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A GENERAL STOCHASTIC MODEL FOR TRADING-DAY VARIATIONS

by

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ABSTRACT

A large class of flows and stocks series related to production, shipments, sales and inventories are affected by trading-day or calendar variations. Trading-day variations represent the "within-month variations" due to the number of times a particular day or days of the week occur in a calendar month. These variations are systematic and may strongly influence month-to-month comparisons. Whenever present they are usually removed together with seasonality and other effects, such as moving-holiday variations, to obtain a clearer signal of the short-term trend-cycle component, which is used for decision making by socio-economic agents.

A deterministic trading-day model developed by Young (1965) is currently used by the X-11-ARIMA (Dagum, 1980) and the Census Method II-X-11 variant (Shiskin, Young and Musgrave, 1967) to estimate trading-day variations. This deterministic model assumes that the daily weights are fixed and estimated using least squares. As long as the relative weight of daily activities is constant throughout the length of the chosen span of the series, this nodel produces reasonable estimates. However, that is not always a realistic assumption. For exemple, in retail trade, store opening hours and, hence, consummer shopping patterns have changed over the last decade; there are nany more retail outlets open on Sundays, and even during the week stores keep longer hours than previously.

The purpose of this paper is to introduce a general stochastic model that allows for gradual changes of the daily activity coefficients used to calculate trading-day variations. The model is presented in a state-space form and estimated using the Kalman filter and fixed interval smoother. Examples of the stochastic and deterministic models are given for real data.

Key words: Fibonacci line search, fixed interval smoother, Kalman filter, maximum likelihood estimation, method of scoring, spectral densities.

RESUMÉ

Un grand nombre de séries de flux et de stock reliés à la production, ventes et inventaires sont affectées par la rotation des jours ou l'effet du calendrier. La composante de rotation des jours représente les variations intra-mensuelles causées par la distribution des jours dans le mois (ex: 4 lundis,..., 4 vendredis, 5 samedis et 5 dimanches dans un mois de 30 jours). Ces variations sont systématiques et peuvent fortement influeçées les comparaisons entre les mois. Ainsi, lorsque ces variations sont présentes dans une série, elles sont généralement soustraites avec la saisonnalité et possiblement d'autres effets, tel que l'effet de Pâques, pour obtenir un estimé plus préçis de la tendance-cycle.

Un modèle déterministe, developpé par Young (1965), est utilisé par les les méthodes X11-ARMMI (Dagum, 1980) et X11 (Shiskin, Young et Musgrave, 1967) pour l'estimation de la composante de rotation des jours . Ce modèle assume que les poids des jours et leur patron hebdomadaire restent constants à travers une période pré-déterminée de la série. Pour certaines séries socio-économiques cette hypothèse peut être discutable.

Cet article présente un modèle stochastique général pour l'estimation de la composante de rotation des jours. La méthode d'estimation est discutée et les modèles déterministe et stochastique sont appliqués à des séries réelles.

I. Introduction.

A large class of flows and stocks series related to production, shipments, sales and inventories are affected by trading-day or calendar variations. Trading-day variations represent the "within-month variations" due to the number of times a particular day or days of the week occur in a calendar month. These variations are systematic and may strongly influence month-to-month comparisons. Whenever present they are usually removed together with seasonality and other effects, such as moving-holiday variations, to obtain a clearer signal of the short-term trend-cycle component, which is used for decision making by socio-economic agents.

A deterministic trading-day model developed by Young (1965) is currently used by the X-11-ARIMA (Dagum, 1980) and the Census Method II-X-11 variant (Shiskin, Young and Musgrave, 1967) to estimate the trading-day component. This deterministic model assumes that the daily weights remain constant throughout the chosen span of the series. Similarly, deterministic models for trading-day variations were discussed by Cleveland and Devlin (1980 and 1982), Hillmer (1982), Cleveland and Grupe (1983), Hillmer, Bell and Tiao (1983), Bell and Hillmer (1983), Kitagawa and Gersch (1984) and Salinas and Hillmer (1987). For some socio-economic time series this assumption might be too restrictive and a stochastic model for trading-day variations be more adequate. This is particularly true for series affected by changes in store opening hours which affect consumer shopping patterns and by more retail outlets open on Sundays.

The main purpose of this study is to introduce a general stochastic model of trading-day variations for gradually changing daily coefficients. Section 2 gives a definition of trading-day variations. The stochastic model together with the deterministic model are discussed in section 3. Section 4 deals with the estimation procedure of the stochastic model. In section 5, the stochastic and deterministic models are applied to two real series affected by trading-day variations. Finally, section 6 gives the conclusions of this study.

2. Definition of Trading-Day Variations.

Let ξ_{it} i=1,2,...,7 represent the effects of daily activity on Monday, Tuesday,..., and Sunday in month t. The effect attributed to the number of times each day of the week occurs in month t defines what is known as trading-day variations or effects. In this study, we define the trading-day effect in month t as:

$$D_t = \sum_{i=1}^{6} (\xi_{it} - \bar{\xi}_t)(X_{it} - X_{7t}), \tag{2.1}$$

where X_{it} i=1,...,7 denotes respectively, the number of Mondays, Tuesdays,..., and Sundays in month t. If all the ξ_{it}'s are equal, there is no trading-day effect. Similarly, for the month of February, except in leap year, $X_{it}=X_7$ i=1,...,6, and there is no trading-day effect. For notational convenience, let $\delta_{it} = \xi_{it} - \bar{\xi}_t$ i=1,...,6 and $T_{it} = X_{it} - X_{7t}$. The δ_{it}'s represent the difference between the Monday, Tuesday,..., and Saturday effects ξ_{it} and the average of the daily effects $\bar{\xi}_t$, for month t. The difference between the Sunday effect and the average of the daily effects is:

