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MONTHLY VERSUS ANNUAL REVISIONS OF

CONCURRENT SEASONALLY ADJUSTED SERIES(1)

by

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Introduction

After Statistics Canada's official policy of using concurrent seasonal adjustment in 1975, gradually other foreign statistical agencies followed it. The old practice for seasonally adjusting a current (monthly or quarterly) observation was to apply year-ahead seasonal factors generated from a series that ended in the month of December of the previous year. Since these projected factors were calculated ahead of the actual time they were applied, they didn't take into account the most recent information incorporated into the series. On the other hand, the use of a concurrent seasonal factor to produce a current seasonally adjusted datum implies the use of all the data in the series up to and including the current month's observation.

The main reason for using concurrent instead of seasonal factor forecasts is that the former are subject to smaller revisions as new observations are added to the series. This important result has been confirmed in several empirical studies (see among several others, Dagum 1978, Bayer and Wilcox, 1981, Kenny and Durbin, 1982, McKenzie, 1982 and 1984 and Dagum and Morry, 1984).

There are two sources influencing the size of the revisions of current seasonally adjusted data: (1) differences in the moving averages or linear filters applied to the same observation as later data become available; and (2) the innovations that enter the series with new observations. Ideally one would like to minimize revisions due to filter discrepancies. Two studies by Dagum (1982.a) and (1982.b) have shown that if the current observation is seasonally adjusted using a concurrent seasonal factor instead of a year-ahead factor, the corresponding concurrent linear filter is subject to smaller revisions than any of the year-ahead seasonal

filters. The same conclusions have been reached in a recent study by Pierce and McKenzie (1985) from a time series analysis viewpoint.

The use of concurrent seasonal factors for current seasonal adjustment poses the problem of how often should the series be revised. In this regard, Kenny and Durbin (1982) recommended that revisions should be made after one month and thereafter each calendar year. Dagum (1982.b) supported these conclusions and furthermore, recommended an additional revision at six months if the seasonal adjustment method is the X-11-ARIMA without ARIMA extrapolation. In this case, the X-11-ARIMA (Dagum, 1980) closely approximates the Census Method II-X-11 version (Shiskin, Young and Musgrave, 1967) except for changes in the treatment of outliers and the use of more accurate end weights for the seasonal moving averages.

Recently, Burridge and Wallis (1984), showed that the X-11 filters are not internally consistent in a signal extraction sense. In fact, they observed that the transfer function of the first year revised concurrent filter differed more from that of the symmetric filter (to which it should converge) than the transfer function of the concurrent filter itself. It would appear as if the transition from asymmetric to symmetric filters was not gradual for all frequencies.

This paper deals with the problem of consistency between successive filters in relation with the revision pattern of the concurrent linear filters of X-11-ARIMA and X-11.

Section 2 introduces two measures of filter revisions given by the root mean square differences between the frequency response functions of the analysed filters.

Section 3 estimates and discusses the time paths of the revision of the concurrent seasonal adjustment filters of X-11 and X-11-ARIMA for

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Section 4 [illegible]

[illegible] revisions of [illegible].

Section 5 gives the main conclusions of this study.

consecutive month-spans.

Section 4 estimates and analyses the time paths of the monthly and annual revisions of the concurrent and remaining asymmetric filters.

Section 5 gives the main conclusions of this study.

2. Measures of Filter Revisions

Under the assumption of an additive decomposition model and no replacement of extreme values, the seasonally adjusted estimates from X-11-ARIMA with and without ARIMA extrapolations are obtained by the application of a set of moving averages or linear filters. For central or middle observations, say $n+1 \leq t \leq T-n$, the filter is always the same and symmetric whereas for the remaining n observations on both ends of the series, the filters are asymmetric and different for each observation.

We can express the seasonally adjusted value, for recent years, from X-11-ARIMA and X-11 by

$$y_t^{(m)} = \sum_{j=-m}^{n} h_{m,j} x_{t-j} = h^{(m)}(B) x_t \qquad (2.1)$$

where $y_t^{(m)}$ is the seasonally adjusted estimate from a series $x_{t-n}, x_{t-n+1}, \ldots, x_t, \ldots, x_{t+m}$; $h_{m,j}$ denotes the moving average weights to be applied to the series and $h^{(m)}(B)$ denotes the corresponding linear filter using the backshift operator B, such that $B^n x_t = x_{t-n}$.

