

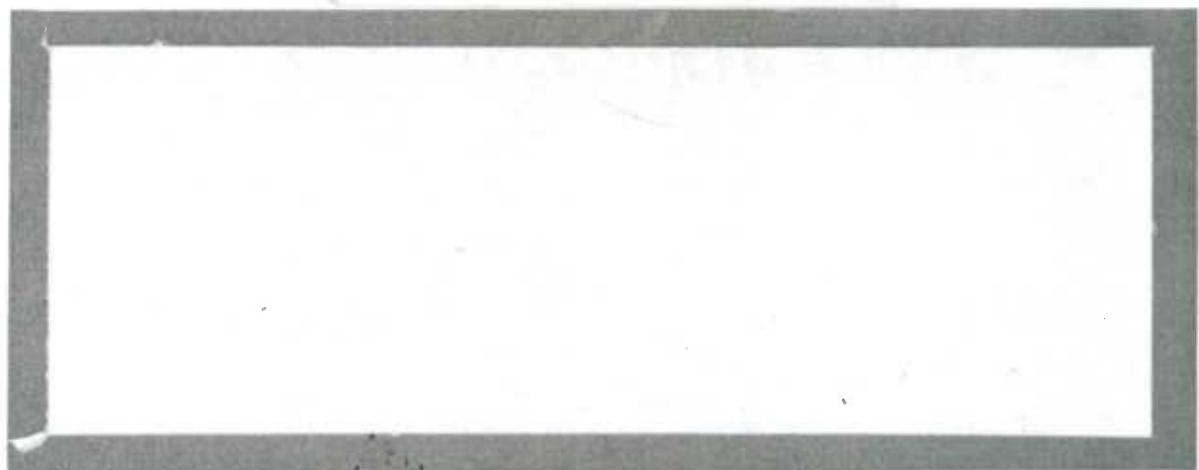
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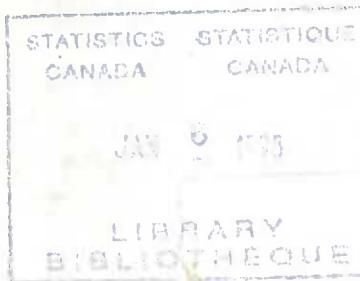
## Direction de la méthodologie

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TIME SERIES RESEARCH & ANALYSIS DIVISION  
METHODOLOGY BRANCH



DESIGN CRITERIA FOR  
13 TERMS HENDERSON END-WEIGHTS

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Comments are welcome.

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## SUMMARY

Criteria for the design of 13 terms Henderson asymmetric end-weight are described. One of these criteria can reproduce exactly the Henderson asymmetric weights of the X-11 seasonal adjustment method. The theoretical bias, variance, expected revision and revision variance of the trend-cycle estimators constructed with the criteria are analyzed. Also, an empirical comparison is done with seven time series. Finally, conclusions are drawn about the choice of a design criteria.

## RESUME

Des critères pour la conception de moyenne-mobiles à poids asymétriques de type Henderson à treize termes sont décrits. Un de ces critères permet la reproduction des poids asymétriques de type Henderson tels qu'on les retrouve dans la méthode de désaisonnalisation X-11. Sont analysés théoriquement, le biais, la variance, la révision espérée et la variance de la révision produit par les estimateurs de tendance-cycle construits à l'aide des critères. Aussi, on compare empiriquement les estimateurs cyclo-tendanciels avec sept séries chronologiques. Finalement, des conclusions sont tirés quant au choix d'un critère de conception de moyenne-mobiles.

## 1. Introduction

In the X-11 (Shiskin, Young and Musgrave, 1967) and X-11-ARIMA procedures (Dagum, 1980) for seasonal adjustment, the 9 terms, 13 terms and 23 terms Henderson moving averages (Henderson, 1916) are used for the estimation of the trend-cyclical movements in monthly time series. For these m.a., symmetric and asymmetric set of weights are available. Preliminary estimates of the smoothed seasonally adjusted series, at its ends, are provided by the asymmetric weights while elsewhere final estimates are given by the symmetric weights. Consequently, as more information becomes available, the smoothed values at the right end of the series are revised until symmetric weights can be applied.

Ideally, the asymmetric set of weights should provide an estimate consistent with the one given by the symmetric weights and minimizing the amplitude of the revisions. The purpose of this study is to compare some design criteria for 13 terms Henderson end weights to see how well the two properties stated are satisfied.

The design criteria compared are described in the next section. A theoretical comparison of the criteria is presented in the third section. An empirical comparison is done in section 4. Finally, section 5 concludes.

## 2. The Criteria

The criteria described here are for the design of asymmetric set of weights for the 13 terms Henderson moving averages, but they can easily be modified for  $2m+1$  terms moving averages.

Before going to the criteria themselves, some notation will be introduced. Let  $y_t$  be the seasonally adjusted monthly series,  $x_t$  its trend-cycle component and  $n_t$  the noise or irregular component. It is assumed that

$$y_t = x_t + n_t \quad (2.1)$$

with  $E[n_t] \stackrel{\Delta}{=} 0$ .

A final estimate of  $x_t$ ,  $\hat{x}_t^{(m)}$ , is obtained by application of the symmetric filter  $h_m(B)$

$$\hat{x}_t^{(m)} = h_m(B)y_t = \sum_{j=-m}^m h_{m,j} y_{t-j} \quad (2.2)$$

where  $B$  is the lag operator and  $h_{m,j}$  ( $= h_{m,-j}$ ) are the weights of the filter. This filter is a moving average of  $2m+1$  terms ( $m=6$  for 13 terms). Preliminary estimates for points near the end of the series are given by asymmetric filters

$$\hat{x}_t^{(i)} = h_i(B)y_t = \sum_{j=-i}^m h_{i,j} y_{t-j}. \quad (2.3)$$

where  $i$  is the number of values future to  $y_t$  entering the moving average.

At times  $t+i$  and  $t+k$ ,  $0 < i < k < m$ , two smoothed values of  $y_t$  can be calculated and thus the revision is defined as

$$r_t^{(i,k)} = \hat{x}_t^{(k)} - \hat{x}_t^{(i)}. \quad (2.4)$$

## 2.1 The B.L.U.E. Criterion

Wallis (1981) has shown, using the approach of Hannan (1970, pp. 186-187), that the Henderson symmetric and some asymmetric filters can be obtained by a generalized least squares polynomial regression subject to an error structure, which gives the best linear unbiased estimator. In that context, the trend-cycle is assumed to follow, locally, a cubic

$$x_t = \sum_{l=0}^3 c_l t^l \quad (2.5)$$

and, the noise, the moving average process

$$n_t = (1-B)^3 a_t, \quad a_t \sim NI(0, \sigma_a^2). \quad (2.6)$$

The fitted values given by the B.L.U.E. are

$$\hat{x}_t^{(i)} = \underline{h}'_i \underline{y} = \underline{z}'_t (\underline{Z}' \underline{V}^{-1} \underline{Z})^{-1} \underline{Z}' \underline{V}^{-1} \underline{y} \quad (2.7)$$

where  $\underline{h}_i$  is a column vector containing the coefficients of  $h_i(B)$  with the coefficients of the less recent data entering first;  $\underline{y} = (y_{t-m}, \dots, y_t, \dots, y_{t+i})'$  is a column vector containing  $m+1+i$  seasonally adjusted values;

$\underline{z}'_t = (1, t, t^2, t^3)$ ;  $\underline{Z}$  is an  $(m+1+i) \times 4$  matrix whose  $j$ th column comprises the  $(j-1)$ th powers of the integers  $1, \dots, m+1+i$ ;  $\underline{V}$  is the variance-covariance matrix of the error (noise) process and is a symmetric band matrix with elements on the main diagonal and first three sub-diagonals equal to 20, -15, 6 and -1 respectively, all other elements being zero.

By setting  $m=6$  and  $t=m+1=7$  and considering in turn  $i=0, 1, \dots, 6$  equation (2.7) gives a set of six asymmetric and one symmetric filters minimizing the criterion of Henderson (1916). The coefficients are in

table A-1 and it can be verified that the symmetric ones correspond to the published values for the 13 terms Henderson moving average.

## 2.2 The MMSR Criterion

The second and new criterion to design asymmetric end weights consists in minimizing the mean squared revision between the final estimate and a preliminary estimate when the trend-cycle is a cubic and the noise is given by (2.6). Which can be expressed by

$$\min E [(r_t(i,m))^2] \quad (2.8)$$

$$\text{subject to } h_i(1) = \sum_{j=-i}^m h_{i,j} = 1.$$

As is, expression (2.8) can lead to an algebraic solution for the  $h_{i,j}$  expressed in terms of the coefficients of the cubic and the variance of the noise. In order to obtain a solution, one must develop a more explicit form for the mean squared revision. By definition one obtains

$$E[r_t(i,m)^2] = E^2[r_t(i,m)] + \text{Var}[r_t(i,m)],$$

$$\text{where } r_t(i,m) = (h_m(B) - h_i(B))(x_t + n_t).$$

Since  $E[n_t] = 0$ , only  $x_t$  contributes to the mean of the revision. Using that  $h_m(B)$  reproduces exactly a cubic and  $h_i(B)$  a constant, one is lead to

$$E[r_t(i,m)] = \sum_{\ell=1}^3 c_\ell [t^\ell - \sum_{j=-i}^m h_{i,j} (t-j)^\ell]. \quad (2.9)$$

The trend-cycle  $x_t$  is considered to be deterministic, this implies that only  $n_t$  contributes to the variance of the revision. Thus the variance can be expressed as

$$\text{Var}[r_t(i,m)] = \sigma_a^2 \sum_{j=-m}^{m+3} g_j^2 \quad (2.10)$$

where  $g_j$  is the coefficient of  $B^j$  in the polynomial operator

$$g(B) = (h_m(B) - h_1(B))(1-B)^3.$$

From equations (2.9) and (2.10), it is clear that the mean squared revision can only be expressed as a function of the parameters  $c_1$ ,  $c_2$ ,  $c_3$  and  $\sigma_a^2$ . This is undesirable in the sense that the purpose of this exercise is to find asymmetric set of weights which can be used with any time series (e.g. for any values of the parameters). A way to partially get around this difficulty is to approximate (2.8) by minimizing the total MSR of three particular cases of (2.8). These are the cases where  $x_t$  is a pure straight line, where  $x_t$  is a pure quadratic and where  $x_t$  is a pure cubic. For each of these cases, the parameter  $c_\ell$  (coefficient of  $t^\ell$ ) may be expressed as a function of  $\sigma_a^2$  in evaluating the I/C ratios as defined in Shiskin et al (1967).

The I/C ratio is the average of the absolute values of the first differenced irregulars (noise) divided by the average of the absolute values of the trend-cycle first differences. Let us first consider the numerator of the ratio, it can be expressed as

$$I = \frac{1}{2m} \sum_{j=-m}^{m-1} |\Delta n_{t-j}| \quad (*)$$

Since the stochastic process  $n_t$  is stationary, ergodic and gaussian, the new process  $|\Delta n_t|$  has the same properties, which means that the above expression can be evaluated using

---

(\*) Only  $2m+1$  time periods appear in the sum, since the hypothesis of MA(3) noise and cubic trend-cycle are valid only for the length of the  $2m+1$  terms Henderson moving average (e.g.  $2m+1$  time periods).

$$I = E[|\Delta_{nt}|] = \left\{ \frac{2}{\pi} \sigma_{\Delta nt}^2 \right\}^{1/2}$$

where  $\sigma_{\Delta nt}^2 = E[(\Delta_{nt})^2] = E[((1-B)^4 a_t)^2] = 70 \sigma_a^2$ ,

thus  $I = \left( \frac{140}{\pi} \sigma_a^2 \right)^{1/2}$  (2.11)

Let us now consider the denominator for the three cases of  $x_t$  mentioned earlier.

i) If  $x_t = c_0 + c_1 t$  the denominator becomes

$$C = \frac{1}{2m} \sum_{j=-m}^{m-1} |x_{t-j} - x_{t-j-1}| = |c_1|$$

ii) If  $x_t = c_0 + c_2 t^2$  the denominator is

$$C = \frac{1}{2m} \sum_{j=-m}^{m-1} |c_2| \cdot |2(t-j)-1|$$

at  $t=0$  (time point for which the estimation is done)  $C=m|c_2|$ .

iii) If  $x_t = c_0 + c_3 t^3$  the denominator is expressed by

$$C = \frac{1}{2m} \sum_{j=-m}^{m-1} |c_3| \cdot |3(t-j)^2 - 3(t-j)+1|$$

at  $t=0$  (time point for which the estimation is done)  $C=m^2|c_3|$ .