$$\xi_{i7} - \bar{\xi}_t = -\sum_{i=1}^{6} \delta_{it}. \tag{2.2}$$

3. Deterministic and Stochastic Models for Trading-day Coefficients or Daily Weights.

The estimation of trading-day variations in X-11-ARIMA and Census Method II X-11 variant is done sequentially after other variations, namely, seasonality and trend-cycle are removed from the data. Other model-based seasonal adjustment methods estimate trading-day variations simultaneously with the remaning components via an iterative process where the seasonality and the trend-cycle are initially assumed to follow very simple stochastic models. Hence, given a time series, say y_t, we assume:

$$y_t = D_t + e_t, \quad t=1,\dots,T \tag{3.1}$$

where $e_t \sim NID(0,\sigma^2)$ and T is the number of observations.

The deterministic trading-day variations model developed by Young (1965) assumes that $\delta_{it} = \delta_i$ for all t. In this case, equation (2.1) reduces to:

$$D_t = \sum_{i=1}^{6} \delta_i T_{it} \qquad (3.2)$$

where the δ_i's denote fixed parameters and are estimated by ordinary least squares (OLS).

A general stochastic model for the trading-day coefficients can be written as follows:

$$D_t = \sum_{i=1}^{6} \delta_{it} T_{it} \qquad (3.3.a)$$

with

$$(1-B)^k \underline{\delta}_t = \underline{\chi}_t \qquad (3.3.b)$$

where

$$\underline{\delta}_t = (\delta_{1t}, \ldots, \delta_{6t})' \qquad (3.4.a)$$

$$\underline{\chi}_t = (\chi_{1t}, \ldots, \chi_{6t})' \qquad (3.4.b)$$

$$\underline{\chi}_t \sim NID(0, \sigma^2_\chi I_6). \qquad (3.4.c)$$

Here, B denotes the backshift operator ($B\underline{\delta}_t = \underline{\delta}_{t-1}$) and I_6 is the identity matrix of order 6.

The rational behind model (3.3.b) is to adjust locally a polynomial of degree k to the trading-day coefficients. The degree of difference k can be selected using Akaike's criterion discussed in section 4.6.

In this study, we will focus our attention on $k=1$ and 2. When $k=1$, the model is the well-known random walk model which has been previously used by Monsell (1983) and Dagum and Quenneville (1988). For $k=2$ the model (3.3.b) assumes that the daily coefficients behave locally as a straight line and will be called here the quadratic model.

The stochastic model (3.3.b) for $k=1$ and $k=2$ is written in state-space form and the estimates of $\underline{\delta}_t$ together with their mean squared error matrices are estimated with the Kalman filter. Smoothed estimates are obtained using the fixed interval smoother. Finally, maximum likelihood estimators are used to estimate the parameters σ^2 and σ^2_χ/σ^2.

4. Estimation of the Stochastic Models.

A brief description of the Kalman filter and the fixed interval smoother follows.

4.1 The Kalman Filter and Fixed Interval Smoother.

The state space model consists of a measurement equation, namely,

$$y_t = \underline{z}'_t \underline{\alpha}_t + \epsilon_t, \quad t=1,\ldots,T \tag{4.1}$$

and a transition equation, namely,

$$\underline{\alpha}_t = G\underline{\alpha}_{t-1} + \underline{\eta}_t, \quad t=1,\ldots,T \tag{4.2}$$

where $\underline{\alpha}_t$ is an mxl state vector, $\underline{z}_t$ is a mxl fixed vector, G is a fixed mxm matrix and the errors ϵ_t and $\underline{\eta}_t$ are independent. It is further assumed that $\epsilon_t \sim NID(0,\sigma^2)$ and $\underline{\eta}_t \sim NID(0,\sigma^2 Q)$ where Q is a fixed mxm matrix and σ^2 is a scalar. Although Q may depend on unknown parameters it is regarded as being fixed and known for the purpose of the Kalman filter.

Let $\underline{a}_{t-1}$ be the minimum mean squared estimator (MMSE) of $\underline{\alpha}_{t-1}$ based on all the information up to and including t-1, and let $\sigma^2 P_{t-1}$ be the MSE matrix of $\underline{a}_{t-1}$, i.e., the covariance matrix of $\underline{a}_{t-1} - \underline{\alpha}_{t-1}$. The MMSE of $\underline{\alpha}_t$, given $\underline{a}_{t-1}$ and P_{t-1}, is then:

$$\underline{a}_{t|t-1} = G\underline{a}_{t-1} \tag{4.3}$$

with MSE matrix:

$$P_{t|t-1} = GP_{t-1}G' + Q. \tag{4.4}$$

Once y_t becomes available, this estimator of $\underline{\alpha}_t$ can be updated as follows:

$$\underline{a}_t = \underline{a}_{t|t-1} + P_{t|t-1}\underline{z}_t v_t / f_t \tag{4.5}$$

with MSE matrix:

$$P_t = P_{t|t-1} - P_{t|t-1}\underline{z}_t \underline{z}_t' P_{t|t-1} / f_t \tag{4.6}$$

where

$$v_t = y_t - \underline{z}_t' \underline{a}_{t|t-1} \tag{4.7}$$

$$f_t = \underline{z}_t' P_{t|t-1} \underline{z}_t + 1. \tag{4.8}$$

Starting values $\underline{a}_0$ and P_0 are needed to implement the Kalman filter given by (4.3) to (4.8).

