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REPORT ON THE COMPILATION OF THE MAP OF VERTICAL CRUSTAL MOVEMENTS IN CANADA

Petr Vaníček, Dept. of Surveying Engg., U.N.B., Fredericton, N.B.
and
Dezsö Nagy, Earth Physics Branch, Dept. E.M.R., Ottawa, Ontario

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Ottawa, Canada, 1980

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ABSTRACT

The first vertical movement map of Canada was compiled from 5046 relevelled segments, from 39 pairs of water gauge data and from trends at 47 tide gauge stations. The country was divided into 9 regions and polynomial surfaces of order 2, 3 and 4 were calculated by the method of least squares for each region to obtain representations of the recent vertical movements. Vertical control for the solution over a region was provided by tide gauge trends, calculated from monthly means; by predicted values obtained from a solution for a strip connecting the Pacific with the Gulf of St. Lawrence and based upon 16 tide gauge trends and 2104 relevelled segments; and by predicted values available from an earlier determination for the overlapping region. The computations commenced at the east and west coasts and progressed inwards to the central region. The final solutions for the various orders of surface were compiled from the 9 overlapping solutions. The 3rd order surface represents the available data best. This solution is in agreement with the results and models presented by other investigators for uplift in the Yukon and northern B.C., for uplift in the Great Lakes region, and for subsidence in the Bay of Fundy. Subsidence in the Prairies, based on the presently available limited data-set requires further investigation.

RESUME

Trente neuf paires de données de jauge de niveau de l'eau ainsi que la tendance de 47 stations à jauge de marée et 5046 segments de renivellement ont servi à dresser la première carte de mouvement vertical du Canada. On obtient les représentations des mouvements verticaux récents en divisant le pays en 9 régions et en calculant pour chaque région, des surfaces de polynôme du 2^{e}, 3^{e} et 4^{e} ordre par la méthode des moindres carrés. Le contrôle vertical de la solution d'une région provient des tendances issues des moyennes mensuelles d'échelles de marée; des prédictions résultant d'une solution pour une bande de terrain joignant le Pacific au golfe du St. Laurent et basée sur 16 tendances d'échelles de marée et 2104 segments de renivellement; et enfin des prédictions venues d'une détermination antérieure pour les régions qui se chevauchent. Les calculs débutent aux côtes de l'est et de l'ouest et avancent vers le région centrale. Les 9 solutions qui se chevauchent ont fourni les résultats finals pour les surfaces d'ordres différents. La surface du 3^{e} ordre représente le vieux les données disponibles. Cette solution s'accorde bien avec les résultats et les modéles présentés par d'autres chercheurs sur le soulévement au Yukon, au nord de la Colombie Brittanique et dans la région des grands lacs, et sur l'affaissement à la baie de Fundy. L'affaissement dans les prairies qui jusqu'a maintenant, est basé sur un groupe limité de données, demande plus d'étude.

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1. INTRODUCTION

For years, the IUGG interunion Commission on Recent Crustal Movements, voicing the wishes of various groups of geoscientists, has been urging its member countries to produce maps of geologically contemporary (i.e. those that occured in the past 100 years) vertical movements in regions under their jurisdiction. In Canada, the work on the map for the country started at the beginning of 1977 in the form of a cooperative project between the Earth Physics Branch, E.M.R. and the Department of Surveying Engineering, U.N.B. It had been well known from previous projects that relevelled data in Canada is very sparse, except for a few more heavily inhabited regions. Thus, the main expectation from this project was that an as complete as possible inventory of existing relevelled segments, sea-tide gauge records and lake-level differences would be compiled.

Some of the data shown here (see Figure 6) had been used in earlier regional studies. Of these we should name the studies of the Laurentide Park - by Frost and Lilly [1966], Gale [1970], and Vaníček and Hamilton [1972]. The movements in Maritime Canada were investigated by Vaníček [1975, 1976] and Grant [1975]. Southern Ontario has been studies quite extensively ever since the end of the nineteenth century by many researchers; from the recent publications we quote the Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data [1977] and Vaníček [1977, 1978b]. The movements in Newfoundland were first reported by Nagy and Vaníček [1978]. None of the data west of Ontario-Manitoba border, however, has, to our knowledge, been exploited before.

2. SOURCES OF INFORMATION ON VERTICAL MOVEMENTS

There are four fundamental kinds of data that can be considered suitable for vertical movement determination: sea level variations, repeated vertical positions, tilt changes and gravity variations. The most direct type of data are the sea-level variations as recorded by automatic tide-gauges; they are recorded basically for oceanographic purpose. The noise in the records due to water movements can be to a certain extent filtered out if we have the auxiliary data to construct the linear filter with. From the crustal movement point of view the signal is composed of the linear term $C_E (\tau - \tau_o)$ - after subtracting the eustatic water raise - and the episodic movements hidden in the residuals. Clearly, the linear term corrected for eustatic raise reflects the linear vertical movement of the land at the locality of the tide-gauge with respect to the mean sea-level. Considering for the moment the eustatic correction perfectly known, the linear movement can be determined with an accuracy (one standard deviation) of about 2 cm per century. This is under the condition that the sea level record of 30 year duration and at least the main kinds of auxiliary data are available [Vanicek, 1978a].

The next kind of data are repeated vertical positions. It should not be very difficult to see that if a point moves vertically between two determinations of its vertical position, this will show directly as a change of position. Conversely, if a change in position is experienced then it is symptomatic of a vertical movement. So far there does not exist any system capable of measuring vertical positions continuously;

only positioning discrete in time is practicable. The existing absolute vertical point positioning techniques have sufficient accuracy to be useful only in studies of the most pronounced vertical movements such as sizeable coseismic displacements. The same holds true also about most of the terrestrial relative positioning techniques. Really, only results from levelling of higher orders appear to be of a general value in this context. Thus the most elementary useful piece of data is the height difference ΔH_{AB}^{M} along a segment connecting two permanent benchmarks, levelled at two different epochs τ_1 and τ_2. The interpretation of such a relevelled segment is shown on fig. 1. These segments may be levelled either for the crustal movement studies or for a variety of other purposes; usually the latter. Hence their distributuion in space and time is usually not optimal for the movement determination. The spatial configuration may be scattered, linear or areal; the time sampling may be random or confined to some sharply defined time bands. All these circumstances have to be taken into account when the appropriate mathematical model for the movement is designed.

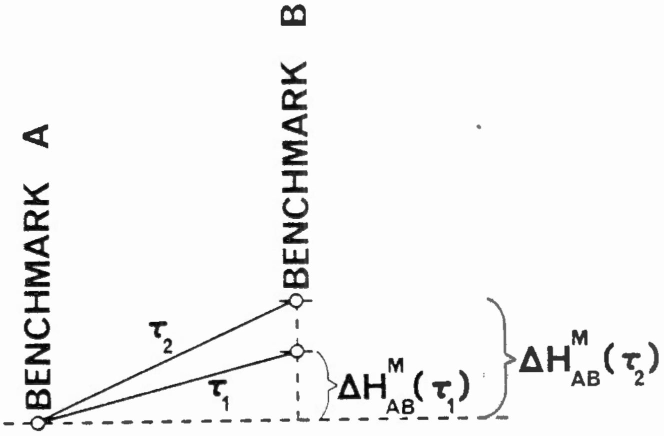

Figure 1

Once relevelled segment

There are several important aspects pertaining to levelling data that should be pointed out here. To obtain proper height differences of any kind from the levelled height differences, we have to account for the effect of actual gravity field. This is done in terms an additive correction to the levelled height difference. Therefore, the value of the height difference in any height system

$$\delta\Delta H_{AB}(\tau_1, \tau_2) = \Delta H_{AB}(\tau_2) - \Delta H_{AB}(\tau_1) \quad , \tag{1}$$

is affected not only by the relative vertical motion of B with respect to A, by systematic and random errors arising from the levelling process, but also by the difference in the two corrections (to $\Delta H_{AB}(\tau_1)$ and to $\Delta H_{AB}(\tau_2)$) for actual gravity effect. One way to eliminate the effect of gravity - for complete discussion see next section - and to reduce some of the other errors is to require the levellings between A and B to follow the same route. When this requirement is followed, errors related to geographic location (e.g. settling of staves and instruments in specific soils), to height and height gradient (e.g. the bulk of residual refraction) and to the direction of levelling (e.g. possible solar radiation effect) get significantly supressed. Even with this requirement in effect there still may be some troublesome errors present and the interpretation of the height difference differences should be done with caution, preferably in conjunction with some geophysical evidence, see e.g. [Reilinger et al., 1979]. More will be said about this later.

Clearly, a relevelled segment may be viewed as giving information, discrete in time, on tilt changes between the two benchmarks involved. There exist other data on tilt variations: the most obvious of those are lake level variations as registered by water level gauges. These instruments, identical to tide gauges used in recording sea level variations, collect continuously data for hydrological purposes. The lake level records cannot be used the same way as the sea level data because the lake levels fluctuate from year to year as well as within the year considerably more than the sea level does. This is mainly in response to precipitations and man activity. The recorded data are usable only in a differential fashion. When couples of gauges on the same lake are paired, the difference of the two corresponding records represents the difference in the vertical displacements of the two water gauges. As in the case of sea level, the records should be rid of as much of the noise originating in water dynamics as possible. However, this requirement is not nearly as important as in the case of sea level because much of the noise is common to the two records and thus gets eliminated through the differencing. The evaluation of the difference in the two certical, displacement, linear in time, is shown conceptually on figure 2. By dividing the obtained difference by the distance of the gauges we obtain the linear tilt variation, referred to local equipotential surface. For an illustration and more detail on the usage of lake level data in crustal movement studies see [The Coordinating Committee on Great Lakes Basic Hydraulic and Hydrological Data, 1977].

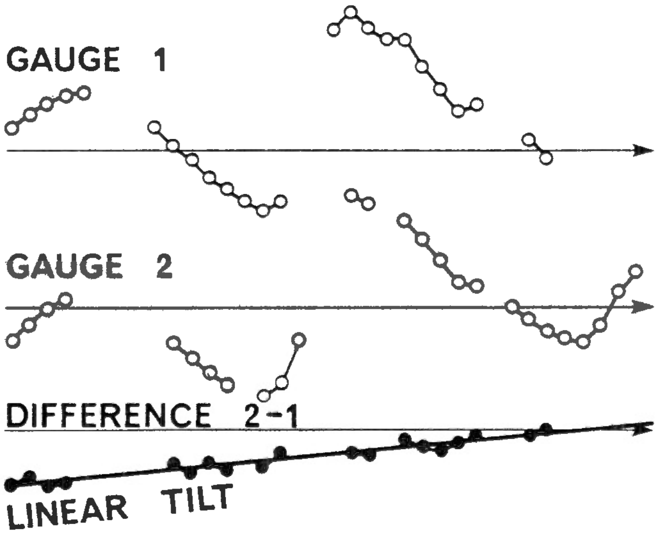

Figure 2
Lake level variations

3. DATUM FOR VERTICAL MOVEMENTS

Observed elevation changes may be taken as fairly representative of the actual vertical displacements; the influence of gravity variations on the observed elevation differences is relatively small. To prove this, let us first show that a change in observed elevations difference ΔH^M is for all practical purposes equivalent to the change in orthometric height difference ΔH^O [Heiskanen and Moritz, 1967]. This equivalence is satisfied even when the accompanying variation in gravity is quite extreme.

Recalling equation 1, the change in the difference of the orthometric heights of two benchmarks AB can be written as

$$\delta\Delta H_{AB} = \delta\Delta H^M_{AB} + \delta OC_{AB} \quad , \tag{2}$$

where δOC_{AB} is the change in the orthometric correction. If the relevelling followed the same route as the original levelling, this change can be only due to a difference in gravity 'δg' and the difference in heights 'δH'. Differentiation of the orthometric correction and substitution for the mean gravity along the plumbline yield [Vanícek et al., 1979]:

$$\delta OC_{AB} \doteq \delta g^* \frac{\Delta H^M_{AB}}{\bar{g}} + \left(\delta g_A + \delta\nabla g_A \frac{H_A}{2} + \frac{\nabla g_A}{2}\,\delta H_A\right)\frac{H_A}{\bar{g}}$$

$$- \left(\delta g_B + \delta\nabla g_B \frac{H_B}{2} + \frac{\nabla g_B}{2}\,\delta H_B\right)\frac{H_B}{\bar{g}} \quad , \tag{3}$$

where δg^* is the average surface gravity change between A and B, ∇g is the vertical gravity gradient, and $\bar{g}$ is some global mean gravity. We can show that even under extreme conditions, δOC_{AB} is at most of the order of one millimetre.

So far nothing has been said about the effect of the gravity change on the geoid. The orthometric height difference treated above must be considered related to an instantaneous geoid. Whether or not the instantaneous geoid departs much from the 'mean' geoid is a different question which shall be addressed presently. To this end, let us take the Stokes formula [Heiskanen and Moritz, 1967] that expresses the height N of the geoid above a geocentric reference ellipsoid as a function of gravity anomalies. Denoting the mean anomaly in a ring at spherical distance ψ from the point of interest by $\overline{\Delta g}$, we have

$$\overline{\Delta g}(\psi) = \frac{1}{2\pi} \int_0^{2\pi} \Delta g(\psi, \alpha)\, d\alpha \,. \tag{4}$$

The use of the F function [Lambert and Darling, 1936] gives

$$N \doteq \frac{R}{2\bar{g}} \int_0^{\pi} \overline{\Delta g}(\psi)\, F(\psi)\, d\psi \quad . \tag{5}$$

Then the change in N due to the change of gravity is

$$\delta N \doteq \frac{R}{2\bar{g}} \int_0^{\pi} \delta\overline{\Delta g}(\psi)\, F(\psi)\, d\psi \;, \tag{6}$$

where $\delta\overline{\Delta g}(\psi)$ is the mean change of the gravity anomaly within the spherical distance ψ.

