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REVISION OF TREND CYCLE ESTIMATORS OF MOVING AVERAGE SEASONAL ADJUSTMENT METHODS

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Estela Bee Dagum and Normand Laniel

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OF MOVING AVERAGE SEASONAL ADJUSTMENT METHODS

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Estela Bee Dagum
Statistics Canada

and

Normand Laniel
Statistics Canada

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ABSTRACT

One of the main purposes of the seasonal adjustment of economic time series is to provide information on current economic conditions, particularly to determine the stage of the cycle at which the economy stands. Since seasonal adjustment means removing seasonal variations thus leaving a seasonally adjusted series consisting of trend-cycle together with the irregular fluctuations, it is often very difficult to detect cyclical turning points for series strongly contaminated with irregulars. In such cases, it is preferable to smooth the seasonally adjusted series using trend-cycle filters which suppress as much as possible the irregulars without affecting the cyclical component.

It is inherent, however, in any moving average procedure that the first and last m points of an original series cannot be smoothed with the same symmetric filters applied to middle values. The current and most recent years of data are smoothed by asymmetric filters which change for each point in time. Hence as new information becomes available, revisions are made because of: (1) the new innovations entering into the series and; (2) the changes in the weight system or frequency response function of the filters.

The purpose of this study is to calculate the size of the revisions of the asymmetric trend-cycle filters of X-11 and X-11-ARIMA and to analyse the pattern of these revisions.

1. Introduction

One of the main purposes of the seasonal adjustment of economic time series is to provide information on current economic conditions, particularly to determine stage of the cycle at which the economy stands. Since seasonal adjustment means removing seasonal variations thus leaving a seasonally adjusted series consisting of trend-cycle together with irregular fluctuations, it is often difficult to detect cyclical turning points for series strongly affected with irregulars. In such cases, it is preferable to smooth the seasonally adjusted series using trend-cycle estimators which suppress as much as possible the irregulars without affecting the cyclical component.

The estimation of trend-cycle variations instead of seasonal adjustment has been recommended by many writers and recently by Moore et als (1981) and Kenny and Durbin (1982).

It is inherent, however, in any moving average procedure that the first and last $\underline{n}$ points of an original series cannot be smoothed with the same symmetric filters applied to central values. The current and most recent years of data are smoothed by asymmetric moving averages or filters which in fact change for each point in time. Hence as new information becomes available, revisions are made because of: (1) the new innovations entering into the series and (2) the changes in the weights or frequency response function of each asymmetric filter. In such situations, one would like to minimize the revisions of the trend-cycle estimates due to filter changes. Since the estimation of the trend-cycle is made concurrently, i.e. using all the data up to and including the most recent value, knowledge of the time path and speed of convergence of the concurrent trend-cycle filter to the central (symmetric) filter gives valuable information on how often the concurrent estimate should be revised and the significance of comparisons for various month-spans.

The properties of the concurrent trend-cycle filters of the X-11-ARIMA seasonal adjustment method (Dagum, 1980) with and without extrapolations, are studied here by analysing their corresponding frequency response functions.

Section 2 introduces the gain and phase shift functions of the symmetric (central), concurrent and first-month revised filters of X-11-ARIMA with and without extrapolation (in this latter case, the symmetric filter is almost equivalent to the Census Method II-X-11 variant symmetric filter of Shiskin, Young and Musgrave, 1967). This section also introduces two measures of filter revisions based on the root mean square differences between the frequency response functions of the analysed filters.

Section 3 estimates and analyses the time path of the direct revisions of the concurrent filters with respect to the asymmetric filters shifted a given number of months later.

Section 4 estimates and analyses the time path of the monthly revisions of the concurrent trend-cycle filter and remaining asymmetric filters.

Section 5 compares the time path of the total revisions of each asymmetric trend-cycle filter with the corresponding seasonal adjustment filter.

Section 6 gives the conclusions of this study.

2. Measures of Filter Revisions

Under the assumption of an additive decomposition model and no replacement of extreme values, the trend-cycle estimates from X-11-ARIMA with and without ARIMA extrapolations are obtained by the application of a set of moving averages or linear filters. For central or middle observations, say $n+1 \leq t \leq T-n$, where T denotes the series length , the filter is always the same and symmetric with 2n+1 weights whereas for the remaining n observations on both ends, the filters are time-varying (different for each consecutive observation) and asymmetric.

We can express the trend-cycle estimates for recent years of a time series x_t obtained from X-11-ARIMA by

$$z_t^{(m)} = \sum_{j=-m}^{n} \gamma_{m,j} x_{t-j} = \gamma^{(m)}(B) x_t \qquad (2.1)$$

where $z_t^{(m)}$ stands for the trend-cycle value from a series $x_{t-n}, x_{t-n+1}, \ldots, x_t, \ldots, x_{t+m}$. The trend-cycle moving average weights are $\gamma_{m,j}$ and $\gamma^{(m)}(B)$ denotes the corresponding linear filter using the backshift operator B such that $B^m x_t = x_{t-m}$.

For m=o, $z_t^{(o)}$ is the concurrent trend-cycle value and $\gamma^{(o)}(B)$ the corresponding concurrent filter; for m = 1, $z_t^{(1)}$ is the first-period revised trend-cycle estimate and for m=n, $z_t^{(n)}$ is the final trend-cycle value in the sense that it is estimated with a symmetric filter $\gamma^{(n)}(B)$ where $\gamma_{n,j} = \gamma_{n,-j}$ for all j. The final trend-cycle value will no longer change when more observations are entered into the series.

For any two points in time, say $t+k$ and $t+\ell$, $(k<\ell)$, the revisions of the trend-cycle value is given by

$$r_t^{(\ell,k)} = z_t^{(\ell)} - z_t^{(k)} \qquad k < \ell \qquad (2.2)$$

This revision reflects: (1) the innovations introduced by the new observations x_{t+k+1}, x_{t+k+2}, ..., $x_{t+k+\ell}$; and (2) the differences between the two asymmetric filters $\gamma^{(\ell)}(B)$ and $\gamma^{(k)}(B)$. Fixing k=o and letting ℓ vary from 1 to n the (2.2) gives a sequence of revisions of the concurrent trend-cycle value for different time-spans or lags. The <u>total revision</u> of the concurrent value is given by $\ell = n$. Fixing the $\ell = k+1$ and letting k take values from 0 to n-1, equation (2.2) gives a sequence of single-period revisions for each trend-cycle estimate and particularly starting at k=o, we obtain the time path of the single-period revisions of the concurrent trend-cycle value.

The part of the revisions we will study here are those introduced by filter discrepancies. These latter will be analysed via the frequency response functions of the corresponding filters.

Equations (2.1) represents a linear system where $z_t^{(m)}$ is the convolution of the input x_t and a sequence of weights $\gamma_{m,j}$ called the <u>impulse response function</u> of the filter. The properties of this latter can be described by its Fourier transform called the <u>frequency response function</u>,

$$\Gamma^{(m)}(\omega) = \sum_{j=-m}^{n} \gamma_{m,j}\, e^{-i2\pi\omega j} \qquad o \leq \omega \leq 1/2 \qquad (2.3)$$

where ω is the frequency in cycle per time period. $\Gamma(\omega)$ fully describes the effect of the linear filter on the given input. In general, the frequency response function may be expressed in polar form by,

$$\Gamma(\omega)=A(\omega)+iB(\omega) = G(\omega)e^{i\phi(\omega)} \qquad (2.4)$$

where $G(\omega) = [A^2(\omega)+B^2(\omega)]^{1/2}$ is called the <u>gain</u> of the filter and $\phi(\omega) =$ artan $[B(\omega)/A(\omega)]$ is called the <u>phase shift</u> of the filter and is expressed in radians. The gain and the phase-shift vary with the frequency ω. For symmetric filters, the phase-shift is zero or $\pm\pi$ and for asymmetric filters it can take any value between $\pm\pi$; being undefined at those frequencies where the gain is zero.

