\Delta x}\left\{\frac{B_{i+1}^2 Q_{i+1}^2}{A_{i+1}^3}(y_{i+1} - y_i) - \frac{B_{i+1} Q_{i+1}}{A_{i+1}^2}(Q_{i+1} - Q_i)\right.$$

$$+ \frac{g}{2}\left[B_{i+1}(y_{i+1} + z_{i+1} - y_i - z_i) + (A_{i+1} + A_i)\right]$$

$$\left. - \frac{1}{2}\left[\left.\frac{dB}{dy}\right|_{i+1}\left(\frac{Q_{i+1}^2}{A_{i+1}^2}\right)(y_{i+1} - y_i) + \frac{B_{i+1} Q_{i+1}^2}{A_{i+1}^2} + \frac{B_i Q_i^2}{A_i^2}\right]\right\}$$

$$+ \frac{\theta}{2}\left\{\left.\frac{dP}{dy}\right|_{i+1}\left(\frac{Q_{i+1}^2}{C_{i+1}^2 A_{i+1}^2}\right) - 2\left(\frac{P_{i+1} Q_{i+1}^2 B_{i+1}}{C_{i+1}^2 A_{i+1}^3}\right) + 2\left(\frac{Q_{i+1}^2 B_{i+1}}{A_{i+1}^3}\right) A_x^y\Big|_{i+1}\right\}$$

$$b_i' = \frac{1}{2\Delta t} + \frac{\theta}{\Delta x}\left[\frac{(2Q_{i+1} - Q_i)}{A_{i+1}} + \frac{Q_i}{A_i} - \frac{Q_{i+1} B_{i+1}}{A_{i+1}^2}(y_{i+1} - y_i) + \right.$$

$$\left. \frac{\theta}{C_{i+1}^2}\left(\frac{P_{i+1} Q_{i+1}}{A_{i+1}^2}\right) - \left(\frac{\theta}{A_{i+1}^2}\right) Q_{i+1} A_x^y\Big|_{i+1}\right]$$

$$c_i' = \frac{\theta}{\Delta x}\left\{\frac{B_i Q_i}{A_i^2}(Q_{i+1} - Q_i) - \frac{B_i^2 Q_i^2}{A_i^3}(y_{i+1} - y_i) - \frac{g}{2}\left[B_i(y_{i+1}\right.\right.$$

$$\left. - z_i - y_i + z_{i+1}) - (A_{i+1} + A_i)\right] - \frac{1}{2}\left[\frac{B_{i+1} Q_{i+1}^2}{A_{i+1}^2} + \frac{B_i Q_i^2}{A_i^2} - \left.\frac{dB}{dy}\right|_i\right.$$

$$\left.\left.\left(\frac{Q_i^2}{A_i^2}\right)(y_{i+1} - y_i)\right]\right\} - \frac{\theta}{2}\left\{\left.\frac{dP}{dy}\right|_i\left(\frac{Q_i^2}{C_i^2 A_i^2}\right) - 2\left(\frac{P_i Q_i^2 B_i}{C_i^2 A_i^3}\right) + 2\left(\frac{Q_i^2 B_i}{A_i^3}\right) A_x^y\Big|_i\right\}$$

$$d_i' = -\left(\frac{1}{2\Delta t}\right) - \frac{\theta}{\Delta x}\left[\frac{Q_{i+1} - 2Q_i}{A_i} - \frac{Q_{i+1}}{A_{i+1}} - \frac{Q_i B_i}{A_i^2}(y_{i+1} - y_i)\right] -$$

$$\frac{\theta}{C_i^2}\left(\frac{P_i Q_i}{A_i}\right) + \left(\frac{\theta}{A_i^2}\right) A_x^y\Big|_i$$

$$e_i' = -\frac{1}{\Delta x}\left\{(Q_{i+1} - Q_i)\left[\frac{Q_{i+1}}{A_{i+1}} + \frac{Q_i}{A_i}\right] - \frac{g}{2}\left[(A_{i+1} + A_i)(y_{i+1} + z_{i+1} - y_i - z_i)\right] - \frac{1}{2}\left[\frac{B_{i+1}Q^2_{i+1}}{A^2_{i+1}} + \frac{B_iQ_i^2}{A_i^2}\right](y_{i+1} - y_i)\right\} -$$

$$\frac{1}{2}\left\{\left(\frac{P_{i+1}}{C^2_{i+1}}\frac{Q^2_{i+1}}{A^2_{i+1}} + \frac{P_i}{C_i^2}\frac{Q_i^2}{A_i^2}\right) - \left(\frac{Q^2_{i+1}}{A^2_{i+1}}\right)A^y_x\Big|_{c+1} - \left(\frac{Q_i^2}{A_i^2}\right)A^y_x\Big|_i\right\} \tag{48}$$

In the derivation of Equation 48, the frictional slope S_f appearing in the momentum equation has been expressed in terms of the flow parameters and the friction coefficient C as follows:

$$gAS_f = (Q^2/A^2)\ (P/C^2). \tag{49}$$

The friction coefficient C stands for the ratio between the average velocity $v(=Q/A)$ and the shear velocity $v_*(= \sqrt{gRs_f}\ )$ and can be related to the Darcy-Weisbach friction factor f by the following expression

$$f = 8/C^2. \tag{50}$$

Furthermore, the velocity U_q of the lateral inflow is assumed to be of the same order of magnitude as v and hence the term $q_\ell\ [U_q - (Q/A)]$ appearing in the momentum equation is also dropped.

Equations 45 and 47 give rise to a system of two (N - 1) linear equations involving 2N unknowns, namely ΔQ_1, ΔQ_2, ΔQ_3 ... ΔQ_N and Δy_1, Δy_2, Δy_3 ... Δy_N, where N is the number of grid points along the length of the river. With two known boundary conditions (one at the upstream boundary and the other at the downstream boundary for the subcritical flows) the number of equations matches the number of unknowns and the system of equations can be solved using any one of the available standard methods. In this work, the "Double Sweep Method" (8) is adopted, which is the fastest of the available methods. The number of elementary operations (and consequently the computer time required) necessary to solve the system of equations by this method is proportional only to the number of points N, whereas the number of operations required by the existing standard methods of matrix inversion is proportional to N^3. A detailed description of the operations involved in the Double Sweep method is given in the following subsection.

Double Sweep Method

If we assume that for any point i (for a particular time step j) the following linear relation holds between Δy_i and ΔQ_i, i.e.

$$\Delta Q_i = E_i \Delta y_i + F_i \tag{51}$$

then it is possible to prove that an analogous linear relationship also exists for the next point i + 1. Indeed, substituting Equation 51 into Equation 45 we get

$$a_i \Delta y_{i+1} + b_i \Delta Q_{i+1} = c_i \Delta y_i + d_i \; (E_i \Delta y_i + F_i) + e_i \tag{52}$$

from which Δy_i can be evaluated as

$$\Delta y_i = (L_i \Delta y_{i+1} + M_i \Delta Q_{i+1}) - K_i \tag{53}$$

$$\left.\begin{aligned} \text{where} \quad & L_i = A_i/(C_i + d_i E_i) \\ & M_i = b_i/(C_i + d_i E_i) \\ \text{and} \quad & K_i = (e_i + d_i F_i)/(C_i + d_i + E_i) \end{aligned}\right\} \tag{54}$$

Similarly, substituting Equation 51 into Equation 47, we get

$$a_i' \; \Delta y_{i+1} + b_i' \; \Delta Q_{i+1} = (c_i' + d_i' E_i) \; \Delta y_i + (d_i' \; F_i + e_i'). \tag{55}$$

When the value of Δy_i as given by Equation 53 is substituted into Equation 55 and the terms rearranged, the latter becomes

$$\Delta Q_{i+1} = E_{i+1} \; \Delta y_{i+1} + F_{i+1} \tag{56}$$

$$\left.\begin{aligned} \text{where} \quad & E_{i+1} = \frac{a_i(c_i' + d_i' E_i) - a_i'(c_i + d_i E_i)}{b_i'(c_i + d_i E_i) - b_i(c_i' + d_i' E_i)} \\ \text{and} \quad & F_{i+1} = \frac{(e_i' + d_i' F_i) \; (c_i + d_i E_i) - (e_i + d_i F_i) \; (c_i' + d_i' E_i)}{b_i'(c_i + d_i E_i) - b_i(c_i' + d_i' E_i)} \end{aligned}\right\} \tag{57}$$

Therefore, by expressing the upstream boundary condition in the form of Equation 51 the values of E_1 and F_1 can be evaluated. If E_1 and F_1 are known, the values of E_2 and F_2 can be found using Equation 57. By repeating this procedure, the values of E_3, E_4, E_5... E_N and F_3, F_4, F_5 ... F_N can be found (Forward Sweep). If we use the downstream boundary condition to evaluate Δy_N and, E_N and F_N are known, the value of ΔQ_N at the downstream boundary can be determined. Since ΔQ_N and Δy_N are known the value of Δy_{N-1} can be evaluated using Equation 53. Since E_{N-1} and F_{n-1} are known, it is possible now to determine ΔQ_{N-1}. By the repeated application of Equation 53 the unknowns Δy_{N-2}, Δy_{N-3}... Δy_1, ΔQ_{N-2}, ΔQ_{N-3}... ΔQ_1 can be computed (Backward Sweep).

Since the initial condition provides the values of y_1, y_2... y_N and Q_1, Q_2, Q_3... Q_N, the water depth and the flow rates at all the grid points along the river at the end of the time step can be obtained by simply adding the above solution to the initial condition. A flow chart description of the Double Sweep method is given in Figure 4.

The application of the Double Sweep method, therefore, requires the evaluation of the coefficients E_1 and F_1 from the upstream boundary condition and Δy_N from the downstream boundary condition. The various possible boundary conditions and the evaluation of E_1, F_1 and Δy_N are considered in the next two subsections.

Evaluation of E_1 and F_1 from Upstream Boundary Condition

There are three possible ways in which the boundary conditions can be prescribed:

(1) the flow depth y_i is known for all time:

$$\text{i.e.} \quad y_1 = f_1\,(t); \tag{58}$$

(2) the flow rate Q_1 is known for all time:

$$\text{i.e.} \quad Q_1 = f_2\,(t); \tag{59}$$

and (3) the flow rate Q_1 is expressed as a known function of the flow depth:

$$\text{i.e.} \quad Q_1 = f_3\,(y_1). \tag{60}$$

Each of the above conditions is considered separately for the evaluation of E_1 and F_1.

