Statistics   Statistique
Canada   Canada
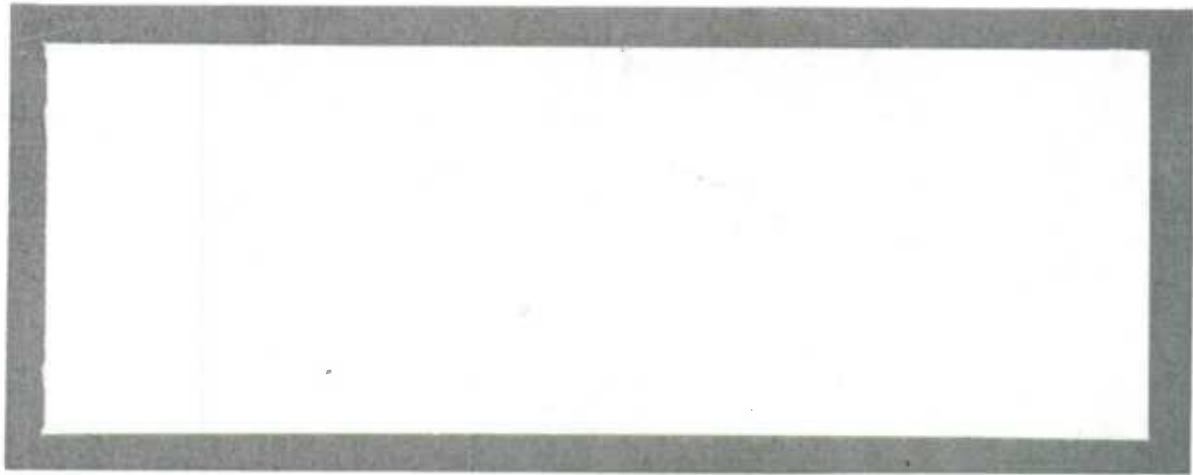
Methodology Branch

Time Series Research and Analysis
Division

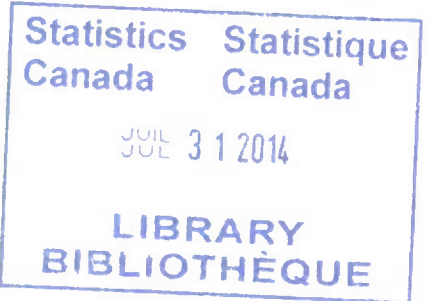# Direction de la méthodologie

Division de la recherche
et de l'analyse des chroniques

Canada

CURRENT ISSUES ON SEASONAL ADJUSTMENT

by

Estela Bee Dagum

ABSTRACT

This paper discusses three problems that have been a major preoccupation among researchers and practitioners of seasonal adjustment in statistical bureaus for the last ten years. These problems are (1) the use of concurrent versus seasonal factor forecasts for current seasonal adjustment, (2) finding an optional pattern of revisions for series seasonally adjusted with concurrent factors and (3) smoothing highly irregular seasonally adjusted data.

RESUME

Cette étude traite de trois problèmes qui ont été une
source de préoccupations majeures au cours des dix
dernières années pour les théoriciens et praticiens
spécialisés en désaisonnalisation. Ces problèmes
sont (1) de choisir entre l'utilisation de filtres
contemporains ou de facteurs saisonniers extrapolés,
(2) d'établir une procédure optionnelle de révisions
des séries désaisonnalisées à l'aide de filtres
contemporains et (3) de lissage de séries désaisonnalisées
quand celles-ci sont très irrégulières.

1.   INTRODUCTION

During the last decade, statistical bureaus, have focussed their attention on three important issues: (1) the seasonal adjustment of a current value; (2) the revisions of concurrent seasonally adjusted data; and (3) the smoothing of highly irregular seasonally adjusted series.

The main purpose of this article is to discuss each of the above problems with respect to the X-11-ARIMA seasonal adjustment program developed by Dagum (1980) and which is applied by Statistics Canada and other statistical bureaus of the world.

Section 2 discusses the four modes in which the X-11-ARIMA computer package can be used to produce a current seasonally adjusted value. Section 3 analyses the revisions of concurrent seasonally adjusted data based on the linear filters of X-11-ARIMA and Section 4 deals with the nature and characteristics of the smoothing (trend-cycle) filters available in X-11-ARIMA.

2. SEASONAL ADJUSTMENT OF CURRENT VALUES

There are four modes in which the X-11-ARIMA computer program can be applied to produce a current (last observation) seasonally adjusted value. These four modes are: (I) using ARIMA extrapolations and concurrent seasonal factors, (II) using ARIMA extrapolations and seasonal factor forecasts; (III) without the use of ARIMA extrapolations and only concurrent seasonal factors and (IV) without the use of ARIMA extrapolations and only seasonal factor forecasts.

Statistical bureaus use the four modes to obtain current seasonally adjusted values but differ in their frequency. Thus, for example, the dominant mode in Statistics Canada is (I) followed by mode (III) whereas in the U.S. Bureau of Labor, the dominant mode is (II) followed by mode (IV). The current seasonally adjusted value produced by each type of seasonal adjustment is varies and is subject to different degrees of error.

A "concurrent" seasonal estimate (factor or effect depending on whether a multiplicative or additive model is assumed) is obtained by seasonally adjusting each time a new observation is available, all the data available up to and including that observation. On the other hand, a seasonal forecast is obtained from a series that ended in the previous year. A common practise is to generate these seasonal forecasts say, for year t+1, from data that ended in December of the previous year t.

