Statistics Canada Statistique Canada

STRUCTURAL AND REDUCED-FORM APPROACHES OF ARIMA MODEL BASED SEASONAL ADJUSTMENT METHODS

by

Estela Bee Dagum

Time Series Research and Analysis Division

Statistics Canada

Methodology Branch

Time Series Research and Analysis Division

Direction de la méthodologie

Division de la recherche et de l'analyse des chroniques

Canada

WORKING PAPER TSRA-87-007E
TIME SERIES RESEARCH & ANALYSIS DIVISION
METHODOLOGY BRANCH

STRUCTURAL AND REDUCED-FORM APPROACHES OF ARIMA MODEL BASED SEASONAL ADJUSTMENT METHODS

by

Estela Bee Dagum

Time Series Research and Analysis Division

Statistics Canada

This is a preliminary version. Do not quote without author's permission. Comments are welcome.

ABSTRACT

Two strategies have been followed for the development of model-based seasonal adjustment methods. One, where each of the unobserved components, trend-cycle, seasonal and irregular is assumed to follow a normal stochastic process of the ARIMA class and the other, where the observed data are assumed to follow an ARIMA process and from it similar kind of models are derived for the components. The first approach is known as "structural" and the second, as the "reduced-form" given their similarities to the problems of identification of structures from the data in econometrics.

This paper discusses the major properties and operational limitations of these two approaches. It also analyses the salient characteristics of the empirical comparisons made between model-based seasonal adjustment methods and the X-11-ARIMA which is used by the majority of government statistical agencies.

Keywords: structural, reduced-form, model based seasonal adjustment, Kalman filter, ARIMA models, X-11-ARIMA.

RESUME

Deux approches ont été utilisées jusqu'à maintenant dans le développement des méthodes paramétriques de désaisonnalisation. La première suppose que chacune des composantes inobservables, la tendance-cycle, la saisonnalité et les irréguliers suit un processus stochastique normal de type ARMMI. La deuxième présume que les données observées suivent un processus ARMMI utilisable pour la déduction de modèles semblables représentant les composantes de la série. Étant données les similitudes avec les modèles économétriques linéaires, la première approche est connue comme étant celle de forme structurelle et la deuxième, de forme réduite.

Cette étude traite des principales propriétées et limitations de ces deux approches. On y analyse également les résultats d'une comparaison empirique entre les deux méthodes paramétriques de désaisonnalisation et X-11-ARMMI, cette dernière étant celle utilisée par la majorité des agences statistiques gouvernementales.

1. Introduction

During the last decade a great effort has been made by researchers to develop seasonal adjustment methods based on the decomposition of univariate time series models. These univariate time series models mainly belong to the class of Gaussian ARIMA (autoregressive integrated moving average) stochastic process or minor variants from it.

Two procedures have been followed for the development of model-based seasonal adjustment methods. One, where each of the classical unobserved components, trend-cycle, seasonal and irregulars, is assumed to follow a Gaussian stochastic model and the other, where the observed series is assumed to follow an ARIMA model and from it similar kinds of models are deduced for the components. Engle (1978) called the first approach "structural" given its similarity to the problem of identification of a class of structures consistent with a given reduced-form in econometric models. The second approach is referred to as the "reduced-form", again, given its analogy to that which is observable or identifiable from the data in econometrics.

The main purpose of this article is to summarize the major properties of the two approaches and problems encountered in their implementation. Section 2 discusses the structural form of model based seasonal adjustment methods. Section 3 analyses the assumptions of the reduced-form approach and discusses results obtained in most empirical studies where comparisons have been made with the X-11-ARIMA method. Finally, section 4 gives the conclusions of this investigation.

2. "STRUCTURAL" APPROACH TO MODEL-BASED SEASONAL ADJUSTMENT METHODS

The first structural model seasonal adjustment methods were based on simple regression models where each unobserved component was assumed to follow a deterministic function of time. The extension of regression methods to include stochastic models for each component was first introduced by Hannan (1967) who filtered the original series to remove the trend and modeled the seasonal component with trigonometric functions multiplied by first-order autoregressive processes. These models were stationary and later were extended to non-stationary processes by several authors, among them, Hannan, Terrell and Tuckwell (1970), Grether and Nerlove (1970) Pagan (1973, 1975), Engle (1978), Cleveland (1979) Akaike (1980), Kitagawa and Gersch (1984), Harvey (1984), Burridge and Wallis (1984), Durbin (1984) and Maravall (1985).

