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Current-Trend-Level Estimation: A DEAD-END STREET?

by
Guy Huot

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Guy Huot, Statistics Canada
7-F, R.H. Coats Bldg, Ottawa, Ontario, K1A 0T6

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1. INTRODUCTION

Economic statisticians and users of time series have found it useful to decompose series into trend-cycle, seasonal, and irregular components for separate study. For instance, those using the series for policy analysis are frequently interested in the month-to-month changes in the current and recent trend. On the one hand, using data from the original series may not be appropriate since the month-to-month changes will likely be dominated by the seasonal effect. On the other hand, the traditionally published seasonally adjusted series may not be a helpful trend indicator when the users are not interested in following the irregular fluctuations in the monthly or quarterly values.

The trend-cycle estimation procedure presently used for official statistics throughout the world is based on the Henderson moving averages in the X11ARIMA seasonal adjustment program (Dagum, 1980). These moving averages are trend-cycle estimators designed to smooth out the irregulars from the seasonally adjusted series. Throughout the paper, the words "trend" and "trend-cycle" are used synonymously, though economists have made a distinction.

The procedure is based on the application of symmetric moving averages to data points in the historical part of the series. A historical trend estimate is usually made after 4 years further data are available. Then historical trend values remain essentially unaltered when the series is subsequently readjusted. However, the end-point trend estimates (i.e. the early trend indicators) are obtained using non-symmetric moving averages. Whenever new observations are added to the series, the initially adjusted end-point data are readjusted and "revised". The "revision" is due to the new observations and the application of moving averages different from the one used to previously adjust these same end-point values. Readjustment means that one can come closer to using the symmetric moving averages. More accurate estimates are then generally obtained.

Let's define "revision" as a measure of the deviation of the end-point estimates from corresponding more accurate estimates (not necessarily their final estimates); and the "lag" as the period between the latest observation and the date of the estimate. Thus, the estimate for time t based on data up to time t is an estimate at lag 0. Lag 0 refers to current estimates. The estimate for time t with data to time t+1 is an estimate at lag 1, and so on.

Important considerations are the revision to the direction of the end-point estimates (early trend-cycle indicators), and the convergence of these estimates to their final (historical) estimates. The trend estimates at lags 0 and 1 often require large revision; thereafter, the further revision as lag increases is much smaller. Thus these first two estimates are considered less accurate, and so some statistical agencies wait 1 or 2 months before publishing trend figures.

The purpose of this paper is to investigate methods of improving the estimation of the current and recent trend. Section 2 looks into a new approach toward reducing the revision of the current trend estimates. This approach consists of combining forecasts of a single ARIMA model made at successive origins. The third section provides a summary of the results. This is a shortened version of the original paper, which also includes a section on data and statistical tests, and a section on an application to different series.

2. PROCEDURES TO REDUCE CURRENT TREND REVISION

The X-11 variant of the Census Method II seasonal adjustment program by Shiskin, Young and Musgrave (1967) was made available in 1967. There were concerns about the fact that the X-11 variant was better at producing measures of historical performances, whereas the greater interest was in the most recent figures.

2.1 Two Major Developments

Dagum (1975) has shown that under certain conditions the extension of the original series using one year of ARIMA forecast values increases the degree of reliability for current estimates, and minimizes the revision of the seasonally adjusted values. The ARIMA extension of the series allows for the use of past observations and acceptable forecast future values. When adjusting the current observations, the symmetric Henderson moving average is used. Thus, the moving average applied to generate the current trend estimate is closer to the moving average used for central observations. The revisions are then related to the forecast error and the use of different non-symmetric seasonal moving averages.

Studies by Dagum (1978), Kenny and Durbin (1982), Pierce and McKenzie (1987), and others, have shown that the current updating of the seasonal factors each time a new observation becomes available improves on the common practice of performing the seasonal adjustment procedure only once or twice a year. In the latter case, 6-month or year-ahead seasonal factors are normally used to adjust the more recent data points. The current updating practice provides more accurate seasonal factors and produces more reliable current estimates.