Under the assumption of an additive decomposition model, the seasonal adjustment of a current value X_t can be obtained by,

(1) $\hat{X}_t^{(\ell)} = X_t - \hat{S}_t^{(\ell)}$;   where $\hat{S}_t^{(\ell)}$ denotes a forward seasonal estimate.

or

(2) $\hat{X}_t^{(0)} = X_t - \hat{S}_t^{(0)}$ where $\hat{S}_t^{(0)}$ denotes a concurrent seasonal estimate.

The current seasonally adjusted value will become "final" in the sense that it will no longer be revised after 42 more observations are added. Thus,

(3) $\hat{X}_t^{(m)} = X_t - \hat{S}_t^{(m)}$;   where $\hat{S}_t^{(m)}$ denotes a final seasonal estimate.

Therefore, the total revision of a concurrent and of a forward seasonal estimate can be written as,

(4) $r_t(0,m) = \hat{S}_t(0) - \hat{S}_t^{(m)}$; $m > 0$

(5) $r_t(\ell,m) = \hat{S}_t^{(\ell)} - \hat{S}_t^{(m)}$; $m > 0 \geq \ell$

Under the assumption of an additive decomposition and no replacement of extreme values, $S_t^{(m)}$ can be expressed by

$$(6) \quad \hat{S}_t^{(m)} = \sum_{j=-m}^{m} h_{m,j} \, X_{t-j} = h^{(m)}(B) X_t;$$

where $\hat{S}_t(m)$ is the final seasonal estimate from a series $X_{t-m},\ldots,X_t,\ldots,X_{t+m}$; and $h_{m,j} = h_{m,-j}$ are the symmetric moving average weights to be applied to the series. $h(m)(B)$ denotes the corresponding linear filter using the backshift operator B, such that $B^n = X_{t-n}$. Following equation (6) we can express a concurrent seasonal estimate $\hat{S}_t^{(0)}$ and a forward seasonal estimate $\hat{S}_t^{(\ell)}$ by

$$(7) \quad \hat{S}_t^{(0)} = \sum_{j=-2m}^{-\ell} h_{0,j} \, X_{t-j} = h^{(0)}(B) X_t; \quad m=42$$

where $h^{(0)}(B)$ denotes the asymmetric <u>concurrent</u> seasonal filter.

$$(8) \quad \hat{S}_t^{(\ell)} = \sum_{j=-2m}^{\ell} h_{\ell,j} \, X_{t-j} = h^{(\ell)}(B) X_t; \quad m=42$$

Where $h^{(\ell)}(B)$ denotes the asymmetric <u>forecasting</u> seasonal filter and $\ell = 1,2,\ldots,12$ for a monthly series.

The revision of a concurrent seasonal estimate depends on the distance between the concurrent and the final filter; that is, $d[h^{(0)}(B), h^{(m)}(B)]$ and the innovations of the new observations $X_{t+1}, X_{t+2}, \ldots, X_{t+m}$.

Similarly, the revision of a forward seasonal estimate depends on $d[h^{(\ell)}(B), h^{(m)}(B)]$ and on the new innovations introduced by $X_{t-\ell}, \ldots, X_t, X_{t+1}, \ldots, X_{t+m}$

Theoretical studies by Dagum (1982.a and 1982.b) have shown that,

(9) $d[h^{(0)}B, h^{(m)}(B)] < d[h^{(\ell)}(B), h^{(m)}(B)]$ for $\ell = 1, 2, \ldots, 12$.

Relation (9) is true whether ARIMA extrapolations are used or not. Furthermore, the two studies also showed that if ARIMA extrapolations are used then

(10) $d[h(0)(B), h(m)(B)]$ using ARIMA extrapolations
       $< d[h(0)(B), h(m)(B)]$ without ARIMA extrapolations

and similarly

(11) $d[h(\ell)(B), h(m)(B)]$ using ARIMA extrapolations
       $< d[h(\ell)(B), h(m)(B)]$ without ARIMA extrapolations.
          for $= 1, 2, \ldots, 12$.

Several studies by Dagum (1978), Bayer and Wilcox (1981);
and Durbin (1982), McKenzie (1984), Dagum and Morry (1984) Pierce
(1980) and Pierce and McKenzie (1985) have shown that

(12) $r(0,m) < r(\ell,m)$

except in a few cases where

(13) $r(0,m) > r(\ell,m)$

The relationship (13) can be observed when the current
observations of the latest year are strongly revised since X_t
gets the largest weight in the estimations of $\hat{S}_t(0)$.

From the viewpoint of the total revisions of the seasonal
estimates, the results of the above empirical studies permit to
rank the four modes as follows: Mode I (ARIMA extrapolations
with concurrent seasonal estimates) gives the smallest total
revision; Mode III (no ARIMA extrapolations with concurrent
seasonal estimates) ranks second; Mode II (ARIMA extrapolations
with forward seasonal estimates) ranks third and Mode IV (ARIMA
extrapolations with forward seasonal estimates) ranks fourth.

3. REVISIONS OF CONCURRENT SEASONALLY ADJUSTED DATA

Statistics Canada's practise of using concurrent seasonal
adjustment was first established in 1975 for the Labour Force
Survey series. Gradually other foreign statistical agencies
followed it. The use of concurrent seasonal factors for current
seasonal adjustment poses the problem of how often should the
series be revised. Kenny and Durbin (1982) recommended that
revisions should be made after one month and thereafter each
calendar year. Dagum (1982.c) supported these conclusions and
furthermore, recommended an additional revision at six months if
the seasonal adjustment method is the X-11-ARIMA without the
ARIMA extrapolation option.

