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NEW DEVELOPMENTS IN THE X-11-ARIMA

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NEW DEVELOPMENTS IN THE X-11-ARIMA*

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Abstract

The X-11-ARIMA Seasonal Adjustment computer program (Dagum 1980) performs three basic functions, namely, (1) forecasting; (2) seasonal adjustment and (3) composition of original and seasonally adjusted series.

Major developments are being carried out in two key functions in order to increase the efficiency of the method and its capability to handle adequately a larger number of series.

New developments related to the forecast functions are (i) a new set of built-in ARIMA models; (ii) variable forecasting horizons; (iii) backcasting only series shorter than seven years; (iv) new acceptance criteria of fitting and extrapolation for the built-in ARIMA models; (v) determining the optimal forecast horizon as a function of both the amount of noise in the series and its trend-cycle pattern; (vi) introduction of other extrapolation methods.

Developments related with the seasonal adjustment function include (i) estimation of Easter effect; (ii) estimation of stochastic trading-day variations; (iii) new replacement of extreme values; (iv) increasing the accuracy of the end-weights of the five Henderson trend-cycle filters; and (v) new diagnostic tools.

The main purpose of this paper is to provide a summary of each one of these major developments and their impact on the two corresponding basic functions of X-11-ARIMA.

KEY WORDS: Forecasting, Seasonal Adjustment, Trading Day Variations, Easter Effect, Extreme Values, Trend-cycle filters.

Introduction

The X-11-ARIMA seasonal adjustment method developed by Dagum (1975 and 1980) basically consists of:

(i) Modelling the original series by Autoregressive Integrated Moving Average Processes (ARIMA models) of the Box and Jenkins (1970) types.

(ii) Extrapolating one year of unadjusted data at each end of the series from an ARIMA model that fit and extrapolated well the original series according to some acceptance criteria. This operation called forecasting and backcasting is designed to extend the observed series at both ends.

(iii) Seasonally adjusting the extended series with moving averages (linear filters) that result from the combination of the ARIMA model extrapolation filters and the seasonal adjustment filters of the Census Method II-X-11 variant (Shiskin, Young and Musgrave, 1967).

For all "historical" values (at least three and a half years from the last available observation), the ARIMA extrapolation filters have practically no effect on the seasonally adjusted figures.

The X-11-ARIMA computer package can be applied in two different modes, namely, (i) with ARIMA extrapolations and (ii) without ARIMA extrapolations. In the latter case, the seasonally adjusted values are close but not necessarily equal to those obtained with the Census Method II X-11 variant. The discrepancies are due to some differences in the process of identification and replacement of extreme values and the fact that the seasonal filters are calculated from their explicit formulas.

The current X-11-ARIMA computer program performs three basic functions, (i) forecasting; (ii) seasonal adjustment and (iii) series composition. This composition can be made with original series and/or seasonally adjusted data by addition, subtraction, multiplication and/or division.

In recent years several economic and institutional events have affected the pattern of the various components of most time series. Among these events, notable ones are the deep recession of 1981-82; the introduction of some government regulations that enabled business stores to be open longer hours and on Sunday and the early arrival of Easter (March 1986) after having fallen on April since 1979.

To cope with the problems generated by these events, a significant amount of research was carried out that led to new developments. Some of these developments have already been fully tested and incorporated in Statistics Canada's version of X-11-ARIMA (not yet available for distribution abroad). Other developments are currently being tested and will be implemented at a later stage.

The main purpose of this paper is to provide a summary of these new developments and their impact on X-11-ARIMA.

2. Major Developments in the Forecasting Function

The new developments in the basic function of forecasting apply mainly to the automatic ARIMA extrapolation option of the program. They are described next.

2.1. A New Set of Built-In ARIMA Models

Currently there are three built-in ARIMA models tested automatically by the X-11-ARIMA method. These ARIMA models were selected according to some criteria of fitting and extrapolation, from a sample of 174 series that ended in 1977 (Lothian and Morry 1978). Given the economic and institutional events of the early 80's, it was important to assess whether these models were still relevant. Chiu, Higginson and Huot (1985) conducted a study on a sample of 190 seasonal series that ended in 1983 from eleven sectors of the Canadian economy. These authors evaluated the performance of a set of 7 ARIMA models (including the 3 available in the program) according to the following eight criteria: mean absolute percentage error of the forecasts for the last three years, the chi-square statistics for the randomness of the residuals, underdifferencing, overdifferencing, stability, invertibility, correlation between parameters and the presence of small parameter values. Although not mutually independent, these criteria were useful to evaluate the goodness of fit and the goodness of forecasting for each model.

