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ADDITIVE VERSUS MULTIPLICATIVE SEASONAL ADJUSTMENT WHEN THERE ARE FAST CHANGES IN THE TREND-CYCLE

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1. INTRODUCTION

1981 and 1982 were atypical years afflicted by a severe recession. This recession has profoundly affected the evolution and structure of economic time series, and consequently their seasonal adjustment. Seasonally adjusted time series are necessary to diagnose the socio-economic health of a country. In turn, social and economic policies founded on these data influence decisions both in the private and public sectors. Thus, this recession raises many questions. One can readily see that a prompt examination of seasonal adjustment is necessary.

The series under consideration here are: initial and renewal claims received (for unemployment benefits) and beneficiaries. It is difficult to see how their trend and cycle components evolve when they are contaminated by seasonal variation, namely intra-annual climatic and institutional factors. Seasonal adjustment permits a better detection of fundamental tendencies, such as turning points, and evaluation of the present performance of the economy.

This article analyses some aspects of the interplay between a severe recession and seasonal adjustment. In just one year, that is in 1981, this recession has nearly doubled the level of beneficiaries. Such a sudden large change prompts questions about the structure of the series, the choice of the X-11-ARIMA decomposition model, the determination of turning points at the end of the series, and the use of ARIMA forecasts for seasonal adjustment.

In section 2, we discuss two important consequences of using a wrong decomposition model, namely a systematic over- and under-adjustment of series and the possibility of having a false turning point at the end of the series. In section 3, we use the lead-lag relationship between the claims and beneficiaries series to help seasonally adjust the latter series.

The ARIMA forecasts generally help to reduce the revision to the seasonal factors and they can help to provide a more accurate recognition of the turning points at the end of the series. Section 4 considers this question.

2. DECOMPOSITION MODELS FOR SEASONAL ADJUSTMENT

Most of the claims and beneficiaries series have similar characteristics, so we have chosen to study one claims series and one beneficiaries series which can clearly illustrate some of the problems peculiar to seasonal adjustment during a severe recesssion. It should be noted that the results of our analysis are equally valid during a sudden strong expansion in the economy. It is the sudden large change in the level of the series caused by the recession or the expansion that is important.

The X-11-ARIMA program (Dagum, 1980) will be used to seasonally adjust these series. The program is applied to the claims and beneficiaries series, using data from January 1973 and May 1975 respectively, up to February 1983.

The X-11-ARIMA program provides three decomposition models for the estimation of the time series components. The program assumes an additive relationship between the components

$$0_t = (TC)_t + S_t + I_t \qquad (2.1)$$

or a multiplicative one

$$0_t = (TC)_t \cdot S_t \cdot I_t \qquad (2.2)$$

or a log additive one

$$0_t = \log\,(TC)_t + \log S_t + \log I_t \qquad (2.3)$$

where 0 stands for the observed and unadjusted series; (TC), the trend-cycle; S and I, the seasonal and irregular components; and t, the time.

Seasonal adjustment means removing the seasonal variations S_t from the raw data 0_t, thus leaving a seasonally adjusted series consisting of TC_t and I_t. In order to know whether a certain series contains a significant amount of seasonality and if so, whether an additive or multiplicative model provides the better fit, one can perform a test for the presence of seasonality and a model test on the series (Higginson, 1977). The first test shows that both series contain a very significant amount of seasonality. According to the second test, the multiplicative model fits the beneficiaries series better when tested from May 1975 to June 1981. When the series is extended to February 1983, taking into account the impact of the recession on the series, the additive model then fits better. On the other hand, the model test favours neither the additive nor the multiplicative model for the claims series.

One usually adjusts the series using only one model, however, figure 1 shows the beneficiaries series adjusted using the two models, both without using the ARIMA option. During 1980 and 1981, the difference between the additive and multiplicative adjustments was small compared with the difference observed in 1982.

The multiplicative model assumes that the seasonal variation is proportional to the level of the trend-cycle. During 1982, the seasonal amplitude did not increase in this way. Consequently, using the multiplicative model is likely to overestimate the seasonal component in the seasonally high months from January to April and underestimate it from June to November, the seasonally low months. As figure 1 shows, in underestimating the number of seasonal beneficiaries, the multiplicative model has drastically overestimated the number of seasonally adjusted beneficiaries. The converse is also true.