For any two points in time t+k, t+ℓ (k<ℓ), the revisions of the
seasonal estimates and consequently of the seasonally adjusted
value is given by

(14) $r_t(\ell, k) = \hat{X}_t(\ell) - \hat{X}_t(k)$, k<$\ell$.

This revision reflects: (1) the innovations introduced by the
new observations $X_{t+k+1}, X_{t+k+2}, \ldots, X_{t+k+\ell}$; and (2)
the differences between the two asymmetric seasonal adjustment
filters $Y^{(\ell)}(B)$ and $Y^{(k)}(B)$. If one fixes k=0 and lets ℓ vary
from 1 to m then the relation (13) gives a sequence of revisions
of the concurrent seasonally adjusted values for different time

spans or lags. The total revision of the concurrent estimate is given for $\ell=m$. If one fixes $\ell=k+1$ and lets k take values from 0 to m-1, then relation (14) gives the sequence of single period revision of each estimated seasonally adjusted value and in particular, if one starts at k=0 one obtains the m-1 successive single-revision of each estimated seasonally adjusted value before it becomes final. If one fixes $\ell=k+12$ and lets k take values from 0 to m-12, then equation (14) gives the sequence of annual revisions.

The revisions in which we are interested here are those introduced by filter discrepancies, and these can be studied by looking at the frequency response functions of the corresponding filters. Similarly to equation (6), we can approximate the seasonally adjusted value, for recent years from the X-11-ARIMA program (with or without ARIMA extrapolations) by,

$$(15) \quad \hat{X}_t^{(n)} = \sum_{j=-n}^{m} Y_{n,j} \, X_{t-j} = Y(n)(B) \, X_t$$

Equation (15) represents a linear system where $\hat{X}_t^{(n)}$ is the convolution of the input X_t and a sequence of weights $Y_{n,j}$ called the impulse response function of the filter. The properties of this function can be studied using its Fourier transform which is called the frequency response function, defined by

(16) $\quad \Gamma^{(n)}(\omega) = \sum_{j=-n}^{m} Y_{n,j} \ e^{-12\pi\omega j}; \quad -1/2 \le \omega \le 1/2$

Where ω is the frequency in cycles per unit time $\Gamma^{(n)}(\omega)$ fully describes the effects of the linear filter on the given input. Monthly and annual revisions of the concurrent filter of X-11-ARIMA with and without the ARIMA extrapolations have been calculated by Dagum (1987) based on the mathematical distance between the various frequency response functions of the filters. The pattern is characterized by a rapid decrease in the size of the monthly revisions of the concurrent filter for $\ell = 1, 2$, and 3; and a slow decrease thereafter till $\ell = 11$; then a large increase occurs at $\ell = 12$ followed by a decrease at $\ell = 13$ and then another large increase at $\ell = 24$ followed by a decrease at $\ell = 25$. Dagum (1987) showed that this pattern of monthly revisions is the same whether ARIMA extrapolations are used or not.

The significant decreases for the first three consecutive revisions are due to the improvement of the Henderson (trend-cycle) filter weights. The reversal of direction in the size of the filter revisions at $\ell = 12$ and $\ell = 13$, is due to the improvements of the seasonal filter that becomes less asymmetrical from year to year until three full years are added to the series. The two largest revisions occur at $\ell = 1$ and $\ell = 12$. Given the non-monotonicity of single monthly revisions, it is not advisable to revise the concurrent estimate any time a new observation is added to the series.

A revision scheme often used by statistical bureaus for their concurrent seasonally adjusted series consists of keeping constant the concurrent estimate from the time it appears until the end of the year and then revising annually the current and earliest years. Therefore, first year revisions due to filter discrepancies are given by R(0,0), R(1,0),..., R(11,0); second year revisions by R(12,0), R(13,1),... RF(23,11), third year revisions by R(24,12), R(25,13) and so on where $R^{(\ell,k)}$ is defined by,

$$(17) \quad R(\ell,k) = [2\int_0^{1/2} \| \Gamma^{(\ell)}(\omega) - \Gamma^{(k)}(\omega) \|^2 d\omega]^{1/2}$$
$$0 \le \omega < 1/2; \quad \ell = 1,2,\ldots,n$$
$$k = 0,1,2,\ldots,n-12.$$

where n=42 for the X-11-ARIMA seasonal adjustment filters.

Table 1 shows the first, second and third year revisions of the concurrent seasonal adjustment filter for X-11-ARIMA without extrapolation and with extrapolations from two ARIMA models (other cases are shown in Dagum 1987).

Table 1 - First-Second and Third Year Revisions of the Concurrent Seasonal Adjustment Filter of X-11-ARIMA

Revisions R(ℓ,k)	Without ARIMA Extrapolations	With ARIMA Extrapolations from a (0,1,1)(0,1,1) Model			
		θ=.40	θ=.80	θ=.80	θ=.80
R(1,0)	.12		.12		.06
R(2,0)	.13		.13		.08
R(3,0)	.13		.13		.08
R(4,0)	.13		.13		.09
R(5,0)	.15		.13		.09
R(6,0)	.17		.13		.09
R(7,0)	.16		.13		.09
R(8,0)	.16		.13		.09
R(9,0)	.16		.13		.09
R(10,0)	.16		.14		.09
R(11,0)	.16		.14		.09
R(12,0)	.29		.28		.26
R(13,1)	.27		.27		.26
R(14,2)	.27		.27		.26
.	.		.		.
.	.		.		.
.	.		.		.
R(23,11)	.27		.26		.26
R(24,12)	.20		.16		.16
R(24,13)	.18		.17		.16
R(36,24)	.16		.17		.16
.	.		.		.
.	.		.		.
.	.		.		.

