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A PROPOSED CANADIAN STANDARD FOR ESTIMATING ATMOSPHERIC DISPERSION OF COMBUSTION SOURCE POLLUTION FROM CHIMNEYS

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A PROPOSED CANADIAN STANDARD FOR ESTIMATING ATMOSPHERIC DISPERSION OF COMBUSTION SOURCE POLLUTION FROM CHIMNEYS

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Abstract—Rising public concern over the persistent nature of air pollution may be expected to result in more stringent ambient air quality criteria. Hence, until air resource management becomes a reality, there is a requirement for a simple yet reliable method of estimating the dispersion of combustion source chimney effluents. To satisfy this need an empirical plume rise equation was developed and together with established diffusion equations, it provides the basis for the graphical stack height calculation method described in this paper. The emissions of gaseous and particulate matter are related to heat flux, stack height and maximum ground level pollution concentrations for both a severe atmospheric inversion and a neutral atmosphere. This enables the selection of a stack height to be made which should meet any stipulated ambient pollution level.

1. INTRODUCTION

WHEN it became necessary in 1966 to write a national code for air pollution control in Canada a CSA (Candian Standards Association) sub-committee on particulate matter

Nomenclature:

C_{omax} = maximum ground level concentration of pollutant with respect to axial distance from the stack ($\mu g/nm^3$);
C'_{omax} = maximum ground level concentration of pollutant with respect to wind velocity at stack top ($\mu g/nm^3$);
C_z, C_y = Sutton's diffusion coefficients defined in equation (4) for vertical and horizontal diffusion respectively (dimensionless);
e = base of natural logarithms = 2·7183;
E = particulate or gaseous pollution emission rate from stack (kg/sec);
F = inversion layer reflection factor defined in equation (7) (dimensionless);
H_e = effective height of emission (m);
H_s = stack height (m);
p, q = Bosanquet diffusion coefficients defined in equation (6) for vertical and horizontal diffusion respectively;
Q = heat emission from stack (kcal/sec);
U = wind velocity at stack top (m/sec);
U_c = wind velocity at stack top under critical conditions (m/sec);
V_s = stack efflux velocity (m/sec);
x = horizontal distance downwind of stack (m);
Z = plume rise above stack top (m);
Z_c = plume rise above stack top under critical conditions (m);
α = constant in Lucas plume rise equation (1);
σ_z, σ_y = Pasquill plume standard deviations defined in equation (5) for vertical and horizontal diffusion respectively (m).

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emissions from combustion sources was faced with the complex problem of relating dust and heat emission from stacks to ground level dust concentrations. Obviously this relationship will be influenced by the ambient conditions of meteorology and topography because of their influence on plume development and dispersion. In the present study the dilution of combustion gases in the atmosphere is considered to occur in two main stages, namely plume rise and dispersion. Initially, the plume axis attains an effective height arbitrarily defined as plume rise, and then subsequently spreads both vertically and laterally in the dispersion process until it reaches ground level. The latter phase of dispersion from an elevated source has been the subject of considerable theoretical and experimental work by BOSANQUET and PEARSON (1936), SUTTON (1947, 1953) and PASQUILL (1961, 1962) to name only a few. Consequently, ground level concentrations from elevated source emissions can be predicted to a reasonable degree of accuracy for simplified terrain and meteorological considerations.

On the other hand plume rise can be estimated with considerably less accuracy than dispersion and it follows that overall predictions can only be as accurate as the combination of the two stages will allow. The development of an empirical plume rise equation from available theoretical and experimental information and its application to a proposed design standard are described in this paper.

In the proposed standard the two phases of the plume development are co-ordinated to relate combustion source heat, dust and gaseous emissions, chimney height and dust collector efficiency to maximum ground level dust and gaseous concentrations in a flexible graphical form which is simple to use. It is possible to optimize the cost of collection equipment against that of additional chimney height above the basic minimum height which may be necessary for other reasons. It is recommended that large plant (> 1000 MW generating capacity) or plant situated in locations having unusual meteorological or topographical conditions be made the subject of a separate more detailed study, while using the present standard only as a rough guide.

2. THE DEVELOPMENT OF PLUME RISE AND DISPERSION EQUATIONS

The dispersion of a hot gaseous plume containing particulate material is extremely complex with efflux characterististics, meteorology, topography and relative motion between particles and gases each playing an important part.

In plumes from combustion sources, it is usually accepted that buoyancy is much more important than momentum in determining the height that the plume will attain and consequently efflux momentum ~~is not considered separately~~ ignored as a significant parameter. This simplification does not imply that efflux momentum has no effect on all stages of plume development and dispersion.

If the plume model is considered to have two main stages, plume rise and subsequent dispersion, each is considered independently in the following sections of this paper. Simplification of the meteorology and topography may be made assuming a two-layer atmosphere after that of SCRIVEN (1967). In the atmospheric layer next to the earth's surface, turbulence is induced due to surface roughness and heat convection. Apart from this the earth's surface is assumed to be flat, with no large-scale topographical features such as valleys or hills. The plume axis is assumed to reach its maximum height in this lower layer. Above it is a layer of more stable air in which the diffusive capabilities are assumed to be less than in the lower layer. It has been shown

by Scriven that the highest ground level concentrations occur when the plume height and the height of the base of the upper atmospheric layer are coincident. This is the condition used for design purposes here.

Relative dispersion between particles and gases is influenced by three main factors. Firstly, if the particle is large enough to have an appreciable settling velocity then relative motion may be expected especially since it is acknowledged that buoyancy forces do not carry large particles to the same height as small particles and gases. However, SMITH (1959) has found that this may be neglected below a free fall velocity of 6 ft/sec corresponding to a particle of 240 μ dia. This is well above the maximum size limitation of 60 μ recommended for particulate emissions in this standard. The second factor which may cause relative diffusion is denoted by YUDINE (1959) as the crossing trajectories effect. Particles tend not to follow turbulent atmospheric eddies which increase gaseous diffusion and hence may reach ground level sooner than gases. The third factor, a continuity effect, is caused by a particle being caught in the backflow to an eddy and thus retarded further. Obviously these last two effects are interrelated and may bring a considerable reduction in the dispersion of the larger particles. A consideration of CSANADY (1963) has established that deviations of less than 5 per cent from gaseous dispersion may be expected if particle diameters are kept below 60 μ. Hence the further simplification of assuming gaseous diffusion laws to apply to particles may be made. The plume model has two main stages, plume rise and dispersion and each will be dealt with in the following sections.

