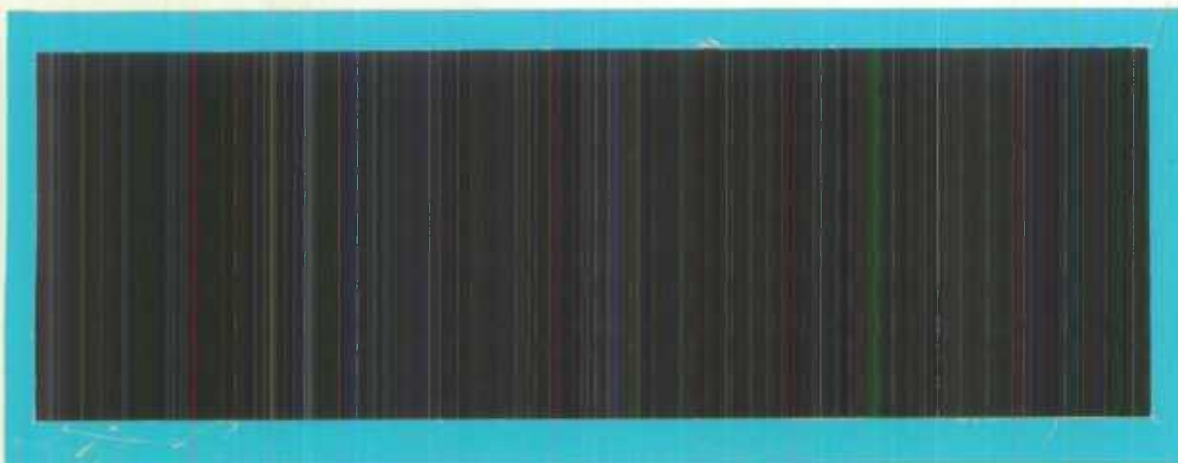




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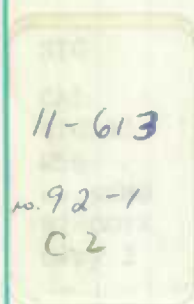


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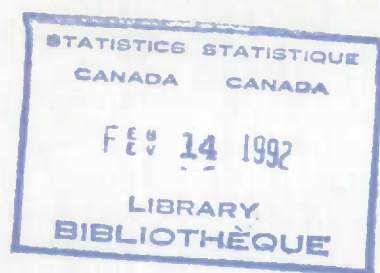
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METHODOLOGY BRANCH

**THE IMPACT OF INCENTIVES ON THE RESPONSE RATES
FOR FAMEX 1990: AN EVALUATION**

S. KUMAR AND A. DURNING





The Impact of Incentives on the Response Rates for FAMEX 1990: An Evaluation

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ABSTRACT

It is well-known that decreases in response rates generally have an adverse effect on the quality of the resulting estimates. In an attempt to improve response rates for the 1990 Family Expenditure Survey, incentives were given to part of the sample. An experiment, evaluating the impact of the incentives, was undertaken. Under the design, some of the sampled households were given one of two incentives. This paper describes the analysis undertaken to evaluate the impact of incentives on response rates. For each city in the sample, the effects of the incentives are investigated using Pearson's chi-square test. Additional analysis at the national level is undertaken, and it is based on the sign (not the magnitude) of differences in the response rates between the incentive and non-incentive (control) groups. Further analysis, using Wald tests for logit models, examines not only the effects of the incentives on the response rates, but the effects of interviewer experience and regional differences as well. Pairwise comparisons are also used to investigate effects that are found to be significant in earlier analysis.

Key words: binary response data, logit model, pairwise comparisons, Pearson's chi-square statistic, response rates, Wald test.

RÉSUMÉ

Il est notoire qu'une baisse des taux de réponse a généralement des répercussions négatives sur la qualité des estimations d'enquête. Dans l'espoir d'accroître les taux de réponse dans l'Enquête sur les dépenses des familles de 1990, on a offert des incitations à une partie de l'échantillon. À cet égard, on a fait une expérience qui visait à évaluer l'effet des mesures incitatives, au nombre de deux. Selon le plan de sondage, on présentait l'une ou l'autre de ces mesures à une partie des ménages échantillonnés. Cet article décrit l'analyse qui a servi à évaluer l'effet des mesures incitatives sur les taux de réponse. Pour les besoins de cette analyse, on se sert du test chi carré de Pearson pour chaque ville de l'échantillon. On fait aussi une analyse au niveau national, laquelle porte sur le signe (et non la grandeur) des différences de taux de réponse entre le groupe de l'échantillon qui s'est vu offrir des incitations et celui auquel on n'en a pas offertes (groupe de contrôle). D'autres analyses, qui utilisent des tests de Wald pour modèles logit, permettent d'évaluer aussi quel effet peuvent avoir sur les taux de réponse l'expérience de l'interviewer et les différences régionales. Enfin, à l'aide de comparaisons par paires, on examine les effets qui ont été déclarés significatifs dans les analyses précédentes.

Mots-clés : données d'enquête binaires, modèle logit, comparaisons par paires, critère chi carré de Pearson, taux de réponse, test de Wald.

The experimental design used for the study is outlined in section 2. It describes the procedure used for assigning each sampled household to one of three groups. All of the units in a group receive the same incentive or do not receive any incentive. In this study, the quantity of interest is the unit response rate and not the item non-response. For our study, we have used two definitions of response rates, which will be referred to as $R_{(1)}$ and $R_{(2)}$. $R_{(1)}$ is the unconditional response rate, and is based on the ratio of the number of response questionnaires (also referred to as usable questionnaires) to the total number of questionnaires which are eligible for inclusion in the sample. $R_{(2)}$ is the conditional response rate, and is based on the ratio of the number of response questionnaires to the number of questionnaires from eligible households with which contact was made. The formal definitions of the two response rates and additional details are given in Appendix II.

In section 3, we describe the methodological approaches used for analyzing the data. The mathematical details of the methodology are given in Appendix III. Within each city, the impact of each incentive is assessed by using Pearson chi-square tests. These results and other analysis of the data by cities, are described in sections 4 and 5. This analysis is based on the frequency distribution of increases and decreases due to the two incentives. Also, a hypothesis about the probability model for the increases is formulated and tested. Additional analysis, using the logit model approach, is presented in section 6. Results of the statistical tests for the effects of the various factors and the tenability of the models are also given. Section 7 summarizes the conclusions based on the results of the analysis.

1. INTRODUCTION

The Survey of Family Expenditures (FAMEX) is conducted approximately every two years either nationally, or, as is the case in 1991, in selected Canadian cities. The main objective of the survey is to obtain detailed accounts of all expenditures of households during the previous calendar year. Information on income and changes in debts and assets is also collected. The survey data relates family expenditure to family income and other characteristics such as family composition, occupation, and education (see [1]).