For m=o, $y_t^{(o)}$ is the concurrent seasonally adjusted value and $h^{(o)}(B)$ the corresponding concurrent filter; for m=1, $y_t^{(1)}$ is the first-period (month, quarter) revised seasonally adjusted figure and for m=n $y_t^{(n)}$ is the final seasonally adjusted value in the sense that it is estimated with a symmetric filter $h^{(n)}(B)$ where $h_{n,j} = h_{n,-j}$ for all j. For any two points in time $t+k$, $t+\ell$, $(k<\ell)$ the revision of the seasonally adjusted value is given by,

$$r_t^{(\ell,k)} = y_t^{(\ell)} - y_t^{(k)} \quad k < \ell \qquad (2.2)$$

This revision reflects: (1) the innovations introduced by the new observations $x_{t+k+1}, x_{t+k+2}, \ldots, x_{t+k+\ell}$; and (2) the differences between the two asymmetric filters $h^{(\ell)}(B)$ and $h^{(k)}(B)$. Fixing k=o and letting ℓ vary from 1 to n, the (2.2) gives a sequence of revisions of the concurrent seasonally adjusted value for different time spans or lags. The <u>total revision</u> of the concurrent estimate is obtained for $\ell = n$. Fixing $\ell = k+1$ and letting k take values from o to n-1, equation (2.2) gives the sequence of <u>single period revisions</u> of each estimated seasonally adjusted value and particularly starting at k=o we obtain the n-1 successive single-period revisions of each estimated seasonally adjusted value and before it becomes final. Fixing $\ell = k+12$ and letting k to take value from o to n-12 equation (2.2) gives the sequence of annual revisions. The revisions in which we are here interested are those introduced by filter discrepancies and these will be studied by looking at the frequency response functions of the corresponding filters.

Equation (2.1) represents a linear system where $y_t^{(m)}$ is the convolution of the input x_t and a sequence of weights $h_{m,j}$ called the <u>impulse response function</u> of the filter. The properties of this latter can be described by its Fourier transform called the <u>frequency response function</u>,

$$H^{(m)}(\omega) = \sum_{j=-m}^{n} h_{m,j}\, e^{-i2\pi\omega j} \qquad o \leq \omega \leq 1/2 \qquad (2.3)$$

where ω is the frequency response function in cycles per time unit. $H(\omega)$ fully describes the effects of the linear filter on the given input. In general, the frequency response function may be expressed in polar form by

$$H(\omega) = A(\omega) + iB(\omega) = G(\omega)\, e^{i\phi(\omega)} \qquad (2.4)$$

where $G(\omega) = [A(\omega) + B(\omega)]^{1/2}$ is called the gain of the filter and

$\phi(\omega)$ = artan $[B(\omega)/A(\omega)]$is called the phase angle of the filter and is expressed in radians. The gain and the phase angle vary with the frequency ω. For symmetric filters, the phase angle is zero or $\pm\pi$ and for asymmetric filters it can take any value between $\pm\pi$; being undefined at those frequencies where the gain function is zero.

Following Dagum (1982.a and 1982.b), we introduce next three measures based on the root mean square revision of different filters over all the frequencies.

The first measure is:

$$R^{(\ell,o)} = [2\int_o^{1/2} \| H^{(\ell)}(\omega) - H^{(o)}(\omega)\|^2 \quad d\omega]^{1/2} \qquad o \leq \omega \leq 1/2 \qquad (2.5)$$
$$\ell = 1, 2, 3, \ldots, n.$$

where $H^{(o)}(\omega)$ is the frequency response function of the concurrent seasonal adjustment filter and $H^{(\ell)}(\omega)$ the frequency response function of a filter shifted ℓ periods with respect to the concurrent. Taking into consideration that for monthly series the symmetric seasonal adjustment filter of X-11 can be well approximated with 7 years of data plus one (see Young (1968) and Wallis (1974) and similarly, for X-11-ARIMA (see Dagum 1983); the $H^{(o)}(\omega)$ corresponds to the filter applied to the last observation of a series of at least 85 data. This filter becomes central or symmetric after the series is extended with forty-two more observations and thus $H^{(42)}(\omega)$ denotes the frequency response function of the symmetric filter.

Equation (2.5) gives the time path of the concurrent filter as it approaches to the symmetric filter for ℓ =1,2,...,42.

The second measure to be used in this study refers to the differences between consecutive filters defined by

$$R^{(k+1,k)} = [2\int_o^{1/2} \| H^{(k+1)}(\omega) - H^{(k)}(\omega)\|^2 \, d\omega]^{1/2} \qquad o \leq \omega \leq 1/2 \qquad (2.6)$$
$$k = o, 1, 2, \ldots, n-1$$

Equation (2.6) gives the time path of single-period revisions of the filters as new observations enter into the series. In the case of monthly data which are those to be discussed in this study, $R^{(k+1,k)}$ gives the time path of the monthly revisions.