2.1 *Plume rise*

The primary mixing zone of the plume (FIG. 1) is defined by buoyancy, momentum, initial plume diameter and the horizontal shear forces applied by the prevailing wind. In practice, provided the plume has sufficient efflux velocity to overcome drag or down-

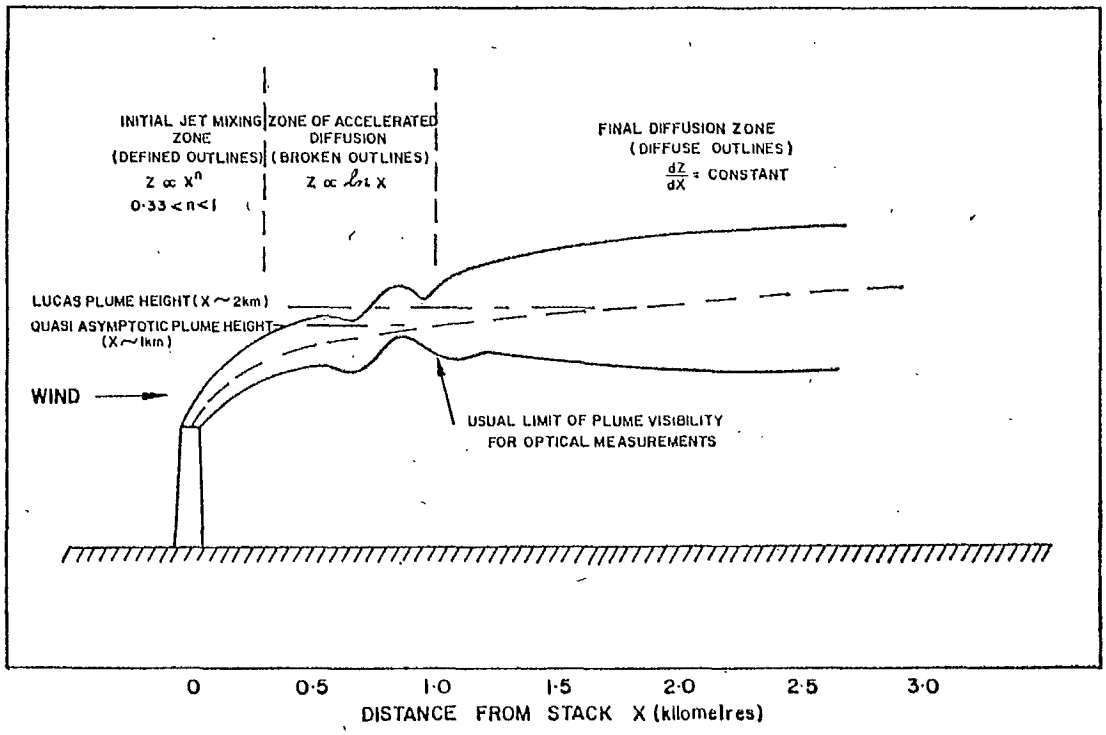


FIG. 1. The physical model of a hot gas plume.

wash, caused by unusual topographical considerations, it may be regarded as a sheared over jet as will be the case in the simplified model. In this region the self-induced turbulence of the byoyant plume is the major diffusing agency and atmospheric turbulence plays a minor role. This phase of plume development is amenable to analytical treatment by adaptation of simple free jet theory to account for transverse flow. Hence, during this phase which is typically between 0–300 m from the stack $Z \propto x^n$ where $0{\cdot}33 < n < 1{\cdot}0$ (after SUTTON, 1950; PRIESTLEY, 1956; SCORER, 1959; and PATRICK, 1967).

With the onset of the second phase of plume development, atmospheric turbulence begins to play a more important role in the diffusion process; the plume boundaries begin to fluctuate giving rise to broken outlines and a very much higher diffusion rate. In this region, sometimes called the accelerated diffusion zone, the plume has lost much of its initial excess temperature and vertical momentum and tends toward an asymptotic height at a distance in the order of 1 km. Since this distance generally coincides with the limit of visibility the asymptotic height is often used as the definition of plume rise and its measurement by optical means is possible. Also, in this region, $Z \propto \ln x$ if the overall plume diameter is regarded as proportional to x, the distance from the stack. It is recognized that in the final diffusion zone, to be considered in the next section, the plume axis is seldom horizontal. CSANADY (1961) indicates it will rise gradually and linearly with distance if a neutral homogeneous atmosphere is assumed or it may be deflected back to ground level if an inversion layer is present. LUCAS, JAMES and DAVIES (1967) described a sophisticated ruby-laser technique (Lidar) which can detect non-visible plumes containing particulate material at a distances more than 2 km from the stack. These measurements indicate that at these distances the plume axis can rise by as much as 60 per cent over the height of the axis at about $\frac{1}{2}$ km from the stack. This result is confirmed by CARPENTER *et al.* (1967) using a photographic technique showing the path of the plume up to 2 km from the stack. They suggest that an arbitrary plume rise should be defined as the point where momentum and buoyancy cease to play a part in the further elevation of the plume axis. This is defined as the point where the rate of rise of the axis with distance from the stack reaches a minimum value or becomes constant. This generally occurs at horizontal distances of between 450 m and 1200 m from the stack. BOSANQUET (1957) selected as his final height the plume rise after a minimum of 200 sec travel time in order to ensure that emission characteristics such as buoyancy and momentum had ceased to contribute to further plume rise. Theoretically it is only under certain meteorological conditions that the plume axis levels off or falls and selection of a definition of plume rise with respect to axial distance is arbitrary under all other conditions.

Of the many plume rise equations published, those by DAVIDSON (1954); LUCAS, MOORE and SPURR (1963); HOLLAND (1953); SCORER (1958, 1959); BOSANQUET, CAREY and HALTON (1950); BOSANQUET (1957) and SUTTON (1950), were considered to be serious contenders but detailed analyses revealed certain limitations in each case. On the premise that each equation represents reliable data, a mean plume rise equation identified as CCRL-1, was developed for a range of heat emissions from 10^2 kcal/sec to 10^5 kcal/sec. This mean equation appeared to offer interesting possibilities in the absence of a more suitable or reliable equation but at this point in time (1966) the ASME published their APS-1 Standard and the Tennessee Valley Authority plume rise measurements of CARPENTER *et al.* (1967) became available to supplement the earlier

European measurements on which the CONCAWE design standard was based, (BRUMMAGE *et al.*, 1966).

The plume rise measurements of LUCAS, MOORE and SPURR (1963), RAUCH (1964), STEWART, GALE and CROOKS (1954) together with those of the Tennessee Valley Authority were felt to be sufficient to justify the development of an empirical equation.

In the mathematical development of this equation the data were grouped into seven heat emission categories ranging from less than 10^3 up to 10^5 kcal/sec. Assuming an inverse relationship between Z and U these groups were plotted using the ordinates Z/Q and U^{-1} and the best regression line was fitted to each group. In this way the power n to which the heat emission Q must be raised to give a unique plot of Z/Q^n against U^{-1} was estimated. This showed that the plume rise dependence on heat emission was less than at first thought, the best overall fit to the data being obtained by a plot of the form $Z/Q^{0.25}$ against U^{-1} as was found by PRIESTLEY (1956) and LUCAS, MOORE and SPURR (1963). However, the plume rise predictions by this equation, designated as CCRL-2, were considerably lower than those predicted by the Lucas equation, most probably due to inclusion of the RAUCH (1964) low heat flux data.

$$\text{Lucas, } Z = \alpha \frac{Q^{0.25}}{U} \tag{1}$$

where $100 < \alpha < 126$ at present, the value depending on site and meteorology in a manner as yet unknown. Here Z is defined as the height at which the mean plume axis levels off whether this be in the visible or non-visible regions of the plume and hence the rate of rise is zero at the point of measurement ($x \sim 2$ km)

$$\text{CCRL-2, } Z = 66{\cdot}4 \frac{Q^{0.25}}{U} \tag{2}$$

where Z is defined as the height attained by the visible plume when buoyancy and initial efflux momentum cease to contribute to any subsequent elevation of the mean plume axis. This definition implies that the rate of rise either becomes constant or attains a minimum value at the point of measurement (usually 450 m $< x <$ 1200 m).