Case 1 - When the boundary condition is expressed as in Equation 58, it is possible to compute Δy_1 as

$$\Delta y_1 = f_1(t + \Delta t) - f_1\,(t). \tag{61}$$

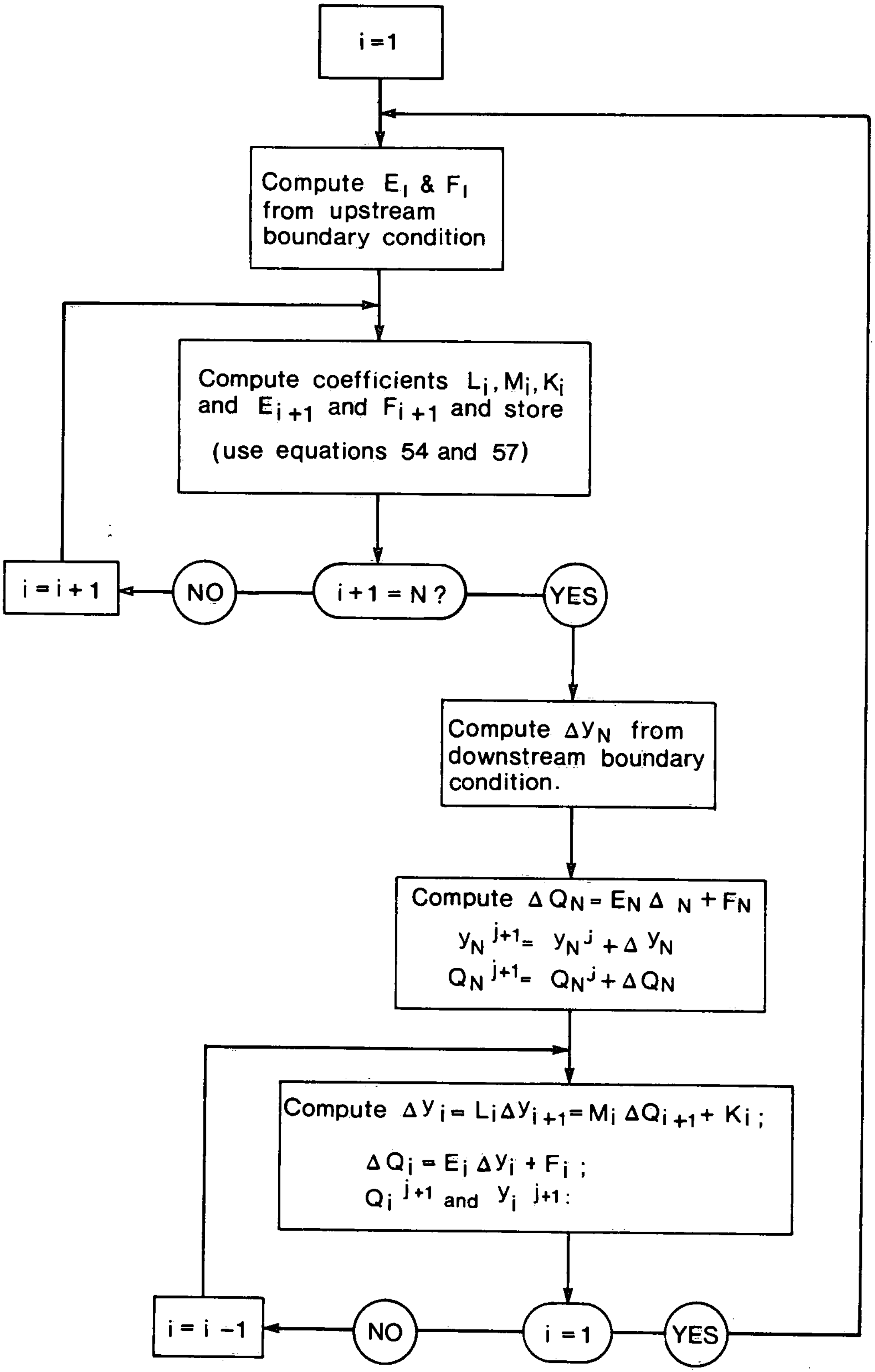

Figure 4. Flow chart for the Double Sweep method.

The Equation 51 corresponding to the upstream boundary can be written as

$$\Delta Q_1 = E_1 \, \Delta y_1 + F_1 \tag{62}$$

which can be rearranged as:

$$\Delta y_1 = (\Delta Q_1/E_1) - (F_1/E_1). \tag{63}$$

Since Δy_1 and ΔQ_1 are, in general, independent parameters, the above equation can be considered to be valid only for large values of E_1 compared to ΔQ_1 so that the first term on the right-hand side of Equation 63 approaches zero. By equating E_1 to a very large value, say α, we can determine the value of F_1 using Equations 61 and 63 as follows.

$$F_1 = -\alpha[f_1(t + \Delta t) - f_1(t)]. \tag{64}$$

In practice the value of α should be of the order of 10^4 to 10^6.

<u>Case 2</u> – When the boundary condition is given by Equation 59, the value of ΔQ_1 can be evaluated as

$$\Delta Q_1 = f_2(t + \Delta t) - f_2(t). \tag{65}$$

Since ΔQ_1 and Δy_1 are independent quantities, the relation connecting them (i.e. Equation 62) could be valid only when $E_1 = 0$. Therefore, F_1 becomes

$$F_1 = \Delta Q_1 = f_2(t + \Delta t) - f_2(t). \tag{66}$$

<u>Case 3</u> – When the boundary condition is given by Equation 60, which is termed as the rating curve and is unique only under special circumstances, ΔQ_1 can be expressed as

$$\Delta Q_1 = Q_1\Big|_{t+\Delta t} - Q_1\Big|_t \tag{67}$$

By evaluating $Q_1\big|_{t+\Delta t}$ and $Q_1\big|_t$ using the rating curve f_3, we can show that

$$\Delta Q_1 = \frac{\partial f_3(y_1)}{\partial y_1}\bigg|_t \Delta y_1 \tag{68}$$

Comparing Equations 62 and 68, we can see that

$$E_1 = \frac{\partial f_3(y_1)}{\partial y_1}\bigg|_t$$

and $F_1 = 0$.

Evaluation of Δy_N from the Downstream Boundary Condition

Case 1 – When the downstream boundary condition is expressed as $y_N = g_1(t)$, the value of Δy_N can be computed as

$$\Delta y_N = g_1(t + \Delta t) - g_1(t). \tag{69}$$

Case 2 – When the downstream boundary condition is given as $Q_N = g_2(t)$, ΔQ_N can be calculated as $\Delta Q_N = g_2\ (t + \Delta t) - g_2(t)$, and since at the downstream boundary ΔQ_N and Δy_N can be related by Equation 51 as

$$\Delta Q_N = E_N \Delta y_N + F_N, \tag{70}$$

the value of Δy_N can be determined as

$$\Delta y_N = \frac{[g_2(t+\Delta t) - g_2(t)]-F_N}{E_N}. \tag{71}$$

Case 3 – When the downstream boundary condition is expressed in the form of a rating curve, i.e. $Q_N = g_3(y_N)$, we have

$$\Delta Q_N = \left.\frac{\partial g_3(y_N)}{\partial y_N}\right|_t \Delta y_N. \tag{72}$$

Using Equation 70, we can evaluate Δy_N from Equation 72 as

$$\Delta y_N = \frac{F_N}{\left.\frac{\partial g_3(y_N)}{\partial y_N}\right|_t - E_N} \tag{73}$$

Solution of Sediment Continuity Equation

Evaluating E_1 and F_1 from the upstream boundary condition and Δy_N from the downstream boundary condition the Double Sweep method described earlier can be used to solve the flow continuity and momentum equations and obtain the values of the flow depths and the flow rates at all the sections along the length of the river at the end of the first time step (i.e. at $t = t_0 + \Delta t$; t_0 corresponding to the time when the initial conditions are given). If the flow conditions at t_0 and at $t_0 + \Delta t$ are known, the sediment continuity equation can be solved, in order to correct the flow condition at $t_0 + \Delta t$ as follows.

The sediment continuity equation can be rearranged as

$$\frac{\partial z}{\partial \tau} = - \frac{1}{Pp} \left\{ \left[\frac{\partial Q_s}{\partial x} + BC_{av}\left(\frac{\partial y}{\partial t}\right)\right] + \left[A\left(\frac{\partial C_{av}}{\partial t}\right) - q_s \right] \right\} . \tag{74}$$

Using the approximations expressed by Equation 41, Equation 74 can be discretized as

$$\Delta z_{i+1} = - 2\left(\frac{\Delta t}{p}\right) \left[\frac{\theta}{2} \left(P^{j+1}_{i+1} + P^{j+1}_{i}\right) + \left(\frac{1-\theta}{2}\right)\left(P^{j}_{i+1} + P^{j}_{i}\right)\right]^{-1} \left\{ \left[\theta \left(\frac{Q^{j+1}_{s_{i+1}} - Q^{j+1}_{s_i}}{\Delta x}\right) + \right.\right.$$

$$(1-\theta) \left(\frac{Q^{j}_{s_{i+1}} - Q^{j}_{s_i}}{\Delta x}\right)\Bigg] + \left[\frac{\theta}{2} \left(B^{j+1}_{i+1} C^{j+1}_{av_i} + B^{j+1}_{i} C^{j+1}_{av_i}\right) + \left(\frac{1-\theta}{2}\right)\right.$$

$$\left(B^{j}_{i+1} C^{j}_{av_{i+1}}\right) + B^{j}_{i} C^{j}_{av_i} \Bigg] + \frac{1}{4\Delta t} \left[\theta \left(A^{j+1}_{i+1} + A^{j+1}_{i}\right) + \left(1-\theta\right)\left(A^{j}_{i+1} + A^{j}_{i}\right)\right.$$

$$\left.\left(y^{j+1}_{i+1} - y^{j}_{i+1} + y^{j+1}_{i} - y^{j}_{i}\right)\right] - \left[\frac{\theta}{2} \left(q_{s_{i+1}}^{j+1} + q^{j+1}_{s_i}\right) + \left(\frac{1-\theta}{2}\right)\left(q^{j}_{s_{i+1}} + q^{j}_{s_i}\right)\right]\Bigg\} - \Delta z_i \tag{75}$$

From the boundary condition for the bed elevation at the upstream the quantity Δz_i can be evaluated as

$$\Delta z_i = z^{j+1}_i - z^j_i \tag{76}$$

and hence the quantities appearing on the right-hand side of Equation 75 are completely specified once the quantities Q_s and C_{av} are known. (The method for the evaluation of Q_s and C_{av} will be taken up in the next subsection.) Since Δz_{i+1} is known, the bed elevations at the time $t_0 + \Delta t$ can be obtained as:

$$z^{j+1}_{i+1} = z^{j}_{i+1} + \Delta z_{i+1} . \tag{77}$$

The flow depth at time $t = t_0 + \Delta t$ is corrected using Δz_{i+1} as

$$y^{j+1\,*}_{i+1} = y^{i+1}_{i+1} - \Delta z_{i+1} ,$$

where $y^{j+1\,*}_{i+1}$ is the corrected flow depth after consideration of the sediment continuity equation. It is assumed that the computed flow rate Q^{j+1}_{i+1} at $t_0 + \Delta t$ does not change significantly due to the consideration of the sediment continuity equation during the interval Δt. However, the change in the flow depth will result in changes in the flow cross-sectional areas A^{j+1}_{i+1}, wetted perimeters P^{j+1}_{i+1}, hydraulic radii R^{j+1}_{i+1}, the widths B^{j+1}_{i+1} and the friction coefficients C^{j+1}_{i+1}. Using these new values for the above parameters, the flow continuity and the momentum equations are solved again for another time step and the procedure outlined above is repeated to correct the solution. This process is continued until the required number of time steps is reached. A flow chart describing the above computational steps is shown in Figure 5. The subroutine "Geom" calculates the geometric parameters A, P, R, B, A_x^y (dB/dy), dP/dy. The subroutine "Frict" calculates the friction coefficient C and the subroutine "Sedi" calculates the sediment transport rate Q_s and the average concentration C_{av}.

Sediment Transport Rate Q_s

The sediment transport rate Q_s has been predicted using a new method proposed by Ackers and White (4). This method has been found to be superior to the most commonly used methods such as those of Einstein, Meyer-Peter and Muller, Bagnold, Toffeleti, Rottner, Engelund and Hansen, Biship, etc. (see Ref. 9). The computations involved in this method are listed below.

1. Since we know the grain size distribution and hence D_{35} (grain size for which 35% (by weight) of the sediments is finer), the submerged specific weight γ_s', the specific weight γ and the kinematic viscosity υ of the fluid, a dimensionless number D_{gr} is calculated as

$$D_{gr} = D_{35} \, (g\gamma_s/\gamma\upsilon^2)^{1/3}. \tag{78}$$

2. Depending on the value of D_{gr}, the sediment transport is considered in two different modes. When D_{gr} is greater than 60, the sediment is considered to move as a bed load and when D_{gr} is in the range between 1 and 60, it is considered to move both as bed load and suspended load. The case when D_{gr} is less than 1 occurs only for cohesive sediments and hence is not considered.