The Kalman filter yields the MMSE of $\underline{\alpha}_t$, given the information available up to time t. However, once all the observations are available, a better estimator can be obtained. One of the techniques for computing such estimators is the fixed interval smoother. The fixed interval smoother is a set of recursions which start with the Kalman filter estimates $\underline{a}_T$ and P_T, and work backwards. If $\underline{a}_{t|T}$ and $\sigma^2 P_{t|T}$, denote the smoothed estimate and its covariance matrix, the smoothing equation is given by:

$$\underline{a}_{t|T} = \underline{a}_t + P^*_t(\underline{a}_{t+1|T} - G\underline{a}_t) \tag{4.9}$$

with

$$P_{t|T} = P_t + P^*_t(P_{t+1|T}-P_{t+1|t})(P^*_t)' \qquad (4.10)$$

where

$$P^*_t = P_tG'(P_{t+1|t})^{-1}. \qquad (4.11)$$

4.2 State Space Representation of the Two Stochastic Models.

A convenient state-space representation of the random walk model for the trading-day coefficients (3.3) and (3.4) along with the observation equation (3.1) is obtained through the following equivalences with the transition equation (4.2) and the measurement equation (4.1):

$$\underline{\alpha}_t = \underline{\delta}_t,\ \underline{z}'_t = (T_{1t},\ldots,T_{6t}),\ \epsilon_t = e_t, \qquad (4.12)$$

$$G = I_6,\ \underline{\eta}_t = \underline{\chi}_t \text{ and } Q = \sigma^2_\chi/\sigma^2 I_6.$$

For the quadratic model, k=2, the equivalences are given by:

$$\underline{\alpha}_t' = [\underline{\delta}_t',\underline{\delta}_{t-1}'],\ \underline{z}'_t = (T_{1t},\ldots,T_{6t},\underline{0}_6'),\ \epsilon_t = e_t,$$

$$G = \begin{bmatrix} 2I_6 & -I_6 \\ I_6 & 0 \end{bmatrix},\ \underline{\eta}_t = \begin{bmatrix} \underline{\chi}_t \\ \underline{0}_6 \end{bmatrix} \text{ and } Q = \begin{bmatrix} \sigma^2_\chi/\sigma^2 I_6 & 0 \\ 0 & 0 \end{bmatrix} \qquad (4.13)$$

It is clear from (4.12) and (4.13) that Q depends on the unknown parameters σ^2_χ/σ^2. The estimation of this parameter and σ^2 as well as the estimation of the initial conditions $\underline{\alpha}_0$ and P_0 are discussed in the next sections.

4.3 Estimation of the Initial Conditions.

For a stationary process, the theoretical mean value and covariance matrix of the state vector can be used to initiate the Kalman filter. However, for a nonstationary process the theoretical mean and covariance matrix cannot be defined. In order to obtain initial estimates of $\underline{\alpha}_0$ and P_0 we apply the Kalman filter over a time-reversed series. First we define a new series $w_t=y_{T-t+1}$. The w series is obtained by reversing the order of the y's series; that is, $w_1=y_T$, $w_2=y_{T-1}$ and so on until $w_T=y_1$. The Kalman filter is applied on this transformed series to predict $\underline{\alpha}_0$. In the random walk model (k=1) the estimate of $\underline{\alpha}_0$ is $\underline{\alpha}^\#_{T+1|T}$ and the estimate of P_0 is $P^\#_{T+1|T}$ where $\underline{\alpha}^\#_t$ and $P^\#_t$ are the estimates of the state vector and covariance matrix of the w series. In the quadratic model (k=2) the estimate of $\underline{\alpha}_0$ and P_0 are obtained by rearranging the sub-matrices of $\underline{\alpha}^\#_{T+2|T}$ and $P^\#_{T+2|T}$. In applying the Kalman filter to the w series, the initial estimate of the state vector $\underline{\alpha}^\#_0$ is taken to be equal to the zero vector, $P^\#_0$ equal to kI where k is a large constant and the remaining parameters of the w series are computed using the Fibonacci line search method described in the next section.

4.4 Estimation of the Parameters $\sigma^2\chi/\sigma^2$ and σ^2.

Maximum Likelihood Estimators (MLE) are used for the estimation of the parameters $\sigma^2\chi/\sigma^2$ and σ^2. Using the prediction error decomposition (Harvey 1981a), the likelihood function, L, can be written as follows:

$$-2\log L = T\log 2\pi + T\log\sigma^2 + \sum_{t=1}^{T} \log f_t + \sigma^{-2}\sum_{t=1}^{T} v^2{}_t/f_t \qquad (4.14)$$

where T is the number of observations and v_t and f_t are defined by (4.7) and (4.8).

Differentiation of (4.14) with respect to σ leads to $\hat{\sigma}^2$, the MLE of σ^2, given by:

$$\hat{\sigma}^2 = T^{-1}\sum_{t=1}^{T} v^2{}_t/f_t \qquad (4.15)$$

The scalar parameter, σ^2, may be concentrated out of the log-likelihood function leaving the concentrated log-likelihood function:

$$-2\log L_c = T\log 2\pi + T + T\log\hat{\sigma}^2 + \sum_{t=1}^{T} \log f_t \qquad (4.16)$$

Numerical optimization has to be carried out with respect to the remaining parameter $\sigma^2{}_\chi/\sigma^2$ to find its MLE. The Fibonacci line search method (Bazaraa and Shetly, 1979) is used to find the maximum between zero and one. This assumes that the noise in the signal $\underline{\alpha}_t$ is less than the noise in the measurements y_t. This estimate is further improved using the method of scoring (Harvey, 1981b). A brief description of the method of scoring follows.