The CCRL-2 plume rise equation is compared with the recent CERL data of HAMILTON (1967) in FIGS. 3, 4 and 5. In FIGS. 3 and 4 the data represents optical height measurements at a distance of approximately $\frac{1}{2}$ km from the stack at the Tilbury Power Station. There is little difference between CCRL-2 and the equation developed by Lucas and subsequently modified for proximity of the measuring location to the stack. However, the modified Lucas and CCRL-2 curves predict plume rises which are higher than observed values reported in FIG. 3. These data represent conditions of low heat emission when the station was on part load and hence the efflux velocity was reduced. In most cases V_s was reported to be less than 0·625 U. MOORE (1967) suggests that, where $V_s < 0{\cdot}5\ U$, any enhanced ground level concentrations measured are possibly due to reduced plume height caused by downwash effects in the lee of the stack and buildings. Only plumes of low exit momentum coupled with short stacks will be affected in this way. NONHEBEL (1960) suggests that if the efflux velocity is $1\frac{1}{2}$ times as great as the wind speed past the stack top and the stack is $2\frac{1}{2}$ times as high as the building then downwash is likely to be avoided. In cases where these figures are impractical, extensive wind-tunnel work may be needed to assess the situation for design purposes.

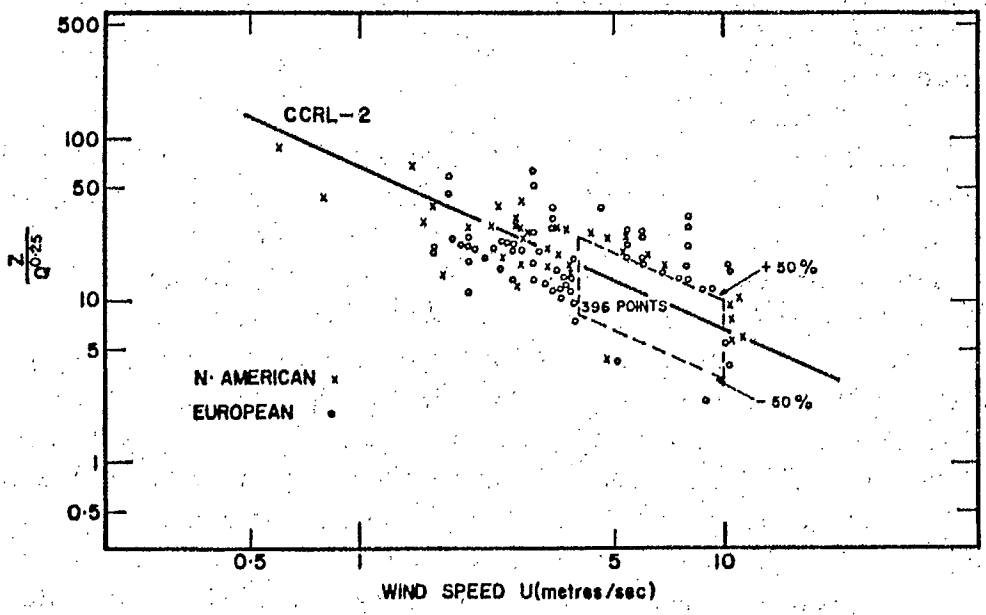


FIG. 2. Empirical derivation of the CCRL-2 plume rise equation.

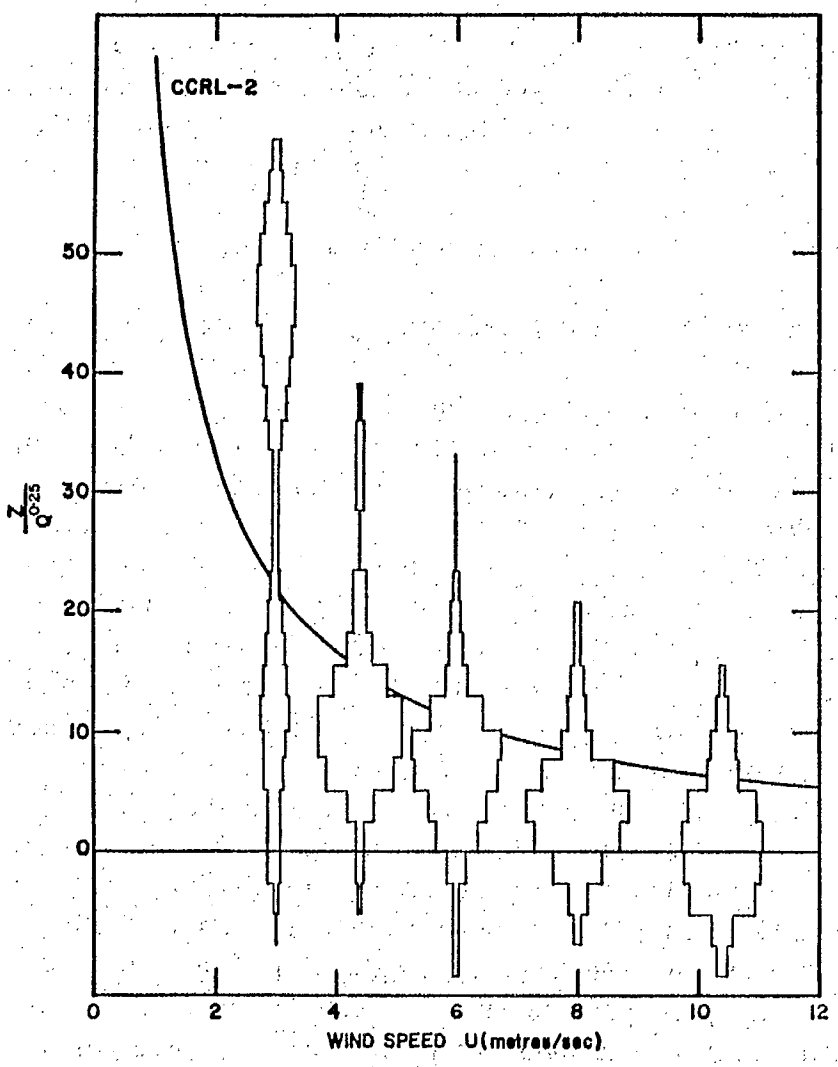


FIG. 3. Comparison of the CCRL-2 plume rise equation with recent CERL data, Tilbury 1200–4800 kcal/sec stack emission.

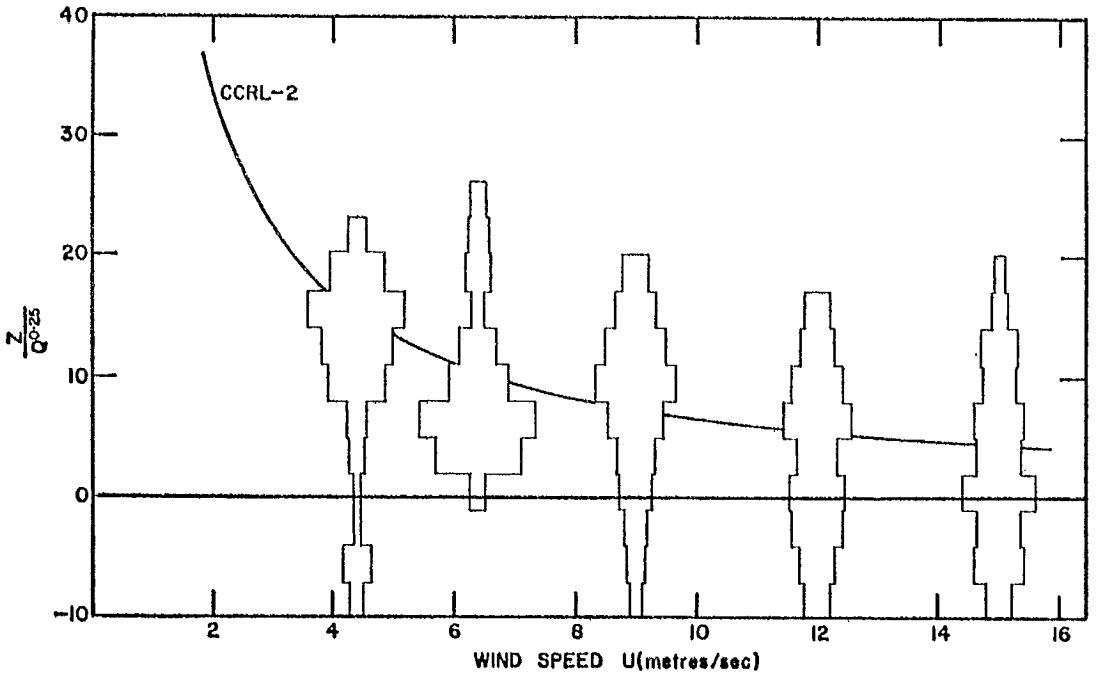


FIG. 4. Comparison of the CCRL-2 plume rise equation with recent CERL data, Tilbury 4800–14,300 kcal/sec stack emission.