The additive model, on the other hand, does not assume that the components of the series evolve proportionately. Figure 1 confirms that the trend cycle increased while the seasonal amplitude remained constant. Thus, the additive model provides the better seasonal adjustment. It performs better in 1982 than the multiplicative model and is acceptable in 1980 and 1981.

BENEFICIARIES

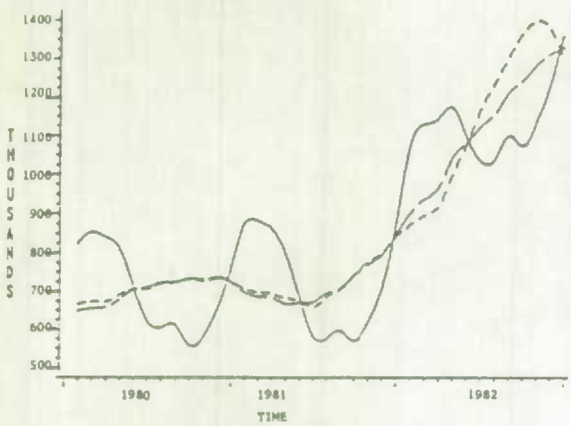


FIGURE 1

By mid-1982, it was not easy to tell which of the additive or multiplicative models would adjust the beneficiaries series better. Since this series was adjusted multiplicatively until June 1981, one would normally continue to do so in 1982. During 1982, were there some clues or pieces of evidence showing that the multiplicative model was no longer adequate?

The acceptance or rejection of a model, given a sudden large change in the level of a series, clearly has to be based on a thorough analysis of the data. The set of quality control statistics included in the X-11-ARIMA program is not meant to detect that kind of problem in the model. In this experiment with the multiplicative model, none of the ten individual control statistics failed the guideline. However, the F test for the presence of moving seasonality showed the presence of increasing moving seasonality during 1982 in the final unmodified SI ratios.

Besides a systematic over and under-adjustment of the series, another consequence of using a wrong decomposition model is the possibility of having a false turning point at the end of the series.

Let us say that a cyclical turning point has occurred if the seasonally adjusted series shows a change in direction that persists for at least 5 months. Once the beneficiaries series has been seasonally adjusted multiplicatively, figure 1 shows the possible presence of a turning point around October 1982, where the upward trend has suddenly changed to a downward trend. This turning point seems to be confirmed when the series ending in December 1982 is extended by one month. The additively adjusted series, on the other hand, shows no turning point. The two results conflict. Thus, either the multiplicative model is signalling a false turn or the additive model is missing the turning point.

It is not that easy to show that the multiplicative model has signalled a false turn. The multiplicative model has created a turning point around October 1982. Table 1 shows that in the very short run, the updating of the series did not reverse this turning point.

Table 1
Multiplicatively adjusted beneficiaries series
('000, July 1982 -- February 1983)

July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.
124	131	140					
124	130	140	142				
124	130	140	141	141			
124	130	140	142	138	131		
123	130	140	142	141	131	121	
123	129	139	142	141	134	121	123

3. LEAD-LAG RELATIONSHIP BETWEEN THE CLAIMS AND BENEFICIARIES SERIES

Leading indicators are sensitive to the evolution of the economic climate. They are measures of anticipations or new commitments, and as such they give an advance indication of changes expected in the trend-cycle of coincident and lagging indicators.

Figure 2 shows the claims series as a leading indicator for the beneficiaries series. The performance of the seasonally adjusted indicators can be tested using the criteria of Klein and Moore (1982). The two series satisfy these criteria. First, the correspondence between the series is one-to-one -- the number of cycles is the same in each series. Second, there is uniformity in timing -- the claims series always lead. Third, these are monthly series and they are current, or up-to-date. Thus, the claims series is likely to predict an upward or a downward change in the trend of the beneficiaries series.