Let now $\delta\overline{\Delta g} = 0$ for $\psi > \psi_{max}$, i.e. let the gravity and height changes occur only within the radius ψ_{max} of the point of interest. Then integration in parts of eq. 6 yields [Vaniček et al., 1979].

$$\delta N \doteq -\frac{R}{2\bar{g}} \int_0^{\psi_{max}} \frac{\partial}{\partial\psi}\, \delta\overline{\Delta g}(\psi)\, \Phi(\psi)\, d\psi \;, \tag{7}$$

where $\Phi(\psi)$ for $\psi < 10°$ may be approximated by $2.3\,\psi$ [Lambert and Darling, 1936]. Moreover, considering the free-air anomaly, we get

$$\delta\overline{\Delta g}(\psi) \doteq \delta\bar{g}(\psi) + 0.31\ \mathrm{mgal}\ \mathrm{m}^{-1}\ \delta\bar{H}(\psi) \quad , \tag{8}$$

where $\overline{\delta g(\psi)}$ is the average surface gravity change and $\overline{\delta H}(\psi)$ the average vertical displacement, both at the distance ψ. Substituting this result back into eq. 7 we obtain finally:

$$\delta N \doteq -\ 7.4\ \mathrm{mgal}^{-1}\ \mathrm{m} \int_0^{\psi_{max}} \psi\ \frac{\partial}{\partial\psi}\ \delta\overline{g(\psi)}\ d\psi$$

$$-2.3 \int_0^{\psi_{max}} \psi\ \frac{\partial}{\partial\psi}\ \delta\bar{H}(\psi)\ \ d\psi \quad . \tag{9}$$

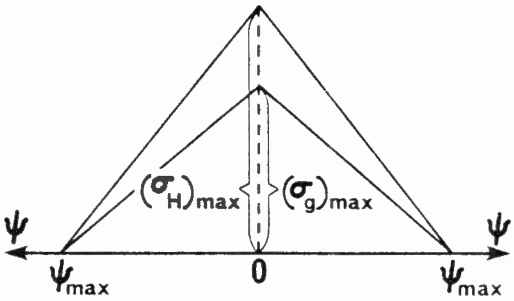

Figure 3

Linear model for vertical displacement and gravity changes.

For illustration, let us assume such a model for $\delta\bar{g}$ and $\delta\bar{H}$ where the maxima $\delta\bar{g}_{max}$, $\delta\bar{H}_{max}$ are reached for the point of interest ($\psi = 0$) and both δg and δH decrease linearly to zero at $\psi = \psi_{max} \leq 10°$ (cf. Fig. 3). Then the above formula reduces to

$$\delta N \doteq (7.4\ \mathrm{mgal}^{-1}\ \mathrm{m}\ \delta\bar{g}_{max} + 2.3\ \delta\bar{H}_{max})\,\psi_{max}. \tag{10}$$

Clearly, for smaller features of the order of 100 km the geoid change is small: for $\delta\bar{g}_{max} = 100\ \mu\mathrm{Gal}$, $\psi_{max} = 1°$ we get $\delta N = 1.3\ \mathrm{cm} + 0.04\ \delta\bar{H}_{max}$.

The last point that should be made here concerns the temporal variations of the geoid due to its definition through the mean sea level. The consequence of the standard definition is that both the shape as well as probably the potential (and thus even the size) of the geoid change with the eustatic water rise thus diminishing all the heights around the world by 0.6 to 1.0 mm every year. Clearly, even fixing for instance the geoid potential, the shape would still change in response to the redistribution of water and ice on the earth. Holding the geoid at a fixed level with respect to the tide-gauges used in vertical datum definition may be convenient in terms of vertical positioning but would make life very difficult for other applications.

4. DETERMINATION OF VERTICAL DISPLACEMENTS

Of the sources of information discussed above, lake level data are scarce. This leaves the relevelled segments, wherever possible combined with sea level records, the prime data for vertical movement detection. In many cases, relevelled segments are strung together making a continuous profile. This situation occurs when a whole levelling line is relevelled and from the common sense point of view, that is the most favourable setup.

If the first levelling was carried out at epoch τ_1 and the second at τ_2, the relative rise or fall of common benchmarks can be simply plotted as the accumulated relative displacement profile - see figure 4. If the relevelled line is connected to a tide-gauge A, for which the record from the period τ_1, τ_2, is available then the relative vertical displacement can be converted to absolute by taking into account the absolute displacement H_A of A, as indicated by the tide-gauge record. This situation is shown on figure 5.

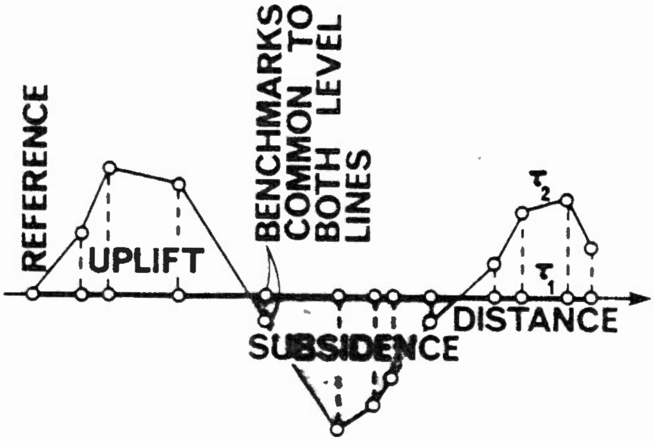

Figure 4
Relative Displacement Profile

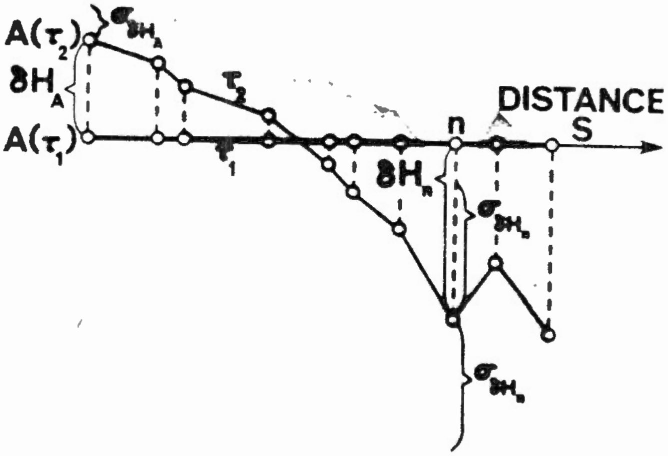

Figure 5
Absolute Displacement Profile with error bounds

It should be clear that the absolute displacement, linear in time, of the tide-gauge during the period τ_1, τ_2 is given as

$$\delta H_A(\tau_1,\tau_2)=-(C_E-r_E)(\tau_2-\tau_1) \qquad (11)$$

where C_E is the observed linear rate of water rise and r_E is the eustatic water rise. Note the negative sign.

The relevelled elevation differences are burdened with observational as well as modelling errors. Thus not all the variations depicted in the profiles are justifiable in terms of vertical displacement of benchmarks; it is therefore recommendable to plot not only the accumulated displacements δH_n but also the accumulated standard deviations $\sigma_{\delta H_n}$ of the displacements. Since δH_n is given by the evident formula (cf. eq. 1):

$$\delta H_n = \delta H_A + \sum_{i=1}^{n-1} \delta\Delta H_{i,i+1} = \delta H_A + \sum_{i=1}^{n-1} \Delta H_{i,i+1}(\tau_1) - \sum_{i=1}^{n-1} \Delta H_{i,i+1}(\tau_2) \qquad (12)$$

then its standard deviation $\sigma_{\delta H_n}$ can be written as

$$\sigma^2_{\delta H_n} = \sigma^2_{\delta H_A} + \underline{u}\,\underline{C}_{\delta\Delta}\,\underline{u}^T = \sigma^2_{\delta H_A} + \underline{u}\,(\underline{C}_{\Delta H(\tau_1)} + \underline{C}_{\Delta H(\tau_2)})\,\underline{u}^T \quad , \qquad (13)$$

where $\underline{u}$ is a vector of (n-1) 1's. When the covariances are expected to be close to zero (statistically independent determinations of ΔH's), which is the assumption mostly adopted, then the equation degenerates to

$$\sigma^2_{\delta H_n} = \sigma^2_{\delta H_A} + \sum_{i=1}^{n-1} (\sigma^2_{\Delta H_{i,i+1}}(\tau_1) + \sigma^2_{\Delta H_{i,i+1}}(\tau_2)) \ . \qquad (14)$$

If, in addition, each levelling was carried out to a specific accuracy of σ_o^2 (corresponding to a standard distance), then we can use the a posteriori estimate $\hat{\sigma}_o^2$ of this σ_o^2 to further simplify the above equation to

$$\sigma^2_{\delta H_m} = \sigma^2_{\delta H_A} + (\hat{\sigma}_o^2(\tau_1) + \hat{\sigma}_o^2(\tau_2)) \sum_{i=1}^{n=1} S_{i,i+1} \quad ,$$

or

$$\sigma^2_{\delta H_n} = \sigma^2_{\delta H_A} + S_n\,(\hat{\sigma}^2_o(\tau_1) + \hat{\sigma}^2_o(\tau_2)) \quad , \qquad (15)$$

where S_n is the accumulated distance of the n-th benchmark from A.

It is of interest to realise that if the results of the two levellings of the same segment ($\Delta H_{i,i+1}(\tau_1)$ and $\Delta H_{i,i+1}(\tau_2)$) are positively statistically dependent then the error in $\delta\Delta H_{i,i+1}$ is smaller. Denoting the covariance between the two results by σ_{12} and the variances of the results by σ_1^2, σ_2^2 we can write for the normalised covariance ρ_{12}:

$$\rho_{12} = \sigma_{12}/(\sigma_1 \cdot \sigma_2) \quad . \tag{16}$$

On the other hand, the covariance law yields

$$\sigma^2_{\delta\Delta h} = \sigma_1^{\ 2} - 2\sigma_{12} + \sigma_2^{\ 2} , \tag{17}$$

Assuming, without any detriment to generality, $\sigma_1 = \sigma_2 = \sigma$, we have

$$\sigma^2_{\delta\Delta h} = 2\sigma^2(1-\rho_{12}) , \tag{18}$$

Which is evidently $(1-\rho_{12})$ - times smaller than if there were no statistical dependence. Remmer [1975] found the correlation between forward and backward runnings (in the Danish geodetic network) to be higher than +0.7. It is reasonable to expect the degree of statistical dependence between two levellings over the same grounds to be also quite high, due to common systematic effects of height, height gradients, type of soil, azimuths of segment, latitude etc. Thus the vertical displacements obtained from relevelled segments should be significantly more accurate than the levelled heights.

Obviously, if only two levellings of the same line are available then the only thing that can be done is to interpret the indicated vertical displacements as being a result of a non-accelerated, or linear movement, that took place during the period between the two epochs τ_1, τ_2.

5. AREAL MODELLING OF VERTICAL MOVEMENTS

Let us, begin by assuming linear movements. Then, the relative velocities δv_{ij} (to be adjusted), may be written as

$$\delta v_{ij} = \frac{\delta \Delta H_{ij}(\tau_1, \tau_2)}{\tau_2 - \tau_1} = \frac{\Delta H_{ij}(\tau_2) - \Delta H_{ij}(\tau_1)}{\tau_2 - \tau_1} \qquad (19)$$

and if they are statistically independent, the inverse of their variances can be used for weights. The variances are given by the obvious formula (cf. eq. 14)

$$\sigma^2_{\delta v_{ij}} = \frac{S_{ij}}{(\tau_2 - \tau_1)^2} (\hat{\sigma}_o^2(\tau_1) + \hat{\sigma}_o^2(\tau_2)) = S_{ij}\, \hat{\sigma}_o^2(v) \quad . \qquad (20)$$

The adjustment of a completely relevelled network can then be carried out. Sea level information, i.e. the velocities v_i of tide-gauge vertical motions (cf. eq. 11), or point velocities:

$$v_i = \frac{\delta H_i(\tau_1, \tau_2)}{\tau_2 - \tau_1} = - (c_E - r_E) \ , \qquad (21)$$

can be also easily merged with the levelling data, when properly weighted through their variances

$$\sigma^2_{v_i} = \sigma^2_{\delta H_i} / (\tau_2 - \tau_1)^2 = \sigma^2_{c_E} + \sigma^2_{r_E} \quad . \qquad (22)$$

A typical example of such a treatment is the Finnish levelling network [Kaariainen, 1953; Korhonen, 1961].

It is easy to see that once the adjusted velocities of both the junction points as well as points within the lines are determined, they can be plotted as a discrete velocity field. If there are physical grounds to believe that the movements are not only linear in time but also varying smoothly with location, then velocities at other intermittent points in the area may be predicted. The usual way of doing it is by hand contouring. An example of such predictions can be found in [Holdahl and Morrison, 1974].

Clearly, the manual prediction (using contouring) can be replaced by an automated process: the contours can be drawn by a computer. More fundamentally, a velocity surface can be fitted to the discrete velocity field. Here we should realise that there is no need for interpolation, i.e. for the requirement that the velocity surface go exactly through all the discrete points (velocities), since these points are determined only to a limited degree of accuracy, characterised by the standard deviations. Hence any approximation method may be used instead of interpolation to predict the velocity surface. The most simple procedure is the surface fitting using least-squares regression.

It is interesting to realise that if a smooth velocity surface is regarded to be good enough representation then it is no longer necessary to require the relevelling segments to be interconnected into lines and lines into a network. Such a smooth velocity surface may be constructed from disconnected, scattered relevelled segments, treating each as a tilt element, i.e. a horizontal gradient of velocity between the two end points. To show how this may be done, let us assume that the velocity surface is wanted in the form

$$v(\phi, \lambda) = \sum_{i=1}^{m} \Phi_i(\phi, \lambda)\, c_i = \underline{\Phi}^T (\phi, \lambda)\, \underline{c} \tag{23}$$

where Φ_i are some selected base functions. Then for each relevelled segment A, B the following observation equations [Vaníček and Christodulides, 1974] can be written:

$$\delta v_{AB} = v(\phi_B, \lambda_B) - v(\phi_A, \lambda_A) = (\underline{\Phi}^T(\phi_B, \lambda_B) - \underline{\Phi}^T(\phi_A, \lambda_A))\, \underline{c} \tag{24}$$

or in a more compact fashion

$$\delta v_{AB} = \underline{\Delta\Phi}^T(\phi_A, \lambda_A, \phi_B, \lambda_B)\ \underline{c} \quad . \tag{25}$$

It can be seen that this formulation is not dissimilar to the formulation of a partial differential equation.

