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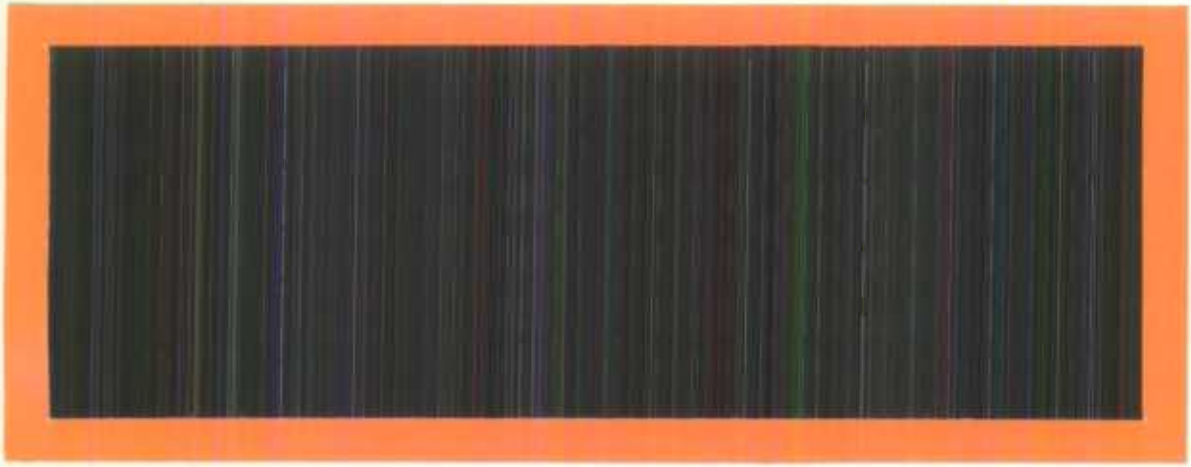

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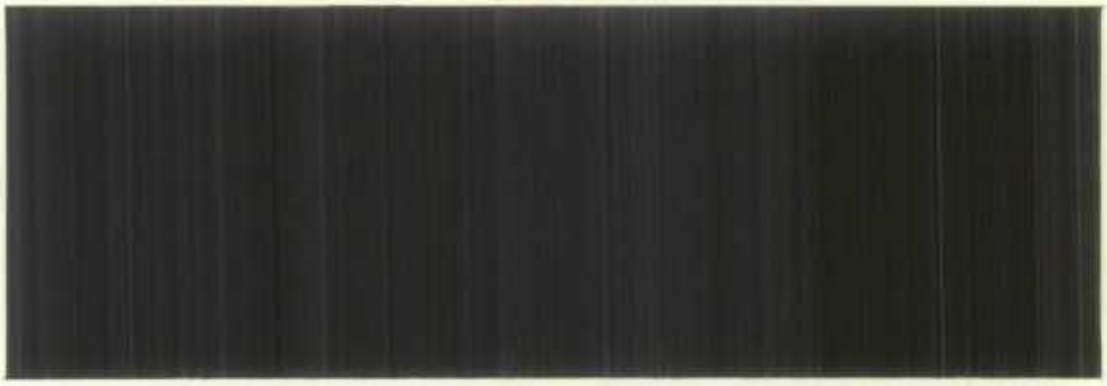

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A STUDY ON THE AUTOCORRELATION OF RESIDUALS FROM THE X11ARIMA METHOD

by

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ABSTRACT

The problem of autocorrelation in the residuals of X11ARIMA has raised many controversial discussions. This paper shows that the presence of significant autocorrelated values, particularly, at lag one and at the lag associated with the seasonal variations is mainly due to the blind use of the standard option. The autocorrelation disappears when the appropriate options available in X11ARIMA are used.

RÉSUMÉ

Le problème d'autocorrélation des résidus du X11ARIMA a généré plusieurs contreverses. Cet article démontre que la présence des valeurs autocorrélés significatives est due à une utilisation erronée de la version standard. L'autocorrélation disparaît chaque fois que les options disponibles dans le X11ARIMA sont appliquées adéquatement.