With the expressions for  $I$  and  $C$  above one can derive a formula to approximate (2.8) by a function of  $I/C$ . This formula, for  $t=0$  and  $m=6$ , is

$$\frac{E[(r_0^{(i,6)})^2]}{\sigma_a^2} = \frac{140}{\pi(I/C)^2} \sum_{l=1}^3 \left| \sum_{j=-i}^6 h_{i,j} (-j)^l \right|^2 + 3 \sum_{j=-6}^9 g_j^2 \quad (2.12)$$

Integrating over  $I/C$  on the domain  $[1.0, 3.5]$ , values for which the 13 terms Henderson m.a. is applied in the X-11 method, one obtains the following integrated criterion

$$\text{IMSR} = \frac{40}{\pi} \sum_{l=1}^3 (36)^{l-1} \left[ \sum_{j=-i}^6 h_{i,j} (-j)^l \right]^2 + 3 \sum_{j=-6}^9 g_j^2 \quad (2.13)$$

which can be minimized to obtain end weights.

End weights has been derived from the minimization of the criterion (2.13) above subject to the constraint that the weights sum to 1. They are presented in table A-2.

### 2.3 The X-11 Criterion

The criterion that will be derived here is a particular case of the MMSR. Its minimization gives the asymmetric weights of the 13-term Henderson moving average as published. This is not necessarily the one used by Shiskin et al (1967), but it is not possible to verify the statement since nothing has been written on such a criterion. The derivation is based on two hypothesis. One is that the trend-cycle is a straight line and the other that the noise is white (a series of normal independent random variables identically distributed). In this particular case the MSR is expressed by (using expressions similar to (2.9) and (2.10)):

$$E[(r_t^{(i,m)})^2] = c_1^2 (t - \sum_{j=-i}^m h_{i,j} (t-j))^2 + \sigma_a^2 \sum_{j=-m}^m (h_{m,j} - h_{i,j})^2 \quad (2.14)$$

where  $h_{i,j} \stackrel{\Delta}{=} 0$  for  $j = -m, \dots, -i-1$ ,  $c_1$  is the slope of the line and  $\sigma_a^2$  is the variance of the noise.

As in section 2.2 the ratio I/C can be calculated and one obtains

$$I/C = (4\sigma_a^2/\pi)^{1/2} / |c_1|.$$

Thus, setting  $t=0$  the following criterion is obtained for  $m=6$ :

$$\frac{E[(r_o^{(i,6)})^2]}{\sigma_a^2} = \frac{4}{\pi(I/C)^2} \left( \sum_{j=-i}^6 h_{i,j} j^2 + \sum_{j=-6}^6 (h_{6,j} - h_{i,j})^2 \right). \quad (2.15)$$

Minimizing this expression with  $I/C=3.5$  (the most noisy situation where the 13 terms Henderson is applied) and subject to the constraint that  $\sum_{j=-i}^6 h_{i,j} = 1$  the X-11 Henderson asymmetric end weights are obtained. They appear in Table A.3 with 7 decimals.

#### 2.4 Other design criteria

Two design criteria found empirically are outlined in the present section. The first one is a suggestion of Kenny and Durbin (1981) about how X-11 13-terms Henderson asymmetric filters may have been constructed. They suggested that the asymmetric weights may have been calculated by assuming that the series to be smoothed by the symmetric weights can be extended by a straight line fitted by ordinary least squares. Following this the weights presented in Table A-4 are obtained.

The second criterion is a modified version of the suggestion of Kenny and Durbin. Cholette (1983) suggested that the straight line should be fitted by generalized least squares, where the error structure is given by model (2.6). He obtained the weights presented in Table A-5.

### 3. Theoretical Comparison

The criteria described in section 2 lead to different estimators for the trend-cycle of a seasonally adjusted series at its last time periods. In this section the bias, variance, expected revision and revision variance of these estimators are analyzed theoretically, under the hypothesis that  $x_t$  is a cubic and  $n_t$  follows model (2.6). Frequency response functions (gain and phase-shift) are also presented for the last period filters.

#### 3.1 The Measures of Comparison

The bias of  $\hat{x}_t^{(i)}$ , since  $E[n_t] = 0$ , is given by

$$E[\hat{x}_t^{(i)} - x_t] = \sum_{\ell=1}^3 c_\ell \left( \sum_{j=-i}^m h_{i,j} (-j)^\ell - t^\ell \right). \quad (3.1)$$

As the  $c_\ell$ 's are unknown, three bias ratios have been used. They are calculated in setting  $t=0$  in (3.1) and in considering in turn that  $x_t$  is a pure straight line, a pure quadratic or a pure cubic. One obtains

$$\frac{E[\hat{x}_t^{(i)} - x_t]}{c_\ell} = \sum_{j=-i}^m h_{i,j} (-j)^\ell, \quad \ell = 1, 2, 3.$$

The variance of  $\hat{x}_t^{(i)}$ , since  $\text{Var}[x_t] = 0$ , is given by

$$\text{Var}[\hat{x}_t^{(i)}] = E[(h_i(B)n_t)^2].$$

The variance of the  $a_t$ 's in model (2.6) is unknown, thus a variance ratio is used to do the comparison which is

$$\frac{\text{Var}[\hat{x}_t^{(i)}]}{\sigma_a^2} = \sum_{j=-i}^{m+3} p_j^2$$

where  $p_j$  is the coefficient of  $B^j$  in the polynomial operator

$$p(B) = h_i(B)(1-B)^3.$$

The expected total revision is given by equation (2.9) which allows to calculate three mean revision ratios

$$\frac{E[r_o^{(i,m)}]}{c_{\ell}} = \sum_{j=-i}^m h_{i,j} (-j)^{\ell} \quad \ell = 1, 2, 3.$$

These are exactly the bias ratios multiplied by minus one.

The total revision variance is expressed by (2.10) from this expression a revision variance is obtained as:

$$\frac{\text{var}[r_t^{(i,m)}]}{\sigma_a^2} = \sum_{j=-i}^{m+3} g_j^2$$

where  $g_j$  is the coefficient of  $B^j$  in the polynomial operator

$$g(B) = (h_m(B) - h_i(B))(1-B)^3.$$

The other measure used to compare the different estimators is the frequency response function of the  $h_i(B)$  filters, this is computed by

$$H_i(f) = \sum_{j=-i}^m h_{i,j} \exp(-z2\pi f j), \quad 0 \leq f \leq 1/2$$

where  $f$  is the frequency in cycle per month and  $z = \sqrt{-1}$ . This function is complex and thus can be expressed by two functions called the gain and the phase-shift, these are respectively given by

$$G_i(f) = ||H_i(f)|| = (\text{Re}(H_i(f))^2 + \text{Im}(H_i(f))^2)^{1/2}$$

and  $\phi_i(f) = \arctan \left[ \frac{\text{Im}(H_i(f))}{\text{Re}(H_i(f))} \right]$ .

$\phi_i(f)$  has its units in radians but it can be expressed in months by the transformation

$$\phi'_i(f) = \phi_i(f)/2\pi f, \quad 0 \leq f \leq 1/2.$$

### 3.2 Discussion of results

The bias ratios and variance ratios, calculated assuming that the trend-cycle follows equations (2.5) and (2.6), are shown in tables B-1 and B-2 respectively. It can be observed that BLUE weights are unbiased; Kenny-Durbin and Cholette weights are unbiased if the trend-cycle is a straight line but they have large bias ratios otherwise; MMSR and X-11 weights are always biased but the first set of weights less than the second.

For the far end-point filter ( $i=0$ ), the different estimators can be ranked as follows according to their variance (beginning with the lowest variance): Cholette, MMSR, X-11, Kenny-Durbin and BLUE. For the filter corresponding to  $i=1$ , X-11 and Kenny-Durbin weights have larger variance ratios than BLUE, MMSR and Cholette weights. Finally for the filters corresponding to  $i=2,3,4$  and 5, MMSR, X-11, Kenny-Durbin and Cholette weights have small variance ratios while BLUE has a large one in comparison.

As stated in subsection 3.1 the expected revisions gives the same results (except for the sign) as the bias ratios. Thus the conclusions about the estimators are the same for this measure.

The revision variance ratios are presented in table B-3. With this measure, one can draw the same conclusions as with the variance ratios since the values tabulated are almost the same except a bit smaller.

In figures B-1.a and B-1.b to B-6.a and B-6.b, the gain and phase shift functions are shown respectively for the symmetric filter, the BLUE, MMSR, X-11, Kenny-Durbin and Cholette asymmetric end-point filters ( $i=0$ ).

As one can observe, the gain and phase-shift functions respectively reflect the variance and bias properties of the trend-cycle estimators.

In table B-4 the main characteristics of the frequency responses are summarized. These are the value of the gain at low frequencies (trend-cyclical frequencies), the peak value of the gain with its corresponding frequency, the cut-off frequency of the gain (the frequency where it equals  $\sqrt{2}/2$ ), the smallest value of the gain at high frequencies (noise frequencies) and the value of the phase-shift at low frequencies.

The small variances of X-11, Kenny-Durbin and Cholette end weights are reflected in their low cut-off frequencies. The small variance of MMSR is mainly due to its very small gain value at high frequencies. The large variance of BLUE is certainly caused by its high peak value of its gain, its high cut-off frequency and its large gain at high frequencies. The unbiasedness of the symmetric and BLUE end-point filters is shown by their zero phase-shift at low frequencies. The unbiasedness of Kenny-Durbin and Cholette filters if the trend-cycle is a straight line is reflected in the zero phase-shift at very low frequencies. Their bias for quadratic or a cubic is shown by the increase in phase-shift with frequency. The phase-shift of X-11 end weights is larger than the one of MMSR which reflects its larger bias.

#### 4. Empirical Comparison

In this section, the proportional revisions (defined below) obtained by the application of the BLUE, MMSR, X-11, Kenny-Durbin and Cholette asymmetric Henderson weights to seven time series, described in table C.1, are analysed.

These series have been additively seasonally adjusted with the X-11 method and their I/C ratio lie in the interval [1.0,3.5]. When a series had a multiplicative seasonal component a natural logarithmic transformation was done before running X-11.

#### 4.1 The Measures of Comparison

Given a series of length N, the proportional total revision has been calculated as

$$pr_t^{(i)} = \frac{r_t^{(6,i)}}{\hat{x}_t^{(6)}} \times 100 = \frac{\hat{x}_t^{(6)} - \hat{x}_t^{(i)}}{\hat{x}_t^{(6)}} \times 100$$

for  $t=7,8,\dots,N-6$  and  $i=0,1,\dots,5$  (six asymmetric filters).

For all series, the mean, standard deviation and root mean square value of  $pr_t^{(i)}$  have been computed as

$$\bar{pr}^{(i)} = \frac{1}{N-12} \sum_{t=7}^{N-6} pr_t^{(i)} \quad \text{for } i=0,1,\dots,5$$

$$s(pr^{(i)}) = \left[ \frac{1}{N-12} \sum_{t=7}^{N-6} (pr_t^{(i)} - \bar{pr}^{(i)})^2 \right]^{1/2} \quad \text{for } i=0,1,\dots,5$$

$$\text{and rms } (pr^{(i)}) = ((\bar{pr}^{(i)})^2 + s^2 (pr^{(i)}))^{1/2} \quad \text{for } i=0,1,\dots,5.$$

#### 4.2 The Results

The statistics described above are presented in tables C-2 to C-8. The following discussion is based on the results for the filters corresponding to  $i=0$  and  $1$ , since the revisions are generally much bigger for those.

Comparing the results for the different estimators; it is seen that 5 times over 7 X-11 has the smallest rms values closely followed by Kenny-Durbin which has smallest rms for 2 series. This is due to the fact that X-11 has the smallest standard deviation for the seven series again closely

followed by Kenny-Durbin which has much smaller bias than X-11 causing its two smallest rms. The three other estimators BLUE, MMSR and Cholette have larger rms due to their large standard deviations, with BLUE always having the largest rms.