Let $x = \sigma_\chi{}^2/\sigma^2$ be the unknown parameter to be estimated. The method of scoring is the iterative scheme:

$$x^{(i)} = x^{(i-1)} + I^{-1}(x^{(i-1)})\, D\log L(x^{(i-1)}) \qquad (4.17)$$

where $x^{(i)}$ is the estimate of x at the i-th iteration, I(x) is the Fisher information number evaluated at x, DlogL(x) is the derivative of the log-likelihood function evaluated at x and $x^{(0)}$ is the estimated parameter $\sigma_\chi{}^2/\sigma^2$ obtainded from the Fibonacci line search.

From (4.14) DlogL is:

$$\frac{d}{dx}\text{Log}(L) = \sum_{t=1}^{T} \frac{d}{dx}\log(p_t). \tag{4.18}$$

The density of the innovation process (4.7) is here symbolized by p_t to simplify the notation. The derivative of the log-likelihood of the t-th innovation is:

$$\frac{d}{dx}\log(p_t) = -.5f_t^{-1}\frac{d}{dx}f_t - \sigma^{-2}f_t^{-1}v_t\frac{d}{dx}v_t + .5\sigma^{-2}f_t^{-2}v_t^{2}\frac{d}{dx}f_t. \tag{4.19}$$

The derivatives of f_t and v_t are computed recursively from the Kalman filter equations (4.3) to (4.8) starting with the derivatives of $\underline{a}_0$ and P_0 equal to zero.

The Fisher information number is given by:

$$I(x) = E(\frac{d}{dx}\text{Log}(L))^2 \tag{4.20}$$

$$= \sum_{t=1}^{T} E\,(\frac{d}{dx}\log(p_t))^2. \tag{4.21}$$

Since the innovations v_t are independent and normally distributed with mean zero and variance $\sigma^2 f_t$ it follows that

$$E\,(\frac{d}{dx}\log(p_t))^2 = .5f_t^{-2}\,(\frac{d}{dx}f_t)^2 + \sigma^{-2}f_t^{-1}\,(\frac{d}{dx}v_t)^2 \tag{4.22}$$

from which (4.20) is easily derived.

The above discussion applies when the log-likelihood (4.14) is maximized. In our context, it is the concentrated log-likelihood (4.16) that has to be maximized. However it can be easily verify that:

$$\frac{d}{dx}\log(Lc) = \frac{d}{dx}\log(L(\hat{\sigma}^2)) \tag{4.23}$$

where the right hand side of (4.23) is the derivative of the log-likelihood evaluated at $\sigma^2 = \hat{\sigma}^2$.

An asymptotic t-statistic can be constructed for the parameter σ_χ^2/σ^2, as follows:

$$\hat{\sigma}_\chi^2/\sigma^2 \;/\; \text{Var}(\hat{\sigma}_\chi^2/\sigma^2)^{1/2}. \tag{4.24}$$

An estimate of the variance of the MLE of this paramater is provided by the inverse of the Fisher information number.

4.5 Outliers detection and accomodation.

In practice, the parameters are estimated from the data and the Kalman filter is used conditional on the estimated values of the parameters. In the application of the stochastic model to real series (section 5), we found that outliers in the data strongly influence the estimation of both the parameters and the state vectors. In both cases, whenever an observation is identified as an outlier, it is treated as if it was missing.

4.6 Tests Procedures for the Selection of the Model.

We use the asymptotic t-statistic (4.24) to distinguish between the random walk model and the deterministic model. A large value of the t-statistic leads to the rejection of the null hypothesis that the trading-day coefficients are deterministic. In the quadratic model (k=2), the null hypothesis is H0: $\delta_t = \delta_{t-1} + (\delta_{t-1} - \delta_{t-2})$ and the alternative is H1: $\delta_t = \delta_{t-1} + (\delta_{t-1} - \delta_{t-2}) + \chi_t$ where χ_t is purely random. In this case, the acceptance of H0 does not imply the acceptance of the random walk model since it is possible that $\delta_{t-1} - \delta_{t-2}$ is not significantly different from zero, which would imply to accept the deterministic model.

The degree of difference k adequate for each series can be selected using Akaike's criterion (AIC) as discussed in Kitagawa and Gersch (1984). The AIC for a particular k is:

$$AIC(k) = -2\log(\text{maximized likelihood}) + 2(6k+1). \qquad (4.25)$$

The value of k for which AIC(k) is smallest is the best model. Here the number 6k refers to the dimension of the initial state vector and 1 is for the parameter σ_χ^2/σ^2, both having to be estimated.

5. Application.

The deterministic and the two stochastic models for the estimation of the trading-day coefficients were applied on a large sample of real series affected by trading-day variations. The series were from the sectors of Retail Trade, Wholesale Trade, Imports and Exports.

All the series were first seasonally adjusted using the X-11-ARIMA method without ARIMA extrapolations and with the default options. Therefore, the input y series is the irregular series I obtained from Table B-13 and transformed by the following equation:

$$y_t = (I_t/100 - 1)N_t \qquad (5.1)$$

where N_t is the number of days in month t.

The estimation of the stochastic trading-day coefficients, in a typical application, is done iteratively as follows:

Step1: A first estimation of the parameter $\sigma_\chi{}^2/\sigma^2$ is obtained with the Fibonacci line search on the time-reversed w series given fixed initial conditions $\underline{a}^{\#}{}_0=\underline{0}$ and $P^{\#}{}_0=kI$ (k=21 in the applications given below).

Step2: The initial conditions $\underline{a}_0$ and P_0 are estimated from the time-reversed w series with the estimated parameter $\sigma_\chi{}^2/\sigma^2$ from step1.

Step3: A second estimation of the parameter $\sigma_\chi{}^2/\sigma^2$ with the Fibonacci line search is done on the original series with the initial conditions from step2.

Step4: A third and final estimation of the parameter $\sigma_\chi{}^2/\sigma^2$ is done with the method of scoring. If the procedure leads to a negative estimate, the initial estimate from step3 is used.