3. The general transport function proposed by Ackers and White (4) is

$$G_{gr} = \alpha[(F_{gr}/A) - 1]^m \tag{79}$$

where $$G_{gr} = \left[\frac{Xy}{(\gamma_s/\gamma) + 1}\right]\left(\frac{V_*^n}{V}\right) \tag{80}$$

and $$F_{gr} = \frac{V_*^n \gamma^{1/2}}{\sqrt{\gamma_s g D}} \left[\frac{V}{2.46 \ln (10y/D_{35})}\right]^{1-n} \tag{81}$$

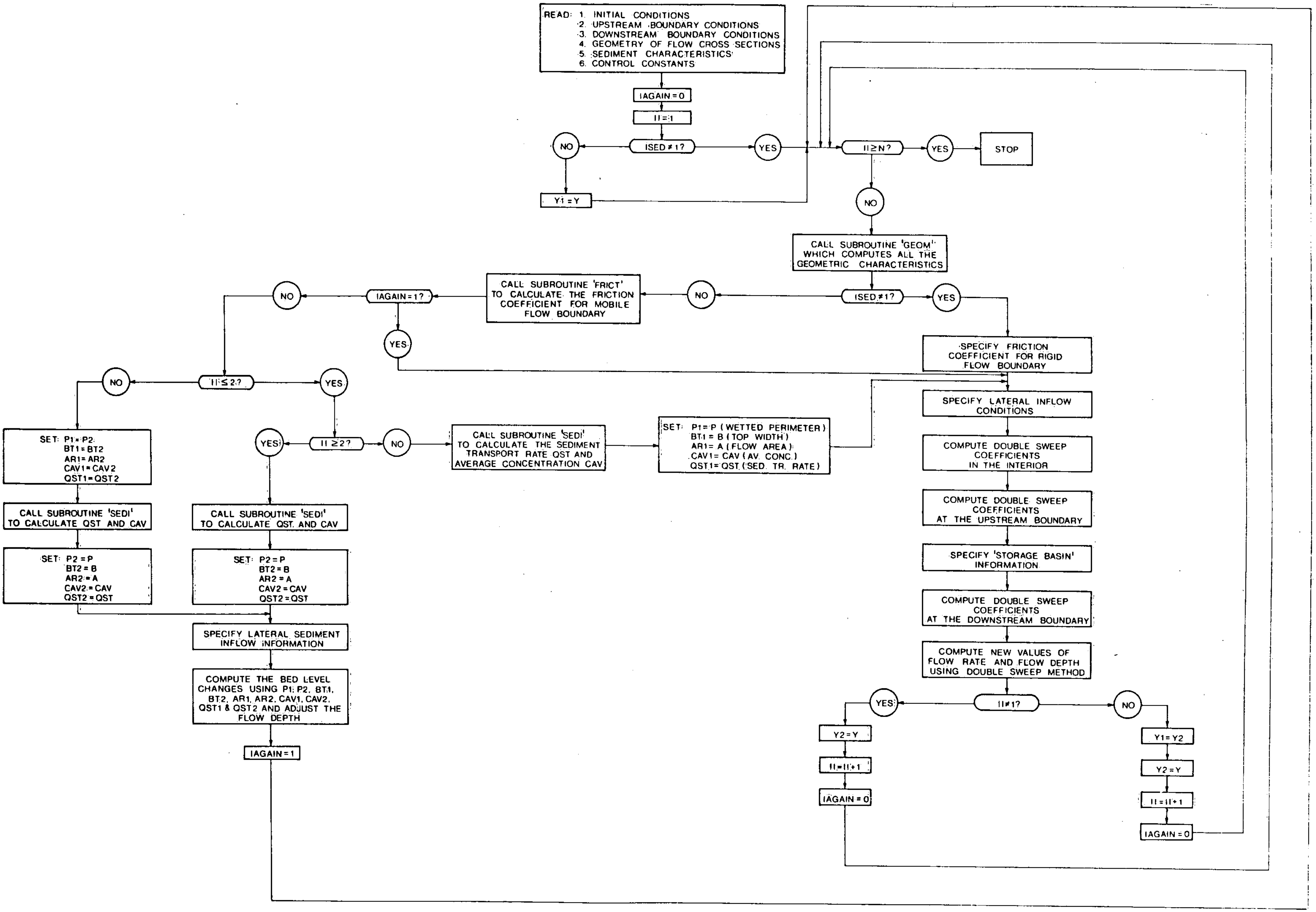

Figure 5. Flow chart of the mathematical model.

and α, A, m are constants. The symbol X in Equation 80 stands for the concentration of sediment by weight, i.e. the mass flux of sediment divided by the mass flow rate. The exponent n appearing in Equations 80 and 81 and the constants α, A, m take the following values, depending on the value of D_{gr}.

When D_{gr} is greater than 60

$$\left.\begin{aligned} n &= 0.00 \\ A &= 0.17 \\ m &= 1.50 \\ \text{and}\quad \alpha &= 0.025 \end{aligned}\right\} \qquad (82)$$

When D_{gr} is in the range between 1 and 60

$$\left.\begin{aligned} n &= 1.00 - 0.24 \ln (D_{gr}) \\ A &= (0.23/D_{gr}) + 0.14 \\ m &= (9.66/D_{gr}) + 1.34 \\ \text{and}\quad \alpha &= \exp \left\{[2.86 \ln(D_{gr}) - \ln(D_{gr})]^2 /(2.303 - 8.130)\right\} \end{aligned}\right\} \qquad (83)$$

Therefore, once D_{gr} is known, the values of n, A, m and α are known and using Equations 79, 80 and 81 the value of X can be calculated. When the volume flow rate and the specific gravity of sediments are known the volume of sediment transported per unit time (Q_s) can be calculated.

Friction Coefficient C

In alluvial streams, the bottom topography changes as the flow changes and the prediction of the friction coefficient in such streams is the most difficult task encountered so far in the field of hydraulics. Many researchers have attempted to solve this important problem, but none of them have succeeded in developing a general method that could be applied over the whole range of flow conditions. Some of the methods available in the literature were developed by Einstein and Barbarossa (10), Garde and Ranga Raju (11), Engelund (12), Alam and Kennedy (13), Kikkawa and Fukuoka (5), and Kishi and Kuroki (5). Among the methods listed above the one by Kishi and Kuroki takes into account all the governing characteristic parameters and compares fairly reasonably with the measurements (see Fig. 6). For the present work the method of Kishi and Kuroki is adopted to predict the friction coefficient.

Kishi and Kuroki considered the bottom topography in terms of six different geometric forms. They are dunes I, dunes II, transition 1, transition 11, flat bed and antidunes.

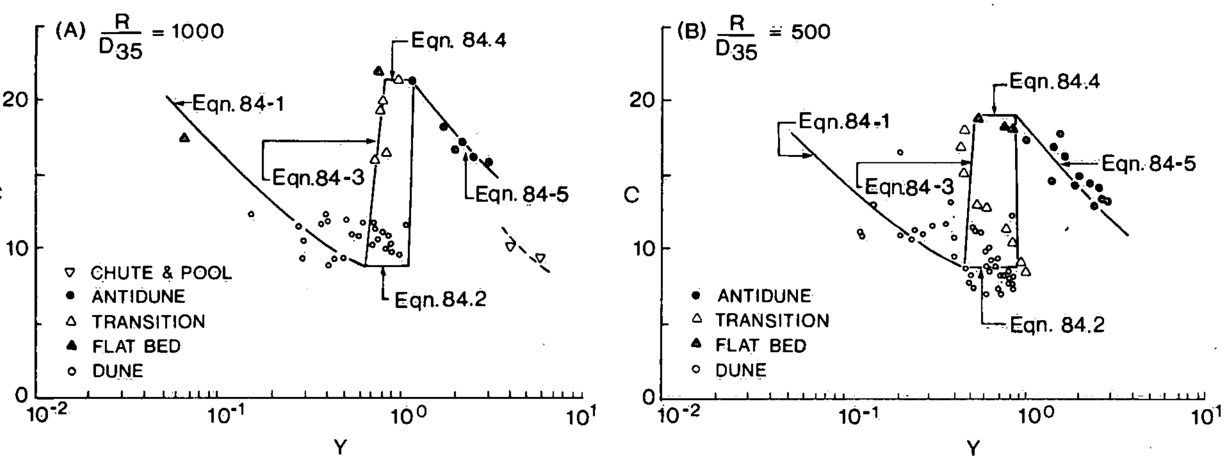

Figure 6. Comparison of calculated values of C with experiments (after Kishi and Kuroki, ref. 5).

The equations for the friction coefficient C for these bed configurations are

$$\left.\begin{array}{lll} (1) & \text{for dunes I,} & C = 2.4Z^{1/6}Y^{-1/3} \\ (2) & \text{for dunes II,} & C = 8.9 \\ (3) & \text{for transition 1,} & C = 1.1 \times 10^{6}Z^{-3/2}Y^{3} \\ (4) & \text{for flat bed,} & C = 6.9Z^{1/6} \\ (5) & \text{for antidunes,} & C = 2.8Z^{3/10}Y^{-1/3} \end{array}\right\} \quad (84)$$

The criteria for the occurrence of the various bed configurations can be stated as follows:

$$\left.\begin{array}{lll} (1) & \text{for dunes I,} & Y < 0.02Z^{1/2} \\ (2) & \text{for dunes II,} & Y = 0.02Z^{1/2} \\ (3) & \text{for transition 1,} & 0.02Z^{1/2} < Y < 0.02Z^{5/9} \\ (4) & \text{for flat bed,} & 0.02Z^{5/9} < Y < 0.07Z^{2/5} \\ \text{and } (5) & \text{for antidunes,} & Y < 0.07Z^{2/5} \end{array}\right\} \quad (85)$$

In the above equations the symbols Z and Y stand for the following dimensionless groups consisting of flow and sediment characteristic parameters:

$$\left.\begin{array}{l} Z = R/D_{35} \\ Y = \rho u_*^2/\gamma_s D \end{array}\right\} \quad (86)$$

The values of Z and Y are evaluated at each time step from the computed flow parameters. Using Equation 84 and the values of Z and Y, we can predict the values of C. These C values are then used to solve the equation for the next time step.

Storage Basins

If the river reach to be modelled includes a storage basin, the coefficients E, F, L, M, and K have to be modified at the sections enclosing the control volume, to which the storage basin is assumed to be connected (see Fig. 7). The modifications required for these coefficients are made as follows.

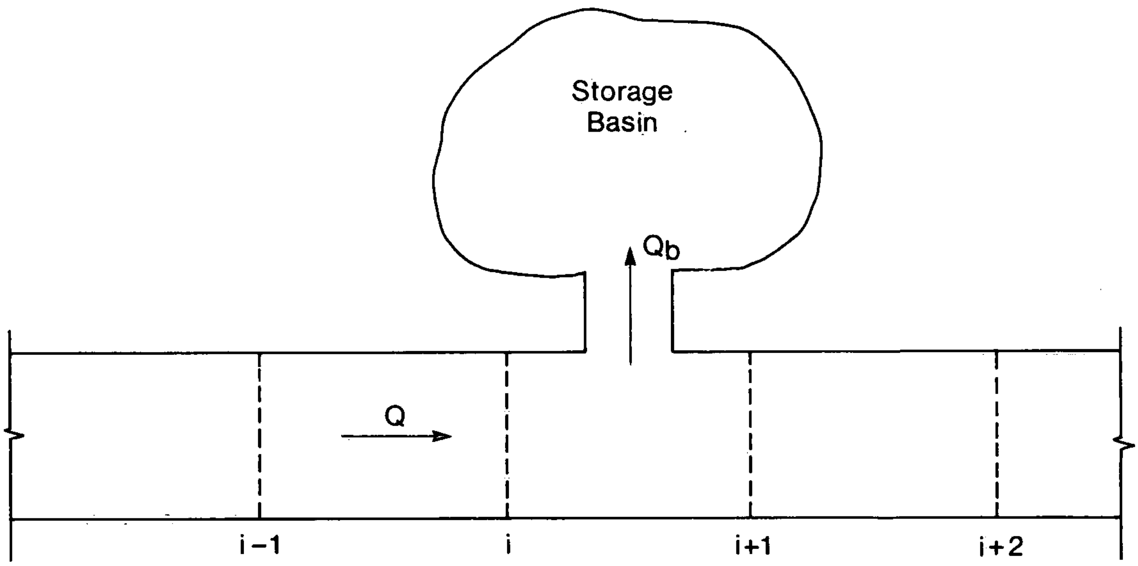

Figure 7. Schematic representation of a storage basin present in a river reach.

