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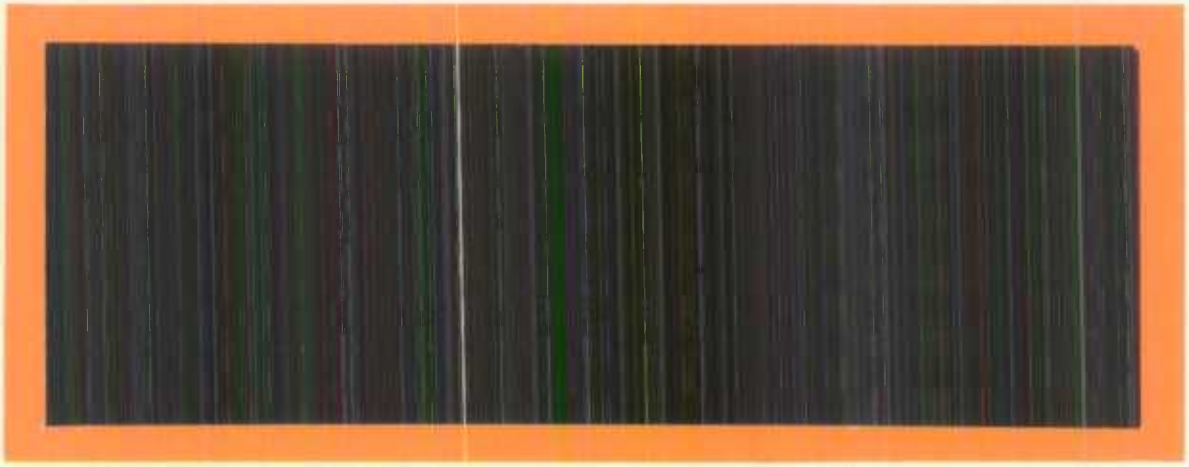

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SELECTING AN OPTIMAL SEASONAL ADJUSTMENT PROCEDURE FOR TEN RETAIL TRADE SERIES

by

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INTRODUCTION

Most of the Canadian Retail Trade Series exhibit a well defined seasonal pattern throughout the year. Unlike some other economic time series where the seasonal fluctuations are governed by climatic factors, the annual retail trade sales patterns basically reflect customer buying habits. For example, it is a well known fact that Department Store sales are highest before Christmas time, or that a large percentage of car purchases take place in the spring.

Another important characteristic of these series is the presence of trading day variations, namely that the monthly sales figures do not only depend on the month of the year but also on the number of Mondays, Tuesday etc that a particular month has. There is usually heavier trading in the second half of the week than in the first, consequently a month with five Fridays and Saturdays would show higher sales figures than the same month in another year where Mondays and Tuesdays outnumber Fridays and Saturdays. Of course, even here there is no general rule, and the trading day variation of some series could be quite different because of the data collection practices. A case in point is the North American passenger car sales series, that gives the number of units sold as recorded by the manufacturer. Here the weekly pattern shows higher number of transactions in the middle of the week, rather than heavy trading at the end of the week as one would expect it based on consumer buying habits.

Because of the presence of pronounced yearly and weekly patterns in the retail trade series, it is very important that both types of fluctuations are removed before undertaking any policy analysis based on the data. The

question is what kind of seasonal adjustment will produce most reliable estimates. This paper sets out to answer that question. In Section 2 the alternative procedures are presented. Section 3 describes the methodology applied to choose among the alternatives. Section 4 contains the result of the analysis in the retail trade series. Finally, Section 5 concludes the study.

2. A SURVEY OF COMMONLY USED SEASONAL ADJUSTMENT TECHNIQUES

The majority of the seasonal adjustment methods applied by government statistical agencies are based on linear smoothing filters, usually known as moving averages. It is inherent to these methods that the estimates of the observations of the most recent years are less accurate than those corresponding to central data because of asymmetry of the end point filters. Among these methods the Method II-X-11 variant developed by Shiskin , Young and Musgrave (1967) and the X-11-ARIMA developed by Dagum (1975 and 1980) are the most widely applied. The seasonal adjustment filters of X-11-ARIMA and X-11 differ for the data of the most recent year, but they agree for central observations positioned at least three and a half years away from the end point. The seasonally adjusted estimates of most recent years keep being revised as more data is added to the series until the data point in question is three and a half years from the end and the estimate reaches its final value. The revisions of current seasonally adjusted values are due to: (1) Differences in the smoothing linear filters applied to the same observations as later data become available; and (2) the innovations that enter into the series with new observations. One would like to see the revisions of the first kind reduced to a minimum.