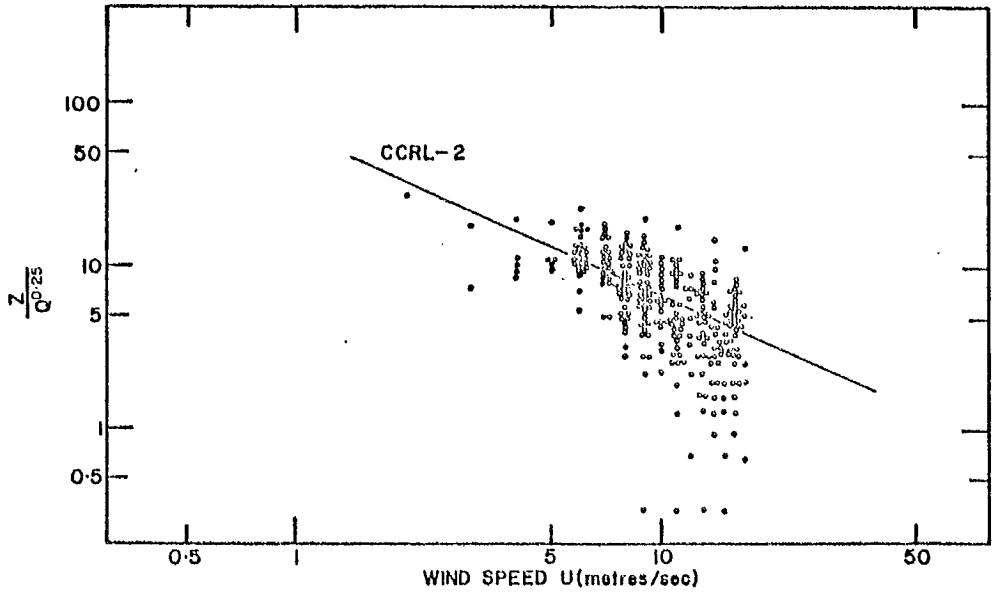


FIG. 5. Comparison of the CCRL-2 plume rise equation with recent CERL data, Northfleet (modified) 2400–14,300 kcal/sec stack emission.

To compare CCRL-2 in FIG. 5 with plume rise data from the Northfleet Power Station where measurements were made by the LIDAR technique at about 2 km from the stack, it was necessary to apply the factor of 0·62 to the data according to LUCAS, MOORE and SPURR (1963). Otherwise CCRL-2 underestimates the data by about 35 per cent and a better fit is obtained by the Lucas equation. This reinforces the argument that the main difference between the plume rise measurements of different research groups is the horizontal distance of the point of plume rise measurement from the stack, i.e. the basic definition of plume rise. Obviously the measurements are influenced by other factors as well; measuring techniques, duration of the measuring

TABLE 1. COMPARISON OF MEASURED AND EXTRAPOLATED PLUME RISE WITH CALCULATED VALUES

Heat flux (Mcal/sec)	Wind vel. U (m/sec)	Measured and extrapolated plume rise: Measured* Z_x (m)	Measured* x † (m)	Extrapolated Z_{1000} (m)	Extrapolated Z_{2000} (m)	Calculated plume rise: Concawe simplified $Z_{2000} = 5{\cdot}53\frac{Q^{0{\cdot}25}}{U^{0{\cdot}75}}$	Lucas $Z_{2000} = 116{\cdot}5\frac{Q^{0{\cdot}25}}{U}$	Moses simplified $Z_{1000} = 5{\cdot}32\frac{Q^{0{\cdot}5}}{U}$	Briggs I $Z_{1000} = 66{\cdot}6\frac{Q^{0{\cdot}33}}{U}$	Briggs II $Z_{2000} = 15{\cdot}2\frac{Q}{U^3}$	Csanady $Z_{1000} = 84{\cdot}5\frac{Q^{0{\cdot}33}}{U}$	CCRL-2 $Z_{1000} = 66{\cdot}4\frac{Q^{0{\cdot}25}}{U}$
1·25	4·0	97	400–1200	97	156	69	173	47	178	297	226	99
2·50	3·5	147	400–1200	91	147	108	235	76	258	886	327	135
4·06	7·2	82	1200–2500	51	82	80	129	47	148	165	187	74
4·06	10·2	107	1200–2500	107	173	62	91	33	104	58	131	52
7·89	4·2	182	1200–2500	113	167	167	261	118	316	1619	401	149
7·89	7·2	182	1200–2500	113	182	112	152	66	184	321	233	87
7·89	10·2	105	1200–2500	65	105	86	108	46	130	113	165	61
7·89	13·0	60	1200–2500	37	60	72	84	36	102	55	129	48
7·89	16·2	68	1200–2500	42	68	61	68	29	82	28	104	39
8·5	4·5	180	400–1200	180	290	165	248	109	302	1418	383	142
8·5	5·2	158	400–1200	158	255	148	215	94	216	919	331	123
8·5	6·1	146	400–1200	146	236	131	183	80	223	569	283	105
8·5	9·5	120	400–1200	120	193	94	118	52	143	150	181	67
9·56	2·0	450	1200–2500	279	450	321	577	260	706	$1{\cdot}86\times10^4$	896	329
11·9	4·2	167	1200–2500	104	167	205	288	138	362	2441	460	164
11·9	7·2	162	1200–2500	100	162	137	168	81	211	485	268	96
11·9	10·2	100	1200–2500	62	100	106	119	57	149	170	189	68
11·9	13·0	85	1200–2500	53	85	88	93	45	117	82	148	53
11·9	16·2	60	1200–2500	37	60	75	75	36	94	43	119	43
15·7	2·0	510	1200–2500	316	510	412	652	333	833	$2{\cdot}983\times10^4$	1056	370
15·7	6·0	220	1200–2500	136	220	181	217	111	277	1105	352	123
15·7	8·0	165	1200–2500	102	165	146	163	83	208	466	264	93
15·7	10·0	96	1200–2500	60	96	123	130	67	167	239	211	74

15·7	12·0	80	1200–2500	50	80	107	109	56	139	138	176	62
15·7	16·0	120	1200–2500	74	120	87	82	42	104	58	132	46
17·6	0·6	1025	400–1200	1025	1653	1074	2232	1177	2890	$1{\cdot}239 \times 10^6$	3667	1273
17·6	1·4	777	400–1200	777	1253	570	957	504	1235	$9{\cdot}749 \times 10^4$	1567	545
17·7	6·6	103	400–1200	103	166	179	203	107	262	936	333	116
18·4	11·3	70	400–1200	70	113	123	121	64	155	194	196	68
24·6	5·2	106	400–1200	106	171	252	282	160	372	2659	472	160
	log mean error $\dfrac{\sum_{1\text{-}n} \left\lvert \dfrac{\ln Z_x \text{ measured}}{Z_x \text{ calculated}} \right\rvert}{n}$					41%	24%	28%	115%	330%	173%	21%

* These data are from GARTRELL, THOMAS and CARPENTER (1964), CARPENTER *et al.* (1967) and HAMILTON (1967).
† Axial distance x from stack in metres.

period, differences in topography and meteorology, stack diameter: height ratio, and efflux momentum cannot be considered of minor importance.