As a result of these two developments, the X11ARIMA program can be applied in four modes. From the point of view of the revision of the current estimates, Dagum, Huot and Morry (1988) have ranked the average performance of the four modes as shown in table 1:

Table 1. Ranking of the Average Performance of the Four Modes of the X11ARIMA Program

	Current updating of the seasonal factors	6-month or year-ahead seasonal factors
ARIMA extrapolation	1	3
No ARIMA extrapolation	2	4

2.2 The Traditional Procedures

Table 1 shows that the revision of the current estimates is reduced the most, by using both current updating and ARIMA extrapolations. The revision is also strongly related to the smoothing operations performed on the seasonally adjusted series. Different smoothing operators as well as the use of trend estimates or smoothed seasonally adjusted data have been discussed by several authors: recently by Moore et al. (1981), Kenny and Durbin (1982), Maravall (1986), Castles (1987), Dagum and Laniel (1987), and Dagum, Huot and Morry (1988).

The general recommendation in order to estimate the trend at lag 0 is to use all of: current updating, forecasts usually of the ARIMA type, and the symmetric Henderson smoothing operator. The alternative procedure is to wait 1 or 2 months before publishing trend figures. Reliability is then obtained at the cost of timeliness. In the rest of this paper we consider only one of the three recommended procedures, namely ARIMA forecasts for which a new approach is investigated.

In figure 1 the impact of the ARIMA forecasts on (mode 1) trend estimates at lags 0, 1 and 2 is illustrated, using an Argentine laminated steel production series. The no-forecast estimates (mode 2 in table 1) are also depicted; they are obtained using current updating and a non-symmetric Henderson smoothing operator. The two short curves plotted for Nov. 81 (lag 2), Dec. 81 (lag 1) and Jan. 82 (lag 0) are the forecast and no-forecast trend estimates when the series ends in Jan. 82; similarly for the other short curves. These estimates are compared with the final trend estimates which show a turning point in Feb. 82. The ARIMA forecasts improve the timing of the recognition of this turning point when the series ends in March and April 82. However, there is no improvement when the series ends in Feb. 82. Consequently, within a three-month period (Feb. 82 to April 82), the trend direction has undergone a 90 degree shift. This is due to an overestimation of the forecast trend level when the series ends in Feb. 82. In fact predicting a turning point is difficult.

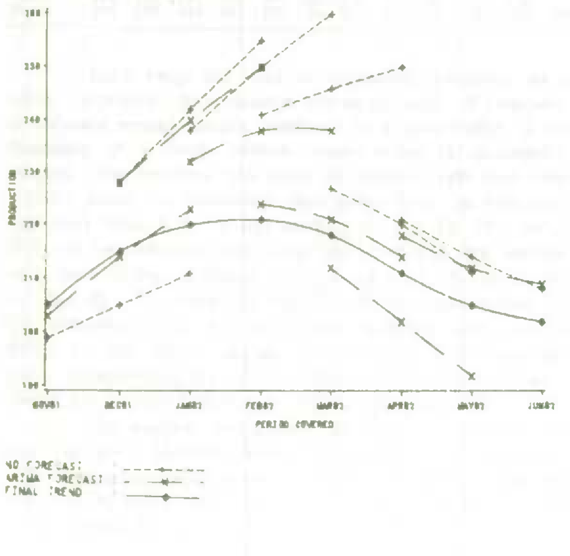

Figure 1. Current and Final Trend-Cycle Estimates

2.3 A New Combined ARIMA Forecasts Procedure

Aggregating two or more forecasts for the same variable often leads to increased accuracy. This conclusion, reported by Clemen (1989) and Mahmoud and Makridakis (1989), is based on major empirical studies about combining forecasts. Increased forecast accuracy results simply from the fact that no ARIMA model consistently outperforms the others. Combining forecasts contributes to smoothing the forecast errors, thus making the errors smaller on the average, as long as the forecasts are unbiased.