INTRODUCTION

The presence of autocorrelation in the residuals from time series decomposition methods has preoccupied researchers and practitioners for a long time and raised controversial discussions. Among several authors, Nerlove (1964), Granger (1978) and Pierce (1978) pointed out the presence of negative autocorrelated values of the residuals at the seasonal lags or equivalently, dips in the seasonal frequency bands of the seasonally adjusted series using either the BLS or the U.S. Bureau of Census X11 Method. These authors then concluded that the two methods overadjust for seasonality. On the other hand, commenting on Granger's paper, Sims (1978) agreed that it is reasonable to have dips given the low signal-to-noise ratio in the neighbourhood of the seasonal frequencies. Similarly, Tukey (1978)and Wecker (1978) sustained that the presence of dips in the spectrum does not indicate inadequacy of the method. Unless the white noise component has zero power at the seasonal frequencies the spectrum of the seasonally adjusted series will have dips at the seasonal frequencies. In fact, almost a decade earlier, Grether and Nerlove (1970) showed that all minimum mean squared error (MMSE) signal extraction estimators will produce dips at the seasonal frequencies.

In a comparative study of various seasonal adjustment methods for labor series, Bordignon (1988) found significant autocorrelated values of the residuals at lag 1 and 4 (for quarterly series) when

using the X11ARIMA (Dagum, 1980), BAYSEA (Akaike and Ishiguro, 1980) and to lesser extent, SIGEX (Burman, (1980). Similarly, Daddi and D'Esposito (1985) in a comparison between X11ARIMA and S.A.B.L. (Cleveland et als, 1978) pointed out that both procedures left significant autocorrelated residuals at the trend and seasonal lags. Bell and Hillmer (1984) also found dips in the spectrum implied by the linear filters of X11 when applied to seasonally adjusted Employed Non-Agriculture-Males aged 20 and over series. On commenting Bell and Hillmer (1984), Ansley and Wecker (1984) showed how it is possible to construct a MMSE seasonal adjustment method that will not have dips in the spectrum but it will no longer be optimal.

This investigation aims to analyse the autocorrelation of the residuals from the X11ARIMA/88 version when the standard option is applied; and to show how, for most cases, the presence of autocorrelation is not a problem of the method but of application of the wrong options.

Section 2 introduces a test for autocorrelation of the residuals at each lag and gives the results obtained for a sample of monthly and quarterly series. Section 3 shows how significant autocorrelation at lag 1 can be corrected; section 4 shows how significant autocorrelation at the seasonal lag can be corrected; section 5 discusses the effect of trading-day variations in the autocorrelation of the residuals. Finally, section 6 concludes this study.

2. TEST FOR THE AUTOCORRELATION OF THE RESIDUALS

The X11ARIMA/88 computer package prints the sample autocorrelations of the residuals in Table F2.G from lag 1 up to and including lag 14, for monthly data; and from 1 up to 6, for quarterly data. The estimate of the kth lag autocorrelation ρ_k is given by

$$(2.1) \quad r_k = 1/N \sum_{t=1}^{N-1} (Y_t-\bar{Y})(Y_{t-k}-\bar{Y})/\sigma^2_y; \; k=0,1,\ldots,k$$

where Y_t, $\bar{Y}$ and σ^2_y denote the observed series, its mean and variance, respectively; and N is the series length.

To test whether the sample autocorrelated values r_k are significant we use Bartlett's approximations (Box and Jenkins, 1970 p. 34-35) according to which, the variance of the estimated autocorrelation r_k, at lags k greater than some value q beyond which the theoretical autocorrelation function may be deemed to have "died out" is given by,

$$(2.2) \quad \text{var } r_k \simeq 1/N(1+2 \sum_{v=1}^{q} \rho_v), \; k>q$$

On the assumption that a series is completely random, we have that q=o. Then, for all lags (2.2) gives

$$(2.3) \quad \text{var } r_k \simeq 1/N$$

If only ρ_1 is expected to be significant, then q=1 and

$$(2.4) \quad \text{var } r_k \simeq 1/N \; (1+2\rho_1)$$

and so on. In practical applications ρ_k is replaced by its estimated value r_k.

We used Bartlett's approximations to test for the presence of significant autocorrelation of the residuals on a sample of 50 monthly and 20 quarterly series for the period 1979-88. The level of significance was fixed at 5% for each lag k, from 1 to 14 for the monthly series; and from 1 to 6 for the quarterly series. All the series were seasonally adjusted using the standard option of the X11ARIMA/88 version, that is, multiplicative decomposition (unless zeroes or negative values are present in which case it applies the additive model), variable selection of the Henderson trend-cycle filters, and variable selection of the seasonal filters. We also removed trading-day variations if significant. The results are shown in Table 1 for those series where at least one significant autocorrelated value was found. For the monthly series, significant autocorrelation is mainly found at lags 1 and 12 for 20 out of the 50 series tested. For the quarterly series significant autocorrelation is shown mainly at lag 1 for 14 out of the 20 series tested. In all cases the values are negative, an indication of overadjustment.