The study ranked the four first models as follows:

1 -(0,1,1)(0,1,1)s

2 - (0,1,2)(0,1,1)s

3 - (2,1,0)(0,1,1)s

4 - (0,2,2)(0,1,1)s

These models are expressed in the classical Box and Jenkins (1970) symbolic notation, where p and P denote the order of the ordinary and seasonal autoregressive parameters, respectively; q and Q denote the order of the ordinary and seasonal moving average respectively; d and D denote the order of the ordinary and seasonal differences, respectively.

The combined rate of success for the first three models varied from 97% for labour series to 21% for external trade series. The rate was considered good, in general, given the fact that during two of the three years tested, Canada suffered a severe recession. Furthermore, it was evident that the rate of success of model (1) was much smaller than the rate obtained by Lothian and Morry (1978) with series ending in 1977. The fourth model (0,2,2)(0,1,1) was found to fit well an important class of series (series with a steep change in trend) that all the other models fit poorly. (Similar results were obtained by Lothian and Morry, 1978.)

The new experiment detected two new models, the (0,1,2)(0,1,1)s and the (2,1,0)(0,1,1)s as good for extrapolation and fitting a large class of series. It was then decided to keep the currently available three ARIMA models and add the two new models. The reason for keeping the (2,1,2)(0,1,1)s model was that it performed the best from the viewpoint of forecasting alone.

The availability of five models instead of three does not increase expenses in running the program because these models are tested sequentially in the order shown plus the (2,1,2)(0,1,1) as the last. In other words, if model (1) passes then, the program does not try the others, but if model (1) fails, it tries model (2) and so on.

2.2 Variable Forecasting Horizons

The automatic ARIMA option of X-11-ARIMA currently extrapolates only one year of data from a model that has passed the acceptance criteria. Since one of the main purpose of extending the series with forecasts is to improve the efficiency of the asymmetric seasonal filters, an important question arises, what is the optimal length of the forecast horizon that minimizes the discrepancy between the symmetric and the asymmetric filters? Of these latter, the filter applied to the last available observation when seasonal adjustment is done contemporaneously (concurrent seasonal adjustment) is the most important for it is the dominant mode of current seasonal adjustment in Statistics Canada. For all practical purposes, when the series is extended with three and a half years of data, the observation that was adjusted by the concurrent filter will be adjusted by a symmetric filter. Therefore, we wanted to know how far the data should be extended into the future for the asymmetric concurrent filter of X-11-ARIMA with extrapolation to improve on the concurrent filter without ARIMA extrapolation. This latter is close to the concurrent seasonal filters of Census Method II-X-11 variant. A study by Dagum (1982) addressed the problem of revisions of the concurrent and other non-symmetric filters when one, two and three years of data are extrapolated from the three built-in ARIMA models in X-11-ARIMA, namely, $(0,1,1)(0,1,1)_{12}$; $(0,2,2)(0,1,1)_{12}$ and $(2,1,2)(0,1,1)_{12}$ with several combinations of parameter values.

Dagum (1982) showed that from the viewpoint of filter revision, the largest gain was obtained with one year of extrapolated values; there was a small incremental gain if two years were used and finally there was no gain going from two to three years. This study was extended for the concurrent filter by Huot et als (1986) using four models, namely, $(0,1,1)(0,1,1)_{12}$, $(2,1,0)(0,1,1)_{12}$, $(012)(011)_{12}$ and $(0,2,2)(0,1,1)_{12}$. These authors found first the set of ARIMA parameter values for which the X-11-ARIMA concurrent seasonal adjustment filters improve on X-11-ARIMA concurrent filter without extrapolation. Then, they estimated the parameter values corresponding to a large sample of real series for which the above models passed the acceptance criteria. Almost all the estimated parameter values fell within the region where using ARIMA extrapolations would reduce the size of the revisions. Huot et als (1986) also investigated the effect of various forecast horizons on the total revisions for selected parameter values. Their results showed that the optimal forecast horizon that minimizes filter revisions changes with the parameter values of the model. Thus for the $(0,1,1)(0,1,1)_{12}$ where $\Theta=.50$ and $\Theta=.90$, the filter revisions are minimized for a forecast horizon of 24 months. On the other hand, for small values of Θ (say $\Theta=.40$) the forecast horizon should be shorter than a year. These results are in agreement with the fact that the larger the value of Θ the more stable the seasonal pattern is assumed to be and consequently, a longer forecast horizon is feasible.

