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Variance of X-11-ARIMA Estimates -A Structural Approach

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ABSTRACT

This paper presents a method for the approximation of the variance of X-11-ARIMA estimates. The method uses structural models in a state space form, the Kalman filter and the fixed interval smoother.

Stochastic models are assumed for the trend-cycle, seasonal and irregular components of a time series. These models are fitted to the X-11-ARIMA components to estimate the parameters needed in the Kalman filter. The Kalman filter and the fixed interval smoother are applied to the original series to obtain stochastic estimates of the trend-cycle, seasonal and irregular components. Confidence intervals are constructed around these stochastic estimates. Finally, the variances of the X-11-ARIMA values are approximated by the variances of the stochastic estimates if the X-11-ARIMA values fall within the confidence intervals of the stochastic estimates.

Key words: Kalman filter, fixed interval smoother, trend-cycle, seasonally adjusted, month to month change and ratio.

RESUME

Cet article présente une méthode pour l'approximation de la variance des données corrigées de leurs variations saisonnières par le progiciel X-11-ARMMI.

La méthode utilise une formulation de vecteur d'état et le filtrage de Kalman. Des modèles stochastiques sont définis pour les composantes cyclo-tendancielle, saisonnière et irrégulière de la série chronologique. Ces modèles sont ajustés à la décomposition du X-11-ARMMI pour obtenir les estimés des paramètres nécessaires au filtrage de Kalman. Le filtrage de Kalman et le lissage sur un intervalle fixe sont appliqués sur la série originale pour obtenir des estimés "stochastiques" des composantes de la série ainsi que leurs variances. Des intervalles de confiance sont construits autour des estimés stochastiques de la série désaisonnalisée. Finalement, la variance de l'estimé "X-11-ARMMI" est approximée par celle de l'estimé stochastique si l'estimé X-11-ARMMI est inclus dans l'intervalle de confiance de l'estimé stochastique.

1. INTRODUCTION.

The need for the development of standard errors of seasonally adjusted data as published by statistical bureaus has a long standing. The President's Committee to Appraise Employment and Unemployment Statistics (1962) recommended: "that estimates of the standard errors of seasonally adjusted data be prepared and published as soon as the technical problems have been solved". Seventeen years later, the National Commission on Employment and Unemployment Statistics (1979) reemphasized the importance of standard errors for seasonally adjusted series and urged the Census Bureau to undertake research to develop them. In response to this goal, Wolter and Monsour (1981) developed a procedure based on the linear filters of the Method II-X-11-variant (Shiskin, Young and Musgrave, 1967) to calculate the variance of seasonally adjusted data. These authors considered two situations, one, where the components were assumed as deterministic and thus only the sample variability contributes to the variance of the seasonally adjusted value; and, two, where the components are assumed to be stochastic processes and the nonstationary part of the time series is removed by fitting a polynomial in time. This procedure offered a simplified approximation to the variance of the X-11 estimates given the two assumptions on the kind of variability that affected the data and the fact that the linear filters themselves are an approximation of what the method really does to actual series. With the same kind of reasoning, Burridge and Wallis (1984) developed ARIMA models that approximate the various filters used by the X-11 variant to estimate the time series components and derived measures of variance using Kalman filters. Similarly, measures of the asymptotic variance could be calculated from the ARIMA model developed by Cleveland and Tiao (1976) as an approximation of the symmetric filters of the X-11 variant. Hillmer (1985) made a major contribution for computing variances of the components estimates from model based procedures such as Hillmer and Tiao (1982) and Burman (1980); and generalized Pierce (1980) results for the revision of current seasonally adjusted data. Hillmer (1985) calculated the total variance as the sum of the conditional asymptotic variance (from the case in which a doubly infinite realization is available) and the variance from the forecasts and backcasts values that are needed to replace the missing observations from the future and the past when dealing with actual series.

The studies, concerned with measures of variance of seasonally adjusted data from the X-11-variant, approached the problem in relation with its linear filters. These linear filters, however, are approximations of what the method really does under the assumptions of: (1) additive decomposition, (2) no treatment of extreme values, (3) no trading-day variations and (4) only the standard option is applied to estimate the seasonal and trend-cycle components.

The main purpose of this paper is to present a new procedure that approximates the variance of the components as really estimated from actual data by the X-11-ARIMA method (Dagum, 1980) with or without ARIMA extapolations. In the latter, the results from X-11-ARIMA are close to those from the X-11 variant.

The new procedure basically consists of assuming structural models for the trend-cycle, the seasonal and the irregular components which are first fitted to the corresponding X-11-ARIMA estimates to obtain the parameters needed for the Kalman filter. The Kalman filter and smoother are then applied to the observed data to obtain structural model estimates of the components. The variance of the X-11-ARIMA components are approximated by the variance of the estimates given by the structural models if the X-11-ARIMA estimates fall within the confidence intervals of the structural estimates.

2. THE X-11-ARIMA METHOD AND THE BASIC STRUCTURAL MODEL.

The X-11-ARIMA seasonal adjustment method assumes that a series Y_t can be decomposed into the trend-cycle C_t, the seasonal S_t and the irregular variations I_t, either in an additive manner:

$$Y_t = C_t + S_t + I_t, \tag{2.1}$$

a multiplicative manner:

$$Y_t = C_t S_t I_t \tag{2.2}$$

or, a logarithmic manner:

$$\log Y_t = \log C_t + \log S_t + \log I_t. \tag{2.3}$$

This method is based on moving averages or linear smoothing filters implying that the time series components are stochastic and thus, cannot be closely approximated by simple functions of time over the entire range of the series. The X-11-ARIMA method consists of extending the original series at each end with extrapolated values from seasonal ARIMA models and then seasonally adjusting the extended series with a combination of the X-11 filters and the ARIMA model extrapolation filters.

The basic structural model as discussed by Harvey and Todd (1983) and Harvey (1984) has the form:

$$Y_t = \mu_t + \gamma_t + \epsilon_t \quad , t=1,\ldots,T \tag{2.4}$$

where μ_t, γ_t and ϵ_t are the trend ,seasonal and irregular components respectively.

The process generating the trend is of the form:

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t, \tag{2.5}$$

and

$$\beta_t = \beta_{t-1} + \zeta_t, \tag{2.6}$$

where η_t and ζ_t are normally distributed independent white noise processes with zero means and variances σ_η^2 and σ_ζ^2 respectively. The essential feature of this model is that it is a local approximation to a linear trend and the level and slope both change slowly over time according to a random walk model.

The process generating the seasonal component is defined by:

$$\gamma_t = -\sum_{j=1}^{s-1} \gamma_{t-j} + \omega_t \tag{2.7}$$

where ω_t is distributed as NID$(0,\sigma_\omega^2)$ and s is the number of "seasons" in the year. The seasonal pattern is thus slowly changing but by a process that ensures that the sum of the seasonal components over any s consecutive time periods has an <u>expected</u> value of zero and a variance that remains constant over time. The disturbances η_t, ζ_t and ω_t are independent of each other and of the irregular component ϵ_t, distributed as NID$(0,\sigma^2)$.

In our context, the errors terms η_t and ζ_t are restricted to be equal for the reasons to be given later.