The plume rise equations of MOSES and CARSON (1967), BRUMMAGE *et al.* (1966), LUCAS, MOORE and SPURR (1963), BRIGGS (1965), SLAWSON and CSANADY (1967) and CCRL-2 are compared in TABLE 1 with experimental data over a range of heat emissions and wind velocities. It is suggested that CCRL-2 is the best representation of these data bearing in mind the definition of plume rise upon which it is based. TABLE 1 presents data largely not included in the derivation of CCRL-2 which were used as an independent check of the validity of the equation; the headings show the plume rise definitions used for comparative purposes in estimating errors. It can be seen that the LUCAS and CCRL-2 equations have similar errors when related to their respective definitions of plume rise. In the derivation of CCRL-2 from the data in references given in TABLE 2 the overall error was found to be 24 per cent which is almost the same as that calculated for this equation in TABLE 1. The error similarly calculated for the Moses equation was 58 per cent which is almost double that given in TABLE 1. This is most likely due to the preponderance of relatively low heat flux data (around 2×10^3 kcal/sec) used for the derivation. The Moses equation tends to underestimate plume rise at low heat fluxes of this order.

TABLE 2. SOURCES OF PLUME RISE DATA

Reference	Number of obervations utilized	Atmospheric category	Efflux velocity range (m/sec)	Wind velocity range (m/sec)	Heat emission range (10^3 kcal/sec)
CARPENTER *et al.* (1967)	62 7	neutral stable	14·8–24·5	0·6–11·3	15·0–24·6
LUCAS, MOORE and SPURR (1963)	13* 14*	neutral stable	2·0–18·0	3·0–10·5	0·7–16·0
STEWART GALE and CROOKS (1954)	26	neutral stable	9·9	3·0–10·5	1·2
RAUCH (1964)	329	neutral stable	2·2–12·0	1·6–10·5	0·6–3·3

* Mean values representing 536 balloon measurements of plume.

2.2 *Dispersion of particulates and gases*

Although plume rise and dispersion are considered separate stages of plume development some dispersion does occur while the plume is rising and continues into the main diffusion zone.

Diffusion from an elevated source has been treated by several workers the most notable being PASQUILL (1961, 1962), SUTTON (1947, 1953) and BOSANQUET and PEARSON (1936) all of whose equations may be simplified to take the form $C_{0\max} \propto E/UH_e^2$, where $C_{0\max}$ is the maximum ground level concentration with respect to axial distance downwind of the source.

The Pasquill–Sutton form of the equation is:

$$C_{0\max} = \frac{2E\,10^9}{\pi U e H_e^2} \tag{3}$$

in which axisymmetrical diffusion is assumed. Sometimes this is not the case, especially near ground level or in very turbulent conditions, and a factor must be applied to the equation to correct for the resulting unevenness of diffusion.

Thus:

$$\text{Sutton} \qquad C_{0\max} = \frac{2E\,10^9}{\pi U e H_e^2}\left(\frac{C_z}{C_y}\right) \tag{4}$$

$$\text{Pasquill} \qquad C_{0\max} = \frac{2E\,10^9}{\pi U e H_e}\left(\frac{\sigma_z}{\sigma_y}\right) \tag{5}$$

$$\text{Bosanquet} \quad C_{0\max} = \frac{4E\,10^9}{\sqrt{2\pi e^2 U H_e^2}}\left(\frac{p}{q}\right). \tag{6}$$

The factor C_x/C_y, σ_x/σ_y or p/q is a ratio of vertical and horizontal diffusion parameters and may be influenced by the vertical temperature gradient, atmospheric stability, degree of turbulence, inversion layers, or the duration of the sampling period.

Equations (4) and (5) are applicable to short-term sampling periods of 3 min, whereas equation (6) is based on a 30-min period. Since p/q varies from 0·5 to 0·625 between neutral and unstable atmospheres and C_z/C_y and σ_z/σ_y are generally unity for neutral conditions, it might appear that Sutton and Pasquill values are almost double those of Bosanquet. However, if the reported p/q value is doubled to account for short-term sampling as indicated by the ASME (1966), LUCAS (1967) and MARTIN and BARBER (1966) then similar values of the concentration are predicted by all the equations. It must also be noted, however, that in a sampling period as short as three minutes H_e may be considerably different from its mean value.

The CONCAWE design standard gives a range of values of C_z/C_y from 0·5 to 0·9 for neutral to turbulent conditions, the sampling period chosen being 30 min. This is in accordance with the accepted values of $C_z/C_y = \sigma_z/\sigma_y = 1$ for neutral conditions and shorter-term sampling. SCRIVEN's (1967) theoretical analysis of a two-layer model of the atmosphere, in which the lower layer has a greater capacity for diffusion than the upper layer, reveals that concentrations may be doubled if the boundary between the two layers is assumed coincident with the effective height of emission. A factor F is defined as the ratio of ground level concentrations with and without the inversion layer present. The value of F rises to a maximum of 2 as the upper layer becomes more stable with respect to the lower layer. Experimental values obtained at Tilbury show that for a neutral upper and turbulent lower layer the mean value of $[F(\sigma_z/\sigma_y)]$ was 0·95, with some values higher than 2 being recorded.

Hence if it is assumed for design purposes that the short-term value of $[F(\sigma_z/\sigma_y)]$ is 2, this may correspond to the extreme conditions mentioned above, with F attaining a maximum value of 2 since $\sigma_z/\sigma_y = 1$.

On the other hand, it may correspond to a more turbulent upper layer, where F is reduced, but σ_z/σ_y is increased with $[F(\sigma_z/\sigma_y)]$ remaining the same.

The equation

$$C_{0\max} = \frac{2E\,10^9}{\pi U e H_e^2}\,[F(\sigma_z/\sigma_y)] \tag{7}$$

becomes

$$C_{0\max} = 4{\cdot}68 \times 10^8 \frac{E}{U H_e^2} \tag{8}$$

when $[F(\sigma_z/\sigma_y)] = 2$, corresponding to an inversion coincident with the effective emission height.

Equation (8) predicts concentrations double those for a uniformly neutral atmosphere, but it does represent a condition which occurs and consequently must be designed for. $F \approx 2$ occurred on eight occasions (SCRIVEN, 1967, FIG. 4, p. 418) out of 100 in strong wind conditions (plume height $\leqslant 225$ m corresponding roughly to wind speeds > 10 m/sec) which in turn represented 24 per cent of all occasions (MOORE, 1967, p. 406). That is to say $F \approx 2$ actually occurred for the Tilbury plume on 2 per cent of all occasions. The condition of an elevated inversion over a turbulent ground-based layer did occur on 15 per cent of the occasions but the inversion must be in the right place with respect to the plume for F to be about 2.