The first basic assumption is that a time series Y_t can be additively decomposed into a trend-cycle, a seasonal and an irregular component. Thus,

$$Y_t = C_t + S_t + I_t \qquad (1)$$

These three components are assumed to be uncorrelated random processes and the signal extraction problem is to estimate S_t and remove it from Y_t to obtain a seasonally adjusted series Ya_t. . From the theory of signal extraction given in Whittle (1963) for stationary processes and extended by Hannan

(1967), Sobel (1967); and Cleveland and Tiao (1976), for non-stationary processes, the minimum mean squared error (MMSE) predictor of S_t, given a complete realization of Y_t is the orthogonal projection (conditional mathematical expectation if Y_t is normal),

$$\hat{S}_t = E(S_t \,|\, \{Y_t\}) = F_S(B) Y_t \qquad (2)$$

$$F_S(B) = \Gamma_S(B) / \Gamma_Y(B) \qquad (3)$$

Where $F_S(B)$ is a linear symmetric filter (a polynomial in the backshift operator B) resulting from the quotient of the two autocovariance functions (equivalently the spectra) $\Gamma_S(B)$ and $\Gamma_Y(B)$ corresponding to the seasonal and original series respectively. In theoretical and empirical works, linear models are postulated for the components and the autocovariances are expressed as functions of the model's parameters.

Since the structural approach refers to modeling the unobserved components with ARIMA processes, Engle (1978) called these latter UCARIMA models. Corresponding to each structural model (which has unobserved endogeneous variables) is a reduced form model that includes only observable variables. This reduced form model will also be an ARIMA model. The problem is to identify the structural models and to estimate the parameters of these models.

Using a general ARIMA representation, the structural models of equation (1) can be written by,

$$\phi_C(B)C_t = \theta_C(B)\xi_{1t} \qquad (4)$$

$$\phi_S(B)S_t = \theta_S(B)\xi_{2t} \qquad (5)$$

$$I_t = \eta_t \qquad (6)$$

Where ξ_{1t}, ξ_{2t} and η_t are uncorrelated normally distributed purely random processes. $\phi_C(B)$, $\phi_S(B)$, $\theta_C(B)$ and $\theta_S(B)$ are polynomials in B of degree p, r, q and s respectively.

In order to calculate $F_S(B)$ of equation (2), Pagan (1975) suggested the use of the Kalman filter. The Kalman filter was developed by Kalman (1960) and later by Kalman and Bucy (1961) for engineering problems. It provides a set of recursive formulas which calculate the mean and variance of the unobserved components at each time conditioned on a particular information set Ω. If Ω includes all past and current data on the observed variables, then the problem is one of filtering. If current data are not included, it is a problem of forecasting and if future data are included it is a smoothing problem. Additional information, generally assumed known, is the mean and variance of the initial conditions. In order to use the Kalman filter, equations (1), (4), (5) and (6) can be put in a state space form as follows,

$$Y_t = H^T X_t + \eta_t \quad ; \ t=0,1,2,... \qquad (7)$$

$$X_{t+1} = FX_t + G\xi_{t+1}; \ t=0,1,2,... \qquad (8)$$

Equation (7) is called the observation or measurement equation. Y_t is the output vector (although in most of applications is a scalar) of observables; H^T is a transposed matrix of known constants.

Equation (8) is called the state vector or transition equation. η_t and ξ_t are uncorrelated normal random variables with zero means and covariance matrices Q and R respectively. The state space representation of equation (1) together with the ARIMA models of (4), (5) and (6) becomes,

$$X_t = (X^t_{1t}, X^t_{2t})^T \tag{9}$$

$$X_{1t} = (C_t, C_{t-1}, \ldots, C_{t-p+1}; \xi_{1t}, \xi_{1,t-1}, \ldots, \xi_{1,t-q+1}) \tag{10}$$

$$X_{2t} = (S_t, S_{t-1} \ldots S_{t-r-1}\ \xi_{2t}, \xi_{2,t-1}) \ldots, \xi_{2,t-s+1}) \tag{11}$$