Since the largest single period revisions occur at $\ell=1$ and $\ell=12$ as mentioned above, a better revision scheme would be to incorporate monthly and annual revisions. It is expected that by (1) adjusting concurrently each month, say from January to November and revising only once when the next month is available, and (2) adjusting concurrently December when it first appears and then revising the first year and earlier years when January is added, should improve the reliability of the filter applied during the current year while maintaining simultaneously the filter's homogeneity for month-to-month comparisons.

The first-year revisions of the first month revised filter would then be $R(1,1)$, $R(2,1)$, ... $R(11,1)$. Table 2 shows these revisions and although the pattern is very similar to that of the concurrent filter, the size of the revisions are much smaller if no extrapolations are used. On the other hand, the improvement is, less important if ARIMA extrapolations are used. Similarly, no major differences were observed for the second and third year revisions.

Table 2.   First-Year revisions of the First-Month Revised
Seasonal Adjustment Filter

Revisions $R(\ell,1)$	Without ARIMA Extrapolations	With ARIMA Extrapolations			
		$\theta=.40$	$\theta=.80$	$\theta=.80$	$\theta=.80=$
$R^{(2,1)}$	.07		.10	.06	
$R^{(3,1)}$	.07		.10	.06	
$R^{(4,1)}$	.07		.10	.07	
$R^{(5,1)}$	.08		.10	.08	
$R^{(6,1)}$	.10		.11	.08	
$R^{(7,1)}$	.11		.11	.08	
$R^{(8,1)}$	.11		.11	.08	
$R^{(9,1)}$	.11		.11	.08	
$R^{(10,1)}$	.12		.11	.08	
$R^{(11,1)}$	.12		.12	.08	

3.1   Estimation of Trading-day Variations and ARIMA Models with
      Concurrent Seasonal Adjustment

Besides the type of revisions scheme to be applied, there are two
other problems posed by concurrent seasonal adjustment associated
with trading-day variations and ARIMA modelling.

For series which are flows in the sense that they result from the accumulation of daily values over the calendar months, there is a systematic effect caused by trading day variations. Trading day variations arise mainly because the activity varies with the days of the week. Other sources are associated with accounting and reporting practices. For example, stores that do their bookkeeping activities on Friday tend to report higher sales in months with five Fridays than in months with four Fridays. The trading day effects are estimated in the X-11-ARIMA program using ordinary least squares on a simple deterministic regression model. Consequently, the weights estimated for each day change anytime a new observation is added to the series. Since regression techniques are very sensitive to outliers, these changes can be sometimes unnecessarily large.

When the series are seasonally adjusted concurrently, the trading day estimates change all the time. In order to avoid unnecessary revisions Statistics Canada's practice is to use the weights calculated by the program at the end of the previous calendar year or the weights provided by the users, as priors for the current year. The weights are then revised on an annual basis.

The effect of trading day variations must be removed from the series before ARIMA modelling for these type of models cannot adequately handle trading-day variations. In other words, if the X-11-ARIMA program is used with ARIMA extrapolations on series with trading day variations, these variations are estimated a priori and if significant, they are removed from the original series before the ARIMA modelling.

Another problem associated with concurrent seasonal adjustment refers to how often should the ARIMA models be identified. The current practice at Statistics Canada is to use the automatic ARIMA model selection option once a year and if the model is accepted, then it is kept constant for a whole year, letting only the parameters change when more observations are added. In order to keep the model constant, the user's supplied model option should be applied. Maintaining the ARIMA model constant avoids unnecessary revisions that may result from changing of models back and forth simply because of the presence of outliers.

4. SMOOTHING OF VOLATILE SEASONALLY ADJUSTED SERIES

One of the main purposes of the seasonal adjustment of economic time series is to provide information on current economic conditions, particularly to determine the stage of the cycle at which the economy stands. Since seasonal adjustment means removing only seasonal variations, thus leaving trend cycle estimates together with irregular fluctuations, it is often difficult to detect the short term trend or cyclical turning points for series strongly affected with irregulars. In such cases, it may be preferable to smooth the seasonally adjusted series using trend-cycle estimators which suppress as much as possible the irregulars without affecting the cyclical component.

The use of trend-cycle estimates or of smoothed seasonally adjusted data instead of a "volatile" seasonally adjusted series have been discussed by several writers and recently by Moore et als (1981), Kenny and Durbin (1982), Maravall (1986), and Dagum and Laniel (1987). Although not yet practised widely, some statistical agencies such as Statistics Canada and the Australian Bureau of Statistics smooth some of their series.

The combined linear filters applied to the original series to generate a central (symmetric) trend-cycle estimate have been calculated by Young (1968) for Census Method II-X-11 variant. This filter is similar to that of X-11-ARIMA with and without ARIMA extrapolations. Dagum and Laniel (1987) extended Young's (1968) results to include the estimation of the asymmetric trend cycle filters of X-11-ARIMA with and without the ARIMA extrapolations.

Figure 1 shows the gain functions of the central (symmetric) seasonal adjustment filters and smoothed seasonally adjusted data (trend-cycle) filters. It is apparent that the trend-cycle filters suppress all the noise present in the series, where the noise is defined as the power present in all frequencies $\omega \leq .166$. This frequency corresponds to the first harmonic of the fundamental seasonal frequency of a monthly series. This pattern results from the convolution of the seasonal adjustment filters with the 13-term Henderson trend-cycle filter.