It is assumed that the water surface elevation at the storage basin (y_b) and that at the river section (y_i) are the same, i.e.

$$y_i^{n+1} = y_{i+1}^{n+1} = y_b^{n+1} . \tag{87}$$

The continuity equation between sections i and i+1 can be written as

$$Q_{i+1}^{n+1} = Q_i^{n+1} - Q_b^{n+1} \tag{88}$$

where Q_b is the discharge rate from river into basin or vice versa. Considering the continuity condition for the storage basin itself, we can write

$$(\Delta_b/\Delta t)\ \Delta y_b = Q_b^{n+1} \tag{89}$$

where A_b is the water surface area of the basin, which is a function of y_b.

Substituting Equation 89 into Equation 88, we get

$$\Delta Q_{i+1} = (Q_i^n - Q_{i+1}^n) + [\Delta Q_i - (A_b/\Delta t)]. \tag{90}$$

Equation 87 can be expressed as

$$y_i^n + \Delta y_i = y_{i+1}^n + \Delta y_{i+1} \tag{91}$$

which can be rearranged as

$$\Delta y_i = \Delta y_{i+1} + (y_{i+1}^n - y_i^n). \tag{92}$$

Using Equation 92, we can express ΔQ_i in Equation 90 as

$$\Delta Q_i = E_i \; [\Delta y_{i+1} + (y_{i+1}^n - y_i^n)] + F_i$$

and hence we can write Equation 90 as

$$\Delta Q_{i+1} = [E_i - (A_b/\Delta t)] \; \Delta y_{i+1} + [E_i (y_{i+1}^n - y_i^n) + F_i + Q_i^n - Q_{i+1}^n]. \tag{93}$$

Thus the values of E_{i+1} and F_{i+1} become

$$E_{i+1} = E_i - (A_b/\Delta t)$$

$$F_{i+1} = E_i (y_{i+1}^n - y_i^n) + F_i + Q_i^n - Q_{i+1}^n \tag{94}$$

The values of the coefficients L_i, M_i and K_i can be obtained by looking at Equation 92, which yields

$$L_i = 1; \; M_i = 0 \text{ and } K_i = y_{i+1}^n - y_i^n. \tag{95}$$

4. APPLICATION OF THE MODEL FOR A HYPOTHETICAL RIVER

To test and debug the computer program performing the various tasks of the model described so far, a river reach with the geometric characteristics shown in Figure 8 is chosen. As can be seen from Figure 8, the river reach includes both storage basin and a tributary. The length of the river reach is 2.44 km. It is divided into 20 equal segments each 122 m long. The initial condition for the flow rate and flow depth is shown in the computer output corresponding to T = 0. A constant inflow of 42.38 m^3/s is taken as the upstream boundary condition, whereas a constant depth of 2.29 m is assumed to yield the downstream boundary condition. The flow cross sections are approximated as trapeziums and hence the bottom widths and side angles are used as input parameters to describe the geometric parameters. The various control constants used in this program are listed below:

INFLOW: o, no lateral inflow from tributaries
n, number of tributaries

IS: o, no storage basins present in the river reach
N, location of the storage

ISED: o, river bottom is considered to be rigid
I, river bottom is composed of sediments

The description of the input data cards, the listing of the computer program and a sample output with line printer plots are given in the Appendix.

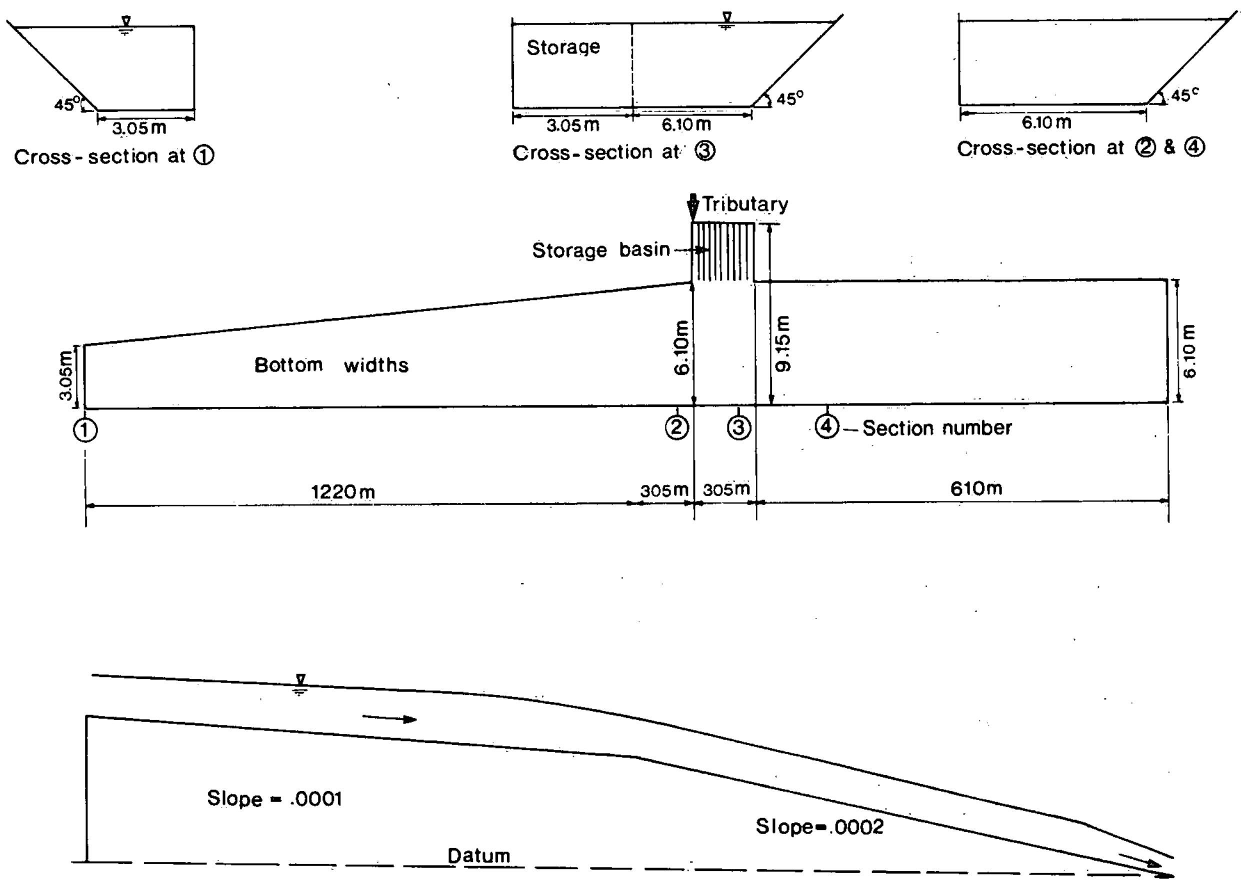

Figure 8. Profile and cross sections of the hypothetical river reach.

5. SUMMARY

A mathematical model describing the flow and sediment transport characteristics in natural streams has been presented in this report. The governing equations are derived from first principles in order to understand the simplifying assumptions better and consequently the limitations of the model. The numerical method and the solution techniques are also elaborated. The methods to predict the sediment transport rates and the friction coefficients in alluvial streams which are used in the present model are described. Finally, the application of the model for a hypothetical river reach is indicated.

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APPENDIX

The data cards are read into the program in the following order.

Card 1

N = 21 - number of points of initial data

INFLOW = 1, indicates presence of tributary

IN = 11, location of tributary, must be equal to 1, ..., N

INT = 800, time tributary inflow begins, s

IS = 12, location of storage basin, can be 1, ..., N

ISED = 1, indicates presence of sediment

Card 2

THETA = .66, weighting coefficient varying $.5 < \underline{\theta < 1}$

DELTAX = 122, distance between sections, m

XLENGH = 2440, total length of channel, m

G = 9.81, gravitational acceleration, m^3/s

QIN = .06, flow rate of tributary inflow, m^3/s

QSED = .0001, flow rate of lateral sediment inflow, m^3/s per unit length

SAR = 930.25, water surface area of storage basin, m^2

Cards 3 - 5

These cards are the values of depth at time zero at evenly spaced points beginning at the upstream boundary. Twenty-one values of depth are specified in metres.

Cards 6 - 8

These cards are the values of velocity at time zero at evenly spaced points beginning at the upstream boundary. Twenty-one values of velocity are specified in metres per second.

Cards 5 - 11

These cards are the values of bottom width at time zero at evenly spaced points beginning at the upstream boundary. Twenty-one values of bottom width are specified in metres.

Cards 12 - 14

These cards are the values of the right slope and left slope of the channel sides at time zero at evenly spaced points beginning at the upstream boundary. Twenty-one values of right slope and 21 values of left slope are specified in degrees.

11. Garde, R.J. and Ranga Raju, K.G., 1966. Resistance relationships for alluvial channel flow. J. Hydraul. Div., Proc. Am. Soc. Civ. Eng., Vol. 92, No. Hy4.

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Cards 15 - 17

These cards are the values of the bed elevation at time zero at evenly spaced points beginning at the upstream boundary. Twenty-one values of bed elevation are specified in metres.