F = Block Diagonal F1,F2 (12)

$$F_1 = \begin{bmatrix} \phi_{c,1}, \phi_{c,2}, \cdots \phi_{c,p-1}, \phi_{cp}, -\theta_{c1}, -\theta_{c2} \cdots -\theta_{c,q-1}, -\theta_{cq} & \\ \begin{matrix} 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & & & & \\ 0 & 0 & \ldots & 1 & 0 \end{matrix} & 0 \\ 0 & \begin{matrix} 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & \ldots & 0 & 0 \\ \vdots & & & & \\ 0 & 0 & \ldots & 1 & 0 \end{matrix} \end{bmatrix} \tag{13}$$

F_2 is equal to F_1 but with parameters from the model for S_t.

$$GT = \begin{bmatrix} 10\ldots010\ldots000\ldots000\ldots0 \\ \\ 00\ldots000\ldots010\ldots010\ldots0 \end{bmatrix} \tag{14}$$

$$H^T = [10\ldots010\ldots0] \qquad (15)$$

$$\xi_t = (\xi_{1t}, \xi_{2t})^T \quad Q = \mathrm{diag}\,[\sigma^2_{\xi_1}, \sigma^2_{\xi_2}] \qquad (16)$$

$$R = \sigma^2_\eta \quad \text{(scalar)} \qquad (17)$$

Usually the state vector X_{t+1} must be augmented to convert the transition equation into a first-order Markov system. The specification is complete under the assumption that the initial vector X_0 is known to be normally distributed with mean $\bar{X}_0$ and covariance $0 \leq P_{0,-1} < \infty$. Usually $\bar{X}_0$ and $P_{0,-1}$ have to be chosen by the researcher and this can be a difficult task. If the series is stationary, this problem is simplified for the optimal choice is then $\bar{X}_0 = 0$, the unconditional mean and $P_{0,-1} = P$, the steady state covariance or $P_{0,-1}$ equal to the unconditional variance of X_t. If no information is available about the levels of the components when the process began, a poor guess of X_0 together with $P_{0,-1}$ causes the filter to attach too much weight to a misleading information and a long time could elapse before the incoming data dominates the initial choice.

The Kalman filter calculates the conditional expectation $\hat{X}_{t+k}$ of X_{t+k} and its covariance matrix $P_{t,t+k}$ given the information set Ω_{t+k} of observed values Y_0, $Y_1, \ldots$ Y_{t+k}, the initial conditions $\bar{X}_0$ and $P_{0,-1}$ and the variances $\sigma^2_\eta, \sigma^2_{\xi_1}$ and $\sigma^2_{\xi_2}$. The essential simplification of the Kalman filter is to assume that the optimal estimate at time t depends only on the optimal estimate at time t-1 and the new observation Y_t.

A set of <u>prediction</u> equations and of <u>updating</u> equations make the Kalman filter. The first set gives the recursions for $\hat{X}_{t,t-1}, \hat{X}_{t,t}$, $P_{t,t-1}$, and $P_{t,t}$ $(t=0,1,2,..)$. The second set permits to update these preliminary values when more observations are added. This is called smoothing and there are three basic smoothing algorithms: fixed point, fixed lag and fixed interval.

EXAMPLE. The majority of the structural models assumed for the components of a given time series, whether in theoretical or empirical studies, are generally very simple. One of the main reasons being that the estimators of these components can be unstable for complex models because the initial conditions X_0, $P_{0,-1}$ and the variances of the transition and measurement equations must be known or estimated a priori by the researcher.

The most commonly used structural models are a random walk for the trend-cycle and a stationary process for the seasonal. That is,

$$Y_t = C_t + S_t + \eta_t \quad (18)$$

$$\Delta C_t = \beta_{t-1} + \varepsilon_t \quad (19)$$

$$\Delta \beta_t = \xi_{1t}$$

$$(1+B+B^2+...+B^{S-1})S_t = \xi_{2t} \quad (20)$$

where η_t, ε_t, ξ_{1t} and ξ_{2t} are independently normally distributed variables with zero means and variances σ^2_η, σ^2_ε, $\sigma^2_{\xi_1}$ and $\sigma^2_{\xi_2}$ respectively. For quarterly data, the transition equations for the structural model above is,