3. KALMAN FILTER AND FIXED INTERVAL SMOOTHER.

In this context and for the case of monthly observations, the state space model consists of a measurement equation:

$$Y_t = \underline{z}'\underline{\alpha}_t + \epsilon_t \tag{3.1}$$

and a transition equation:

$$\underline{\alpha}_t = C\underline{\alpha}_{t-1} + D\underline{r}_t \tag{3.2}$$

where

$$\underline{z}' = (1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0) \tag{3.3}$$

is a fixed vector.

$$\underline{\alpha}_t' = (\mu_t\ \beta_t\ \gamma_t\ \gamma_{t-1}\ \cdots\ \gamma_{t-10}) \tag{3.4}$$

is the state vector and

$$C = \begin{matrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{matrix} \qquad D = \begin{matrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \tag{3.5}$$

are fixed matrices. The signal variance vector

$$\underline{r}_t = (\eta_t\ \omega_t) \tag{3.6}$$

is normally distributed with mean zero and covariance matrix $\sigma^2 Q$,

$$Q = \mathrm{diag}(\sigma^2_\eta/\sigma^2 \quad \sigma^2_\omega/\sigma^2)$$

and σ^2 is the variance of ϵ_t, distributed NID$(0,\sigma^2)$ independently of $\underline{r}_t$.

Let $\underline{a}_t$ be the minimum mean square estimate (MMSE) of $\underline{\alpha}_t$ and $\sigma^2 P_t$ its covariance matrix, i.e. $\sigma^2 P_t = E[\underline{a}_t - \underline{\alpha}_t][\underline{a}_t - \underline{\alpha}_t]'$. The MMSE of $\underline{\alpha}_{t+1}$ given $\underline{a}_t$ and P_t is then given by:

$$\underline{a}_{t+1|t} = C\underline{a}_t \tag{3.7}$$

with MSE matrix:

$$P_{t+1|t} = CP_tC' + DQD'. \tag{3.8}$$

Once Y_{t+1} becomes available, the estimate of $\underline{\alpha}_{t+1}$ can be updated as follows:

$$\underline{a}_{t+1} = \underline{a}_{t+1|t} + \underline{K}_{t+1}v_{t+1} \tag{3.9}$$

$$P_{t+1} = (I-\underline{K}_{t+1}\underline{z}')P_{t+1|t} \tag{3.10}$$

$$v_{t+1} = Y_{t+1} - \underline{z}'\underline{a}_{t+1|t} \tag{3.11}$$

$$\underline{K}_{t+1} = P_{t+1|t}\underline{z}/f_{t+1} \tag{3.12}$$

$$f_{t+1} = \underline{z}'P_{t+1|t}\underline{z} + 1. \tag{3.13}$$

Starting values $\underline{a}_0$ and P_0 and knowledge of the covariance matrix Q are needed to implement the Kalman filter given by (3.7) to (3.13).

The Kalman filter yields the MMSE of $\underline{\alpha}_t$ given the information available up to time t. However, once all the observations are available, a better estimator can be obtained. One of the techniques for computing such an estimator is the fixed interval smoother. The fixed interval smoother is a set of recursions which start with the Kalman filter estimates $\underline{a}_T$ and P_T, and works backwards. If $\underline{a}_{t|T}$ and $\sigma^2 P_{t|T}$ denote the smoothed estimates and its covariance matrix, the smoothing equations are given by:

$$\underline{a}_{t|T} = \underline{a}_t + P^*_t(\underline{a}_{t+1|T} - C\underline{a}_t) \tag{3.14}$$

with

$$P_{t|T} = P_t + P^*_t(P_{t+1|T} - P_{t+1|t})P^*_t{}' \tag{3.15}$$

where

$$P^*_t = P_tC'(P_{t+1|t})^{-1}. \tag{3.16}$$

4. ESTIMATION OF $\underline{a}_0$, P_0 AND Q.

In its additive variant, X-11-ARIMA decomposes Y_t into the sum of a trend-cycle C_t, a seasonal S_t and an irregular I_t. Similarly, the structural model decomposes Y_t into the sum of a trend μ_t, a seasonal γ_t and an irregular ϵ_t.

The structural models for the trend μ_t and the slope β_t are fitted to the X-11-ARIMA trend-cycle estimates C_t to estimate σ_η^2 and σ_ζ^2. This is done by making the structural slope (β_t) equal to the month to month change in the X-11-ARIMA trend-cycle (C_t-C_{t-1}) and the structural trend (μ_t) to the X-11-ARIMA trend-cycle (C_t). Under these assumptions for μ_t and β_t, equations (2.5) and (2.6) reduce to:

$$C_t = C_{t-1} + (C_{t-1} - C_{t-2}) + \eta_t \tag{4.1}$$

$$C_t - C_{t-1} = C_{t-1} - C_{t-2} + \zeta_t \tag{4.2}$$

It follows from (4.1) and (4.2) that $\eta_t = \zeta_t$. The estimates of σ_η^2

and σ_ζ^2 are now derived.

From equations (2.5) and (2.6) we have:

$$\hat{\beta}_t = C_{t-1}-C_{t-2} \tag{4.3}$$

$$\hat{\mu}_t = C_{t-1}+(C_{t-1}-C_{t-2}) \tag{4.4}$$

from which it is deduced:

$$\hat{\eta}_t = C_t-2C_{t-1}+C_{t-2} \tag{4.5}$$

and

$$\hat{\zeta}_t = C_t-2C_{t-1}+C_{t-2}. \tag{4.6}$$

From (4.5) and (4.6) $\hat{\eta}_t$ equals $\hat{\zeta}_t$, and therefore:

$$\hat{\sigma}_\eta^2 = \hat{\sigma}_\zeta^2 = (T-2)^{-1} \sum_{t=3}^{T} (C_t-2C_{t-1}+C_{t-2})^2 \tag{4.7}$$

Similarly, the structural model for the seasonal component γ_t is fitted to the X-11-ARIMA seasonal estimates S_t to estimate σ_ω^2. This is done by approximating the structural seasonal component (γ_t) by the X-11-ARIMA seasonal component (S_t). From (2.7):

$$\hat{\gamma}_t = -\sum_{j=1}^{11} S_{t-j} \tag{4.8}$$

from which it is derived:

$$\hat{\omega}_t = \sum_{j=1}^{12} S_{t-j} \tag{4.9}$$

and therefore:

$$\hat{\sigma}_\omega^2 = (T-11)^{-1} \sum_{t=11}^{T} \hat{\omega}_t^2. \tag{4.10}$$

Finally, the structural model for the error ϵ_t is fitted to the X-11-ARIMA irregular estimates I_t, from which:

$$\hat{\sigma}^2 = T^{-1} \sum_{t=1}^{T} I_t^2. \tag{4.11}$$

The estimate of the initial state vector $\underline{a}_0$ requires knowledge of $(\mu_0, \beta_0, \gamma_0, \gamma_{-1}, \ldots, \gamma_{-10})$. Since S_{-10} to S_0 are not readily available from X-11-ARIMA the first eleven month of data are used to estimate $\underline{a}_{11}$ by $\underline{a}_{11}=(C_{11}, C_{11}-C_{10}, S_{11}, S_{10}, \ldots, S_1)'$ and the Kalman filter is started at time $t=12$. The initial covariance matrix P_{11} is taken to be kI_{13} where k is a large constant and I_{13} is the identity matrix of order 13.

5. VARIANCE OF X-11-ARIMA SEASONALLY ADJUSTED DATA.

5.1 Additive Decomposition.

The Kalman filter and fixed interval smoother as described by equations (3.7) to (3.16) are applied using the above initial state vector, covariance matrix of the initial state vector and the estimates of the noise variances σ_η^2, σ_ζ^2, σ_ω^2 and σ^2.

The X-11-ARIMA seasonally adjusted data is defined by:

$$Y_t - S_t = C_t + I_t \tag{5.1}$$

Similarly, the structural seasonally adjusted data is defined by:

$$Y_t - \hat{\gamma}_t = \hat{\mu}_t + \hat{\epsilon}_t \tag{5.2}$$

where $\hat{\gamma}_t$, $\hat{\mu}_t$ and $\hat{\epsilon}_t$ are the structural estimates of γ_t, μ_t and ϵ_t.