A subroutine that enables the user to select the length of the forecast horizons is already available in the new X-11-ARIMA computer program one year being the default option.

2.3. Determining the Optimal Forecast Horizon for ARIMA models

Having incorporated the variable forecast routine into the program, it became necessary to investigate which factors should determine the length of the forecast horizon to be used in practise to produce the best current seasonally adjusted figure (best in the sense of being the one with smallest revisions). It is obvious that the revisions in the current figure depend on the horizon of the forecast and the forecast errors. Extending the series with perfect forecast for three years and a half would result in an estimate that does not get revised at all. As a first stage toward the final objective of producing guidelines on the number of forecasts necessary to yield seasonally adjusted estimates of minimal revisions, Dagum et als (1986 and 1987) investigated the optimal time horizons of the four ARIMA models listed in section 2.1 as a function of the noise content of the series and their trend-cycle patterns. These authors used a sample of 120 business and economic series classified according to the amount of irregular variation (as identified by the X-11-ARIMA method) into 5 classes ranging from less than 5% to less than 50%. The results showed that the forecast errors increased fast with the increased irregularity in the series when the amount of irregularity was larger than 10%. Furthermore, for a fixed amount of irregularity the dispersion of the forecast errors was very small among the four time horizons analyzed, namely, 6, 12, 18 and 24 months.

In order to evaluate the effect of the trend-cycle pattern on the forecasting performance, Dagum et als (1986) introduced a measure of the degree of monotonicity of the trend-cycle component. The results showed that for the four models the mean absolute forecast errors tend to be more sensitive to the presence of cyclical variations than to either the amount of irregularity in the series or the length of the forecast horizon. This was particularly evident for those series where the irregulars contributed more than 10% to their total variance.

The next step of this major study is to determine how much the total revisions of the current seasonally adjusted values are reduced when the series are extended with an optimal number of forecasts.

2.4. Backcasting

Another modification being introduced into the automatic ARIMA extrapolation option concerns backcasting. The current version of this program backcasts one year of data for all series shorter than 15 years. Although backcasting improves the seasonal adjustment, in general, for series of 7 years or longer it has the disadvantage of generating permanent revisions. These revisions are on early and historical seasonally adjusted values. Furthermore, backcasting also introduces revisions of current seasonally adjusted values in 8, 9 and 10-year series. This is due to the fact that the backcast values will change whenever the parameter values of the ARIMA models change. An optimal trade-off in the sense that the advantages of backcasting dominate its disadvantages can be found in series shorter than 7 years for the use of backcasting enables the application of a symmetric filter to observations in the middle.

The option of backcasting only series of less than seven years is now being incorporated into the X-11-ARIMA package.

2.5. Other Modifications in the Forecasting Function

The criteria of fitting and extrapolation for the built-in ARIMA models introduced by Dagum (1981) have been relaxed. The mean absolute percentage of the forecast errors (M.A.P.E.) has been raised to 15% from ther current 12% and the level of significance of the chi-squared distribution of the Ljung and Box (1978) test for the randomness of the residuals is 5% instead of 10%. These changes in the level of the acceptance criteria do not affect the advantages of using the ARIMA extrapolations but enable a more frequent application of the automatic ARIMA options.

We are currently considering the incorporation of other extrapolation methods besides the ARIMA models as an option for the users. These new methods are expected to be optimal for shorter and longer forecast horizons than a year. They should also be of easy estimation and interpretation.

Another modification already introduced into the program is the automatic removal of trading day variations (if present) before the ARIMA modelling for these models cannot adequately handle trading-day variations.

3. Major Developments in the Seasonal Adjustment Function

The major developments being carried out on the seasonal adjustment function of X-11-ARIMA include (i) estimation of Easter effects; (ii) estimation of stochastic trading day variations; (iii) new replacement of extreme values; (iv) increasing the accuracy of the end-weights of all the Henderson trend-cycle filters available in the program and (v) new diagnostic tools. A description of these developments follow.