Card 18

GAM = 1000. specific weight (submerged) of sediment, N/m^3

GAMS = 1650. specific weight of sediment, N/m^3

D_{35} = .0003 grain size of average sediment, m

YCR = .04 critical mobility number

ANU = $.1 \times 10^{-6}$ viscosity, m^2/s

Card 19

PORS = .8 porosity

```
      COMMON/A/ Y(41),RANG(41),LANG(41),BW(41),II,V(41)
      COMMON/B/ GAM,GAMS,D35,YCR,ANU,G
      COMMON /E/ DELTAX,DELTAT,N
      DIMENSION Z(41),B(41),A(41),P(41),R(41),DERB(41),DERP(41),QL1(41),
     1C(41),AYX(41),A1(41),B1(41),C1(41),D1(41),E1(41),A2(41),B2(41),C2(
     241),D2(41),E2(41),E(41),F(41),L(41),M(41),K(41),TY(100),TQ(100),
     3DELY(41),DELQ(41),Q(41),D(41),T(41),QL2(41)
      DIMENSION QS1(41),QS2(41),QST1(41),QST2(41),ZZ(41),VS(41)       ,D
     1ELZ(41),CAV1(41),CAV2(41),Y1(41),Y2(41)
      DIMENSION AVY(41)
      DIMENSION CAV(41),QST(41)
      INTEGER H
      REAL LANG,L,M,K
C       READ   CONSTANTS
      READ(60,104)  N,INFLOW,IN,INT,IS,ISED
      READ(60,105)  THETA,DELTAX,DELTAT,XLENGH,G,QIN,QSED,SAR
  104 FORMAT(6I5)
  105 FORMAT(8F8.4)
C     READ INITIAL CONDITIONS
      READ(60,100)  (Y(I),I=1,N)
      READ(60,101)  (V(I),I=1,N)
  100 FORMAT(8F10.3)
  101 FORMAT(8F10.4)
C    READ GEOMETRY CONDITIONS
      READ(60,100)  (BW(I),I=1,N)
      READ(60,102)  (LANG(I),RANG(I),I=1,N)
      READ(60,101)  (Z(I),I=1,N)
  102 FORMAT(16F5.2)
C          READ FRICTION CONDITIONS
      READ(60,107)  GAM,GAMS,D35,YCR,ANU
  107 FORMAT(5E10.4)
C           READ SEDIMENT CONDITIONS
      READ(60,103)  PORS
  103 FORMAT(F5.2)
      IAGAIN =0
      II=1
      IF(ISED.NE.1) GO TO 300
      DO 400 I=1,N
  400 Y1(I)=Y(I)
      GO TO 300
  300 IF(II.GE.16) GO TO 305
      CALL GEOMET(A,P,R,B,DERB,DERP,Q,AYX,AVY)
      IF(ISED.NE.1) GO TO 550
      CALL FRICT(Y,Z,Q,A,R,C,VS)
      IF(IAGAIN.EQ.1) GO TO 450
      IF(II.LE.2) GO TO 451
      DO 452 I=1,N
      CAV1(I)=CAV2(I)
  452 QST1(I)=QST2(I)
      GO TO 405
  451 IF(II.GE.2) GO TO 405
C
C                 TOTAL SEDIMENT RATE AND AVERAGE CONCENTRATION CALCULATIONS
C
      CALL SEDI(Q,A,AVY,C,CAV,QST)
      DO 401 I=1,N
      CAV1(I)=CAV(I)
  401 QST1(I)=QST(I)
      GO TO 450
  405 CALL SEDI(Q,A,AVY,C,CAV,QST)
      DO 404 I=1,N
      CAV2(I)=CAV(I)
  404 QST2(I)=QST(I)
C
C                 LATERAL SEDIMENT INFLOW
C
      DO 454 I=1,N
      QS1(I)=QSED
  454 QS2(I)=QSED
C
C                FLOW DEPTH AND BED ELEVATION CORRECTION
C
      DO 407 I=2,H
  407 DELZ(I)=.25/PORS*(DELTAT*(QS2(I-1)+QS2(I+1)+QS1(I-1)+QS1(I+1))-DEL
     1TAT/DELTAX*(QST2(I+1)-QST2(I-1)+QST1(I+1)-QST1(I-1))-(Y2(I+1)*CAV2
     2(I+1)+2.*Y2(I)*CAV2(I)+Y2(I-1)*CAV2(I-1)-Y1(I+1)*CAV1(I+1)-2.*Y1(I
     3)*CAV1(I)-Y1(I-1)*CAV1(I-1)))
      DELZ(1)=DELZ(2)
```

```
      DELZ(N)=DELZ(H)
      DO 408 I=1,N
      ZZ(I)=Z(I)+DELZ(I)
      Y(I)=Z(I)+Y2(I)-ZZ(I)
  408 Z(I)=ZZ(I)
      IAGAIN=1
      GO TO 300
  550 DO 551 I=1,N
  551 C(I)=10.0
C
C                    LATERAL INFLOW ADJUSTEMANTS
C
  450 DO 221 I=1,N
      QL1(I)=0.0
  221 QL2(I)=0.0
      IF(INFLOW.EQ.0) GO TO 199
      IT=II*DELTAT-DELTAT
      IIT=IT+DELTAT
      DO 198 I=1,N
      IF(I.NE.IN) GO TO 198
      IF(IT.LT.INT.AND.IIT.GE.INT) GO TO 195
      IF(IT.LT.INT) GO TO 198
      QL1(I)=QIN
      QL2(I)=0.0
      GO TO 198
  195 QL1(I)=0.0
      QL2(I)=QIN
  198 CONTINUE
  199 H=N-1
C
C                    DOUBLE SWEEP COEFFICIENTS
C
      DO 200 I=1,H
      A1(I)=(B(I)+B(I+1))/(2.*DELTAT)-(2.*THETA/DELTAX)*((Q(I+1)-Q(I))/(
     1B(I+1)+B(I))*DERB(I+1))+THETA*((QL1(I+1)+QL1(I))/(B(I+1)+B(I))*DER
     2B(I+1))
      B1(I)=2.*THETA/DELTAX
      C1(I)=(B(I)+B(I+1))/(2.*DELTAT)-(2.*THETA/DELTAX)*((Q(I+1)-Q(I))/
     1(B(I+1)+B(I))*DERB(I))+THETA*((QL1(I)+QL1(I+1))/(B(I+1)+B(I))*DERB
     2(I))
      C1(I)=-C1(I)
      D1(I)=-2.*THETA/DELTAX
      D1(I)=-D1(I)
      E1(I)=2./DELTAX*(Q(I+1)-Q(I))-(QL1(I+1)+QL1(I))-THETA*(QL2(I+1)+QL
     22(I))
  200 E1(I)=-E1(I)
      DO 201 I=1,H
      A2(I)=THETA/DELTAX*((Y(I+1)-Y(I))*Q(I+1)*Q(I+1)*B(I+1)*B(I+1)/(A(I
     1+1)**3)-(Q(I+1)-Q(I))*Q(I+1)*B(I+1)/(A(I+1)*A(I+1))+G/2.*(A(I+1)+
     2A(I)+B(I+1)*((Y(I+1)-Y(I))-(Z(I+1)-Z(I))))-.5*((B(I+1)
     3*Q(I+1)*Q(I+1)/(A(I+1)*A(I+1)))+B(I)*Q(I)*Q(I)/(A(I)*A(I))+(Y(I+1)
     4-Y(I))*Q(I+1)*Q(I+1)/(A(I+1)*A(I+1))*DERB(I+1)))+(THETA/(2.*C(I+1)
     5*C(I+1)))*Q(I+1)*Q(I+1)/(A(I+1)*A(I+1))*DERP(I+1)-THETA/(C(I+1)*C(
     6I+1))*P(I+1)*Q(I+1)*Q(I+1)*B(I+1)/A(I+1)**3+THETA*Q(I+1)*Q(I+1)
     7*AYX(I+1)*B(I+1)/A(I+1)**3
      B2(I)=1./(2.*DELTAT)+THETA/DELTAX*((2.*Q(I+1)-Q(I))/A(I+1)+Q(I)/A(
     1I)-(Y(I+1)-Y(I))*Q(I+1)*B(I+1)/(A(I+1)*A(I+1)))+(THETA/(C(I+1)*C(I
     2+1)))*P(I+1)*Q(I+1)/(A(I+1)*A(I+1))-THETA*Q(I+1)*AYX(I+1)/(A(I+1)*
     3A(I+1))
      C2(I)=THETA/DELTAX*((Y(I+1)-Y(I))*Q(I)*Q(I)*B(I)*B(I)/A(I)**3-(Q(I
     1+1)-Q(I))*Q(I)*B(I)/(A(I)*A(I))+G/2.*((Y(I+1)-Y(I))*B(I)-(A(I+1)+A
     2(I))-(Z(I+1)-Z(I))*B(I))+.5*(B(I+1)*Q(I+1)*Q(I+1)/(A(I+1)*A(I+1))+
     3B(I)*Q(I)*Q(I)/(A(I)*A(I))-(Y(I+1)-Y(I))*Q(I)*Q(I)*DERB(I)/(A(I)*A
     4(I))))+THETA/(C(I)*C(I))*(.5*Q(I)*Q(I)*DERP(I)/(A(I)*A(I))-P(I)*Q(
     5I)*Q(I)*B(I)/A(I)**3)+THETA*Q(I)*Q(I)*AYX(I)*B(I)/A(I)**3
      C2(I)=-C2(I)
      D2(I)=1./(2.*DELTAT)+THETA/DELTAX*((Q(I+1)-Q(I))/A(I)-(Q(I+1)/A(I+
     11)+Q(I)/A(I))-(Y(I+1)-Y(I))*Q(I)*B(I)/(A(I)*A(I)))+THETA*(P(I)*Q(I
     2)/(C(I)*C(I)*A(I)*A(I))-Q(I)*AYX(I)/(A(I)*A(I)))
      D2(I)=-D2(I)
      E2(I)=1./DELTAX*((Q(I+1)-Q(I))*(Q(I+1)/A(I+1)+Q(I)/A(I))+G/2.*((Y(
     1I+1)-Y(I))-(Z(I+1)-Z(I)))*(A(I+1)+A(I))-.5*(Y(I+1)-Y(I))*(B(I+1)*Q
     2(I+1)*Q(I+1)/(A(I+1)*A(I+1))+B(I)*Q(I)*Q(I)/(A(I)*A(I))))+.5*(P(I+
     31)*Q(I+1)*Q(I+1)/(C(I+1)*C(I+1)*A(I+1)*A(I+1))+P(I)*Q(I)*Q(I)/(C(I
     4)*C(I)*A(I)*A(I))-Q(I+1)*Q(I+1)*AYX(I+1)/(A(I+1)*A(I+1))-Q(I)*Q(I)
     5*AYX(I)/(A(I)*A(I)))
  201 E2(I)=-E2(I)
C
```

```
C                      UPSTREAM BOUNDARY CONDITIONS
C
      DO 202 I=1,100
  202 TQ(I)=42.475
      K1=II
      E(1)=0.0
      F(1)=TQ(K1+1)-TQ(K1)
C
C                      STORAGE ADJUSTEMENTS
C
      DO 211 I=2,N
      J=I-1
      E(I)=(A1(J)*(C2(J)+D2(J)*E(J))-A2(J)*(C1(J)+D1(J)*E(J)))/(B2(J)*(C
     11(J)+D1(J)*E(J))-B1(J)*(C2(J)+D2(J)*E(J)))
      F(I)=((E2(J)+D2(J)*F(J))*(C1(J)+D1(J)*E(J))-(E1(J)+D1(J)*F(J))*(C2
     1(J)+D2(J)*E(J)))/(B2(J)*(C1(J)+D1(J)*E(J))-B1(J)*(C2(J)+D2(J)*E(J)
     2))
      IF(I.NE.IS) GO TO 211
      E(I)=E(J)-SAR/DELTAT
      F(I)=Q(J)-Q(J+1)+F(J)+E(J)*(Y(J+1)-Y(J))
  211 CONTINUE
      DO 212 I=1,H
      L(I)=A1(I)/(C1(I)+D1(I)*E(I))
      M(I)=B1(I)/(C1(I)+D1(I)*E(I))
      K(I)=(E1(I)+D1(I)*F(I))/(C1(I)+D1(I)*E(I))
      IF(I.NE.IS) GO TO 212
      L(I)=1.0
      M(I)=0.0
      K(I)=0.0-(Y(I+1)-Y(I))
  212 CONTINUE
C
C                      DOWNSTREAM BOUNDARY CONDITIONS
C
      DO 216 I=1,100
  216 TY(I)=2.29
      DO 3 I=1,N
      T(I)=I*DELTAT-DELTAT
    3 D(I)=I*DELTAX-DELTAX
      IF(II.NE.1) GO TO 8
      WRITE(61,14)
   14 FORMAT(1H1,1X,≠TIME = SECONDS≠,6X,≠DISTANCE = METRES≠,6X,≠SLOPE =
     1DEGREES≠)
      WRITE(61,1)
    1 FORMAT(1H ,1X,≠GEOMETRIC PROPERTIES≠)
      WRITE(61,2)
    2 FORMAT(1H0,1X,≠DISTANCE≠,7X,≠BOTTOM WIDTH≠,7X,≠SLOPE OF RIGHT BANK
     1≠,7X,≠SLOPE OF LEFT BANK≠)
      WRITE(61,4)   (D(I),BW(I),RANG(I),LANG(I),I=1,N)
    4 FORMAT(1H ,F7.0,9X,F10.4,18X,F3.0,23X,F3.0)
      WRITE(61,5)
    5 FORMAT(1H1,1X,≠UPSTREAM BOUNDARY CONDITIONS≠)
      WRITE(61,6)
    6 FORMAT(1H0,1X,≠TIME≠,6X,≠FLOW RATE≠)
      WRITE(61,7)  (T(I),TQ(I),I=1,12)
    7 FORMAT(1H ,F5.0,4X,F10.4)
      WRITE(61,9)
    9 FORMAT(1H0,1X,≠DOWNSTREAM BOUNDARY CONDITIONS≠)
      WRITE(61,10)
   10 FORMAT(1H0,1X,≠TIME≠,6X,≠FLOW DEPTH≠)
      WRITE(61,7)  (T(I),TY(I),I=1,12)
    8 WRITE(61,11)
   11 FORMAT(1H1,1X,≠SOLUTION AT TIME T=     SECONDS≠)
      WRITE(61,12)
   12 FORMAT(1H0,1X,≠DISTANCE≠,2X,≠FLOW RATE≠,2X,≠FLOW DEPTH≠,2X,≠SEDIME
     1NT RATE≠,2X,≠FRICTION≠,2X,≠BOTTOM ELEVATION≠,2X,≠TOP WIDTH≠,2X,≠FL
     2OW AREA≠,2X,≠WETTED PERIMETER≠,2X,≠HYDRAULIC RADIUS≠)
      IF(II.NE.1) GO TO 15
      WRITE(61,13)(D(I),Q(I),Y(I),QST1(I),C(I),Z(I),B(I),A(I),P(I),R(I),
     1I=1,N)
   13 FORMAT(1H ,1X,F6.0,2X,F10.4,1X,F10.4,2X,F10.4,4X,F10.4,4X,F10.4,
     14X,F10.4,2X,F10.4,4X,F10.4,8X,F10.4)
      GO TO 16
   15 WRITE(61,13)(D(I),Q(I),Y(I),QST2(I),C(I),Z(I),B(I),A(I),P(I),R(I),
     1I=1,N)
   16 DELY(N)=TY(K1+1)-TY(K1)
      DELQ(N)=E(N)*DELY(N)+F(N)
      Y(N)=Y(N)+DELY(N)
      Q(N)=Q(N)+DELQ(N)
```

```
C
C                        NEW FLOW DEPTH AND DISCHARGE
C
      DO 213 I=1,H
      J=N-I
      DELY(J)=L(J)*DELY(J+1)+M(J)*DELQ(J+1)-K(J)
      Y(J)=Y(J)+DELY(J)
      DELQ(J)=E(J)*DELY(J)+F(J)
  213 Q(J)=Q(J)+DELQ(J)
      DO 453 I=1,N
      IF(II.EQ.1) GO TO 453
      Y1(I)=Y2(I)
  453 Y2(I)=Y(I)
      II=II+1
      IAGAIN=0
      GO TO 300
  305 CONTINUE
      STOP
      END
```

```
;ASI FORTRAN DIAGNOSTIC RESULTS FOR FTN.MAIN

    NO ERRORS

IG ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE

    B              E
```

```
      SUBROUTINE GEOMET(A,P,R,B,DERB,DERP,Q,AYX,AVY)
      COMMON/A/ Y(41),RANG(41),LANG(41),BW(41),II,V(41)
      COMMON /E/ DELTAX,DELTAT,N
      DIMENSION A(41),P(41),R(41),B(41),DERB(41),DERP(41),Q(41),X1(41),
     1X2(41),AYX(41),AVY(41)
      REAL LANG
      INTEGER H
      DO 3 I=1,N
      X1(I)=LANG(I)*0.017453293
      X2(I)=RANG(I)*0.017453293
      A(I)=Y(I)*(BW(I)+Y(I)*(TAN(X1(I))+TAN(X2(I)))/2.)
      P(I)=BW(I)+Y(I)*(SEC(X1(I))+SEC(X2(I)))
      R(I)=A(I)/P(I)
      IF(II.GE.2) GO TO 2
      Q(I)=A(I)*V(I)
    2 B(I)=BW(I)+Y(I)*(TAN(X1(I))+TAN(X2(I)))
      AVY(I)=A(I)/B(I)
      DERB(I)=TAN(X1(I))+TAN(X2(I))
    3 DERP(I)=SEC(X1(I))+SEC(X2(I))
      H=N-1
      DO 4 I=1,H
    4 AYX(I)=1./(2.*DELTAX)*((Y(I)+Y(I+1))*(BW(I+1)-BW(I))+(Y(I
     1+1)+Y(I))**2*(TAN(X1(I+1))+TAN(X2(I+1))-TAN(X1(I))-TAN(X2(I)))/4.)
      AYX(N)=AYX(H)
      RETURN
      END
```

```
;ASI FORTRAN DIAGNOSTIC RESULTS FOR GEOMET

    NO ERRORS

IG ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE

    E
```

```
      SUBROUTINE FRICT(Y,Z,Q,A,R,C,VS)
      COMMON/B/ GAM,GAMS,D35,YCR,ANU,G
      COMMON/E/ DELTAX,DELTAT,N
      DIMENSION C(41),Y(41),Z(41),VS(41),YD(41),YR(41),S(41),SF(41),
     1Q(41),A(41),R(41)
      K=N-1
      DO 10 I=2,K
      SF(I)=((Z(I-1)+Y(I-1)+((Q(I-1)/A(I-1))*(Q(I-1)/A(I-1))/(2.*G)))
     1-(Z(I+1)+Y(I+1)+((Q(I+1)/A(I+1))*(Q(I+1)/A(I+1))/(2.*G))))/(2.*DEL
     2TAX)
      IF(SF(I).LT.0.00) SF(I)=0.000001
   10 CONTINUE
      SF(N)=SF(K)
      SF(1)=SF(2)
      DO 11 I=1,N
      VS(I)=SQRT(G*R(I)*SF(I))
      YD(I)=GAM*SF(I)*R(I)/(GAMS*D35)
      YR(I)=R(I)/D35
   11 S(I)=VS(I)*D35/ANU
      DO 17 I=1,N
      YCR1=0.02*SQRT(YR(I))
      YCR2=0.01844*YR(I)**(5./9.)
      YCR3=0.07*YR(I)**(2./5.)
      IF(YCR2-YCR3) 19,19,18
   18 YCR2=YCR3
   19 CONTINUE
      IF(YD(I).LT.YCR1) GO TO 12
      IF(YD(I).EQ.YCR1) GO TO 13
      IF((YD(I).GT.YCR1).AND.(YD(I).LT.YCR2)) GO TO 14
      IF((YD(I).GT.YCR2).AND.(YD(I).LT.YCR3)) GO TO 15
      IF(YD(I).GT.YCR3) GO TO 16
   12 C(I)=2.40*YR(I)**(1./6.)/YD(I)**(1./3.)
      GO TO 17
   13 C(I)=8.9
      GO TO 17
   14 C(I)=1.1E+6*YD(I)**3/YR(I)**(3./2.)
      GO TO 17
   15 C(I)=6.9*YR(I)**(1./6.)
      GO TO 17
   16 C(I)=2.8*YR(I)**(3./10.)/YD(I)**(1./3.)
   17 CONTINUE
      RETURN
      END
```

```
;ASI FORTRAN DIAGNOSTIC RESULTS FOR FRICT

    NO ERRORS

IG ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE

    E
```

```
      SUBROUTINE SEDI(Q,A,AVY,C,CAV,QST)
      COMMON/B/ GAM,GAMS,D35,YCR,ANU,G
      COMMON /E/ DELTAX,DELTAT,N
      DIMENSION Q(41),A(41),AVY(41),C(41),CAV(41),QST(41)
C
C                       SEDIMENT CONSTANTS
C
      DGR=D35*(G*GAMS/(ANU*ANU*GAM))**(1.0/3.0)
      IF(DGR.GT.1.0.AND.DGR.LE.60.0) GO TO 402
      RN=0.0
      RA=.17
      RM=1.5
      RC=0.025
      GO TO 10
  402 RN=1.-.56*ALOG10(DGR)
      RA=.23/SQRT(DGR)+.14
      RM=9.66/DGR+1.34
      RCC=2.86*ALOG10(DGR)-(ALOG10(DGR))**2-3.53
      RC=10.0**(RCC)
   10 DO 11  I=1,N
      FGR=(Q(I)/A(I))*(SQRT(32.0)*ALOG10(10.0*AVY(I)/D35))**(RN-1.0)/
     1(C(I)**RN*SQRT(G*D35*(GAMS/GAM)))
      ZX=RC*((FGR-RA)/RA)**RM
      XX=ZX*((GAMS/GAM)+1)*D35*C(I)**RN/AVY(I)
      CAV(I)=XX/((GAMS/GAM)+1)
       QST(I)=ZX*D35*(Q(I)/A(I))*C(I)**RN
   11 CONTINUE
      RETURN
      END
```

```
SASI FORTRAN DIAGNOSTIC RESULTS FOR SEDI

    NO ERRORS

IG ARE COMMON BLOCK NAMES OR NAMES NOT ASSIGNED STORAGE

    E
```