Step5: The trading-day coefficients are estimated from the original series with the Kalman filter and the fixed interval smoother using the parameter $\sigma_\chi{}^2/\sigma^2$ from step4 and the initial conditions from step2.

To illustrate we use two exemples with real data. The first example is the series (I) of Total Retail Trade Sales for All Stores in Nova-Scotia from January 1977 to December 1986 and the second example is the series (II) of Total Retail Trade Sales for Department Stores in Canada from January 1977 to December 1986. The input data as obtained from equation 5.1 for both series are given in Tables 1A and 1B.

Table 2 gives a summary of the estimation method for the signal to noise ratio $\sigma_\chi{}^2/\sigma^2$. For both series the AIC(k) is smallest with k=1 suggesting that the random walk model is preferred over the quadratic model. The results of the t-test further suggest that the random walk model is appropriate for the series (I) whereas the deterministic model is more appropriate for the series (II).

Graphs of the daily coefficients for both series are provided in figures 1A and 1B. Clearly the stochastic model allows for a moving behaviour of the daily coefficients. For the series (I), the graphs of the coefficients under the stochastic models have similar paths but the quadratic model (k=2) exagerates the amplitude. For the series (II), the graphs of the coefficients under the random walk model are almost straight lines. This agree with the result from the t-test suggesting that the deterministic model is appropriate for this series.

Next a frequency domain analysis was conducted on both the original series and the trading-day adjusted series which is defined as the original series minus the estimated trading day component. Estimates of the trading-day component from five procedures are compared, namely , the deterministic (Det), the random walk without smoothing (Ka(k=1)), the random walk with smoothing (Sm(k=1)), the quadratic model without smoothing (Ka(k=2)) and the quadratic model with smoothing (Sm(k=2)). Tables 3A and 3B provides the estimates of the spectral densities using the Parzen window for a band of frequencies between .3278 and .3722. The spectral densities of the input series are charactarized by a peak at the frequencies around .35. This is typical of series affected by trading-day variations as shown by Cleveland and Devlin (1980). For both series, the largest reduction in power is given by the random walk model with smoothing. However in series (II) the difference beteween the deterministic and random walk model is very small, in accordance with the result obtained from the t-test.

6. Conclusions.

We have introduced two stochastic models for trading-day coefficients and compared them with the deterministic model developed by Young (1965). The stochastic models are given in a state-space form and the parameters are estimated by the Kalman filter and fixed interval smoother. We have provided a solution to the problem of estimating the parameters and initial conditions of the Kalman filter and applied tests statistics to distinguish between alternative models. Two real case studies are thoroughly discussed to illustrate when the deterministic and the stochastic models are adequate.

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TABLE 1A

Total Retail Trade Sales - All Stores - Nova Scotia (I)
Transformed Irregular Series[1]

Year	Jan	Fev	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
77	161	-221	-300	574	-641	-363	480	-583	447	-223	342	693
78	-883	-34	243	-559	-11	558	-182	-99	585	-503	-155	-36
79	272	-428	935	-1113	171	1062	-921	1344	-1287	-230	1136	-954
80	44	712	-756	-136	1212	-903	-62	488	-275	696	-333	-1167
81	720	499	-657	695	409	-651	233	11	-817	514	-239	490
82	84	-443	-230	1251	-1097	0	682	-732	293	-168	-177	874
83	-563	-796	1613	-640	-1307	1451	73	-513	639	-739	-631	958
84	-865	638	407	-552	330	795	-1350	383	-206	-337	910	-928
85	693	-288	285	186	128	-475	-139	1008	-865	30	530	-1289
86	1514	161	-1202	705	608	-438	-184	-788	393	777	-640	-194

(1) Entries have to be multiplied by 10^{-3}

TABLE 1B

Total Retail Trade Sales - Department Stores -Canada (II)
Transformed Irregular Series[1]

Year	Jan	Fev	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
77	-225	270	-243	871	-721	-765	234	288	490	-562	309	-173
78	-338	-65	810	-315	81	-31	-827	-202	1473	-652	78	217
79	-396	-731	1833	-1233	168	119	-655	1230	-598	-12	102	-585
80	962	470	-780	-440	1181	-1680	938	-8	170	248	-600	-565
81	1955	-437	-1076	352	-358	611	-149	-273	-317	1014	-362	298
82	-315	105	-569	603	-273	-642	623	-270	421	-370	-157	481
83	-240	-24	1476	-1735	-1455	1630	104	8	215	-353	-649	886
84	-719	897	63	-941	-27	1143	-1006	-222	293	-213	830	-514
85	118	-504	537	257	528	-1401	-090	1451	-995	-116	756	-676
86	738	-603	-87	492	1390	-1917	453	100	-191	436	-405	214

(1) Entries have to be multiplied by 10^{-3}

TABLE 2

Summary of the estimation procedure for $\sigma_\chi{}^2/\sigma^2$.