Table 1 - AUTOCORRELATED VALUES r_k OF RESIDUALS FROM X11ARIMA STANDARD OPTION

Monthly Series	r_k
Retail Trade -	
Canada All Stores - Chain (D65000)	r^*_1=-.20 r^*_2=-.26 r^*_{13}=.37
Combination Stores (D650001)	r^*_1=-.21 r^*_{13}=.21
Grocery-Confect. Sundries (D650002)	r^*_1=-.23 r^*_2=-.26 r^*_3=.20 r^*_{12}=-.30
General Merchandise Stores - Chain (D650005)	r^*_1=-.20 r^*_4=-.20
All Stores - Canada (D650058)	r^*_1=-.31 r^*_{13}=.23
All Dept. Stores (D650062)	r^*_1=-.20 r^*_2=-.28
Service Station All Stores (D650068)	r^*_2=-.28 r^*_5=-.39 r^*_7=.24 r^*_{10}=-.25
Florist All Stores (D650082)	r^*_1=-.35 r^*_{11}=-.23 r^*_{12}=-29
All Stores Nova Scotia (D650350)	r^*_1=-.24
All Stores Ontario (D650702)	r^*_{10}=.22
All Stores Alberta (D651142)	r^*_1=-.23 r^*_8=-.20 r^*_{12}=-.21
Imports B of P - Fresh Vegetables (0397775)	r^*_2=-.26 r^*_{10}=.20
Non-Metal Minerals (D397809)	r^*_6=.20 r^*_{11}=-.28 r^*_{12}=-.20
Agriculture Machine Tractors (D397817)	r^*_3=-.21
Furniture (D397831)	r^*_2=-.20 r^*_{11}=.29

Table 1 - Cont'd.

Employment - Total Men 25 years and over (D767386)	r^*_1=-.25
Unemployment - Women 25 years and over (D767519)	r^*_2=-.30 r^*_3=-.28 r^*_4=-.20 r^*_{12}=-.26
Employment Total PEI (767976)	r^*_{12}=-25
Unemployment Total PEI (D767977)	r^*_8-.28 $r3^*_{12}$=-.25 r^*_3=-.20 r^*_4=-.31 r^*_5=-.20
Employed Men Quebec (D768421)	r^*_{12}=-.25
Unemployed Women - B.C. (D769184)	r^*_{12}=-.25 r^*_{14}=.26

Quarterly Series	r_k
Wages and Labour Income (D10002)	r^*_4=-.44
Company Profits (D10003)	r^*_1=-.43
Farm Net Income (D10005)	r^*_1=-.33
Non-Farm Net Income (D10006)	r^*_1=-.39
Indirect Taxes (D10008)	r^*_1=-.45
Capital Cons. Allowance (D10009)	r^*_1=-.50
Personal Expenditures (D10012)	r^*_1=-.57
Government Expenditures (D10013)	r^*_1=-.51
Business Fixed Capital (D10036)	r^*_1=-.56
Imports - Foods - Alcohol (D397844)	r^*_1=-.53
Imports B of P - Transportation (D397900)	r^*_1=-.39

Table 1 - concluded

Imports - B of P. End Products D397907	r^*_1=-.57
Imports - B. of P Printed Matter D397904)	r^*_1=-.55
Corporate Claim (D151986)	r^*_1=-.70 r_4=-.44

* Values significant at 5%.

3. CORRECTING FOR SIGNIFICANT AUTOCORRELATION AT LAG ONE

For all the series of table 1 where r_1 is significantly different from zero, the estimated value is negative; an indication of overadjustment due to the length of the automatically chosen Henderson Trend-cycle filter. The steps involved in the selection of the trend-cycle filter by the X11ARIMA/88 program are as follows:

1. As a preliminary estimate of the trend-cycle C, a 13-term Henderson moving average of the seasonally adjusted series is computed.

2. As a preliminary estimate of the irregulars I, the 13 term moving average is divided (subtracted) into (from) the seasonally adjusted series.

3. The average month-to-month percent change (difference) without regard to sign of the preliminary C and I is calculated to obtain

an I/C ratio.