It has been shown by Young (1968) that the long symmetric trend-cycle filter of the Census Method II-X-11 variant can be well approximated with 97 points for a monthly series.(1) This symmetric filter is almost the same for X-11-ARIMA with and without extrapolations.

Figure 1 shows the gain functions of the symmetric (central) trend-cycle filter $\gamma^{(48)}(B)$ the concurrent $\gamma^{(o)}(B)$ and the first-month revised filters $\gamma^{(1)}(B)$ of X-11-ARIMA without extrapolation (X-11). We observe contrary to the usual definition of the trend-cycle which is associated with high power only at the low frequencies (generally $o \leq \omega \leq 0.055$; i.e. cycles of periodicities equal to and longer than 18 months) that the implicit definition of the symmetric trend-cycle estimator of X-11-ARIMA (or X-11) includes cycles with power at all frequencies $0 \leq \omega < 0.166$. In fact, it only excludes the 12 month-cycle since the gain at the fundamental seasonal frequency $\omega = 0.083$ is zero. This pattern results from the convolution of the symmetric seasonal adjustment filter with the 13-term Henderson trend-cycle filter. Because of the presence of relatively short cycles (i.e., from 11 months to nearly 6 months of periodicity) we will refer to the output from this filter indistinctively as trend-cycle estimates or "smoothed" seasonally adjusted estimates (the latter in the sense of being less affected by the irregulars).

We can also observe that the gain for all $\omega > 0.166$ is much larger for the two asymmetric filters as compared to the central filter. Furthermore, there are large amplifications for certain frequencies near the fundamental seasonal. All this means that the concurrent and first revised estimates will have more noise than the final estimate which will be

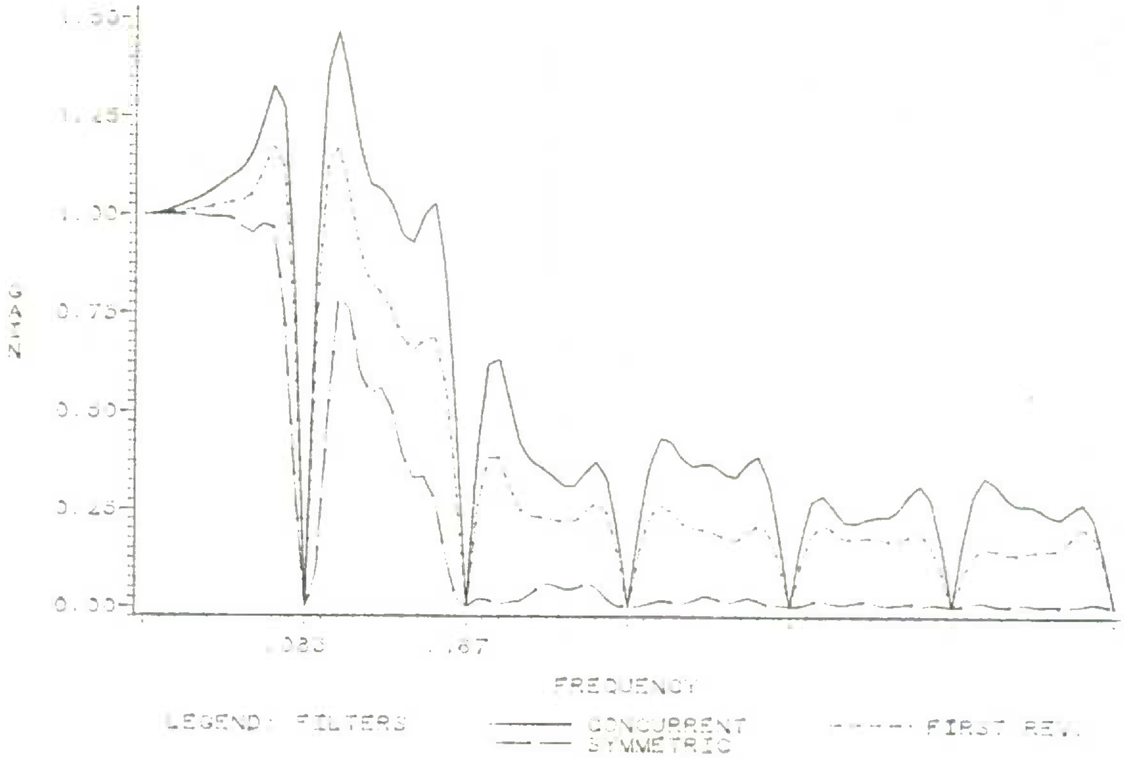


FIGURE 1: GAIN FUNCTIONS OF X11 TREND-CYCLE FILTERS

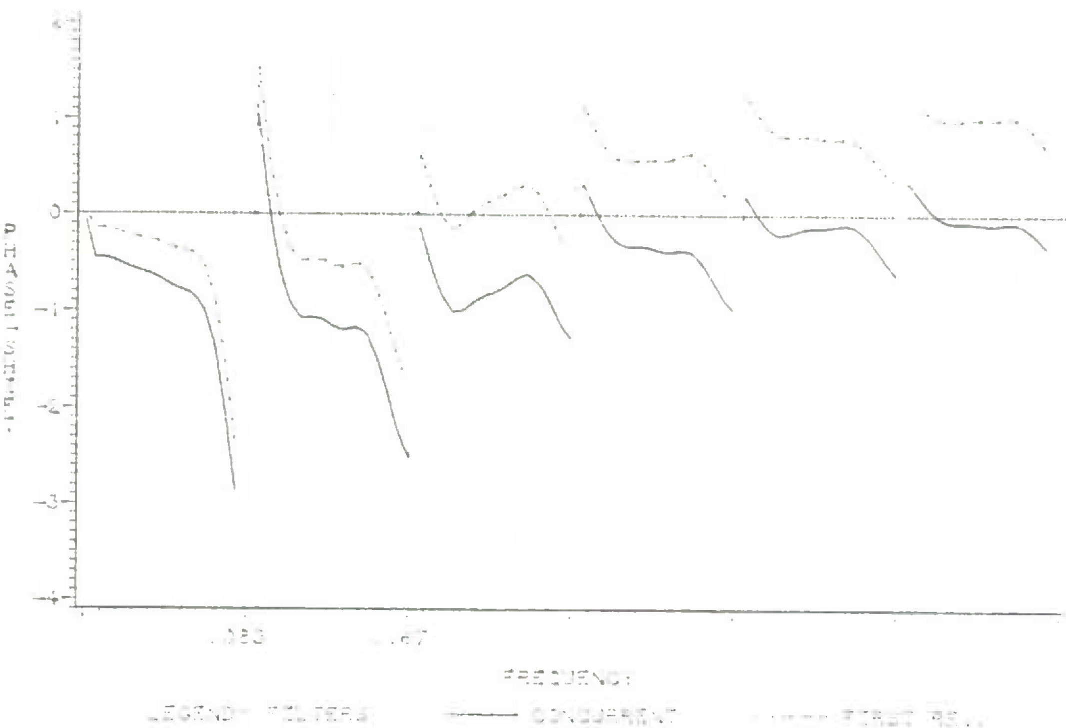


FIGURE 2: PHASE-SHIFT FUNCTIONS OF X11 TREND-CYCLE FILTERS

reached after 48 more observations have entered into the series.