The lead-lag relationship between the two series can help to seasonally adjust the beneficiaries series. It reduces the likelihood of mistaking an irregular turn for a cyclical turning point. Figure 2 shows September 1982 to be a turning point in the multiplicatively adjusted claims series. This is also true for the additive adjustment of the series. Since the cross-correlations between the two series shows a lead-lag relationship of 5 to 6 months, the September 1982 turning point in the claims series indicates that the multiplicative model applied to the beneficiaries series has signalled a false turn around October 1982. However, the leading indicator predicts a turning point around March 1983 in the beneficiaries series.

4. ARIMA EXTRAPOLATIONS

An optimal seasonal adjustment procedure has to minimize the revision to the current seasonal factors and also has to produce reliable estimates of the trend-cycle, particularly of turning points, at the end of the series (Dagum, 1979). The analysis carried on in the previous sections is based on seasonally adjusted data without using the ARIMA option. In this section, we shall

focus on the use of the ARIMA forecasts as a variable that can provide an accurate recognition of the turning points.

The automatic X-11-ARIMA program proceeds as follows:

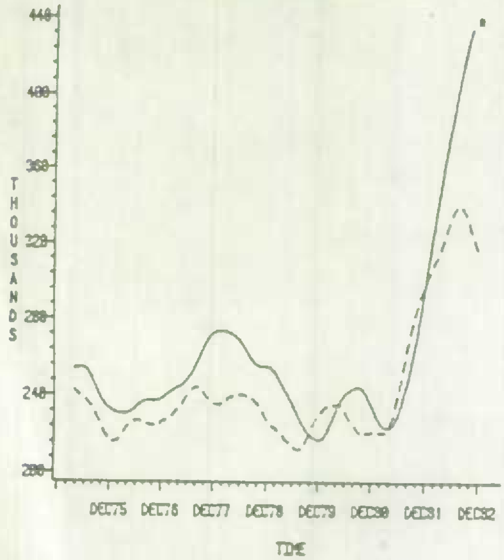
1. Three univariate ARIMA models of the general multiplicative form $(p,d,q)(P,D,Q)_s$ (Box and Jenkins, 1970) are fitted to the monthly or quarterly series that is to be seasonally adjusted. The models are
   $(0,1,1)(0,1,1)_s$
   $(0,2,2)(0,1,1)_s$
   $(2,1,2)(0,1,1)_s$

   when the series is seasonally adjusted additively. For series adjusted multiplicatively, the same models are used and the log transform is applied to the data for the first two models.
2. The series is extrapolated one year in advance; and
3. provided the extrapolations are acceptable, the ordinary X-11 method is then applied to the series thus extended.

Figure 3 shows the beneficiaries series seasonally adjusted both additively and multiplicatively, using the automatic X-11-ARIMA options. The ARIMA models that best fit and forecast the series ending in December 1982 are $(0,2,2)(0,1,1)_{12}$ when the series is seasonally adjusted additively and log $(0,2,2)(0,1,1)_{12}$ when adjusted mutliplicatively. The log $(0,2,2)(0,1,1)_{12}$ model has forecast a decrease in the series, while the $(0,2,2)$ $(0,1,1)_{12}$ model has maintained the upward trend.

Figure 3 shows the multiplicative seasonal adjustment of the beneficiaries series using both the upward trend and the downward trend extrapolations. One can see from the comparison of