Increased forecast accuracy can only be obtained at an extra cost. Is it worthwhile to combine forecasts in the context of X11ARIMA? What is the expected gain? In order to answer these questions, let's introduce an additional dimension to the analysis: "perfect forecasts". "Perfect forecasts" means that the series are extrapolated using twelve actual observations instead of twelve forecast values. Thus the maximum possible benefit of increasing forecast accuracy can be measured. Table 2 compares the performance of the X11ARIMA mode 2 (no forecast) trend estimates with mode 1 (perfect forecasts) estimates. An average root mean-square percentage error (RMSPE) was calculated for a set of 23 economic time series. "Error" refers to trend estimates at lag 0 minus their corresponding final estimates at lag 48. Perfect forecasts have reduced the average error from 95% to 29%. This suggests that current trend accuracy is strongly related to forecast accuracy. An average RMSPE was also calculated for the seasonally adjusted figures. The comparison suggests that the current trend estimates are more sensitive to forecast accuracy than the seasonally adjusted values are (i.e. 69% versus 30% potential gain). Forecast accuracy would then be a key factor in current trend estimation.

Table 2. Benefits of Increasing Forecast Accuracy

RMSPEs no forecast		RMSPEs "perfect forecasts"	
Current seasonally adjusted figures	Current trend-cycle estimates	Current seasonally adjusted figures	Current trend-cycle estimates
63	95	44	29

A common practice to increase forecast accuracy is to combine the forecasts of different models and/or methods to average out their forecasting errors. But can such a procedure help predict more accurately both a continuing pattern and a turning point? The overestimation of the Feb. 82 mode 1 current trend value in figure 1 is associated with an overextended trend forecast. In order to deal accurately with the Feb. 82 turning point, the combined forecast procedure implies that the other forecast (assuming for instance forecasts from two models only) should underestimate the trend. So the overextention and underestimation of the trend might cancel out, and produce a more accurate forecast. However, it was pointed out by Lawrence and al. (1986), and Mahmoud and Makridakis (1989) that when the trend pattern changes, the forecast errors of various methods generally have the same sign. Thus this combining procedure is not likely to improve accuracy much at turning points.

What is needed is a set of forecasts that, when averaged, would better estimate the trend direction as well as recognize the timing of the turning points earlier. Table 3 can be examined in two ways: in columns and in rows. The forecasts, used for the estimation of the mode 1 current trend values in figure 1, are displayed in columns underneath their origin. Table 3 is cut off at Sept. 82 for convenience. The column presentation lends itself to a seasonal (and trend level) display. Accordingly, the first column shows a seasonal peak around Oct. 81 and a trough around Feb. 82. On the other hand, the row presentation seems to reflect the way the trend-cycle evolves. There are, for instance on the last row, twelve different forecasts of Sept. 82 with lead times 12, 11, ..., 1. Their values range from 190 (lead time 12) to 294 (lead time 7; Feb. 82 - already identified as a turning point - being the origin). The forecasts made at the Feb. 82 origin are peak values which tend to reflect the underlying trend at the origin. Conversely, the forecasts in the rows, with peak values underneath the Feb. 82 origin, suggest that Feb. 82 might be a turning point. Feb. 82, identified by coincidence as a seasonal trough in columns and a trend-cycle peak in rows, also emphasizes the difference between, and the complementary character of, column and row analyses. The significance of all this is that different pieces of information are available from the forecasts of a single ARIMA model made at successive origins.

Table 3. ARIMA Forecasts Made at
12 Different Time Origins

Month being forecast	origins Sep 81	Oct 81	Nov 81	Dec 81	Jan 82	Fev 82	Mar 82	Apr 82	May 82	Jun 82	Jul 82	Aug 82
Oct81	202											
Nov81	204	208										
Dec81	158	158	160									
Jan82	124	127	128	150								
Feb82	103	105	106	124	151							
Mar82	171	174	175	205	248	266						
Apr82	180	183	184	193	234	251	229					
May82	170	173	175	204	247	268	241	217				
Jun82	152	155	156	184	222	239	218	185	17[illegible]			
Jul82	178	179	181	211	255	273	248	223	18[illegible]	221		
Aug82	182	185	187	218	264	283	257	231	20[illegible]	230	223	
Sep82	190	193	195	227	274	294	287	240	21[illegible]	238	231	240

Let's keep the idea of combining forecasts as a basis. However, let's replace the aggregation of forecasts of different models and/or methods by a combination of the forecasts of a single ARIMA model made at successive origins. For instance, the Sept. 82 forecast with lead time 7 (294) could be combined with other Sept. 82 forecasts with lead times 8, 9, 10, and possibly 11 and 12. Obviously, 294 will be reduced to a value closer to 218 (the sample realization of the stochastic process for Sept. 82, not shown in table 3). The expected benefits are an improvement of the estimated Feb. 82 trend-cycle direction and level in figure 1. For this to happen, the forecasts should contain some independant trend information. In general, forecasts based on no new information will not bring benefits.