A possible explanation for the disagreements between the empirical and theoretical comparisons could be that a straight line plus white noise is a very good approximation to a 13 months series of seasonally adjusted values. The two best estimators according to rms use a straight line and white noise as design hypothesis.

## 5. Conclusions

This study has shown that BLUE and MMSR asymmetric weights are fully consistent estimators in the sense that they are designed using the same model for seasonally adjusted data as used for symmetric Henderson weights. X-11, Kenny-Durbin and Cholette asymmetric weights are only partially consistent since they use the hypothesis that the trend-cycle is a straight line apart from using the symmetric weights in their design.

It has been seen that X-11 asymmetric weights are very good or the best ones to minimize revisions while the BLUE weights (suggested by Wallis (1981) ) are the worst. The Kenny-Durbin weights did also very well for revisions. This suggests that it could be worth trying to estimate the trend-cycle by symmetric weights designed to reproduce a straight line in white noise.

The actual asymmetric weights for the 13 terms Henderson found in X-11 should be kept since they minimize revisions very well.

In designing asymmetric sets of weights for other length of Henderson moving averages, it is expected that the X-11 approach or the one of Kenny-Durbin will produce weights with very good properties.

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## APPENDIX 'A'

Asymmetric set of weights by design criterion

TABLE A.1 BLUE Weights

$i/j$	$h_{i,j}$												
	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
0	0.0	0.0	0.0	0.0	0.0	0.0	0.8181820	0.4895100	-.2447600	-.2797200	0.1398600	0.1818180	-.1049000
1	0.0	0.0	0.0	0.0	0.0	0.2159610	0.4078570	0.3743320	0.1511720	-.0637600	-.1120900	-.0197400	0.0462770
2	0.0	0.0	0.0	0.0	0.0131980	0.1967640	0.3929800	0.3887290	0.1727680	-.0637600	-.1288900	-.0264600	0.0546760
3	0.0	0.0	0.0	-.0416800	0.0583470	0.2561220	0.3784560	0.3205310	0.1329860	-.0372400	-.0802700	-.0176200	0.0303640
4	0.0	0.0	-.0448900	-.0071400	0.1274110	0.2686790	0.3203790	0.2546060	0.1219990	0.0067120	-.0363200	-.0176200	0.0061920
5	0.0	-.0330200	-.0283800	0.0436610	0.1528140	0.2421210	0.2678420	0.2222750	0.1329680	0.0436610	-.0109200	-.0227000	-.0103200
6	-.0193498	-.0278638	0.0	0.0654918	0.1473565	0.2143367	0.2400572	0.2143367	0.1473565	0.0654918	0.0	-.0278638	-.0193498

TABLE A.2 - MMSR Weights

$i/j$	$h_{i,j}$												
	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
0	0.0	0.0	0.0	0.0	0.0	0.0	0.4115136	0.5713122	0.2906449	-.0912048	-.2070245	-.0507960	0.0755546
1	0.0	0.0	0.0	0.0	0.0	0.2191137	0.3940269	0.3521102	0.1496015	-.0370256	-.0848398	-.0210131	0.0280262
2	0.0	0.0	0.0	0.0	0.0712708	0.2163722	0.3221137	0.2957977	0.1552890	0.0061985	-.0550283	-.0250967	0.0130831
3	0.0	0.0	0.0	-.0170224	0.0741652	0.2343751	0.3335118	0.2921578	0.1450520	0.0007170	-.0535296	-.0224588	0.0130319
4	0.0	0.0	-.0393794	-.0023531	0.1242629	0.2582646	0.3103770	0.2525647	0.1289391	0.0165007	-.0314513	-.0198943	0.0022169
5	0.0	-.0322002	-.0275117	0.0434329	0.1512870	0.2402059	0.2668006	0.2227619	0.1345667	0.0452194	-.0103863	-.0232214	-.0109548
6	-.0193498	-.0278638	0.0	0.0654918	0.1473565	0.2143367	0.2400572	0.2143367	0.1473565	0.0654918	0.0	-.0278638	-.0193498

TABLE A.3 - X-11 Weights

i	j	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
0	0.0	0.0	0.0	0.0	0.0	0.0	0.4211310	0.3531464	0.2439022	0.1197734	0.0120176	-.0581103	-.0918604	
1	0.0	0.0	0.0	0.0	0.0	0.2791025	0.2922340	0.2539245	0.1743553	0.0799016	0.0018209	-.0386319	-.0427069	
2	0.0	0.0	0.0	0.0	0.1481012	0.2154030	0.2414450	0.2160461	0.1493874	0.0678443	0.0026740	-.0248683	-.0160327	
3	0.0	0.0	0.0	0.0448341	0.1302402	0.2007618	0.2300237	0.2078446	0.1444058	0.0660825	0.0041322	-.0201902	-.0081348	
4	0.0	0.0	-.0169417	0.0510800	0.1354746	0.2049847	0.2332351	0.2100446	0.1455943	0.0662595	0.0032976	-.0220363	-.0109924	
5	0.0	-.0340088	-.0053209	0.0609950	0.1436838	0.2114881	0.2380327	0.2131363	0.1469802	0.0659395	0.0012718	-.0257679	-.0164298	
6	-.0193498	-.0278638	0.0	0.0654918	0.1473565	0.2143367	0.2400572	0.2143367	0.1473565	0.0654918	0.0	-.0278638	-.0193498	

TABLE A.4 Kenny- Durbin Weights

i	j	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
0	0.0	0.0	0.0	0.0	0.0	0.0	0.4646983	0.3821913	0.2584246	0.1197734	-.0025049	-.0871551	-.1354276	
1	0.0	0.0	0.0	0.0	0.0	0.2891958	0.2994435	0.2582502	0.1757972	0.0784597	-.0025049	-.0458414	-.0528002	
2	0.0	0.0	0.0	0.0	0.1478950	0.2152483	0.2413419	0.2159945	0.1493874	0.0678958	0.0027771	-.0247136	-.0158265	
3	0.0	0.0	0.0	0.0429756	0.1287947	0.1997293	0.2294042	0.2076381	0.1446123	0.0667020	0.0051646	-.0187448	-.0062763	
4	0.0	0.0	-.0180481	0.0501949	0.1348108	0.2045422	0.2330139	0.2100446	0.1458155	0.0667020	0.0039614	-.0211512	-.0098860	
5	0.0	-.0343137	-.0055704	0.0608009	0.1435452	0.2114049	0.2380049	0.2131640	0.1470633	0.0660782	0.0014659	-.0255184	-.0161248	
6	-.0193498	-.0278638	0.0	0.0654918	0.1473565	0.2143367	0.2400572	0.2143367	0.1473565	0.0654918	0.0	-.0278638	-.0193498	

TABLE A.5 Cholette Weights

		$h_{i,j}$														
		-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6		
i	j	0.0	0.0	0.0	0.0	0.0	0.0	0.3587284	0.4418412	0.3638444	0.1540632	-.0614879				
0	0	0.0	0.0	0.0	0.0	0.0	0.0	0.2473949	0.3108066	0.2958968	0.2038621	0.0765528	-.0275068	-.0661813	-.0408252	
1	0	0.0	0.0	0.0	0.0	0.0	0.1471794	0.2142912	0.2407994	0.2164222	0.1508360	0.0697689	0.0040297	-.0250914	-.0182354	
2	0	0.0	0.0	0.0	0.0	0.0	0.0579410	0.1291326	0.1892544	0.2162146	0.1996719	0.1458432	0.0754674	0.0151774	-.0150876	-.0136149
3	0	0.0	0.0	0.0	0.0579410	0.1291326	0.1892544	0.2162146	0.1996719	0.1458432	0.0754674	0.0151774	-.0150876	-.0136149		
4	0	0.0	0.0	-.0051448	0.0524004	0.1279454	0.1935717	0.2237699	0.2067235	0.1493690	0.0746040	0.0113998	-.0190452	-.0155937		
5	0	0.0	-.0294480	-.0042001	0.0589123	0.1397338	0.2074943	0.2355959	0.2130777	0.1490916	0.0691015	0.0039042	-.0250548	-.0182083		
6	0	-.0193498	-.0278638	0.0	0.0654918	0.1473565	0.2143367	0.2400572	0.2143367	0.1473565	0.0654918	0.0	-.0278638	-.0193498		

**APPENDIX B**

Theoretical results

TABLE B.1 Bias Ratios

Criterion	Filter's index	Straight line	Quadratic	Cubic
BLUE	0	.0000399999999998	-0.0002	0.00112
	1	-0.000037	0.000173	-0.000799
	2	-0.000021	0.000113	-0.000561
	3	-0.000011	-0.000011	-0.000245
	4	.0000089999999999	0.000125	-.000086999999995
	5	0.000018 0.000101999999998		0.000294
	6	0 0.0001860000000001		0
MMSR	0	-0.2502372	-0.9492778	2.8453326
	1	-0.0448552	-0.0374216	1.6726082
	2	0.0010407	0.4373169	2.9140185
	3	-0.0045535	0.3078579	2.3145131
	4	-0.00575626	0.10012576	0.95977424
	5	-0.0021417	0.0179725	0.2508321
	6	0 .0000021999999998	2.22044604925E-16	
X-11	0	-0.407	-2.161	20.809
	1	-0.123	-0.517	10.383
	2	0.003	1.079	4.545
	3	0.053	1.711	3.779
	4	0.018	1.406	3.246
	5	0.007	0.565	1.939
	6	0 0.0001860000000001		0
KENNY - DURBIN	0	2.08166817117E-16	-4.6004992	34.6235928
	1	-9.9999999751E-08	-1.1301485	13.8014879
	2	2.00000000034E-07	1.0882742	4.4841176
	3	1.9999999997E-07	1.8761606	2.593184
	4	9.00000000017E-07	1.4791009	2.8314363
	5	-9.9999999890E-08	0.5869475	1.7608301
	6	0 .0000021999999998	2.22044604925E-16	
CHOLETTE	0	1.0000000294E-07	-5.2337859	37.7213179
	1	4.0000000025E-07	-1.5166332	15.1050166
	2	-3.0000000009E-07	1.0314109	4.9032265
	3	-1.0000000003E-07	2.0650171	3.2380997
	4	1.38777878078E-17	1.7151692	3.905997
	5	1.38777878078E-17	0.7051946	2.5096134
	6	0 .0000021999999998	2.22044604925E-16	

TABLE B.2 Variance Ratios

Filter's Index	BLUE	MMSR	X-11	Kenny-Durbin	Cholette
0	7.83222965124	1.098262179915	1.272258	1.639887392423	0.6203161847037
1	0.273549509918	0.2289945562666	0.433648	0.4764149843363	0.2726048607578
2	0.256753111044	0.06200366414974	0.07918	0.07956805240074	0.07893914403914
3	0.146251750402	0.0746777426644	0.009996	0.01006004122852	0.00935220884408
4	0.055206405154	0.04570762493596	0.021584	0.02170266169016	0.01154315498952
5	0.018257541456	0.01767643063646	0.017082	0.01742424066658	0.0143554400793
6	0.008335025628	0.00833531269388	0.008335025628	0.00833531269388	0.00833531269388

TABLE B.3 - Revision Variance Ratios

Filter's Index	BLUE	MMSR	X-11	Kenny-Durbin	Cholette
0	7.823886044988	1.087685750295	1.258774115628	1.620595029769	0.5995159682066
1	0.265214667572	0.2205700429531	0.424080051628	0.465389502952	0.2606578147975
2	0.248418326774	0.05470979167016	0.073415293628	0.07382449008572	0.07306016309026
3	0.137918735344	0.06707557140924	0.005724903628	0.00619283466404	0.00593477641372
4	0.04686703492	0.03760999281218	0.016593921628	0.01688926135296	0.00729255755444
5	0.00992322422	0.00938387831212	0.010094927628	0.0104868954032	0.00769950692672