Theoretical studies by Dagum (1982.a and 1982.b) have shown that the revisions of current seasonally adjusted values due to filter changes can be reduced significantly if: (1) the original series is extended with ARIMA extrapolated values; and (2) concurrent seasonal factors are used

instead of year-ahead seasonal factors. The latter are obtained from the seasonal adjustment of a series that ended one year before. The former are generated by seasonally adjusting, each month, all the data available up to and including that month. The conclusions drawn from the theoretical studies conform to the results given in empirical works (see e.g. Dagum, 1978, Dagum and Morry, 1982; Kuiper, 1978 and 1981, Pierce, 1980, Kenny and Durvin 1982; McKenzie, 1982). The purpose of this study is to examine which of the four available options of X-11-ARIMA (concurrent seasonal factors with or without ARIMA extrapolations or forecast seasonal factors with or without ARIMA extrapolation) is preferable for the current seasonal adjustment of retail trade series. Although theoretical results strictly favour one method it is always important to conduct empirical studies to find out if theory applies to the series in question. It is especially important when the seasonal behaviour of a series deviates significantly from the prototype that the theory was based on, as is the case with the series "Jewellery Store Sales". Here the seasonal movement is characterized by basically one very large peak in December, a highly uncommon pattern.

As was mentioned earlier these series display significant fluctuations corresponding to the number and type of trading days present in each month. The removal of this variation introduces four further options per the above mentioned procedures, yielding altogether 16 alternative adjustments. The four trading day options are the following:

(1) Do not remove trading day variation

(2) Estimate trading day variation from the data and remove if significant (X-11-ARIMA trading day option '2')

(3) Use prior daily weights and modify then if necessary (X-11-ARIMA trading day option '2')

(4) Use fixed prior daily weights.

These 16 seasonal adjustment options were applied to ten retail trade series selected by subject matter experts as either typical or as very important series. A list of them is given in Appendix A.

3. DEVELOPMENT OF A CRITERION FOR ASSESSING SEASONAL ADJUSTMENT QUALITY AND DESIGN OF THE EXPERIMENT BASED ON THE CRITERION.

Both the X-11 and X-11-ARIMA seasonal adjustment methods have been known to give reliable historical estimates.
These values are often referred to as final estimates, because they are not subject to revisions as further data is added to the series. The reliability of current seasonal adjustment estimates can be measured by the size of revision the estimate undergoes from the time it is published for the first time to the time when it becomes final three and a half years later. Thus the difference between the concurrent (or projected) seasonal factor and the corresponding final estimate (the final revision) will serve as a measure of reliability. The method giving minimal revisions will be considered optimal.

The data involved in this study ranged from 1971 to 1982 for eight out of the ten series. The remaining two series 2368 and 2370 were much longer starting in 1960. Given the length of the first eight series there were two years 1978 and 1979 for which final revisions of projected factors were available and three years: 1977, 1978 and 1979 for which final revisions of concurrent factors were obtained. In the case of the new Motor Vehicle Sales series the years involved ranged from 1967 to 1979 and from 1966 to 1979 respectively.

These revisions were summarized in two statistics; the average revision

$$\bar{R} = \underset{ij}{\Sigma\Sigma} \left| \hat{S}^{c}_{ij} - \hat{S}^{f}_{ij} \right| / N$$

where $\hat{S}_{ij}^{c}$ denotes the current seasonal factor (either concurrent or projected) in year i and month j

and $\hat{S}_{ij}^{f}$ denotes the corresponding final seasonal factor estimate in year i and month j and the maximum revision: $R^{max} = \max_{ij} (\hat{S}_{ij}^{c} - \hat{S}_{ij}^{f})$

In order to calculate these statistics it was necessary to process each of the eight shorter series through X-11-ARIMA four times, one ending in 1977, one in 1978, one in 1979 and one in 1982 for each of the 16 seasonal adjustment procedures. The two longer series involved ten further runs per each of the 16 options.

4. THE ANALYSIS OF REVISIONS IN THE RETAIL TRADE SERIES

The revision statistics described in Section 3 were recorded in four tables (tables 1 to 9) corresponding to the four possible trading day variation options. Apart from the average and maximum revision values for the four alternative procedures, the tables also contain the arithmetic and percentage revision difference between ARIMA and NO-ARIMA for both the concurrent and the projected factors.