When there are sufficiently many relevelled segments available, distributed so that the selected functions are linearly independent, the coefficients $\underline{c}$ can be estimated using the least-squares technique. The weight matrix $\underline{P}_t$ (diagonal), is assembled from the inverse values of variances computed for each relevelled segment from eq. 20. The normal equations then read

$$\underline{\Delta\Phi}\ \underline{P}_t \underline{\Delta\Phi}^T\ \underline{c} = \underline{\Delta\Phi}\ \underline{P}_t\ \underline{\delta v} \quad . \tag{26}$$

The question now arises as to how to combine the relevelled segments with other kinds of information within this scheme. Obviously, the lake level tilts can be used in precisely the same manner as the relevelled segments, i.e. as tilt elements. They yield observation equations of the same type. The only difference is in weighting; the variance of the lake tilt element is given simply as

$$\sigma^2_{\delta v} = \sigma^2_{v_A} + \sigma^2_{v_B} \tag{27}$$

where σ_{v_A}, σ_{v_B} are standard deviations of the linear trends determined from the two gauge records.

The situation is different with sea level records. Every (sea) tide gauge supplies one observation equation of the kind shown earlier (eq. 23), where the observed velocity is given by eq. (21). The variance of the observation is spelled out by eq. (22). The two system of observation equations, i.e. for tilt elements and point velocity, are then combined, to arrive at

$$[\underline{\Delta\Phi}^T \vdots \underline{\Phi}^T]^T \underline{c} = [\underline{\delta v} \vdots \underline{v}]^T \quad . \tag{28}$$

The appropriate weight matrix $\underline{P}$ for the corresponding system of normal equations is evidently

$$\underline{P} = \begin{bmatrix} \underline{P}_t & \underline{0} \\ \underline{0} & \underline{P}_v \end{bmatrix} \tag{29}$$

where P_v is the weight matrix for the point velocities. The normal equations then read.

$$[\underline{\Delta\Phi}^T \vdots \underline{\Phi}^T] \underline{P} [\underline{\Delta\Phi}^T \vdots \underline{\Phi}^T]^T \underline{c} = [\underline{\Delta\Phi}^T \vdots \underline{\Phi}^T] \underline{P} [\underline{\delta v} \vdots \underline{v}]^T . \tag{30}$$

The covariance matrix $\underline{C}_c$ of the resulting coefficients $\underline{c}$ is obtained simply as a properly scaled inverse of the matrix of normal equations. It can be then used in the evaluation of the standard deviation of predicted velocity at any point (ϕ, λ). Referring to eq. 23, the covariance law yields.

$$\sigma_v(\phi, \lambda) = \surd(\underline{\Phi}^T(\phi, \lambda)\ \underline{C}_c\ \underline{\Phi}(\phi, \lambda)) . \tag{31}$$

Two-dimensional algebraic functions up to degree 4 are used for the base. Other base functions may be, of course, used; Holdahl and Hardy [1979], for instance use quadrics. When the movements cannot be assumed linear, e.g. in tectonically active areas, the above approach gives only some average velocities. If in addition the survey epochs are too much scattered then the results are practically useless.

6. DATA USED

(A) Sea level data

The basic data were provided by Paul-Andre Bolduc, Marine Environmental Data Services Branch; it consisted of monthly means for 136 stations. The earliest record dates back to 1897, the latest available records extended to Dec. 31, 1975. The file consists of 3,824 records which, after some format changes, was put on permanent file and used in all computations. As this file does not contain the latitude and longitude of the station, these were obtained separately from Mr. Bolduc and also put on permanent file. As the first step in deciding which stations to use, a program to plot all records was developed. Parallel to this, a program for trend computation (least squares line fitting) has also been developed. For the record, the expressions used in the trend computation are listed below. If

$$y = a + b\tau \qquad (32)$$

represents the trend, then the various quantities, including standard deviations are calculated as follows:

$$b = \frac{n\,\Sigma\tau_i\,y_i - \Sigma\tau_i\,\Sigma\,y_i}{n\,\Sigma\tau_i^2 - (\Sigma\,\tau_i)^2}\,, \qquad (33)$$

$$a = \frac{\Sigma\,y_i - b\,\Sigma\tau_i}{n} \qquad (34)$$

where n is the number of monthly averages available for the particular station $\tau_i = t_i - 1897$ where t_i is the time, in years, of the recording (middle of month) y_i = the change in readings relative to the first recordings, i.e. $= y_i - y_o$.

$$s^2_{y\tau} = \frac{\Sigma(y_i - \bar{y})^2}{n-2} = \frac{\Sigma\, y_i^2 - a\,\Sigma\, y_i - b\,\Sigma\tau_i\, y_i}{n-2} \tag{35}$$

where $s^2_{y}\tau$ is an estimate of the variance σ^2 of the observations y_i. The estimate of σ_b^2 was calculated from:

$$s_b^2 = \frac{s^2_{y\tau}}{S(\tau^2)} = \frac{n(\Sigma\, y_i^2 - a\,\Sigma\, y_i - b\,\Sigma\tau_i\, y_i)}{(n-2)\ \{n\,\Sigma\tau_i^2 - (\Sigma\,\tau_i)^2\}} \tag{36}$$

The next step was to select the stations to be used as control stations in the computations to provide the "absolute" movement control. Based upon visual inspection of the plots, location with respect to the relevelled segment, length of recordings available, etc. a total of 47 stations were selected to be used (see figure 6). Note that the running numbers of the original file are used. Monthly averages for each station have been plotted in centimetres as a function of time (figs. 7 to 12). For the trend calculation all available data were used, even when the recordings were interrupted for various lengths of time. The reference level for vertical movement was always the first available record. The calculated trend is also shown for each diagram as a slope with respect to a long-dashed horizontal line. The 1σ error in the slope is indicated by the two short-dashed lines on either side of the slope. Table 1 gives a summary of all the 47 stations with their sequence number which corresponds to that on fig. 6. Thus figs. 6 to 12 can be correlated by the use of Table 1 (name vs. sequential No.).

For some of the 47 stations Dohler and Ku [1970] also obtained slopes from least squares computation using raw data. These computations were later repeated by Ku [1970] including recordings for 1968-9. There