CLAIMS AND BENEFICIARIES*

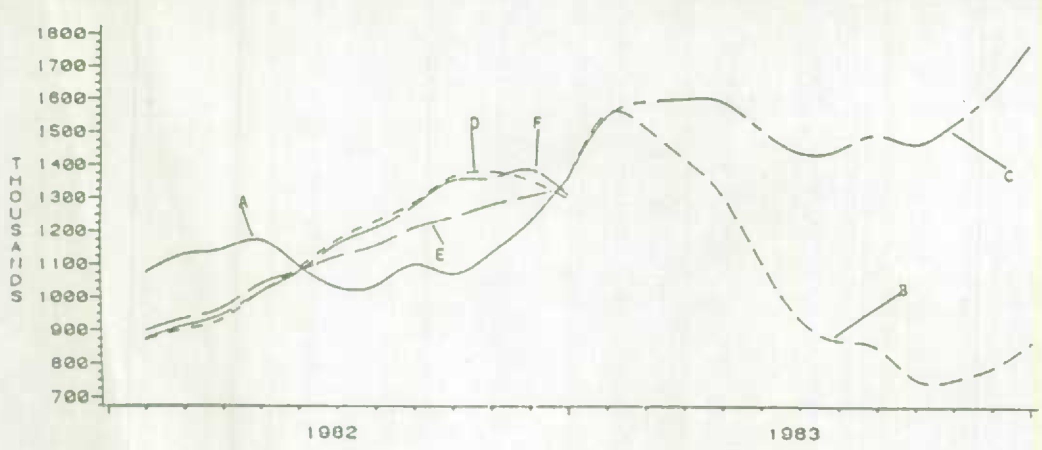


---- CLAIMS
—— BENEFICIARIES

* THE NUMBER OF BENEFICIARIES HAS BEEN DIVIDED BY 3 IN ORDER TO MAKE THE SCALE OF BOTH SERIES COMPATIBLE

FIGURE 2

BENEFICIARIES SERIES SEASONALLY ADJUSTED ADDITIVELY AND MULTIPLICATIVELY WITH DIFFERENT ARIMA EXTRAPOLATIONS



A. ORIGINAL SERIES
B. EXTRAPOLATIONS USING LOG (0,2,2)(0,1,1)
C. EXTRAPOLATIONS USING (0,2,2)(0,1,1)
D. MULTIPLICATIVE ADJUSTMENT USING ARIMA EXTRAPOLATIONS WITH LOG (0,2,2)(0,1,1)
E. ADDITIVE ADJUSTMENT USING ARIMA EXTRAPOLATIONS WITH (0,2,2)(0,1,1)
F. MULTIPLICATIVE ADJUSTMENT USING ARIMA EXTRAPOLATIONS WITH (0,2,2)(0,1,1)

FIGURE 3

figure 1 with figure 3 that the ARIMA extrapolations did not modify the multiplicative estimates of the trend-cycle in the last year. The multiplicative model is still signalling a turning point around October (downward trend, log transform) or November of 1982 (upward trend, no transform). The multiplicative model applied to either the non-extended beneficiaries series (figure 1) or the extended series is questionable.

By the end of 1983, one could see that the true turning point has actually occurred around February 1983. Thus, the October or November 1982 turning point can hardly be corrected by extrapolation when it is due to the wrong selection of the decomposition model.

Over and under-adjustment and problems of identifying the turning points occurred in other series as well. Figure 4 shows for instance, the series of "benefits paid" when seasonally adjusted multiplicatively with actual data available to the end of 1984. The seasonally adjusted series tends to oscillate systematically around the trend-cycle curve at the turning point, thus over- and underestimating the benefits paid. After the turning point, the oscillation decays to the trend-cycle curve; showing that the multiplicative model is doing poorly around the turning point. Note that this series has strong trading-day-variation which has also been removed.

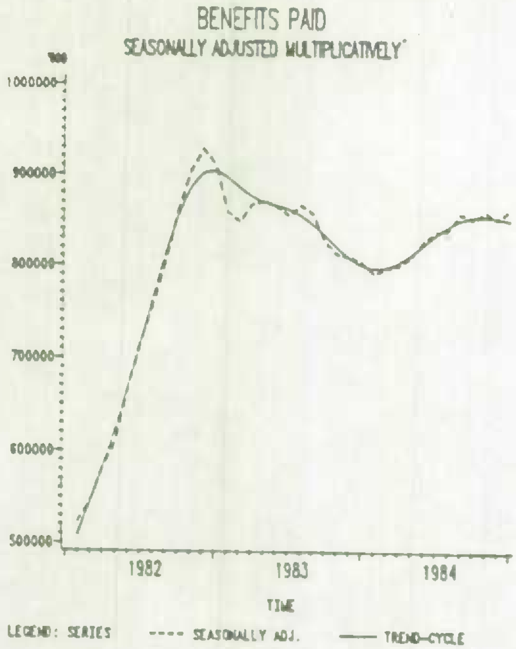


FIGURE 4

5. CONCLUSION

The seasonal adjustment of a time series is not a straightforward procedure particularly when the level of a series nearly doubles in just one year. The 1981-82 recession had a very sudden great impact not only on the structure of the series but on the estimation of the trend-cycle and seasonal components at the end of the series. Consequently, serious seasonal adjustment problems are expected to occur. The analysis of the beneficiaries series has shown that the selection of the wrong decomposition model produces a drastic overestimation of the seasonally adjusted figures in the seasonally low months. The converse is also true. Moreover, a false turning point has been signalled.