A third measure gives the time path of annual revisions and is defined by:

$$R^{(k+12,k)} = [2\int_o^{1/2} \| H^{(k+12)}(\omega) - H^{(k)}(\omega) \|^2 d\omega]^{1/2} \qquad o \le \omega \le 1/2 \qquad (2.7)$$
$$k = o,1,2,3,\ldots,n-12$$

Equations (2.6), and (2.7) are useful to assess the frequency of revisions of the concurrent seasonal adjustment filter as new observations enter into the series.

3. Time Path of the Concurrent Seasonal Adjustment Filters of X-11 and X-11-ARIMA

The $R^{(\ell,o)}$ $\ell=1,2,\ldots,42$ measure given in equation (2.5) has been calculated for the X-11 and X-11-ARIMA concurrent filters. The ARIMA extrapolation model applied is the classical $(0,1,1)(0,1,1)_{12}$ IMA type (Box and Jenkins, 1970) of the following form:

$$(1-B)(1-B^{12})X_t = (1-\theta B)(1-\Theta B^{12})a_t \qquad (3.1)$$

Since the extrapolations affect significantly the concurrent filter depending on the parameter values of θ and Θ, we selected some combinations of values often found when modelling economic time series. These are:

$\theta=0.40$	$\Theta=0.40$	$\theta=0.60$	$\Theta=0.40$	$\theta=0.80$	$\Theta=0.40$
$\theta=0.40$	$\Theta=0.60$	$\theta=0.60$	$\Theta=0.60$	$\theta=0.80$	$\Theta=0.60$
$\theta=0.40$	$\Theta=0.80$	$\theta=0.60$	$\Theta=0.80$	$\theta=0.80$	$\Theta=0.80$

The smaller the value of θ, the more flexible or changing the trend-cycle component is assumed to be. Similarly, the smaller the value of Θ, the more flexible or moving the seasonal component is assumed to be.

Table I below gives a summary of the values of: (1) the total revisions of the concurrent filters $R^{(42,0)}$; (2) the revisions of the concurrent filters after a year of observations has been incorporated into the series, $R^{(12,0)}(\omega)$ and (3) the revisions after 13 months i.e. $R^{(13,0)}(\omega)$.

(Place Table I about here)

We note from Table I that the total revisions of the concurrent filters are always smaller if ARIMA extrapolation are used. These observations conform to those given in Dagum 1982.a and 1982.b although these latter studies referred only to the revisions of the seasonal frequency bands whereas here we are analysing the revisions over all the frequencies. The root mean square total revision reduction ranges from 20% for $\Theta = .80$ to 6% for $\Theta = .40$.

Second, the speed of convergence of the concurrent seasonal adjustment filter to the symmetric filter is faster for X-11-ARIMA with extrapolations than without extrapolations (X-11). After 13 months $R^{(13,0)}$ represents between 88% to 100% of the total revision $R^{42,0)}$ depending on the values of Θ. For $\Theta = .40$ which implies a fast moving seasonality, the total revision is completed after the first year whereas for $\Theta = .80$ which corresponds to a more rigid or stable seasonal pattern only 88% of the total revision is corrected during the same period.

TABLE I - ROOT MEAN SQUARE REVISIONS OVER ALL FREQUENCIES OF THE CONCURRENT SEASONAL ADJUSTMENT FILTER FOR SELECTED MONTH-SPANS

Method		Total Revisions $R^{(42,0)}$	First Year Revision $R^{(12,0)}$	13 month-span Revision $R^{(13,0)}$
X-11-ARIMA-without Extrapolations (X-11)		.36	0.29	.30
X-11-ARIMA-with Extrapolations from IMA Model parameter values)				
θ=.40	Θ=.40	.34	.30	.34
θ=.40	Θ=.60	.32	.27	.31
θ=.40	Θ=.80	.30	.25	.27
θ=.60	Θ=.40	.34	.32	.34
θ=.60	Θ=.60	.32	.28	.30
θ=.60	Θ=.80	.30	.25	.26
θ=.80	Θ=.40	.34	.33	.34
θ=.80	Θ=.60	.32	.29	.30
θ=.80	Θ=.80	.30	.26	.26

It is important to point out here that the revisions from the concurrent filter to the 12 month lag filter are not monotonic for each frequency ω. In fact although $R^{(12,0)} < R^{(42,0)}$ overall the ω's, the reverse occurs for the revisions associated with the frequencies ω that fall between 0 and 0.050 which generally are attributed to trend and cyclical variations. This remark agrees with that of Burridge and Wallis (1984) who showed the presence of inconsistencies between the concurrent and the 12 month lag filters. Table II shows the revision measures $R^{(12,0)}$, $R^{(24,0)}$ and $R^{(42,0)}$ for two frequency bands, namely, $0 \leq \omega \leq 0.050$ and $0.050 < \omega < 0.,50$ which are often attributed to the trend-cycle and irregular variations respectively.