The FAMEX sample is selected from the Labour Force Survey (LFS) sampling frame, and the sample size in 1991 was about 7500 households (see [5]). Although the survey took place in 1991, the data collected refers to the year 1990; therefore, the survey is referred to as FAMEX 1990. In comparison to the LFS, response rates for FAMEX are low. Generally, low response rates (unit or item) have adverse effects on data quality and hence on the estimates.

It was observed that the overall response rate for the 1990 Food Expenditure Survey had decreased noticeably as compared to previous years. It was felt that FAMEX 1990, like the 1990 Food Expenditure Survey, might experience a similar drop in response rates that could appreciably affect the quality of the estimates. As one of the possible solutions to the problem of declining response rates, it was decided to give gifts (incentives) to some of the sampled households and evaluate their impact on the FAMEX 1990 response rates. Generally, the use of incentives is designed to motivate the households to respond, i.e., to have a beneficial effect on the response rates. However, the incentives can also have adverse effect, i.e., they may result in the decrease of response rates. This drop may be induced by the attitudes of the individuals of the households to the incentive and/or the sponsoring agency. An experiment was designed to evaluate the impact of two incentives and this report documents the results of the study.

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In section 3, we describe the methodological approaches used for analyzing the data. The mathematical details of the methodology are given in Appendix III. Within each city, the impact of each incentive is assessed by using Pearson chi-square tests. These results and other analysis of the data by cities, are described in sections 4 and 5. This analysis is based on the frequency distribution of increases and decreases due to the two incentives. Also, a hypothesis about the probability model for the increases is formulated and tested. Additional analysis, using the logit model approach, is presented in section 6. Results of the statistical tests for the effects of the various factors and the tenability of the models are also given. Section 7 summarizes the conclusions based on the results of the analysis.

2. EXPERIMENTAL DESIGN

The sampled households (or units) were divided into three groups at the interviewer assignment level. Using systematic sampling, 40% of the interviewers were assigned to the control group. The households to be interviewed were approached by these interviewers for data collection in the usual manner and were not given a gift from Statistics Canada. The households in the control (non-incentive) group will be referred to as receiving incentive O. Another 30% of the interviewers were assigned to the incentive P, which was the Statistics Canada publication "A Portrait of Canada". The sampled households in this group were given one book per household. The remaining 30% of the interviewers were assigned to the incentive X (a clipboard bearing the Statistics Canada logo) and similarly, the sampled households in this group were given one clipboard per household. The use of the letters O, P, and X to represent the three incentives is consistent with the labels used for the incentives in the field. The assignment of households to the various incentive groups, at the interviewer assignment level, was done strictly for operational and not methodological reasons. The fact that each interviewer had to work with only one type of incentive allowed less chance of contamination, i.e., of a "non-incentive" household receiving an incentive from an interviewer.

Generally, the interviewers' work loads were balanced. As such, it was expected that the above 40% - 30% - 30% distribution would be approximately maintained at the household level as well. In practice, however, this distribution was about 42% - 29% - 29%, which is reasonably close to the targeted distribution. The incentives were given to the households at the time of introduction by the interviewers. Note that the giving of an incentive was not conditional on a household's later decision to respond (fully or partially) or not to respond.

Although the primary and initial goal of this study was to investigate the impact of the two incentives on the response rates, it was decided later to examine the regional differences and the effect of the interviewers' survey experience on the FAMEX response rates as well.

3. METHODOLOGY

Three factors are analyzed for their effect on the response rates. However, the actual number of factors for a certain type of analysis may be less than three and will vary from analysis to analysis.

The three factors are:

- (1) Incentive (to be denoted by I);
- (2) Interviewer's Experience (to be denoted by E); and
- (3) Area or Region (to be denoted by A, since R has already been used as a symbol).

The factor I has three levels, which are: (i) level 1: incentive O (no gift), (ii) level 2: incentive P (publication), and (iii) level 3: incentive X (clipboard). For example, when we say that a household is at level 1 of factor I, we mean that this household is in the "incentive O" group and therefore did not receive a gift.

The factor E also has three levels. These are: (i) level 1: interviewers with LFS experience, (ii) level 2: interviewers with other (non-LFS) survey experience, and (iii) level 3: interviewers with no previous survey experience. For example, the statement that a household is at level 2 of factor E means that this household was contacted by an interviewer with other (non-LFS) survey experience.

The factor A has five levels, which correspond to five regions in Canada. They are: (i) level 1: Atlantic region, (ii) level 2: Quebec, (iii) level 3: Ontario, (iv) level 4: Prairie region, and (v) level 5: B.C. For example, a household at level 4 of factor A means that this household is in the Prairie region.

The analysis will be done using three different approaches. The first one examines the effect of each incentive on the response rates within each city. The effect of the various levels of the factors E and A will be ignored, i.e., it is equivalent to the tacit assumption that the effects of E and A are equal to zero. This analysis is based on a probability model postulated for the household response. The second approach is an analysis at the national level, and it is based on the frequency of increases/decreases in the response rates (relative to the control group) due to the incentives in each city. As before, it is based

on the assumption that the factors E and A have no effect on either of the two response rates. The third approach, involving simultaneous consideration of the three factors, is based on the use of logit models for binary response variables in categorical data analysis. The three approaches are described in sections 3.1, 3.2, and 3.3, respectively.

3.1. Effect of an Incentive on the Response Rates $R_{(1)}$ and $R_{(2)}$ at the City Level

For simplicity, we restrict attention to a particular city, the response rate $R_{(1)}$, and incentive I at levels 1 and 2. We are interested in assessing the effect of level 2 relative to level 1 for $R_{(1)}$, i.e., the effect of the publication relative to the control group. For level $i = 1, 2$ for factor I, let

n_i = count of eligible households (in the sample) at level i of factor I.

y_i = count of respondent households among the above n_i .

$X_{(1)ij} = \begin{cases} 1, & \text{if the } j^{\text{th}} \text{ household is labelled as a respondent, where } j = 1, \dots, n_i, \\ 0, & \text{otherwise.} \end{cases}$

$p_{(1)i}$ = true probability that the j^{th} household will respond, i.e., it will be labelled as a respondent.

We will use these X's for defining the city response rates in the next section.

We assume that (1) for a particular i , the probability $p_{(1)i}$ is the same for each household in the sample and is not dependant on j , although $p_{(1)i}$ may be different for different levels of factor I, and (2) the decision of a household to respond or not to respond is independent of any other household's decision.