Step(1)	(I) k=1	(I) k=2	(II) k=1	(II) k=2
Step1	.0153	.00031	.00031	.00031
Step3	.0103	.000046	.00031	.000046
Step4	.0103	.000026	NA(2)	.000043
AIK(3)	-211	-170	-179	-132
t-test(4)	2.18	1.79	.049	1.77

(1) Refer to section 5 for definition

(2) Non-Available: Method of scoring leads to a negative estimate

(3) Ref.: equation (4.25)

(4) Ref.: equation (4.24)

Table 3A
Total Retail Trade Sales - All Stores - Nova Scotia (I)
Spectral densities for the original and trading-day adjusted series for selected frequencies.

freq	Y	Det	Ka(k=1)	Sm(k=1)	Ka(k=2)	Sm(k=2)
.3278	1.1695	9.1319	6.4164	5.7778	7.3873	7.0939
.3333	0.6797	3.6927	2.9862	2.3540	4.3460	6.0100
.3389	5.3868	1.5177	0.9303	1.2743	1.4574	6.1753
.3444	23.8114	2.0735	2.0121	1.6289	0.7589	4.1351
.3500	37.6602	6.9724	5.2330	3.5292	3.0434	4.2020
.3556	22.5998	9.6120	7.1576	5.2519	6.4855	4.7701
.3611	5.5827	4.8960	5.2180	3.9993	5.8694	2.7787
.3667	1.4694	1.5090	1.9696	1.7508	2.2158	1.1023
.3722	1.5816	2.6025	1.2286	1.9509	0.5728	1.6986
sum:	99.9411	42.0077	33.1518	27.5171	32.1365	37.9660

Y: original, Det: Trading-day adjusted by the deterministic model
Ka(k=1): Trading-day adjusted with the coefficients from the random walk model without smoothing
Sm(k=1): Same as Ka(k=1) but with smoothing
Ka(k=2): Same as Ka(k=1) but with the quadratic model(k=2)
Sm(k=2): Same as Sm(k=1) but with k=2

Table 3B
Total Retail Trade Sales - Department Stores - Canada (II)
Spectral densities for the original and trading-day adjusted series for selected frequencies.

freq	Y	Det	Ka(k=1)	Sm(k=1)	Ka(k=2)	Sm(k=2)
.3278	0.6511	2.0879	1.7716	1.8740	2.1924	2.9300
.3333	0.7347	1.1348	0.8986	1.0175	1.6366	3.8936
.3389	4.5313	1.4962	1.0203	1.3066	3.0590	6.3690
.3444	15.7876	1.9041	1.4534	1.5555	3.5352	5.4069
.3500	21.1031	1.4258	1.5010	1.2729	2.0801	3.0818
.3556	11.6191	1.3611	1.5608	1.5225	1.6078	3.7332
.3611	5.1573	1.7703	1.7976	1.9902	2.2078	3.6070
.3667	3.8195	3.2686	3.1939	3.3355	2.5901	2.3057
.3722	3.0257	5.1427	4.7810	4.9885	2.3659	2.2241
sum:	66.4294	19.5915	18.9782	17.8632	21.2749	33.5513

Y: original, Det: Trading-day adjusted by the deterministic model
Ka(k=1): Trading-day adjusted with the coefficients from the random walk model without smoothing
Sm(k=1): Same as Ka(k=1) but with smoothing
Ka(k=2): Same as Ka(k=1) but with the quadratic model(k=2)
Sm(k=2): Same as Sm(k=1) but with k=2