4. If $0 \leq I/C \leq .99$, the program selects a 9-term Henderson

If $1 \leq I/C \leq 3.49$, the program selects 13-term Henderson

if $I/C \geq 3.50$, the program selects 23 term Henderson

For quarterly series only the 5-term and 7-term Henderson filters are available and applied as follows:

if $0 \leq I/C \leq 3.49$, the program selects a 5-term Henderson

if $I/C \geq 3.50$, the program selects a 7-term Henderson

We applied a longer Henderson trend-cycle filter to all those series where r_1 was significant and negative. As shown in Table 2, increasing the length of the Henderson filter eliminates the significant autocorrelation at lag 1 and sometimes at other lags, for all the monthly series except one. For the quarterly series, only 5 out of the 14 series still have significant autocorrelation at a 5% level of significance.

Although not shown here, we also looked at the quality of the seasonally adjusted series as measured by the Q statistics and the values of the F tests for stable and moving seasonality. In most cases, the use of a longer trend-cycle filter decreased both the F values for stable and moving seasonality. The Q statistic changed very little and did not show any systematic increase or decrease.

Table 2. AUTOCORRELATION OF THE RESIDUALS USING HENDERSON'S FILTERS LONGER THAN THOSE SELECTED BY THE STANDARD OPTION

Monthly series	r_k	Henderson Trend-cycle Filter
Retail Trade -		
All Stores - Chain Canada (D650000)	r_1=-.10; r_2=-.18; r^*_{13}=.30	H[23]
Combination Stores (D650001)	r_1=-.11	H[23]
Grocery-Confectionery Sundries (D650002)	r_1=-.04; r_{12}=-.14	H[23]
General Stores (650005)	r_1=-.13; r_4=-.12	H[23]
All Stores-Canada (D650058)	r_1=.03 r_{13}=.12	H[23]
All Dept. Stores (D650062)	r_1=-.11 r^*_2=-.21	H[13]
Florists All Stores (650082)	r^*_1=-.24; r^*_{11}=-.24 r^*_{12}=-.30	H[23]
All Stores-Nova Scotia (D650350)	r_1=.01	H[23]
All Stores - Alberta (D651142)	r_1=-0.11; r_8=-.05; r_{12}=-.16	H[23]
Employment Total - Men 25 years old & over (D767386)	r_1=-.04; r_6=-.16	H[13]

Table 2. - cont'd.

Quarterly series	r_k	Henderson Trend cycle filter
Company Profits (D10003)	r_1=-.32	H[7]
Farm Net Income D10005)	r_1=-.22	H[7]
Non-Farm Net Income (D10006)	r_1=-.21	H[7]
Indirect Taxes (D10008)	r_1=-.28	H[7]
Capital Consumer Allowance (D10009)	r^*_1=-.47	H[7]
Personal Expenditures (D10012)	r^*_1=-.42	H[7]
Government Expenditures (D10013)	r^*_1=-.40	H[7]
Business Fixed Capital (D10036)	r_1=-.28	H[7]
Imports B of P - Transportation (D397900)	r_1=-.23	H[7]
Imports B of P - Printed Matter (D397904)	r_1=-.31	H[7]
Imports B of P - End Products (D397907)	r_1=-.27	H[7]
Imports - Food & Alcohol (D397844)	r^*_1=-.41	H[7]
Corporate Claims (D151986)	r^*_1=-.43; r^*_4=-.44	H[7]

4. CORRECTING FOR SIGNIFICANT AUTOCORRELATION AT SEASONAL LAGS

The presence of negative autocorrelation at lag 12 and 4 for monthly and quarterly series, respectively, can be interpreted as an overadjustment. These negative values may be caused either by:

(1) the application of too short moving averages; or

(2) the use of the wrong decomposition model.

To estimate the seasonal factors, the X11ARIMA/88 version uses a variable seasonal moving average routine in the last iteration. The steps involved are:

1. An average I/S ratio is calculated using complete years for the entire series, that is, up to and including year N where N must be greater than five. The selection of the seasonal filter is based on the value of the I/S_N ratio as follows:

(a) if $I/S_N \leq 2.5$ select a 3x3 m.a.

if $3.5 \leq I/S_N < 5.5$ select a 3x5 m.a.

if $I/S_N \geq 6.5$ select 3x9 m.a.

(b) if $2.5 < I/S_N < 3.5$ or

if $5.5 < I/S_N < 6.5$

redo (a) using the I/S_{N-1} ratio.