We are interested in testing the hypothesis that the factor I at level 2 had no effect (beneficial or adverse) relative to level 1 on the response rate $R_{(1)}$, against the alternative that it did have an effect. Mathematically, this is equivalent to testing the hypothesis $H_0: p_{(1)1} = p_{(1)2}$ against the alternative that $H_A: p_{(1)1} \neq p_{(1)2}$. For this purpose we use the well-known Pearson's chi-square test statistic X^2 (Freeman, [3]) for testing the homogeneity of two binomial populations. Let

$$\hat{p} = (y_1 + y_2) / (n_1 + n_2)$$

$$m_1 = n_1 \hat{p}$$

$$m_2 = n_2 \hat{p}$$

The test statistic X^2 is defined by

$$X^2 = x^2 \left(\frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{n_1 - m_1} + \frac{1}{n_2 - m_2} \right)$$

where $x = y_1 - m_1$.

The value of X^2 is compared to the upper α (where α is the level of significance) value of the χ^2 distribution with 1 degree of freedom to conclude whether the hypothesis of no effect due to the incentive P (level 2 of factor I) should be accepted or not. Note that, when we are dealing with $R_{(2)}$, the sample count n_i will be based on the eligible households for which contact was made. Similar computations will be done to investigate the effect of the clipboard (level 3 of factor I) relative to the control group. The results of the analysis are presented and discussed in section 5.1.

3.2. Changes in Response Rates due to the Incentives

The approach presented in section 4.2 examines the data at both the city (not within cities) and the national level, as well as analyzing the effects of the incentives. As mentioned earlier, the differences due to the interviewer's experience are ignored. For simplicity, we will use $R_{(1)}$ in the following definitions. The concepts for $R_{(2)}$ are similar. Let us restrict attention to a particular city. For level i of factor I, let

$$\begin{aligned} R_{(1)i} &= 100 \times [\sum_j E(X_{(1)ij}) / n_i] = 100 \times [p_{(1)i}] \\ &= \text{true response rate } R_{(1)} \text{ (unknown) for level } i \text{ of incentive I for the city under} \\ &\quad \text{consideration.} \end{aligned}$$

$$\begin{aligned} C_{(1)i1} &= R_{(1)i} - R_{(1)1} \\ &= \text{change in } R_{(1)} \text{ from } R_{(1)1} \text{ due to the } i^{\text{th}} \text{ level of I.} \end{aligned}$$

Note that for $i = 1$, $C_{(1)1} = 0$, and is not of interest for the analysis. In the following part of this subsection, we are concerned with $i = 2$ or 3 only ($I = P$ or X). If the i^{th} level of the incentive I has no effect on the response rate, then $C_{(i)1} = 0$. The observed values (estimates) of $R_{(i)}$ and $C_{(i)1}$ will be denoted by $\hat{R}_{(i)}$ and $\hat{C}_{(i)1}$, respectively. We assume that $\hat{C}_{(i)1} = C_{(i)1} + \epsilon_i$ where ϵ_i is a continuous random error term with a symmetric distribution about the origin. Thus, $P\{\hat{C}_{(i)1} = 0\} = P\{C_{(i)1} + \epsilon_i = 0\} = 0$, since $C_{(i)1}$ is a constant. Define $p_{(i)}^* = \text{Probability}\{\hat{C}_{(i)1} > 0\}$, $i = 2$ or 3 .

If the incentive I has no effect (beneficial or adverse) on the response rates, then the observed increases and decreases are equally likely. This means that, mathematically, the statement $p_{(i)}^* = 1/2$ is equivalent to the statement that the i^{th} level of the incentive I had no effect on the response rates. Assume that (i) the outcome (increase/decrease) in each city has a binary outcome, and (ii) the outcomes in various cities are statistically independent. We can define the quantity C_N (similar to the C 's defined at the city level) at the national level. For the preliminary analysis, we can regard the \hat{C} 's (at the city level) as realizations of the national level \hat{C}_N and use the sign test for inference. Let $p^* = P\{\hat{C}_N > 0\}$, where \hat{C}_N is an estimate of C_N . For this analysis, we test the hypothesis $p^* = 1/2$ versus the alternative $p^* \neq 1/2$ (at the national level). Note that $p \neq 1/2$ means that the level i of the incentive I did have an effect at the national level. However, we cannot say whether an effect is beneficial or adverse. The test statistic for a binomial distribution is given in most elementary statistics books and is also included in Appendix III for reference.

3.3. Logit Model

Given the sample, and for $i, j = 1, 2, 3$, and $k = 1, 2, \dots, 5$, let

$\pi_{ijk} = \text{Probability}\{\text{that an eligible household at factor I-level } i, \text{ factor E-level } j, \text{ and factor A level } k \text{ will be labelled as a respondent}\};$

$I_i = \text{main effect - } i^{\text{th}} \text{ level, factor I};$

$E_j = \text{main effect - } j^{\text{th}} \text{ level, factor E};$

A_k = main effect - k^{th} level, factor A;

IE_{ij} = interaction effect - i^{th} level, factor I, and j^{th} level, factor E.

Similar notation may be used for the other interaction terms. Note that the above definition of π_{ijk} is appropriate for the consideration of $R_{(1)}$. For $R_{(2)}$, the definition of π_{ijk} is given by the following expression.

π_{ijk} = Probability {that an eligible household at factor I-level i , factor E-level j , and factor A-level k will be labelled as a respondent given that the contact was made}.

Henceforth, the definition of π_{ijk} will be evident from the type of response rate under consideration. Note that π_{ijk} is the true (unknown) probability of response. It is reasonable to assume that (i) a household's decision to respond or not to respond is independent of another household's response decision, and (ii) the interaction terms IA_{ik} (between factors I and A) and EA_{jk} (between factors E and A) are equal to zero. That is why we do not have these interaction terms in the model. The assumption that $IA_{ik} = 0$ is motivated by the fact that an incentive will generally have the same impact on the decision to respond in all regions, i.e., it is not likely that a combination of an incentive and a region will have a special effect on the probability to respond. Similar justification can be given for the assumption $EA_{jk} = 0$. Finally, although we expect $IE_{ij} = 0$, this may not be the case. The reasoning for this is as follows: the incentives were given during the interviewer's first visit to a sampled household. Whether the incentives were given by an interviewer with or without experience should not affect the response outcome, since the incentives were given at the time of initial contact with an individual of the household. However, it is possible that some of the more experienced interviewers may have influenced the response outcome by increasing respondents' perception of the importance of the survey and the incentives on subsequent visits. Hence, we will include the term IE_{ij} in the model defined below.

