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A Note on the Consistency of Seasonally Adjusted Systems of Series with Particular Reference to the Canadian Employment Series

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Remarques sur la cohérence de systèmes de séries désaisonnalisées

- résumé -

Pour un mois considéré, le chiffre non désaisonnalisée de l'emploi au niveau canadien est identiquement à égal à la somme des chiffres mensuels provinciaux correspondants; ainsi qu'à la sommes des chiffres des quatre groupes age-sexe correspondants (hommes 15-24 ans, femmes 15-24 ans, hommes 25 ans et plus, et femmes 25 ans et plus). Après désaisonnalisation, cette cohérence du système de série ne se vérifie généralement plus: le total canadien obtenu de la somme des chiffres provinciaux diffère légèrement de celui obtenu de la somme des groupes age-sexe. Le document signale certaines strategies pour restaurer la cohérence de système de séries après désaisonnalisation ou après certains autres traitement statistiques, tels la prévision et l'étalonnage. Ces stratégies se basent sur l'ajustement par moindres carrés généralisés.

A Note on the Consistency of Seasonally Adjusted Systems of Series

- abstract -

For a given month the seasonally un-adjusted figure for employment at the Canada level is identically equal to the sum of the corresponding provincial figures; and identically equal to the sum of the corresponding figures of the four age-sex groups (Males 15-24 years, Males 25 years and over, Females 15-24 years, Females 25 years and over). After seasonal adjustment, the consistency of the system non longer prevails: the total obtained by summing over the provincial figures generally differs slightly from that obtained by summing over the age-sex groups. This documents points out strategies to restore the consistency of a system of series, after seasonal adjustment or after any other statistical processing such as forecasting and benchmarking. Those strategies are based on generalized least square adjustment.

1. Introduction

The seasonally un-adjusted Canadian employment series are consistent over the attributes of the labour force (e.g. age-sex groups) and over time. Indeed, taking the sum of the employment figures over the age-sex groups yields the national total value for each month; and taking the annual sums of any of the series in the system yields the yearly values of the series. This consistency over attributes and time no longer prevails after seasonal adjustment (or any other statistical treatment, e.g. benchmarking, forecasting). This note outlines a strategy by which consistency could be restored.

2. Consistency over time

In the case of seasonal adjustment, the mechanics are already in place to insure consistency over time. Indeed, the benchmarking option of the X-11-ARIMA programme adjusts the seasonally adjusted series to the yearly totals of the un-adjusted series.

3. Consistency over attributes

The indirect adjustment option of the X-11-ARIMA programme also allows consistency for systems of series which are one-way classified. This would be the case of the labour force series, if they were classified only by age-sex groups. (*This is a one-way classification in the sense that the individual age-sex groups are considered as individual components of the labour force.*) The indirect option of the programme then defines the national seasonally adjusted series as being the sum of the seasonally adjusted age-sex groups (and the age group totals and the sex group totals are then the sum of the appropriate seasonally adjusted component).

However the employment series are also classified by job-duration groups, full-time males, full-time females, part-time males and part-time females, which are also seasonally adjusted at that level. The system is furthermore classified by provinces (all groups and durations), which are also seasonally adjusted. As a result, for a given month, one obtains three different national totals depending on whether the sum is carried out over provinces, over job-duration groups or over the age-sex groups. Although the latter provides the official national total, this situation can be misleading to the public and embarrassing to Statistics Canada, especially now-a-days when the job-duration is the focus of much public attention.

4. Raking (Iterative Proportional Fitting)

In order to restore consistency in such circumstances, one procedure commonly applied is "raking". Assuming that the age-sex classification is the soundest and continues to provide the official national total, raking could impose the chosen national employment total onto the provincial series. A separate round of raking could impose the male and the female national (indirect) totals onto the job-duration series. If carried out two-dimensionally, raking could also preserve the yearly totals of each series.

The raking algorithm is so simple that the cost would be very minor. Furthermore raking is available in SAS as a function within the IML procedure. This computer package is by far the most

familiar to the largest number of employees in Statistics Canada and is likely to be more the case in the future.

5. Generalized Least Square Adjustment

A more general way to restore consistency to a system of series is to apply Generalized Least Squares adjustment. This method would allow direct seasonal adjustment of all series in a system (if this is optimal). Consistency can be restored by "imposing" the direct total onto the components, or by imposing the components onto the total (current practice). One could also compromise between those two extremes: If for a month the discrepancy between the total and the sum of one-classified components is +0.50% for instance, then each component could be raised by 0.25% and the total lowered by 0.25%. That sort of compromise would seem to me more logical - or at least more flexible - than either of the two extremes. If done two-dimensionally, G.L.S. could preserve the yearly totals.