Figure 2 shows the phase shifts of the two asymmetric filters. In order to have a better interpretation, the phase shifts are given in months instead of radians.(2) It is apparent that the phase-shift of the asymmetric filters are rather small, less than one month for the most important cyclical frequencies $\omega \leq 0.055$. Furthermore, the phase shifts of the first-month revised filter are smaller than those of the concurrent filter.

Figures 3 and 4 show the gains and phase shifts respectively of the symmetric, concurrent and first-month revised filters of X-11-ARIMA with extrapolations from the IMA seasonal model $(0,1,1)(0,1,1)_{12}$ expressed by $(1-B)(1-B^{12})x_t = (1-\theta B)(1-\Theta B^{12})a_t$ with $\theta=.40$ and $\Theta=.60$. Whereas the symmetric filter is similar to the one shown in figure 1, there are significant differences for the other two asymmetric filters. These asymmetric filters result from the convolution of the Census Method II-X-11 trend-cycle filters with the ARIMA extrapolation filters as described by Dagum (1983). First, we observe that the given function of the two asymmetric filters are closer to the central filter than those of X-11-ARIMA without extrapolation (X-11). There are no amplifications around the fundamental and similar attenuation at higher frequencies. On the other hand, there is more phase-shift (being near to one month) for almost all low frequencies for the concurrent filter and, in general, less phase-shift for all high frequencies.

In this study we are mainly interested in: (1) the time path of the concurrent filter revisions given its importance for the current estimation of the trend-cycle smoothed seasonally adjusted value; and (2) a comparison of the time paths of the total revisions of the trend-cycle filters versus the seasonal adjustment filters.

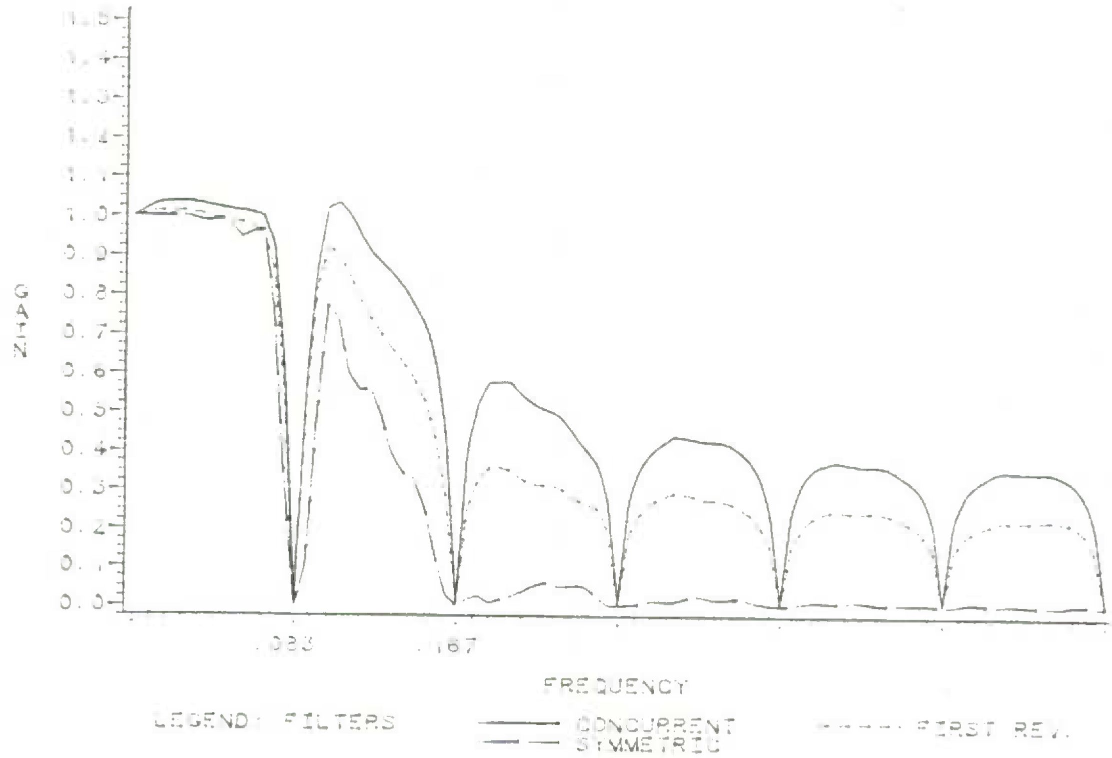


FIGURE 3: GAIN FUNCTIONS OF [illegible] TREND-CYCLE FILTERS [illegible]

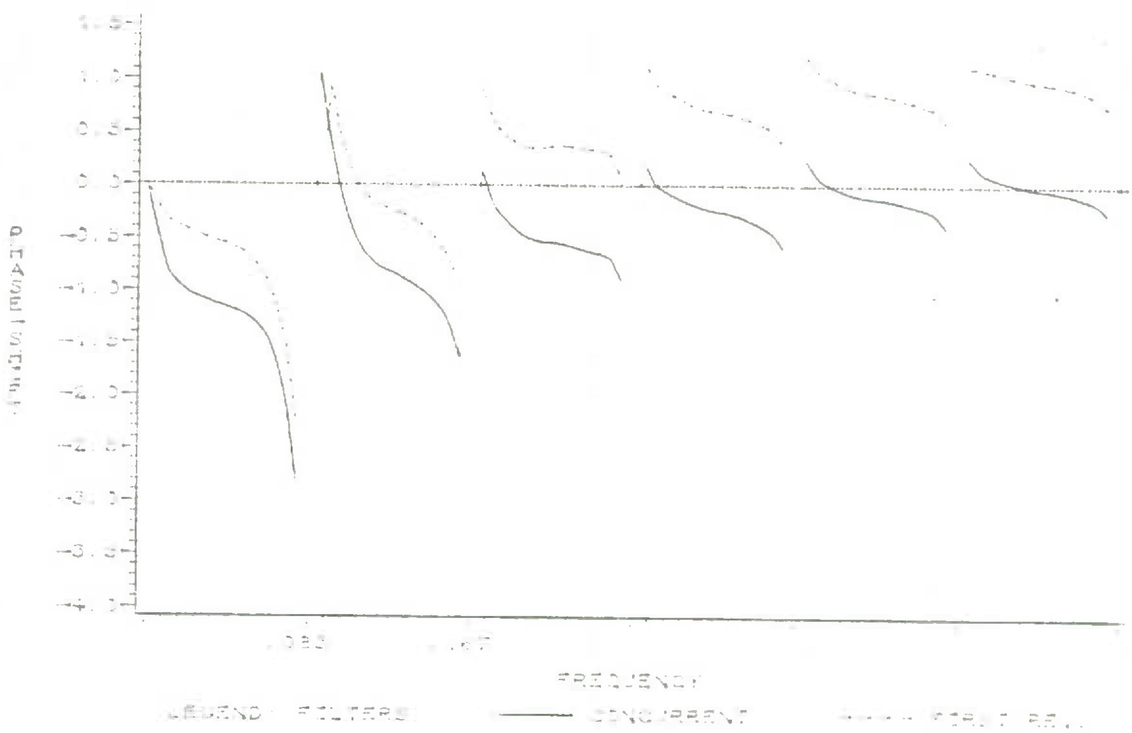


FIGURE 4: PHASE-SHIFT FUNCTIONS OF [illegible] TREND-CYCLE FILTERS [illegible]

Following Dagum (1982(a) and (b)), we introduce next two measures based on the root mean square revision of asymmetric filters over all frequencies.