We assume the following model (called model M1) for the analysis:

$$\text{Ln} [\pi_{ijk} / (1 - \pi_{ijk})] = M + I_i + E_j + A_k + IE_{ij} , \quad (2.1)$$

where M = overall mean (also referred to as the intercept in later tables), and

$$\sum_i I_i = \sum_j E_j = \sum_k A_k = \sum_i IE_{ij} = \sum_j IE_{ij} = 0.$$

The justification for using the logit model (2.1) for binary response variables is well documented in the literature (see Fienberg [2], Freeman [3]). We are interested in testing four hypotheses. Three of them concern the main effects of each factor, while the fourth one pertains to the interaction term. They are:

- H_1 : The various incentives have no effect on the response probabilities. Mathematically, it means that $I_i = 0$ for all i .
- H_2 : The interviewer's prior survey experience (LFS, other, or none) has no effect on the response probabilities, i.e., $E_j = 0$ for all j .
- H_3 : There are no regional differences in response rates, i.e., $A_k = 0$ for all k .
- H_4 : There is no effect due to the interaction of the incentive (I) and interviewer experience (E), i.e., $IE_{ij} = 0$ for all i, j .

Note that, if we reject H_1 : $I_i = 0$ for all i , then it means that the incentives do have an effect upon the response. However, it does not necessarily indicate that the effect is beneficial, i.e., that it results in higher response rates. Also note that the hypothesis about the interaction term must be examined for significance first, before drawing conclusions about the significance of the various main effects.

The model M1 will be used for analyzing data from the four regions other than B.C., due to the fact that interviewer experience data was not available for the B.C. region. We will ensure that (2.1)

does conform to the data although it may not be the best model (in terms of the minimum number of parameters) for conformity to the data.

When the data is analyzed for all five regions, we will use the model M2, which is given by:

$$\text{Ln} [\pi_{ik} / (1 - \pi_{ik})] = M + I_i + A_k \quad (2.2)$$

where π_{ik} , M , I_i , and A_k have similar interpretations as before. The model M1 differs from M2 in the sense that there are no E_j and IE_{ij} terms in M2. This is due to the fact that the interviewer experience data was not available for M2, and thus the parameters E_j and IE_{ij} cannot be analyzed, even if they are non-zero. Note that M , I_i , and A_k of M1 and M2 are defined using the same concepts. Also,

$\sum_i I_i = \sum_k A_k = 0$, and we assume that the interaction term $IA_{ik} = 0$, for all i and k .

Here we will be interested in hypotheses H_1 and H_3 only. Although the testing of the hypotheses is important, we will ensure that the appropriate model (M1 or M2) is tenable for each type of response rate. If the model is untenable for a particular type of response rate, we will examine the individual main effects and interactions. In both cases, the analysis will be done using the Wald approach, which is described in Appendix III.

4. PRELIMINARY EXAMINATION OF THE DATA

In this section, we look at the incompleteness of the design and how it affects the proposed methods of analysis. We also examine the response rates within and between cities. Finally, we undertake the analysis based on the direction of the changes (increase or decrease) in the response rates under each of the two incentives.

4.1. Incompleteness of the Design

A design is said to be incomplete if, for a particular triplet level of factors (I,E,A), there are no structural counts available. The structural zeros are different than the sampling zeros in the sense that, in the case of the sampling zeros, the observed count for a particular category is zero. This situation could arise either due to the design used for the study or the concepts defining various factors. However, in the case of structural zeros, certain categories simply do not exist, and therefore have no corresponding sample counts.

The sample counts, by the three factors at the various levels, are presented in Appendix I. No counts are available for:

- (1) Quebec region, incentives P and X, and the factor E at the "other experience" level;
- (2) Ontario region, incentive P, and the factor E at the "no experience" level; and
- (3) Ontario region, incentive X, and the factor E at the "LFS experience" level.

This incompleteness of design was not decided upon due to methodological considerations, but was a consequence of the operational functions. In spite of the incompleteness of this design, we can undertake all the analyses outlined in section 3.

Furthermore, we observe that the data for the B.C. region are collapsed for the factor E, i.e., we do not have the sample counts by various levels of E. This situation requires us to do the analysis in section 5 under two different scenarios. In the first case, the data from the B.C. region will be

excluded and all of the three factors will be considered simultaneously. In the second case, the data from B.C. will be included, but the factor E will be excluded from the analysis.

4.2. Frequency Distribution of Changes

Table 1 gives, by city, the estimates $\hat{R}_{(r)i}$, $\hat{C}_{(r)i}$ for $r = 1, 2$ (type of response rate) and $i = 2, 3$.

Table 1

Observed Response Rates under factor I by City and Changes due to levels 2 and 3

City	Response Rate 1			Response Rate 2		
	$\hat{R}_{(1)1}$	$\hat{C}_{(1)21}$	$\hat{C}_{(1)31}$	$\hat{R}_{(2)1}$	$\hat{C}_{(2)21}$	$\hat{C}_{(2)31}$
St. John's	79.7	-2.5	4.5	80.3	-1.7	4.7
Charlottetown /Summerside	87.1	5.5	1.9	89.1	7.1	0.0
Halifax	75.5	-12.2	-3.4	78.6	-12.0	-3.3
Saint John	68.6	0.0	9.7	70.0	1.1	11.7
Quebec City	81.1	N.A	7.4	82.6	N.A	7.8
Montreal	71.0	9.2	13.4	76.2	10.7	10.8
Ottawa	70.6	-2.3	0.5	76.2	-3.2	0.6
Toronto	64.6	5.7	1.1	69.8	3.5	1.5
Thunder Bay	63.4	1.3	-6.5	67.5	1.3	-5.3
Winnipeg	66.1	9.5	10.1	69.6	6.6	9.4
Regina	78.3	-4.2	3.3	79.8	-4.3	3.7
Saskatoon	76.4	-7.4	-4.2	79.6	-7.0	0.2
Calgary	77.5	-6.4	-2.2	81.7	-4.9	-1.9
Edmonton	76.1	7.1	8.2	79.7	5.0	4.6
Vancouver	63.6	-16.8	3.2	70.1	-4.8	1.1
Victoria	66.8	-0.2	-1.6	69.2	-1.1	-1.1
CANADA	72.4	-1.3	2.8	76.0	0.0	2.8

If we look at the magnitude of these changes, we have some extreme increases and decreases. For $R_{(1)}$, $\hat{C}_{(1)21}$ is -12.2 for Halifax, and it is -16.8 for Vancouver. On the increase side, $\hat{C}_{(1)21}$ is 9.2 for Montreal. Similarly, $\hat{C}_{(1)31}$ is -6.5 for Thunder Bay, while it is 8.2 for Edmonton. Looking at $R_{(2)}$ as well, we conclude that there is no clear pattern of the magnitude and sign of the \hat{C} 's.

Alternatively, we can disregard the magnitudes of the \hat{C} 's and simply look at the sign of the \hat{C} 's, i.e., whether $\hat{C}_{(r)} > 0$, $\hat{C}_{(r)} < 0$, or $\hat{C}_{(r)} = 0$. Note that $\hat{C}_{(r)il} > 0$ means that an increase was observed in the response rate $R_{(r)}$ at the i^{th} level of I , whereas $\hat{C}_{(r)il} < 0$ means that a decrease was observed. Table 2 summarizes the increase/decrease data of Table 1.