(Please place Figure 1 about here)

Figure 2.a shows the gain functions of the concurrent and first-month revised trend-cycle filters of X-11-ARIMA without ARIMA extrapolations. Figure 2.b shows their corresponding phase-shift functions expressed in months instead of radians. We can observe that the gain for all $\omega < .167$ is much larger for these two asymmetric filters as compared with the central filter. Furthermore, there are large amplifications for frequencies near the fundamental seasonal. All this means that the concurrent and first revised smoothed seasonally adjusted values will have more noise than the final estimates. On the other hand, it is apparent that the phase shifts are very small, less than one month for the most important cyclical frequencies $0 < \omega < .055$ (i.e. cycles of periodicities equal to and longer than 13 months)

(Please place Figures 2.a and 2.b about here)

Figure 3.a and 3.b show the gain and phase-shift functions of the concurrent and first-month revised trend-cycle filters of X-11-ARIMA with ARIMA extrapolations. The extrapolations are obtained from an IMA model $(0,1,1)(0,1,1)_{12}$

with θ=.40 and θ=.60. The gain functions are closer to the symmetric (central) filter than those of X-11-ARIMA without the ARIMA extrapolations. There are no amplifications around the fundamental seasonal frequency and a similar attenuation of power at higher frequencies. On the other hand, there is more phase-shift (being near to one month) for low frequencies and less phase shift for all high frequencies.

(Please place Figures 3.a and 3.b about here)

Dagum and Laniel (1987) studied the time path of the revisions of the trend-cycle filters and compared with those of the seasonal adjustment filters. Their results, as summarized in Table 3 show that the total revisions of the trend-cycle asymmetric filters converge to 0 much faster than those of the corresponding seasonal adjustment filters. In fact, the total revision of the trend-cycle filter 3 months after the concurrent filter is only .1 whereas a close value is achieved for the seasonal adjustment filter only after 24 months have been added to the series. Except for the total revisions of the concurrent filter which is larger for the trend-cycle filters compared with the corresponding seasonal adjustment filter, in all the other cases the total revisions are smaller for the trend-cycle filters. Furthermore, the trend-cycle filter revisions converge much faster to zero as compared with those of the seasonal adjustment filters.

Table 3. Time Path of the total Revisions of the Trend-Cycle;
and the Seasonal Adjustment Asymmetric Filters of
X-11-ARIMA

Revisions $R(\ell,k)$*	Without Extrapolations		With Extrapolations $\theta=.40 \quad \theta=.60$	
	Trend-Cycle Filters	Seasonal Adjustment Filters	Trend-Cycle Filters	Seasonal Adjustment Filters
R(48,0)	.45	.36	.41	.32
R(48,1)	.27	.33	.26	.32
R(48,2)	.15	.32	.15	.32
R(48,3)	.11	.32	.11	.31
R(48,4)	.12	.32	.11	.31
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
R(48,12)	.10	.23	.09	.20
R(48,24)	.07	.13	.05	.10
R(48,36)	.03	.05	.02	.04
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
R(48,47)	.01	.01	.01	.01

* ℓ=48 for the "final" trend-cycle filter and ℓ=42 for the final
seasonal adjustment filter. However, the values shown for the
revision of the seasonal adjustment filters are also calculated
for ℓ=48 since after ℓ=42 the values are final and, thus, do not
change.

REFERENCES

Bayer, A. and Wilcox D. (1981): "An Evaluation of Concurrent Seasonal Adjustment" Technical Report, Washington,m D.C. Board of Governors of the Federal Reserve System, Special Studies Section.

Dagum, E.B. (1978): Comparison and Assessment of Seasonal Adjustment Methods for Labour Force Series, Washington, D.C.: U.S. Government Printing Office, Stock No. 052-003-00603-1

Dagum, E.B. (1980): The X-11-ARIMA Seasonal Adjustment Method, Ottawa: Statistics Canada Catalogue No. 12-564E.

Dagum, E.B. (1982.a): "Revisions of Time Varying Seasonal Filters" Journal of Forecasting, 1, 173-187

Dagum, E.B. (1982.b): "The Effects of Asymmetric Filters on Seasonal Factor Revisions" Journal of the American Statistical Association, Vol. 77, No. 380, pp. 732-738.

Dagum, E.B. (1982.c): "Revisions of Seasonally Adjusted Data Due to Filter Changes" Proceedings of the Business and Economic Section, American Statistical Association, pp. 39-45

Dagum, E.B. and Morry, (1984): "Basic Issues on the Seasonal Adjustment of the Canadian Consumer Price Index" Journal of Business and Economic Statistics, vol. 2, No. 3, pp. 250-259

Dagum, E.B. (1987): "Monthly versus Annual Revisions of Concurrent Seasonally Adjusted Series" in Time Series and Econometric Modelling (I.B. MacNeill and G.J. Umphrey, Editors) D. Reidel Publishing Co. 131-196.

Dagum E.B. and N. Laniel (1987): "Revisions of Trend-Cycle Estimators of Moving Average Seasonal Adjustment Method" Journal of Business and Economic Statistics, (forthcoming).

Kenny, P. and Durbin, J. (1982): "Local Trend Estimation and Seasonal Adjustment of Economic Time Series" Journal of the Royal Statistical Society, Ser. A. vol. 145, Part 1, pp 1-41.

Maravall, A. (1986): "An Application of Model-Based Estimation of Unobserved Components" International Journal of Forecasting, 2, N.3, 305-318.