It is noted at this point that the variation of concentration with distance has a very flat profile so the position of the maximum ground level concentration is difficult to determine, especially since the experimental values do not lie on a smooth curve. The flatness of the profile makes it possible to measure the maximum ground level concentration to a reasonable degree of accuracy. The variation of concentrations with sampling period duration has been observed by LUCAS (1967) and MARTIN and BARBER (1966). TABLE 3 gives the relationship of concentration with time, the reference sampling duration being taken as 3 min. This table gives the concentration decay for

TABLE 3. VARIATION OF CONCENTRATION WITH SAMPLING PERIOD

Sampling period	Concentration ratio 3 min = 1
3 min	1·00
30 min	0·50
1 h	0·33
1 day	0·08
1 month	0·02

sampling periods from 3 min to 1 month and is in good agreement with the theory of OGURA (1959) and data presented by HINO (1968). Essentially after an initial period, usually of a few minutes, in which the concentration-time decay obeys a $-\frac{1}{5}$ power law as advocated by NONHEBEL (1960) and WIPPERMAN (1961), the decay then becomes more rapid, obeying a $-\frac{1}{2}$ power law after that of HINO and OGURA. The range over which each power law is applicable seems to vary with meteorological conditions and whether the measurement is made either at the axis or the fringes of the plume. LUCAS (1967) suggests that as conditions become less steady the ratio of 1 hr to 3 min concentrations can vary from $\frac{3}{4}$ to $\frac{1}{3}$ on the plume axis. At the plume fringes this value can

be much less. This range embraces all the data yet presented. Diffusion theories and formula are usually based on short-term measurements assuming a steady wind and constant emission characteristics. These assumptions may become progressively less realistic as the sampling time is extended. Therefore, it is possible that long-term concentrations may be considerably less than Nonhebel and Wipperman suggest.

3. DEVELOPMENT OF STACK DESIGN STANDARD

It has been shown that the primary and secondary mixing zones of a hot gas plume are generally responsible for the greatest proportion of the height attained by the plume. In the final zone, which is mainly the diffusion process, there is sometimes a further rise of the plume axis which may be important in estimating concentrations at large distances from the stack. This may be neglected in predicting the maximum ground level concentrations for locations closer to the stack when critical conditions prevail.

In order to determine the maximum ground level concentration with respect to wind speed and atmospheric condition the CCRL-2 plume rise equation (2) and the diffusion equation (7) have to be optimized by use of the usual mathematical procedures. Since plume rise is inversely proportional to wind speed it has been shown by THOMAS (1965) that when

$$Z = H_s \tag{9}$$

the maximum ground level concentrations are obtained, other conditions being assumed constant.

Denoting the critical wind speed as U_c substitution in equation 2 gives

$$U_c = 66{\cdot}4\,\frac{Q^{0{\cdot}25}}{H_s}. \tag{10}$$

Whence from equation (7)

$$C'_{0\max} = \frac{2\times 10^9 . E}{66{\cdot}4\,Q^{0{\cdot}25}\,4H_s . \pi . e}\left(\frac{F.\sigma_z}{\sigma_y}\right) \text{ since } 2H_s = H_e$$

$$= \frac{8{\cdot}8\times 10^5\,E}{Q^{0{\cdot}25}\,H_s}\left(\frac{F.\sigma_z}{\sigma_y}\right). \tag{11}$$

Equation (2) (CCRL-2 Plume rise) is used to plot FIG. 6, equation (10) is used to plot FIG. 7, and equation (11) is the basis of the design standard given in FIG. 8.

At the present time most air pollution control regulations specify limits that may not be exceeded in neutral conditions. In this case the F factor would be unity and the value of the ratio of diffusion coefficients σ_z/σ_y would depend on sampling time. A short-term sampling time such as 3 min is more likely to reflect peak concentrations from single source emissions, but it is recognized that dust sampling periods of much less than 30 min are not feasible. Consequently 3 min samples are recommended in the standard for gaseous concentrations and 30 or 60 min for particulates. Sampling durations longer than this tend to yield concentrations contaminated by background pollution levels and are not considered satisfactory for single source design purposes.

It is hoped that eventually air pollution control authorities will specify levels not to be exceeded in more adverse than neutral conditions such as a severe inversion con-

dition when $F = 2$. This seems a logical step, since the use of a design condition which it is known can fail under certain more severe conditions is of limited value. Thus when designers select a value of the parameter $F.\sigma_z/\sigma_y$ it is intended that a reasonable degree of caution should be exercised before selecting any value less than unity to establish that inversions or unstable conditions rarely occur at the proposed site.

Examples of the FIG. 8 for design purposes are given in the Appendix. FIGURE 7 may be used to estimate the critical wind speed for a particular stack height and heat emission. MOORE (1967) observed that the highest concentrations were measured at Tilbury when $U > 10$ m/sec ($H_s = 100$ m $Q = 1{\cdot}43 \times 10^4$ kcal/sec, near neutral conditions). FIGURE 7 predicts a critical wind speed of 7·2 m/sec for these conditions. MARTIN and BARBER (1966) found a similar result for High Marnham and FIG. 7 gives a critical wind speed of 7 m/sec for this plant under full load conditions. Thus, FIG. 7 tends to underestimate critical winds for plant of this size in neutral conditions. MOORE (1967) indicates that high concentrations due to inversions will likely occur in light to moderate winds, so that although FIG. 7 is based on data of all weather conditions it predicts critical wind speeds which are reasonable in the context of the meteorological conditions recommended in the preceding sections. It is possible that

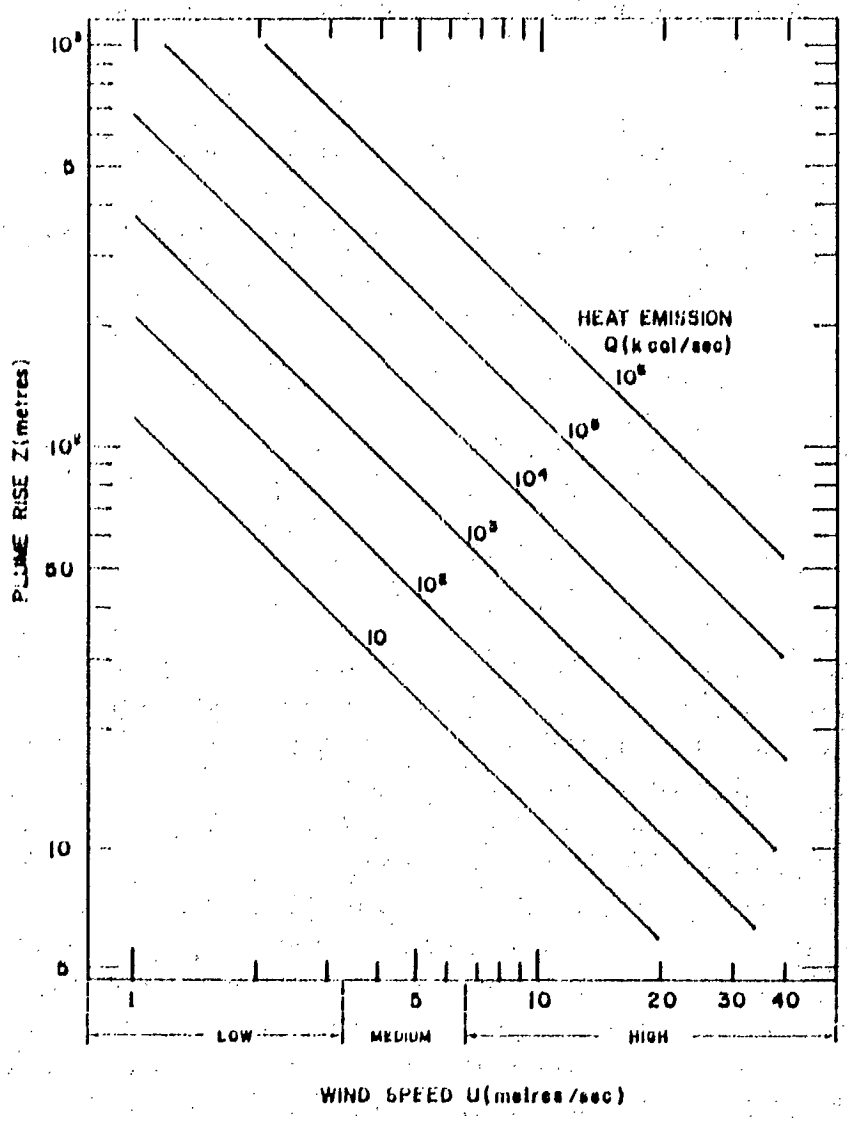


FIG. 6. CCRL-2 plume rise dependence on heat emission and wind speed.