Table 2
Frequency Distribution (based on city counts)
of Increases/Decreases by Incentive

Change in Magnitude	Response Rate 1		Response Rate 2	
	Level 2 (P)	Level 3 (X)	Level 2 (P)	Level 3 (X)
Increase	6	11	8	10
Decrease	8	5	7	5
No Change	1	0	0	1

Under P, and both types of response rates, the number of increases and decreases are very close. However, under X, the number of increases is slightly higher than the corresponding number of decreases. Incentive X (clipboard) appears to perform better than incentive P (publication) if we base our statement on the ratio of increases to decreases in each column in the above table.

5. ANALYSIS USING THE PEARSON'S CHI-SQUARE AND SIGN TESTS

In this section, we examine the response rates for statistically significant differences between the control and the incentives at the city and national level.

5.1. Testing of the Incentive Effects for each City

Here we examine the Pearson's X^2 values for the differences between the control group and each of the two incentives, using the formulas given in section 3.1. Table 3 presents the results of the chi-square tests for each of the 16 sampled cities.

Table 3
Values of X^2 statistic for Testing Incentives vs. Control

City	Response Rate 1		Response Rate 2	
	Control-Publication X^2	Control-Clipboard X^2	Control-Publication X^2	Control-Clipboard X^2
St. John's	0.25	0.86	0.11	0.96
Charlottetown /Summerside	1.40	0.18	4.19*	0.00
Halifax	4.33*	0.28	4.67*	0.35
Saint John	0.00	1.73	0.03	9.35*
Quebec City	N.A.	1.85	N.A.	0.15
Montreal	4.95*	10.69*	7.89*	8.28*
Ottawa	0.17	0.01	0.32	0.01
Toronto	1.73	0.06	0.69	0.11
Thunder Bay	0.05	1.18	0.05	0.77
Winnipeg	3.61	3.93*	1.74	3.48
Regina	0.66	0.51	0.69	0.66
Saskatoon	1.86	0.59	1.72	0.00
Calgary	1.54	0.18	1.00	0.15
Edmonton	1.97	2.75	1.05	0.93
Vancouver	10.41*	0.37	0.75	0.05
Victoria	0.00	0.07	0.03	0.03

* - significant at 5% level of significance.

For the values in presented in Table 3, the X^2_{α} value, with a significance level of $\alpha = 0.05$, is 3.84. Therefore, any value which is larger in magnitude than 3.84 indicates that the difference between the pertinent levels of factor I is statistically significant. For both $R_{(1)}$ and $R_{(2)}$, there were five occurrences of significant X^2 values in the 16 cities. For each type of response rate, there were six significant differences between the control and publication groups, and four significant differences between the control and clipboard groups. Further examination of tables 1 and 3 gives the following summary of incentive effects within each city in Table 4.

Table 4
Summary of Incentive Effects by City

Effect	Publication		Clipboard	
	R_1	R_2	R_1	R_2
Adverse	Halifax Vancouver	Halifax	-	-
Beneficial	Montreal	Charlottetown/ Summerside Montreal	Montreal Winnipeg	Saint John Quebec City Montreal

The impact on response rates due to incentives in other cities was statistically insignificant. Note that no adverse effect has been found for the incentive "clipboard".

5.2. Evaluating the Effects of the Incentives - Using the Sign Test

Instead of examining the frequency distribution of the increases/decreases in the response rates, we have postulated a probability model about the increases, which was described in section 3.2.

For each type of incentive and response rate, we test the hypothesis $p = \frac{1}{2}$ against the alternative $p \neq \frac{1}{2}$. This will be done for each of four cases, i.e., two incentives by two response rates. Note that, from Table 5, we have a total number of increases and decreases, say T, of 15 for $R_{(2)}$, 14 for $R_{(1)}$ under

P, and 16 for $R_{(1)}$ under X. However, for simplicity, we ignore the case when there is no change. Using $p = 1/2$, we compute the rejection region for certain α , with α being the level of significance. The rejection region, along with α , is also given in Table 5.

Table 5
Probabilities for the Rejection Region when testing $p = 1/2$ vs. $p \neq 1/2$

	Response Rate 1		Response Rates 2	
	Level 2 (P)	Level 3 (X)	Level 2 (P)	Level 3 (X)
T	14	16	15	15
C	≤ 4 or ≥ 10	≤ 4 or ≥ 12	≤ 4 or ≥ 11	≤ 4 or ≥ 11
α	0.180	0.077	0.119	0.119

Note: T = total number of increases and decreases;

C = number of increases necessary for statistical significance¹;

$\alpha = \text{Prob}\{C \leq 4 \text{ or } C \geq 10 \mid p = 1/2\}$, where α is found using binomial probabilities, with $p = 1/2$ and $N = T$. This approach of computing α avoids the problem of randomization at the boundary point.

We take the desired level of significance $\alpha = 0.05$. Comparing the number of increases (see Table 2) with the corresponding entry (response rate and incentive) in Table 5, we can say that the data fails to contradict the hypothesis that $p = 1/2$. In other words, the effect of each incentive (P or X) is insignificant.

¹ The actual number of increases are given in Table 2.

6. ANALYSIS BASED ON THE LOGIT MODEL

We describe the analysis using the models M1 and M2 mentioned in section 3. The mathematical details of the theory underlying the analysis are given in Appendix III. The statistical tests for the effects of the various factors, and the validity of the models, are based on Wald statistics and the appropriate chi-square values. The SAS procedure CATMOD was used to carry out the necessary computations and analysis (see [4]). As mentioned earlier in section 2, we look at two cases, i.e., data excluding the B.C. region, and data including the B.C. region.