The weights can simply be designed to reflect the fact that the probability limit about the forecasts increases with increasing lead times. Smaller weights would be assigned to larger lead times. The moving averages can be of the simple form:

$$w_r = r \Big/ \sum_{i=1}^{q} i \quad \text{for } r = 1, \ldots, q$$

with weights w_r and length q. The procedure is to use this family of moving averages to convert the table 3 row forecasts (f) into combined forecasts (cf), by the following linear operation:

$$(cf)_t = \sum_{r=1}^{q} w_r f_{t-q+r} .$$

Figure 2 shows, for example, the gain function for the 3, 5 and 7-term moving averages (filters). Their cut-off properties vary with the number of weights used. The more irregular the series, the longer the moving average should be. Accordingly, part of the irregular fluctuations in the row forecasts are smoothed out in order to avoid arratic changes in the trend direction. Some of the forecasts for Sept. 82 are quite far from the sample realization value (i.e. 218). Smoothing out is also expected to replace some of the overextended and/or underestimated trend forecasts at turning points by values that are likely to be closer to the forthcoming sample realization. In such a case, the variance of the row forecasts would be reduced.

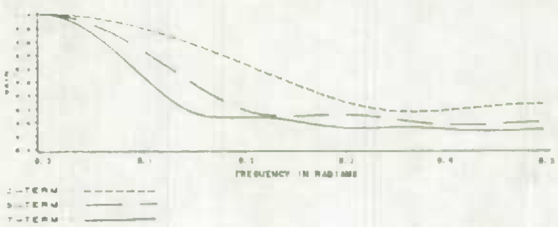

Figure 2. Gain Function for the 3, 5
and 7-Term Moving Averages

Table 4 displays the combined forecasts. Given the volatility of the row forecasts in table 3, a 7-term moving average was selected. However, only the first 6 forecasts underneath an origin can be smoothed out using a 7-term moving average. Shorter moving averages had to be used for lead times 7 to 12, and the forecast value with lead time 12 is the same in both tables (i.e. Sept. 82). For instance, let's assume that the series ends in Aug. 82. The Sept. 82 forecast with lead time 1 can be combined with Sept. 82 forecasts with lead times 2 to 12. But the second forecast, Oct. 82, can be combined only with forecasts with lead times 3 to 12, and so on. However, the lack of adequate smoothing of some forecasts happens to be compensated by the fact that the weights assigned to the more distant forecasts are very small in the X11ARIMA program.

Table 4. Combined ARIMA Forecasts

Month being forecast	Last origins in the combination Sep 81	Oct 81	Nov 81	Dec 81	Jan 82	Fev 82	Mar 82	Apr 82	May 82	Jun 82	Jul 82	Aug 82
Oct81	194											
Nov81	195	199										
Dec81	149	152	155									
Jan82	119	121	124	131								
Feb82	98	101	103	109	120							
Mar82	164	167	170	180	198	218						
Apr82	154	156	158	168	188	205	215					
May82	166	166	170	179	198	218	228	228				
Jun82	150	151	153	161	177	195	205	206	201			
Jul82	174	176	178	187	204	224	235	236	230	228		
Aug82	181	183	185	198	215	232	243	244	238	237	232	
Sep82	190	192	194	207	229	248	253	254	247	246	241	238

Figure 3.a and 3.b illustrate the interaction that takes place between the forecasts and the lag 0 trend estimates. The broken line connects the Sept. 82 forecasts with lead times 8 to 3. These forecasts (or any other forecasts) are assumed to reflect the underlying trend pattern present at the Jan. 82 to June 82 origins (or any other origins). For convenience, only these are shown. The continuous line joins the lag 0 trend estimates obtained when the series ends successively in Jan. 82, Feb. 82, ..., June 82. One can observe that the combined ARIMA forecasts (figure 3.b) have a smaller variance than the ARIMA forecasts, and so do the corresponding trend estimates. Forecasts and trend estimates also tend to move in the same direction.