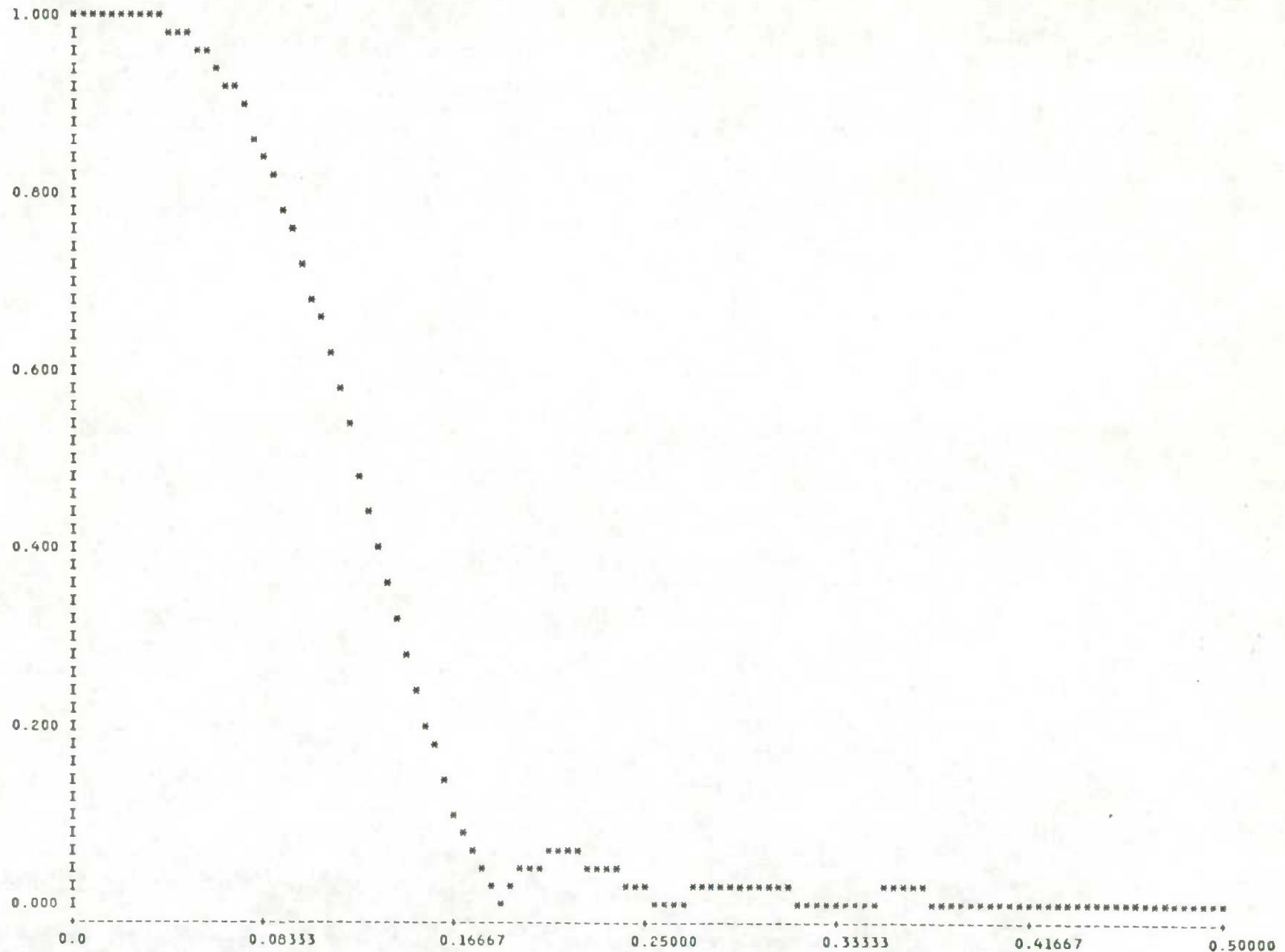
TABLE B.4 FREQUENCY RESPONSES CHARACTERISTICS

Criterion	Gain					Phase Shift in months at low frequencies
	at low frequen- cies	peak	peak frequen- cy	cut-off frequen- cy	smallest at high frequencies	
Symmetric	1.0	n/a	n/a	0.1	0.0	0.0
BLUE	1.0	1.676	0.228	0.328	0.217	0.0
MMSR	1.0	1.287	0.144	0.244	0.062	$\approx -0.25$
X-11	1.0	1.105	0.072	0.155	0.170	$\approx -0.50$
Kenny- Durbin	1.0	1.237	0.083	0.163	0.170	0.0 to -0.25
Cholette	1.0	1.323	0.092	0.183	0.121	0.0 to -0.25

## HENDERSON SYMMETRIC FILTER

## GAIN FUNCTION

FIGURE B.1a



HENDERSON SYMMETRIC FILTER

PHASE-SHIFT IN TIME INTERVALS

FIGURE B.1b

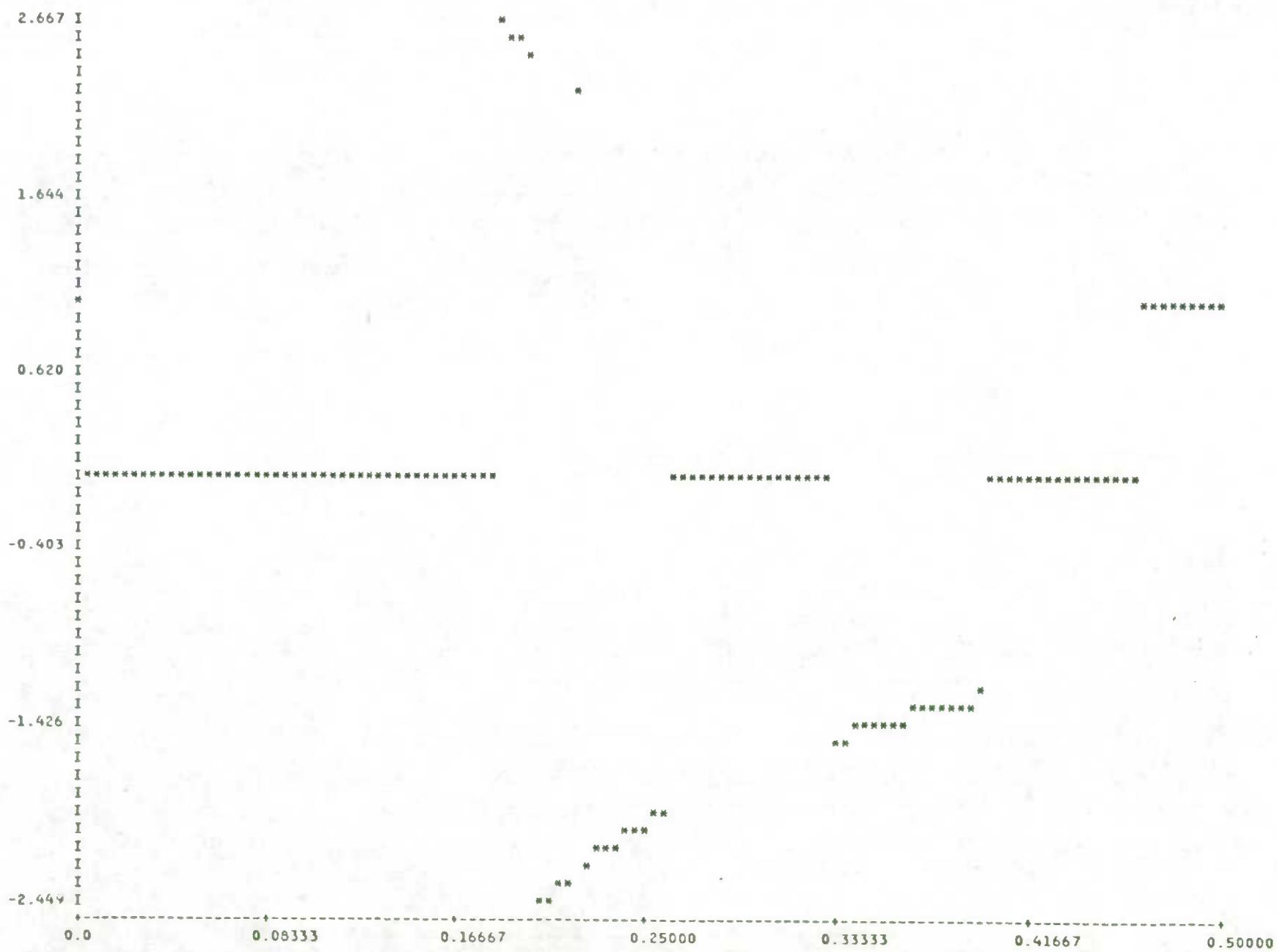
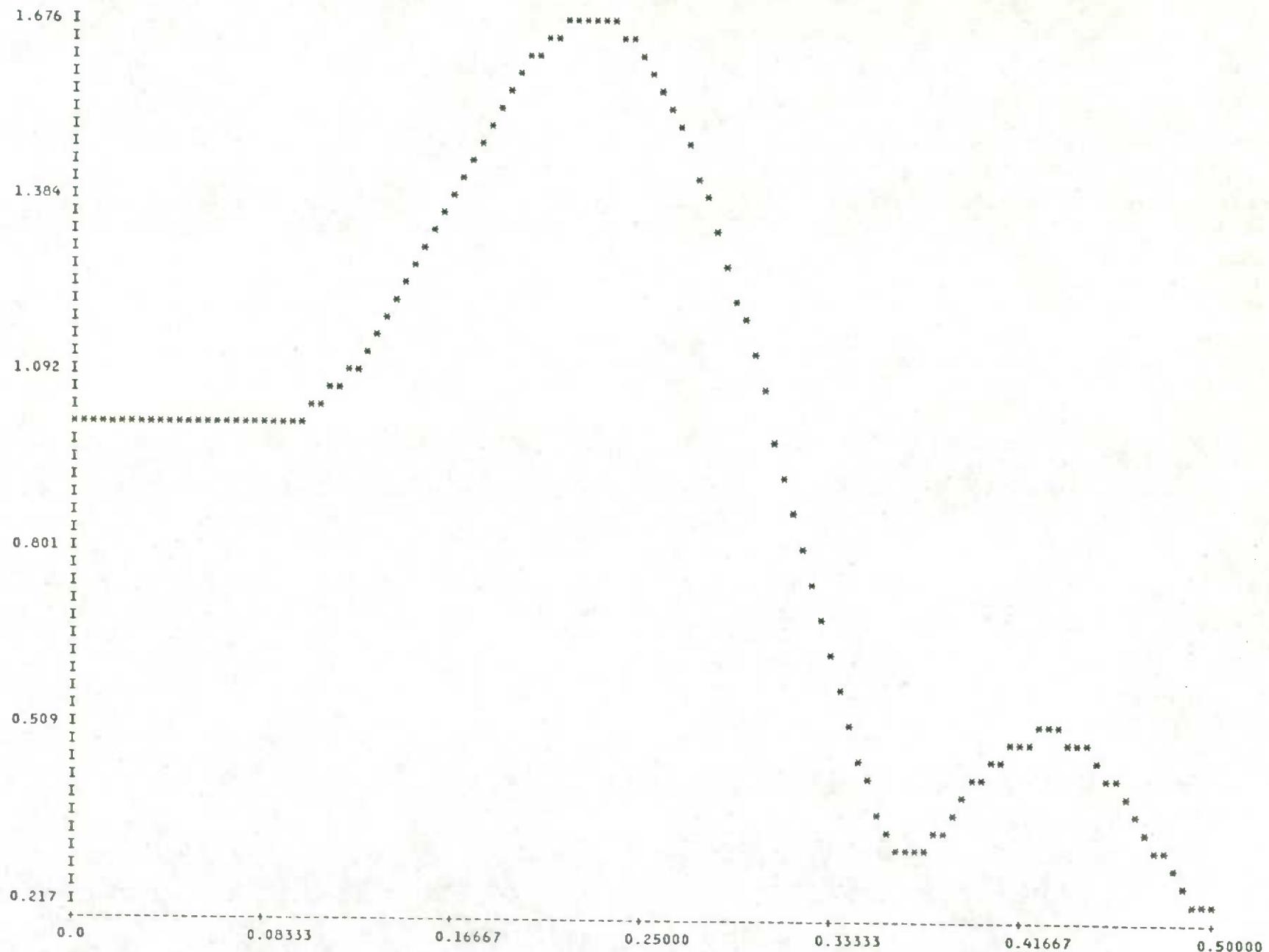