6.1. All Regions except B.C.

The model of interest, M1 (described in section 3), is the one that includes the constant term, the main effects of the three factors, and an interaction term. This model is used with each of the two types of response rates. The analysis is used to answer the questions outlined previously concerning the effects of (i) the incentives, (ii) the interviewers' experience, and (iii) the regions, on the response rates. The ANOVA table produced by the SAS procedure CATMOD is as follows:

Table 6
Analysis of Variance for the Logit Model M1

Source	DF	Response Rate 1		Response Rate 2	
		Chi-Square	Prob	Chi-Square	Prob
Intercept	1	662.20	0.0001	778.62	0.0001
Incentive	2	2.88	0.2365	2.80	0.2468
Experience	2	1.47	0.4797	1.01	0.6023
Region	3	55.07	0.0001	40.54	0.0001
Incent*Exp	4	3.82	0.4303	2.62	0.6241
Residual	20	30.51	0.0620	29.65	0.0758

When we look at the chi-square value, or at the p-value, for the residuals (for both of the response rates) in Table 6, we can draw a conclusion about the goodness of fit of the model M1. Using 20 degrees of freedom (DF), the upper $\alpha = 0.05$ region of the χ^2 distribution is 31.41, i.e., $\chi^2_{.05,20} = 31.41$. Since the chi-square values for the residuals for the two response rates are less than 31.41, we may say that the model is tenable for both of the response rates. Alternatively, the same conclusion is reached by examining the corresponding p-values, since each p-value is greater than 0.05 in each case. We make the following observations about the effects of the three factors:

- (1) Incentives: The corresponding p-values are greater than 0.05 for the two response rates. We conclude that the effect of the incentives on the response rates is statistically insignificant, i.e., the data conforms to the view that the incentives have no effect upon the response rates.
- (2) Interviewer Experience: The conclusions about this factor are the same as the conclusions for the incentives.
- (3) Area/Region: Since the p-values are less than 0.05 for both of the response rates, we may conclude that the regions do have an effect on the response rates, i.e., there are regional differences in the response rates. For both $R_{(1)}$ and $R_{(2)}$, the region showing the highest response rate was Quebec, while the region with the lowest response rate was Ontario.
- (4) Interaction (Incentive*Interviewer Experience): Because the p-values were greater than 0.05 for the two response rates, we conclude that the interactions are not significant in the model.

Summarizing the above analysis, we may say that the effects due to the incentive and interviewer experience factors were insignificant for both of the response rates, whereas the regional differences were found to be statistically significant.

It should be pointed out that the model was fitted again, this time using only the three main factors, since the interactions (Incentive*Interviewer Experience) were not significant. Conclusions drawn from this model were the same as those drawn from the previous version of M1 above, i.e., region was

the only factor which had a significant effect on the response rates. Because of this, the details of the additional work are not included here.

6.2. All Regions, including B.C.

The model of interest in this case is M2, which includes the constant term and the main effects of the two factors, incentive and region. As before, this model is used with each of the two types of response rates. Here, the analysis is used to answer the questions concerning the effects of (i) the incentives, and (ii) the regions, on the response rates. The results of the ANOVA are as follows:

Table 7
Analysis of Variance for the Logit Model M2

Source	DF	Response Rate 1		Response Rate 2	
		Chi-Square	Prob	Chi-Square	Prob
Intercept	1	1156.72	0.0001	1447.59	0.0001
Incentive	2	9.04	0.0109	6.76	0.0341
Region	4	135.08	0.0001	88.12	0.0001
Residual	8	19.94	0.0106	12.63	0.1252

When we examine the above table, we look at the chi-square value, or at the p-value, for the residuals for the two response rates. The upper 0.05 region of the χ^2 distribution, for 8 degrees of freedom (DF), is 15.51, i.e., $\chi^2_{.05,8} = 15.51$. For response rate 2, the χ^2 value for the residual is 12.63, which is less than 15.51. For response rate 1, the χ^2 value is 19.94, which is greater than 15.51. This shows that the model is tenable for response rate 2, but not for response rate 1. Additional analysis for M2 under $R_{(1)}$ will be undertaken in section 6.3.

The following analysis pertains to $R_{(2)}$ only.

(1) Incentives: The corresponding p-value is less than 0.05 for $R_{(2)}$; therefore, we may conclude that the effect of the incentives upon $R_{(2)}$ is statistically significant, i.e., we reject the view that the incentives have no effect upon the response rates. The clipboard was the incentive that resulted in the highest response estimates of 78.8%, while the control and publication groups had nearly identical response estimates of 76.0% and 75.9%, respectively.

(2) Region: The p-value is less than 0.05 for $R_{(2)}$, therefore we may conclude that the regions do have an effect upon response rate 2, i.e., there are regional differences for $R_{(2)}$. In fact, the region with the highest response estimate was again Quebec, at 83.5%, while the lowest was B.C. at 69.0%.

To summarize the above analysis, we may state that the effects due to the incentive factor and the region factor are significant for $R_{(2)}$.

6.3. Analysis of $R_{(1)}$ (All Regions)

As mentioned in section 6.2, the model M2 was found to be untenable for $R_{(1)}$. We will look at a new model which includes not only the main effects of the incentives and regions, but also includes all of the interaction terms. However, we are unable to test whether this model is tenable or not, since the number of parameters (main effects and interactions) here will not leave any degrees of freedom to estimate the residuals. The estimates of the main effects, with other appropriate quantities, are presented in Table 8 on the following page. However, the estimates and related quantities of the interaction terms are not presented in order to keep the table manageable in size. We note that, at the 5% level of significance, the publication and clipboard effects are statistically significant. This change in conclusion from that described in section 6.1 can be attributed to the effect of the observations from B.C. All of the region effects are also statistically significant at the $\alpha = 0.05$ level. In fact, similar to the results observed for $R_{(2)}$, the region with the highest response rate under $R_{(1)}$ was Quebec, while the lowest was B.C. The conclusions about the interactions will not be described here.

Table 8
Parameters for $R_{(1)}$ - Including B.C.

Analysis of Individual Parameters					
Factor	Effect	Estimates	s.e.	χ^2	Prob
REGION MAIN EFFECTS	Atlantic	0.246	0.0577	18.22	0.0001
	Quebec	0.465	0.0739	39.64	0.0001
	Ontario	-0.329	0.0523	39.59	0.0001
	Prairies	0.137	0.0502	7.50	0.0062
	B.C.	-0.520	0.0604	74.09	0.0001
INCENTIVE MAIN EFFECTS	Control	-0.046	0.0385	1.45	0.2293
	Publication	-0.104	0.0438	5.62	0.0178
	Clipboard	0.150	0.0437	11.74	0.0006
INTERACTIONS	NOT PRESENTED HERE				

6.4. Examination of Differences in Incentive Effects

We now conduct pair-wise comparisons on the incentives since it was shown above that the effects of the incentives are significant for response rates 1 and 2 under M2. We do this to explore the differences between the incentives. In this case, the pair-wise contrasts are based on these treatment effects:

1. I_1 = Effect of Control
2. I_2 = Effect of Publication
3. I_3 = Effect of Clipboard.

Table 9 on the following page shows the results of the pair-wise comparisons.