TIME = SECONDS DISTANCE = METRES SLOPE = DEGREES

GEOMETRIC PROPERTIES

DISTANCE	BOTTOM WIDTH	SLOPE OF RIGHT BANK	SLOPE OF LEFT BANK
0.	3.0500	45.	0.
122.	3.2940	45.	0.
244.	3.5380	45.	0.
366.	3.7820	45.	0.
488.	4.0260	45.	0.
610.	4.2700	45.	0.
732.	4.5140	45.	0.
854.	4.7580	45.	0.
976.	5.0020	45.	0.
1098.	5.2460	45.	0.
1220.	5.4900	45.	0.
1342.	5.7340	45.	0.
1464.	5.9780	45.	0.
1586.	6.1000	0.	45.
1708.	6.1000	0.	45.
1830.	6.1000	0.	45.
1952.	6.1000	0.	45.
2074.	6.1000	0.	45.
2196.	6.1000	0.	45.
2318.	6.1000	0.	45.
2440.	6.1000	0.	45.

UPSTREAM BOUNDARY CONDITIONS

TIME	FLOW RATE
0.	42.4750
244.	42.4750
488.	42.4750
732.	42.4750
976.	42.4750
1220.	42.4750
1464.	42.4750
1708.	42.4750
1952.	42.4750
2196.	42.4750
2440.	42.4750
2684.	42.4750

DOWNSTREAM BOUNDARY CONDITIONS

TIME	FLOW DEPTH
0.	2.2900
244.	2.2900
488.	2.2900
732.	2.2900
976.	2.2900
1220.	2.2900
1464.	2.2900
1708.	2.2900
1952.	2.2900
2196.	2.2900
2440.	2.2900
2684.	2.2900

SOLUTION AT TIME T= 0 SECONDS

DISTANCE	FLOW RATE	FLOW DEPTH	SEDIMENT TRANSPORT RATE	BOTTOM ELEVATION	TOP WIDTH	FLOW AREA	WETTED PERIMETER	HYDRAULIC RADIUS
0.	42.3792	4.8000	0.0024	0.3620	7.8500	26.1600	14.6382	1.7871
122.	42.4429	4.7000	0.0023	0.3498	7.9940	26.5258	14.6408	1.8118
244.	42.4306	4.6000	0.0022	0.3376	8.1380	26.8548	14.6434	1.8339
366.	42.3446	4.5000	0.0020	0.3254	8.2820	27.1440	14.6460	1.8533
488.	42.5920	4.4100	0.0020	0.3132	8.4360	27.4787	14.6727	1.8728
610.	42.3684	4.3100	0.0018	0.3010	8.5800	27.6918	14.6753	1.8870
732.	42.4890	4.2200	0.0018	0.2888	8.7340	27.9533	14.7020	1.9013
854.	42.5503	4.1300	0.0018	0.2766	8.8860	28.1790	14.7287	1.9132
976.	42.7006	4.0300	0.0018	0.2644	9.0320	28.2785	14.7313	1.9196
1098.	42.5088	3.9300	0.0016	0.2522	9.1760	28.3392	14.7339	1.9234
1220.	42.5361	3.8500	0.0017	0.2400	9.3400	28.5478	14.7847	1.9303
1342.	42.3704	3.7600	0.0015	0.2190	9.4940	28.6285	14.8114	1.9329
1464.	42.7230	3.6900	0.0015	0.1950	9.6680	28.8669	14.8864	1.9391
1586.	43.0909	3.6100	0.0017	0.1708	9.7100	28.5371	14.8153	1.9262
1708.	42.7351	3.5100	0.0020	0.1464	9.6100	27.5711	14.5739	1.8918
1830.	42.1668	3.4000	0.0021	0.1220	9.5000	26.5200	14.3093	1.8535
1952.	42.6509	3.2800	0.0028	0.0976	9.3800	25.3872	14.0186	1.8110
2074.	43.0186	3.1200	0.0035	0.0732	9.2200	23.8992	13.6323	1.7531
2196.	42.5086	2.9400	0.0046	0.0488	9.0400	22.2558	13.1978	1.6863
2318.	42.4573	2.6900	0.0072	0.0244	8.7900	20.0271	12.5942	1.5902
2440.	42.4731	2.2900	0.0118	0.0000	8.3900	16.5911	11.6285	1.4268