If none of the conditions in (a) are satisfied when using I/S_{N-1}, redo (a) using I/S_{N-2} and so on. If none of the I/S ratios satisfy the conditions in (a), then the program uses the 3x5 m.a.

For the series analysed, the automatic option of X11ARIMA/88 always selected the 3x5 m.a. and used the multiplicative decomposition model. Table 3 shows how the significant autocorrelated values are corrected. For two series, the use of an additive decomposition is preferable to increasing the length of the seasonal moving average whereas for the remainder, the application of the 3x9 m.a. is adequate. Finally, for two series, namely Retail Trade All Stores Alberta and Retail Trade Grocery, confectionary and Sundries, the use of a longer Henderson filter corrected the significant autocorrelated values at lag 12 as already shown in table 2.

Table 3. AUTOCORRELATION OF THE RESIDUALS USING THE 3X9 SEASONAL MOVING AVERAGE OR THE ADDITIVE DECOMPOSITION MODEL

Monthly Series	r_k when 3x9 m.a. is applied	r_k when additive decomposition model is applied
Retail Trade - Grocery Confec. and Sundries (D650002)	r_{12}=-13	worsen
Retail Trade - Florists All stores (650082)	r^*_1=-.24 r_{12}=-.08 r_{11}=0.13 (Easter option applied)	worsen
Retail Trade - All Stores, Alberta (D651142)	r_{12}=-.13; r^*_8=-.20	worsen
Imports - B of P. Non-Metal Minerals (D397809)	r^*_6=-.20; r^*_{11}=-.28; r_{12}=-.15 (Easter option applied)	worsen
Unemployment - Women 25 years & over (D767519)	r_{12}=-.14	worsen

Employment Total,PEI (D767976)	r^*_{12}=-.23	r_{12}=-.19
Unemployment, Total PEI (767977)	r^*_{12}=-.17	worsen
Employed - Men Quebec (D768421)	r_{12}=-.15	worsen
Unemployed Women, B.C. (D769184)	r_{12}=-.18; r_{14}=-.06	worsen
Quarterly Series		
Wages & Labour, Income (D100002)	r^*_4=-.28	r_4=-.17
Corporate Claims (D151986)	r_4=-.13	worsen

*Values significant at 5%.

5. AUTOCORRELATION OF THE RESIDUALS DUE TO THE PRESENCE OF TRADING-DAY VARIATIONS

Among the monthly series, a sample of 15 was selected to analyse the autocorrelation of the residuals before and after removal of trading-day variations. Table 4 shows that when trading-day variations are present, the autocorrelation of the residuals is significant for a large number of lags. Positive autocorrelation is often found at lags 3, 6, 9, 11 and 14. Negative autocorrelation is found at lags 1, 4, 7, 10 and 13. The removal of trading-day variations corrects for most of the significant autocorrelated values. The autocorrelation at lag one that still remains for some series, can be corrected using a longer

Henderson trend-cycle filter as shown in Table 2. For the series, namely Import Balance of Payments-Non-Metal Minerals and Retail Trade - Florists All Stores, the autocorrelation at lag 12 was removed after applying the Easter option.

Table 4. AUTOCORRELATIONS OF RESIDUALS WITH AND WITHOUT TRADING-DAY VARIATIONS

Series Title	r_k(Trading-Day Variations Present)	r_k(Trading Day Variations Removed)
Retail Trade All Stores - Department Stores (D650062)	r^*_1=-.37; r^*_3=.20 r^*_7=-.34; r^*_6=-.27 r^*_{10}=-.30; r^*_{11}=.20 r^*_{13}=-.24; r^*_{14}=.24	r^*_2=-.28
Retail Trade - All Stores - Canada (650058)	r^*_1=-.37; r^*_3=.42 r^*_4=-.42; r^*_6=.24 r^*_7=-.46; r^*_9=.25 r^*_{10}=-.40; r^*_{11}=.37 r^*_{13}=-.39; r^*_{14}=.40	r^*_1=-.31 r^*_{13}=.23
Retail Trade - All Stores Nova Scotia (D650350)	r^*_1=-.32; r^*_3=.44 r^*_4=-.41; r^*_6=.24 r^*_7=-.38; r^*_9=.27 r^*_4=.27 r^*_{10}=-.41; r^*_{14}=.35 r^*_{13}=-.46	r^*_1=-.24
Retail Trade - All Stores- Alberta (D651142)	r^*_1=-.43; r^*_3=.34 r^*_4=-.36 r^*_7=-.36; r^*_{11}=.26 r^*_{10}=-.32; r^*_{14}=.27 r^*_{13}=-.31	r^*_1=-.23 r^*_{12}=-.21
All Stores - Chain (D650000)	r^*_1=-.37; r^*_3=.3 r^*_4=-.33; r^*_6=.28 r^*_7=-.44; r^*_9=.30 r^*_{10}=-.45; r^*_{11}=.21 r^*_{13}=-.40; r^*_{14}=.42	r^*_2=-.26 r^*_{13}=.37