Table 9

Analysis of Variance for the Contrasts

Contrast	DF	Response Rate 1		Response Rate 2	
		Chi-Square	Prob	Chi-Square	Prob
Control - Publication ($\hat{I}_1 - \hat{I}_2$)	1	0.68	0.4096	0.02	0.8908
Control - Clipboard ($\hat{I}_1 - \hat{I}_3$)	1	5.39	0.0202	5.96	0.0147
Publication - Clipboard ($\hat{I}_2 - \hat{I}_3$)	1	8.29	0.0040	4.40	0.0359

Because the χ^2 values for the pair-wise comparisons "Control - Clipboard" ($\hat{I}_1 - \hat{I}_3$) and "Publication - Clipboard" ($\hat{I}_2 - \hat{I}_3$) are greater than the test value of $\chi^2_{.05,1} = 3.84$ for the two response rates, we conclude that the clipboard incentive is significantly different from both the publication incentive and the control for response rates 1 and 2. However, there is no significant difference between the control and the publication incentive for either of the response rates.

Under both $R_{(1)}$ and $R_{(2)}$, it was observed that the clipboard group had the highest response estimates. The response estimates for the control and publication groups were nearly identical in both cases, and were about three percentage points lower than the response estimates for the clipboard group.

7. CONCLUSIONS

When the data was analyzed for the effects of the incentives in each city, it was found that the incentives had a significant effect upon the response rates in some of the cities, while no statistically significant effects were observed in the other cities. Significant differences between the control and incentive groups existed in only six of the cities, which were: Charlottetown/Summerside, Halifax, Saint John, Montreal, Winnipeg, and Vancouver. Within these cities, the clipboard proved to have a beneficial

effect upon the response rates for all six cities, while the publication had a beneficial effect on the response rates in three of the cities and it had an adverse effect upon response rates in the other three cities. This conclusion is of interest for those household surveys which may be conducted in certain cities rather than nation-wide. However, these conclusions (significant effect of incentives in some cities) may not be valid if we are interested in a national policy (i.e., a policy which is the same for all regions) concerning the distribution of incentives.

The preliminary analysis at the national level consisted of an examination, by each incentive, of the frequency of increases and decreases in response rates in the cities. Note that the interviewer experience is not part of this analysis; only the incentives are studied here. It was observed empirically that the clipboard group had a slightly higher ratio of increases to decreases (relative to the control group) than did the publication. However, the result of the sign test (using a binomial probability model) showed that the effect of each incentive upon the two response rates was not statistically significant at the national level.

When the raw data (for use under the logit model M1) was examined (only four regions were studied, since B.C. was excluded), it was observed that (i) the response rates for the interviewer experience was slightly higher for interviewers with LFS experience, as compared to interviewers with other or no experience; (ii) for the incentives, the response rates were slightly higher for the clipboard than for the control and publication groups; and (iii) the response rates were highest in Quebec, followed by the Atlantic and Prairie regions, while they were lowest in Ontario. However, when the data was analyzed using the logit model it was found that both the interviewer experience and the incentives did not have a statistically significant effect upon the response estimates for FAMEX 1990. Still, the region effect was found to be significant, i.e., the response estimates were not the same for all regions.

When data for the B.C. region was included in the logit model analysis (under M2), the results changed somewhat. Note that the results of the two models (with and without B.C.) are not strictly comparable, since in M1 there are three factors under study, while in M2 only two factors are examined.

Nothing could be concluded about the effect of interviewer experience using the model M2, as the interviewer experience data was unavailable for the B.C. region. However, it was found that the incentives had a significant effect upon the response estimates for both response rates 1 and 2. This change in conclusion from that which was observed under M1 can be attributed to the fact that the observed response was extremely low in Vancouver and Victoria. However, this indicates that the significance of the effects of the incentives, detected under model M2, is really a regional effect, as it is caused by the inclusion of unusual observations from the B.C. region. Pair-wise comparisons between incentive effects showed, for both response rates, that it was the clipboard which was significantly different from both the control and the publication groups, while no significant differences were observed between the publication and the control groups. As under M2, there were differences between the regions for both of the response rates. However, no additional analysis of regional differences was done in this study since a policy on incentives for a national survey must be national in character and not based on regional differences. In other words, it is unlikely that incentives would be given in one region and not in another.

In summary, we conclude that the two incentives under study had no significant effect nationally upon the 1990 FAMEX response rates. However, in some cities, the incentives did have a significant effect on the response rates. It should be pointed out that this result refers to these two incentives only, and it is not conclusive for incentives in general.

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APPENDIX I

Table I
Counts by Region, Incentive and Interviewer Experience

Region	Incentive	Interviewer Experience	Response	Non-response	
				NR1	NR2
ATLANTIC St. John's Summerside/ Charlottetown Halifax Saint John	Control	LFS	120	37	36
		Other	101	25	25
		None	238	72	59
	Publication	LFS	47	15	14
		Other	75	23	20
		None	202	76	65
	Clipboard	LFS	103	26	23
		Other	71	7	7
		None	100	33	27
QUEBEC Quebec City Montreal	Control	LFS	157	44	40
		Other	36	13	12
		None	214	67	52
	Publication	LFS	90	29	18
		None	76	11	7
	Clipboard	LFS	130	19	13
None	129	24	22		
ONTARIO Ottawa Toronto Thunder Bay	Control	LFS	51	26	21
		Other	15	10	7
		None	320	161	126
	Publication	LFS	23	6	6
		Other	269	131	108
	Clipboard	Other	67	43	35
None	219	112	85		
PRAIRIES Winnipeg Regina Saskatoon Calgary Edmonton	Control	LFS	202	57	42
		Other	28	9	5
		None	236	72	66
	Publication	LFS	90	34	66
		Other	38	8	29
		None	188	66	6
	Clipboard	LFS	68	16	15
		Other	17	1	1
		None	229	70	55
B.C. Vancouver Victoria	Control		261	139	114
	Publication		141	121	71
	Clipboard		195	100	83
TOTALS			4,546	1,703	1,381

Note 1: In the text, Control = 'O', Publication = 'P', and Clipboard = 'X'.

Note 2: NR1 refers to the non-response observed under $R_{(1)}$, while NR2 refers to the non-response under $R_{(2)}$.

APPENDIX II

Two different definitions of response rates are used for this study. The reason for the use of two types of response rates was to determine whether the use of incentives had an impact on response by those people who were actually contacted by an interviewer. Response rate 1 is the overall response rate, while response rate 2 is based on the households for which actual contact was made by the interviewer.

According to the definition of a household as used by the Labour Force and FAMEX surveys, each dwelling contains only one household. Some of these dwellings may not be eligible for inclusion in the sample, such as those dwellings which are vacant or under construction, and thus do not contain a household. Household eligibility codes, described below, are assigned to each (eligible or ineligible) FAMEX questionnaire. These codes are used when defining the response rates. Each questionnaire from a sampled household is given a household eligibility code on the FAMEX database. The household eligibility codes are defined as follows:

U = usable full-year data

P = usable part-year data

* U and P constitute *usable* (or respondent) questionnaires and thus are responses.

M = refusal - no Household Composition data

R = refusal - with Household Composition data

* M and R are designated as *official refusals* and are non-responses.