3.1. Estimation of Easter Effects

Currently the X-11-ARIMA does not estimate the impact of Easter on the total variation of a series. This type of moving holiday associated with the calendar can cause serious distortions in month-to-month movements when it occurs in March or at the beginning of April.

There has been a renewed interest in Easter adjustment recently due to the early arrival of Easter (March 28) in 1986. In international trade series, particularly imports, a drop was observed in March followed by a rise in April caused by processing the end of March custom forms only in April because of the Easter closing of customs offices. This is an example of series in which the presence of Easter is accompanied by a decrease of level. The opposite effect can be observed in other series; e.g. marriages always show a marked increase during Easter. In these two examples the <u>impact</u> of Easter is <u>immediate</u> in the sense that only the holiday period displays a change of activity.

There is another type of Easter effect which affects not only the holiday period but days (sometimes weeks) before it. This type of <u>gradual</u> impact occurs in retail trade series such as chocolates, flowers, women's clothing.

To take into account both kinds of Easter effect, the following models are now being incorporated into the X-11-ARIMA program (Dagum, Huot and Morry 1987.a).

(a) <u>Immediate Impact Model for Easter Effect.</u>

$$E_i = 1/2\, f(Z_i) \left[\frac{\sum_{i \in M} (I_{i,j+1} - I_{i,j})}{n_M} - \frac{\sum_{i \in A} (I_{i,j+1} - I_{i,j})}{n_A} \right] \qquad (3.1.1)$$

where

Z_i = number of days between Easter Sunday in year i and March 22 (the earliest possible Easter date)

$f(Z_i) = 1$ if $Z_i \leq 9$ (Easter falls in March)

$f(Z_i) = 0$ if $Z_i > 9$ (Easter falls in April)

I_{ij} = residuals estimated in first iteration of X-11-ARIMA and assumed to be affected by Easter effect (E_i); i denotes year and j month of March (consequently j+1 denotes April)

n_M = number of years when Easter fell in March;

n_A = number of years when Easter fell in April.

(b) <u>Gradual Impact Model for Easter Effect.</u>

$$E_i = 1/2\, f(Z_i)\left[\frac{\sum_{i \in M}(I_{i,j+1} - I_{i,j})}{n_M} - \frac{\sum_{i \in A}(I_{i,j+1} - I_{i,j})}{n_{LA}}\right] \qquad (3.1.2)$$

where now the indicator function $f(Z_i)$ is defined by

$f(Z_i) = 1$ if $Z_i \leq 9$

$f(Z_i) = \dfrac{k+9-Z_i}{k}$ if $9 < Z_i < k$

$f(Z_i) = 0$ if $Z_i \geq k$

The subscript LA refers to late April Easter defined as an Easter date occurring on or after the k-th of April.

The build-up period k can be automatically chosen by the program according to the characteristics of the series or provided by the user. The process of estimating k is fully described in Dagum, Huot and Morry (1987.a).

3.2. Estimation of Trading-Day Variations

The trading-day variations are estimated in the X-11-ARIMA program using ordinary least squares (OLS) on a regression model developed by Young (1965). It is assumed that,

$$I_t = TD_t + \varepsilon_t \qquad (3.2.1)$$

where I_t denotes the residuals after trend-cycle and seasonality have been removed from the original series, TD_t denotes the trading-day variations and ε_t is a purely random irregular, i.i.d $(0, \sigma_\varepsilon^2)$.

The trading day variations can be expressed by,

$$TD_t = \sum_{i=1}^{6} \delta_i T_{it} \qquad (3.2.2)$$

where $\delta_i = (\xi_i - \bar{\xi})$ and $T_{it} = X_{it} - X_{7t}$. The δ_i's represent the difference between the Monday, Tuesday,..., Saturday effects and the average daily effect $\bar{\xi}$. X_{it} denotes the number of times a given day of the week occurs in months t and X_{7t} denotes the number of times that Sunday occurs in month t. As long as the relative weight of daily activities is stable throughout the length of the time series, this approach produces reliable estimates.