Moore, G.H., Box, G.E.P., Kaitz, H.,B., Stephenson, J.A. and Zellner, A. (1981): "Seasonal Adjustment of the Monetary Aggregates" Report of the Committee of Experts on Seasonal Adjustment Techniques, Washington, D.C. Board of Governors of the Federal Reserve System.

McKenzie, S. (1984): "Concurrent Seasonal Adjustment with Census X-11" Journal of Business and Economic Statistics, Vol. 2, No. 3, pp 235-249.

Pierce, D.A. (1980): "Data Revisions with Moving Average Seasonal Adjustment Procedures" Journal of Econometrics, 14, pp 95-114.

Pierce, D. and McKenzie, S. (1985): "On Concurrent Seasonal Adjustment" Special Studies Paper, No. 164, Federal Reserve Board, Washington, D.C.
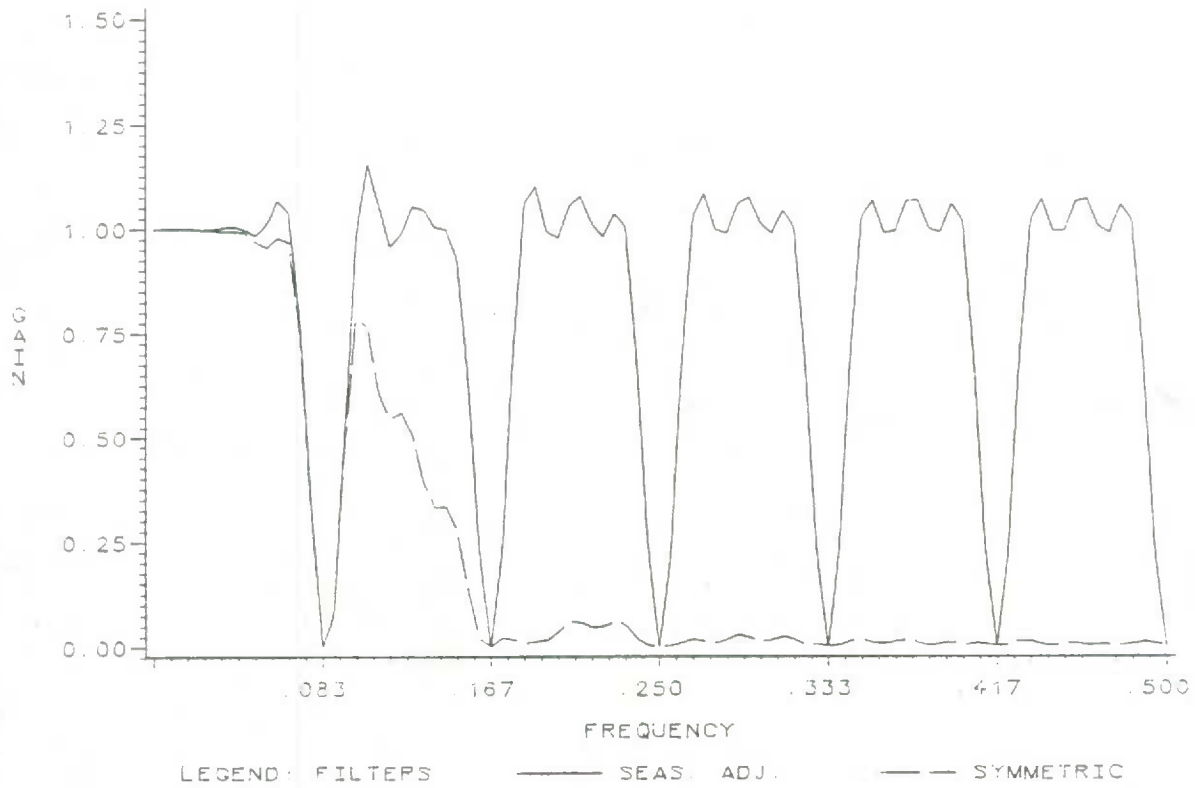
FIGURE 1: Gain Functions of the Central (Symetric) Trend—Cycle and Seasonal Adjustment Filters OF X-11--ARiMA.
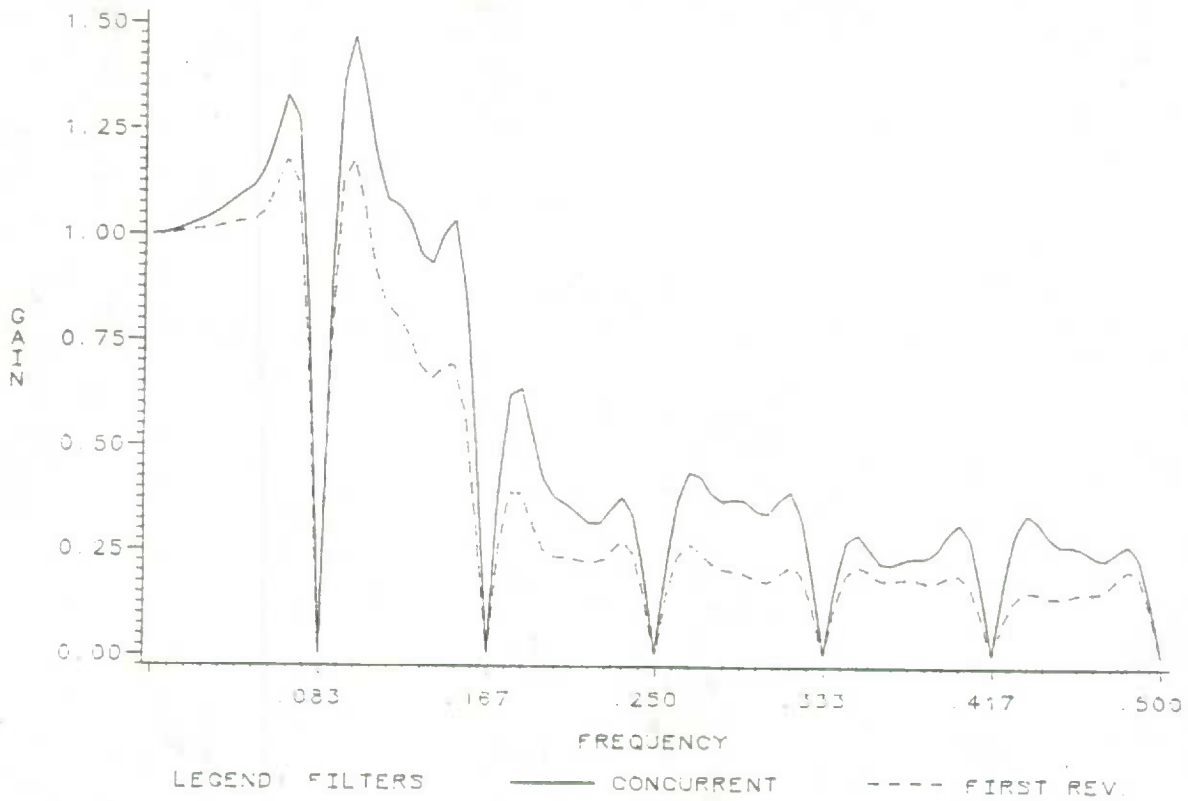
FIGURE 2.a: Gain Functions of the Concurrent and First-Month Revised Filters of X-11-ARIMA without ARIMA Extrapolations.
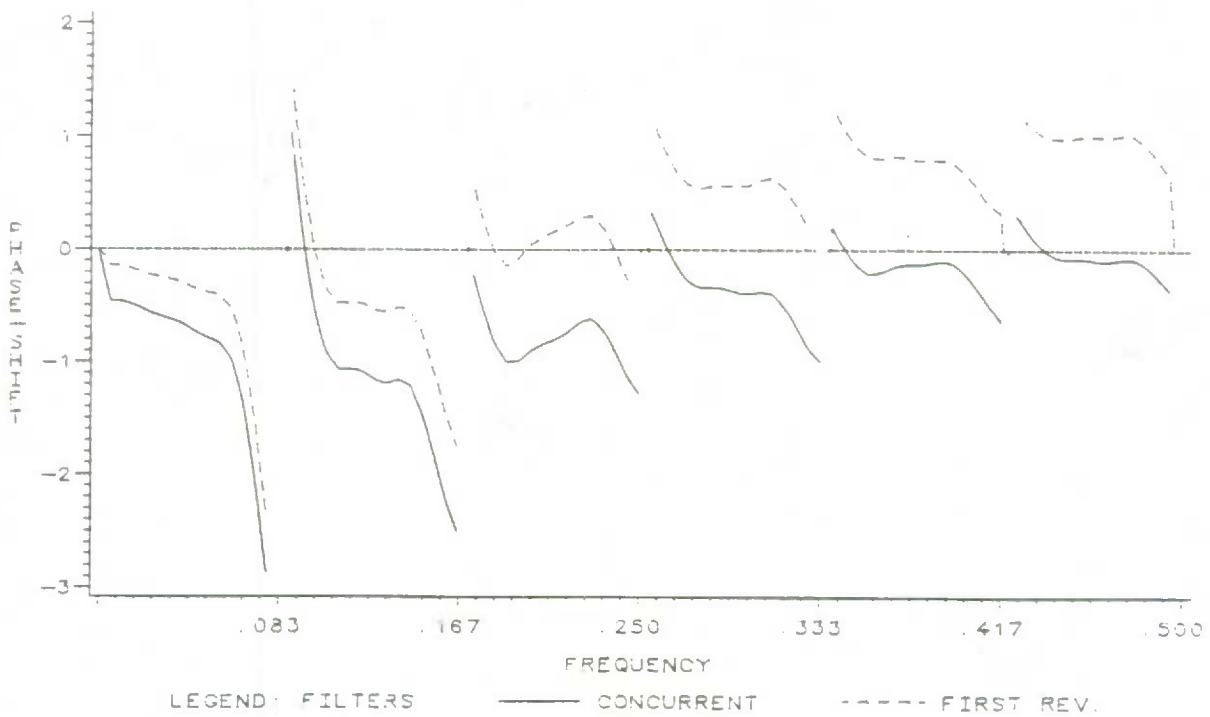