The first measure is:

$$R^{(\ell,o)} = [2\int_o^{1/2} \| \Gamma^{(\ell)}(\omega) - \Gamma^{(o)}(\omega) \|^2 d\omega]^{1/2} \qquad o<\omega<1/2 \qquad (2.5)$$
$$\ell = 1,2,3,...,48.$$

where $\Gamma^{(o)}(\omega)$ denotes the frequency response function of the trend-cycle concurrent filter $\gamma^{(o)}(B)$ and $\Gamma^{(\ell)}(\omega)$ stands for the frequency response function of a filter shifted ℓ periods with respect to the concurrent; i.e., $\gamma^{(\ell)}(B)$.

The $R^{(\ell,o)}$ measure gives the time path of the concurrent filter as it approaches to the central filter for month-spans ℓ =1,2,3,..., 48. Note that for ℓ = 48, $R^{(48,o)}$ gives the root mean square total revision of the concurrent filter.

The second measure is:

$$R^{(k+1,k)} = [2\int_o^{1/2} \| \Gamma^{(k+1)}(\omega) - \Gamma^{(k)}(\omega) \|^2 d\omega]^{1/2} \qquad o\leq\omega\leq 1/2 \qquad (2.6)$$
$$k=o,1,2,...,47.$$

the $R^{(k+1,k)}$ measure gives the single-period revision of the asymmetric filters any time a new observation enters into the series. It is also a measure of the average distance between consecutive asymmetric filters. Starting at k=o, equation (2.6) gives the time paths of the monthly revisions of the concurrent trend-cycle filter.

3. Time Path of Concurrent Trend-Cycle Filters of X-11 and X-11-ARIMA

The $R^{(\ell,o)}$ measure given in equation (2.5) has been calculated for the X-11 and X-11-ARIMA trend-cycle filters for $\ell = 1,2,3,\ldots,48$. The ARIMA extrapolation model used is the classical $(0,1,1)(0,1,1)_{12}$ IMA type (Box and Jenkins, 1970) of the following form:

$$(1-B)(1-B^{12})x_t=(1-\theta B)(1-\Theta B^{12})a_t \qquad (3.1)$$

Since the extrapolations affect significantly, the concurrent trend-cycle filter depending on the parameter values of θ and Θ, we selected some combinations often found when modelling economic time series. These are:

$\theta=0.40$	$\Theta=0.40$	$\theta=0.60$	$\Theta=0.40$	$\theta=0.80$	$\Theta=0.40$
$\theta=0.40$	$\Theta=0.60$	$\theta=0.60$	$\Theta=0.60$	$\theta=0.80$	$\Theta=0.60$
$\theta=0.40$	$\Theta=0.80$	$\theta=0.60$	$\Theta=0.80$	$\theta=0.80$	$\Theta=0.80$

The smaller the values of θ and Θ the more flexible the trend-cycle and seasonal component are respectively. The opposite occurs for large values of θ and Θ.

The time path of the concurrent filter of X-11-ARIMA without extrapolation (X-11) for the various month-spans $\ell=1,2,3,\ldots,48$ is shown in Figure 5.

The time path of the concurrent filters corresponding to X-11-ARIMA with extrapolation using the parameter values combination given above are shown in Figures 6 to 8. First, we observe that whether ARIMA extrapolations are used or not, the distance between the concurrent filter and a filter shifted 6 months or more is kept almost constant and equal to the total distance $R^{(48,o)}$. This is due to the fact that the weights of the 13-term Henderson trend-cycle filter are asymmetric only for the last six observations of the series; being symmetric for the remaining data.

FIGURE 5: TIME PATH OF THE CONCURRENT TREND-CYCLE FILTERS OF X11.

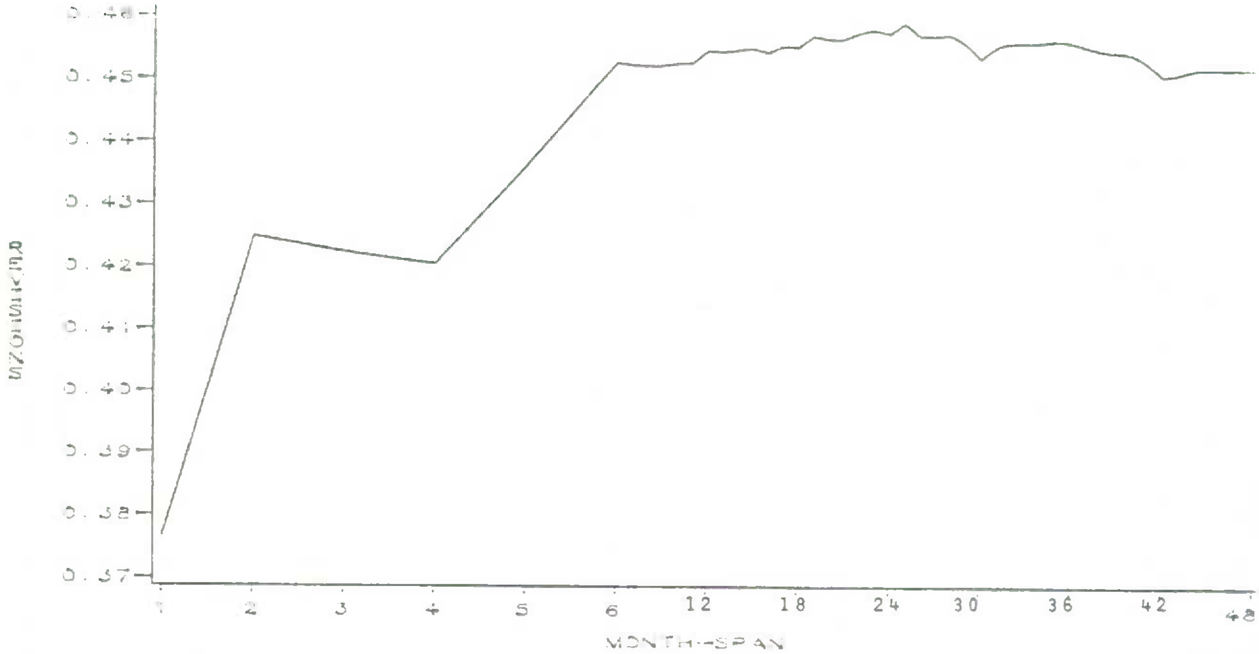


FIGURE 6: TIME PATH OF THE CONCURRENT TREND-CYCLE FILTERS OF X11-ARIMA.