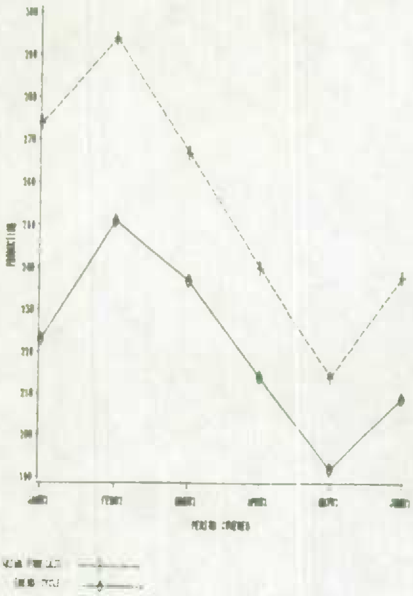

Figure 3.a. ARIMA Forecasts and Lag 0 Trend Estimates

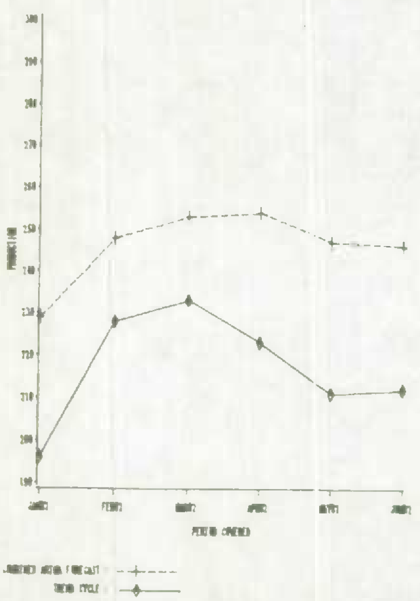

Figure 3.b. Combined ARIMA Forecasts and Lag 0 Trend Estimates

Figure 4 shows the trend estimates at lags 0, 1 and 2 obtained using both the ARIMA forecasts and the combined ARIMA forecasts. The combined forecasts have improved the recognition of the timing of the turning point by reducing the level and direction errors. The trend estimates are also closer in direction. Unfortunately, combined forecasts are not "perfect forecasts", so a level error was introduced when the series ends in Jan. 82. It indicates that combined forecasts may be affected by phase shifts, since non-symmetric moving averages are used. However, it seems that economists and users of time series prefer to live with level errors rather then direction errors whenever the series are used as trend indicators.

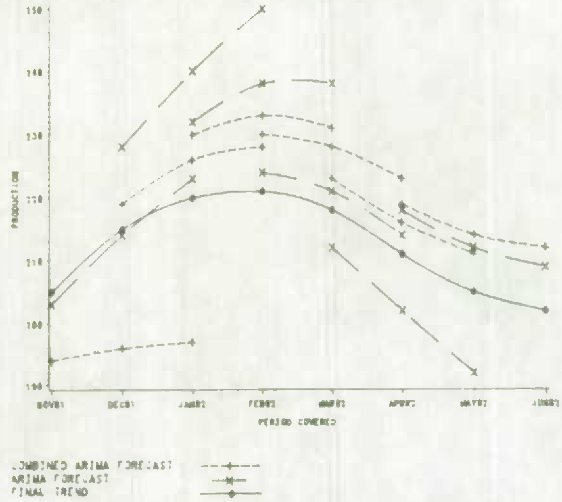

Figure 4. Current Trend-Cycle Estimates Obtained Using ARIMA and Combined ARIMA Forecasts