(place Table II about here)

We can see that the revisions of the low frequency band are larger after 12 months than when 24 or 42 months have been added to the series. Although not shown, smaller discrepancies are also observed for $R^{(13,0)}$ and $R^{(14,0)}$. These discrepancies disappear for most cases after 16 months and, in all cases, after 24 months where the concurrent filter revisions are equivalent to those obtained from the final filter. These results imply that second year revisions would suffice from the viewpoint of filter changes.

We also observe larger revisions of the frequencies attributed to trend and cyclical variations when the ARIMA extrapolations are used. In practice, however, these revisions would not be observed for they correspond mainly to short cycles (3 years or less) which would not be present if series is well represented by an IMA model of the $(0,1,1)(0,1,1)_s$ type used for the extrapolations.

Summarizing the above observations, the time path of the various concurrent filters shows that the use of ARIMA extrapolations is highly

beneficial from the viewpoint of: (1) the size of the total revisions which is significantly decreased; and (2) the period of time required for the concurrent filter to converge to the final symmetric filter which is also significantly decreased.

TABLE II - ROOT MEAN SQUARE REVISION OF THE CONCURRENT SEASONAL ADJUSTMENT FILTER FOR SELECTED FREQUENCY BANDS AND SELECTED MONTH-SPANS

X-11-ARIMA METHOD	12 month-span Revisions $R(12,0)$		24 month-span Revisions $R(24,0)$		Total Revisions $R(42,0)$	
	$o<\omega<.05$ Trend-cycle	$.05<\omega<.5$ Irregular	$o<\omega<.05$ Trend-cycle	$.05<\omega<.5$ Irregular	$o<\omega<.05$ Trend-cycle	$.05<\omega<.5$ Irregular
Without Extrapolations(X-11)	.09	.31	.05	.39	.05	.38
With Extrapolations from IMA Model (Parameter Values)						
$\theta=.40$ $\Theta=.40$	.24	.31	.14	.36	.15	.36
$\theta=.40$ $\Theta=.60$	.21	.27	.12	.34	.12	.34
$\theta=.40$ $\Theta=.80$	.18	.24	.11	.32	.11	.32
$\theta=.60$ $\Theta=.40$	.28	.32	.17	.37	.17	.36
$\theta=.60$ $\Theta=.60$	.24	.28	.15	.35	.15	.34
$\theta=.60$ $\Theta=.80$	.21	.25	.13	.32	.13	.32
$\theta=.80$ $\Theta=.40$	.34	.33	.20	.36	.20	.35
$\theta=.80$ $\Theta=.60$	.29	.29	.18	.34	.18	.34
$\theta=.80$ $\Theta=.80$	.26	.26	.16	.32	.16	.32

4 - Time Path of Monthly and Annual Revisions of the Seasonal Adjustment Asymmetric Filters of X-11 and X-11ARIMA

4.1 - Monthly Revisions

The $R^{(k+1,k)}$ measure of equation (2.6) has been calculated for the X-11-ARIMA concurrent filters with extrapolations from the ARIMA models discussed in the previous section and for X-11-ARIMA without extrapolations (X-11). The monthly revisions $R^{(k+1,k)}$ also measure the distance between consecutive asymmetric filters.

One important set of single-period revisions is that corresponding to the first year, that is, for time lags $\ell=k+1=1,2,\ldots,11$ where $\ell=k+1$. These eleven monthly revisions should improve the seasonal adjustment filter because of the improvement in the weight system of the 13-term Henderson trend-cycle filter which becomes symmetric after six observations have been added to the series. We will discuss later whether it is advisable or not to revise eleven times the concurrent filter in order to improve the seasonally adjusted estimates.

Another set of consecutive filter revisions of interest includes $\ell=12$ and $\ell=24$. These revisions reflect the improvement in the 3x5(7-term) seasonal moving average weights which change from year to year (being constant within the year) until they become symmetric (after three years). Finally, the revisions at $\ell=13$ and $\ell=25$ are important because they are due to the fact that in the X-11-ARIMA the seasonal estimates are forced to sum to zero (12) over each calendar year if an additive (multiplicative) decomposition model is applied.

The observations drawn from Table III can be summarized as follows. First, the pattern of monthly revisions is the same whether ARIMA extrapolations are used or not. This pattern is characterized by a rapid decrease in the

(place Table III about here)

TABLE III. Monthly Root Mean Square Revisions over all Frequencies of the Concurrent and Asymmetric Seasonal Filters of X-11-ARIMA with and without Extrapolations (X-11)