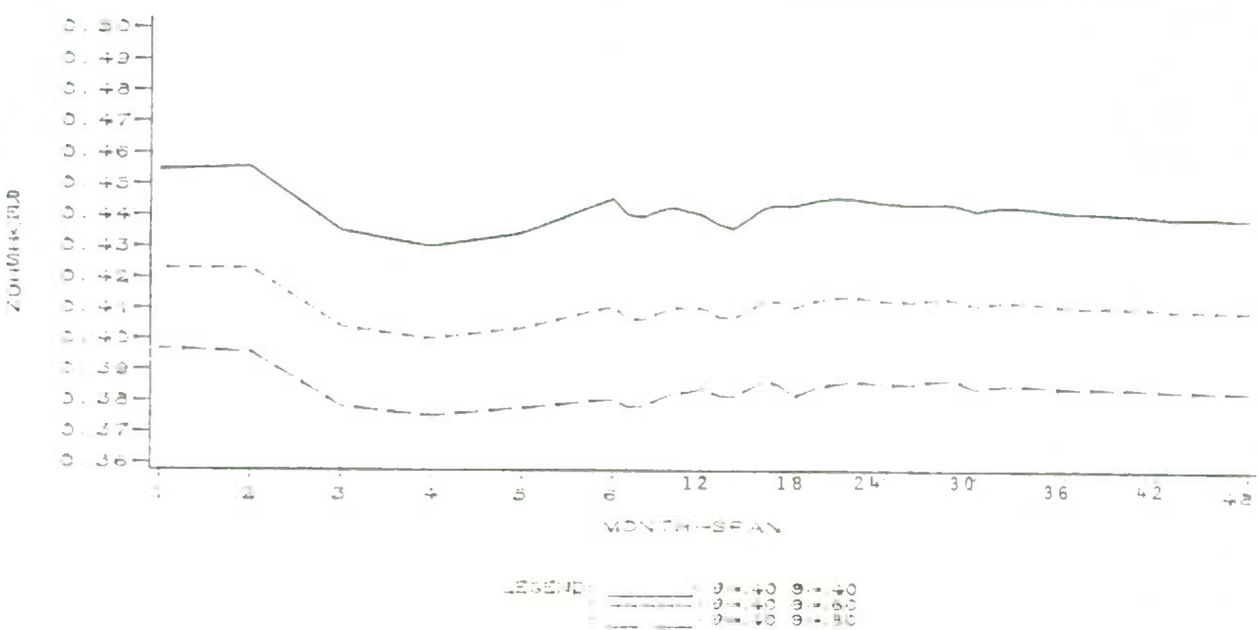


FIGURE 7: TIME PATH OF THE CONCURRENT TREND-CYCLE FILTERS OF X11-ARIMA.

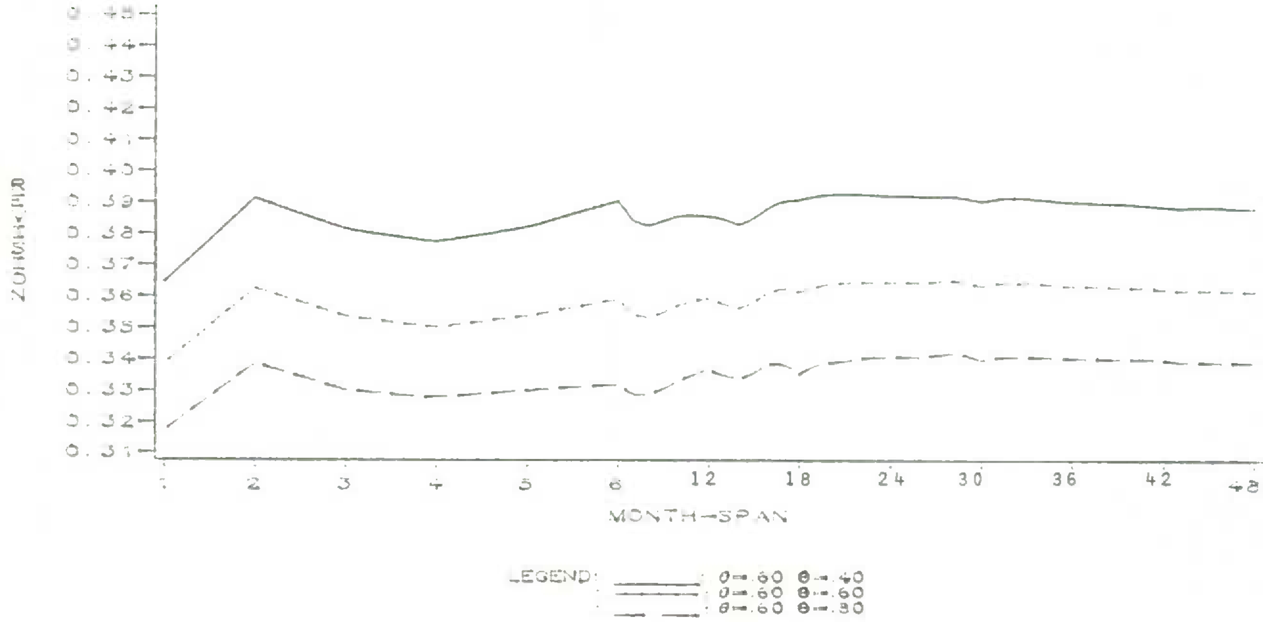
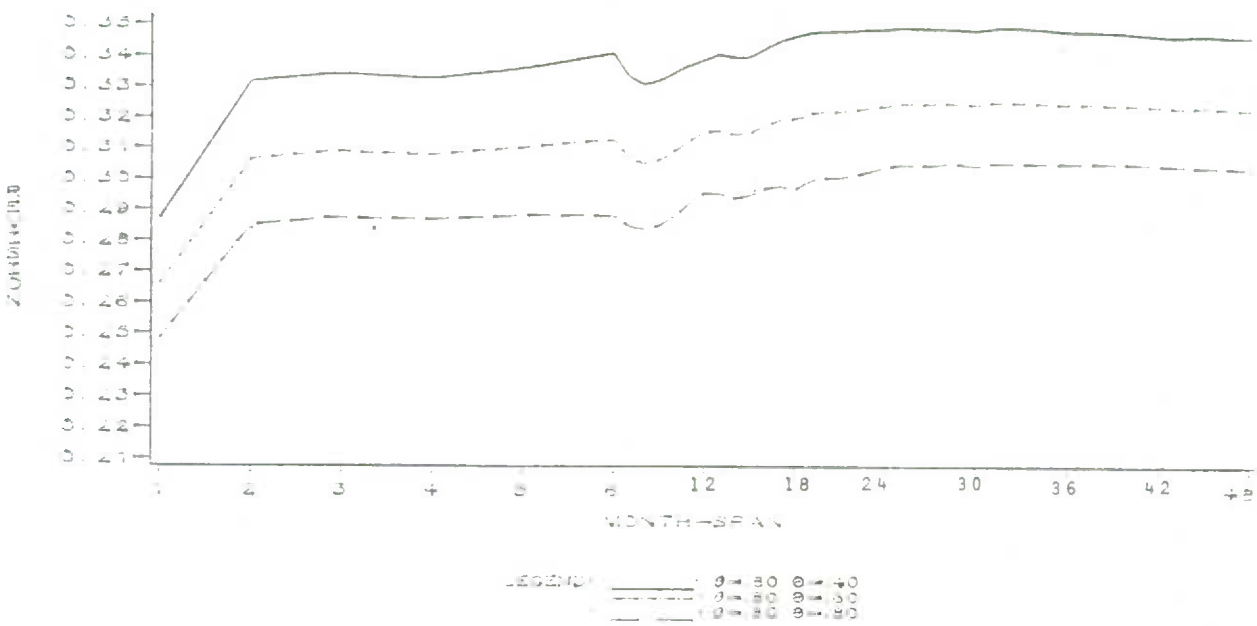


FIGURE 8: TIME PATH OF THE CONCURRENT TREND-CYCLE FILTERS OF X11-ARIMA.



The first six month-spans, $\ell =1,2,3,..,6$, are the most important to assess the stage of the cycle at which the economy stands in the current year. Therefore, Table 1 shows the values of $R^{(\ell,o)}$ for $\ell=1,2,3,..6$ and $R^{(48,o)}$; this latter corresponds to the total revision of the concurrent trend-cycle filter.