The variance of the seasonally adjusted structural estimate is defined by:

$$E[(Y_t - \hat{\gamma}_t) - (Y_t - \gamma_t)]^2 = E[\hat{\gamma}_t - \gamma_t]^2 \tag{5.3}$$

and is given by $\sigma^2 P_{t|T}(3,3)$, where $P_{t|T}(3,3)$ is the third row, third column element of $P_{t|T}$. Confidence intervals at the 95% level for the seasonally adjusted structural estimates are obtained by adding plus or minus twice the standard deviation of the estimates. Finally, the variances of the X-11-ARIMA estimates are approximated by the variances of the structural estimates if the X-11-ARIMA estimates fall within the confidence intervals of the structural estimates.

5.2 Logarithmic and Multiplicative Decompositions.

For the logarithmic and multiplicative decompositions, the structural models for the trend μ_t are fitted to log C_t, for the seasonal γ_t to log S_t and for the irregular ϵ_t to log I_t. Confidence intervals are constructed for log Y_t - log $\hat{\gamma}_t$ and by taking the antilog of the results, confidence intervals around the structural estimates are obtained. Final estimates of the variances are obtained by squaring the length of the confidence intervals divided by 4.

6. VARIANCE OF X-11-ARIMA SEASONALLY ADJUSTED MONTH TO MONTH CHANGES.

In the analysis of seasonally adjusted data, comparisons of month to month changes are often done to assess the direction and magnitude of the short-term trend.

The method discussed here allows the estimation of the variance of changes between any two months included in the state vector. The change in the structural seasonally adjusted data betwen month t and t-a, for a=1,.,10 is given by:

$$(Y_t-\hat{\gamma}_t)-(Y_{t-a}-\hat{\gamma}_{t-a}) \tag{6.1}$$

with variance:

$$E[\{(Y_t-\hat{\gamma}_t)-(Y_{t-a}-\hat{\gamma}_{t-a})\}-\{(Y_t-\gamma_t)-(Y_{t-a}-\gamma_{t-a})\}]^2 =$$

$$E[(\hat{\gamma}_{t-a}-\gamma_{t-a})-(\hat{\gamma}_t-\gamma_t)]^2=$$

$$E(\hat{\gamma}_{t-a}-\gamma_{t-a})^2 + E(\hat{\gamma}_t-\gamma_t)^2 -2E[(\hat{\gamma}_{t-1}-\gamma_{t-1})(\hat{\gamma}_t-\gamma_t)] \tag{6.2}$$

which is:

$$\sigma^2(P_{t|T}(3+a,3+a)+P_{t|T}(3,3)-2P_{t|T}(3,3+a)) \tag{6.3}$$

for the smoothed structural estimate.

In the multiplicative and logarithmic decompositions, month to month changes in the structural estimates become month to month ratios when the data is transformed back to its normal scale by taking the antilog of the results.

7. VARIANCE OF THE X-11-ARIMA TREND-CYCLE LEVELS AND MONTH TO MONTH CHANGES.

In recent years it has become also important to provide estimates of the trend-cycle as complement to the seasonally adjusted data. This is very important for series that are highly volatile and for which month to month comparisons do not give clear signals of the short-term trend (Dagum, Huot, Morry, 1988). For an additive decomposition, the variance of the trend-cycle estimates from X-11-ARIMA are approximated by those of the corresponding structural model. Month-to-month changes in the structural trend-cycle are given by:

$$\mu_t - \mu_{t-1} = \beta_{t-1} + \eta_t = \beta_{t-1} + \zeta_t = \beta_t. \qquad (7.1)$$

An estimate of the change in the structural trend-cycle is thus provided by the structural estimate of the change with variance:

$$V(\hat{\beta}_t) = E[\beta_t - \hat{\beta}_t]^2 = \sigma^2 P_{t|T}(2,2) \qquad (7.2)$$

for the smoothed structural estimate.

8. APPLICATIONS.

The results of this paper are applied to the Canada Total of Unemployed Male, Aged 25 and Over (UM) for the period January 1975 to December 1986. The official X-11-ARIMA decomposition for this series is of the additive type with one year of forecast from an ARIMA model $(0,1,2)(0,1,1)_{12}$ on the log-transformed series. The estimates of the noise variances and ratios of noise variances are provided in Table 1 where (A), (L), (M) denote the additive, logarithmic and multiplicative decomposition models respectively. For the logarithmic and multiplicative models the variances shown are associated with the log-transformed components. By comparing the estimates of σ_ω^2 it can be seen that the structural model fits better the logarithmic than the multiplicative X-11-ARIMA decomposition model. This is due to the fact that in the logarithmic decomposition model of X-11-ARIMA the seasonal factors for a given year are forced to add up to zero whereas in the multiplicative decomposition model their arithmetic mean is forced to 1 instead of their geometric mean. The estimated ratios of σ_η^2/σ^2 and σ_ω^2/σ^2 indicate how smooth the structural seasonally adjusted series are. A small estimate of σ_η^2/σ^2 with a large estimate of σ_ω^2/σ^2 give a smooth structural seasonally adjusted series since a large amount of noise is passed to the seasonality. On the other hand, a large estimate of σ_η^2/σ^2 with a small estimate of σ_ω^2/σ^2 gives a more erratic structural seasonally adjusted series.

Figure 1A.1 shows the original series and the seasonally adjusted series. Figure 1A.2 shows the X-11-ARIMA seasonally adjusted series with the smoothed seasonally adjusted structural estimates. Figure 1A.3 shows the seasonally adjusted X-11-ARIMA series and the confidence intervals constructed around the smoothed seasonally adjusted structural series.

Figure 1A.4 shows the confidence intervals for the difference between the X-11-ARIMA seasonally adjusted series and the smoothed seasonally adjusted structural series. Whenever one of the two lines cross the horizontal line drawn at zero, the null hypothesis of no difference between the estimates has to be rejected at level 95%. Figure 1A.5 shows the relative difference in percentage of the smoothed seasonally adjusted structural to the seasonally adjusted X-11-ARIMA estimates (the relative difference is defined as: 100 (Structural - X-11-ARIMA)/X-11-ARIMA). Figure 1A.6 shows the variances of the smoothed seasonally adjusted structural estimates. In all figures the first two years of data are not shown, the first year is excluded because it is used in the estimation of the initial state vector and the second year, because the non-smoothed structural estimates have too high variance caused by setting the initial covariance matrix as 100000 I.

The structural estimates give an excellent fit of the X-11-ARIMA estimates as shown by figure 1A.2 and figure 1A.3. In figure 1A.4 there are only 4 time points where the null hypothesis has to be rejected, a quantity smaller than 5% of the observations. The relative differences between the smoothed seasonally structural estimates and the X-11-ARIMA seasonally adjusted estimates (figure 1A.5) are very small (maximum is -2.78%) but autocorrelated. It would be quite unrealistic to expect pure white noise in the relative differences (or in the differences) as it would mean that the X-11-ARIMA decomposition would have been perfectly modelled. This means that the exact model behind the convolution of the X-11-ARIMA filters and the innovations of the series would have been perfectly identified. In figure 1A.6 the graph of the smoothed variances versus time has a concave shape with jumps every year. The variances are the smallest in the middle of the series which is not only intuitive but also in accordance with the results obtained by Wolter and Monsour (1981) for other series.

All figures 1B show the X-11-ARIMA trend-cycle estimates and structural trend-cycle estimates equivalent to the figures in 1A. In figure 1B.4 there are only 4 time points where the null hypothesis has to be rejected. As in the case of the seasonally adjusted series, the relative difference in the trend-cycle estimates are small and autocorrelated.