FIGURE B.2a

BLUE END-POINT FILTER

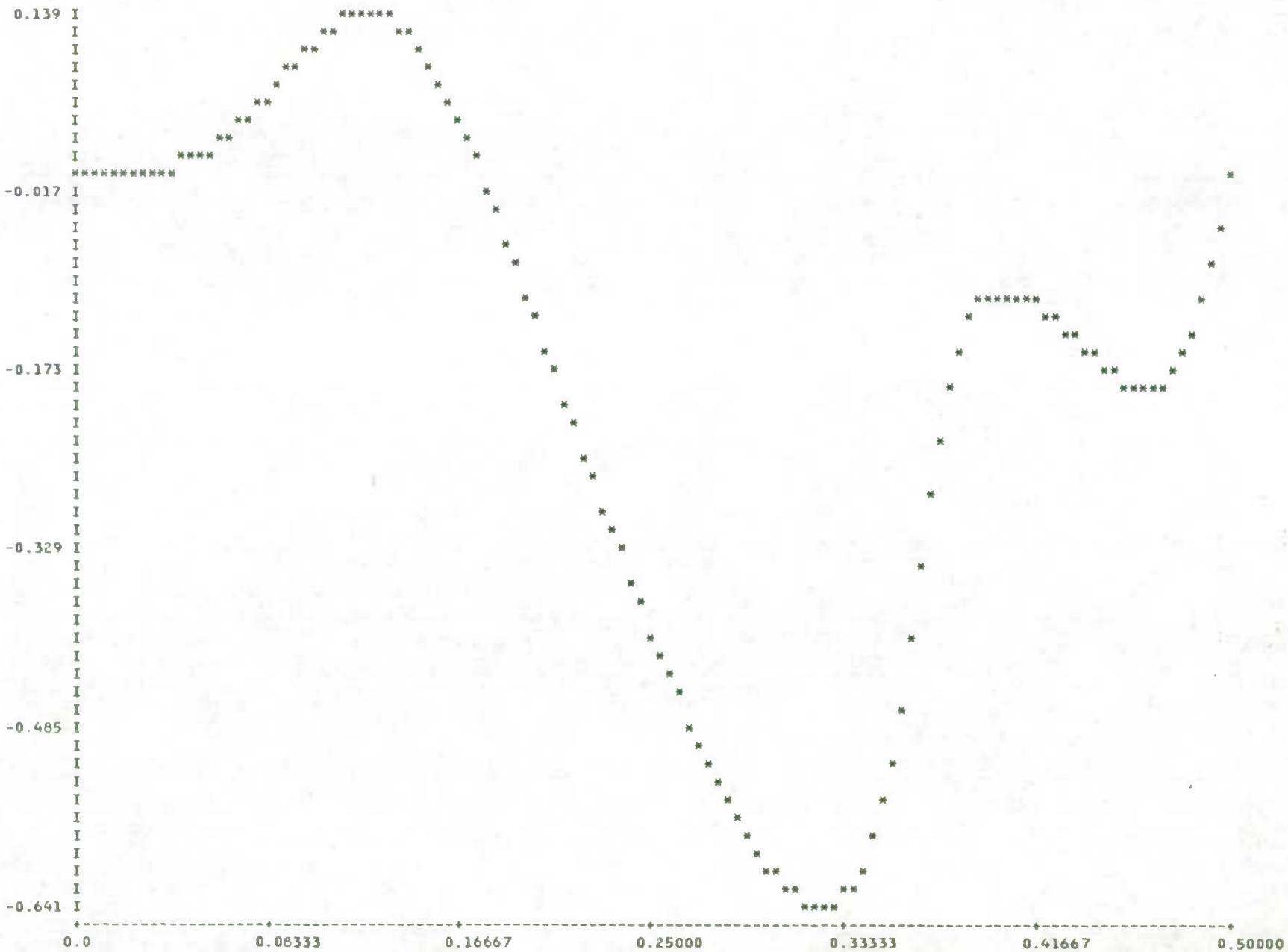
GAIN FUNCTION



BLUE END-POINT FILTER

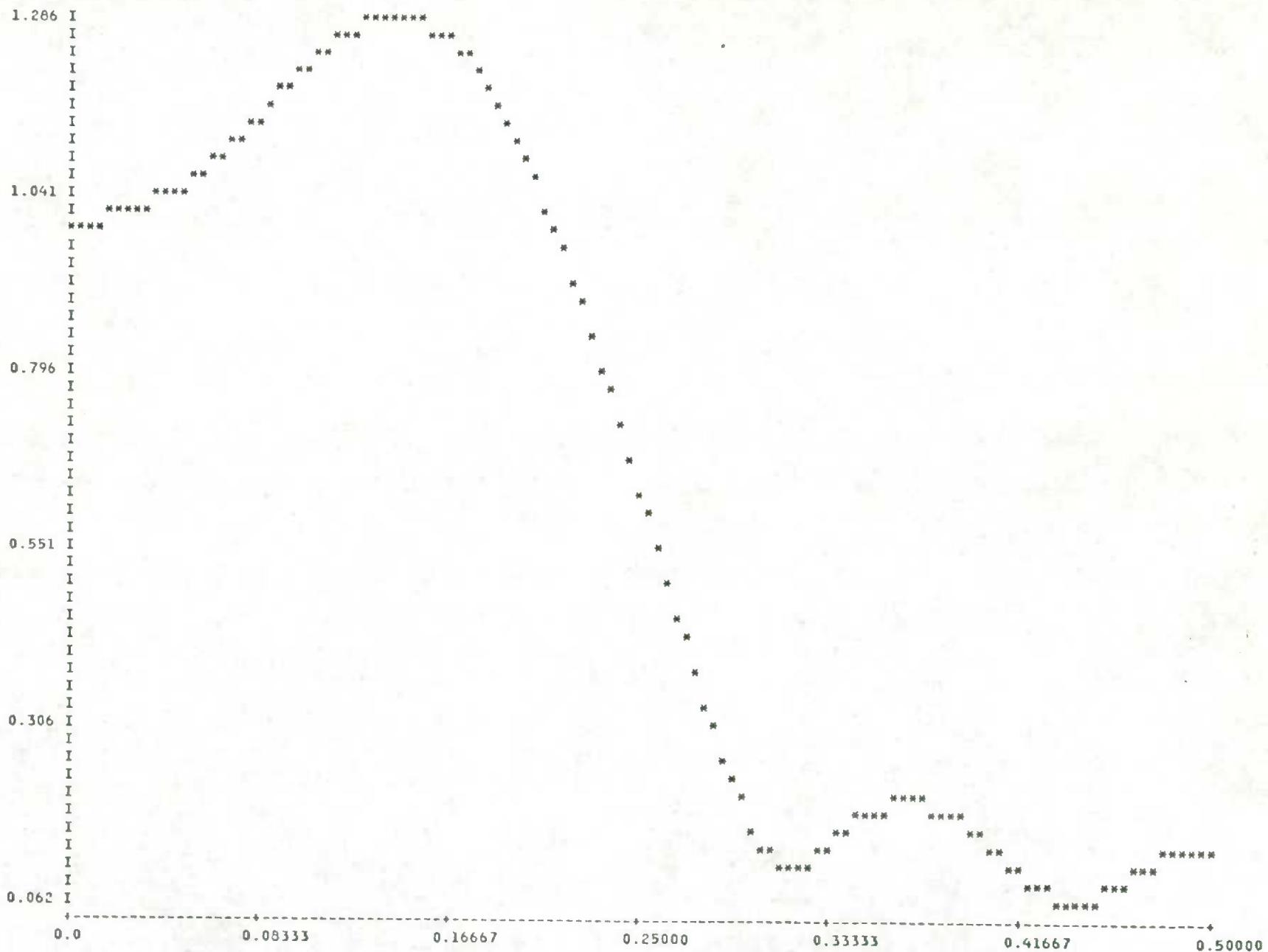
PHASE-SHIFT IN TIME INTERVALS

FIGURE B.2b



## GAIN FUNCTION

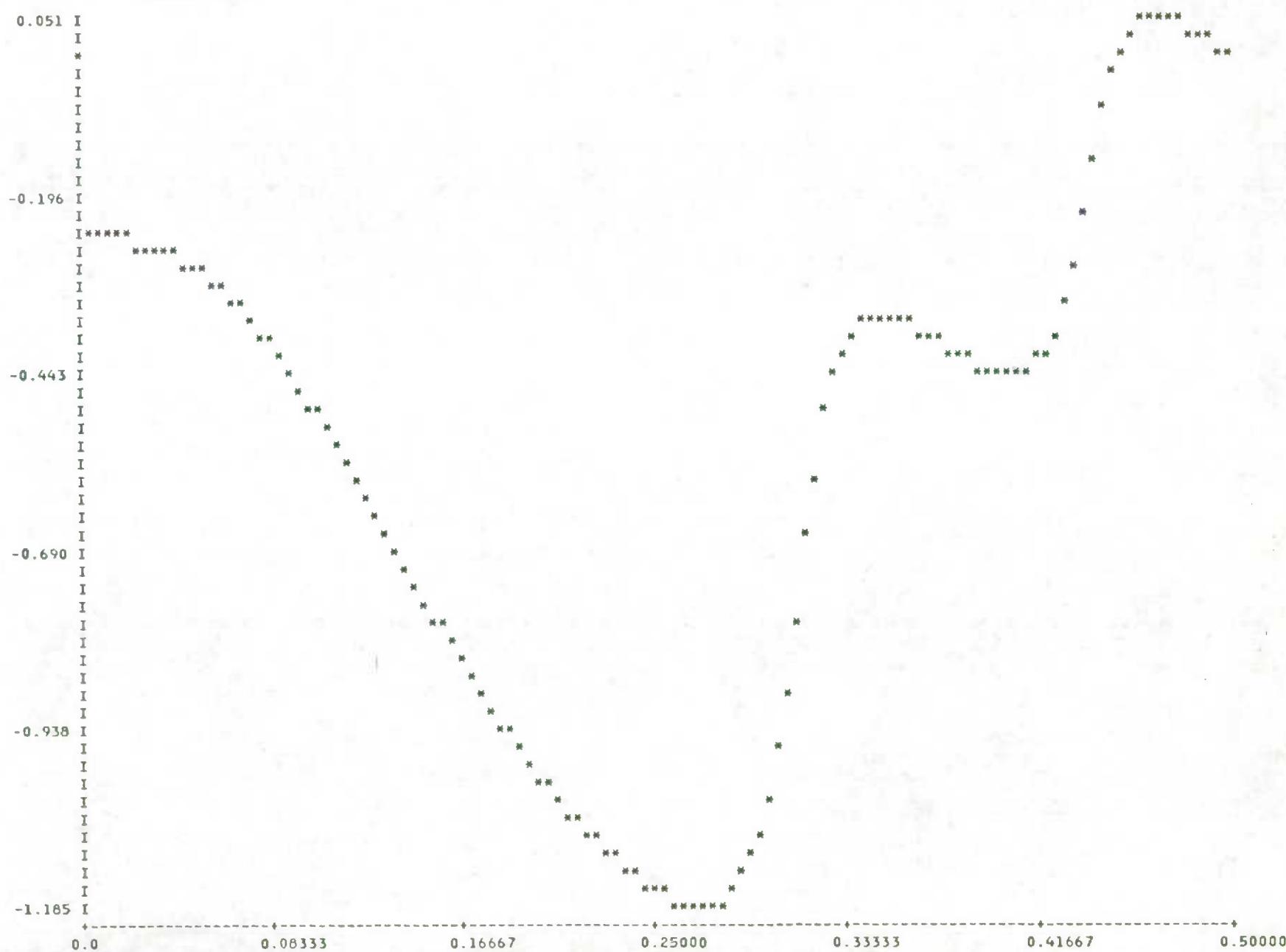
FIGURE B.3a



HMSR END-POINT FILTER

PHASE-SHIFT IN TIME INTERVALS

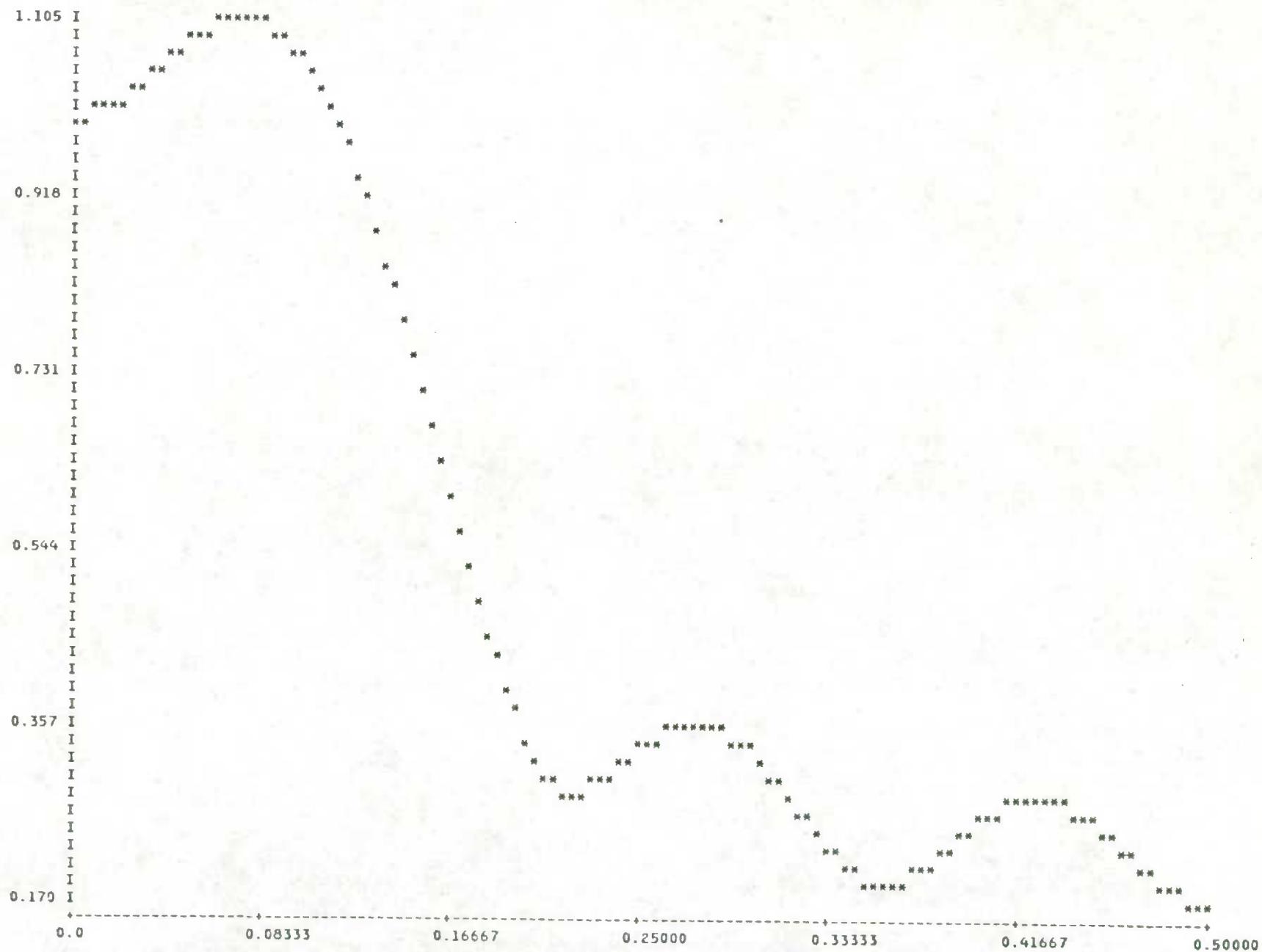
FIGURE B3.b



## X-11 END-POINT FILTER

## GAIN FUNCTION

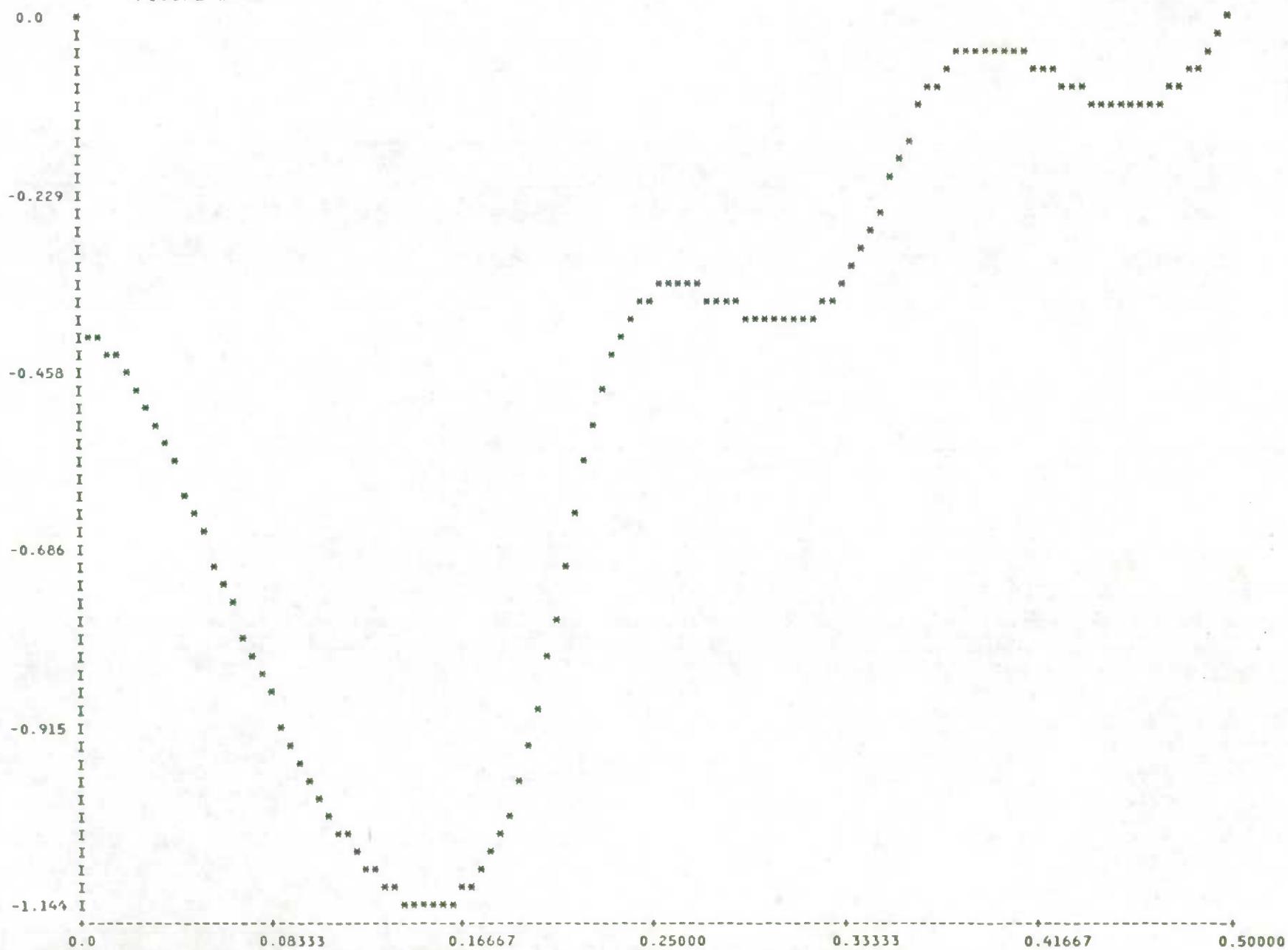
FIGURE B.4a



X-11 END-POINT FILTER

PHASE-SHIFT IN TIME INTERVALS

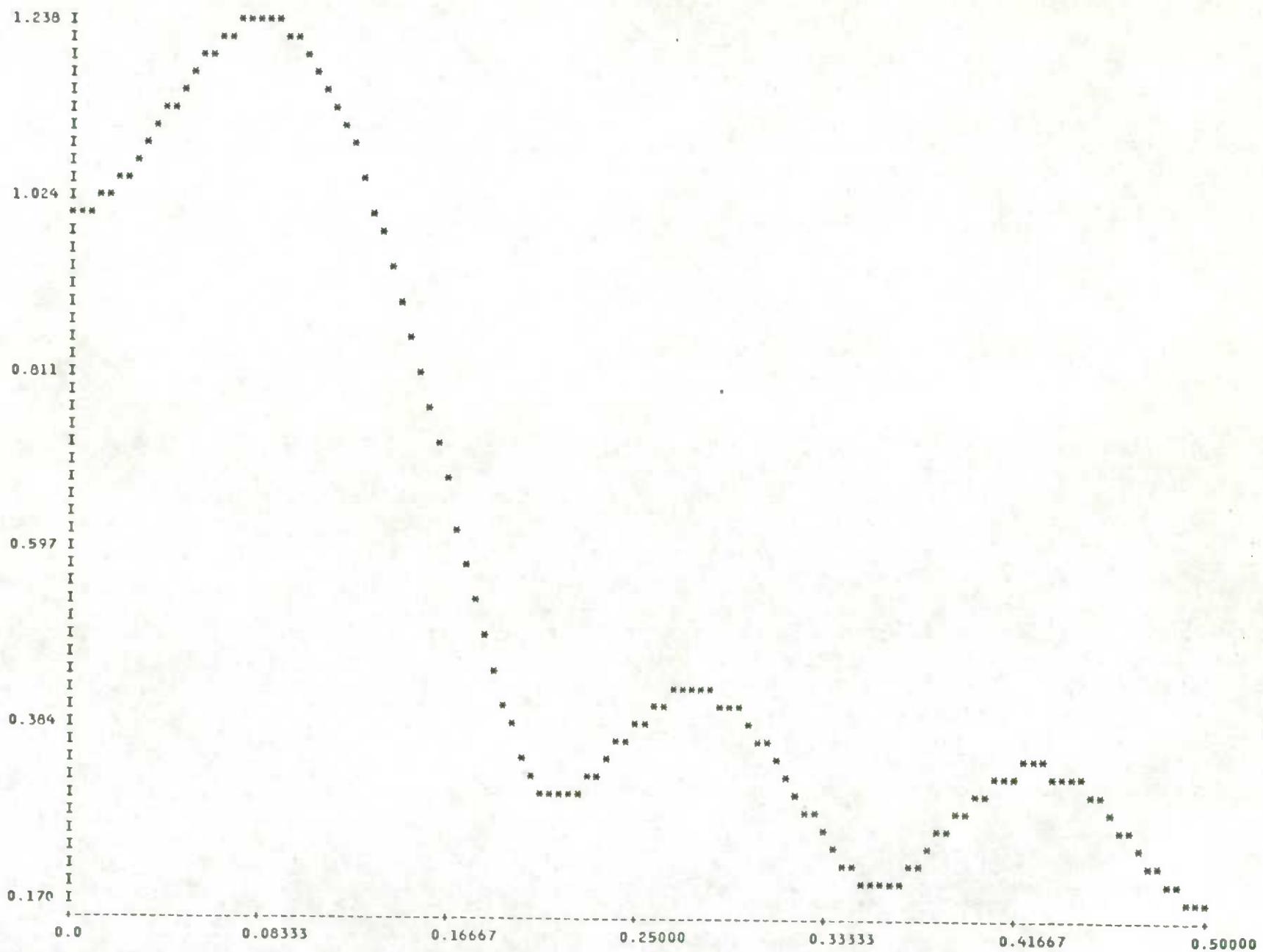
