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OPTIMAL TIME HORIZON FOR ARIMA EXTRAPOLATION

by

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SELECTION DE L'HORIZON DE PREVISION ARMMI OPTIMAL

L'utilisation d'un horizon de prévision de 12 mois réduit les révisions associées aux facteurs saisonniers concourrants du programme de désaisonalisation X-11-ARMMI. Cependant du point de vue de la révision associée aux filtrés, l'horizon de prévision optimal n'est pas toujours de 12 mois. Cette étude projette de déterminer pour chacune des 120 séries choisies un horizon de prévision optimal en fonction de leurs composantes irrégulière et de tendance-cycle.

Des M.A.P.E. associés à différent horizons de prévision sont calculés pour chaque série. Nous sommes ainsi en mesure d'établir la relation qui existe entre chaque horizon de prévision, l'erreur de prévision et la contribution des composantes irrégulière et de tendance-cycle à la variance totale de la série.

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1. INTRODUCTION

It has been proven theoretically through the study of filters (Dagum 1982.a; 1982.b and 1983) and corroborated by several empirical studies, among others, in Kuiper (1978), Kenny and Durbin (1982), Dagum and Morry (1984) that the use of one year ahead ARIMA extrapolations reduces the revisions of the concurrent seasonal factors obtained from the X-11-ARIMA Program (Dagum, 1980).

A set of four ARIMA models ($(011)(011)_{12}$, $(012)(011)_{12}$, $(022)(011)_{12}$ and $(210)(011)_{12}$) identified by Chiu, Higginson and Huot (1985) fitted and forecasted well a large variety of economic time series. These models were ranked on the basis of the one year ahead forecast error (as well as some other criteria concerning fit, parsimony, etc.) The inclusion of the four models in the X-11-ARIMA program in the order of their ranking is to ensure that a good set of extrapolated values can be produced even if the user has no expertise in ARIMA model identification.

From the viewpoint of minimizing the filter revisions of the X-11-ARIMA method, the maximum forecast horizon should not be restricted to one year only for all series. Thus, it is necessary to investigate which factors would determine the length of the forecast horizon to be used in order to produce the best seasonally adjusted estimates (the ones with the smallest revision). Furthermore, it is also of interest to know how the varying forecast horizon affects the ranking of the models.

The ultimate objective of a larger study in preparation by the authors is to produce guidelines based on certain characteristics of the series regarding the number of forecast values necessary to yield seasonally adjusted estimates of minimal revision. It is obvious that the revisions in these estimates will depend on the forecast error. (A perfect three-year forecast would result in estimates that do not get revised at all). Thus, as a first stage in this direction, this paper examines the relationship between the forecast error at different time horizons and certain characteristics of the series namely, the amount of irregular variation present and, the pattern of the trend-cycle component. Other authors (Makridakis and Hibon, 1979) before us examined the relationship between forecast errors and the randomness present in a series. It should be pointed out, however, that the randomness in the context of X-11-ARIMA is the variation in the irregular component as identified by the seasonal adjustment method, while in the quoted study, it referred to the variance of residuals left in the series after fitting a model.

Section 2 describes the design of the experiment; section 3 deals with the relationship between the forecast error and the noise in the series; section 4 investigates the relationship between the forecast error and the pattern of the trend-cycle component and; finally, section 5 gives the conclusions of this paper.

2. THE DESIGN OF THE EXPERIMENT

In designing the experiment our main objectives are: (1) to have a representative sample of economic time series in terms of the amount of irregular fluctuations present in them; (2) to eliminate the dependence of the forecast error on the particular month of the year chosen as forecast origin; (3) to minimize the number of ARIMA fitting and forecasting necessary in order to reduce costs.

To achieve these objectives, the following design is adopted. A sample of 120 series is selected from five sectors of the economy (Labour, External Trade, Manufacturing, Finance and Agriculture). These series fall into five classes (of 24 series each) according to the amount of irregular variation present as identified by the X-11-ARIMA program in table F.2.B. The five classes are:

Class 1	0.0 - 5.0%	irregular variation
Class 2	5.1 - 10.0%	" "
Class 3	10.1 - 20.0%	" "
Class 4	20.1 - 30.0%	" "
Class 5	30.1 - 50.0%	" "

All the series begin in January 1970 but the last value of the series (which becomes the forecast origin) varies between June 1977 to May 1978, for the first set of forecasts, to satisfy our second objective. Thus in each class there are two series that end in the same month. There are two ARIMA forecasts of 24 observations obtained for each series, with the forecasting origins one year apart. For example, the series that ends in June 1977 has a set of forecasts ending in June 1979. The series is then extended with one more year of real data and a new set of 24 forecasts (based on the newly fitted ARIMA model) is produced with June 1978 as the forecasting origin. Thus the last time point for which a forecast is available is May 1981 coming from a series that ends in May 1979.