Since the original purpose of this analysis was to find the most suitable adjustment technique for all retail trade series we needed an additional table to facilitate us in selecting the best method. Table 5 summarizes the results of the previous four tables by presenting the averages of the ten series from all 16 seasonal adjustment procedures. From Table 5 it is evident that the procedure giving minimal revisions both in terms of average and the maximum value is the one using concurrent seasonal factors and ARIMA extrapolation combined with fixed prior daily weight applied to the ten series are given in Appendix B.

The worst results are obtained with no trading day treatment (Table 1) NO-ARIMA and projected seasonal factors revision sizes when using these options are almost doubled as compared to the optimal method. While ARIMA extrapolation alone reduces average revision sizes by 10% from 1.44 to 1.30 the gain introduced by switching from forecast to concurrent factors is especially important and is in the neighbourhood of 30% for all the methods as indicated by the last two columns of Table 5. We can now

switch back to Table 4 to examine the revisions originating from the proposed methodology in greater detail.

Ignoring absolute differences of less than .05 (in which case the two methods ARIMA and NO-ARIMA are considered equally good) it can be seen from column 9 that in five out of ten series ARIMA concurrent outperformed NO-ARIMA concurrent in terms of minimal revisions while being equally acceptable for the rest of the series. In terms of percentages the reductions introduced with the use of ARIMA extrapolation range from 4% to 26%. The maximum revision per series is also diminished significantly with this option for 6 out of ten series.

TABLE 5. FINAL REVISION OF SEASONAL FACTORS AVERAGED OVER TEN RETAIL TRADE SERIES OBTAINED FROM 16 ALTERNATIVE SEASONAL ADJUSTMENTS

X-11 ARIMA Procedures / Treatment of trading day variation	NO ARIMA				ARIMA				Forecast÷Concurrent	
	concurrent		forecast		concurrent		forecast		NO ARIMA	ARIMA
	$\Sigma\bar{R}_{i/10}$	$\Sigma R^{max}_{i/10}$	$\Sigma R_{i/10}$	$\Sigma R^{max}_{i/10}$	$\Sigma R_{i/10}$	$\Sigma R^{max}_{i/10}$	$\Sigma\bar{R}_{i/10}$	$\Sigma R^{max}_{/10}$	$\frac{\Sigma\bar{R}^p}{\Sigma\bar{R}^c}$	$\frac{\Sigma\bar{R}^p}{\Sigma\bar{R}^c}$
1. No trading day (X-11-ARIMA trading day option = 'Y')	1.70	3.35	2.16	4.80	1.62	3.02	2.13	4.97	1.27	1.31
2. Trading day coefficients estimated during the X-11 run (X-11-ARIMA trading day option = '3')	1.43	3.02	1.90	4.18	1.42	2.94	1.87	4.51	1.33	1.31
3. Prior daily weights revised in the X-11 run (7 prior daily weights T.D. option = '2')	1.41	3.14	1.93	4.43	1.48	2.89	1.93	4.42	1.36	1.30
4. Prior daily weights kept fixed in the X-11 run (7 prior daily weights T.D. option = 'Y')	1.44	2.96	1.92	4.48	1.30	2.48	1.73	4.24	1.33	1.33

TABLE 4. FINAL REVISION OF SEASONAL FACTORS IN TEN RETAIL TRADE SERIES USING FIXED PRIOR DAILY WEIGHTS WITH FOUR ALTERNATIVE SEASONAL ADJUSTMENT PROCEDURES

Method / IDENT	NO ARIMA				ARIMA				ARIMA VS NO ARIMA			
	Concurrent		Forecast		Concurrent		Forecast		Concurrent		Forecast	
	$\bar{R}$	Rmax	$\bar{R}$	Rmax	$\bar{R}$	Rmax	$\bar{R}$	Rmax	Difference	% diff.	Difference	% diff.
2369	2.93	6.28	4.03	7.66	2.75	5.22	3.81	6.76	-.18	-6.02	-.21	-5.50
2370	2.37	3.48	3.12	4.83	2.28	3.30	2.91	4.27	-.09	-3.69	-.21	-6.78
650058	.34	.68	.47	.90	.37	.56	.47	.75	+.04	+8.62	0	0
650059	.40	.86	.49	1.10	.38	.86	.50	1.23	-.02	-4.81	+.01	2.38
650062	1.07	2.49	1.49	4.95	.80	1.52	1.18	5.02	-.27	-25.55	-.31	-20.50
650066	1.52	2.16	2.15	4.55	1.29	2.03	1.64	5.43	-.23	-14.90	-.51	-23.59
650067	2.91	8.11	3.81	13.00	2.26	5.68	3.25	10.90	-.65	-22.44	-.56	-14.51
650068	.59	1.01	.72	1.10	.64	1.42	.59	1.05	+.05	+8.4	-.13	-17.56
650077	1.39	2.43	1.81	3.74	1.34	2.19	1.90	4.49	-.05	-4.01	+.09	4.92
650083	.89	2.08	1.18	2.97	.92	2.00	1.11	2.49	+.03	3.75	-.08	-5.91
Average	1.44	2.96	1.92	4.48	1.30	2.48	1.75	4.24	-.14	-10.0	-.19	-11.1