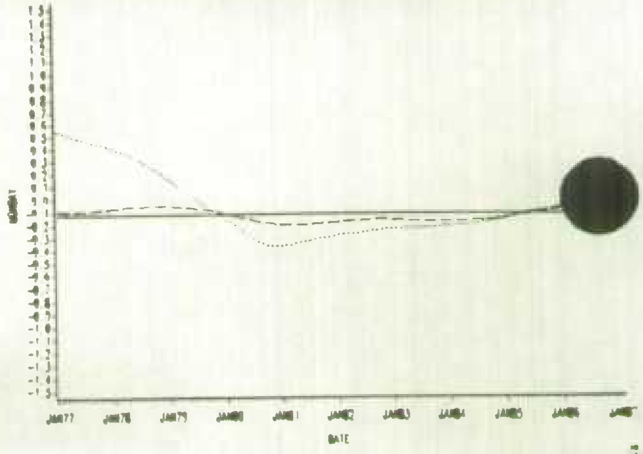
FIGURES 1A

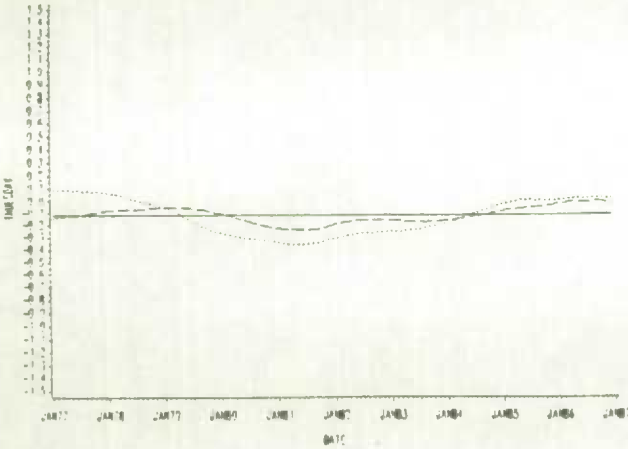
TOTAL RETAIL TRADE SALES ALL STORES – NOVA SCOTIA

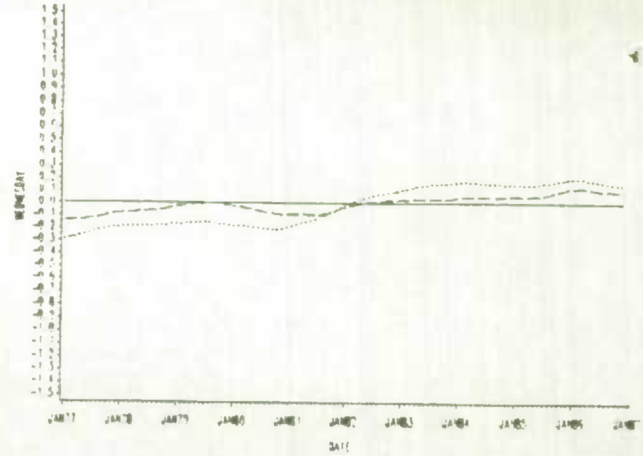
LABEL ——— DETERMINISTIC

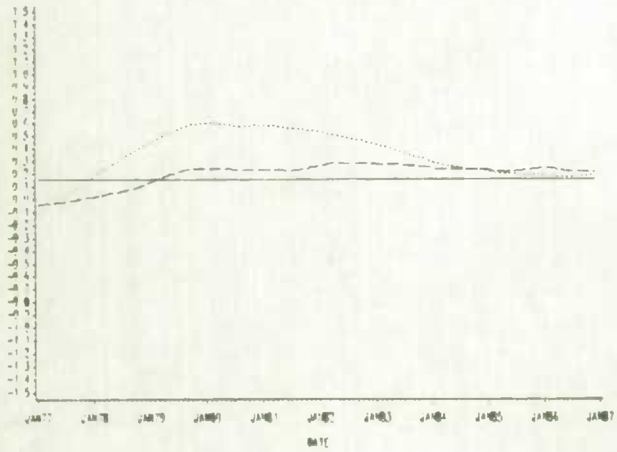
— — RANDOM WALK

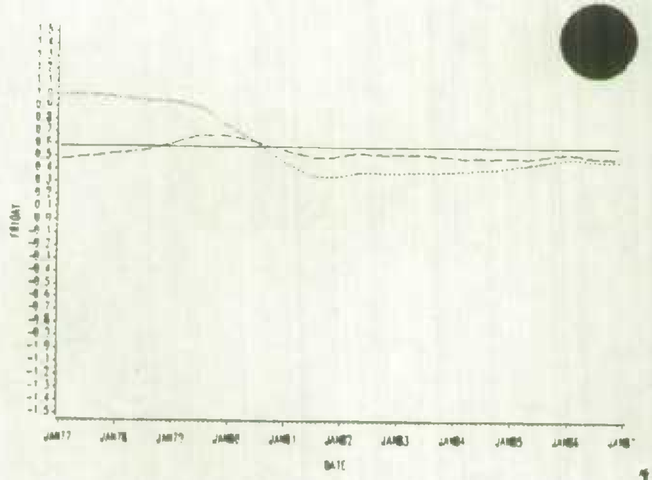
.... QUADRATIC

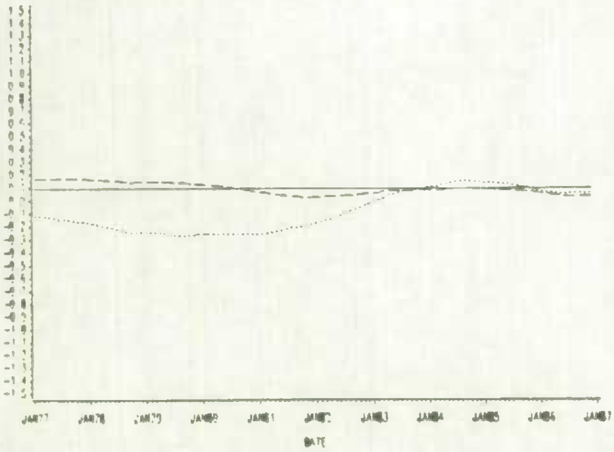


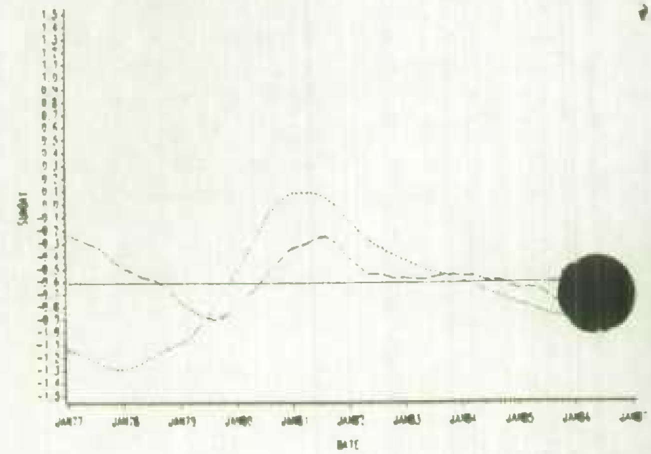