The forecast errors for the ARIMA and combined ARIMA forecasts are displayed in tables 5.a and 5.b. Figures for Oct., Nov. and Dec. 82 have been inserted to better illustrate the forecast error pattern on the right-hand side of the tables. The sample realization for the months listed in the first column of tables 3 and 4 are subtracted from every number in their corresponding row. The column averages in the bottom row of the tables show that the average error underneath the Feb. 82 origin (turning point) is reduced from 41% to 0. However, the total algebraic mean is somewhat larger in table 5.b, so biases in prediction were not reduced. In practice, this result suggests that the average level error is somewhat increased or, at best, remained about the same. That is, reductions in level error are offset by increases. Cancelling out reductions and increases in level error is only one aspect of the analysis. Another question is "where in the pattern of the series do these reductions and increases occur?" In figure 4, a reduction in the level error is observed precisely at the turning point. Of course, other scenarios are possible. A reduction in level error may occur away from a turning point, and the level error may possibly be increased at a turning point. The other issue which is, given the circumstances, more important then level errors is, whether direction errors can be avoided. In fact, the combined forecasts have reduced the standard deviation in table 5.a from 41.2 to 33.8. Accordingly, trend estimates are closer in direction. It seems that the level error is associated with bias while the direction error is related to variance.

Table 5.a. ARIMA Forecast Errors

Month being forecast	Origins											
	Sep 81	Oct 81	Nov 81	Dec 81	Jan 82	Fev 82	Mar 82	Apr 82	May 82	Jun 82	Jul 82	Aug 82
Oct81	- 7											
Nov81	- 7	- 3										
Dec81	-58	-56	-54									
Jan82	-82	-79	-76	-56								
Feb82	-66	-64	-63	-45	-18							
Mar82	-55	-52	-51	-21	22	40						
Apr82	-30	-27	-26	3	44	61	39					
May82	-11	- 8	- 6	23	66	85	60	36				
Jun82	-58	-55	-54	-28	12	29	8	-15	-35			
Jul82	-35	-32	-30	0	44	82	37	12	-12	10		
Aug82	-56	-53	-51	-20	26	45	18	- 7	-32	- 8	-15	
Sep82	-28	-25	-23	8	56	76	49	22	- 4	20	13	22
Oct82		-92	-90	-58	-11	9	-18	-43	-70	-48	-52	-44
Nov82			-99	-66	-20	0	-27	-54	-78	-55	-61	-53
Dec82				-51	-11	7	-17	-40	-82	-41	-47	-40
	-41	-46	-52	-28	19	41	17	-11	-42	-20	-32	-28

Table 5.b. Errors in the Combined ARIMA Forecasts

Month being forecast	Last origins in the combination											
	Sep 81	Oct 81	Nov 81	Dec 81	Jan 82	Fev 82	Mar 82	Apr 82	May 82	Jun 82	Jul 82	Aug 82
Oct81	-15											
Nov81	-16	-12										
Dec81	-65	-63	-60									
Jan82	-87	-85	-82	-75								
Feb82	-71	-68	-66	-60	-49							
Mar82	-62	-59	-56	-46	-29	- 8						
Apr82	-36	-34	-31	-21	- 4	15	25					
May82	-15	-13	-11	- 2	17	37	47	48				
Jun82	-60	-59	-57	-49	-33	-15	- 5	- 4	- 9			
Jul82	-37	-35	-33	-24	- 7	13	24	25	19	17		
Aug82	-57	-55	-53	-42	-23	- 6	5	8	0	- 1	- 6	
Sep82	-28	-26	-24	-11	11	30	35	36	29	28	23	20
Oct82		-92	-91	-74	-49	-30	-26	-31	-38	-39	-43	-46
Nov82			-99	-77	-48	-29	-28	-36	-47	-48	-52	-55
Dec82				-51	-24	- 5	-12	-21	-33	-35	-36	-41
	-46	-50	-55	-44	-22	0	5	3	-11	-13	-23	-30

3. CONCLUSION

Some statistical agencies do not publish trend estimates at lags 0 and 1 because of large level and direction errors. The analysis has shown that forecast accuracy is a key factor in current trend estimation. Accordingly, a new approach which consists of combining the forecasts of a single ARIMA model made at successive origins has been used to hopefully reduce both level and direction errors. The direction error has been successfully reduced. In fact, the application of the procedure to different series shows an average decrease of 24% in the direction error and a 3% increase in the average level error. Only the procedure is discussed in this shortened version of the original paper because of a lack of space. This result is interesting since users of time series seems to prefer to live with level errors rather than direction errors, whenever the series are used as trend indicators.

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