All figures 1C show the month to month changes in X-11-ARIMA and structural estimates. Figure 1C.1 shows the changes in X-11-ARIMA seasonally adjusted estimates $[(Y_t - S_t) - (Y_{t-1} - S_{t-1})]$ with the change in the smoothed seasonally adjusted structural estimates (see equation (6.1)). Confidence intervals (equation (6.2)) are provided in figure 1C.2. Values falling above (below) the zero line indicate increasing (decreasing) changes in the seasonally adjusted UM series. Particularly, it stands out the period from September 1981 till November 1982 with the only exception of October and November 1981 and January 1982. May 1981 till December 1982 corresponded to the deep Canadian recession. Figure 1C.4 shows the variance of the change in the smoothed seasonally adjusted structural series.

Figure 1C.5 shows the change in X-11-ARIMA trend-cycle estimates (C_t-C_{t-1}) with the smoothed structural slope estimates (β_t) (see equation (7.1)). Confidence intervals are provided in figure 1C.6. It should be noted that according to these confidence intervals, the trend-cycle changes were different from zero and increasing during the period May 1981 - December 1982. The trend-cycle changes are different from zero and decreasing particularly, during March - October 1983. Figure 1C.7 shows the confidence intervals for the difference between the change in the X-11-ARIMA and the smoothed structural slope estimates. There are are only 3 time points outside the confidence intervals. Variances of the smoothed structural slope estimates are provided in figure 1C.8.

Policy and decision makers have always been concerned with the turning points in the business cycle. Consequently, it is important to assess if a change of direction in either a current seasonally adjusted value or a current trend-cycle estimate indicate the presence of a turning point.

Given the whole series span from January 1975 to 1986, the month to month change in the seasonally adjusted series where different from zero and increasing for May 1981 and the whole period September 1981-November 1982 with the exception of October and November 1981 and January 1982. Using the series from January 1975 till May 1981 and adding one month at a time, we wanted to identify when these changes would have been detected.

Table 2A provides the confidence intervals constructed around the month to month changes of the smoothed seasonally adjusted structural estimates as an approximation to the confidence intervals of the corresponding X11-ARIMA estimates. It can be seen that the change from April to May 1981 was significantly different from zero and remained so when more data were added to the series. The month to month changes of the current seasonally adjusted values since September 1981 till November 1982 were good estimators of the corresponding "historical" values obtained when the series ended in December 1986. Given the amount of irregularity in the UM series, we used the MCD (Month for Cyclical Dominance) measure of X-11-ARIMA as an indicator of the length of the month-span where the contribution of the cyclical variations surpassed those of the irregulars. For the UM series the MCD is equal to 2 indicating that to assess the direction of the short term trend comparisons must be made between the current seasonally adjusted values and 2 months before.

Table 2B shows the confidence intervals for the 2-month span changes of the UM series. The results show that these changes were significantly different from zero and positive since June 1981 with only two exceptions, August-June and November-September 1981. The historical estimates of the trend-cycle indicated that the month-to-month changes increased during the whole period since May 1981 till December 1982 in agreement with what was observed with the 2-month span of the seasonally adjusted estimates.

On the other hand, as shown in Table 2.C, the current trend-cycle estimates performed poorly. Their month-to-month changes failed to detect the increases for the months of May, June, July and August and only when September 1981 became available all the revised trend-cycle estimates conformed to the corresponding historical estimates of change. Dagum and Laniel (1987) have shown the need for revising the current trend-cycle estimate when more data become available for the corresponding filters of X-11-ARIMA leave a considerable amount of irregularity in the most recent estimates. We reached the same conclusions in this study. On the other hand, as shown in Table 2A and 2B, the need for revising the current seasonally adjusted series during the current year is not so strong. These results conform with those given by Dagum (1982.a and 1982.b) concerning the revisions of the seasonal adjustment filters of X-11-ARIMA.

9. CONCLUSIONS

This study has introduced a method for the approximation of the variance of the X-11-ARIMA estimates. The method basically consists of fitting simple structural models to the X1-11-ARIMA estimates to obtain the initial state sector, its covariance matrix and the signal-noise ratios variances needed for the Kalman filter.

The Kalman filter and smoother are applied to the observed values to obtain structural model estimates of the components. The variances of the X-11-ARIMA components are approximated by the variances of the structural model estimates if the former fall within the confidence interval of the latter. We illustrated with the series Canada Total Unemployed Male - Aged 25 years and over and calculated the variances for the seasonally adjusted values and the trend-cycle estimates of X-11-ARIMA. Furthermore, given the importance of providing variances for the changes in the components, we calculated and analysed the results from month-to-month changes in the current and historical (final) values of the May 1981 - December 1982 period corresponding to the deep Canadian recession. Our results indicated that the month-to-month changes of the current seasonally adjusted series are better predictors of the final changes than those of the current trend-cycle estimates. However, in order to get a better assessment of the final short trend-cycle, it was necessary to calculate the variances of 2-month span comparisons for the current seasonally adjusted values.

This method was applied to other series and performs better whenever the decomposition is additive or log-additive and the series can be extrapolated with simple ARIMA models. Although not shown here, it can also be applied to series affected by trading-day variations.

ACKNOWLEDGEMENTS

The authors wish to thank Normand Laniel who provided helpful comments.

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TABLE 1

Canada Total Unemployed Male - Aged 25 and Over.
Estimate of the noise variances and ratios.

Decomposition model	σ_η^2	σ_ω^2	σ^2	σ_η^2/σ^2	σ_ω^2/σ^2
(A)	6.06	4.34	53.59	.1132	.0809
(L)	$3.40\text{x}10^{-5}$	$3.21\text{x}10^{-4}$	$6.57\text{x}10^{-4}$	.0512	.4885
(M)	$3.31\text{x}10^{-4}$	$2.86\text{x}10^{-2}$	$6.67\text{x}10^{-4}$	.0494	42.68

TABLE 2A

Canada Total Unemployed Male - Aged 25 and Over.
95% Confidence Intervals for $(Y_t-S_t)-(Y_{t-1}-S_{t-1})$ using the
Confidence Intervals for the Change in the
Smoothed Seasonally Adjusted Structural Estimates.

date	May 81	Jun. 81	Jul. 81	Aug. 81	Sep. 81	Oct. 81	Nov. 81	Dec. 81
May 81	(10.63,28.42)							
Jun.81	(10.75,29.02)	(-2.52,15.77)						
Jul.81	(10.79,29.06)	(-2.05,16.23)	(-6.83,11.49)					
Aug.81	(10.66,29.04)	(-2.15,16.24)	(-7.02,11.37)	(-17.44,0.99)				
Sep.81	(11.33,29.64)	(-2.03,16.28)	(-6.60,11.71)	(-16.75,1.58)	(18.25,36.60)			
Oct.81	(10.84,29.44)	(-2.01,17.40)	(-7.06,12.34)	(-16.75,2.66)	(17.64,37.06)	(-3.22,16.24)		
Nov.81	(8.94,29.42)	(-2.33,18.15)	(-7.71,12.77)	(-17.14,3.35)	(16.69,37.18)	(-3.78,16.78)	(-16.40,4.14)	
Dec.81	(8.86,30.04)	(-2.53,18.65)	(-7.79,13.38)	(-17.23,3.95)	(16.77,37.95)	(-3.71,17.47)	(-15.41,5.77)	(27.12,48.23)
May 82	(9.55,28.94)	(-0.88,19.81)	(-7.79,13.42)	(-17.49,3.73)	(17.61,38.83)	(-3.71,17.51)	(-15.21,6.01)	(28.43,49.45)