Figure 5 summarizes the criteria for seasonal adjustment that have been taken into account to overcome the problems due to the interplay between the 1981-82 recession and seasonal adjustment of the beneficiaries and claims series. The selection of the best seasonal adjustment procedure was primarily based on the first criterion.

In order to avoid over- and underestimation and false turning points in the seasonally adjusted figures, the appropriate decomposition model has to be selected. A thorough analysis of the data should be conducted by:

1. performing a model test on the series.
2. adjusting the series both additively and multiplicatively if the effort is justified. If the difference between the two adjustments becomes significant as in figure 1, one has to check for underadjustment in the seasonally high months and for overadjustment in the seasonally low months. One can also look in table D8 of the X-11-ARIMA program at the F tests for the presence of stable and moving seasonality. The decomposition model that better adjusts the series will show the higher F value for stable seasonality and the lower F value for moving seasonality. The F values for the additive decomposition model applied to the beneficiaries series are 1231.9 and 1.17 for the stable and moving seasonality respectively while the multiplicative model shows values of 709.8 and 7.9. The last value is significantly high. Therefore, the multiplicative model is not accurately estimating seasonality, especially moving seasonality.
3. Checking for turning points. For the claims series, both decomposition models have signalled a turn in August or September 1982. On the other hand, for the beneficiaries series, only the multiplicative model has signalled a turn in October 1982. Thus, either the multiplicative model is signalling a false turn or the additive model is missing the turning point. The analysis has shown this turn to be a false one resulting from the drastic overestimation of the number of seasonally adjusted beneficiaries in the seasonally low months as shown in Figure 1.
4. using a bi- or multivariate approach to accurately estimate the turning points at the end of the series. The lead-lag relationship between the claims and beneficiaries series can help to seasonally adjust the beneficiaries series. It reduces the likelihood of mistaking an irregular turn for a cyclical turning point. Since the lead is about 5 to 6 months, the September 1982 turning point in the claims series confirms that the multiplicative model applied to the beneficiaries series has signalled a false turn in October 1982. However, the leading indicator is predicting a turning

OPTIMAL SEASONAL ADJUSTMENT PROCEDURE

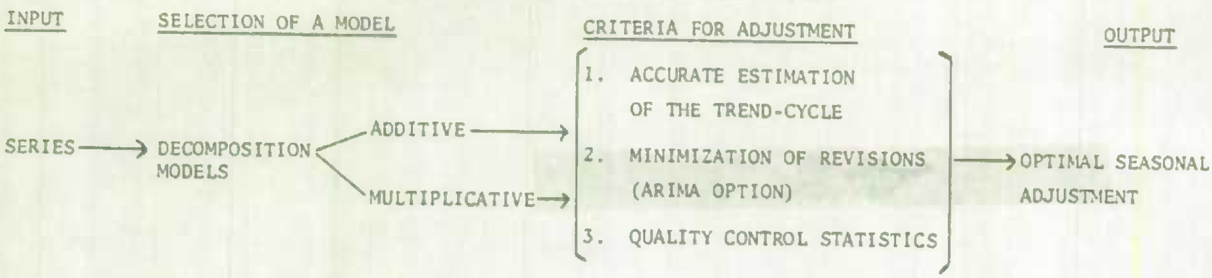


FIGURE 5

point around March 1983 in the beneficiaries series.

5. using the ARIMA option with concurrent seasonal factors. It usually gives smaller revisions to the seasonal factors whether an additive or a multiplicative seasonal adjustment is made. However, a false turning point can hardly be corrected by extrapolations when it is due to the wrong selection of the decomposition model.
6. checking both the raw and seasonally adjusted data. One cannot only rely on tests. For instance, the set of quality control statistics included in the X-11-ARIMA program is not meant to detect under- or overestimation of the series or false turning points.

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7. All the above recommendations apply if the series is not strongly affected by trading-day-variation. If trading-day-variation is present, then it must be removed before the ARIMA option is used.