SOLUTION AT TIME T= 244 SECONDS

DISTANCE	FLOW RATE	FLOW DEPTH	SEDIMENT TRANSPORT RATE	BOTTOM ELEVATION	TOP WIDTH	FLOW AREA	WETTED PERIMETER	HYDRAULIC RADIUS
0.	42.3792	5.1685	0.0025	0.3930	8.2185	29.1209	15.5279	1.8754
122.	40.2747	4.9608	0.0021	0.3808	8.2548	28.6453	15.2703	1.8759
244.	38.7576	4.7756	0.0017	0.3687	8.3135	28.2993	15.0673	1.8782
366.	37.6847	4.6209	0.0015	0.3564	8.4029	28.1525	14.9373	1.8846
488.	36.7749	4.4753	0.0014	0.3442	8.5013	28.0319	14.8304	1.8902
610.	36.3702	4.3433	0.0012	0.3321	8.6133	27.9776	14.7555	1.8961
732.	35.9826	4.2243	0.0012	0.3197	8.7383	27.9906	14.7123	1.9025
854.	35.8195	4.1034	0.0012	0.3075	8.8614	27.9428	14.6644	1.9055
976.	35.7810	3.9832	0.0011	0.2954	8.9852	27.8570	14.6183	1.9056
1098.	36.0138	3.8758	0.0011	0.2835	9.1218	27.8432	14.6030	1.9067
1220.	36.2324	3.7579	0.0006	0.2715	9.2479	27.6919	14.5524	1.9016
1342.	36.5462	3.7423	0.0006	0.2502	9.4763	28.4604	14.7586	1.9271
1464.	33.6634	3.7426	0.0007	0.2258	9.7206	29.3772	15.0135	1.9567
1586.	32.9387	3.6505	0.0007	0.2016	9.7505	28.9315	14.9132	1.9400
1708.	32.5665	3.5507	0.0008	0.1772	9.6507	27.9632	14.6722	1.9059
1830.	32.3296	3.4431	0.0009	0.1526	9.5431	26.9300	14.4123	1.8685
1952.	31.5936	3.3133	0.0011	0.1279	9.4133	25.6997	14.0939	1.8228
2074.	30.9547	3.1548	0.0014	0.1032	9.2548	24.2205	13.7163	1.7658
2196.	30.8798	2.9435	0.0019	0.0774	9.0435	22.2872	13.2062	1.6876
2318.	30.7435	2.6801	0.0030	0.0503	8.7801	19.9400	12.5703	1.5863
2440.	30.6833	2.2641	0.0049	0.0259	8.3641	16.3738	11.5659	1.4157

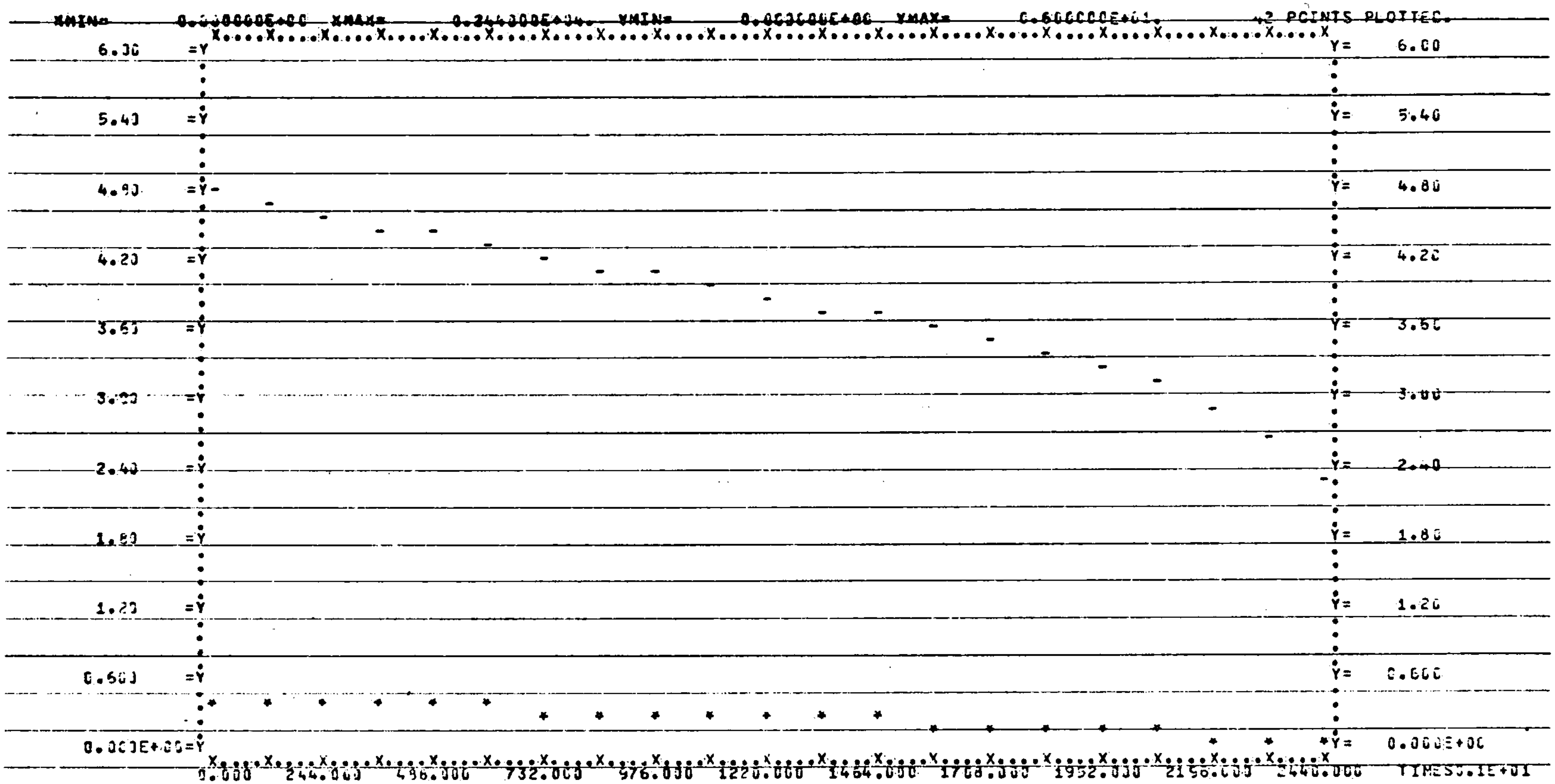
SOLUTION AT TIME T=488 SECONDS

DISTANCE	FLOW RATE	FLOW DEPTH	SEDIMENT TRANSPORT RATE	BOTTOM ELEVATION	TOP WIDTH	FLOW AREA	WETTED PERIMETER	HYDRAULIC RADIUS
0.	42.3792	5.2704	0.0019	0.4241	8.3204	29.9636	15.7740	1.8996
122.	42.4061	5.1222	0.0019	0.4119	8.4162	29.9908	15.6600	1.9151
244.	41.8441	4.9683	0.0019	0.3995	8.5063	29.9196	15.5325	1.9263
366.	40.9848	4.8039	0.0018	0.3871	8.5859	29.7070	15.3796	1.9316
488.	40.1189	4.6411	0.0017	0.3748	8.6671	29.4553	15.2307	1.9339
610.	39.1535	4.4760	0.0016	0.3625	8.7460	29.1299	15.0760	1.9322
732.	38.4339	4.3085	0.0015	0.3501	8.8225	28.7303	14.9157	1.9262
854.	37.8759	4.1504	0.0014	0.3380	8.9084	28.3601	14.7779	1.9191
976.	37.4349	4.0116	0.0014	0.3260	9.0135	28.1127	14.6869	1.9141
1098.	37.0072	3.8753	0.0011	0.3150	9.1213	27.8386	14.6018	1.9065
1220.	36.5485	3.7865	0.0003	0.3029	9.2765	27.9568	14.6315	1.9107
1342.	35.8823	3.8035	0.0005	0.2805	9.5375	29.0429	14.9165	1.9470
1464.	33.3809	3.8035	0.0007	0.2561	9.7815	29.9710	15.1605	1.9769
1586.	33.1193	3.7035	0.0007	0.2320	9.8035	29.4495	15.0411	1.9579
1708.	32.7013	3.5982	0.0008	0.2074	9.6382	28.4222	14.7367	1.9221
1830.	32.3177	3.4741	0.0010	0.1826	9.5741	27.2265	14.4872	1.8794
1952.	32.3406	3.3206	0.0012	0.1576	9.4206	25.7693	14.1167	1.8254
2074.	32.4974	3.1367	0.0016	0.1324	9.2367	24.0530	13.6726	1.7592
2196.	32.4597	2.9263	0.0022	0.1055	9.0263	22.1319	13.1647	1.6812
2318.	32.5023	2.6515	0.0035	0.0762	8.7515	19.6891	12.5012	1.5750
2440.	32.5422	2.2382	0.0060	0.0518	8.3382	16.1574	11.5034	1.4046