On the other hand, changes in shopping patterns or store opening hours introduce modifications in trading-day variations that cannot be adequately estimated with the existing deterministic model. To overcome this limitation, Dagum, Quenneville and Cholette, (1987) are presently investigating two other types of model that assume a stochastic behaviour of trading-day variations. One model, already discussed by Monsell (1983) assumes that the δ's follow a random walk model. That is,

$$TD_t = \sum_{i=1}^{6} \delta^1_{it} \; T_{it} \qquad (3.2.3)$$

where the process generating $\underset{\sim}{\delta}^{1'}_t = (\delta^1_{1t}, \ldots, \delta^1_{6t})$ is given by

$$\underset{\sim}{\delta}^1_t = \underset{\sim}{\delta}^1_{t-1} + \underset{\sim}{\chi}_t ; \quad \underset{\sim}{\chi}_t \quad N.I.D.(0, \sigma^2_\chi I_6) \qquad (3.2.4)$$

The second stochastic model assumes that the vector of daily coefficients follows a random walk with a random drift. That is,

$$TD_t = \sum_{i=1}^{6} \delta^2_{it} \; T_{it} \qquad (3.2.5)$$

$$\underset{\sim}{\delta}^2_t = \underset{\sim}{\delta}^2_{t-1} + \underset{\sim}{\rho}_t + \underset{\sim}{\chi}_t \qquad (3.2.6)$$

$$\underset{\sim}{\rho}_t = \underset{\sim}{\rho}_{t-1} + \underset{\sim}{\phi}_t \qquad (3.2.7)$$

where $\underset{\sim}{\chi}_t$ and $\underset{\sim}{\phi}_t$ are mutually independent and distributed $N(0,\sigma^2_\chi I_6)$ and $N(0,\sigma^2_\phi \; I_6)$ respectively.

These two stochastic models for trading-day variations are written in state-space form and the estimates $\underset{\sim}{\hat{\delta}}^1_t$ and $\underset{\sim}{\hat{\delta}}^2_t$ are calculated with the Kalman filter and Kalman fixed interval smoother. The hyper-parameters σ^2, σ^2_χ/σ^2 and σ^2_ϕ/σ^2 are estimated with maximum likelihood estimators.

3.3. New Replacement of Extreme Values

Currently the X-11-ARIMA method replaces an extreme seasonal-irregular value say SI_{t_o}, receiving less than full-weight, with an average of the corresponding SI_{t_o} times its weight and the <u>two nearest preceding</u> and the <u>two nearest following</u> full-weight SI_{t_i} for that month (quarter). That is,

$$SI_{t_o}^{(m)} = \left(\sum_{i=1}^{4} SI_{t_i} + \omega_o SI_{t_o}\right) / \left(4+\omega_o\right) \qquad (3.3.1)$$

where $t_1<t_2<t_o<t_3<t_4$ but not necessarily equally spaced and ω_o is the weight for the extreme value. In general, the ω_i's such that $o\leq\omega_i\leq 1$ are calculated in a linearly graduated way, being zero for values of the irregulars I_{t_i} falling at or outside 2.5σ and equal to one for values falling at 1.5σ or less. The equation (3.3.1) is changed for SI extreme values in either of the two beginning or ending years. In such cases, the $SI_{t_o}^{(m)}$ is the average of the corresponding SI_{t_o} times its weight ω_o and the four nearest full-weight ($\omega_i=1$) seasonal-irregular values for that month (quarter). If enough full-weight SI_{t_i} are not present, then the $SI_{t_o}^{(m)}$ is simply the arithmetic average of all the SI's for that month (quarter).

There are two limitations with the current procedure; one, for borderline extremes (ω_o near to one) the modified seasonal-irregular can change abruptly from an average of five SI's to its actual value SI_{t_o} just simply when one more observation is added. Two, if full-weight seasonal-irregulars can be found only very far from the extreme value, serious distortions are introduced when seasonality changes rapidly. In order to avoid these limitations, a new formula is being tested on a large sample of series. The proposed formula (Dagum and Chiu, 1987) is

$$SI_{t_o}^{(m)} = \left(\sum_{i=1}^{4} \alpha_i SI_{t_i} + \omega_o SI_{t_o}\right) / \left(\Sigma\alpha_i + \omega_o\right) \qquad (3.3.2)$$

where $t_1<t_2<t_o<t_3<t_4$ and not necessarily equally spaced.