Table 1 - Root Mean Square Revisions of the Concurrent Trend. Cycle Filters of X-11 and X-11-ARIMA for Selected Month-Spans

Month-span ℓ	X-11 $R^{(\ell,o)}$	X-11-ARIMA - $R^{(\ell,o)}$								
		θ =.40 Θ =.40	θ=.40 Θ=.60	θ=.40 Θ=.80	θ=.60 Θ=.40	θ=.60 Θ=.60	θ=.60 Θ=.80	θ=.80 Θ=.40	θ=.80 Θ=.60	θ=.80 Θ=.80
1	0.38	0.45	0.42	0.40	0.37	0.34	0.32	0.29	0.27	0.21
2	0.43	0.45	0.42	0.40	0.39	0.36	0.34	0.33	0.31	0.29
3	0.42	0.44	0.41	0.38	0.38	0.35	0.34	0.34	0.31	0.29
4	0.42	0.43	0.40	0.38	0.38	0.35	0.34	0.35	0.31	0.29
5	0.44	0.44	0.41	0.38	0.39	0.36	0.34	0.35	0.31	0.29
6	0.46	0.44	0.41	0.38	0.39	0.36	0.34	0.35	0.31	0.29
48	0.45	0.44	0.41	0.38	0.39	0.36	0.34	0.35	0.32	0.30

The values of Table 1 indicate that:

First, the total revision tends to be smaller, the higher the values of the parameters θ and Θ (which implies more rigid trend-cycle and seasonal components, respectively). The decrease, however, is more sensitive to changes in the values of the trend-cycle parameter θ than of the seasonal parameter Θ. In fact, using Θ =.60, the total revisions decrease from .41 to .32 (28.1%) when the trend-cycle parameters change from .40 to .80 whereas fixing θ=.60 the total revisions decrease only from .39 to .34 (14.7%) when the seasonal parameters vary from Θ =.40 to

$\Theta=.80$.

Second, the distances between the concurrent filter and the asymmetric filters become constant much earlier if the ARIMA extrapolations are used. For most cases analysed here, this is true for $\ell\geq 3$. On the other hand, if ARIMA extrapolations ar not used, the values of the $R^{(\ell,0)}$ measure change significantly for $\ell=1,2,5$ and 6.

If the time path of the total revision of the concurrent filter were constant for all the month-span $\ell=1,2,\ldots,6,\ldots,48$, this would indicate that in performing month-to-month or any number of month-span comparisons of trend-cycle estimates, the observed changes can then be attributed only to the innovations entering into the series. If ARIMA extrapolations are used, comparisons of three month-span or more have this property.

Third, we observe that the largest revision occurs after one month has entered into the series. This is true whether ARIMA extrapolations are used or not.

4. Time Path of the Monthly Revisions of the Concurrent and Other Asymmetric Filters of X-11 and X-1-ARIMA

The $R^{(k+1,k)}$ measure of equation (2.6) has been calculated for the X-11-ARIMA concurrent and remaining asymmetric filters corresponding to the ARIMA extrapolation models discussed in the previous section and also for X-11 (X-11-ARIMA without extrapolation). This statistic provides the time path of the monthly revisions of the asymmetric filters; being that of the concurrent the one that interests the most.

Table 2 shows the values of $R^{(k+1,k)}$ for the first twelve consecutive filters.

Table 2 - Root Mean Square Monthly Revisions of the Concurrent and Other Asymmetric Trend-Cycle Filters of X-11 and X-11-ARIMA

$\ell=k+1$	X-11 $R^{(k+1,k)}$	X-11-ARIMA $R^{(k+1,k)}$								
		$\theta=.40$ $\Theta=.40$	$\theta=.40$ $\Theta=.60$	$\theta=.40$ $\Theta=.80$	$\theta=.60$ $\Theta=.40$	$\theta=.60$ $\Theta=.60$	$\theta=.60$ $\Theta=.80$	$\theta=.80$ $\Theta=.40$	$\theta=.80$ $\Theta=.60$	$\theta=.80$ $\Theta=.80$
1	0.38	0.45	0.42	0.40	0.37	0.34	0.32	0.29	0.27	0.25
2	0.21	0.27	0.25	0.24	0.23	0.21	0.20	0.20	0.18	0.17
3	0.08	0.12	0.11	0.11	0.11	0.10	0.09	0.10	0.09	0.09
4	0.02	0.03	0.03	0.03	0.04	0.03	0.03	0.04	0.03	0.03
5	0.05	0.02	0.01	0.01	0.02	0.01	0.01	0.02	0.01	0.01
6	0.04	0.02	0.01	0.01	0.02	0.01	0.01	0.02	0.02	0.02
7	0.03	0.02	0.01	0.01	0.02	0.01	0.01	0.02	0.02	0.02
8	0.02	0.03	0.03	0.03	0.01	0.01	0.01	0.01	0.01	0.01
9	0.01	0.05	0.05	0.04	0.03	0.03	0.03	0.02	0.01	0.01
10	0.02	0.06	0.06	0.05	0.05	0.04	0.04	0.04	0.03	0.03
11	0.03	0.06	0.05	0.05	0.05	0.05	0.04	0.05	0.04	0.04
12	0.03	0.03	0.03	0.03	0.04	0.04	0.03	0.04	0.04	0.04

First, we note that the single-period revisions decrease very fast and monotonically for the first seven filters if the ARIMA extrapolations have been used. This is due to the improvement in the weight system of the 13-term Henderson trend-cycle filter which becomes symmetric after six observations have been added to the series. There are some reversals of direction in the time path of the monthly revision after the seventh month filter but since the size of the revisions are already very small, these reversals have no practical significance. For X-11, the time path is similar except that the first reversal occurs after the fourth month filter.

Second, we observe that independently of the use of ARIMA extrapolations the largest single-period revisions occur at $\ell = 1$, 2 and 3. This suggests that by revising the current estimate only three times, we can reach the final estimate for all practical purposes. This agrees with the previous section observations where the direct distance between the concurrent estimate and any filter shifted three or more months later is almost constant and equal to the total revision.

Given the importance of this conclusion for a revision policy of trend-cycle estimates, we looked at the improvement introduced by each monthly revision for specific band of frequencies instead of over all the frequencies as given by the measure of equation (2.6).

The values in Table 3 show that for the frequency band $0 \leq \omega \leq 0.055$ associated with cycles equal to or greater than 18 months, the root mean square single revisions approach monotonically to the direct revision of the concurrent filter three months later, i.e., $R^{(3,0)}$. For the remaining frequencies associated with short cycles and mainly attributed to the irregulars, the root mean square single revisions do not approach monotonically to the direct revisions of the concurrent filter three months later. This shows an inconsistency between the asymmetric trend-cycle filters in the sense that they do not approach monotonically to the final filter for the frequencies corresponding to the irregulars.

However, since most of the power of economic time series is concentrated at the low frequency band, there will always be a significant improvement if the current estimate is revised monthly instead of waiting three months before its revision. In all cases, a large improvement will occur at each frequency band if the filter is revised once whenever a new observation is available. If instead of the root mean square we calculate the mean square revision, the discrepancies at the irregular frequencies are much smaller and, similarly, over all the frequencies.

We use the root mean square, however, because we are interested in measuring the effect of filter changes in the same dimension of the series.