FIGURE 2.b: Phase-Shift Functions of the Concurrent and First-Month Revised Filters of X-11-ARIMA without ARIMA Extrapolations.
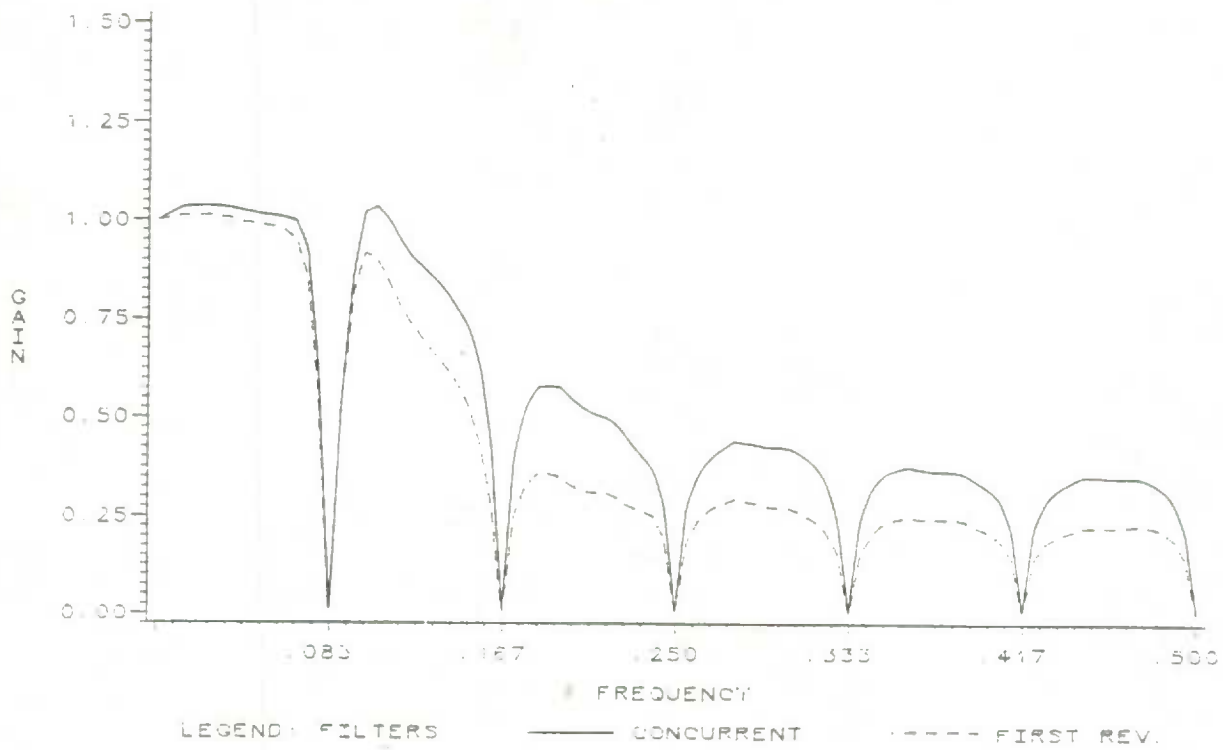
FIGURE 3.a: Gain Functions of the Concurrent and First-Month Revised Filters of X—11—ARIMA with ARIMA extrapolations. (θ=.40 Θ=.60).
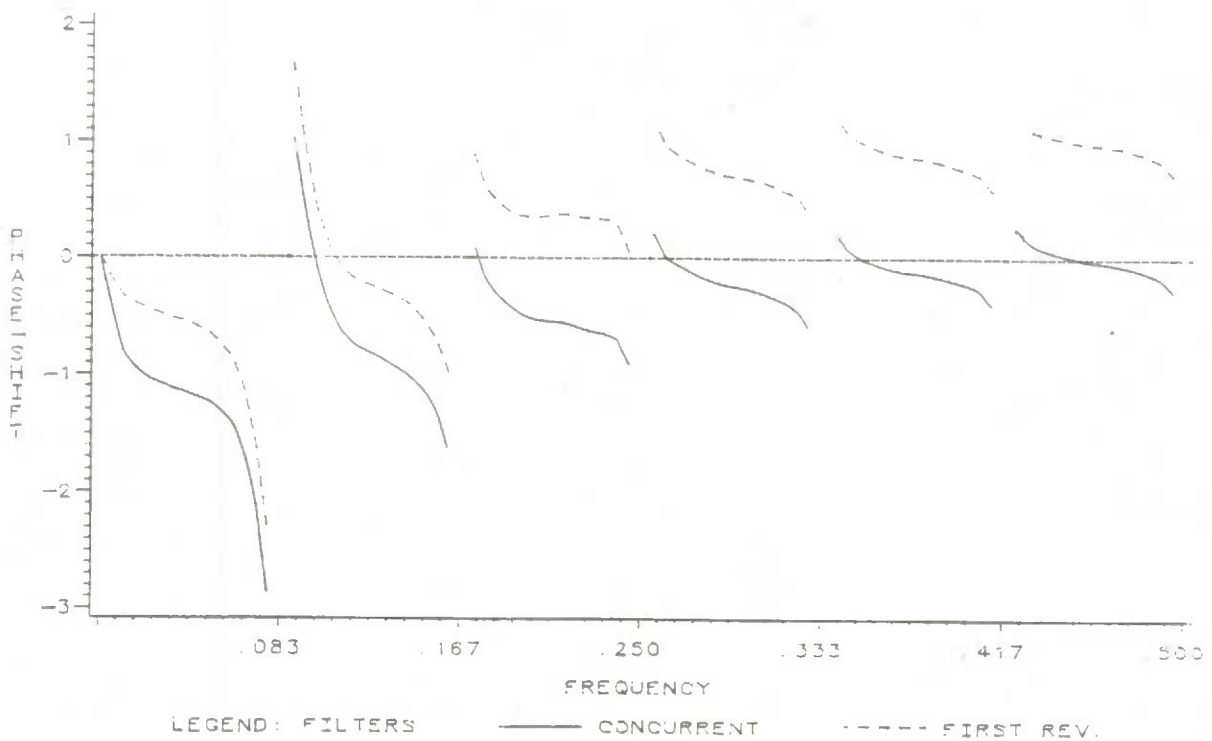

FIGURE 3.b: Phase—Shift Functions of the Concurrent and First—Month Revised Filters of X—11—ARIMA with ARIMA Extrapolations. (θ=.40 Θ=.60).