$$X_t = \begin{bmatrix} C_t \\ \beta_t \\ S_t \\ S_{t-1} \\ S_{t-2} \end{bmatrix} = \left[\begin{array}{cc|ccc} 1 & 1 & & 0 & \\ 0 & 1 & & & \\ \hline & & -1 & -1 & -1 \\ & 0 & 1 & 0 & 0 \\ & & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} C_{t-1} \\ \beta_{t-1} \\ S_{t-1} \\ S_{t-2} \\ S_{t-3} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \xi_{1t} \\ \xi_{2t} \\ 0 \\ 0 \end{bmatrix} \quad (21)$$

The corresponding measurement equation is

$$Y_t = (1,0,1,00)\ X_t + \eta_t \quad (22)$$

with the covariance matrices of the disturbances in the transition equation being

$$Q = \begin{bmatrix} \sigma^2_\varepsilon & & & & & \\ & \sigma^2_{\xi_1} & & & 0 & \\ & & \sigma^2_{\xi_2} & & & \\ 0 & & & 0 & & \\ & & & & 0 & \end{bmatrix} \quad (23)$$

and $R = \sigma^2_\eta$ (a scalar)

The reduced form of this structural model is a particular case of the (0,1,1)(0,1,1) ARIMA class that has been found often adequate for many economic time series.

3. "REDUCED-FORM" APPROACH TO MODEL-BASED SEASONAL ADJUSTMENT METHODS

The "Reduced-Form" approach to ARIMA model-based seasonal adjustment methods is operationally older than the structural approach and has reached a much more advanced stage of development. In fact, there exist already computer packages that enable the use of this procedure for seasonal adjustment. The two notable examples are the MSX (signal extraction) developed by Burman (1980) and the model-based seasonal adjustment method by Hillmer and Tiao (1982). Their basic principle is to find an ARIMA model that adequately fits the observed series and then to derive from it ARIMA models for the seasonal and non-seasonal components which are uniquely determined under certain conditions.

Several authors have been preoccupied with the development of this type of seasonal adjustment method. Pierce (1978) suggested using a combination of ARIMA and deterministic models for the estimation of the trend-cycle and the seasonal components. Box, Hillmer and Tiao (1978) started with a (0,1,1)(0,1,1) ARIMA model and derived models for the components consistent with this overall model. This approach was extended by Burman (1980) and Hillmer and Tiao (1982) to include other ARIMA models.

Similar to the structural approach, the reduced form approach to model-based seasonal adjustment methods assumes that a time series, usually denoted by Z_t can be decomposed into S_t and N_t which are mutually independent seasonal and non-seasonal components (N_t can be further decomposed into trend-cycle and noise). the estimation of S_t and N_t can be obtained from the signal extraction theory as discussed in section 2.

Assuming,

$$Z_t = S_t + N_t \tag{24}$$

and supposing the component models

$$\phi_s(B)S_t = \theta_s(B)b_t \tag{25}$$

$$\phi_n(B)N_t = \theta_n(B)C_t \tag{26}$$

where the pairs of polynomials in the backshift operator B, $\{\phi_s(B), \theta_s(B)\}$, $\{\phi_n(B), \theta_n(B)\}$ and $\{\phi_s(B), \phi_n(B)\}$ have no common roots and b and c are mutually independent, iid $N(0,\sigma_b^2)$ and $N(0,\sigma_c^2)$ respectively, then the corresponding model for Z_t is (Cleveland, 1972)

$$\phi^*(B)Z_t = \theta^*(B)a_t \tag{27}$$

where $\phi^*(B) = \phi_s(B)\phi_n(B)$ and $\theta^*(B)$ and σ_a^2 are determined from

$$\Gamma_z(B) = \Gamma_s(B) + \Gamma_n(B) \tag{28}$$

where Γ_z, Γ_s and Γ_n are the autocovariance generating functions such that,

$$\frac{\theta^*(B)\theta^*(F)\sigma_a^2}{\phi^*(B)\phi^*(F)} = \frac{\theta_s(B)\theta_s(F)\sigma_b^2}{\phi_s(B)\phi_s(F)} + \frac{\theta_n(B)\theta_s(F)\sigma_c^2}{\phi_n(B)\phi_n(F)}$$