Monthly Revisions ℓ = k+1	WITHOUT EXTRAPOLATIONS(X-11) $R^{(k+1,k)}$	X-11-ARIMA WITH EXTRAPOLATIONS, $R^{(k+1,k)}$								
		Model θ=.40 Θ=.40	Model θ=.40 Θ=.60	Model θ=.40 Θ=.80	Model θ=.60 Θ=.40	Model θ=.60 Θ=.60	Model θ=.60 Θ=.80	Model θ=.80 Θ=.40	Model θ=.80 Θ=.60	Model θ=.80 Θ=.80
1	0.122	0.176	0.148	0.123	0.136	0.115	0.090	0.100	0.079	0.063
2	0.066	0.130	0.113	.098	0.102	0.087	0.074	0.078	0.065	0.054
3	0.024	0.089	0.081	.073	0.071	0.065	0.058	0.056	0.051	0.045
4	0.022	0.056	0.054	0.051	0.048	0.046	0.044	0.041	0.039	0.038
5	0.037	0.034	0.033	0.033	0.033	0.033	0.033	0.033	0.32	0.033
6	0.041	0.022	0.018	0.018	0.024	0.021	0.021	0.026	0.024	0.024
7	0.033	0.019	0.015	0.014	0.016	0.013	0.012	0.018	0.015	0.015
8	0.018	0.022	0.019	0.018	0.014	0.009	0.008	0.009	0.009	0.008
9	0.014	0.029	0.027	0.026	0.012	0.011	0.010	0.007	0.007	0.006
10	0.025	0.041	0.038	0.037	0.019	0.017	0.016	0.013	0.011	0.010
11	0.030	0.057	0.053	0.051	0.026	0.024	0.023	0.015	0.012	0.012
12	0.210	0.259	0.228	0.199	0.277	0.245	0.215	0.293	0.262	0.232
13	0.108	0.136	0.118	0.101	0.104	0.088	0.073	0.075	0.062	0.050
14	0.059	0.103	0.091	0.080	0.080	0.069	0.060	0.060	0.051	0.043
15	0.022	0.071	0.065	0.059	0.057	0.052	0.047	0.045	0.040	0.036
16	0.019	0.046	0.043	0.041	0.038	0.037	0.035	0.033	0.031	0.025
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
24	0.122	0.150	0.134	0.119	0.161	0.145	0.129	0.172	0.155	0.139
25	0.066	0.079	0.069	.060	0.058	0.050	0.042	0.040	0.034	0.028
26	0.036	0.063	0.057	.050	0.048	0.042	0.037	0.030	0.030	0.026
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
36	0.054	0.062	0.058	0.054	0.067	0.062	0.059	0.072	0.067	0.063
37	0.031	0.032	0.029	0.027	0.021	0.020	0.019	0.013	0.012	0.011

monthly revisions for ℓ=1,2, and 3; and slow thereafter till ℓ=11; then a large increase (reversal of direction) at ℓ=12 follow by a rapid decrease for ℓ=13 then another large increase at ℓ=24 followed by a rapid decrease at ℓ=25.

The significant decreases for the first three consecutive revisions are due to the improvement of the Henderson filter weights. What looks like a reversal of direction in the size of the filter revisions at ℓ=12 and ℓ=13, is due to the improvement in the seasonal weights which become less asymmetric from year to year until three full years are added to the series.

Second, the effect of the ARIMA extrapolations can be observed in the monthly revisions during the first year, particularly at ℓ =1,2 and 3 where the revisions tend to be larger for small θ and Θ whereas the opposite occurs for large θ and Θ.

Third, we note that the consecutive single-period revisions do not decrease monotonically within the year. Although the revision values are very small for $\ell \geq 4$; there are reversals of direction at ℓ =5,6 and 10 when no extrapolations are used. There is only one reversal and at a later lag if ARIMA extrapolations are used, being at ℓ=7 for θ =.40 and at ℓ =10 for θ =.60 and Θ=.80. This pattern repeats for the second year after a large jump at ℓ=12 and again during the third year after lag ℓ=24.

Since the monthly revisions during the first year are not monotonically decreasing, it is not advisable to revise any time a new observation enters into the series. Revising eleven times the concurrent filter will introduce unwanted revisions because the distance between consecutive

asymmetric filters does not decrease monotonically as the filters approach to $\ell=12$. Although not shown here, this inconsistency of the distance between asymmetric filters is mainly due to the phase angle of the filters and affects particularly the high frequencies ω associated with the irregulars. This type of inconsistency between the distances of consecutive asymmetric filters does not imply however that the time path of the total distance of each asymmetric filter with respect to the final is inconsistent. In fact, the total distances of each asymmetric filter, i.e. $R^{(42,k)}$, $k=0,1,2,\ldots,41$ decrease monotonically with increasing k.

Finally, we observe that the two largest single period revisions occur at $\ell = 1$ and $\ell=12$.

4.2. Annual Revisions

The $R^{(k+12,k)}$ measure of equation (2.7) has been calculated for the X-11-ARIMA asymmetric filters with and without ARIMA extrapolations.