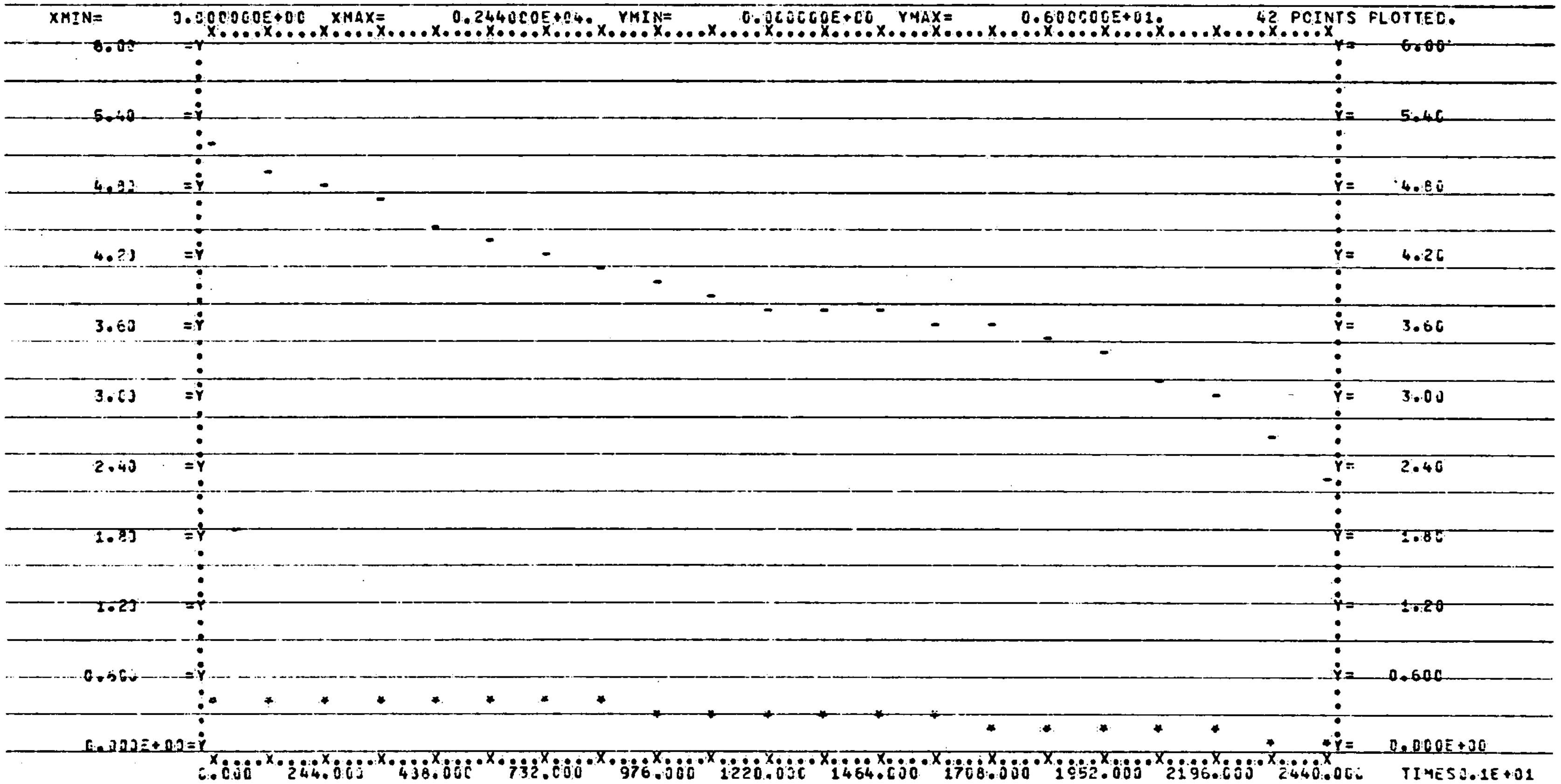
SOLUTION AT TIME T=3416 SECONDS

DISTANCE	FLOW RATE	FLOW DEPTH	SEDIMENT TRANSPORT RATE	BOTTOM ELEVATION	TOP WIDTH	FLOW AREA	WETTED PERIMETER	HYDRAULIC RADIUS
0.	42.3792	5.7281	0.0013	0.7930	8.7781	33.6763	16.8789	2.0073
122.	42.1996	5.6023	0.0012	0.7808	8.8963	34.1468	16.8191	2.0302
244.	42.0136	5.4802	0.0011	0.7684	9.0182	34.4054	16.7684	2.0518
366.	41.8215	5.3617	0.0011	0.7558	9.1437	34.6517	16.7262	2.0717
488.	41.6236	5.2467	0.0010	0.7434	9.2727	34.8871	16.6926	2.0900
610.	41.4234	5.1350	0.0009	0.7313	9.4050	35.1107	16.6670	2.1066
732.	41.2166	5.0262	0.0009	0.7190	9.5402	35.3201	16.6484	2.1215
854.	41.0018	4.9199	0.0009	0.7066	9.6779	35.5119	16.6358	2.1347
976.	40.7821	4.8154	0.0008	0.6935	9.8174	35.6807	16.6274	2.1459
1098.	40.5564	4.7108	0.0008	0.6776	9.9568	35.8089	16.6189	2.1547
1220.	43.9940	4.5808	0.0012	0.6706	10.0708	35.6466	16.5491	2.1536
1342.	47.4039	4.4303	0.0008	0.6474	10.1643	35.2173	16.4297	2.1435
1464.	43.8129	4.4309	0.0011	0.6164	10.4989	36.3048	16.6752	2.1772
1586.	43.5640	4.3022	0.0012	0.5942	10.4022	35.4977	16.4364	2.1532
1708.	43.3114	4.1598	0.0013	0.5675	10.2598	34.0265	16.1426	2.1079
1830.	43.0586	4.0037	0.0016	0.5400	10.1037	32.4374	15.7658	2.0575
1952.	42.8060	3.8277	0.0020	0.5098	9.9277	30.6750	15.3410	1.9995
2074.	42.5600	3.6195	0.0027	0.4713	9.7195	28.6296	14.8383	1.9294
2196.	42.3323	3.3512	0.0045	0.4136	9.4512	26.0579	14.1936	1.8363
2318.	42.1951	2.9006	0.0077	0.2698	9.0006	21.9002	13.1026	1.6714
2440.	42.1652	2.0446	0.0224	0.2454	8.1446	14.5627	11.0362	1.3195

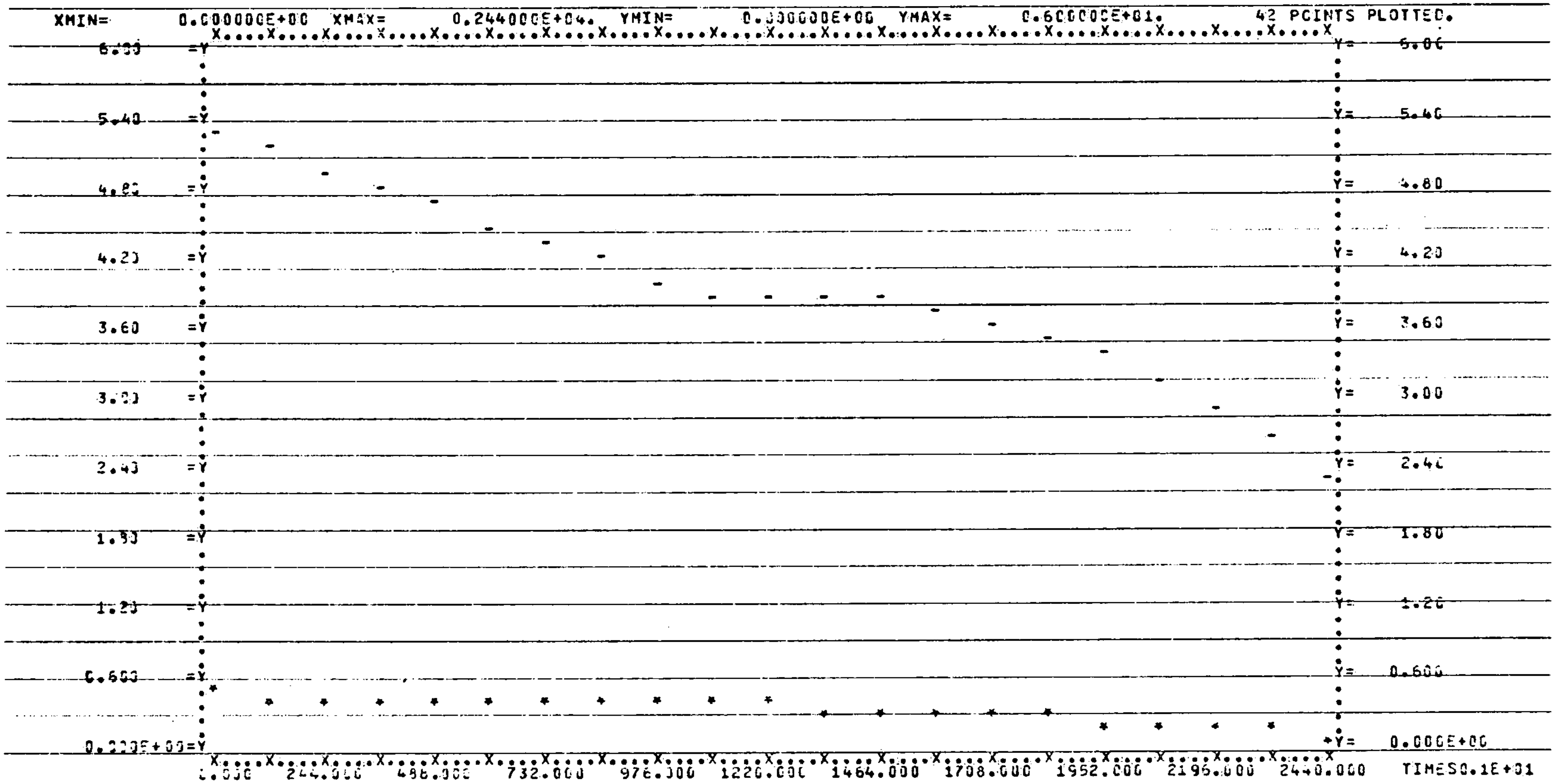
STOP



PLOT OF BED ELEVATION(*),DEPTH(-) AGAINST DISTANCES

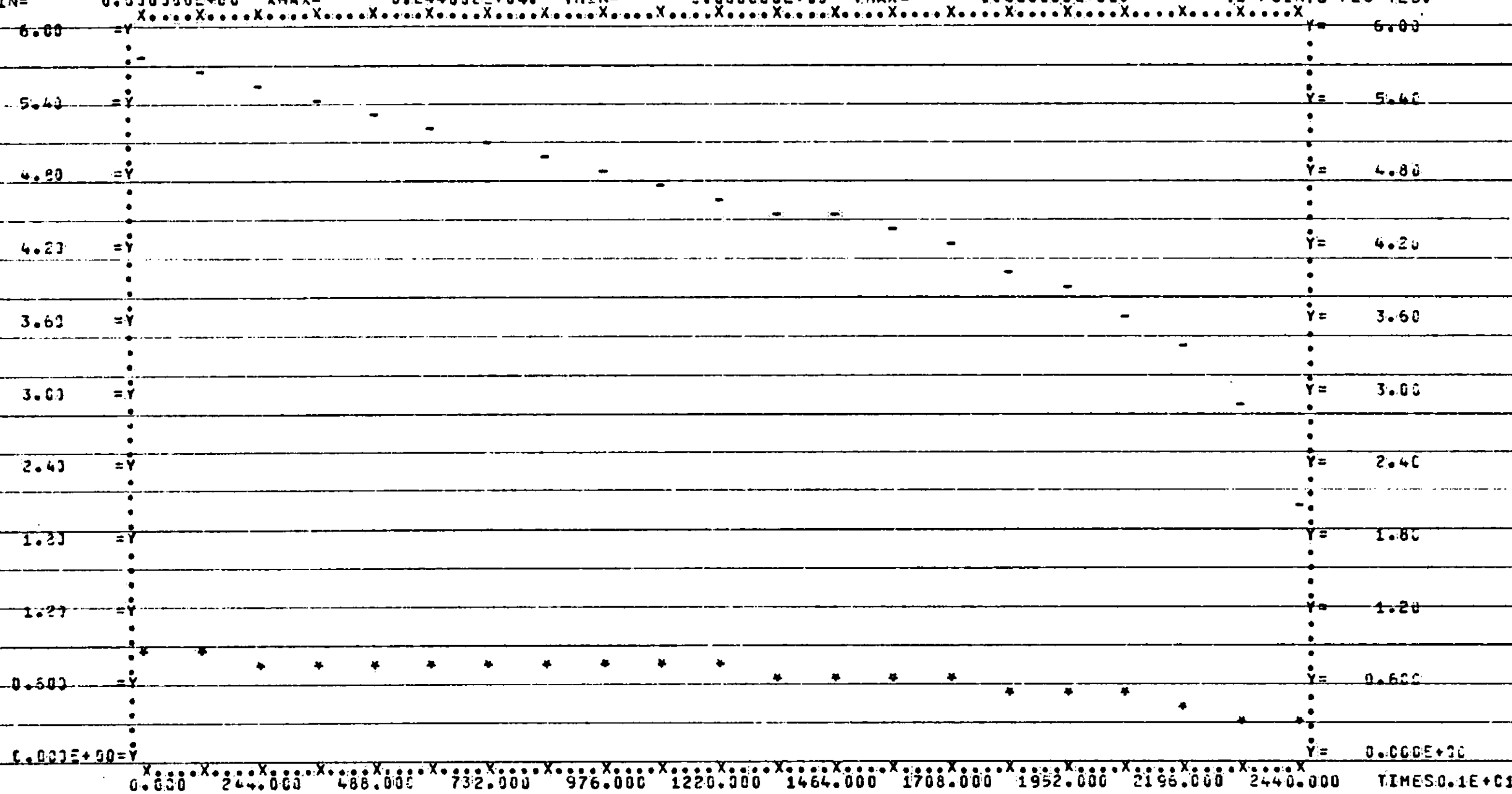


PLOT OF BED ELEVATION(*),DEPTH(-) AGAINST DISTANCES



PLOT OF BED ELEVATION(*),DEPTH(-) AGAINST DISTANCES

XMIN= 0.000000E+00 XMAX= 0.244000E+04 YMIN= 0.000000E+00 YMAX= 0.600000E+01 42 POINTS PLOTTED.



PLOT OF BED ELEVATION(*),DEPTH(-) AGAINST DISTANCES

STOP