TABLE 2B

Canada Total Unemployed Male - Aged 25 and Over.
95% Confidence Intervals for $(Y_t-S_t)-(Y_{t-2}-S_{t-2})$ using the
Confidence Intervals for the Change in the
Smoothed Seasonally Adjusted Structural Estimates.

late	Mar. to May 81	Apr. to Jun. 81	May. to Jul. 81	Jun. to Aug. 81	Jul. to Sep. 81	Aug. to Oct. 81	Sep. to Nov. 81	Oct. to Dec. 81
lay 81	(-4.63,13.16)							
un.81	(-4.11,14.15)	(17.35,35.66)						
ul.81	(-3.84,14.44)	(17.88,36.15)	(0.26,18.57)					
ug.81	(-3.97,14.41)	(17.70,36.08)	(0.03,18.42)	(-15.26,3.16)				
ep.81	(-2.44,14.87)	(18.45,36.76)	(0.52,18.84)	(-14.19,4.13)	(10.67,29.02)			
ct.81	(-3.93,15.47)	(17.73,37.14)	(0.63,20.03)	(-14.10,5.30)	(10.60,30.01)	(24.13,43.59)		
ov.81	(-4.61,15.88)	(16.85,37.33)	(0.20,20.68)	(-14.60,5.88)	(9.79,30.38)	(23.15,43.65)	(- 9.99,10.61)	
ec.81	(-4.67,16.51)	(16.91,38.10)	(0.27,21.45)	(-14.43,6.75)	(10.13,31.31)	(23.65,44.83)	(- 8.54,12.65)	(22.30,43.40)
ay 82	(-5.94,13.44)	(18.36,39.05)	(1.67,22.88)	(-14.68,6.54)	(10.73,31.95)	(24.51,45.73)	(- 8.31,12.91)	(23.83,44.85)

ABLE 2C

anada Total Unemployed Male - Aged 25 and Over.
5% Confidence Intervals for C_t-C_{t-1} using the
onfidence Intervals for the Smoothed Structural Slope β_t.

ate	May 81	Jun. 81	Jul. 81	Aug. 81	Sep. 81	Oct. 81	Nov. 81	Dec. 81
ay 81	(-5.49,1.84)							
n.81	(-2.85,3.49)	(-3.16,4.32)						
l.81	(-1.14,4.45)	(-1.05,5.48)	(-1.47,6.28)					
g.81	(-1.17,3.67)	(-1.03,4.42)	(-1.34,4.97)	(-2.02, 5.49)				
p.81	(0.20,4.77)	(0.96,5.87)	(1.28,6.82)	(1.27, 7.72)	(1.03, 8.64)			
t.81	(0.79,5.28)	(1.93,6.58)	(2.72,7.72)	(3.20, 8.83)	(3.40, 9.97)	(3.02,10.79)		
v.81	(0.70,5.09)	(1.79,6.24)	(2.57,7.19)	(3.07, 8.05)	(3.29, 8.90)	(2.95, 9.49)	(2.32,10.01)	
c.81	(0.94,5.51)	(2.38,6.96)	(3.63,8.28)	(4.76, 9.58)	(5.73,10.92)	(6.18,12.03)	(6.25,13.07)	(6.05,14.08)
y 82	(0.39,5.34)	(1.99,6.96)	(3.50,8.47)	(5.04,10.02)	(6.67,11.65)	(8.07,13.05)	(9.40,14.39)	(10.72,15.78)

FIGURES 1A
CANADA TOTAL OF UNEMPLOYED MALE - AGED 25 AND OVER
SEASONALLY ADJUSTED VALUES

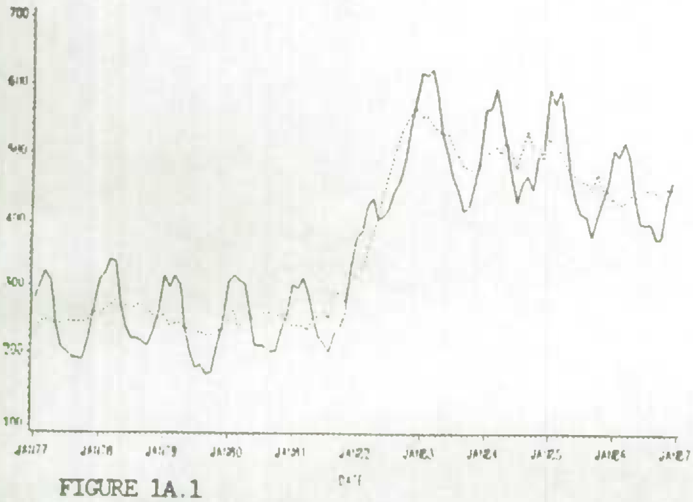

FIGURE 1A.1
___: ORIGINAL SEIRES
...: X-11-ARIMA SEASONALLY ADJUSTED

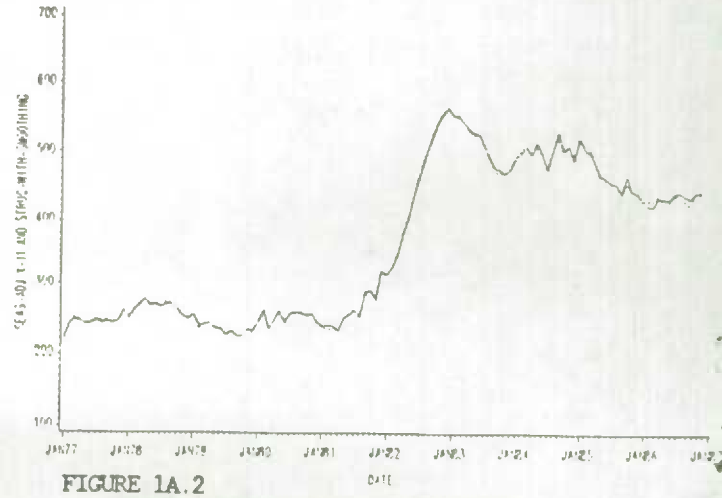

FIGURE 1A.2
___: X-11-ARIMA SEASONALLY ADJUSTED
...: SMOOTHED STRUCTURAL SEASONALLY ADJUSTED

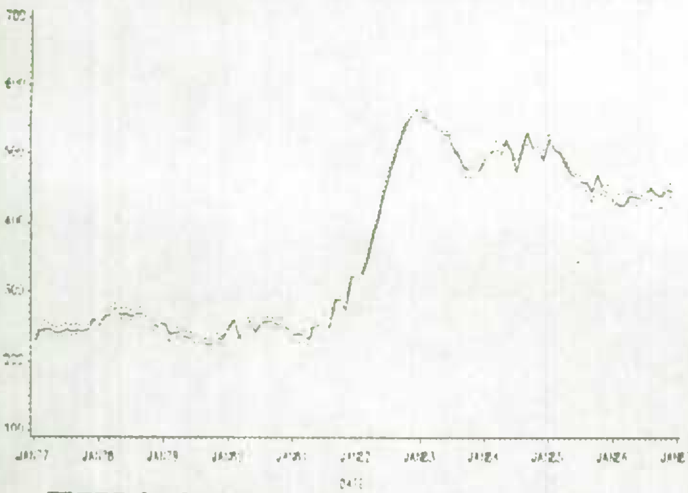

FIGURE 1A.3
___: X-11-ARIMA SEASONALLY ADJUSTED
...: CONFIDENCE INTERVALS FOR THE SMOOTHED STRUCTURAL SEASONALLY ADJUSTED