Table IV shows that for each asymmetric filter, $k=0,1,2,\ldots,30$, its corresponding annual revision converges monotonically and very fast to zero. For example, the revision of the concurrent filter of X-11 after 12 months (first annual revision) is 0.29, its second annual revision is 0.20 and its third annual revision is 0.11. Similar pattern is followed by the remaining asymmetric filters. This monotonic convergence holds for the root mean square revisions over all frequencies. For the band of low frequencies associated with cyclical variations this monotonic convergence is not observed at $\ell=12$ as discussed in Section 3.

Table IV also shows that the size of the annual revisions are rather constant for consecutive filter within each year that is, from $\ell=12$ to $\ell=23$, $\ell=24$ to $\ell=35$ and $\ell=37$ to $\ell=42$. This pattern of annual

TABLE IV - ANNUAL ROOT MEAN SQUARE REVISIONS OVER ALL FREQUENCIES OF THE CONCURRENT AND ASYMMETRIC FILTERS OF X-11-ARIMA WITH AND WITHOUT ARIMA EXTRAPOLATIONS (X-11)

Annual Revisions $\ell=k+12$	(k)	Without Extrapolation (X-11) $R^{(k+12,k)}$	X-11-ARIMA with Extrapolations, $R^{(k+12,k)}$								
			Model $\theta=.40$ $\Theta=.40$	Model $\theta=.40$ $\Theta=.60$	Model $\theta=.40$ $\Theta=.80$	Model $\theta=.60$ $\Theta=.40$	Model $\theta=.60$ $\Theta=.60$	Model $\theta=.60$ $\Theta=.80$	Model $\theta=.80$ $\Theta=.40$	Model $\theta=.80$ $\Theta=.60$	Model $\theta=.80$ $\Theta=.80$
12	(0)	0.29	0.30	0.27	0.23	0.31	0.28	0.25	0.33	0.29	0.26
13	(1)	0.27	0.35	0.31	0.27	0.34	0.30	0.26	0.34	0.30	0.26
14	(2)	0.26	0.35	0.31	0.26	0.34	0.30	0.26	0.34	0.30	0.26
15	(3)	0.26	0.34	0.30	0.26	0.34	0.30	0.26	0.34	0.30	0.26
16	(4)	0.26	0.33	0.30	0.26	0.33	0.29	0.26	0.33	0.29	0.26
17	(5)	0.26	0.33	0.29	0.26	0.33	0.29	0.26	0.33	0.29	0.26
18	(6)	0.28	0.34	0.30	0.26	0.34	0.30	0.26	0.34	0.30	0.26
19	(7)	0.27	0.34	0.30	0.26	0.34	0.30	0.26	0.34	0.30	0.26
20	(8)	0.27	0.34	0.30	0.26	0.34	0.30	0.26	0.34	0.30	0.26
21	(9)	0.27	0.34	0.30	0.26	0.34	0.30	0.26	0.34	0.30	0.26
22	(10)	0.27	0.34	0.30	0.26	0.34	0.30	0.26	0.34	0.30	0.26
23	(11)	0.27	0.34	0.31	0.26	0.34	0.30	0.26	0.34	0.30	0.26
24	(12)	0.20	0.19	0.17	0.15	0.20	0.18	0.16	0.20	0.18	0.16
25	(13)	0.18	0.22	0.19	0.17	0.20	0.18	0.16	0.20	0.18	0.16
26	(14)	0.16	0.21	0.19	0.17	0.20	0.18	0.16	0.20	0.18	0.16
27	(15)	0.16	0.20	0.18	0.16	0.20	0.18	0.16	0.19	0.17	0.16
28	(16)	0.16	0.20	0.17	0.16	0.19	0.18	0.16	0.20	0.17	0.16
29	(17)	0.16	0.19	0.17	0.16	0.19	0.18	0.16	0.20	0.17	0.16
30	(18)	0.16	0.20	0.17	0.16	0.20	0.18	0.15	0.20	0.17	0.15
31	(19)	0.17	0.20	0.17	0.16	0.20	0.18	0.15	0.20	0.17	0.15
32	(20)	0.16	0.20	0.17	0.16	0.20	0.18	0.15	0.20	0.17	0.15
33	(21)	0.16	0.20	0.18	0.16	0.20	0.18	0.16	0.20	0.17	0.15
34	(22)	0.16	0.20	0.18	0.16	0.20	0.18	0.16	0.20	0.17	0.15
35	(23)	0.16	0.20	0.18	0.16	0.20	0.18	0.16	0.20	0.17	0.15
36	(24)	0.11	0.10	0.09	0.08	0.09	0.09	0.08	0.09	0.09	0.08
37	(25)	0.09	0.10	0.09	0.08	0.09	0.09	0.08	0.09	0.08	0.08
38	(26)	0.08	0.10	0.09	0.08	0.09	0.09	0.08	0.09	0.08	0.08
39	(27)	0.07	0.09	0.08	0.08	0.09	0.09	0.08	0.09	0.08	0.08
40	(28)	0.07	0.08	0.07	0.07	0.08	0.08	0.07	0.08	0.07	0.07
41	(29)	0.07	0.08	0.07	0.07	0.08	0.08	0.07	0.08	0.07	0.07
42	(30)	0.07	0.08	0.07	0.07	0.08	0.08	0.07	0.08	0.07	0.07

revisions implies that all changes observed in month-to-month comparisons within the same year are attributed mainly to the innovations entering into the series.