Figure 6

Control Tide Gauges

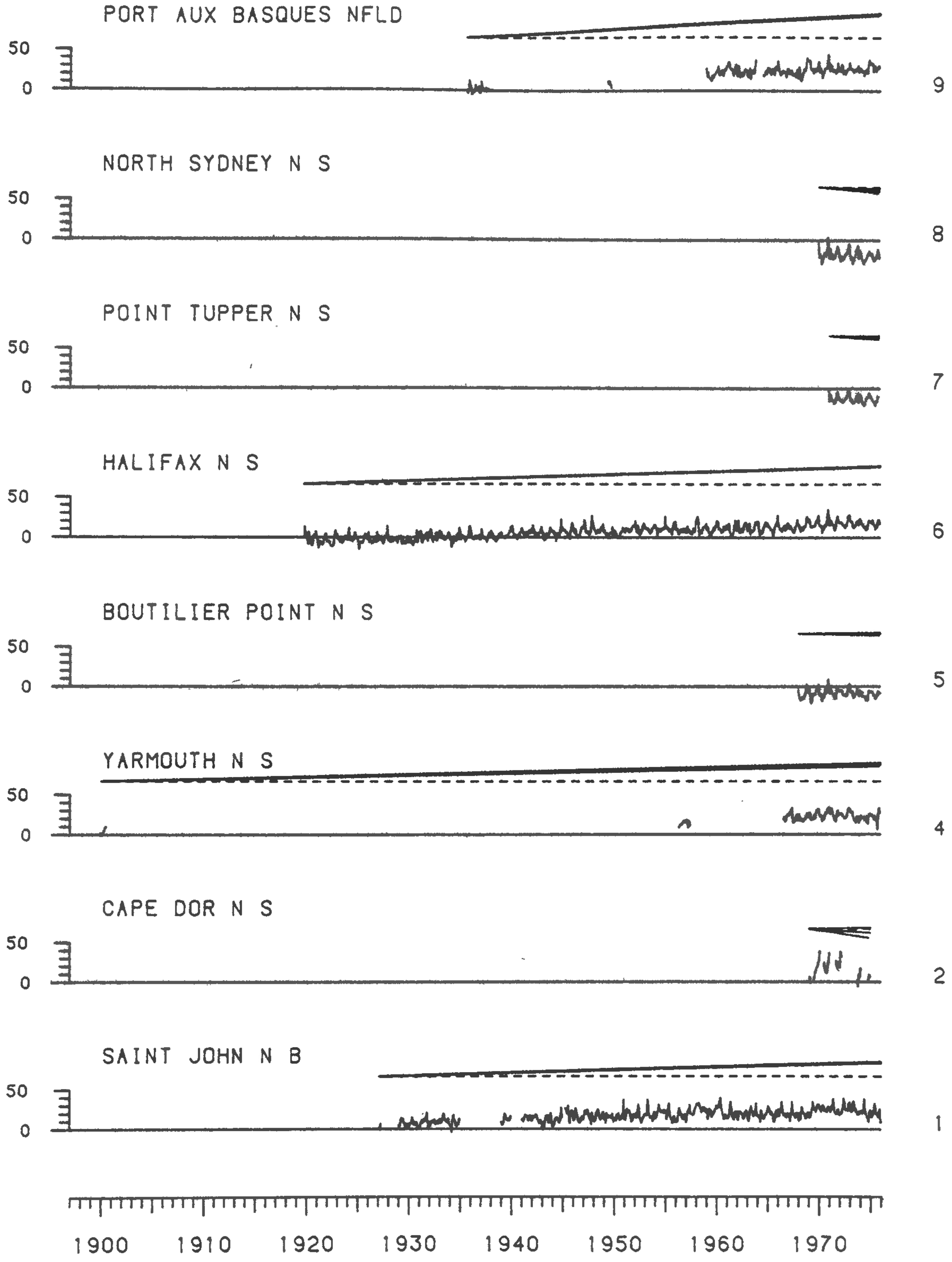

Figure 7

Plots of monthly sea-level values from Canadian ports

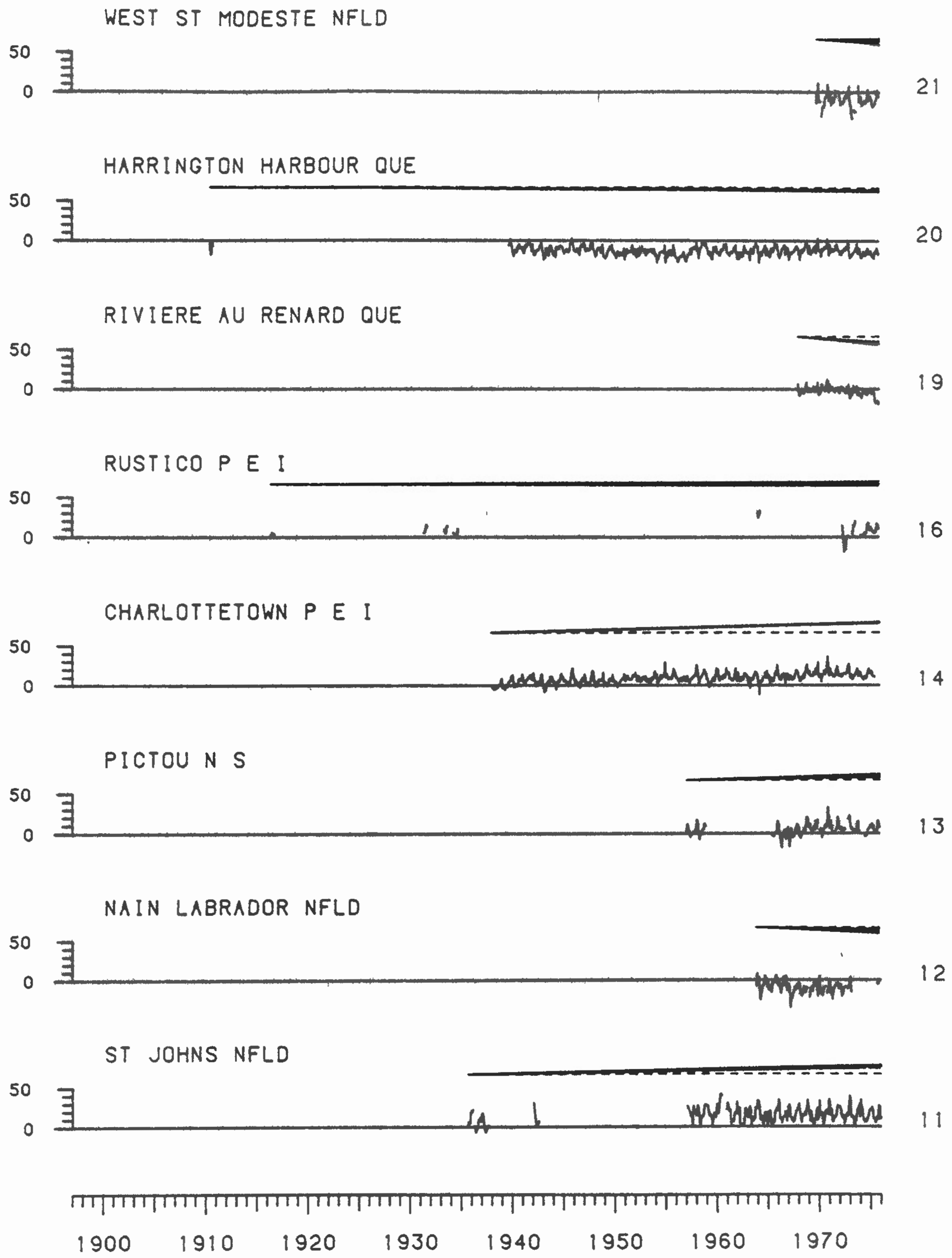

Figure 8

Plots of monthly sea-level values from Canadian ports

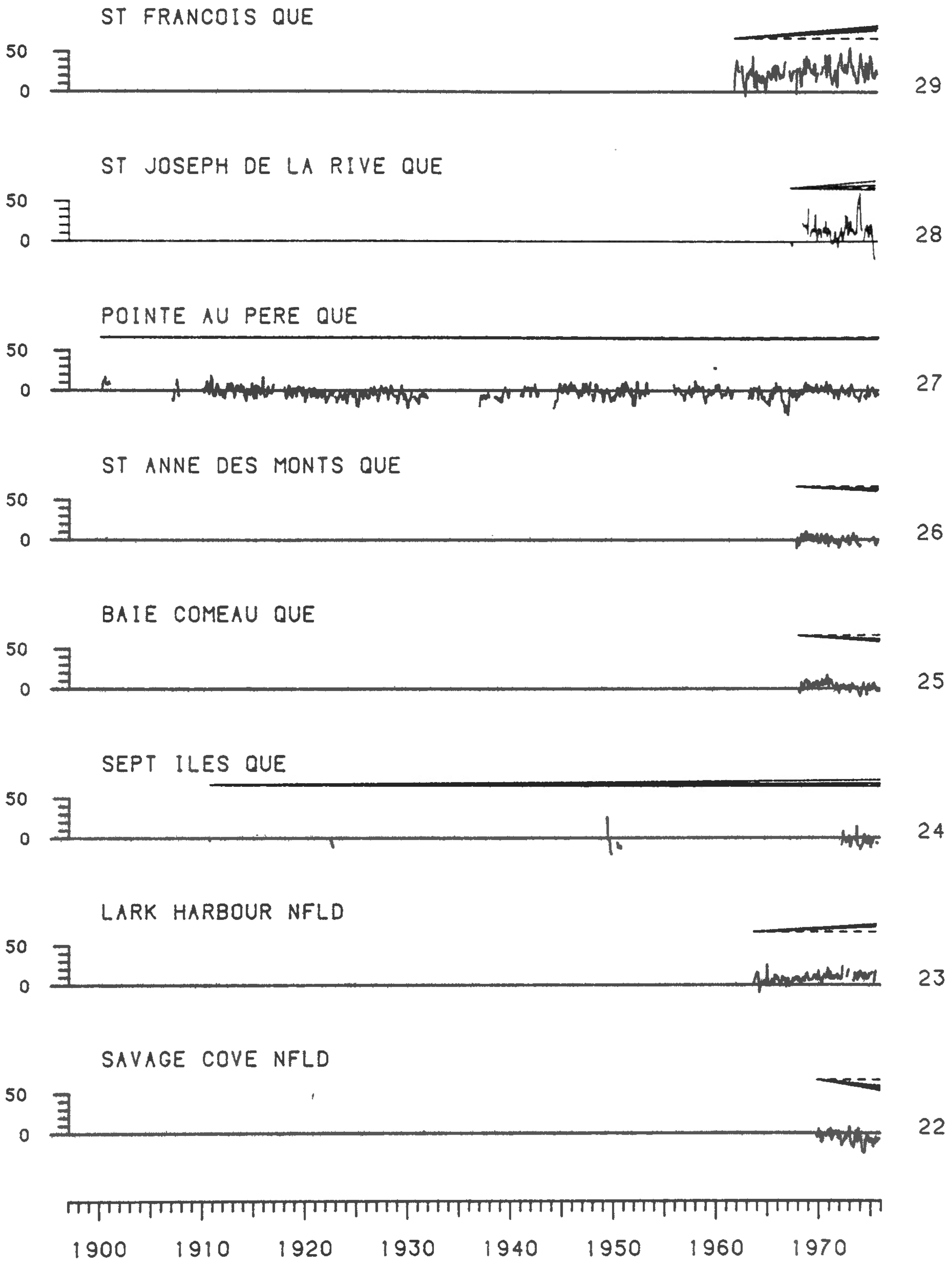

Figure 9

Plots of monthly sea-level values from Canadian ports

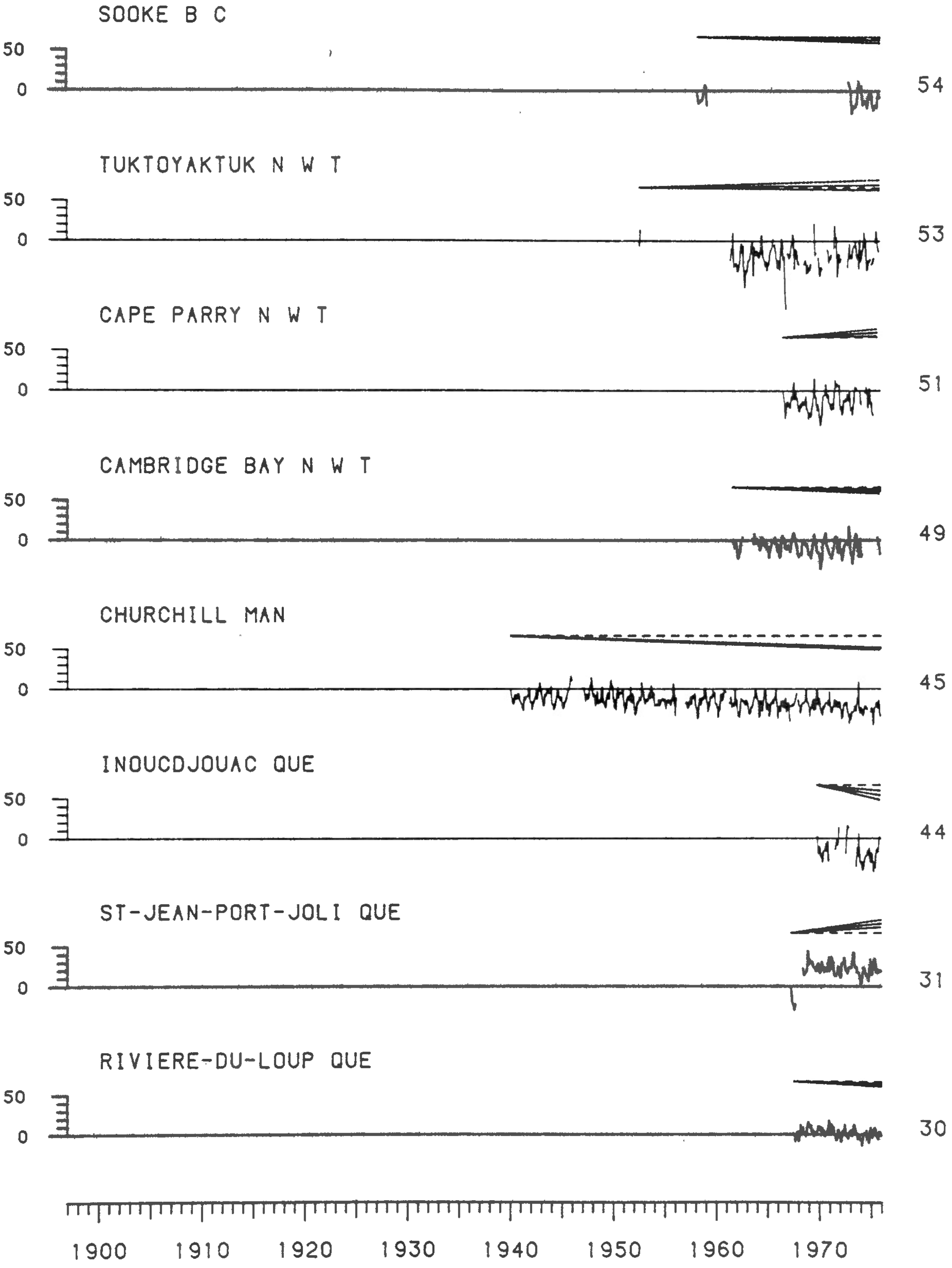

Figure 10

Plots of monthly sea-level values from Canadian ports

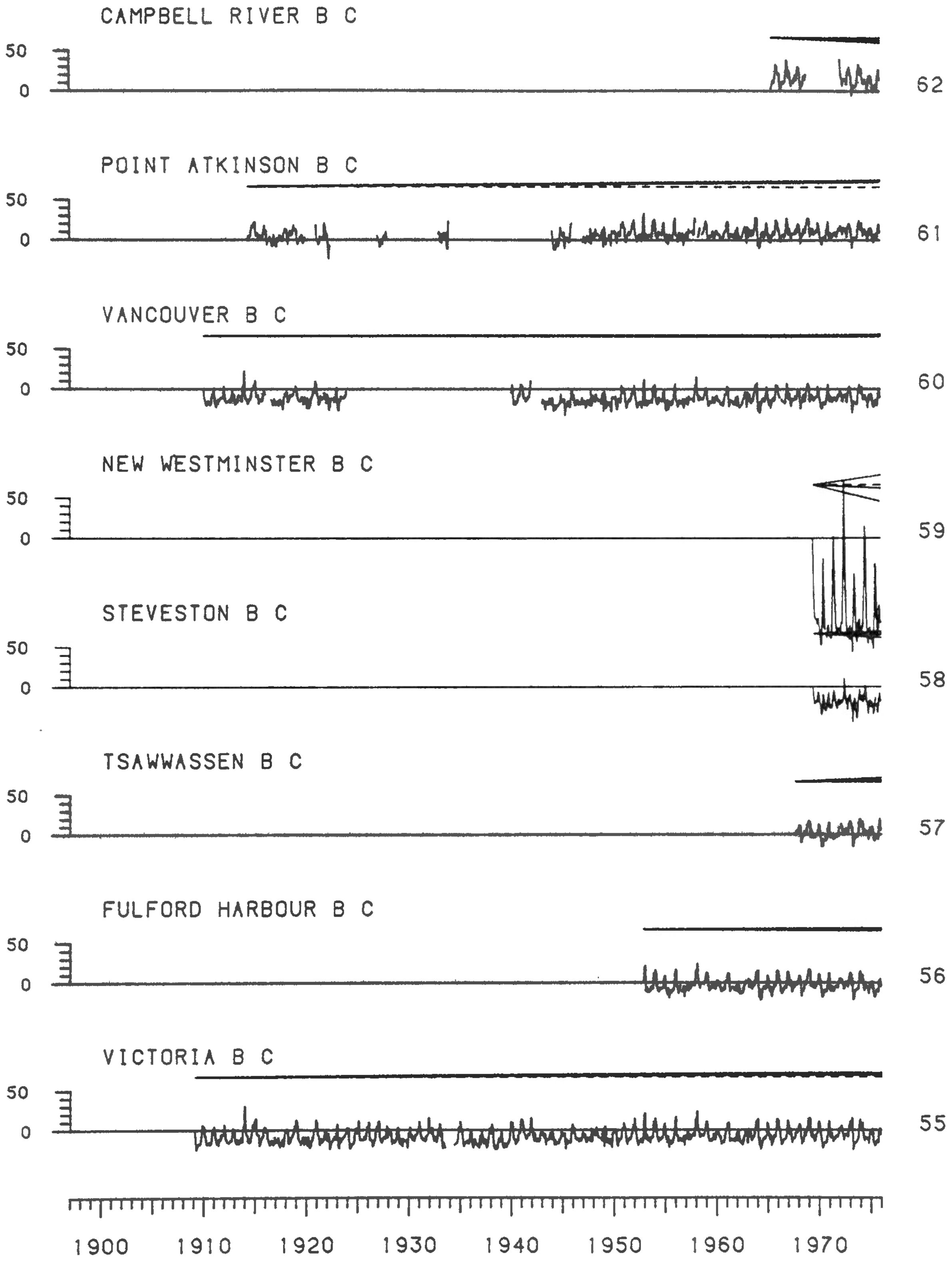

Figure 11

Plots of monthly sea-level values from Canadian ports

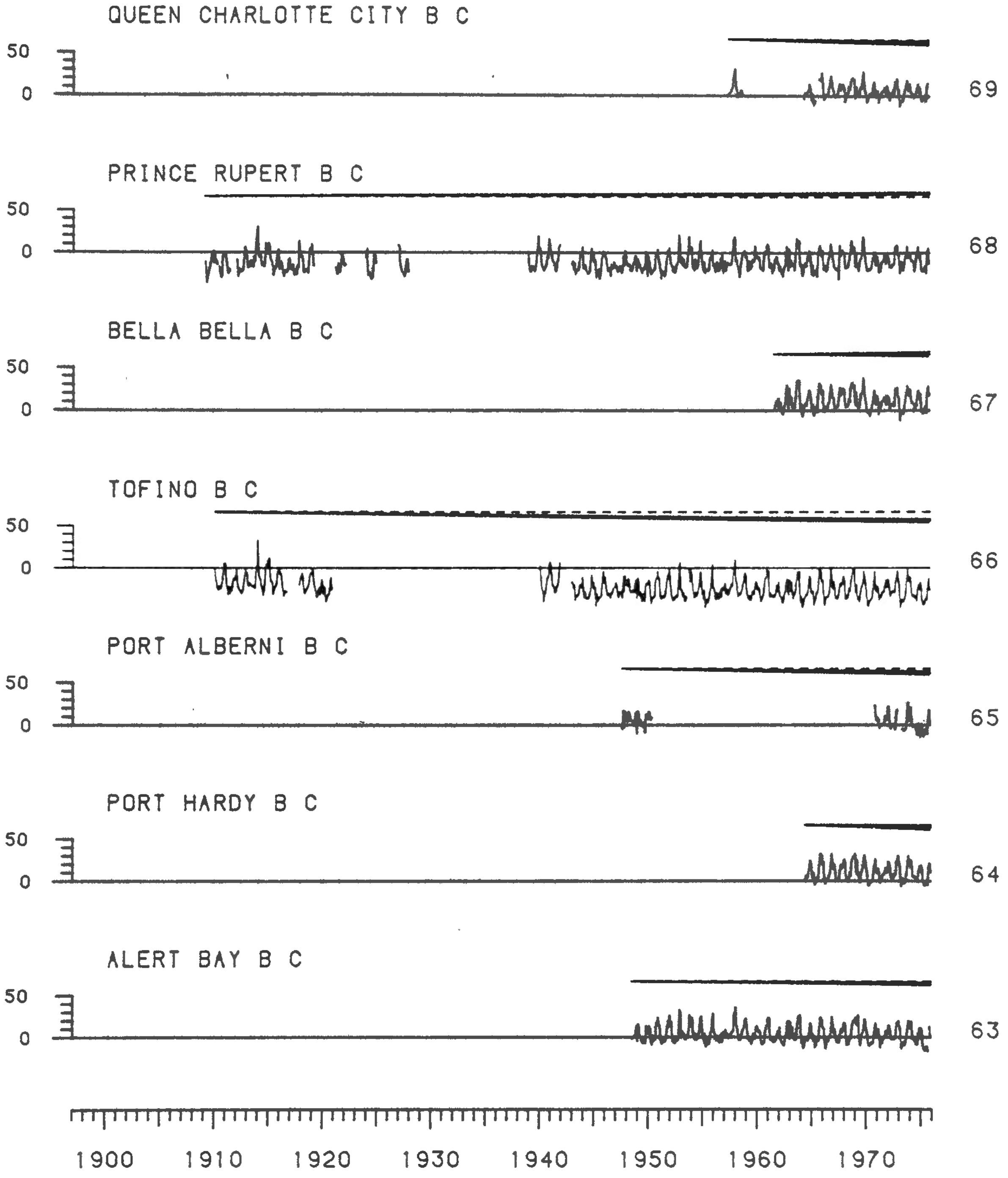

Figure 12

Plots of monthly sea-level values from Canadian ports

NAME	FIRST	LAST	N	B †	S(B) †	SEQ
SAINT JOHN N B	1927	1975	504	.36	.02	1
CAPE DOR N S	1969	1974	33	-.82	1.12	2
YARMOUTH N S	1900	1975	133	.28	.03	4
BOUTILIER POINT N S	1968	1975	96	-.08	.26	5
HALIFAX N S	1919	1975	675	.40	.01	6
POINT TUPPER N S	1971	1975	59	-.56	.48	7
NORTH SYDNEY N S	1970	1975	72	-.79	.47	8
PORT AUX BASQUES NFLD	1935	1975	227	.72	.03	9
ST JOHNS NFLD	1935	1975	255	.23	.05	11
NAIN LABRADOR NFLD	1963	1975	115	-.42	.25	12
PICTOU N S	1957	1975	143	.31	.12	13
CHARLOTTETOWN P E I	1938	1975	451	.33	.02	14
RUSTICO P E I	1916	1975	54	.01	.05	16
RIVIERE AU RENARD QUE	1968	1975	96	-1.06	.23	19
HARRINGTON HARBOUR QUE	1910	1975	440	-.05	.02	20
WEST ST MODESTE NFLD	1969	1975	71	-.37	.60	21
SAVAGE COVE NFLD	1969	1975	73	-1.75	.46	22
LARK HARBOUR NFLD	1963	1975	134	.68	.13	23
SEPT ILES QUE	1910	1975	59	.02	.06	24
BAIE COMEAU QUE	1968	1975	94	-.85	.22	25
ST ANNE DES MONTS QUE	1967	1975	87	-.61	.23	26
POINTE AU PERE QUE	1900	1975	650	-.02	.01	27
ST JOSEPH DE LA RIVE Q.	1967	1975	87	.44	.65	28
ST FRANCOIS QUE	1962	1975	158	.93	.21	29
RIVIERE-DU-LOUP QUE	1967	1975	101	-.55	.25	30
ST-JEAN-PORT-JOLI QUE	1967	1975	96	1.39	.53	31
INOUCDJOUAC QUE	1969	1975	52	-2.10	.95	44
CHURCHILL MAN	1940	1975	401	-.45	.04	45
CAMBRIDGE BAY N W T	1961	1975	146	-.37	.21	49
CAPE PARRY N W T	1966	1975	100	.64	.49	51
TUKTOYAKTUK N W T	1952	1975	149	.16	.27	53
SOOKE B C	1958	1975	50	-.18	.21	54
VICTORIA B C	1909	1975	793	.06	.02	55
FULFORD HARBOUR B C	1952	1975	276	.00	.08	56
TSAWWASSEN B C	1967	1975	99	.28	.34	57
STEVESTON B C	1969	1975	78	-.21	.59	58
NEW WESTMINSTER B C	1969	1975	78	-.68	2.54	59
VANCOUVER B C	1910	1975	581	.01	.02	60
POINT ATKINSON B C	1914	1975	478	.12	.02	61
CAMPBELL RIVER B C	1965	1975	89	-.31	.28	62
ALERT BAY B C	1948	1975	330	-.11	.07	63
PORT HARDY B C	1964	1975	139	-.25	.27	64
PORT ALBERNI B C	1947	1975	89	-.19	.08	65
TOFINO B C	1910	1975	534	-.16	.03	66
BELLA BELLA B C	1961	1975	173	-.01	.20	67
PRINCE RUPERT B C	1909	1975	582	.07	.02	68
QUEEN CHARLOTTE CITY B C	1957	1975	151	-.19	.14	69

TABLE 1

Numerical value of the trend of sea level.

† cm/century

NAME	OURS YRF	YRL	N	B	S(B)	KU (1970) YRF	YRL	N	B	S(B)	SEQ
SAINT JOHN N B	1927	1974	504	.36	.02	1929	1969	429	.17	.03	1
HALIFAX N S	1919	1975	675	.40	.01	1919	1969	603	.39	.01	6
ST JOHNS NFLD	1935	1975	255	.23	.05	1935	1969	165	.29	.07	11
NAIN LABRADOR NFLD	1963	1975	115	-.42	.25	1963	1969	66	-1.24	.54	12
CHARLOTTETOWN P E I	1938	1975	451	.33	.02	1938	1969	367	.32	.03	14
HARRINGTON HARBOUR QUE	1910	1975	440	-.05	.02	1910	1969	356	-.07	.03	20
POINTE AU PERE QUE	1900	1975	650	-.02	.01	1900	1969	557	-.03	.02	27
CHURCHILL MAN	1940	1975	401	-.45	.04	1940	1969	313	-.36	.06	45
CAMBRIDGE BAY N W T	1961	1975	146	-.37	.21	1961	1969	88	-.26	.38	49
TUKTOYAKTUK N W T	1952	1975	149	.16	.27	1961	1969	81	.30	.77	53
VICTORIA B C	1909	1975	793	.06	.02	1909	1969	721	.07	.02	55
VANCOUVER B C	1910	1975	581	.01	.02	1910	1969	499	0.00	.02	60
POINT ATKINSON B C	1914	1975	478	.12	.02	1914	1969	398	.13	.02	61
ALERT BAY B C	1948	1975	330	-.11	.07	1948	1969	259	.05	.10	63
TOFINO B C	1910	1975	534	-.16	.03	1910	1969		458	-.17	66
PRINCE RUPERT B C	1909	1975	582	.07	.02	1909	1969	505	.07	.03	68

Table 2