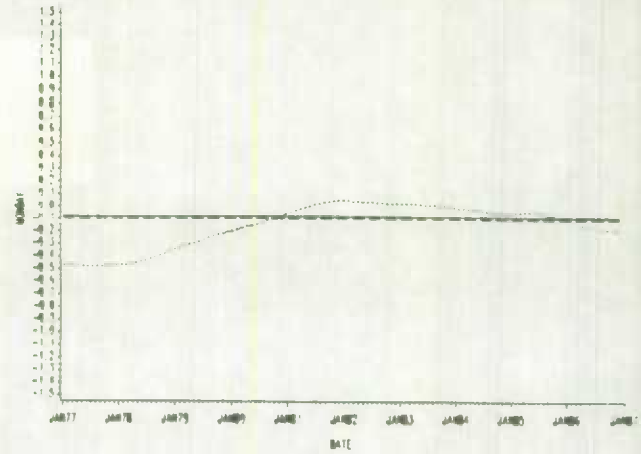
FIGURES 1B

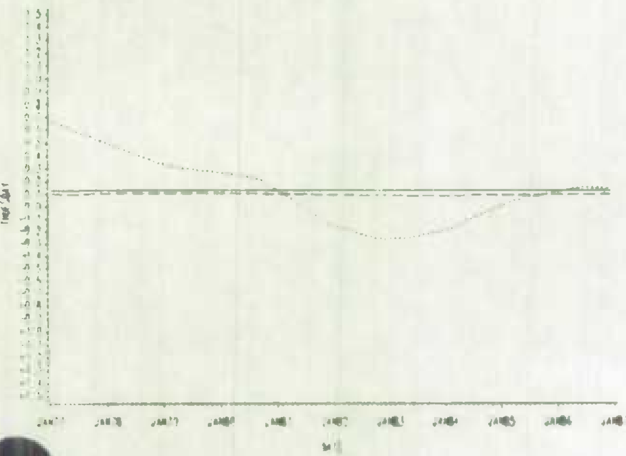
EAL RETAIL TRADE SALES - DEPARTMENT STORES - CANADA

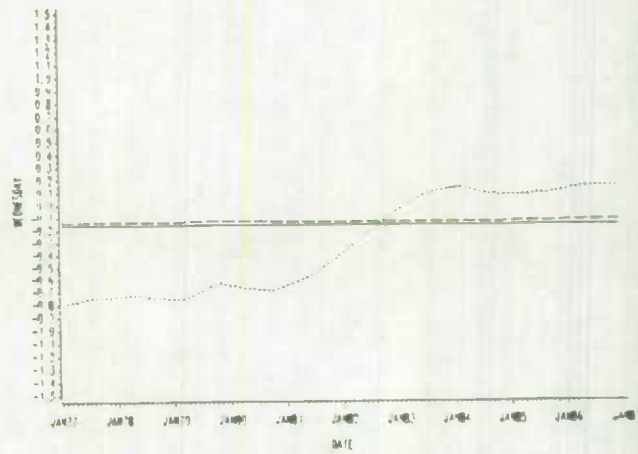
LABEL — DETERMINISTIC

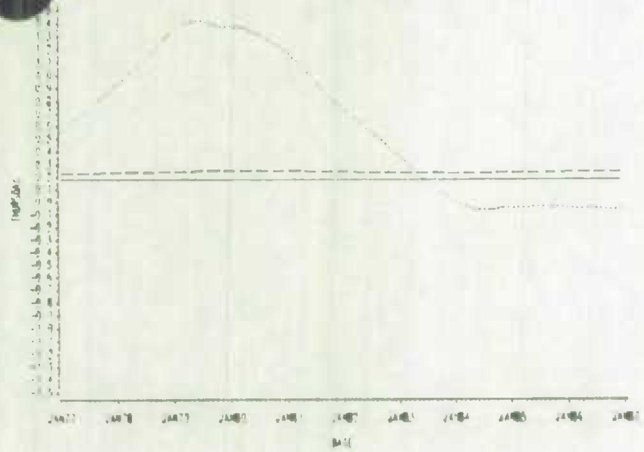
— — RANDOM WALK

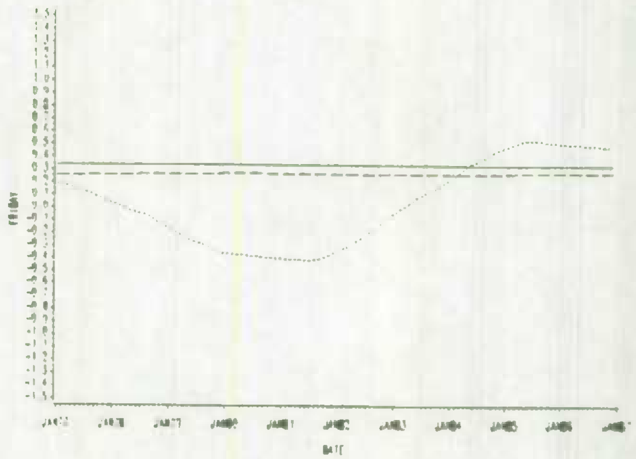
QUADRATIC

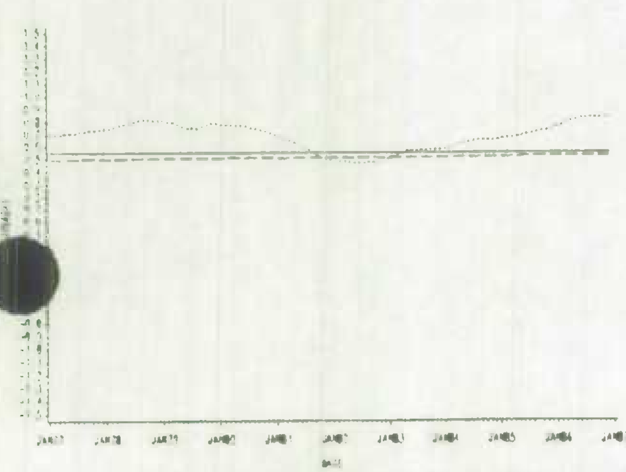


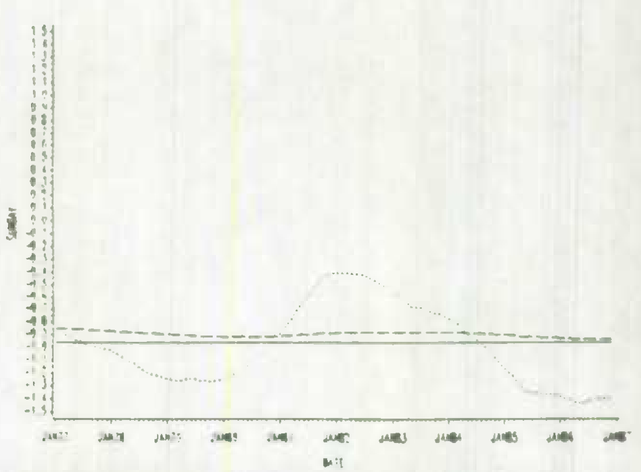












c. 2