However, the most common practise of revising current seasonally adjusted data consists of keeping constant the concurrent estimate from the time it appears until the end of the year and then revising annually the current and earliest years, generally up to three. Consequently, first-year revisions are given by $R^{(0,0)}$, $R^{(1,0)}$, $R^{(2,0)},\ldots,R^{(11,0)}$. Second-year revisions by $R^{(12,0)}$, $R^{(13,1)}$, $R^{(14,2)},\ldots,$ $R^{(23,11)}$; and third-year revisions by $R^{(24,12)}$, $R^{(25,13)}$, $R^{(26,14)},\ldots,$ $R^{(35,23)}$. Table IV shows the second and third-year revisions and Table V, first-year revisions. Given that these latter are generally the most relevant for decision making, the revisions are shown for two frequency bands and over all frequency. Since the pattern is similar when using ARIMA extrapolation only two of the cases are shown.

(place Table V about here)

We can observe that there is a monotonic increase in the revisions of the low frequencies from $\ell=1$ to $\ell=3$ or 4 and thereafter remaining rather constant. On the other hand, the revision of the high frequencies seems to be rather constant for $\ell\geq 2$ if ARIMA extrapolations are used and for $\ell\geq 7$ if no extrapolations are made.

The advantage of this common scheme of revisions is that when doing month to month comparisons, all changes within the first year are due mainly to the innovations for the filter applied during the current year is always the same, i.e. the <u>concurrent seasonal adjustment filter.</u> Furthermore, the filters of the previous years are modified by almost the same amount within each year with the only exceptions of $\ell=1$, 2, 12 and 13 in most cases.

TABLE V - FIRST YEAR REVISIONS OF THE CONCURRENT SEASONAL ADJUSTMENT FILTER FOR SELECTED FREQUENCY BANDS AND OVERALL FREQUENCIES

Root Mean Square Revisions $R^{(\ell,o)}$	X-11-ARIMA								
	Without Extrapolation (X-11)			With Extrapolations					
				θ =.40 Θ=0.80			θ=.80 Θ=.80		
ℓ	$o\leq\omega\leq.05$ Trend-cycle	$.05<\omega\leq.5$ Irregu-lar	$o\leq\omega\leq.5$ Total	$o\leq\omega\leq.05$ Trend-cycle	$.05<\omega\leq.5$ Irregu-lar	$o\leq\omega\leq.5$ Total	$o\leq\omega\leq.05$ Trend-cycle	$.05<\omega\leq.5$ Irregu-lar	$o\leq\omega\leq.5$ Total
1	.02	.13	.12	.04	.13	.12	.05	.06	.06
2	.03	.14	.13	.07	.13	.13	.08	.08	.08
3	.04	.14	.13	.09	.13	.13	.11	.08	.09
4	.04	.14	.13	.10	.13	.13	.13	.08	.09
5	.04	.16	.15	.11	.13	.13	.14	.08	.09
6	.04	.18	.17	.11	.13	.13	.15	.08	.09
7	.05	.17	.16	.11	.13	.13	.15	.08	.09
8	.05	.17	.16	.11	.13	.13	.15	.08	.09
9	.05	.17	.16	.11	.13	.13	.15	.08	.09
10	.05	.17	.16	.11	.14	.14	.15	.08	.09
11	.05	.17	.16	.11	.14	.14	.15	.08	.09

4.3. Combining Monthly and Annual Revisions

The results discussed in sections 4.1 and 4.2 suggest that a better scheme of revisions than the common practice should include monthly as well as annual revisions since the largest single period revisions occur at $\ell=1$ and 12. It is expected that: (1) adjusting concurrently each month, say from January to November and revising only once when the next month is available and (2) adjusting concurrently December when it first appears and then revising the first year and earlier years when January is added, should improve the reliability of the filter applied during the current year while maintaining simultaneously the filters homogeneity for month to month comparisons.

The first-year revisions would then be $R^{(1,1)}$, $R^{(2,1)}$, $R^{(3,1)}$,..., $R^{(10,1)}$ and $R^{(11,1)}$. These revisions are shown in Table VI.