are 16 such stations for which computations from two sources are available. The comparison is shown in Table 2. Considering that an additional six years of recording is available for our computations, the agreement is very good. It is within one standard deviation at 12 stations, within two σ's for 3 stations and poor at one station (Saint John, N.B.). Changes due to the additional recordings available now, account for the differences which can be seen easily from the plots of recordings (Fig. 7-12).

A summary of changes in the sea level of tide gauges is presented on Fig. 13. For each of the 47 stations the length of the horizontal line (centered at the station) represents the month of recorded data. The length of the vertical line is proportional to the slope: positive upward, negative otherwise.

(B) Lake-level data

39 pairs of water gauges from the great lakes were used. These were identical to those used in the earlier study of Ontario [Vaníček, 1977] in treatment as well as values and will not be repeated here. They make part of the data base under numbers 4900-4938.

(C) Relevelled segments

Total number of relevelled segments, very probably incomplete, is 7387. From those 1-st order 6200, mixed 1st and 2nd order 434, 3rd order 16 and 737 undefined including 16 segments from the U.S.

These segments range in length from 100 m to 53.9 km and cover the period 1904 to 1978. The shortest time elapsed between the two levellings is lyear, the longest is 74 years.

The total of 556 level lines was searched (for relevellings) representing about 23,000 km of levelling. Rod corrections were applied

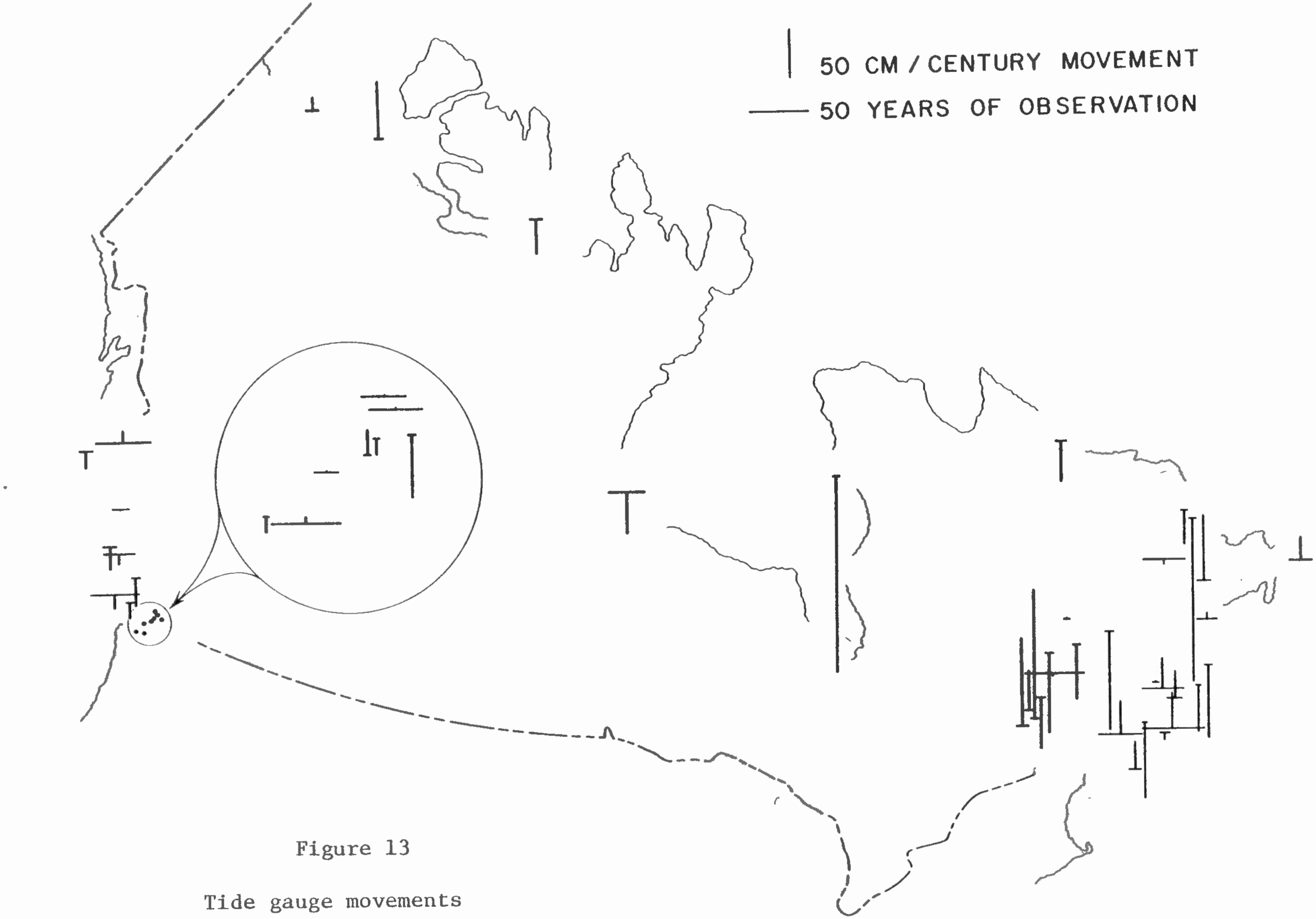

Figure 13

Tide gauge movements

to all the results whenever available.

All the discovered segments were numbered according to their geographical location as follows:

Nova Soctia	1 - 292
P.E.I.	701 - 794
New Brunswick	1002 - 1414
Newfoundland	1501 - 1625
Quebec	1801 - 3296
	9501 - 9888
Ontario	3300 - 4411
Western Canada	5001 - 8160

They appear under these numbers in the data base.

As the information about relevelled segments were being put on cards, it became obvious that various procedures must be developed to ensure a maximum integrity of data. Due to very rigid and limited facility for displaying data in the required projection at UNB, it was decided that most procedures for checking and especially producing maps will be developed at EPB/EMR.

The first step was to simply display the relevelled segments in the projection and scale of existing maps (the 2,000,000 provincial series was used for base maps). Then inspecting the lines for continuity and location (e.g. benchmarks in water), some errors were discovered. Comsulting then the original documents corrections were made wherever possible, otherwise the segments were rejected. In addition to this procedure, some other checks were implemented at UNB such as checking for the length of the line (zero length, too long of a line,) which without plotting pointed to "suspect" data. The program listing is in the external appendices. As the data collection progressed, these checks were carried out and the end result of this is that the data set used in this study is in a reasonably good shape. The total number of segments retained is 5046 and these are displayed in figure 14. For the format of data in the data base see the specifications spelled out in the main program listed in the external appendices.

The levelling data were obtained partly from earlier conducted studies [Vaníček, 1976; Vaníček, 1977] and mostly through a 2 year long search in levelling files of the following organisations:

Vertical Control Section
Surveys & Mapping
Geodetic Survey of Canada
Ottawa, Ontario

Surveys & Plans Office
Ministry of Transport and Communication
Toronto, Ontario

Ministère des Terres et Forêts
Direction Générale du Domaine territorial
Direction des relevés techniques
Service de la Géodésie
Québec, Québec

Control Surveys
Land Registration and Information Service
Summerside, P.E.I.