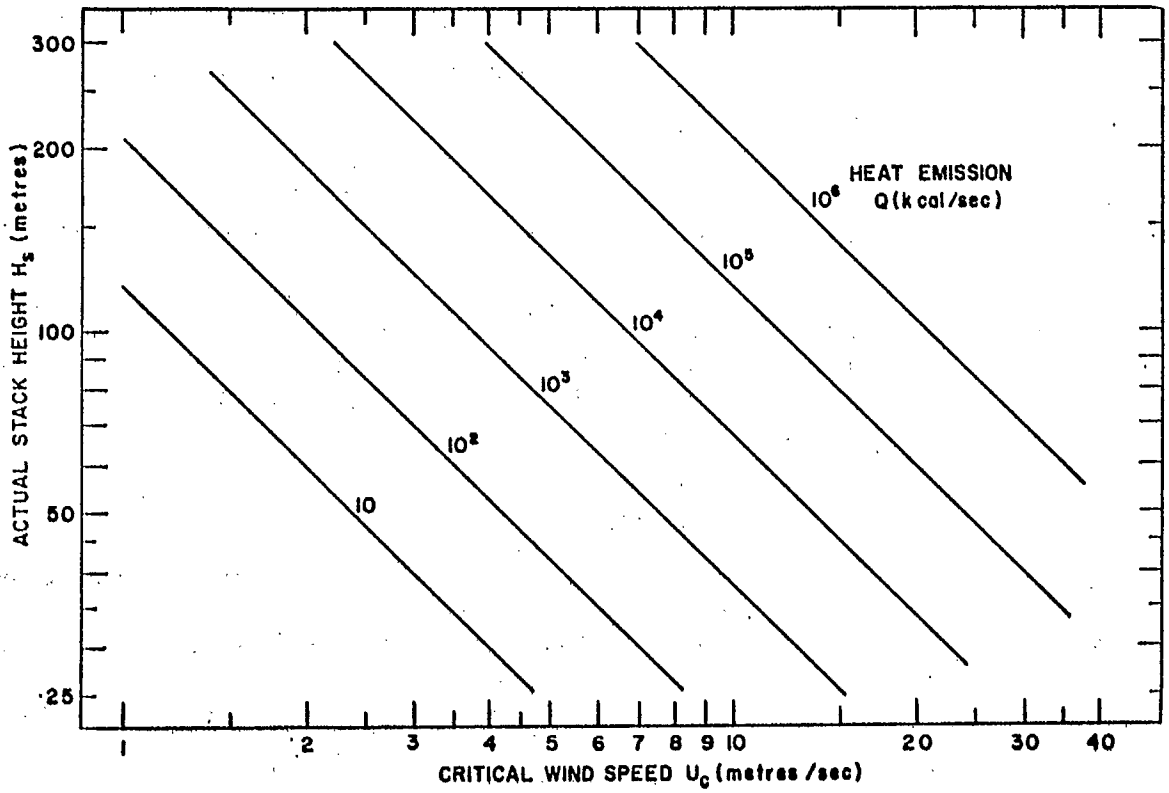
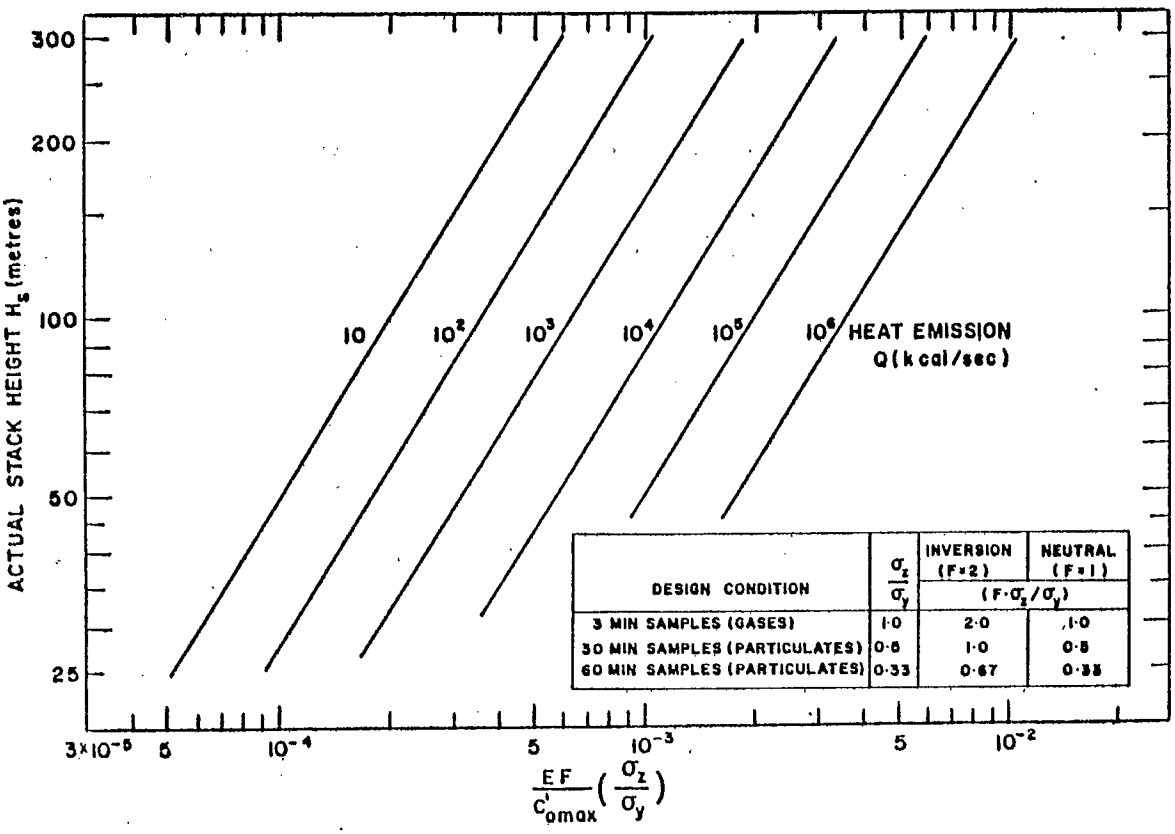


FIG 7. Variation of critical wind speed with heat emission and stack height.



DESIGN CONDITION	$\frac{\sigma_z}{\sigma_y}$	INVERSION (F=2)	NEUTRAL (F=1)
		$(F\cdot\sigma_z/\sigma_y)$	
3 MIN SAMPLES (GASES)	1·0	2·0	1·0
30 MIN SAMPLES (PARTICULATES)	0·5	1·0	0·5
60 MIN SAMPLES (PARTICULATES)	0·33	0·67	0·33

FIG. 8. Proposed Canadian standard for estimating stack height and dust collector efficiency.

the value of U_c given by FIG. 7 may be higher than might reasonably be expected to occur at the planned location of the stack. Hence a reasonable lower value of U than U_c should be selected from meteorological records and by use of FIG. 6 and equation (7) a reduced stack height may be calculated.