TABLE 1. FINAL REVISION OF SEASONAL FACTORS IN TEN RETAIL TRADE SERIES USING NO TRADING DAY ADJUSTMENT WITH FOUR ALTERNATIVE SEASONAL ADJUSTMENT PROCEDURES

Method / IDENT	NO ARIMA				ARIMA				ARIMA VS NO ARIMA			
	Concurrent		Forecast		Concurrent		Forecast		Concurrent		Forecast	
	$\bar{R}$	Rmax	$\bar{R}$	Rmax	$\bar{R}$	Rmax	$\bar{R}$	Rmax	Difference	% diff.	Difference	% diff.
2369	2.99	4.98	4.00	6.16	2.93	5.20	3.94	6.82	-.06	-2.20	-.06	-1.42
2370	2.47	3.98	3.23	5.73	2.50	3.95	3.15	5.23	+.03	1.45	-.08	-2.43
650058	.83	2.14	1.01	2.72	.89	1.37	1.07	2.75	+.06	6.83	+.06	6.11
650059	2.06	3.29	2.10	4.07	1.91	3.19	2.61	4.47	-.15	-7.63	+.51	24.23
650062	1.23	3.21	1.61	5.44	.92	1.88	1.19	3.18	-.31	-25.59	-.42	-25.91
650066	1.77	4.44	2.49	6.38	1.61	5.34	2.14	8.70	-.16	-8.59	-.35	-14.01
650067	2.56	5.34	3.36	7.70	2.20	5.16	3.09	7.70	-.36	-14.04	-.27	-8.09
650068	.67	1.50	.77	1.99	.65	1.36	.58	10.20	-.02	-2.99	-.19	-24.95
650077	1.33	2.74	1.61	4.85	1.33	2.90	1.67	4.64	0	4	+.06	+.25
650083	1.12	1.93	1.44	2.95	1.29	1.86	1.89	5.21	+.07	15.71	-.45	31.08
Average	1.70	3.35	2.16	4.80	1.62	3.02	2.13	4.97	-.08	5.0	-.03	1.5

TABLE 2. FINAL REVISION OF SEASONAL FACTORS IN TEN RETAIL TRADE SERIES USING TRADING DAY OPTION '3' WITH FOUR ALTERNATIVE SEASONAL ADJUSTMENT PROCEDURES

Method / IDENT	NO ARIMA				ARIMA				ARIMA VS NO ARIMA			
	Concurrent		Forecast		Concurrent		Forecast		Concurrent		Forecast	
	$\bar{R}$	Rmax	$\bar{R}$	Rmax	$\bar{R}$	Rmax	$\bar{R}$	Rmax	Difference	% diff.	Difference	% diff.
2369	3.07	6.27	4.16	7.77	2.83	5.30	3.95	7.63	-.24	-7.31	-.21	-4.83
2370	2.20	3.82	3.06	5.10	2.38	3.42	2.99	4.68	+.18	+2.81	-.07	-2.50
650058	.39	.66	.44	.85	.51	.82	.60	1.32	+.12	43.54	+.16	36.62
650059	.62	1.17	.56	1.05	.75	1.79	1.07	1.98	+.13	61.44	+.51	89.38
650062	.90	3.13	1.35	4.97	.80	1.95	1.13	4.37	-.10	-9.42	-.22	-16.00
650066	1.64	3.95	2.31	5.26	1.50	4.55	2.10	8.26	-.14	-2.27	-.21	-8.89
650067	2.61	5.75	3.50	8.08	2.18	4.56	3.00	8.11	-.43	-16.39	-.50	-14.22
650068	.73	1.48	.84	1.75	.73	1.19	.69	1.24	0	-.57	-.15	-18.38
650077	1.43	2.95	1.79	5.09	1.45	3.02	1.82	4.87	+.02	1.75	+.03	1.81
650083	.66	1.04	1.04	1.93	1.11	2.77	1.38	2.67	+.45	38.92	+.34	32.40
Average	1.43	3.02	1.90	4.18	1.42	2.94	1.87	4.51	-.01	-.5	-.03	-1.5

TABLE 3. FINAL REVISION OF SEASONAL FACTORS IN TEN RETAIL TRADE SERIES USING REVISED PRIOR DAILY WEIGHTS WITH FOUR ALTERNATIVE SEASONAL ADJUSTMENT PROCEDURES.