Another (simpler) alternative would be to produce 12 sets of forecasts per series each starting at a different month of the year. This design, however, requires a much larger number of ARIMA fits and would increase substantially the cost since the number of series cannot be reduced without jeopardizing the representativeness of the sample.

Finally, May 1981 is chosen as the last forecast time point to avoid the effect of the 1981 recession on the forecast errors.

The four ARIMA models described before are applied to each series and forecasts are generated from two time origins; i.e. eight sets of forecasts are obtained from each series.

The information obtained for each series includes the forecast errors for 24 time points for all eight sets, the ARIMA parameter values, the goodness of fit statistic, the forecasting origin, the class according to the amount of irregularity and the series identifier. This information is merged with data collected previously from each series during the X-11-ARIMA seasonal adjustment run concerning the amount of irregular, cyclical and seasonal variation in the series.

4. RELATIONSHIP BETWEEN M.A.P.E.'s OF ARIMA EXTRAPOLATIONS AND THE PATTERN OF THE TREND-CYCLE COMPONENT OF THE SERIES

In order to evaluate the effect of the trend-cycle on the forecasting performance of the four ARIMA models, we compare here the M.A.P.E. of the forecasts with a measure M that provides information on the pattern of the trend-cycle component in the series. The measure M is defined by

$$M = \frac{\displaystyle\sum_{i=2}^{n} \frac{(T_i - T_{i-1})}{T_{i-1}}}{\displaystyle\sum_{i=2}^{n} \left| \frac{T_i - T_{i-1}}{T_{i-1}} \right|} \qquad (1)$$

where T_i is the annual total of the original series for year i. M can take values between -1 and 1. When M is equal to one, the series has a monotonically increasing annual trend. For values between 0 and 1, the series has reversals of direction in its annual rates of change (negative values). The series is then assumed to be affected by the business cycle.

Values between -1 and 0 will have a similar interpretation but the sample of series chosen for this study has very few cases where M is smaller than .60. Table 1 shows the frequency distribution of the sample according to the values of M.

Table 1
Frequency Distribution of the Sample
According to the Measure M

Intervals	Frequency	% of Total
-1.0	0	0.00
-.99 to -.80	1	0.83
-.79 to -.60	0	0.00
-.59 to -.40	1	0.83
-.39 to -.20	3	2.50
-.19 to 0	2	1.67
.01 to .20	1	0.83
.21 to -.40	7	5.83
.41 to .60	13	10.83
.61 to .80	11	9.17
.81 to .99	34	28.33
1.0	47	39.18
Total	120	100.00%

Figure 4 shows the M.A.P.E. of the forecasts from a $(0,1,1)(0,1,1)_{12}$ ARIMA model for the four time horizons versus several values of M. It is apparent that the average forecast error decreases with increasing M. For the case of M=1, a perfectly monotonic trend, the M.A.P.E. is very small. There are no major differences for the four time horizons when M is close to 1. Series with more cyclical movements ($.6<M<.8$) however, tend to have higher M.A.P.E.'s at longer time horizons i.e. the models predictive power diminishes with time if the series are affected by cycles. The other models showed similar forecast error pattern according to the M measure.