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APPENDIX

Examples showing the use of the design graphs.

(i) It is required to design a multiflue stack for a proposed power station having a capacity of 500 MW.

Design data:—

Total coal utilization	50 kg/sec
Ash content of coal	10 per cent
Efficiency of dust collection	98·5 per cent
Sulphur content of coal	3 per cent
Heat emission from stack	2×10^4 kcal/sec.

The regulations in the area where the proposed station is to be located stipulate that the plant shall not add more than 50 $\mu g/nm^3$ (30 min) of dust and 1·0 ppm (3 min) of sulphur dioxide to background concentrations even during inversions.

(a) Calculation of stack height to give adequate dust dispersion.

$$\text{Rate of emission of dust } E = 0{\cdot}015 \times 0{\cdot}1 \times 50$$
$$= 0{\cdot}075 \text{ kg/sec}$$

$$\therefore \frac{E.F}{C'_{0\max}}\left(\frac{\sigma_z}{\sigma_y}\right) = \frac{0{\cdot}075}{50} = 1{\cdot}50 \times 10^{-3} \text{ since } \left(\frac{F.\sigma_z}{\sigma_y}\right) = 1.$$

From FIG. 8 interpolating for $Q = 2 \times 10^4$ kcal/sec

$$H_s = 111 \text{ m.}$$

(b) Calculation of stack height to give the required SO_2 dispersion.

$$\text{Rate of emission of } SO_2 = 50 \times 0{\cdot}03 \times 2$$
$$= 3{\cdot}0 \text{ kg/sec}$$

$$\therefore C'_{0\max} = 1{\cdot}0 \text{ ppm or } 2860 \ \mu\text{g/m}^3 \text{ (3 min)}$$

$$\frac{E.F}{C'_{0\max}}\left(\frac{\sigma_z}{\sigma_y}\right) = 2{\cdot}1 \times 10^{-3} \text{ since } \left(\frac{F.\sigma_z}{\sigma_y}\right) = 2.$$

From FIG. 8 interpolating for $Q = 2 \times 10^4$ kcal/sec

$$H_s = 155 \text{ m.}$$

So, in order to avoid both dust and SO_2 pollution at the levels stipulated a stack height of 155 m must be chosen.

(ii) A 1000 MW power station having a 200 m stack is in an industrial area where new legislation on pollution levels limits maximum dust and SO_2 concentrations during inversions to 50 μg/m^3 (30 min) and 1 ppm (3 min) respectively. It must be decided whether the plant will require additional collection equipment or a taller stack to meet these new requirements.

Input and Emission data:—

Total coal utilized	100 kg/sec
Ash content	14 per cent
Present collection efficiency	98·5 per cent
Sulphur content of coal	2 per cent
Heat emission from stack	$3{\cdot}5 \times 10^4$ kcal/sec.

(a) Estimated maximum dust concentration.

$$\text{Rate of emission of dust} = 0{\cdot}015 \times 0{\cdot}14 \times 100$$
$$= 0{\cdot}21 \text{ kg/sec}$$

From FIG. 8 for a 200 m stack at $Q = 3{\cdot}5 \times 10^4$ kcal/sec:

$$\frac{E.F}{C'_{0\max}}\left(\frac{\sigma_z}{\sigma_y}\right) = 3{\cdot}1 \times 10^{-3}$$

$$\therefore \quad C'_{0\max} = \frac{0{\cdot}21}{3{\cdot}1} \times 10^{-3}$$

$$= 67{\cdot}5 \ \mu\text{g/nm}^3 \text{ since } \left(\frac{F.\sigma_z}{\sigma_y}\right) = 1.$$

(b) Estimated maximum SO_2 concentration.

$$\text{Rate of emission of } SO_2 = 100 \times 2 \times 0{\cdot}02 \text{ kg/sec}$$
$$= 4{\cdot}0 \text{ kg/sec}$$

As in (a) from FIG. 8

$$\frac{E.F}{C'_{0\max}}\left(\frac{\sigma_z}{\sigma_y}\right) = 3{\cdot}1 \times 10^{-3}$$

$$\therefore \quad C_{0\max} = \frac{2 \times 4{\cdot}0}{3{\cdot}1} \times 10^3$$

$$= 2580 \ \mu\text{g/nm}^3$$

$$\text{or } 0{\cdot}9 \text{ ppm since } \left(\frac{F.\sigma_z}{\sigma_y}\right) = 2.$$

Hence the plant will not contravene the new SO_2 limitation but will have to install dust collection equipment of at least 99 per cent efficiency or select a coal with an ash content of less than 10 per cent to avoid contravening the new dust concentration requirements.

(iii) A chimney height is to be selected for a 350 MW plant burning 21,500 gal/hr of 3 per cent residual fuel oil. The ambient concentration of SO_2 from the plant must not exceed 0·5 ppm (3 min) in neutral conditions.

Specific gravity of fuel	0·98
Calorific value (net)	9800 kcal/kg
Heat loss to stack	5 per cent heat input

$$\text{Firing rate} = \frac{21{,}500}{3600} \times 4{\cdot}546 \times 0{\cdot}98 \text{ kg/sec}$$

$$= 26{\cdot}6 \text{ kg/sec}$$

$$\text{Sulphur dioxide emission} = 26{\cdot}6 \times 2 \times 0{\cdot}03 \text{ kg/sec}$$

$$= 1{\cdot}596 \text{ kg/sec}$$

$$\text{Heat emission} = 26{\cdot}6 \times 9800 \times 0{\cdot}05 \text{ kcal/sec}$$

$$= 1{\cdot}3 \times 10^4 \text{ kcal/sec}$$

$$\text{Now } C'_{0\max} = 0{\cdot}5 \text{ ppm or } 1430 \ \mu\text{g/m}^3$$

$$\therefore \text{ The design parameter } \frac{E.F}{C'_{0\max}}\left(\frac{\sigma_z}{\sigma_y}\right)$$

$$= 1{\cdot}12 \times 10^{-3} \text{ since } \left(\frac{F.\sigma_z}{\sigma_y}\right) = 1.$$

From FIG. 8 interpolating for the heat emission $1{\cdot}3 \times 10^4$ kcal/sec.

$$H_s = 92 \text{ m}$$

Hence a stack of 92 m should not exceed the prescribed ambient level during neutral or more stable conditions.

(iv) Application to the High Marnham Power Station SO_2 measurements, after MARTIN and BARBER (1966 and 1967).

Consider an average station load of 600 MW

$$H_s = 137 \text{ m}$$

$$Q = 2{\cdot}6 \times 10^4 \text{ kcal/sec (estimated)}$$

$$E = 2{\cdot}0 \text{ kg/sec } SO_2$$

From FIG. 8 for the above stack height and heat emission

$$\frac{E.F}{C'_{0\max}}\left(\frac{\sigma_z}{\sigma_y}\right) = 1{\cdot}98 \times 10^{-3}$$

$$\therefore \quad C'_{0\max} = 2020 \ \mu\text{g/nm}^3$$

$$\textit{or } 0{\cdot}70 \text{ ppm when } \left(\frac{F.\sigma_z}{\sigma_y}\right) = 2.$$

This is in agreement with measured values reported by MARTIN and BARBER (1967) when during convective instability with winds around 5 m/sec, concentrations up to 0·9 ppm were recorded. In the above estimation the critical wind speed from FIG. 7 would be 6·2 m/sec. It must be noted that the measured values apply to two stacks and the interaction of these is not known.