Method / IDENT	NO ARIMA				ARIMA				ARIMA VS NO ARIMA			
	Concurrent		Forecast		Concurrent		Forecast		Concurrent		Forecast	
	$\bar{R}$	Rmax	$\bar{R}$	Rmax	$\bar{R}$	Rmax	$\bar{R}$	Rmax	Difference	% diff.	Difference	% diff.
2369	2.99	6.53	4.14	8.35	2.76	5.02	3.84	7.16	-.23	-8.65	+.30	-7.07
2370	2.28	3.88	3.06	4.63	2.39	3.78	2.96	4.17	+.11	1.40	-.10	-3.32
650058	.39	.73	.46	1.02	.72	1.23	.76	1.75	+.33	115.98	+.30	65.82
650059	.35	.56	.43	.88	1.27	1.73	1.41	2.73	+.92	299.08	+.98	230.35
650062	1.08	4.32	1.48	3.49	1.05	3.36	1.35	1.90	-.03	0	-.13	-8.65
650066	1.43	2.76	2.12	4.19	1.25	3.13	1.61	5.99	-.18	-7.95	-.51	-24.43
650067	2.98	7.23	4.07	12.33	2.40	5.72	3.53	11.05	-.58	-16.76	-.54	-13.24
650068	.61	.86	.79	1.45	.58	1.09	.61	1.50	-.03	1.27	-.18	-22.92
650077	1.46	3.09	1.85	5.73	1.37	2.23	2.00	5.89	-.09	6.06	+.15	7.86
650083	.57	1.47	.92	2.21	1.03	1.62	1.25	2.07	+.46	59.06	+.33	35.71
Average	1.41	3.14	1.93	4.43	1.48	2.89	1.93	4.42	+.07	+6.1	Ø	Ø

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PRIOR DAILY WEIGHTS AND ARIMA MODELS TO BE USED FOR THE SEASONAL ADJUSTMENT OF TEN RETAIL TRADE SERIES

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Model
2369	1.327	.901	1.725	1.180	.702	1.420	.100	(011)(011)1of
2370	.532	1.431	1.361	1.775	.399	.711	.100	(011)(011)1of
650058	.779	1.070	1.011	1.252	1.531	1.241	.100	(212)(011)*
650059	.432	.538	.703	1.428	2.002	1.841	.057	(011)(011)1of
650062	.799	1.089	.925	1.194	1.353	1.538	.101	(011)(011)1of
650066	1.166	1.203	1.415	1.245	1.368	.598	.100	(212)(011)
650067	1.565	.561	1.321	1.971	.420	1.152	.100	(212)(011)
650068	1.072	1.085	1.318	.975	1.353	1.120	.100	(011)(011)1of*
650077	.569	1.284	1.061	1.285	1.195	1.504	.100	(011)(011)1of
650083	1.120	1.070	1.151	1.236	1.226	1.120	.100	(011)(011)1of*

APPENDIX A

Ident.	Description
2369	N.M.V.S. Passenger Car Sales, North American Manufactured (units)
2370	N.M.V.S. Passenger Car Sales - overseas
650058	Retail Trade Canada: all stores
650059	Combination stores: all stores
650062	Department Stores: all stores
650066	Retail Trade Motor Vehicle Dealers: all stores
650067	Retail Trade: Used car dealers
650068	Retail Trade: Service Stations
650077	Retail Trade: Household Furniture stores
650083	Jewellery Stores

5. CONCLUSIONS

Sixteen different seasonal adjustment alternatives were applied to ten retail trade series with the objective of selecting an optimal procedure.

The seasonal adjustment that produced the smallest revisions on the average was the X-11-ARIMA method with ARIMA extrapolation using concurrent seasonal factors, and including fixed prior daily weights to remove trading day variation.

The reduction in revision size introduced by the ARIMA extrapolation was in the neighbourhood of 10% on average. More significant gains (30%) resulted from replacing projected seasonal factors by concurrent ones, i.e. including all available raw data in the seasonal adjustment of the series each month.

On the basis of the above evidence it is highly recommended that the Retail Trade Series be adjusted with concurrent seasonal factors. The use of ARIMA extrapolation is also beneficial for most of the series.