F = incomplete expenditure full-year data

G = incomplete income full-year data

H = questionnaire temporarily out of balance

K = interview prevented

O = questionnaire out of balance

Q = incomplete expenditure part-year data

W = incomplete income part-year data

* F,G,H,K,O,Q,W are known as *partial refusals* and may be considered non-responses.

A = interview cancelled
 L = interview prevented by weather
 N = not at home
 T = absent

* A,L,N,T are considered non-responses. Also, N and T are known as *no contacts*.

B = not to be interviewed
 C = dwelling under construction
 D = demolished dwelling
 E = member of another household elsewhere at Dec. 31, 1990
 I = immigrant or infant with Household Composition data
 V = vacant dwelling

* B,C,D,E,I,V constitute data that is ineligible for inclusion in the sample. They are not to be used in any of the definitions of response rate.

Note that the term "eligible questionnaires" includes questionnaires that are labelled as usable, official refusal, partial refusal, no contacts and cancellations, and excludes ineligible questionnaires (see the corresponding codes above).

The two definitions of the response rates, using the household eligibility codes, are as follows:

$$\begin{aligned} \text{Response Rate 1 (R}_{(1)}\text{)} &= \text{Usable questionnaires} / \text{Eligible questionnaires} \\ &= (U + P) / (U + P + (M, R, F, G, H, K, O, Q, W, A, L, N, T)) \end{aligned}$$

$$\begin{aligned} \text{Response Rate 2 (R}_{(2)}\text{)} &= \text{Usable questionnaires} / (\text{Usable questionnaires} + \text{questionnaires coded as} \\ &\quad \text{official and non-official refusals}) \\ &= (U + P) / (U + P + M + R + F + G + H + K + O + Q + W) \end{aligned}$$

Note that the concept for $R_{(2)}$ is a conditional one. It is based on those households for which contact was made, i.e., it excludes those households for which no contact was made or a cancellation of the interview occurred.

The numerator is the same in each of the two definitions of the response rates, as it is composed of usable questionnaires, which always include codes U and P. However, the definition of the denominator differs for the two response rates, and this is what causes the response rates themselves to differ. Response rate 1 uses all household eligibility codes to calculate non-response, except for those designated as ineligible. Response rate 2 uses both official and partial refusals as non-responses, but excludes the no contacts and cancellations. Therefore, Response Rates 1 and 2 use the same response codes in their denominators, except that $R_{(1)}$ uses the codes A,L,N,T, while $R_{(2)}$ does not.

APPENDIX III

We give the mathematical details of the tests used in sections 4 and 5.

IIIA. Test for the Frequency of Increases/Decreases

Let N = number of cities,

T = number of increases and decreases in those cities

$C_i = \begin{cases} 1 & \text{if there is an increase (relative to the control group) in the } i^{\text{th}} \text{ city response rate,} \\ 0, & \text{otherwise.} \end{cases}$

$C = \sum_{i=1}^N C_i$ = total number of increases,

p = Prob{the incentive will result in an increase in the response rate}

Note that $C \leq T$.

The rejection region, corresponding to the significance level α , for testing $p = 1/2$ vs. $p \neq 1/2$ is of the form $C \leq C_0$ or $C \geq T - C_0$, where α , shown in Table 5, is given by

$$\alpha = \text{Prob} \{ X \leq c_0 \text{ or } X \geq T - c_0 \}$$

and

$$\alpha = \sum_{x=c_0}^T \binom{T}{x} \left(\frac{1}{2}\right)^{T-1}$$

By comparing the desired level of significance with the tabulated α , we can draw the relevant conclusion.

IIIB. Logit Model Analysis based on Wald Statistics

We give the outline of the methodology used for the analysis under M2 as it uses only double subscripts and is less cumbersome. If the reader is interested in the mathematical proofs and other details, refer to Freeman [3].

Let n_{ik} be the number of observations (responses and non-responses) in each (i,k) cell, and let r_{ik} be the number of responses. Then

$$\begin{aligned} p_{ik} &= r_{ik} / n_{ik} \\ &= \text{sample proportion of responses.} \end{aligned}$$

$$\begin{aligned} v(p_{ik}) &= p_{ik}(1 - p_{ik}) / n_{ik} = v_{ik} \\ &= \text{variance of } p_{ik} \end{aligned}$$

If we denote p_{ik} as the column vector \mathbf{p} , then the covariance of \mathbf{p} , $V(\mathbf{p})$, is given by a diagonal matrix whose diagonal elements are v_{ik} .

The model $\ln[\pi_{ik} / (1 - \pi_{ik})]$ can be written in the matrix form as $F(\pi)$ where π is similar to \mathbf{p} . The variance of $F(\mathbf{p})$ is given by

$$\mathbf{V} = \mathbf{F}^* \mathbf{V}(\mathbf{p}) \mathbf{F}^{*'}$$

where \mathbf{F}^* is the matrix of $[\partial F(\mathbf{p}) / \partial \mathbf{p}]$.

The model concerning the main effects and interactions can be written as

$$F(\pi) = \mathbf{X}\beta$$

where β is the matrix of the unknown parameters, and \mathbf{X} is the known design matrix. The estimate of β is given by

$$\mathbf{b} = (\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1} (\mathbf{X}'\mathbf{V}^{-1}\mathbf{F}).$$

Note that $\hat{\pi} = \mathbf{X}'\mathbf{b}$.

The Residual Chi-Square (Wald Statistic) is used for testing the goodness of fit of the model and is given by $(\mathbf{F}'\mathbf{V}^{-1}\mathbf{F} - \hat{\mathbf{F}}'\mathbf{V}^{-1}\hat{\mathbf{F}})$ where $\hat{\mathbf{F}} = \mathbf{X}'\mathbf{b}$.

Here, the degrees of freedom = # cells - # parameters estimated in the model.

The chi-square (Wald Statistic) for testing $H_0: L\beta = 0$ is given by

$$Q = (Lb)' (L C^{-1} L')^{-1} (Lb), \text{ where } C^{-1} \text{ is the covariance matrix of } b.$$

Here, L is an appropriate matrix of constants, used for defining the contrasts (in β) of interest.

REFERENCES

1. Family Expenditures in Canada: Selected Cities, 1984. Statistics Canada (1986).
2. Fienberg, S.E. The Analysis of Cross-Classified Categorical Data, 2nd ed. Cambridge, Mass.: MIT Press (1980).
3. Freeman, D.H. Applied Categorical Analysis, New York: Marcel Dekker, Inc. (1987).
4. SAS User's Guide: Statistics Version 5 Edition. SAS Institute Inc. (1985).
5. Survey of Family Expenditures in 1990: Interviewer's Manual. Statistics Canada (1990).

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