In the case of the employment series, one could first adjust (with G.L.S) the direct national seasonally adjusted total against the directly seasonally adjusted age-sex series. The national series thus obtained could then be imposed onto the provincial series (either by G.L.S. or by raking). Similarly, the sex group totals could be imposed onto the job duration series. Note that the direct seasonal adjustment of the national total is likely to provide more reliable seasonally adjusted figures that the current indirect practice.

G.L.S. adjustment minimizes an objective function which is easy to interpret: G.L.S. typically minimizes the sum of squared percentage corrections to each of the series in the system. Each term in the minimization may have an alterability coefficient (Federal Reserve 1962), which determines by how much a series is adjusted. A coefficient of zero insures that the series considered - whether total or component - will not be altered. Coefficients equal for all series causes all the series to be equally modified, like in the 0.25% example given above.

6. Relation between Raking and G.L.S. Adjustment

Raking (Iterative Proportional Fitting) is a particular case of G.L.S. adjustment if the latter is performed under the following conditions

1) the objective function specifies percentage corrections,
2) the classification is one-way,
3) the total is imposed onto the components (with an alterability coefficient of zero), and
4) each component has the same alterability coefficient.

If for instance the discrepancy between the total and the sum of the component is +0.5%, then each component is raised by 0.5% under both methods. Raking also has an implicit objective function. However it is not easy to interpret and is not as flexible as in G.L.S. (no alterability coefficients). Furthermore raking is not guaranteed to converge in two-ways classification. Finally raking was developed as an attempt to approximate to G.L.S. (Bankier, 1986).

7. Computational Requirements of G.L.S.

The computational burden of G.L.S. is heavier than that of raking. It requires the inversion of matrices of dimensions equal to the number of constraints (minus a few since some are redundant). For example in a two-way classification, with one total and 10 components with yearly

totals to be preserved, there would be 11 constraints on yearly totals and 11 aggregation constraints for each month of the year (the 12th being redundant). The inversion would be of dimension 22 by 22. In a true one-way situation, the exact algebraic solution to G.L.S. is:

$$z_j = x_j + \delta_j x_j a_j^2 \left[\left(x_1 - \sum_{k=2}^{J} x_k\right) \Big/ \left(\sum_{m=1}^{J} x_m a_m^2\right)\right], \quad j=1,\ldots,J, \qquad (1)$$

where x and z respectively stand for the J original and adjusted components of the system (for a given month), where j=1 for the total and δ_j=-1 for j=1 and 1 otherwise, and where $a_j \geq 0$ are the alterability coefficients. The ratio of the corrections z_j/x_j to the *square* of the alterability coefficients a_j is constant over components.

Two-way (or three-way) G.L.S. adjustment can be accomplished in SAS, although SAS does not supply a specific function. One has to use the matrix algebra procedure of SAS, IML, which I have done for the redesigned Retail Trade series (Cholette, 1989). Two-way G.L.S. could also be accomplished by iterative applications of one-way G.L.S. (using (1)), over attributes, then over time, then over attributes, and so on, like raking, until convergence. This scheme would allow easy processing on micro computers.

8. Conclusion

It has now become very feasible to produce systems of seasonally adjusted series which are consistent both over attributes and over time. Furthermore it is no longer necessary to resort to indirect seasonal adjustment to insure consistency over attributes (only), which in some situations is bound to be sub-optimal. This conclusion is also applicable to forecasting (e.g. demographic inter-census projections), benchmarking and to any statistical procedures applied to time series.

REFERENCES

Bankier, M. (1989), "Generalized Least Squares Estimation Under Poststratification", **Statistics Canada**, Social Survey Methods Division, Working Paper.

Cholette, P.A. (1989) "Benchmarking and Linking the Redesigned Wholesale and Retail Trade Series: preliminary report", **Statistics Canada**, Time Series Research and Analysis Division, internal document.

Copeland, K.R, Peitzmeier, F.K. and Hoy, C.E. (1989), "An Alternative Method of controlling current Pupulation Survey Estimates to Population Counts", U.S. Bureau of Labor Statistics, **Proceeding of the American Statistical Association**, pp. 332-339.

Federal Reserve Bulletin (1962), "Flow of Funds Seasonally Adjusted", **Federal Reserve**, pp.1393-1407.