$\alpha_i = \beta_i \omega_i (1-\omega_o^4)$ results from a combination of the current weights ω_i calculated by the program and a set of β_i weights which penalize SI_{t_i} values more than two years away from SI_{t_o}. The further the SI_{t_i} value the smaller is β_i. The average is taken over the four largest α_i's. In the case of an SI_{t_o} to be replaced with two full-weighted values present at the two nearest preceding and two nearest following years, then $\beta = 1$ and $\omega_i = 1$; and the equation (3.3.2) reduces to

$$SI_{t_o}^{(m)} = \left((1-\omega_o^4) \sum_{i=1} SI_{t_i} + \omega_o SI_{t_o}\right) / \left(4(1-\omega_o^4)+\omega_o\right) \qquad (3.3.3)$$

If $0<\omega_o<1/2$ equation (3.3.3) gives results similar to the current procedure but as ω_o tends to one, the modified seasonal-irregular tends to its observed value. (This formula (3.3.3) was suggested to me in a letter by Mr. Andrew Sutcliffe of the Australian Bureau of Statistics.)

3.4. <u>End-Weights of Henderson Trend-Cycle Filters</u>

The weights of all asymmetric trend-cycle filters in X-11-ARIMA are those of the Census X-11-Variant. These weights are given with three digits only and, thus, the degree of accuracy of the estimates is rather limited. The reason for not using weights with higher precision is that it was not known how these weights were obtained by Shiskin, Young and Musgrave (1967).

Laniel (1985) has recently found a criterion for the design of the 13-term Henderson end weights that gives exactly the same values as those incorporated into X-11-ARIMA and consequently, they can now be calculated to any degree of precision desired.

The formula used to obtain these weights is based on the minimization of the mean squared revision (MSR) between the final estimate (obtained by the application of a symmetric filter) and the preliminary estimate (obtained by the application of an asymmetric filter) subject to the constraint that the sum of the weights is equal to one. The assumption made is that at the end of the series the seasonally adjusted values are equal to a linear trend-cycle plus a purely random irregular NID$\sim(0,\sigma^2_a)$. The equation used by Laniel (1985) is,

$$E[r_t^{(i,m)}]^2 = c_1^2 \left(t-\sum_{j=-i}^{m} h_{ij}(t-j)\right)^2+\sigma_a^2 \sum_{j=-m}^{m} (h_{mj}-h_{ij})^2 \qquad (3.4.1)$$

where h_{mj} and h_{ij} are the weights of the symmetric (central) filter and the asymmetric filters, respectively; $h_{ij}=0$ for $j=-m,\ldots,-i-1$, c_1 is the slope of the line and σ_a^2 denotes the noise variance. There is a relation between c_1 and σ_a^2 such that,

$$I/C= (4\sigma_a^2/\pi)^{1/2}/|c_1| \qquad (3.4.2)$$

the I/C noise to signal ratio in the Census X-11 variant and X-11-ARIMA as well, determines the length of the Henderson trend-cycle filter to be applied. Thus, setting t=o and m=6 for the end weights of the 13-term Henderson, we have,

$$\frac{E\, r_o^{(i,6)}}{\sigma_a^2} = \frac{4}{\pi(I/C)^2}\left((\sum_{j=-i}^{6} h_{ij})^2 + \sum_{j=-6}^{6} (h_{6j} - h_{ij})^2 \right) \tag{3.4.3}$$

Making I/C=3.5 (the most noisy situation where the 13-term Henderson is applied), Laniel (1985) obtained the same set of end weights as those of Census X-11 variant. These end weights have been calculated for the remaining Henderson filters using, for quarterly series I/C=3.5 for the 5-term filter and I/C=7 for the 7-term filter; for monthly series I/C=.99 for the 9-term filter and I/C=7 for the 23-term filter. These weights are now incorporated into the X-11-ARIMA program.

3.5. New Diagnostic Tools

The major developments concerning the various tests and statistics used to control the quality of the seasonally adjusted series are described below.

(a) A Modified F-test for the Presence of Stable and Moving Seasonality.

The most important test applied by X-11-ARIMA to assess the presence of both stable and moving seasonality is the F-test. This test is based on standard ANOVA methods and do not take into account the likely possibility that the seasonal-irregular (SI) estimates to which they are applied can be autocorrelated.

Sutradhar and MacNeill (1987) have investigated the effect on the F statistic assuming the following model for stable seasonality.

$$Y_i(t) = \mu + \alpha_i + Z_i(t)$$
$$i = 1,\ldots,k \; ; \; t=1,\ldots,n \qquad (3.5.1)$$

Where $Y_i(t)$ denotes the final unmodified SI estimates corresponding to year t and season i; α_i is the effect due to the ith season; and $Z_i(t)$ follows an ARMA process (p,q)(P,Q)s. As in standard ANOVA, the season or group effect is constrained by the relation $\sum_{i=1}^{k} \alpha_i = 0$.