7. ACTUAL COMPUTATIONS

Because of the size of the Canadian territory it could not be treated in one solution with the existing software. A piece-wise approach had to be adopted. The territory in which there were either some tide-gauges or relevelled segments (including the lake level data) was divided into 9 overlapping areas: (1) Yukon, (2) British Columbia, (3) Alberta, (4) Prairies, (5) Ontario, (6) Quebec, (7) Maritimes, (8) North Quebec, (9) Newfoundland. The extent of these areas is shown in figure 15. For the first runs the input data for these areas were selected manually by

Figure 14

Relevelled Segments

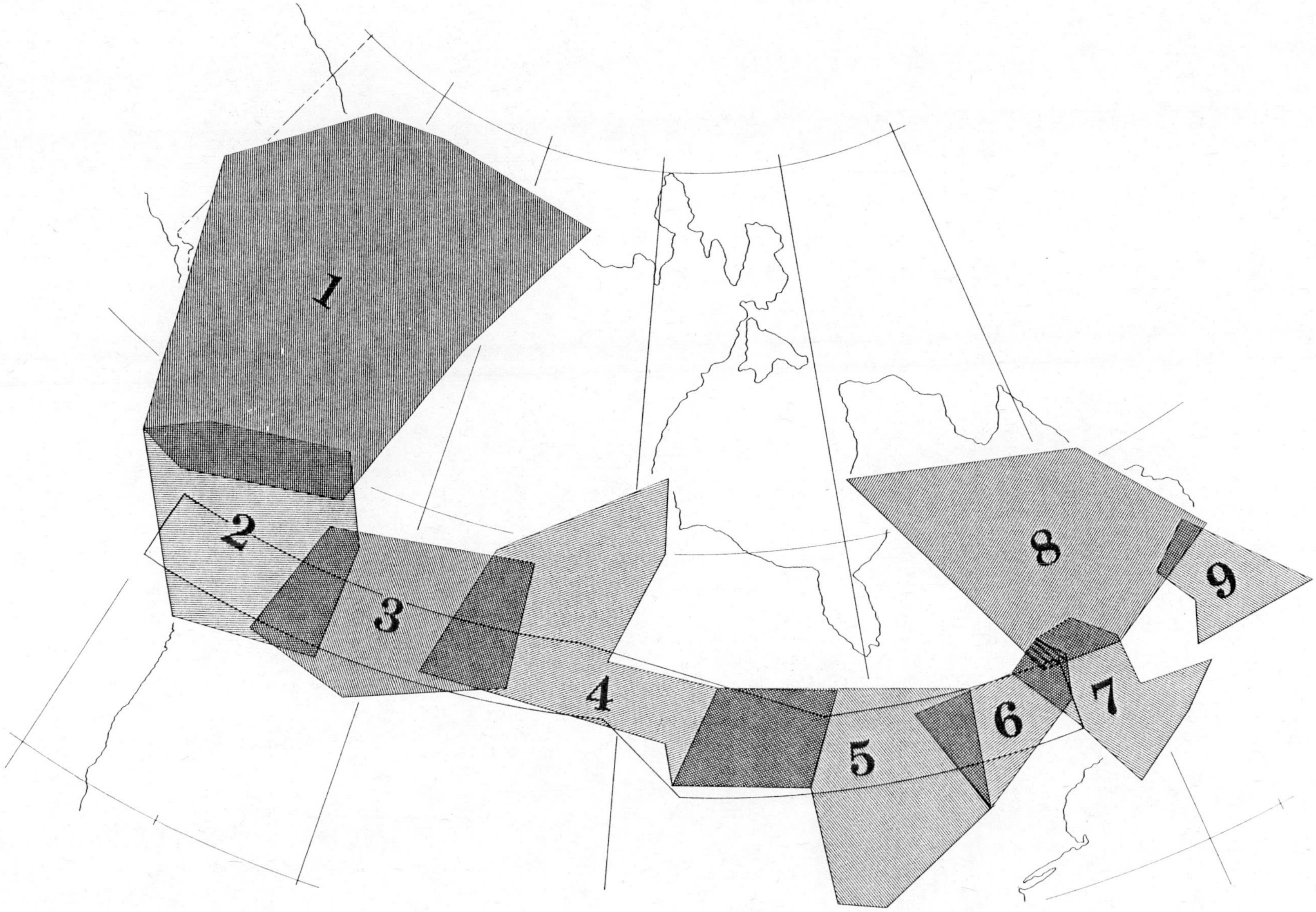

Figure 15

Areas for Partial Solutions.

pulling out the cards required for a particular run. Soon it became obvious that this will cause intolerable delays in computation and possibly even mix-ups of the cards. Therefore a small program was developed at EPB to select data within a given polygon and this was implemented at UNB.

To ensure the continuity of solutions along the area boundaries, the predicted vertical displacements from one area were used (as point displacements, i.e. the same way as the tide-gauge information) in the adjacent area hence the need for overlap. To get the "absolute" displacements this process of carrying forward of weighted constrains had to start in the areas well endowed with tide-gauges. Thus the order of solutions was chosen as follows:

$$\left\{\begin{array}{l}2\\6\\9\end{array}\right. \qquad \left\{\begin{array}{l}1\\3\\5\\7\end{array}\right. \qquad \left\{\begin{array}{l}4\\8\end{array}\right. .$$

Clearly, area 4 (Prairies) was a cause for anxiety since except for Churchill there are no tide-gauges and the predicted absolute displacements depend only on the outcome from areas 3 and 5. Churchill itself is too far removed to the north a isolated from the rest of the zone by the lack of data. Hence, the accumulated errors from both west (1 and 2) and east (6 and 5) were feared to have a significant effect on the results in area 4. It was then decided to first predict the displacements in one continuous long strip (about 200 km wide) connecting the Pacific coast with the Gulf of St. Lawrence. The prediction surface was selected to be of 2nd order in ϕ and 7th in λ. The strip contains 16 tide-gauges on both coasts and 2104 relevelled segments, and predictions for this strip served as weighted point constrains for all the hinterland areas (3, 4, 5). This approach alleviated, to a large extent, the uncertainties in the hinterland areas. The numerical results, including those for the strip are given in the external appendices.

The next phase was to produce a single contour map for each degree of the surface. For this purpose for each degree 18 separate maps were plotted displaying the uplift and the standard deviation values for the 9 areas. The maps then were hand contoured and by putting them together appropriately, a single map of uplift and standard deviation was compiled. Since there was sifficient overlap from area to area continuity should have been no problem. Actually in some cases it was very hard to make the connection due to the lack of control at the boundaries. Then contour lines were drawn examining in detail the predicted values, the standard deviations, the data distribution. The properties of the particular surface (2nd degree must be smoother than 3rd degree, etc.) was also taken into account.

After all contour lines were connected, a final graphic smoothing was carried out consulting again other information if needed. All 3 maps (figs. 16 to 18) were prepared this way. Parallel to hand contouring, we have started to use the contouring package available on the Departmental Computer (GPCP) and all 54 maps were contoured as well. There was no difference between hand and machine contoured versions. The machine contouring was done mostly so that scale changes or changes in projection could be introduced if needed.

Thus in summary the 3 x 18 x 2 = 108 detailed maps were used to produce the final 6 maps (i.e. 3 of the novements and 3 of the standard deviations). The standard deviations of the vertical movements pieced

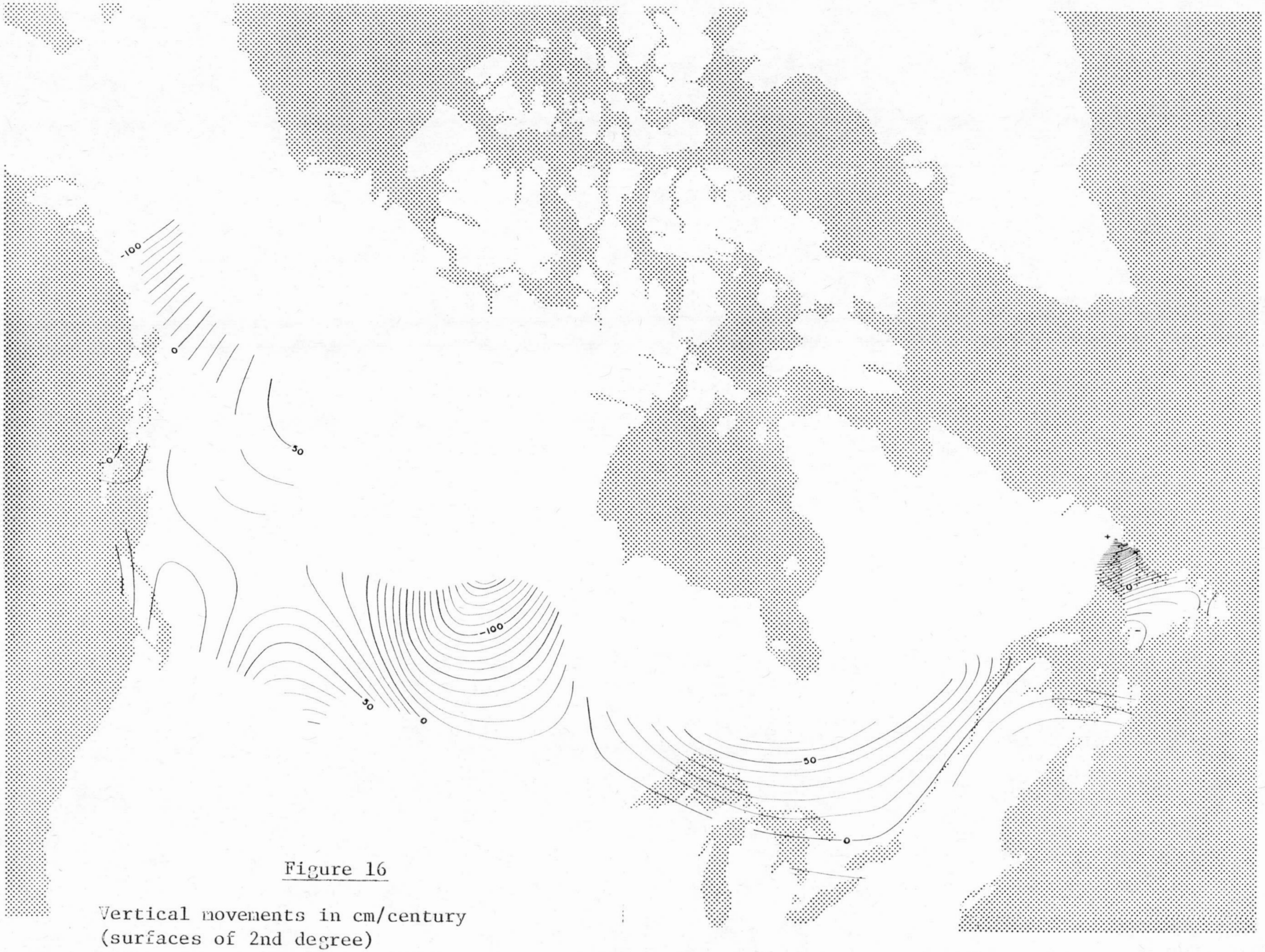

Figure 16

Vertical movements in cm/century
(surfaces of 2nd degree)

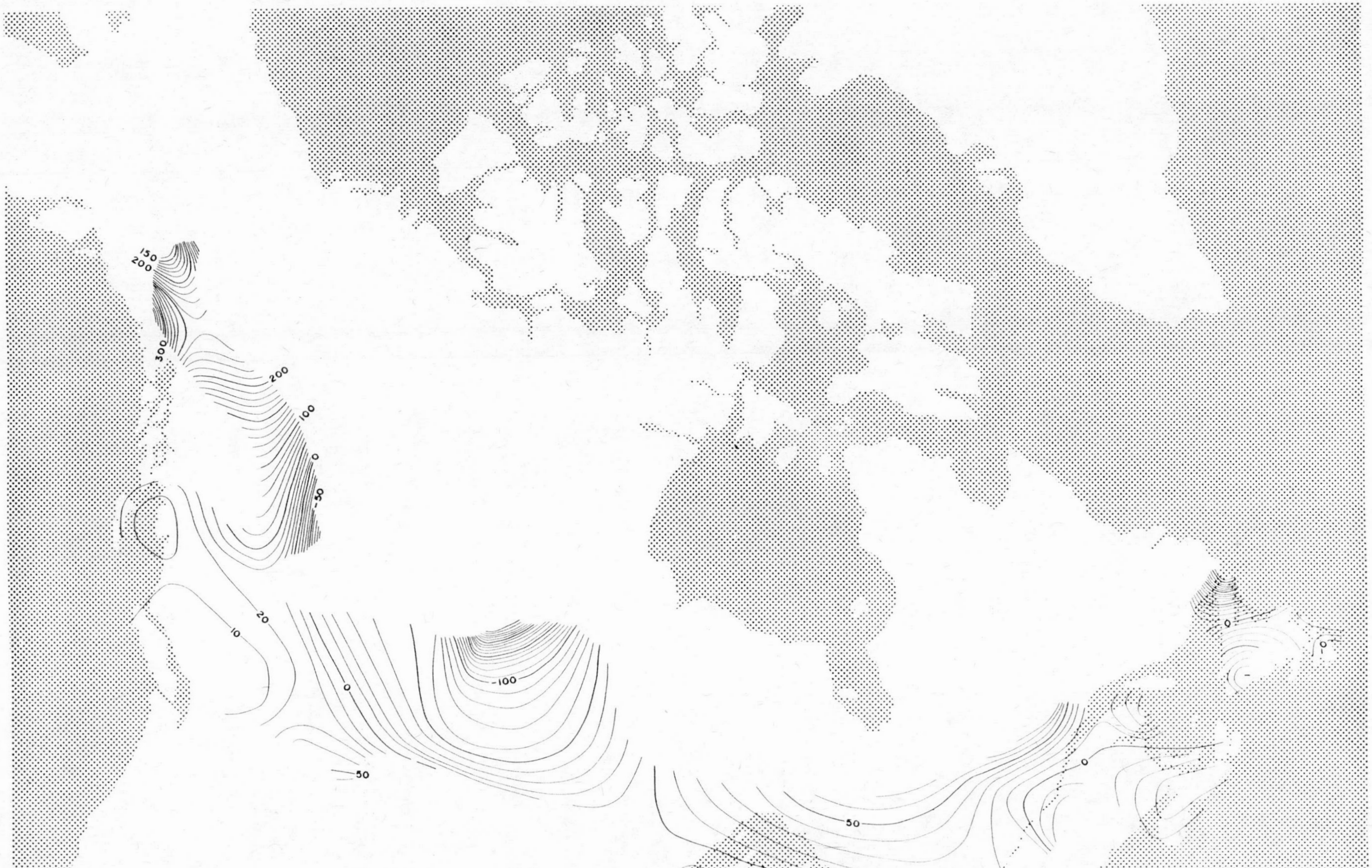

Figure 17

Vertical movements in cm/century
(surfaces of 3rd order)

Figure 18

Vertical movements in cm/century
(surfaces of 4th order)

together from the partial solutions are shown on figures 19 to 21. It is interesting to note how the accuracy of partial determinations degenerates towards boundaries of adjacent zones. The standard deviation also clearly shows the effect of relative abundance of data in the east and the lack of them in the west.

The standard deviation reflects only the contribution of random errors. Any systematic errors that may be present in the levelling, or more precisely present in the tilts obtained from the relevellings, will be aliased as vertical movements. The systematic errors in tilt (derived from levelling) are, however, likely to be much less important than those in heights derived from levelling [Vaníček et al., 1979]. There appears to be, for instance, no trace here of the trans-Canada levelling misclosure evident in heights as discussed by Lachapelle et al. [1977] or Anderson [1979]. Any such effect would have showed up in a poor fit of area no. 4 as mentioned above and it does not.

Before closing this section let us get back to the program which was used first to aid the checking of data, then to display the tide gauges and/or relevelled segments. Some of its capabilities are shown here.