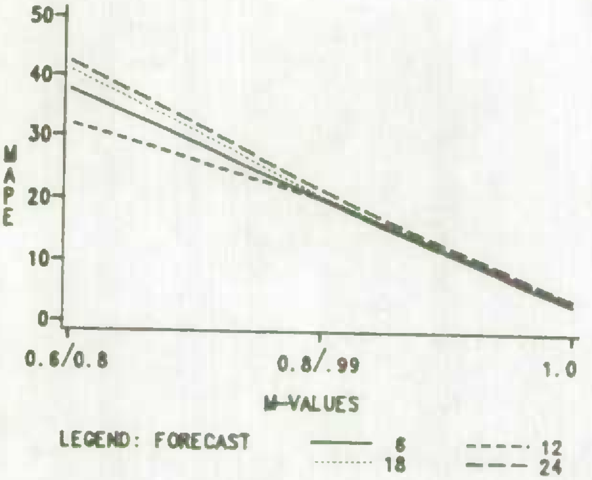

Figure 4. Mape of forecasts from a (011)(011)12 ARIMA model for different time horizons as a function of the M-values of the series

Finally, we looked at the relationship between the M.A.P.E. of the ARIMA extrapolations and the various amount of irregularity in the series for fixed values of M. Figure 5 shows the M.A.P.E.'s for the $(0,1,1)(0,1,1)_{12}$ model with a 12 month time horizon and M=1 and $M \leq .99$.

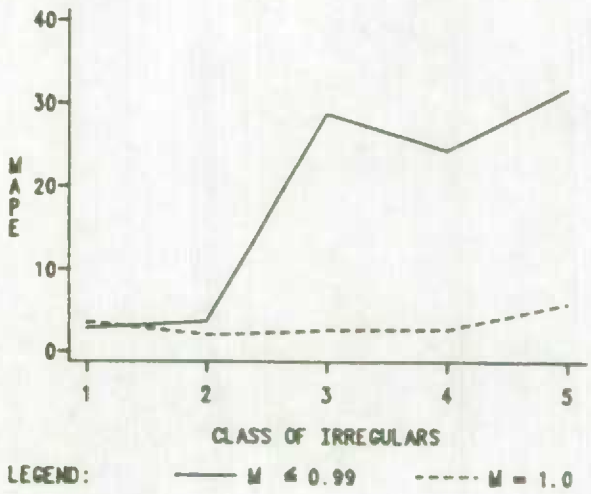

Figure 5. Mape of the 12 month ahead forecasts from a (011)(011)12 ARIMA model for various amounts of irregularity, M ≤ 0.99 and M = 1.0

When M=1, the M.A.P.E.'s of the forecasts are small and change very little with increasing values of the irregular component. On the other hand, when the series have been cyclically affected such as for $M \leq .99$, the M.A.P.E.'s increase with increasing irregularity in the series.

3. THE RELATIONSHIP BETWEEN THE M.A.P.E.'s of ARIMA EXTRAPOLATIONS AND THE NOISE IN THE SERIES

We analyse here the relationship between the forecasting performance of the ARIMA models and the amount of irregularity present in the series. Figure 1 shows the mean absolute percentage error (M.A.P.E.) of the forecasts from a $(0,1,1)(0,1,1)_{12}$ ARIMA model for four time horizons against five classes of irregular variations.

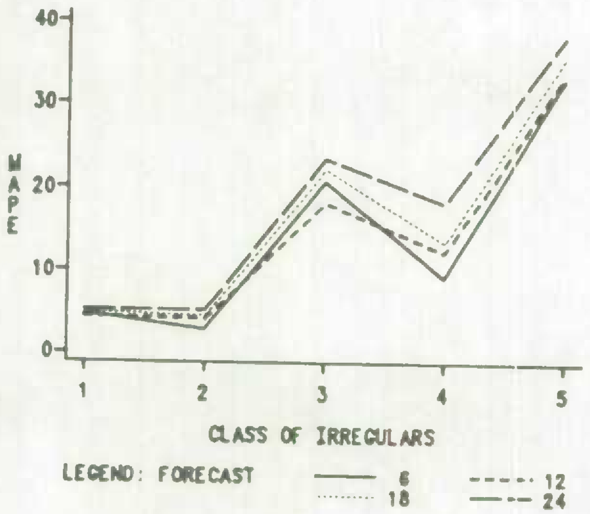

Figure 1. Mape of forecasts from a (011)(011)12 ARIMA model for different time horizons for various amounts of irregularity in the series

It is apparent that the M.A.P.E. increases with the amount of noise present for each of the four time horizons of 6, 12, 18 and 24 months. The increase is very large as we move from class 2 to 3, that is for series with a maximum of 10% of irregularity to a maximum of 20% and similarly from 30% to 50%. On the other hand, a decrease is observed between classes 3 to 4. This unexpected behaviour can only be explained by the characteristics of the sample series that fall in class 4.