with $F=B-1$. If the roots of $\phi_s(B)$ and $\phi_n(B)$ are greater or equal to 1, then in accordance with the signal extraction theory,

$$\hat{S}_t = F_s(B)Z_t \tag{30}$$

and

$$\hat{N}_t = F_n(B)Z_t \tag{31}$$

where

$$F_s(B)=\Gamma_s(B)/\Gamma_z(B) = \frac{\sigma_b^2\theta_s(B)\theta_s(F)\phi^*(B)\phi^*(F)}{\sigma_a^2\theta^*(B)\theta^*(F)\phi_s(B)\phi_s(F)}$$

$$= \frac{\sigma_b^2\theta_b(B)\theta_s(F)\phi_n(B)\phi_n(F)}{\sigma_a^2\theta^*(B)\theta^*(F)} \tag{32}$$

and

$$F_n(B) = \frac{\sigma_c^2\theta_n(B)\theta_n(F)\phi^*(B)\phi^*(F)}{\sigma_a^2\theta^*(B)\theta^*(F)\phi_n(B)\phi_n(F)}$$

$$= \frac{\sigma_c^2\theta_n(B)\theta_n(F)\phi_s(B)\phi_s(F)}{\sigma_a^2\theta^*(B)\theta^*(F)} \tag{33}$$

In practise, S_t and N_t are non-observables but an adequate model can be obtained for the data. In order to derive consistent models for each component, assumptions have to be made. Bell and Hillmer (1984) make the following assumptions for models of S_t and N_t to be uniquely determined from a model for Z_t.

(1) $Z_t = S_t+N_t$

(2) $\{S_t\}$ and $\{N_t\}$ are mutually independent

(3) Z_t follows a known ARIMA model $\phi^*(B)Z_t= \theta^*(B)a_t$

(4) S_t follows an unknown ARIMA model $\phi_s(B)S_t - \theta_s(B)b_t$

(5) N_t follows an unknown ARIMA model $\phi_n(B)S_t = \theta_n(B)c_t$

(6) $\phi_s(B)$ and $\theta_n(B)$ have no common roots.

(7) $\phi_s(B) = 1+B+...+B^{11}$

(8) the order of $\theta_s(B) \leq 11$

(9) σ^2_b = $var(b_t)$ is as small as possible consistent with assumptions 1-8.

Given assumptions 1-8, Hillmer and Tiao (1982) showed that σ^2_b must lie within a given interval $[\bar{\sigma}^2_b, \tilde{\sigma}^2_b]$ and then the models for S_t and N_t are uniquely determined once a σ^2_b is chosen. They called this decomposition, an admissible decomposition. If $\bar{\sigma}^2_b$ (the minimum value within the interval) is selected, the decomposition is called canonical. This canonical decomposition minimizes the variance of $(1+B+B^2+-+B^{11})S_t$ making the seasonal component as stable as possible.

Burman (1984), Dagum and Laniel (1984), Maravall (1984) and others, have argued that the definitions of the seasonal is too restrictive. In fact, Dagum and Laniel (1984) showed that if the series for Z_t follows a $(0,1,1)(0,1,2)_{12}$ ARIMA model, then the canonical decomposition gives the unreasonable result of a non-seasonal component with seasonality. The amount of seasonality will be greater, the greater the value of the Θ_2

parameter relative to the θ_1. The assumptions 1-8 limit seriously the class of ARIMA models that can be decomposed to the one with a seasonal factor equal to (0,1,1). This restriction is relaxed in the method developed by Burman (1980) that enables a seasonal factor $(P,D,Q)_s$ where $P,Q \leq 2$.

To satisfy assumption 6 and let say P=1, Burman's method transfers the non-seasonal root to the non seasonal operator. Maravall (1984) pointed out that if the ARIMA model contains a factor $(1+\phi B)$ with $\phi < 0$ or AR(2) factors with complex roots for seasonal periods, the decomposition done with Bell and Hillmer's criteria would also have seasonality in the non-seasonal component. Furthermore, Pierce and Maravall (1984) showed that the variances of the revisions of the seasonal and trend-cycle component are maximized with the canonical decomposition.