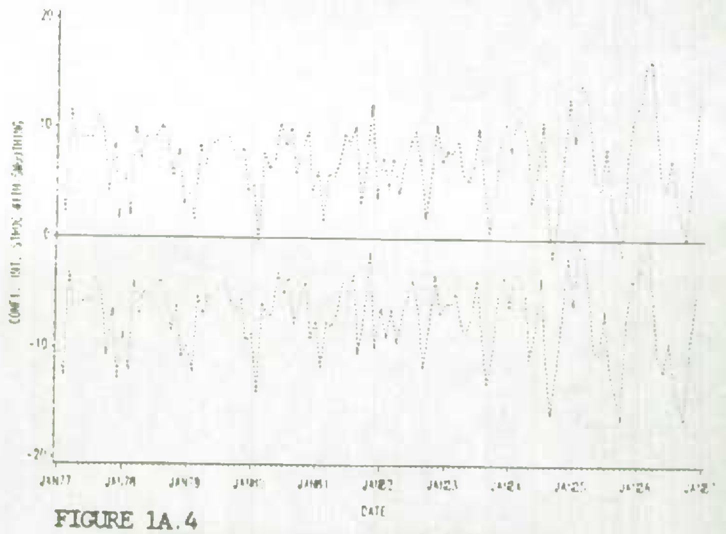

FIGURE 1A.4
...: CONFIDENCE INTERVALS FOR THE DIFFERENCE BETWEEN X-11-ARIMA AND SMOOTHED STRUCTURAL

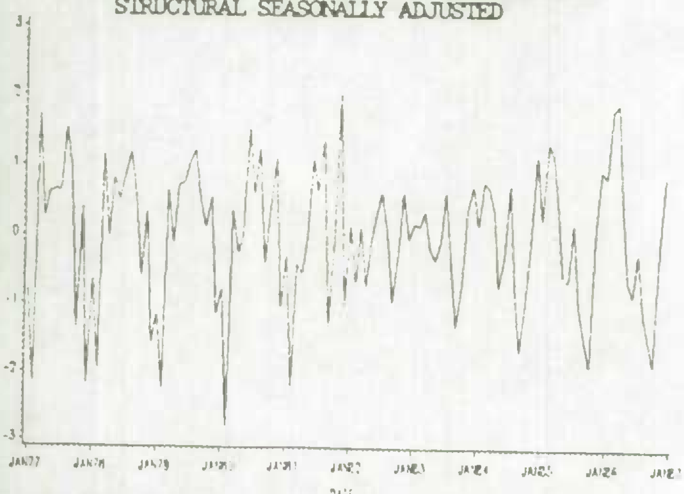

FIGURE 1A.5
___: RELATIVE DIFFERENCE (IN PERCENTAGE) BETEWEEN X-11-ARIMA AND SMOOTHED STRUCTURAL

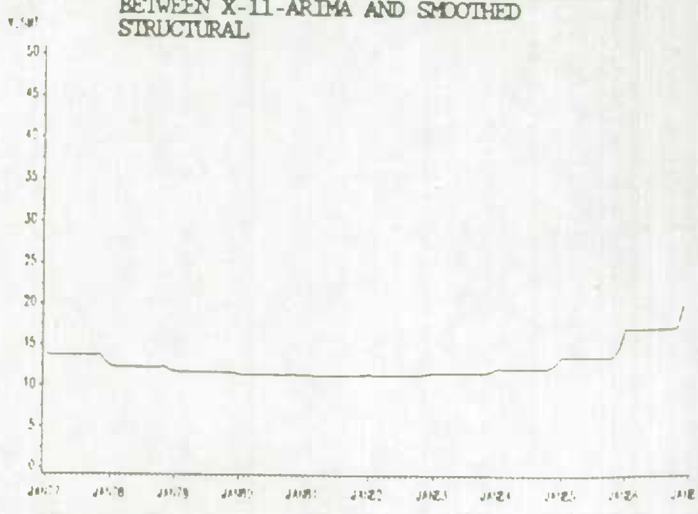

FIGURE 1A.6
___: VARIANCE OF THE SMOOTHED STRUCTURAL ESTIMATES VERSUS TIME

FIGURES 1B
CANADA TOTAL OF UNEMPLOYED MALE - AGED 25 AND OVER
TREND-CYCLE VALUES

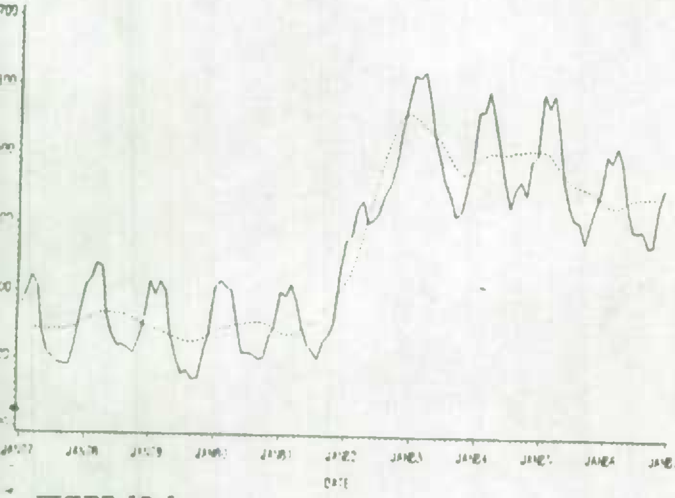

FIGURE 1B.1
___: ORIGINAL SEIRES
...: X-11-ARIMA TREND-CYCLE

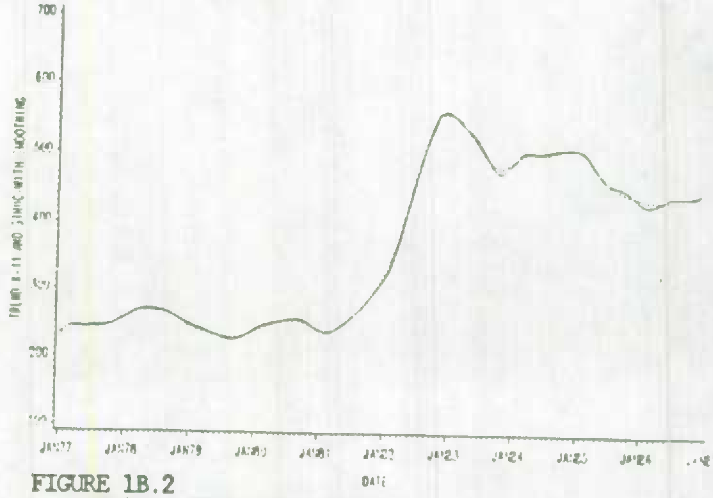

FIGURE 1B.2
___: X-11-ARIMA TREND-CYCLE
...: SMOOTHED STRUCTURAL TREND-CYCLE

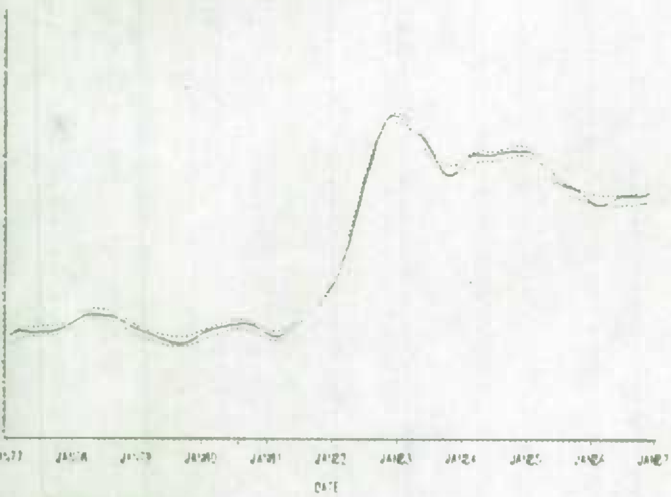

FIGURE 1B.3
___: X-11-ARIMA TREND-CYCLE
...: CONFIDENCE INTERVALS FOR THE SMOOTHED STRUCTURAL TREND-CYCLE