FIGURE B.4b



## KENDY-DURBIN END-POINT FILTER

FIGURE B.5a

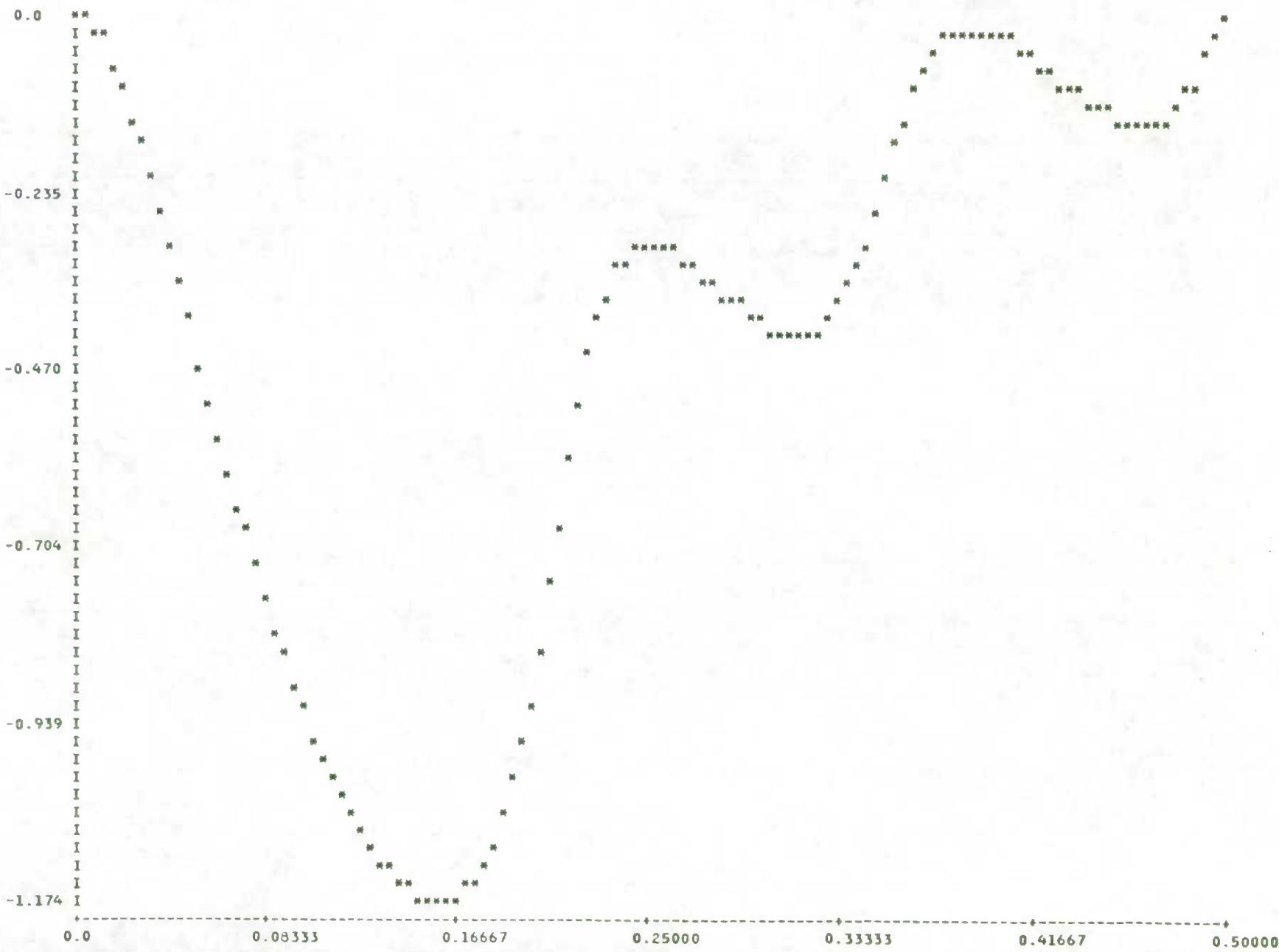
## GAIN FUNCTION



KENNY-DURBIN END-POINT FILTER

PHASE-SHIFT IN TIME INTERVALS

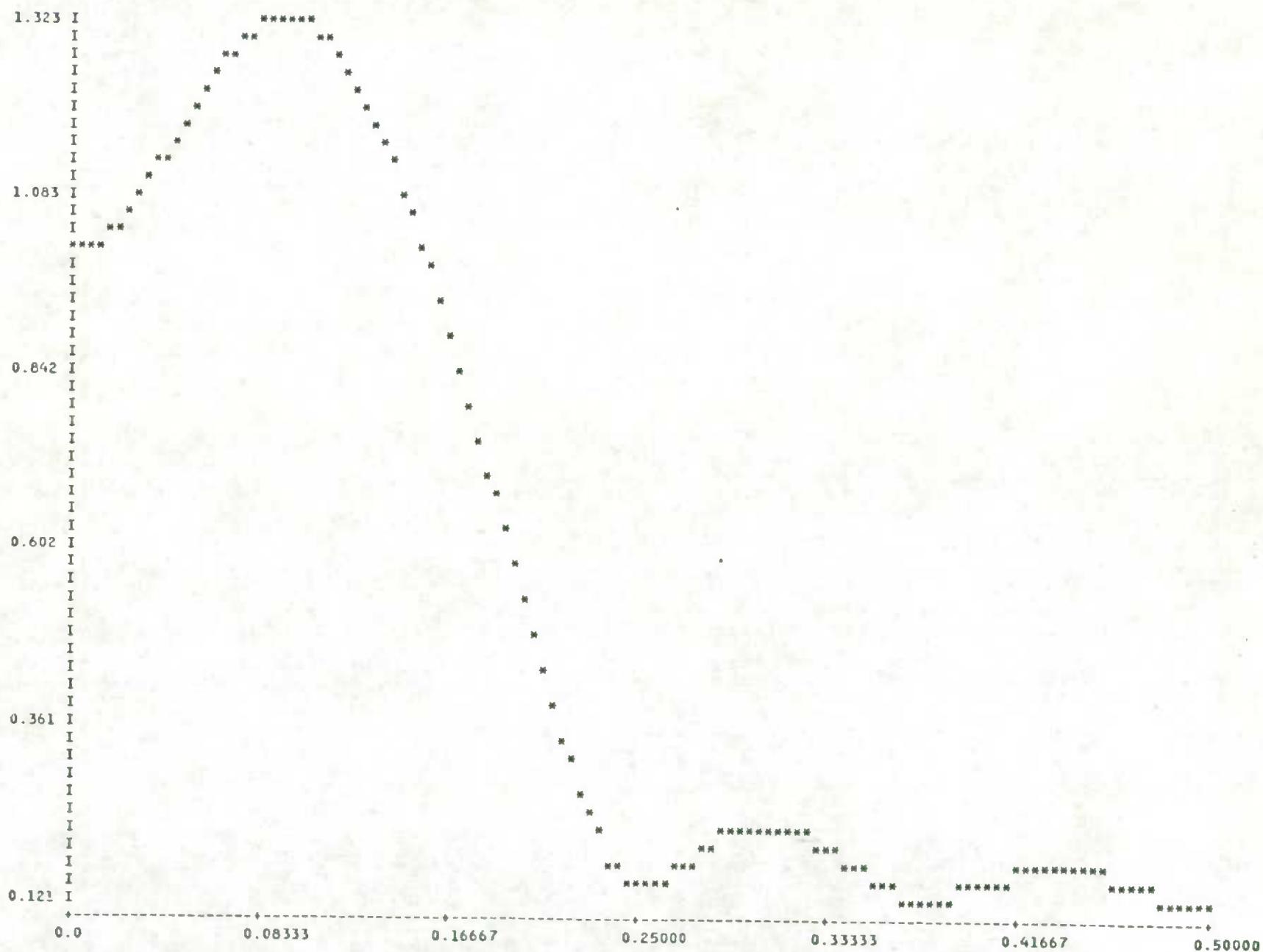
FIGURE B.5b



## CHOLETTE END-POINT FILTER

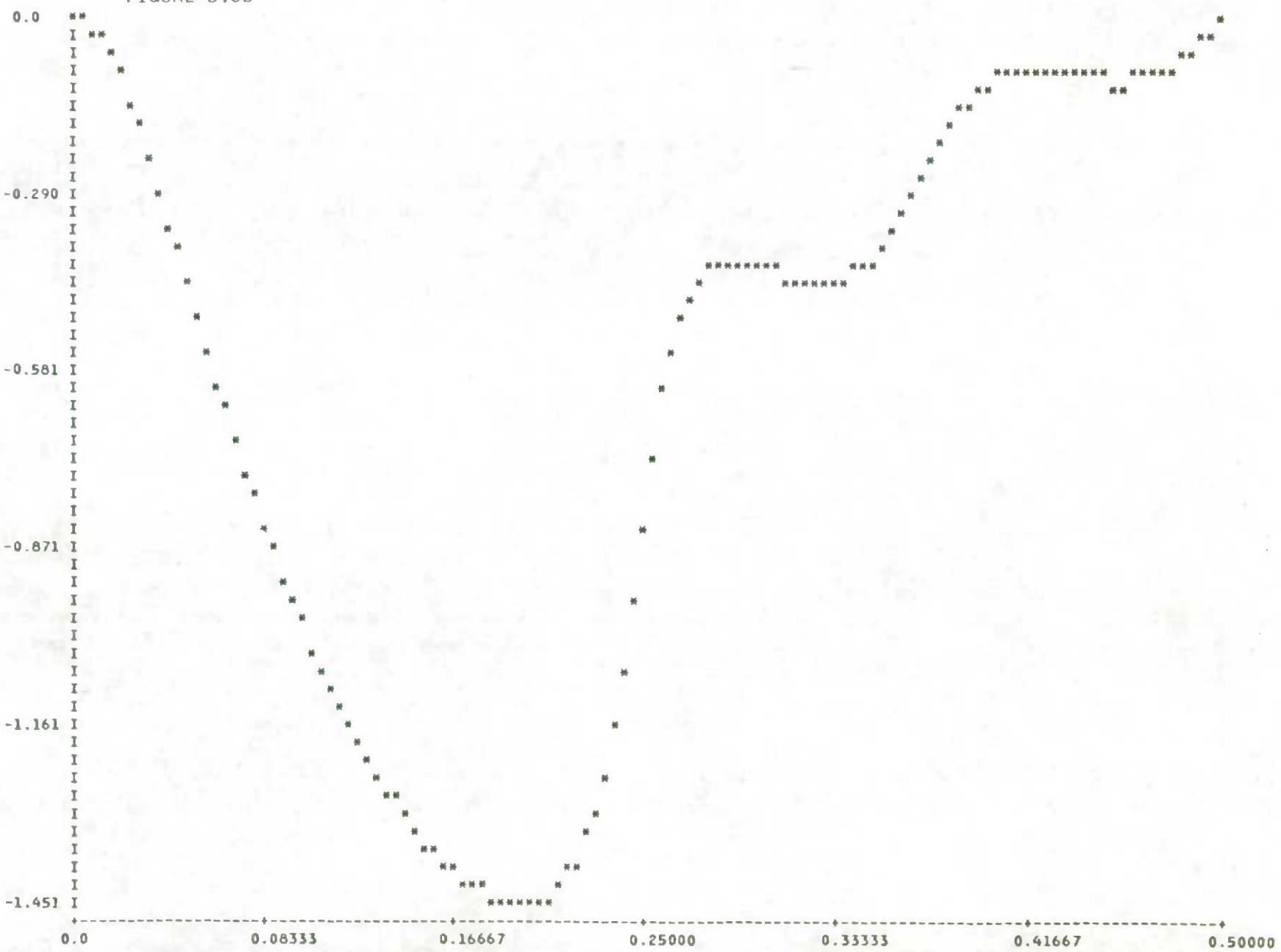
## GAIN FUNCTION

FIGURE B.6a



CHOLETTE END-POINT FILTER  
PHASE-SHIFT IN TIME INTERVALS

FIGURE B.6b



**APPENDIX C**  
**Empirical results**

TABLE C.1 DESCRIPTION OF THE TIME SERIES

Identifier	Title	Period	Length
D500002	Slaughtering and Meat Processors (log.)	Jan.71 to Dec.84	168
D310000	Shipments - Canada Total (log.)	Jan.71 to Dec.84	168
D144336	Gross Domestic Product (log.)	Jan.71 to Dec.84	168
AIRLNSAD	International airline passengers - Box-Jenkins, Series G. (log.)	Jan.49 to Dec. 60	144
D655161	Total stocks all departments- Department stores (log.)	Jan.71 to Dec.84	168
D2437	Total generation - Hydraulic Power - Electric Power (log.)	Jan.71 to Dec.84	168
D5300	Total revenue cars loaded - On Canadian railways	Jan.71 to Dec.84	168

TABLE C.2 - Revisions Statistics for  
Series D500002 (I/C = 1.0)