Table 3 - Root Mean Square Monthly Revisions and Direct Revisions of the Concurrent Trend-Cycle Filter for Selected Bands of Frequencies

Method	Month-lag $\ell=k+1$	Monthly Revisions $R^{(k+1,k)}$		Direct Revisions $R^{(\ell,o)}$	
		Trend-cycle frequency band $0\leq\omega\leq0.055$	Irregular frequency band $0.055\leq\omega\leq0.50$	Trend-cycle frequency band $0.\leq\omega\leq0.055$	Irregular frequency band $0.055\leq\omega\leq0.50$
X-11	1	0.094	0.398	0.094	0.398
	2	0.059	0.224	0.152	0.448
	3	0.025	0.084	0.175	0.444
X-11-ARIMA (IMA Model parameters)					
	1	0.122	0.447	0.122	0.447
$\theta=.40$ $\Theta=.60$	2	0.074	0.268	0.195	0.443
	3	0.035	0.122	0.220	0.421
	1	0.131	0.333	0.131	0.333
$\theta=.60$ $\Theta=.80$	2	0.083	0.209	0.212	0.351
	3	0.040	0.099	0.250	0.339

5 - Time Paths of the Total Revisions of the Asymmetric Trend-Cycle Filters versus the Seasonal Adjustment Filters of X-11 and X-11-ARIMA

To calculate the total revisions of each consecutive asymmetric filter we use the common formula,

$$R^{(48,k)} = \left[2\int_{0}^{1/2} \left\| \Gamma^{(48)}(\omega) - \Gamma^{(k)}(\omega) \right\|^{2} d\omega \right]^{1/2} \qquad 0 \leq \omega \leq 1/2 \qquad (2.7)$$

$$k = 0,1,2,\ldots,47.$$

For the seasonal adjustment filters, however, the total revision is attained after 42 observations are added into the series instead of 48 because the symmetric (central) seasonal adjustment filter requires only 85 monthly data (see Young, 1968, Wallis, 1974, Dagum, 1983). One more year of data is necessary for the symmetric trend-cycle filter. In order to compare the total revision of both types of filters, we calculate them assuming we have a series of 97 points. The total revision of the asymmetric seasonal adjustment filters are equal for $R^{(42,k)}$ and $R^{(48,k)}$ because after 42 points the asymmetric filters reduce to the same symmetric filter. Figure 9 shows the time path of the total revisions of the trend-cycle versus the seasonal adjustment asymmetric filters of X-11. Similarly, Figures 10 and 11 show the time path of the total revisions for the X-11-ARIMA filters when extrapolations have been used.

In all cases it is apparent that the trend-cycle total revisions converge to zero much faster than those of the seasonal adjustment filters. These latter follow a step function, being rather constant within the year and decreasing sharply every twelve months.

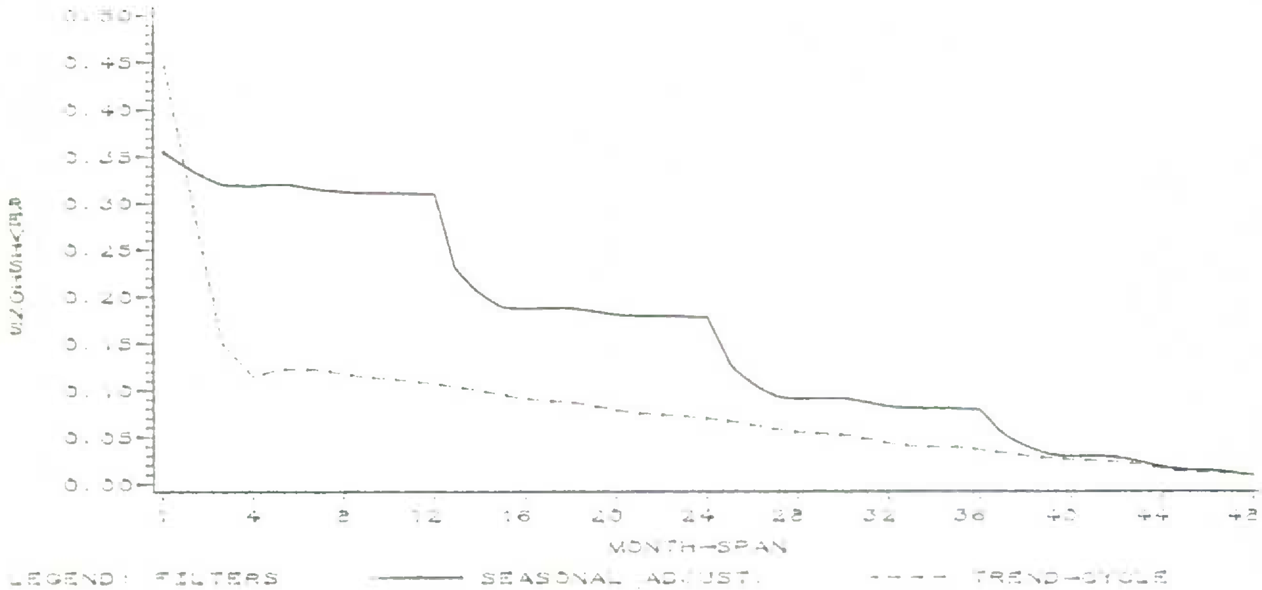


FIGURE 9: TIME PATH OF THE TOTAL REVISIONS OF THE X11 ASYMMETRIC FILTERS.

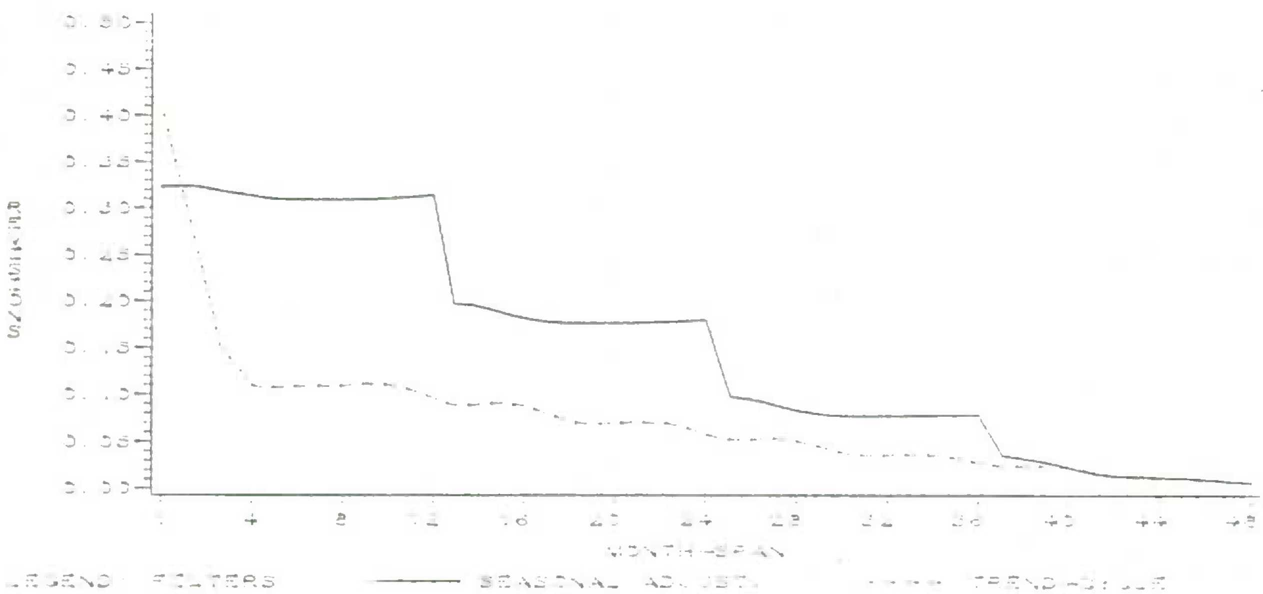


FIGURE 10: TIME PATH OF THE TOTAL REVISIONS OF THE X11ARIMA ASYMMETRIC FILTERS (ARIMA MODEL θ=.40 Θ=.60)