3.1 Results from Empirical Applications and Comparisons with X-11-ARIMA

Several authors have applied the reduced-form of the ARIMA model-based seasonal adjustment method either using Burman's MSX or the Hillmer and Tiao (1982) versions. Among the various empirical studies we can mention, Burman (1980), Hillmer, Bell and Tiao (1983), Bilongo and Carbone (1985), Laniel (1985), den Butter, Coenen and van de Gevel (1985), and Scott (1986). In each of these studies the results obtained were compared with those given by X-11-ARIMA (Dagum, 1980) with and without the extrapolation option. In this latter case, the X-11-ARIMA closely approximates the census Method II-X-11-Variant (Shiskin, Young and Musgrave, 1967).

A thorough analysis of the comparisons made between the reduced-form model-based procedures and X-11-ARIMA leads to the following conclusions.

1. The majority of the series used in these various empirical studies were highly aggregated macroeconomic series which followed the simple (0,1,1)(0,1,1) ARIMA model. In rare occasions the non-seasonal operator showed p=2 and/or q=2, and in only one occasion p=3. For the seasonal operator, P was always equal to 0 and Q equal to 1. When Q was equal to 1, was generally greater than 0.80, indicating highly stable seasonality. For example, in Hillmer, Bell and Tiao, (1983) of the 76 series analyzed, 56 series have $\Theta>0.80$ and 67 have $\Theta\geq 0.70$.

   Scott (1986) showed that for rather irregular series the MSX package often reduced the seasonal operator to zero and calculated a deterministic seasonality.

2. From the viewpoint of the size of the revisions of the seasonal estimates, the various authors used different definitions and loss functions making it almost impossible to draw conclusions outside their particular cases.

   It was also observed that: a) if the series were affected by trading day variations and these were not a prior removed from the ARIMA modeling of X-11-ARIMA, this method would produce larger revisions as compared with the model-based procedure. The reason for this being the fact that ARIMA models cannot adequately pick up trading day variations. Hillmer, Bell and Tiao (1983) and den Butter, Coenen and van de Gevel (1985) did not remove these variations, when present, before ARIMA modeling by X-11-ARIMA and consequently the results were unfavorable to the latter. The X-11-ARIMA enables the user to eliminate a prior trading day's variations but this is not done automatically when using the default option.

In a current experimental new version, the automatic removal of trading day variations is available as a default option if a user asks for ARIMA modeling. This modification is being introduced to avoid wrong applications of this program as done by the above authors or others not familiarized with it.

b) the total revision in all these empirical studies was measured under the assumption that a "final" estimate is obtained after three or four years are added to the last observations. While this is true for X-11-ARIMA (default option) since it takes three and a half years for the concurrent asymmetric filter to become almost symmetric; it is not valid for the model-based procedure. This is particularly relevant when the value of Θ is large, as was the case for most of the series analyzed. In fact, for $\Theta =$ 0.80 the annual consecutive revisions are small but it takes at least 8 years more for the concurrent value to become final. By truncating the total revisions to three or four years only, the results will always show smaller revisions as compared to the X-11-ARIMA default option.

3. Assuming that the estimated Θ is accurate, values of $\Theta \geq 0.80$ or the presence of deterministic seasonality suggest that the default option of X-11-ARIMA should not be applied. In such cases, longe. seasonal moving averages available in this program can be used, e.g. the 3 x 9 m.a.. However, the problem of whether the estimated parameter valueis correct still remains. In fact, it is already well known that we can obtain very different values depending on the estimation procedure. Laniel (1985) obtained for the series he analysed a Θ equal to 0.77 with unconditional least squares and equal to 0.88 with maximum likelihood. For these two parameter values the model based decomposition method produced significantly different seasonal patterns for the same series.

4. CONCLUSIONS

The "structural" approach to ARIMA model-based seasonal adjustment is still in its infancy. Its greater contribution has been the introduction of the Kalman filter to estimate the state-space representations of the ARIMA models assumed for each component. However, it requires a good deal of exploratory work and subjective judgement to assess the level of the components before the process begins. Poor initial conditions causes the filter to attach too much weight to this misleading information and a long time may elapse before the incoming data dominates the initial choice. The estimation of the relative variances of the disturbances corresponding to the component models in the transition equations poses a serious challenge, particularly, when the component models depart from simple processes. Because of this, in most cases the models assumed are the random walk or the random walk with a random drift. These assumptions lead to the simple (0,1,1)(0,1,1) model which can already be adequately estimated by X-11-ARIMA.