The modified F-test statistic, F_m, for testing H_o: α_i=0 versus H_1: $\alpha \neq 0$ for at least one i is

$$F_m = \frac{n \sum_{i=1}^{k} (\bar{Y}_{i.} - \bar{Y}_{..})^2 \quad k(n-1)c_2(\phi,\Phi,\theta,\Theta)}{\sum_{i=1}^{k} \sum_{t=1}^{n} (Y_i(t)-\bar{Y}_{i.})^2 \quad (k-1)c_1(\phi,\Phi,\theta,\Theta)} \qquad (3.5.2)$$

where c_2 and c_1 are given by Sutradhar and MacNeill (1987) for various combinations of the parameter values ϕ, Φ, θ and Θ.

If the seasonal pattern is assumed to change over time, then the model (3.5.2) becomes,

$$Y_i(t) = \mu+\alpha_i+\beta_t+Z_i(t) \qquad (3.5.3)$$

where now β_t denotes the tth time effect and $\sum_{t=1}^{n} \beta_t = 0$.

The hypotheses to be tested are as follows:

$$H_o: \alpha_1 = \alpha_2 = \ldots = \alpha_k = 0$$

versus

$$H_1: \ \alpha_i \neq \alpha_j \quad \text{for some } (i,j)$$

and

$$H_o': \ \beta_1 = \beta_2 = \ldots = \beta_n = 0$$

versus

$$H_1': \ \beta_t \neq \beta_{t'} \quad \text{for some } (t, t').$$

For testing H_o versus H_1, the modified F-test statistics is

$$F_{M(g)} = \frac{n \sum_{i=1}^{n} (\bar{Y}_{i.} - \bar{Y}_{..})^2}{\sum_i \sum_t (Y_i(t) - \bar{Y}_{i.} - \bar{Y}_{.}(t) + \bar{Y}_{..})^2} \, d^*_{31}(\phi, \Phi, \theta, \Theta) \qquad (3.5.4)$$

where $d^*_{31}(\phi, \Phi, \theta, \Theta) = \dfrac{(n-1)\ c^*_3(\phi, \Phi, \theta, \Theta)}{c^*_1(\phi, \Phi, \theta, \Theta)}$.

Similarly for testing H_o' versus H_1',

$$F_{M(t)} = \frac{k \sum_{t=1}^{n} (\bar{Y}_{.}(t) - \bar{Y}_{..})^2}{\sum_{i=1}^{k} \sum_{t=1}^{n} (Y_i(t) - \bar{Y}_{i.} - \bar{Y}_{.}(t) + \bar{Y}_{..})^2} \, d^*_{32}(\phi, \Phi, \theta, \Theta) \qquad (3.5.5)$$

where

$$d^*_{32}(\phi, \Phi, \theta, \Theta) = \frac{(k-1)\ c^*_3(\phi, \Phi, \theta, \Theta)}{c^*_2(\phi, \Phi, \theta, \Theta)}$$

Values of d^*_{31} and d^*_{32} are calculated by Sutradhar and MacNeill (1987) for various combinations of ϕ, Φ, θ and Θ.

Currently research is being carried out by Dagum, Huot and Morry (1987.b) with a large sample of real series to: (1)

determine the type of ARMA model most often found for the residuals of the SI estimates; (2) calculate the values of F_m for stable and moving seasonality and (3) determine the relationship between the existing F as calculated by the program and F_m.

(b) Charts of Trading-Day-Irregular residuals by types of Month.

Optional charts that plot the trading-day-irregular residuals for the 22 types of months are incorporated into the program. These charts permit to evaluate the adequacy of the daily weights estimated by the program to produce reasonable trading-day variations for each month. They also detect those cases where there have been a break in the pattern of the trading day variations.

(c) Spectral Subroutine

The existing set of tests and statistical measures available in the X-11-ARIMA for analytical purposes will be expanded with the inclusion of a subroutine that calculates the spectra of both the original series and the seasonally adjusted series. This tool will mainly supplement the information provided by the F-tests on the presence of stable and moving seasonality as well as the F-test on the presence of trading-day variations. In general, it will be useful to assess the effect of the seasonal adjustment on the original series.

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