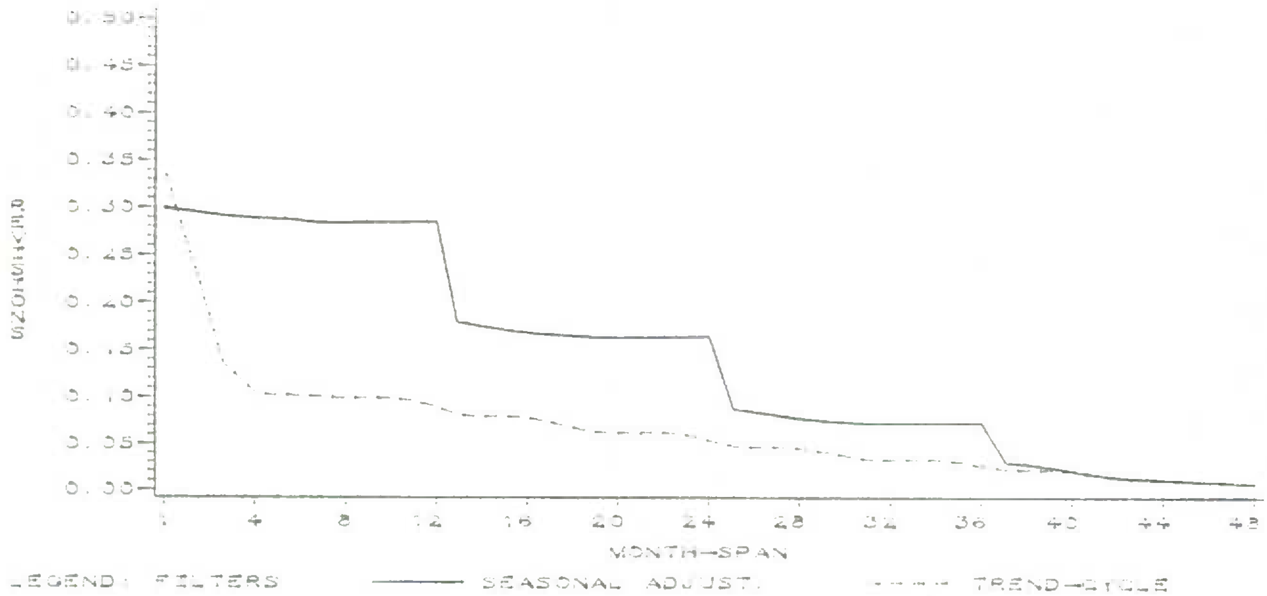


FIGURE 11: TIME PATH OF THE TOTAL REVISIONS OF THE X11-ARIMA ASYMMETRIC FILTERS (ARIMA MODEL θ=.60 Θ=.90)

Table 4 - Time Path of the Root Mean Square Total Revisions of the Trend-Cycle versus the Seasonal Adjustment Asymmetric Filters of X-11 and X-11-ARIMA

Month-lag	X-11		X-11-ARIMA			
			θ=.40 Θ=.60		θ=.60 Θ=.80	
k	Trend-cycle Filters	Seasonal Adjustment Filters	Trend-cycle Filters	Seasonal Adjust. Filters	Trend-Cycle Filters	Seasonal Adjustment Filters
0	0.45	0.36	0.41	0.32	0.34	0.30
1	0.27	0.33	0.26	0.32	0.22	0.30
2	0.15	0.32	0.15	0.32	0.13	0.29
3	0.11	0.32	0.11	0.31	0.10	0.29
4	0.12	0.32	0.11	0.31	0.10	0.29
12	0.10	0.23	0.09	0.20	0.08	0.18
24	0.07	0.13	0.05	0.10	0.05	0.09
36	0.03	0.05	0.02	0.04	0.02	0.03
47	0.01	0.01	0.01	0.01	0.01	0.01

Table 4 shows the total revisions of both sets of filters for selected months. The results indicate: First, the trend-cycle filter total revisions are very small, about .10, only after three more months are added into the series whereas it takes two more years for the total revisions of the seasonal adjustment filters to reach similar values.

Second, the total revision of the concurrent trend cycle filter k=o is always larger than that of the concurrent seasonal adjustment filter. This is due to the fact that the asymmetric weights of the 13-term Henderson trend-cycle filter are very poor for the last observation. However, the total distance of the first-month revised trend-cycle filter (k=1) is always much smaller than that corresponding to the seasonal adjustment filter. It takes a full year for the latter to achieve a total revision of nearly the same order of magnitude.

Third, we observe that the use of the ARIMA extrapolation brings closer the size of the total revisions of the concurrent trend cycle and seasonal adjustment filter for large θ and Θ.

All the above observations suggest that instead of seasonally adjusted data it would be preferable to use smoothed seasonally adjusted data if the values are revised at least once and up to three times whenever new observations are added to the series.

6 - Conclusions

This paper has studied the properties of the trend-cycle filters of X-11-ARIMA with and without extrapolation (in this latter case, the symmetric and asymmetric filters are almost equivalent to those of the Census Method II-X-11 variant). The trend-cycle filters suppress most of the irregulars without distorting the trend-cycle of a seasonally adjusted series and thus leaving a much smoother seasonally adjusted series. The use of trend-cycle filters are mainly recommended for seasonally adjusted data strongly contaminated by irregulars and, in general, for all seasonally adjusted series if the data are revised at least once and no more than three times when a new observation enters into the series.

The nature of the trend cycle filters is studied by estimating and analysing: (1) the time-path of the revisions of the concurrent filter for consecutive month-spans $\ell = 1,2, \ldots, 48$; where the distance between the concurrent and the filter shifted 48 months later defines the total revision; (2) the time path of the monthly revisions of the concurrent and remaining asymmetric trend-cycle filters and (3) the time path of the total revisions of each one of the asymmetric trend-cycle and seasonal adjustment filters.

The results obtained show:

(1) If ARIMA extrapolations are used, the total revision of the concurrent trend cycle filters are smaller the larger the values of the trend-cycle and seasonal parameters θ and Θ respectively. Furthermore, the distance between the concurrent filter and any asymmetric filter shifted 3 months or more from the concurrent is constant and equal to the total revision. This means that if the trend-cycle estimates are revised after three more months are added to the series, changes in the month-span

comparisons will be due to the innovations and not to filter changes.

If ARIMA extrapolations are not used, the distance between the concurrent and any asymmetric filter is constant and equal to the total revisions only after six months have entered into the series.

(2) The pattern followed by the monthly revisions of the concurrent trend-cycle filter indicates that it is always beneficial to revise it at least once, and no more than three times. This is due to the fact that the monthly revisions converge monotonically to the total revision for all the frequencies associated with the trend-cycle component whenever new observations are added. (3) The time path of the total revisions of the trend-cycle asymmetric filters converge to zero much faster than that of the corresponding seasonal adjustment filters. The total revision of the trend-cycle filters shifted three months with respect to the concurrent is only about 0.10 whereas, this value is achieved for the seasonal adjustment filters only after twenty-four months.

Given all the above observations, we recommend instead of concurrent seasonally adjusted data to produce "smoothed" seasonally adjusted data whenever the series are strongly affected by irregulars. The concurrent smoothed seasonally adjusted estimates should be revised at least once and up to three times whenever new observations enter into the series.

The use of smoothed seasonally adjusted data subject to the preceding revision policy is recommended as long as there are at least 97 points in the series.

FOOTNOTES

(1) This filter and others discussed later correspond to the standard option of the program which uses both the 3x3-term and 3x5-term seasonal moving averages and the 13-term Henderson trend-cycle filter.

(2) The phase-shift in months is given by $\phi'(\omega)/2\pi\omega$ for $\omega \neq 0$.

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