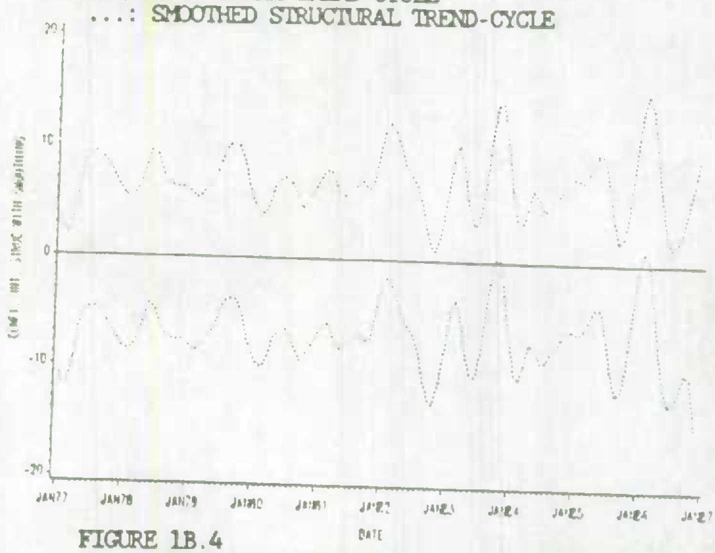

FIGURE 1B.4
...: CONFIDENCE INTERVALS FOR THE DIFFERENCE BETWEEN X-11-ARIMA AND SMOOTHED STRUCTURAL

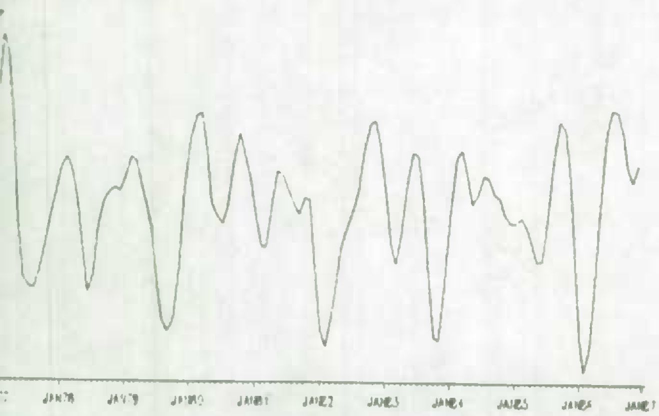

FIGURE 1B.5
___: RELATIVE DIFFERENCE (IN PERCENTAGE) BETEWEEN X-11-ARIMA AND SMOOTHED STRUCTURAL

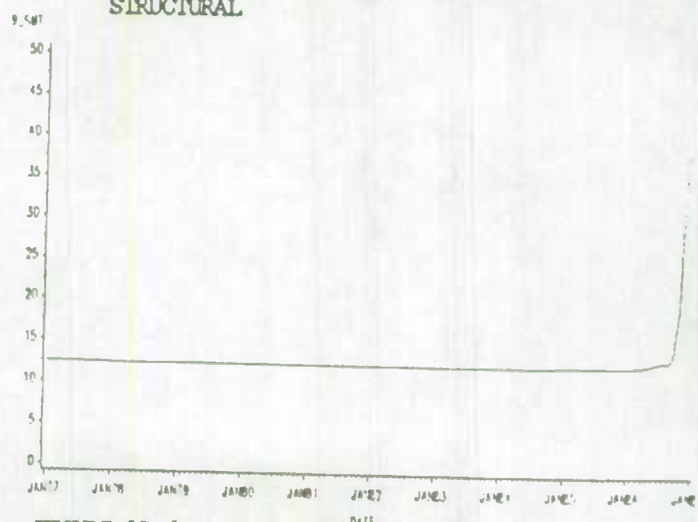

FIGURE 1B.6
___: VARIANCE OF THE SMOOTHED STRUCTURAL ESTIMATES VERSUS TIME

FIGURES 1C
CANADA TOTAL OF UNEMPLOYED MALE - AGED 25 AND OVER
MONTH TO MONTH CHANGES OF SEASONALLY ADJUSTED VALUES

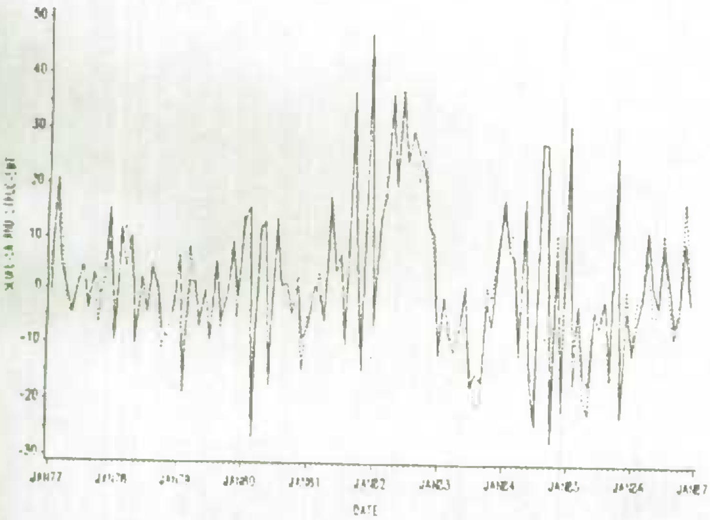

FIGURE 1C.1
___: CHANGE IN X-11-ARIMA SEASONALLY ADJUSTED
...: CHANGE IN SMOOTHED STRUCTURAL
SEASONALLY ADJUSTED

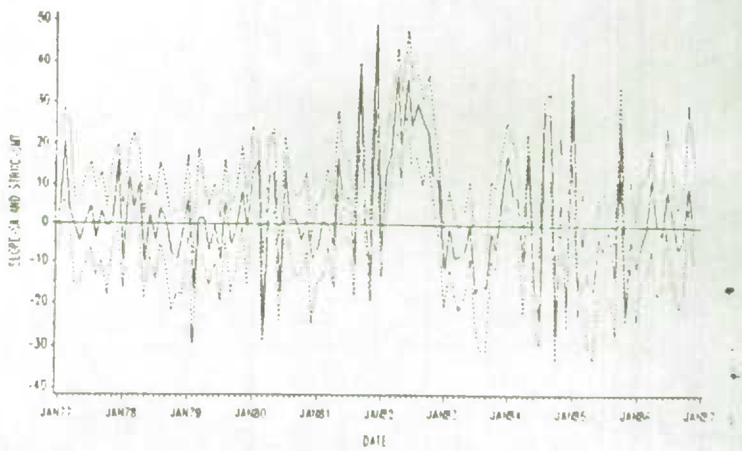

FIGURE 1C.2
___: CHANGE IN X-11-ARIMA SEASONALLY ADJUSTED
...: CONFIDENCE INTERVALS FOR THE SMOOTHED
STRUCTURAL

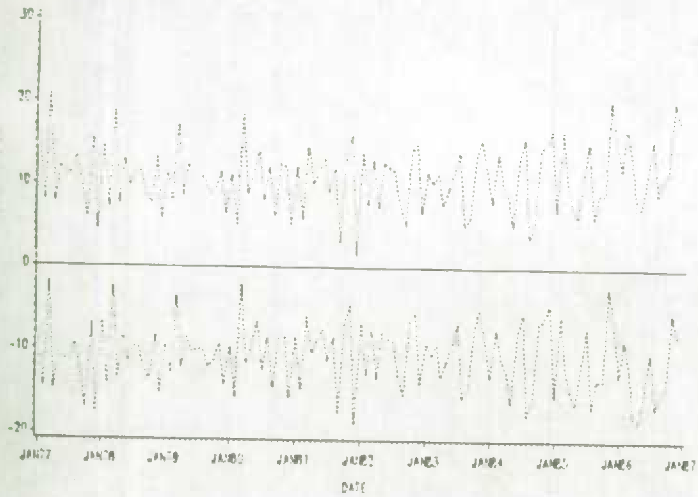