The tide gauge data file can be displayed by a symbol at the location of the station or the way it is displayed on figure 13(or figure 22 that shows only the B.C.). The display of the relevelled segments has many options. Segments can be joined by straight lines and displayed alone (on figure 23) or with the tide gauges (see figure24). Optionally, end points can be marked by circles (see fig. 25). Vectors whose magnitude is proportional to the amount of tilt can be drawn from the center of segment

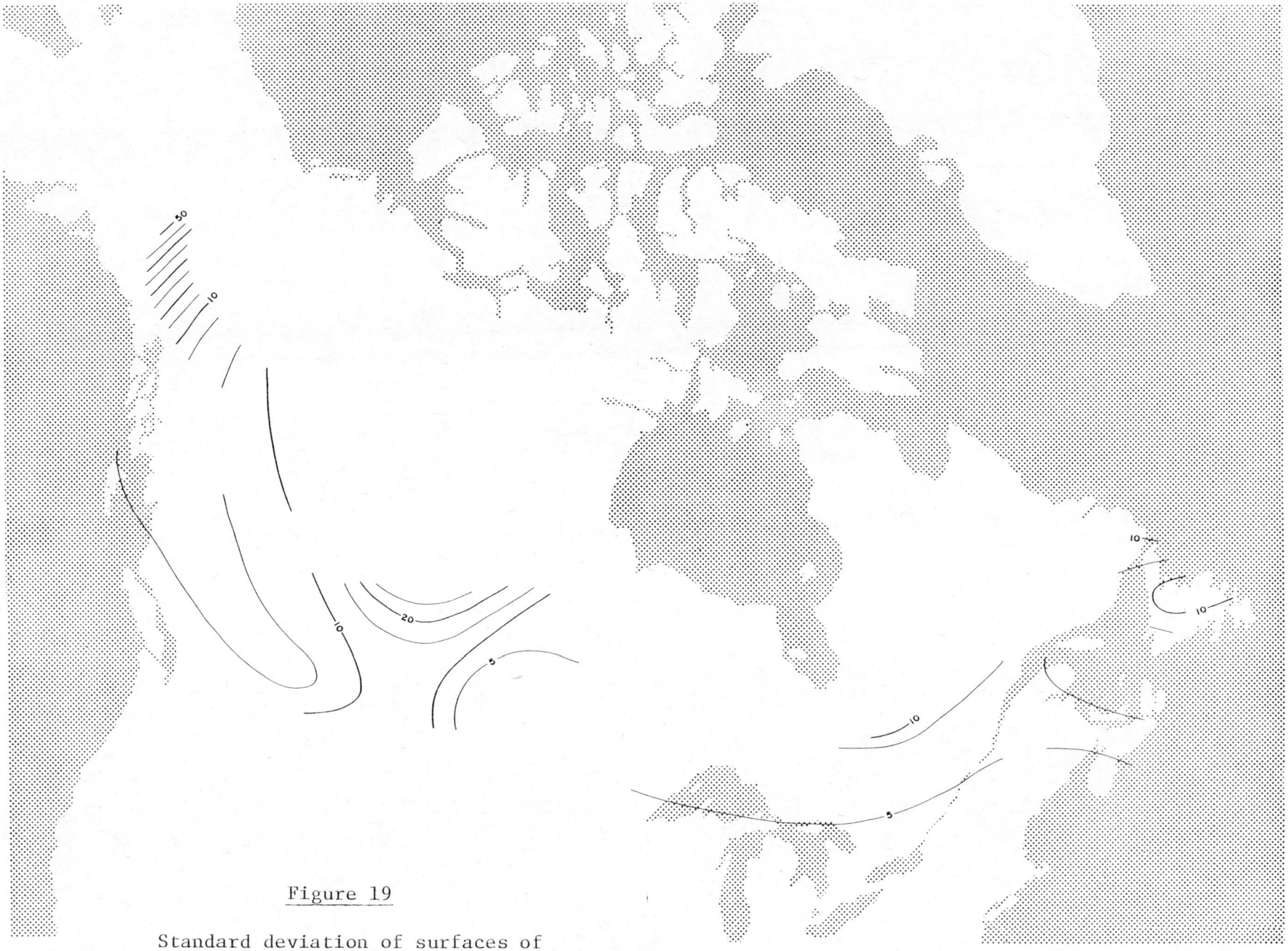

Figure 19

Standard deviation of surfaces of 2nd order.

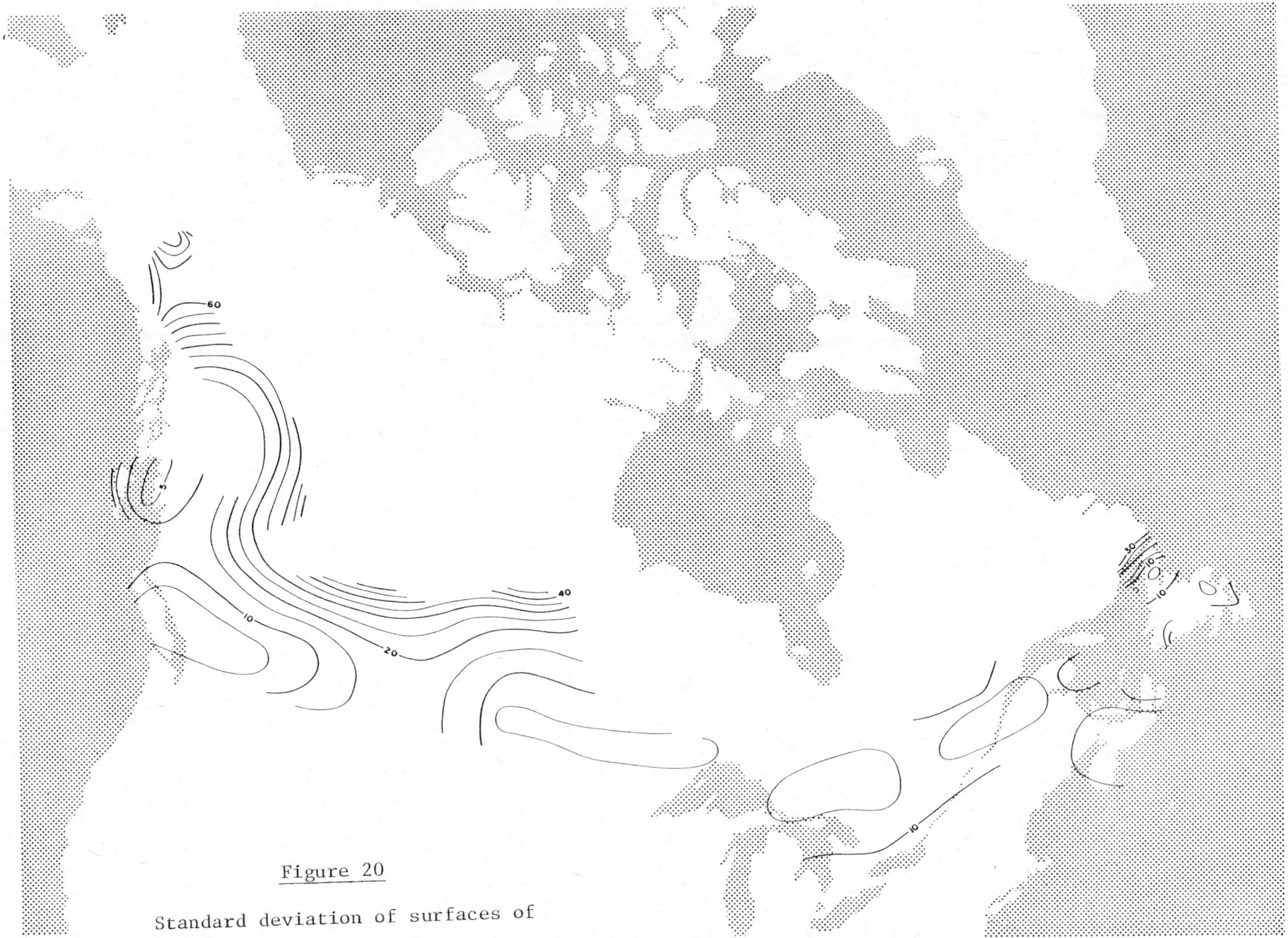

Figure 20

Standard deviation of surfaces of 3rd degree.

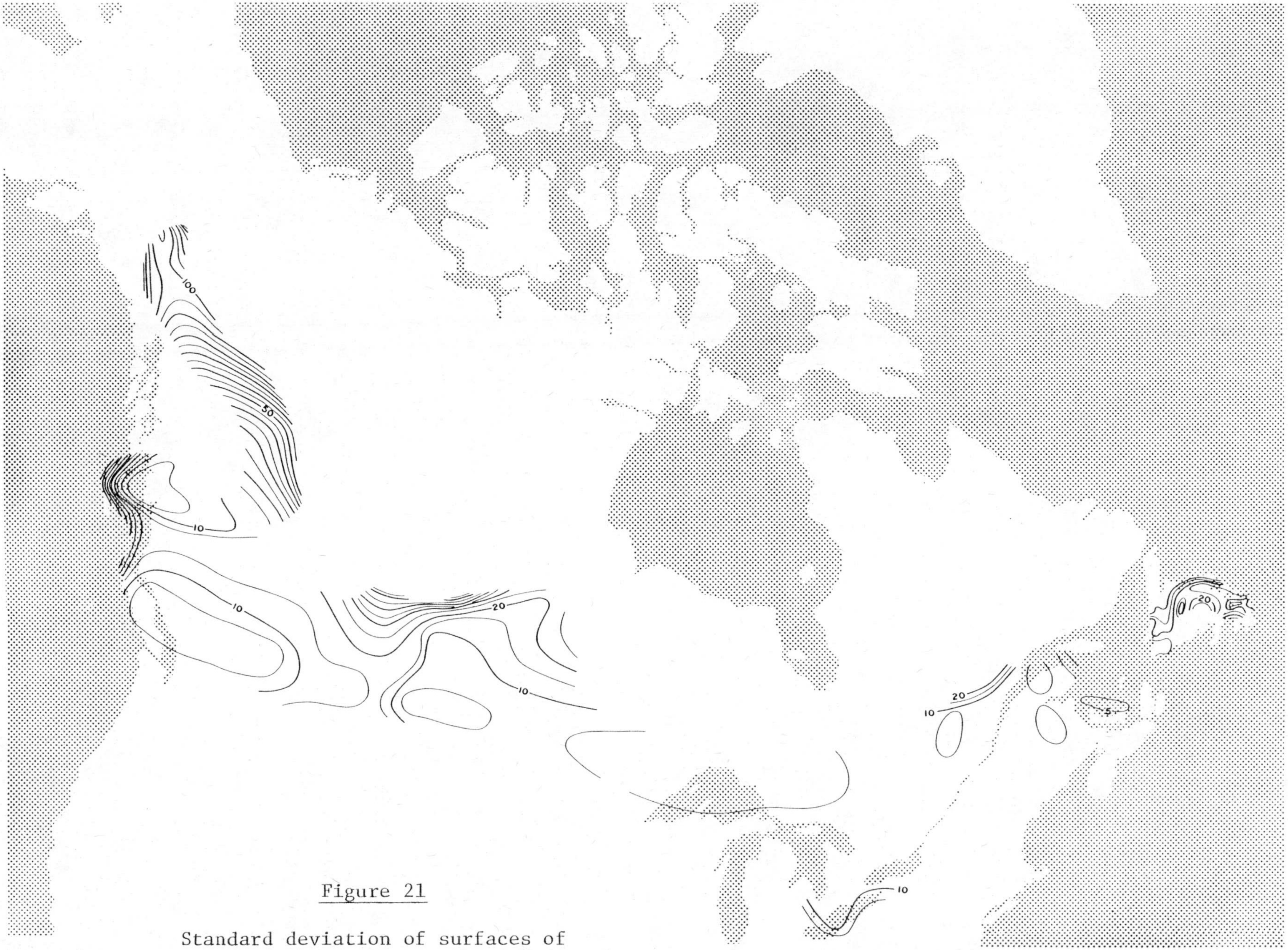

Figure 21

Standard deviation of surfaces of 4th degree.

Figure 22

Tide gauges in British Columbia

Figure 23

Relevelled segments in British Columbia

Figure 24

Relevelled segments and tide-gauges in British Columbia

Figure 25

Relevelled segments in British Columbia

(fig. 26). Finally a circle whose radius is proportional to the weight of the segment, can be also shown (fig. 27).

The selection of data for display can be done either by the polygon or by numbers assigned to each segment. In addition the polygon used for the selection itself can be displayed. If required the inside of the polygon can be shaded. Also if needed a grid of 1° x 1° in the proper projection can be drawn. All these options work in any reasonalbe combination producing almost any desired display of data (cf. figs. 28 and 29).

8. DISCUSSION OF RESULTS

In trying to interpret the results, one must bear in mind the fact that the used modelling technique has been designed to treat only linear movements, i.e. movements with vertical velocity that remains constant in time. Yet, some regions of Canada are very unlikely to be undergoing such a steady vertical movement and the application of this technique may have thus produced spurious movements. One example of such a region is the Pacific coast, another may be the Saint Lawrence river valley.