(Place Table VI about here)

We can see that the pattern is very similar to that of the concurrent filter but the size of the revisions are smaller in all cases which agrees with our expectations.

This scheme of combining monthly and annual revisions has shown to produce smoother seasonally adjusted series and smaller revisions when applied to real data (Kenny and Durbin, 1982 and Dagum and Morry, 1984).

TABLE VI - FIRST-YEAR REVISIONS OF THE FIRST-MONTH REVISED SEASONAL ADJUSTMENT FILTER FOR SELECTED BANDS AND OVERALL FREQUENCIES

Root Mean Square Revision $R^{(\ell,1)}$	X-11-ARIMA								
	without extrapolations (X-11)			with extrapolations					
				$\theta = .40$ $\Theta = .80$			$\theta = .80$ $\Theta = .80$		
ℓ	$0\leq\omega\leq.05$ Trend-cycle	$.05<\omega\leq.5$ Irregu-lars	$0\leq\omega\leq.5$ Total	$0\leq\omega\leq.05$ Trend-cycle	$.05<\omega\leq.5$ Irregu-lars	$0\leq\omega\leq.5$ Total	$0\leq\omega\leq.05$ Trend-cycle	$.05<\omega\leq.5$ Irregu-lars	$0\leq\omega\leq.5$ Total
2	.01	.07	.07	.03	.10	.10	.04	.06	.06
3	.02	.07	.07	.05	.10	.10	.07	.07	.07
4	.02	.07	.07	.07	.10	.10	.09	.07	.07
5	.02	.08	.08	.07	.10	.10	.11	.07	.08
6	.03	.10	.10	.08	.11	.11	.11	.07	.08
7	.03	.11	.11	.08	.11	.11	.12	.07	.08
8	.03	.11	.11	.08	.11	.11	.12	.07	.08
9	.03	.11	.11	.08	.11	.11	.12	.07	.08
10	.03	.12	.11	.08	.11	.11	.12	.07	.08
11	.03	.12	.12	.09	.12	.12	.12	.07	.08

5. Conclusions

This study has addressed the problem of how often the concurrent seasonal adjustment filter of X-11-ARIMA with and without extrapolations should be revised. It is shown that:

(I) The time path of the concurrent filter for consecutive month-spans, $\ell=1,2,\ldots,41$ approaches nearly monotonically to the final symmetric filter ($\ell=42$). The use of the ARIMA extrapolation option decreases the size of the total revision while increasing the concurrent filter's speed of convergence to the final symmetric filter. However, it has been observed that the revisions of the low frequencies are larger after 12 months than when 24 or 42 have been added to the series. This inconsistency was already noted by Burridge and Wallis (1984) when fitting ARIMA models to the transfer functions of the concurrent and first-year revised filters. This inconsistency disappears for most cases after 16 months and, in all cases, the revisions of the concurrent filter are equivalent to those obtained from the final filter after 24 months have been added to the series. These results imply that second-year revisions should suffice from the viewpoint of filter changes.

(II) The monthly revisions of the concurrent filter do not approach monotonically neither to the annual nor to the final filters. The larger one-single period revisions occur at $\ell=1,2,3,12,13,24$, and 25. There are significant decreases for the first three consecutive revisions due to the improvement of the end weights of the Henderson trend-cycle filter. There is a reversal of direction in the size of the filter revision at $\ell=12$ and $\ell=24$ due to an improvement in the seasonal weights which become less asymmetric from year to year until three full years are added to the series. There are two large decreases at $\ell=13$ and $\ell=25$

which are due to the fact that, the seasonal estimates are forced to add to zero (12) over each calendar year if an additive (multiplicative) decomposition model is assumed.

The annual revisions of the concurrent filter and remaining monthly asymmetric filters of the first year approach monotonically to the final filter in root mean square over all the frequencies but not for each frequency; particularly, those frequencies associated with the trend-cycle component are revised more as compared to the total revision (distance between concurrent and final filter). It is also observed that the annual revisions are rather constant for each filter within the same year.

Taking into consideration the patterns of monthly and annual revisions, the <u>best combination of frequency of revision of the concurrent filter</u> would be to revise when a new month appears, keep the estimate constant for the remainder of the year and then, revise annually when the first month of the next year is available. This scheme offers the following advantages: (1) by revising each month once, the reliability of the concurrent filter increases significantly and since the revised filter is kept constant during the first year, changes in month-to-month comparisons are due only to the innovations; and (2) by revising annually, the reliability of the filters improve while maintaining the comparability of consecutive filters since they are all revised by almost a constant amount (without introducing frequency distortions) within each year. This scheme has shown to produce smoother seasonally adjusted series and smaller revisions when applied to real data by Kenny and Durbin (1982) and Dagum and Morry (1984).

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