On the other hand, the reduced-form approach of ARIMA model-based seasonal adjustment has reached a stage where it could be implemented. Two important computer programs have been developed, MSX by Burman (1980) and another by Hillmer and Tiao (1982) but, the assumptions that enable a unique decomposition from a given ARIMA model fitted to the observed data are highly restrictive. They limit the class of ARIMA models that can be fitted to real series to the $(p,d,q)(P,D,Q)_S$ type where p and $q \leq 3$ and $P, Q, \leq 1$.

Furthermore, as discussed in section 3.1, the majority of the empirical studies have been done on series that belong mostly to the simple (0,1,1)(0,1,1) class or minor variants of it and where the value of Θ is usually greater than 0.70 which implies highly stable seasonality.

The comparisons were made with the default option of X-11-ARIMA but a more appropriate selection of options already available in this program would give similar results for most cases. In fact, Cleveland and Tiao (1976) have shown that in an additive decomposition, the standard (default option) symmetric filter of census Method II-X-11 variant (similar to that of X-11-ARIMA) follows a model of the (0,1,1,)(0,1,1) type. But different sets of seasonal moving averages and trend-cycle filters can be chosen by the users and thus, the X-11-ARIMA method can deal adequately with more complex stochastic models than the (0,1,1)(0,1,1) type. Furthermore, this program enables the application of different seasonal filters for each month which implies that the seasonal models for each month are not equal. Series with this characteristic cannot be adequately estimated with the current ARIMA model - based procedures which assume that the seasonal model is the same for each month.

In conclusion, unless the class of ARIMA models that can be estimated by model-based seasonal adjustment methods is enlarged to cover a broader spectrum of series, and some of its estimation problems are solved, these seasonal adjustment methods are still far from being a substitute of X-11-ARIMA as it is currently used by statistical agencies.

Bibliography

Akaike, H. (1980): "Seasonal Adjustment by a Bayesian Modelling" Journal of Time Series Analysis, Vol. 1, p 1-13.

Bell, W. and Hillmer, S. (1984): "Issues Involved with the Seasonal Adjustment of Economic Time Series" Journal of Business and Economic Statistics, Vol. 2, pp 291-320.

Bilongo, R. and Carbone, R. (1985): "Adaptive Model-Based Seasonal Adjustment of Time Series: An Empirical Comparison with X-11-ARIMA and SIGEX" International Journal of Forecasting, Vol. 1, No. 2, pp. 165-177.

Box, G.E.P., Hillmer, S. and Tiao, G.C. (1978): "Analysis and Modelling of Seasonal Time Series" in Seasonal Analysis of Economic Time Series (A. Zellner, Editor), Washington, D.C. U.S. Bureau of Census pp. 309-344.

Burman, J.P. (1980): "Seasonal Adjustment by Signal Extraction" Journal of the Royal Statistical Society, Ser. A. Vol. 143, pp 321-337.

Burman, J.P. (1984): "Comment" in Journal of Business and Economic Statistics, Vol. 2, No. 4, pp. 325-327.

Burridge, P. and Wallis, K.F. (1984): "Unobserved-Components Models for Seasonal Adjustment Filters", Journal of Business and Economic Statistics, Vol. 2, No. 4, pp. 350-359.

Cleveland, W.P. (1972): "Analysis and Forecasting of Seasonal Time Series" Ph.D Dissertation, University of Wisconsin, Madison, Dep. of Statistics.

Cleveland, W.P. and Tiao, G.C. (1976): "Decomposition of Seasonal Time Series: A Model for the X-11 Program", Journal of the American Statistical Association, Vol. 71, pp. 581-587.

Cleveland, W.P. (1979): "Report to the Review Committee of the ASA/Census Project on Time Series" U.S. Bureau of Census, Statistical Research Division.

Dagum, E.B. (1980): The X-11-ARIMA Seasonal Adjustment Method, Ottawa: Statistics Canada Catalogue No. 12-564E.