Criterion	Index	Filter's		
		pr	s(pr)	rms
BLUE	0	-0.00449843	0.37449968	0.37452670
	1	-0.00160879	0.19002669	0.19003350
	2	-0.00195434	0.19318397	0.19319395
	3	-0.00275820	0.15542367	0.15544814
	4	-0.00100427	0.09609953	0.09610478
	5	-0.00104242	0.03997441	0.03998800
MMSR	0	-0.03484016	0.32929452	0.33113247
	1	-0.00738128	0.16997750	0.17013770
	2	-0.00072533	0.10191351	0.10191609
	3	-0.00156916	0.10901597	0.10902745
	4	-0.00153045	0.08411151	0.08412543
	5	-0.00061847	0.03801986	0.03802489
X-11	0	-0.05589646	0.30151500	0.30665243
	1	-0.01804053	0.13169554	0.13292545
	2	-0.00056551	0.04715523	0.04715862
	3	0.00632679	0.05434224	0.05470930
	4	0.00163574	0.04407778	0.04410812
	5	0.00006225	0.01729189	0.01729200
Kenny- Durbin	0	-0.00238492	0.32673940	0.32674810
	1	-0.00122011	0.13407026	0.13407581
	2	-0.00008177	0.04746845	0.04746852
	3	0.00028014	0.05908388	0.05908455
	4	0.00020636	0.04645327	0.04645373
	5	0.00004464	0.01794257	0.01794263
Cholette	0	-0.00236529	0.38173921	0.38174654
	1	-0.00139670	0.15975089	0.15976499
	2	-0.00007232	0.04842999	0.04843004
	3	0.00049890	0.07256868	0.07257039
	4	0.00033403	0.06138604	0.06138695
	5	0.00008801	0.02459211	0.02459226

TABLE C.3 - Revisions Statistics for  
Series D310000 (I/C = 1.31)

Criterion	Index	Filter's		
		pr	s(pr)	rms
BLUE	0	-0.00193138	0.14814416	0.14815675
	1	0.00011295	0.06205457	0.06205469
	2	-0.00031319	0.06301016	0.06301093
	3	-0.00106681	0.05024036	0.05025169
	4	0.00019262	0.03080281	0.03080941
	5	-0.00055112	0.01272060	0.01273253
MMSR	0	-0.02610624	0.10730311	0.11045214
	1	-0.00453659	0.05523802	0.05542399
	2	0.00019799	0.03319921	0.03319981
	3	-0.00031547	0.03543264	0.03543404
	4	-0.00032047	0.02708281	0.02708471
	5	-0.00003008	0.01212988	0.01213015
X-11	0	-0.04261744	0.08924028	0.09889425
	1	-0.01318324	0.03960763	0.04174401
	2	-0.00060371	0.01459458	0.01460706
	3	0.00437570	0.01507177	0.01569411
	4	0.00078691	0.01245596	0.01248079
	5	-0.00020900	0.00512177	0.00512604
Kenny- Durbin	0	0.00215120	0.09602476	0.09604885
	1	0.00090961	0.04034640	0.04035665
	2	-0.00004852	0.01466239	0.01466247
	3	-0.00040078	0.01621229	0.01621724
	4	-0.00023040	0.01304777	0.01304980
	5	-0.00006106	0.00528715	0.00528751
Cholette	0	0.00234509	0.11018360	0.11020856
	1	0.00099590	0.04686163	0.04687221
	2	-0.00000381	0.01480695	0.01480695
	3	-0.00043375	0.01955459	0.01955940
	4	-0.00030665	0.01661678	0.01661961
	5	-0.00006369	0.00679623	0.00679653

TABLE C.4 - Revisions Statistics for Series D144336 (I/C = 1.50)

Criterion	Filter's Index	$\bar{pr}$	$s(pr)$	rms
BLUE	0	-0.00206460	0.13841316	0.13842856
	1	0.00035640	0.04975532	0.04975660
	2	-0.00005308	0.05027991	0.05027994
	3	-0.00060963	0.03898744	0.03899221
	4	0.00065269	0.02314212	0.02315132
	5	-0.00032232	0.00934419	0.00934975
MMSR	0	-0.01312113	0.08926258	0.08923255
	1	-0.00207350	0.04415538	0.04420404
	2	0.00024745	0.02562795	0.02562914
	3	0.00013793	0.02727996	0.02728031
	4	0.00032904	0.02025705	0.02025974
	5	0.00024397	0.00888101	0.00888437
X-11	0	-0.02219582	0.06998395	0.07341990
	1	-0.00697596	0.03148494	0.03224849
	2	-0.00076782	0.01142409	0.01144986
	3	0.00137860	0.01175992	0.01184045
	4	-0.00032411	0.00984415	0.00984948
	5	-0.00063536	0.00411794	0.00416667
Kenny-Durbin	0	0.00156368	0.07469139	0.07470776
	1	0.00078469	0.03205970	0.03206930
	2	-0.00005664	0.01146317	0.01146332
	3	-0.00065388	0.01289410	0.01291067
	4	-0.00041621	0.01038881	0.01039715
	5	-0.00012612	0.00426607	0.00426794
Cholette	0	0.00182470	0.08273265	0.08275277
	1	0.00087618	0.03513464	0.03514556
	2	-0.00002286	0.01150083	0.01150085
	3	-0.00080840	0.01537163	0.01539287
	4	-0.00061854	0.01295518	0.01296994
	5	-0.00017676	0.00531597	0.00531891

TABLE C.5 - Revisions Statistics for Series AIRLNSAD (I/C = 1.87)

Criterion	Filter's Index	$\bar{pr}$	$s(pr)$	rms
BLUE	0	-0.00104238	0.36988409	0.36988556
	1	-0.00088225	0.16444420	0.16444657
	2	-0.00142852	0.16631939	0.16632553
	3	-0.00185491	0.13197064	0.13198368
	4	-0.00030659	0.07937689	0.07937748
	5	-0.00071380	0.03202918	0.03203713
MMSR	0	-0.04953277	0.27360971	0.27788074
	1	-0.00905893	0.14579001	0.14607118
	2	-0.00024704	0.08783979	0.08784014
	3	-0.00124048	0.09362066	0.09352088
	4	-0.00122390	0.07011164	0.07012232
	5	-0.00042043	0.03062124	0.03062413
X-11	0	-0.07320466	0.19011927	0.20557547
	1	-0.02427657	0.09144513	0.09461271
	2	-0.00017550	0.03306207	0.03306253
	3	0.00927317	0.02703537	0.02855151
	4	0.00272269	0.02343036	0.02352672
	5	0.00052620	0.01024662	0.01026013
Kenny-Durbin	0	-0.00403601	0.21125826	0.21129681
	1	-0.00115328	0.09490802	0.09491502
	2	0.00015171	0.03312610	0.03312645
	3	0.00051235	0.02901948	0.02902400
	4	0.00037444	0.02416151	0.02416441
	5	0.00014969	0.01039426	0.01039534
Cholette	0	-0.00497230	0.24629969	0.24634937
	1	-0.00165111	0.10810419	0.10811630
	2	0.00013556	0.03351352	0.03351379
	3	0.00064495	0.03506817	0.03507410
	4	0.00044583	0.03049942	0.03050268
	5	0.00017013	0.01286814	0.01286927

TABLE C.6 - Revisions Statistics for  
Series D655161 (I/C = 2.07)

Criterion	Filter's Index	pr	s(pr)	rms
BLUE	0	-0.00399200	0.12222501	0.12229019
	1	-0.00077821	0.05136192	0.05136782
	2	-0.00114342	0.05193969	0.05195227
	3	-0.00196964	0.04143235	0.04147914
	4	-0.00051962	0.02531940	0.02532473
	5	-0.00089203	0.01042006	0.01045817
M1SR	0	-0.01733211	0.08682913	0.08054208
	1	-0.00354356	0.04571067	0.04585566
	2	-0.00035595	0.02766851	0.02767084
	3	-0.00076293	0.02947597	0.02948584
	4	-0.00073937	0.02237937	0.02239158
	5	-0.00032803	0.00996714	0.00997254
X-11	0	-0.02744679	0.06726092	0.07264542
	1	-0.00897976	0.03146078	0.03271722
	2	-0.0006193	0.01196275	0.01199376
	3	0.00230315	0.01127204	0.01152121
	4	0.00006684	0.00964325	0.00964348
	5	-0.00054234	0.00420394	0.00423878
Kenny-Durbin	0	0.00142545	0.07166820	0.07168237
	1	0.00031435	0.03192131	0.03192286
	2	-0.00019058	0.01200518	0.01200669
	3	-0.00035143	0.01209561	0.01210072
	4	-0.00025010	0.01002361	0.01002673
	5	-0.00012222	0.00431436	0.00431609
Cholette	0	0.00187324	0.08112460	0.08114622
	1	0.00040973	0.03566900	0.03567136
	2	-0.00014533	0.01201408	0.01201496
	3	-0.00028175	0.01408565	0.01408846
	4	-0.00022951	0.01209677	0.01209394
	5	-0.00010219	0.00514265	0.00514367

TABLE C.7 - Revisions Statistics for  
Series D2437(I/C = 2.33)

Criterion	Filter's Index	pr	s(pr)	rms
BLUE	0	-0.00388069	0.16892803	0.16897259
	1	-0.00121589	0.06181656	0.06182851
	2	-0.00164902	0.06264282	0.06266452
	3	-0.00207464	0.04942777	0.04947129
	4	-0.00035083	0.03006554	0.03006759
	5	-0.00073103	0.01238199	0.01240355
M1SR	0	-0.01169804	0.10973658	0.11035833
	1	-0.00281813	0.055119687	0.05526877
	2	-0.00059751	0.03273415	0.03273961
	3	-0.00076988	0.03491745	0.03492591
	4	-0.00047090	0.02647843	0.02649262
	5	-0.00012143	0.01181653	0.01181715
X-11	0	-0.01800903	0.09168704	0.09343895
	1	-0.00620781	0.04229227	0.04271181
	2	-0.00078169	0.01659515	0.01661355
	3	0.00111483	0.01717262	0.01720377
	4	-0.00021329	0.01441910	0.01442068
	5	-0.00063824	0.00597978	0.00591432
Kenny-Durbin	0	-0.00233446	0.09908146	0.09910895
	1	-0.00080859	0.04288379	0.04289141
	2	-0.00001995	0.01666216	0.01666218
	3	0.00002059	0.01879398	0.01879999
	4	0.00002876	0.01515199	0.01515202
	5	-0.00000252	0.00608466	0.00608466
Cholette	0	-0.00249654	0.11117815	0.11120618
	1	-0.00105630	0.04777067	0.04778235
	2	-0.00001369	0.01679072	0.01679072
	3	0.00009339	0.02225379	0.02225398
	4	0.00002091	0.01899320	0.01899321
	5	-0.00000414	0.00781760	0.00781760

TABLE C.8 Revisions Statistics for  
Series D5300 (I/C = 2.84)

Criterion	Filter's Index	$\bar{pr}$	$s(pr)$	rms
BLUE	0	-0.01013198	4.09116194	4.09117448
	1	-0.00368116	1.66656770	1.66657177
	2	-0.00389259	1.68402992	1.68403442
	3	-0.00391341	1.33404090	1.33404664
	4	-0.00204458	0.80139626	0.80138987
	5	-0.00238730	0.32282611	0.32283494
MMSR	0	0.00035125	2.80385575	2.80385577
	1	-0.00134804	1.47814140	1.47814202
	2	0.00171878	0.88425653	0.88425921
	3	0.00090708	0.94361676	0.94361720
	4	-0.00063910	0.70683825	0.70683854
	5	-0.00137594	0.30842804	0.30843111
X-11	0	0.00429661	1.96764260	1.96764729
	1	0.00325374	0.94229506	0.94230058
	2	0.00687790	0.34103509	0.34110444
	3	0.00829224	0.27147672	0.27160333
	4	0.00727213	0.23852020	0.23863104
	5	0.00175223	0.10532495	0.10533953
Kenny-Durbin	0	-0.01958057	2.19880604	2.19889322
	1	-0.00276365	0.98143088	0.98143477
	2	0.00705778	0.34135308	0.34144420
	3	0.01163997	0.29368440	0.29391498
	4	0.00917117	0.24629073	0.24646143
	5	0.00295966	0.10743780	0.10747856
Cholette	0	-0.02069073	2.50497726	2.50506271
	1	-0.00514664	1.09835510	1.09836716
	2	0.00775002	0.34389896	0.34398627
	3	0.01323598	0.35495720	0.35520389
	4	0.01125700	0.30694841	0.30715476
	5	0.00409860	0.13002288	0.13008746

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