The next point that should be made is that the solutions are evidently very generalised. This smoothness mostly cannot be helped being caused by the lack of data. There are, however, regions, where the existing data distribution would support a more detailed solution. For instance, the Laurentide Park is known to be subsiding [Vaníček and Hamilton, 1972] and there is sufficient data available to document the subsidence. But the geographical extent of the subsidence is too small to show up in any of our solutions. The obvious way to surmount this

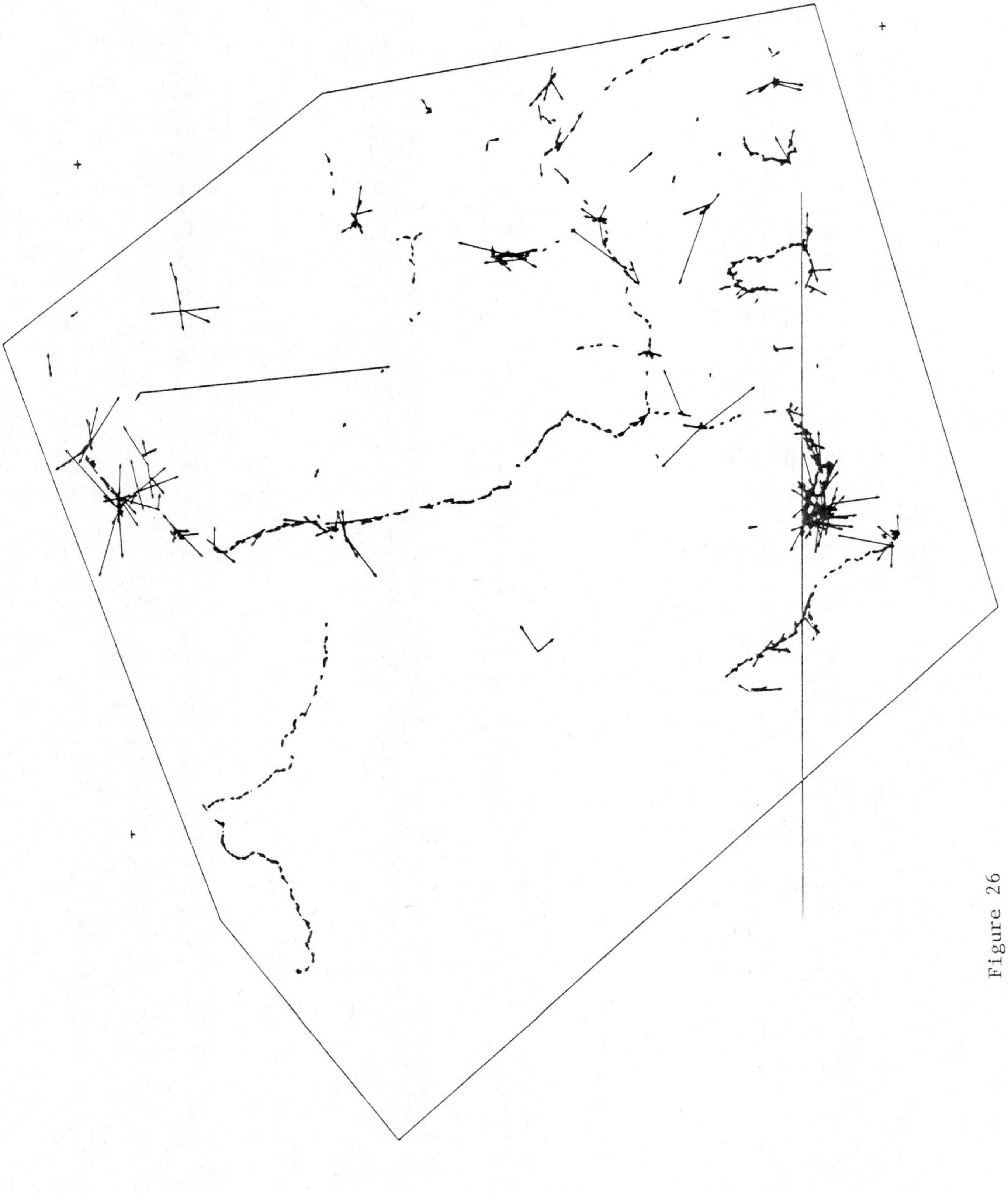

Figure 26

Tilts from relevelled segments in British Columbia

Figure 27

Weights of relevelled segments in British Columbia

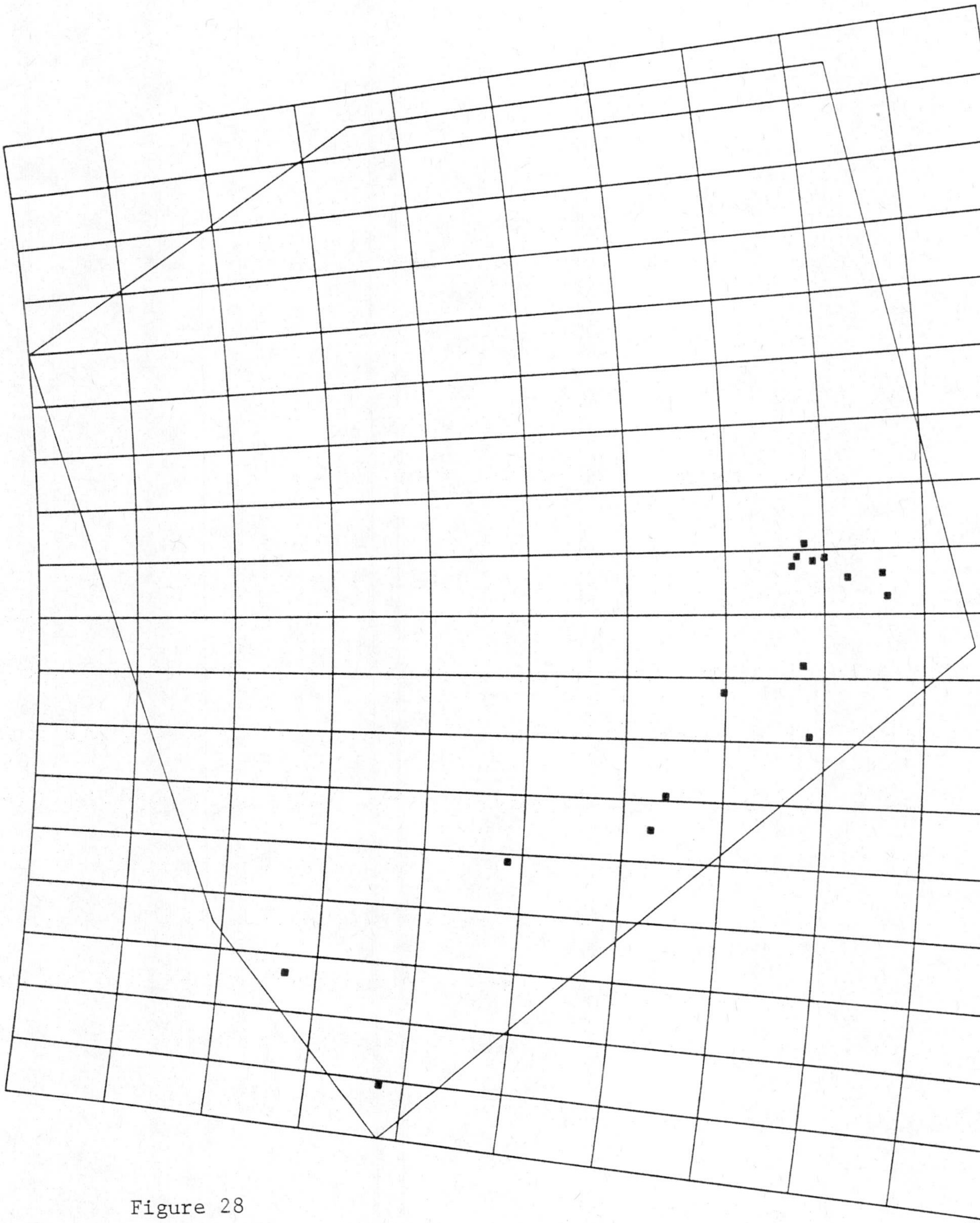

Figure 28

$1^{\circ} \times 1^{\circ}$ grid and tide gauges in British Columbia

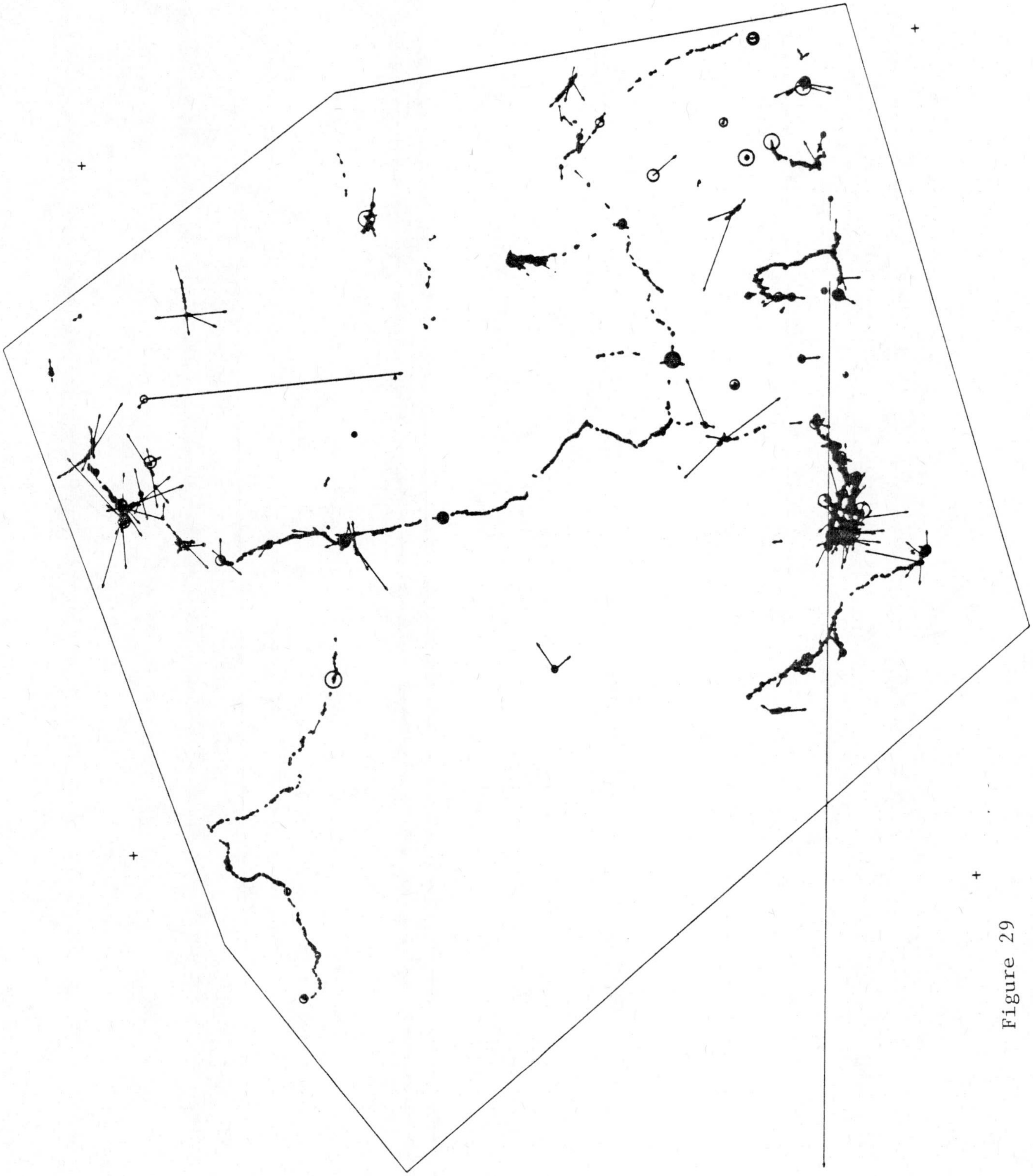

Figure 29

Weights and tilts of relevelled segments in British Columbia obstacle would have been to change the degree of the velocity surface from area to area according to the density of data; we have, however, decided against this strategy in favour of keeping the map uniformly smooth throughout Canada. Generally, the smoothness of the map is advantageous so far that on can see better the regional trends in the movements.

Comparing the three solutions it appears that the 2nd order surfaces (fig. 16) are really too smooth to express the features adequately. Clearly the average data density supports a more detailed solution. On the other hand, the 4th order surfaces are possibly too pliable, particularly in the west. While the 3rd and 4th order surfaces agree very well east from Ontario - Manitoba border, the disagreement in the west is substantial. Upon closer inspection one discovers that the 4th order solution saggs and peaks in the areas where there are sizeable holes in the data coverage: simply the data distribution in the west does not support a higher than 3rd order of the solution. Thus in the discussion here we shall limit ourselves to 3rd order solution, that can be considered to be the most compatible with the existing data coverage.

Turning now to individual features shown on the map, we note that the uplift along the Alaska highway is not incompatible with the tectonic picture of that area. Comparison with Whitten's [1970] results for the central part of the Alaska Panhandle, based solely on U.S. tide-gauge records, show a suprisingly good agreement in both trend and magnitude. The agreement is even more surprising when we realise that the width of the corridor shown on the map is quite optimistic considering the linear character of the given data. The values of standard deviation associated with this result show this point clearly. Also, the three solutions five quite different results for this part of Canada.

The hint of the subsidence in the Prairies is also based on little data (cf. figure 14) and as a is thus determined with a relatively low level of confidence (cf. figures 19 to 21). This feature should be definitely further investigated and if the rates shown on the maps are at all representative of what is really happening, it should not be too difficult to either confirm or disprove the subsidence through other available geophysical and geodetic techniques.

The northern tip of the U.S. Rockies shows a sign of uplifting. This is again derived from scanty data and relies heavily on 16 relevelled segments from Montana supplied by the U.S. National Geodetic Survey - the only U.S. data in the whole data base. Whether one chooses to believe it or not, the uplift agrees with the conviction of some earth scientists that the Rockies may still up up-thrusting. It should also be noted that the north-west trend in the western uplift pattern resembles very closely the shape of the geoid, as the interested reader can see by comparing fig. 17 to e.g. fig. 1 in the publication of Vaníček and Merry [1973].

Further east, one begins to distinguish the patter of post-glacial uplift in the region of Great Lakes. The pattern has been well established by a number of investigators as mentioned earlier, and appears to be in a good agreement with the pattern throughout the Quaternay as published, for example, by Walcott [1972] as well as with the general shape of the geoid in that region. This pattern is clearly discernible throughout southern Quebec with the Laurentide subsidence smoothed out of existence.

It is of interest to note the south-east trending uplift ridge east of Lake Ontario. It may reflect the Adirondack doming reported by Isachsen [1975] with which it agrees fairly well even in magnitude.

In the Maritimes the subsidence of the Bay of Fundy discovered and discussed in some of the above mentioned papers is easily distinguishable. Its magnitude appears to be somewhat diminished, presumably due to the smoothing power of the used technique. Significantly more data have been employed in this compilation, compared with the author's original solution published in 1976. As a result, the originally reported uplift of north-eastern New Brunswick and the Gaspé penisula got more sharply defined and seems to be focused more on the island of Anticosti rather than the Gaspé. Here, once more, the similarity of the movement pattern with the shape of the geoid should be pointed out. The reader can verify this on figure 2 in Merry and Vaníček [1974]. Physical reason for the uplift is not known. Neither is anything known to us about the geophysics of Newfoundland; the movement pattern there is rather complex.

9. SOURCE DATA AVAILABILITY

Copy of the relevelled segments data can be obtained at cost either on card or on tape. Enquiries should be addressed to the:

Director
Gravity and Geodynamics Division
Earth Physics Branch
1 Observatory Crescent
Ottawa, Ontario
K1A 0Y3

Those who are interested in obtaining copies of the monthly mean tide gauge data used in this study should send their inquiries directly to:

Marine Environmental Data Services Branch
Marine Information Directorate
Ocean and Aquatic Studies
Environment Canada
580 Booth Street
Ottawa, Ontario.

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