Combination Stores - Canada (D650001)	r^*_1=-.41; r^*_3=.46 r^*_4=-.41; r^*_6=.28 r^*_7=-.49; r^*_9=.30 r^*_{10}=-.46; r^*_{11}=.21 r^*_{13}=-.47; r^*_{14}=.45	r^*_1=-.21 r^*_{13}=.21
Grocery, Confectionary and Sundries (D650002)	r^*_1=-.46; r^*_3=.47 r^*_4=-.45; r^*_6=.36 r^*_7=-.41; r^*_9=.41 r^*_{10}=-.56; r^*_{11}.26 R^*_{13}=-.59; r^*_{12}=.27 r^*_{14}=.52	r^*_1=-.23 r^*_2=-.26 r^*_3=-.20 r^*_{12}=-.30
Imports B of P Agriculture Machine-Tractors (D397817)	r^*_4=-.23; r^*_6=.21	r^*_3=-.21
Imports B of P Non-Metallic Minerals (D397809)	r^*_1=-.29; r^*_6=-.26 r^*_7=-.30; r^*_{11}=.22 r^*_{13}=-.28	r^*_{11}=-.28 r^*_{12}=-.20
Retail Trade - Florists All Stores (650082)	r^*_1=-.32; r^*_{11}=.25 r^*_{12}=-.23	r^*_1=-.35 r^*_{11}=.23 r^*_{12}=-.29
Retail Trade - Service Station All Stores (D650068)	r^*_2=-.40; r^*_3=.27 r^*_5=-.27; r^*_6=.21 r^*_{13}=-.19; r^*_{14}=.24	r^*_2=-.38 r^*_5=-.39 r^*_7=-.24 r^*_{10}=.25
Retail Trade - All Stores Ontario (D650702)	r^*_1=-.24; r^*_3=.26 r^*_2=-.21; r^*_6=.20 r^*_4=-.33; r^*_9=.19 R^*_7=-.39; r^*_{11}=.33 r^*_{10}=-.30; r^*_{14}=.39 r^*_{13}=-.29	r^*_{10}=.22
Imports B of P - Furniture, Utensils & Goods (D397831)	r^*_1=-.22; r^*_5=.21 r^*_7=-.36 r^*_{10}=-.24; r^*_{14}=.25	r^*_2=-.20 r^*_{11}=.29
Retail Trade - General Merchandise Stores Chain (D650005)	r^*_1=-.35; r^*_3=.23 r^*_4=-.33; r^*_{14}=.23	r^*_1=-.20
Imports B of P - Fresh Vegetables (D397775)	r^*_2=-.27; r^*_4=-.21 r^*_{13}=-.22	r_2=-.26

6. CONCLUSION

This study has shown that the significant autocorrelation of the residuals produced by X11ARIMA for certain lags are not an indication of the inadequacy of the method but result from the inappropriate use of the standard option. In most cases, negative autocorrelation at lag one, associated to an overadjustment of the trend, can be corrected using a longer Henderson filter. Similarly, negative correlation at the seasonal lag can be corrected either with a longer seasonal moving average or changing to an additive model. Finally, the presence of positive autocorrelation at lags 3, 6, 9 and 14 with negative autocorrelation at lags 1, 4, 7, 10 and 13 are mainly due to the presence of trading-day variations and are corrected once these variations are removed.

Since the residuals of X11ARIMA are also affected by extreme values, the occurrence of two or more extreme values in the same month or in the same year can cause significant autocorrelated values at certain lags. These significant autocorrelated values can be corrected only if the extreme values are a-priori permanently modified.

We have also observed that significant negative autocorrelation at lag 12 or 4 which could not be eliminated by changing the decomposition model and/or using longer seasonal filters, disappears after the removal of Easter effects if present in the series.

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