Dagum, E.B. and Laniel, N. (1984): "Comment" in Journal of Business and Economic Statistics, Vol. 2, No. 4, 328-332.

den Butter, F.A.G., Coenen, R.L., and Van de Gebel F.J.J. (1985): "The Use of ARIMA Models in Seasonal Adjustment", Empirical Economics, Vol. 10, No. 4, pp. 209-230.

Durbin, J. (1984): "Present Position and Potential Developments. Some Personal Views in Time Series Analysis", Journal of the Royal Statistical Society, Series A, pp 161-173.

Engle, R.F. (1978): "Estimating Structural Models of Seasonality" in Seasonal Analysis of Economic Time Series (A. Zellner, Editor), Washington, D.C.: U.S. Bureau of Census, pp 281-297.

Grether, D.M. and Nerlove, M. (1970): "Some Properties of 'Optimal' Seasonal Adjustment", Econometrica, Vol. 38, pp 682-703.

Hannan, E. (1967): "Measuring of a Wandering Signal and Noise", Journal of Applied Probability, Vol. 4, pp. 90-102.

Hannan, E., Terell, R.D. and Tuckwell, N., (1970): "The Seasonal Adjustment of Economic Time Series", International Economic Review, Vol. 11, pp. 24-52.

Harvey, A.C. (1984): "A Unified View of Statistical Forecasting Procedures", Journal of Forecasting, Vol. 3 N.3 pp. 245-275.

Hillmer, S.C. and Tiao, G. (1982): "An ARIMA Model-Based Approach to Seasonal Adjustment", Journal of the American Statistical Association, vol. 77, pp. 63-70.

Hillmer, S.C., Bell, W.R., and Tiao, G.C. (1983): "Modelling Considerations in the Seasonal Adjustment of Economic Time Series" in Applied Time Series Analysis of Economic Data (A. Zellner, Editor), Washington, D.C.: U.S. Bureau of Census, pp. 74-100.

Kalman, R.E. (1960): "A New Approach to Linear Filtering and Prediction Problems", Transactions ASME, Journal of Basic Engineering, Vol. 82.

Kalman, R. and Bucy, R.S. (1961): New Results in Linear Filtering and Prediction Theory", Journal of Basic Engineering, Vol. 83, pp. 95-108.

Kitagawa, G. and Gersch, W. (1984): "A Smoothness Prior-State Space Modeling of Time Series with Trend and Seasonality", Journal of the American Statistical Association, vol. 79, pp. 378-389.

Laniel, N. (1985): "Comparaison des méthodes de désaisonnalisation MSX et X-11-ARIMA", Methodology Branch Research Paper, Statistics Canada.

Maravall, A. (1984): "Comment" in Journal of Business and Economic Statistics, Vol. 2, No. 4, pp. 337-339.

Maravall, A. (1985): "On Structural Time Series Models and the Characterization of Components", Journal of Business and Economic Statistics, Vol. 3, No. 4, pp. 350-355.

Pagan, A. (1973): "Efficient Estimation of Models with Composite Disturbance Terms", Journal of Econometrics, vol. 1, pp. 329-340.

Pagan, A. (1975): "A Note on the Extraction of Components from Time Series", Econometrica, vol. 43, 163-168.

Pierce, D. (1978): "Seasonal Adjustment when Both Deterministic and Stochastic Seasonality are Present" in Seasonal Analysis of Economic Time Series (A. Zellner, Editor) Washington, D.C. U.S. Bureau of Census, pp. 242-269.

Pierce, D.A. and Maravall, A. (1984): "Advances in Model-Based Seasonal Adjustment", Proceedings of the Bureau and Economic Statistics Section of the Annual Meetings of the American Statistical Association.

Scott, S. (1986): "A Comparison of Model-Based and X-11-ARIMA Seasonal Adjustment", Proceedings of the Business and Economic Statistics Section of the Annual Meeting of the American Statistical Association.

Shiskin, J. Young, A.H. and Musgrave, J.C. (1967): "The X-11-Variant of the Census Method II Seasonal Adjustment Program" Technical Paper 15, Washington, D.C.: U.S. Bureau of the Census.

Sobel, E.L. (1967): "Prediction of a Noise-Distorted Multivariate Non-Stationary Signal", Journal of Applied Probability, Vol. 4, pp. 330-342.

Whittle, P. (1963): Prediction and Regulation by Linear Least-Squares Methods, London: English Universities Press.