If the amount of irregularity is fixed, the dispersion of the M.A.P.E.'s is very small among the four time horizons of 6, 12, 18 and 24 months. A similar pattern was observed for the M.A.P.E's of forecasts from the ARIMA models $(0,1,2)(0,1,1)_{12}$ and $(2,1,0)(0,1,1)_{12}$.

On the other hand, Figure 2 shows a different pattern for the M.A.P.E.s of the extrapolations from a $(0,2,2)(0,1,1)_{12}$ model. The dispersion of the M.A.P.E.'s of the forecasts for the four time horizons is large within each class of irregular variation. However, similarly to the other 3 models, the M.A.P.E. increases with increasing amount of noise in the series.

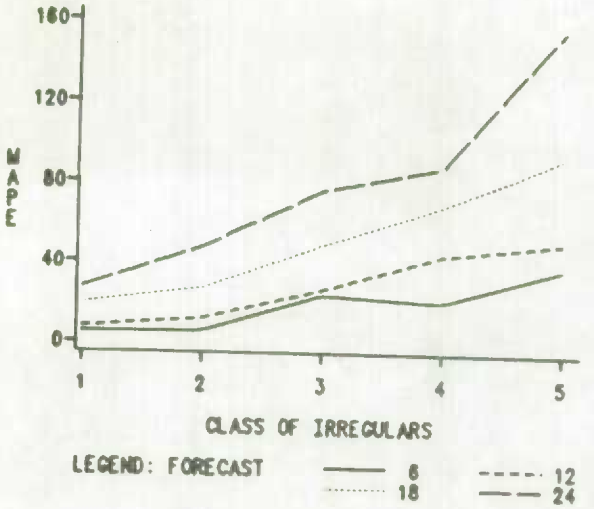

Figure 2. Mape of forecasts from a (022)(011)12 ARIMA model for different time horizons for various amounts of irregularity in the series

Figure 3 shows the predictive performance of the four ARIMA models for a time horizon of 12 months (which is currently the only one included in the X-11-ARIMA program). It can be seen that the M.A.P.E.'s $(0,2,2)(0,1,1)_{12}$ model produces M.A.P.E. values which are much higher than those of the other three remaining models at each class of irregular. In fact, the (0,2,2)(0,1,1) ARIMA model M.A.P.E.'s ranked first in the highest proportion (33%) of the sample series with M.A.P.E.'s smaller than 5% but when it is ranked fourth, the M.A.P.E.'s of the forecasts ranged between 50% and 70%, much higher than the M.A.P.E.'s of forecasts from the other models.

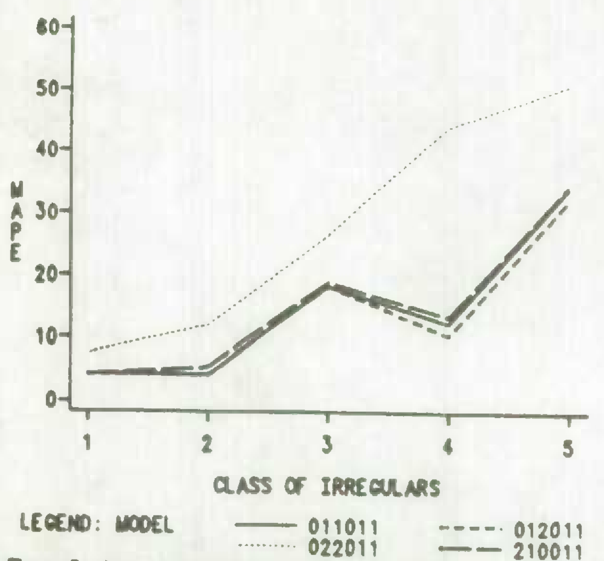

Figure 3. Mape of the 12 month ahead forecasts from four ARIMA models for various amounts of irregularity in the series

In fact, when the amount of irregularity in the series is larger than 10%, the presence of the business cycle seems to be the main cause that affects the forecasting performance of the ARIMA models.

5. CONCLUSIONS

This paper has shown that the average forecast error from ARIMA models seems to be more sensitive to the presence of the business cycle in the series than to either the amount of irregularity or the length of the forecast horizon. This is particularly evident when the contribution of the irregulars to the total variance of the series is larger than 10%.

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