FIGURE 1C.3
...: CONFIDENCE INTERVALS FOR THE DIFFERENCE
BETEWEEN X-11-ARIMA AND SMOOTHED
STRUCTURAL

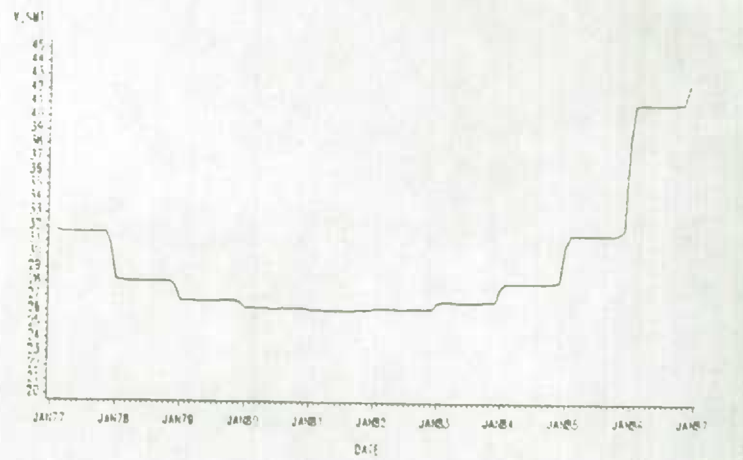

FIGURE 1C.4
___: VARIANCES OF THE SMOOTHED STRUCTURAL
ESTIMATES VERSUS TIME

FIGURES 1C

CANADA TOTAL OF UNEMPLOYED MALE - AGED 25 AND OVER

MONTH TO MONTH CHANGE OF TREND-CYCLE VALUES

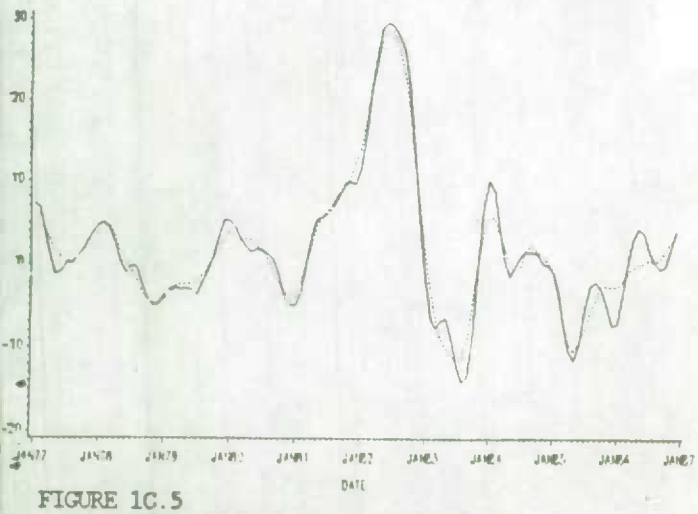

FIGURE 1C.5

___: CHANGE IN X-11-ARIMA TREND-CYCLE

...: SMOOTHED STRUCTURAL SLOPE

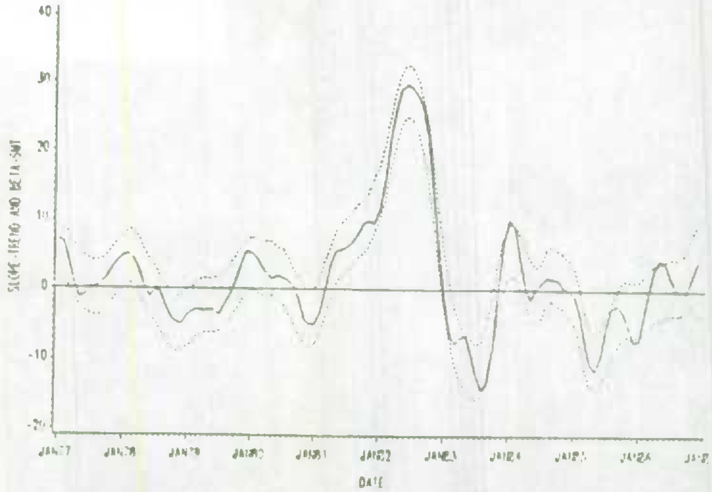

FIGURE 1C.6

___: CHANGE IN X-11-ARIMA TREND-CYCLE

...: CONFIDENCE INTERVALS FOR THE SMOOTHED STRUCTURAL SLOPE

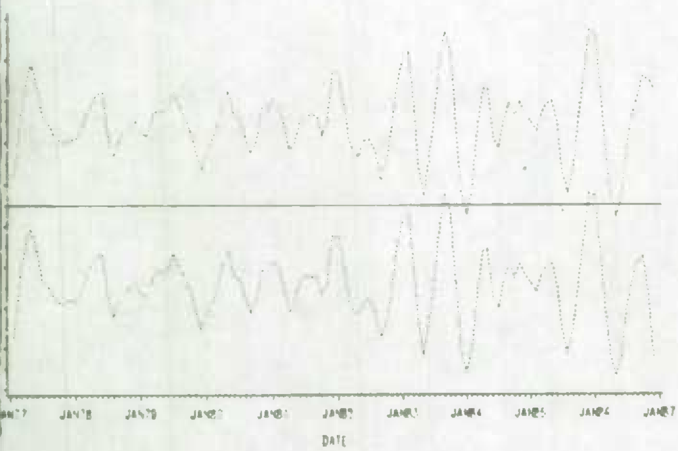

FIGURE 1C.7

...: CONFIDENCE INTERVALS FOR THE DIFFERENCE BETEWEEN X-11-ARIMA AND SMOOTHED STRUCTURAL

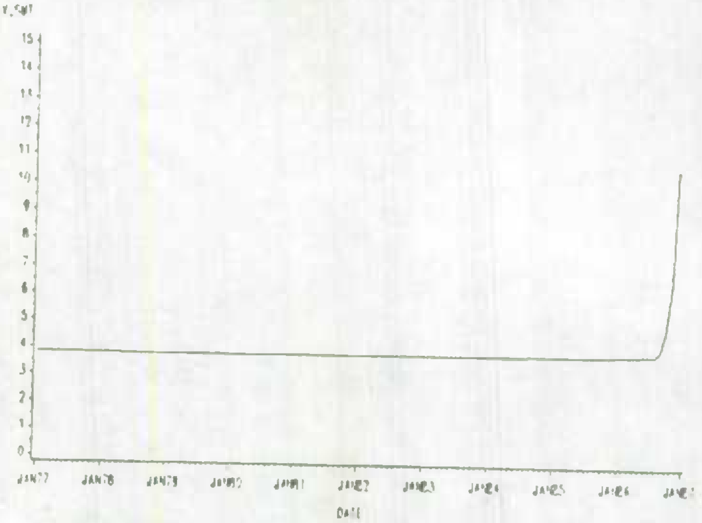

FIGURE 1C.8

___: VARIANCES OF THE SMOOTHED STRUCTURAL ESTIMATES